Exhaustive Gaussian Approach for Performance Evaluation of Direct-Detection OFDM Systems Employing Square and Cross QAM

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***Abstract*—An exhaustive Gaussian approach (EGA) is proposed to evaluate, through numerical simulation, the bit error ratio (BER) of direct-detection orthogonal frequency-division multiplexing systems employing square and cross quadrature amplitude modulation. Excellent agreement between the BER estimates from the direct error counting (DEC) and the EGA is shown for different levels of optical signal-to-noise ratio and signal distortion. It is shown that the EGA requires about three orders of magnitude less in computation time than the DEC method for BER levels around 10^{-6}, with the difference getting higher for lower BER levels.**

***Index Terms*—Bit error ratio, direct-detection, optical communications, orthogonal frequency-division multiplexing, performance evaluation.**

I. Introduction

In the past few years, orthogonal frequency-division multiplexing (OFDM) has received remarkable attention as a promising technology for optical communications [1]–[4]. Combined with direct-detection (DD), it provides a low-cost and simple solution for optical transmission systems [5]. To evaluate the performance of DD-OFDM systems, the bit error ratio (BER) is mainly used as figure of merit [6], [7]. For BER assessment, the most common techniques are using Monte Carlo simulation with direct error counting (DEC), an analytical Gaussian approach (AGA) and using the error vector magnitude (EVM) [6]. With DEC, the OFDM signal is generated through numerical simulation and transmitted along the DD-OFDM system, obtaining at the receiver the OFDM waveform composed by the original OFDM signal corrupted by the noise and distortion introduced along the system. The BER is then easily obtained by direct counting the received erroneous bits. DEC is a good solution in terms of results accuracy. However, the number of transmitted OFDM symbols required to achieve BER values of less than 10^{-6} is unbearable in terms of computation time, as a large number of runs of the DD-OFDM system with different noise samples in each run is required. The BER obtained from the AGA and the EVM overcomes the measurement time drawback as closed-form expressions are used to evaluate the BER, providing fast BER estimates. Nevertheless, inaccurate BER estimates can be obtained as the distortion is implicitly assumed as Gaussian-distributed.

In [8], an exhaustive Gaussian approach (EGA) method has been proposed for BER evaluation through numerical simulation of the mean and standard deviation of each OFDM subcarrier of each OFDM symbol, and in [9] its application has been extended to experimental direct-detection OFDM setups. The great advantage of EGA is that fast and accurate BER estimates for each subcarrier can be obtained, where the noise and distortion effects are accounted subcarrier by subcarrier. Two different types of symbol mapping were considered for BER evaluation in [8], [9]: binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK).

In this paper, the EGA method is proposed to evaluate, through numerical simulation, the BER for M-ary quadrature amplitude modulation (M-QAM) formats with $M \in \{16, 32, 64, 128\}$. To evaluate EGA's accuracy, its BER estimates are compared with the ones obtained with the DEC method. For the square QAM constellations with $M = 16$ and $M = 64$, perfect Gray coding is considered. For the cross QAM constellations with $M = 32$ and $M = 128$, Smith-style Gray coding is used [10]. The reason for considering cross QAM constellations for $M = 32$ and $M = 128$ instead of rectangular QAM is related with its better performance results when comparing with rectangular QAM.

II. Exhaustive Gaussian Approach

The EGA is a method for performance evaluation of the BER, through numerical simulation, that provides fast and accurate estimates independently of the BER levels. The EGA assumes that the received in-phase (I) and quadrature (Q) components of each OFDM subcarrier are well described by a Gaussian distribution, as confirmed in [8]. Therefore, the BER of the received I or Q component of a subcarrier belonging to a specific OFDM symbol is evaluated from the subcarrier mean and standard deviation values. In the EGA context, these values are computed from a set of different noise runs.

To illustrate the application of EGA to high order QAM formats, let us focus the attention on the application of EGA to 16-QAM constellations. For other M-QAM constellations, the procedure is similar. The Gray-mapped 16-QAM constellation used to evaluate the BER estimates with the EGA and DEC methods is shown in Fig. 1. Variables l and c index the position (row and column, respectively) of a QAM symbol on the M-QAM constellation.

The key idea for EGA computation is explained as follows. A certain subcarrier, which has suffered from noise and/or distortion, is received with a mean value and

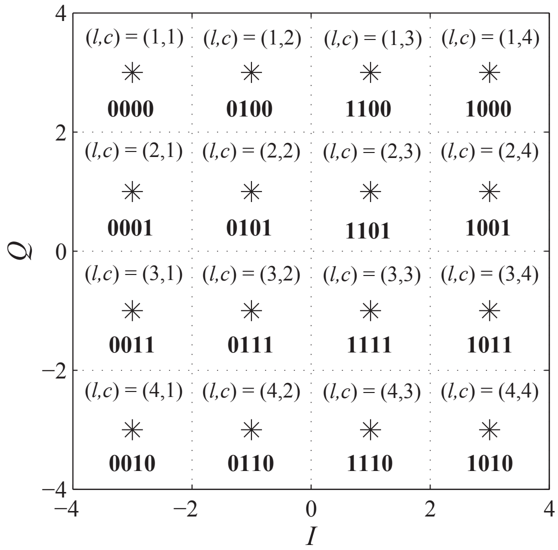


Fig. 1. Square QAM constellation with $M = 16$. The Gray-mapped bits of each QAM symbol represented by a star are highlighted in bold.

a standard deviation for its I and Q components. First, given a certain transmitted subcarrier, the number of different bits between that one and a received subcarrier, for all the possibilities, is obtained. Second, the probability following a Gaussian distribution of a received subcarrier with mean value and standard deviation for its I and Q components to be in all decision regions, is also obtained. Finally, by multiplying both the number of different bits and probabilities for all decision regions, we obtain the BER for that certain subcarrier. Mathematically, the BER of the *k*-th OFDM subcarrier of the *i*-th OFDM symbol, $BER^{(i)}[k]$, calculated with the EGA method is given by:

$$BER^{(i)}[k] = \frac{1}{\log_2 M} \sum_{l=1}^{N_l} \sum_{c=1}^{N_c} \left(N_{E,(l,c)}^{(i)}[k] \times P_{(l,c)}^{(i)}[k] \right) \quad (1)$$

with

$$P_{(l,c)}^{(i)}[k] = P_{I,(l,c)}^{(i)}[k] \times P_{Q,(l,c)}^{(i)}[k] \quad (2)$$

where N_l and N_c are the number of rows and columns of QAM symbols in the constellation, respectively, $N_{E,(l,c)}^{(i)}[k]$ is the number of erroneous bits of the *k*-th OFDM subcarrier of the *i*-th OFDM symbol when falling in the (l,c) decision region, and $P_{(l,c)}^{(i)}[k]$ is the product of $P_{I,(l,c)}^{(i)}[k]$ and $P_{Q,(l,c)}^{(i)}[k]$ that denote the probabilities of the I and Q components of the received *k*-th OFDM subcarrier of the *i*-th OFDM symbol to fall in the (l,c) decision region, respectively. Note that it is assumed that the I and Q components are uncorrelated and that Gray mapping is used for all the M-QAM constellations. Each probability (I or Q) is obtained through the Q_F function, $Q_F(z) = (1/\sqrt{\pi}) \int_{z/\sqrt{2}}^{\infty} \exp(-x^2)\,\mathrm{d}x$, and the mean $m_{(I,Q)}^{(i)}[k]$ and standard deviation $\sigma_{(I,Q)}^{(i)}[k]$ of the received *k*-th OFDM subcarrier of the *i*-th OFDM symbol, given by:

$$\begin{aligned} m_{(I,Q)}^{(i)}[k] &= \frac{1}{N_r} \sum_{n=1}^{N_r} y_{(I,Q),n}^{(i)}[k] \\ \sigma_{(I,Q)}^{(i)}[k] &= \left[\frac{1}{N_r} \sum_{n=1}^{N_r} \left(y_{(I,Q),n}^{(i)}[k] - m_{(I,Q)}[k] \right)^2 \right]^{\frac{1}{2}} \end{aligned} \quad (3)$$

where N_r is the number of noise runs and $y_{(I,Q),n}^{(i)}[k]$ is the amplitude of the I or Q component of the received *k*-th OFDM subcarrier of the *i*-th OFDM symbol in the *n*-th noise run.

For the constellation of Fig. 1, $P_{I,(l,c)}^{(i)}[k]$ is given by:

$$P_{I,(l,c)}^{(i)}[k] = \begin{cases} Q_F\left[\dfrac{2+m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right], c=1 \\ Q_F\left[+\dfrac{m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right] - Q_F\left[\dfrac{2+m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right], c=2 \\ Q_F\left[-\dfrac{m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right] - Q_F\left[\dfrac{2-m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right], c=3 \\ Q_F\left[\dfrac{2-m_I^{(i)}[k]}{\sigma_I^{(i)}[k]}\right], c=4 \end{cases} \quad (4)$$

for all values of $l \in \{1,2,3,4\}$. For $P_{Q,(l,c)}^{(i)}[k]$ we get:

$$P_{Q,(l,c)}^{(i)}[k] = \begin{cases} Q_F\left[\dfrac{2-m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right], l=1 \\ Q_F\left[-\dfrac{m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right] - Q_F\left[\dfrac{2-m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right], l=2 \\ Q_F\left[+\dfrac{m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right] - Q_F\left[\dfrac{2+m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right], l=3 \\ Q_F\left[\dfrac{2+m_Q^{(i)}[k]}{\sigma_Q^{(i)}[k]}\right], l=4 \end{cases} \quad (5)$$

for all values of $c \in \{1,2,3,4\}$. The BER of each subcarrier, $BER[k]$, is then obtained as follows:

$$BER[k] = \frac{1}{N_s} \sum_{i=1}^{N_s} BER^{(i)}[k] \quad (6)$$

where N_s corresponds to the number of OFDM symbols per noise run. The BER obtained from EGA, while considering a Gaussian distribution for the noise, allows describing correctly the statistical distribution of the distortion through the average over different occurrences of symbols of each subcarrier performed by Eq. (6). This is achieved by not assuming that the distortion on all the subcarriers is Gaussian-distributed, as the EVM and AGA methods consider. This means that the degradation induced by distortion, that can appear due to the electro-optic modulator characteristic or due to optical filtering, is correctly accounted by Eq. (6). The overall BER is then evaluated averaging the BER over all N_{sc} information subcarriers, as follows:

$$BER = \frac{1}{N_{sc}} \sum_{k=1}^{N_{sc}} BER[k]. \quad (7)$$

III. System Setup

To analyze and validate the EGA proposed for high-order QAM, the BER estimates from the DEC and EGA methods will be compared for the DD-OFDM system in optical back-to-back illustrated in Fig. 2.

Fig. 2 shows that the signal at the OFDM transmitter (Tx) output is amplified by an electrical amplifier (EA) and biased. The OFDM signal at the Tx output has 128 OFDM subcarriers carrying information, and its

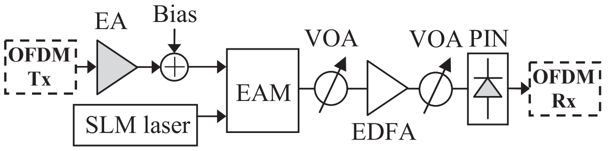


Fig. 2. DD-OFDM system in optical back-to-back. EA - electrical amplifier, EAM - electro-absorption modulator, EDFA - erbium-doped fiber amplifier, PIN - positive-intrinsic-negative, Rx - receiver, SLM - single-longitudinal mode, Tx - transmitter, VOA - variable optical attenuator.

spectra (approximately rectangular) is centered at 5 GHz with a bit rate R_b of 10 Gb/s. The bandwidth B_w of the OFDM signal is variable and dependent on M: $B_w = R_b/\log_2 M$. As M increases, B_w decreases and the spectral efficiency increases. The chirpless electro-absorption modulator (EAM) performs the electrical-to-optical conversion. The EAM output power characteristic as a function of the bias voltage is shown in Fig. 3. The EAM is fed by a single-longitudinal mode (SLM) laser with output power of 5 mW (7 dBm).

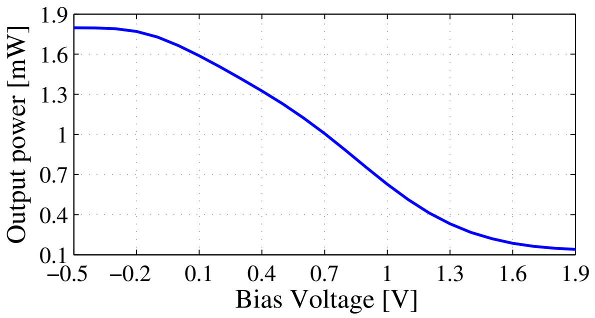


Fig. 3. EAM output power for a SLM laser input power of 5 mW.

Fig. 3 shows that a good compromise between linear behavior and EAM power loss can be the bias voltage of 0.7 V. This results in an average power at the output of the EAM of 1 mW (0 dBm) for OFDM signals with root-mean-square (RMS) voltages much smaller than 0.7 V, meaning that the EAM introduces approximately 7 dB insertion loss. In this paper, the optical center frequency ν_0 is 193.1 THz. After electrical-to-optical conversion, a variable optical attenuator (VOA) and an erbium-doped fiber amplifier (EDFA) are used to adjust the optical signal-to-noise ratio (OSNR) that is defined in a reference optical bandwidth of 0.1 nm. The second VOA imposes a fixed power of 0 dBm at the positive-intrinsic-negative (PIN) input. The PIN, modeled by an ideal square-law detector with responsivity of 1 A/W, performs the optical-to-electrical conversion, and the BER evaluation is performed after demodulation at the OFDM receiver.

IV. Results and Discussion

In order to assess the accuracy of the EGA under different transmission situations, different levels of the optical noise generated by the EDFA and distortion induced by the EAM and PIN are considered. The main advantage of EGA is that it provides fast and accurate estimates of the system performance independently of the BER levels. One of the key parameters of EGA is the number of runs needed to achieve good BER estimates and particularly good estimates of mean and standard deviation for the different transmitted symbols. In the DEC method, different runs of symbols are transmitted (each run with 250 different OFDM symbols) and for each set of system parameters, the BER is estimated when 100 errors occur on the subcarrier with worst performance [9]. This indicates that as BER levels decrease, a higher number of runs is necessary to calculate the BER level. The BER of each subcarrier obtained through the EGA method considers the evaluation of the BER of the received subcarriers of 250 OFDM symbols over 200 noise runs. In all situations considered in this paper with the EGA method, this number of runs was enough to get a stabilized BER estimate with increasing number of runs.

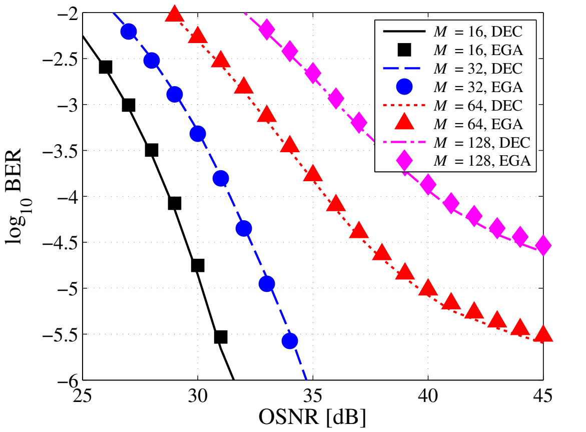
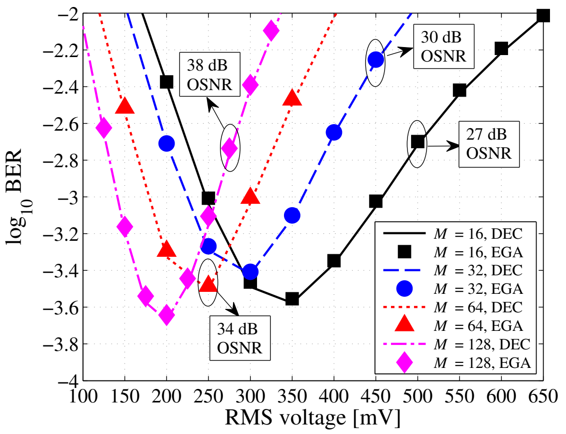


Fig. 4. BER estimates of the EGA and DEC methods as a function of the OSNR, with $M \in \{16, 32, 64, 128\}$ and with a RMS voltage of 250 mV at the EAM input.

Fig. 4 presents the BER estimates of EGA and DEC as a function of the OSNR, with $M \in \{16, 32, 64, 128\}$ and with a RMS voltage of 250 mV at the EAM input. Fig. 4 shows an excellent agreement between the BER estimated by both methods, independently of the OSNR levels. The different behavior that the curves show for different values of M is related to noise and distortion effects. With $M = 16$ and $M = 32$, optical noise is the dominant effect affecting the performance for a RMS voltage of 250 mV. With $M = 64$ and $M = 128$, the BER starts to stabilize for OSNR values higher than 40 dB. This is due to a higher influence of distortion with the increase of M. In addition, the OSNR has to increase to achieve the same BER value as M increases. This is because we have smaller decision regions as M increases, and therefore the errors increase while maintaining the OSNR level.

Besides varying the OSNR for a fixed RMS voltage, it is also important to verify the accuracy of EGA for a fixed OSNR while varying the RMS voltage, as in this case, we can obtain the optimum value of RMS voltage that achieves the minimum BER level. Fig. 5 presents the BER estimates with both methods (EGA and DEC) as a function of the RMS voltage of the signal at the EAM input, with OSNR $= \{27, 30, 34, 38\}$ dB for $M = \{16, 32, 64, 128\}$, respectively. Fig. 5 shows an excellent agreement between the estimates obtained with the EGA



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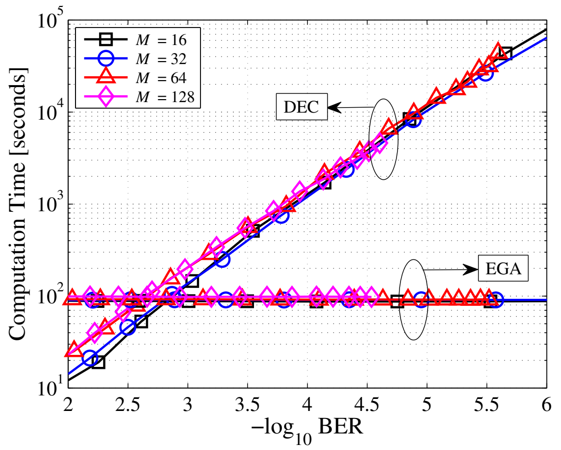

Fig. 6. Computation time, in seconds, of the BER estimates obtained with the DEC and EGA methods, as a function of $-\log_{10}$BER, for $M \in \{16, 32, 64, 128\}$.

and DEC methods, for different RMS voltages of the OFDM signal applied to the EAM input. We stress that for RMS voltage higher than the one corresponding to the minimum BER, the performance is dominantly impaired by the distortion due to EAM nonlinear characteristic (depicted in Fig. 3) and PIN square-law detection. As a consequence of this remark, the results of Fig. 5 show also that the EGA method provides accurate estimates of BER in the presence of nonlinearities, such as the ones imposed by the EAM and PIN. Note that in order to obtain similar BER levels for different values of M, different OSNR values were imposed, and it is verified a decrease in the optimum RMS voltage (the RMS voltage with lowest BER) as M increases. This is related to the higher influence of distortion as M increases.

Other important metric is the computation time, that validates how time spending EGA is for obtaining accurate BER estimates, in comparison with DEC. Fig. 6 presents the computation time, in seconds, of the BER estimates obtained with the DEC and EGA methods, as a function of $-\log_{10}$BER, with $M \in \{16, 32, 64, 128\}$. A 3.5 GHz Intel Core i7-4770K PC with 32 GB of RAM was used in the computation of both methods. Fig. 6 presents that the computation time of the DEC method increases substantially with the BER decrease whereas, with EGA, almost the same computation time is obtained for all the BER levels considered. Fig. 6 shows also that the DEC method is more time spending than EGA for BER levels lower than about 10^{-3} and that EGA requires, in comparison with DEC, three orders of magnitude less in computation time for BER levels around 10^{-6}, with the difference increasing for lower BER levels. In addition, we also note that the computation time required by both methods is almost independent of M.

V. Conclusions

The EGA for DD-OFDM systems employing square and cross QAM has been presented. Excellent agreement has been shown between the BER estimated by the DEC and EGA methods, for all the OSNR levels, for situations with low distortion (mainly impaired by noise) and for the cases where the distortion due to the nonlinear components (EAM and PIN) is significant. A complete study on the EGA accuracy considering also other effects, for instance fiber nonlinearity, will be reported elsewhere. The computation time of both methods has been compared. For BER levels lower than 10^{-3}, the DEC method is more time spending than EGA, reaching three orders of magnitude more for BER $\approx 10^{-6}$.

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