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INTRODUCTORY OFFERS IN TWO-PART TARIFFS: TIME-INCONSISTENCY AND SIGNALING APPROACH

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Abstract

Certain pairs of goods are such complements that a basic good exclusively enables the usage of a consumable good, which permits the firm to design the prices of these pair of goods in the form of two-part tariffs. Studies have been made regarding introductory offers in repeated purchases of a product, while we discuss about this topic with respect to the fixed fee in an aforementioned two-part tariff, examples including the pricing strategy in markets of printers, capsule coffee machines, etc. We take two approaches to model the pricing strategy of a firm. First we consider a rational firm facing two types of time-inconsistent consumers, which is a typical setting in behavioral IO. We show that firms generally make introductory offers accompanied by a raise in per-unit price in response to time-inconsistent consumers, which *ex post* grants the consumers more surplus and a loss in the firm's profit. Nevertheless, when time-inconsistency and second-degree price discriminating contract design interact with each other, the result becomes more nuanced: fixed fee may rise and per-unit price may decrease under some conditions. Secondly we explore the signaling effect of an introductory offer. We show that a firm providing high-quality products may use an introductory offer as a signal of its quality by lowering the expected profit of the low-quality firm from imitating such pricing. We also explored whether our theory is robust in different conditions of time frame or competition.

Key Words

Introductory Offer, Time-inconsistency, Contract Design, Signal.

JEL Classification

D99, L11.

Resumo

Alguns pares de mercadorias são complementos que uma mercadoria básica permite exclusivamente o uso de um produto consumível, que permite a empresa a projetar os preços destes pares de mercadorias sob a forma de tarifas de duas partes. Estudos têm sido feitos sobre ofertas iniciais nas compras repetidas de um produto, enquanto discutimos sobre este tópico no que diz respeito à taxa fixa em uma tarifa em duas partes acima, exemplos incluindo a estratégia de preços nos mercados de impressoras, máquinas de café de cápsula , etc. Tomamos duas abordagens para modelar a estratégia de preços de uma empresa. Primeiro consideramos uma empresa racional enfrentando dois tipos de consumidores tempo-inconsistentes, um cenário típico em IO comportamental. Mostramos que as empresas geralmente fazem ofertas iniciais acompanhado por um aumento no preço por unidade em resposta aos consumidores tempo-inconsistentes, que *ex post* concede ao consumidor mais superávit e uma perda nos lucros da empresa. No entanto, quando o tempo-inconsistência e a discriminação em preço do segundo grau interagir uns com o outro, o resultado torna-se mais sutil: taxa fixa pode subir e preço por unidade pode diminuir em algumas condições. Em segundo lugar, exploramos o efeito de sinalização de uma oferta introdutória. Mostramos que uma empresa fornecendo produtos de alta qualidade pode usar uma oferta introdutória como um sinal da sua qualidade através da redução do lucro esperado da empresa de baixa qualidade de imitar tais preços. Também exploramos se as duas abordagens são robustas em diferentes condições de prazo ou competição.

Palavras-chave

Oferta Introdutória, Tempo-inconsistência, Desenho de Contrato, Signal.

Classificação JEL

D99, L11.

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List of Abbreviations

IC: Incentive Compatibility Constraint

ICH: Incentive Compatibility Constraint of the High Type

ICL: Incentive Compatibility Constraint of the Low Type

IO: Industrial Organization

IR: Individually Rational Constraint

IRH: Individually Rational Constraint of the High Type

IRL: Individually Rational Constraint of the Low Type

Contents

1 Introduction

It is very common that discounts are made upon goods that are new to consumers so that they would be induced to make a try. Meanwhile, when a firm makes such an introductory offer, we would often wonder what is the incentive behind it and whether it will benefit the consumers or it eventually implies exploitation, knowing that a firm always tries to maximize its profit based upon the consumers' preferences.

Studies have shown that in repeat purchases, introductory offers can be made by the company as a pricing strategy, which not only induces the consumers to try out the quality of the product, but also serves as a signaling device of quality, because a high-quality producer is more willing to sacrifice current profits to attract consumers (Tirole, 1988). Meanwhile, cases in various industries demonstrate that introductory offers are also made when the price of a product or service consists of two parts: the fixed and the variable part, in which the fixed fee usually serves as a relation-specific setup cost, allowing the consumers to utilize the consumable goods they'll continue to purchase. Their feature as complements to each other permits the firm to design a two-part tariff as a whole: the utility of either the basic good or the consumable good cannot be realized without the other good. Examples include printers and capsule coffee machines, which are often sold at a relative low price while the supplies and coffee capsules can be expensive and are compatible only with machines of the same brand. Conventionally, theory of price discrimination suggests that the per-unit price is set equal to the marginal cost while the fixed fee is equal to gross consumer surplus, so as to extract the maximum profit. Yet the aforementioned examples challenging this theory. By making introductory offers, the firm reduces its fixed fee, sometimes to a significant degree that it is lower than the marginal cost for the firm to provide the setup service/equipment, and that this loss has to be compensated by a per-unit price much higher than the marginal cost of the repurchasable goods.

We are interested in two mechanisms bringing about introductory offers, among others. The first is the consumers' time-inconsistent preferences, which makes consumers attribute different weights to utilities and prices in current and future periods. Through a model with time-inconsistent consumers and a time-consistent firm, we'll show that time-inconsistency of the consumers causes distortion to the firm's contract design, leading to the pricing behavior described above. The second mechanism involves the consumers' preference for quality and the effect of signaling quality of the pricing pattern. We model the situation with firms providing high and low quality in the market of a single type of product, where the consumers have incomplete information about the quality and have to surmise it from the prices they observe.

Past works in behavioral IO on time-inconsistency in the consumers' preferences sheds their merit on our topic. DellaVigna formalizes that when the consumer has time-inconsistency but is not fully aware of it, the firm may take advantage of the consumers' naiveté in the contract design (DellaVigna and Malmendier, 2004). Cases in industries like health clubs demonstrate that consumers are often time-inconsistent in their preferences, in most cases unaware themselves (DellaVigna and Malmendier, 2006). Further, DellaVigna contributed to the literature of application of psychology with economics, or behavioral economics, covering time-inconsistent preferences of the consumers (DellaVigna, 2009). In the laboratory, individuals are found to be time-inconsistent (Thaler, 1981). Experiments on intertemporal choice, summarized in Loewenstein and Prelec (1992) and Frederick, Loewenstein and O'Donoghue (2002) also challenged the traditional assumption that consumers' preferences are time-consistent.

Laibson (1997) and O'Donoghue and Rabin (1999a) formalized these preferences using (β, δ) preferences. When consumers estimate their payoffs, in addition to the common discounting factor δ between each successive period, there is an extra discounting factor β between the present period and future periods. DellaVigna and Malmendier (2004) and DellaVigna (2009) supposed a good with immediate payoff b_1 at present and delayed payoff b_2 in the future. An investment good, like exercising or searching for a job, has the features $b_1 < 0$ and $b_2 > 0$, which requires effort at present and delivers happiness tomorrow, while conversely a leisure good has the features $b_1 > 0$ and $b_2 < 0$, providing an immediate reward at a future cost. The setup cost in our case is pretty much like the concept of investment good, in the form that it permits consumers to enjoy utility from the goods they'll buy in the future.

This discrepancy between the consumer's preferences through time periods generates decisions different from those under common assumptions. We can refer to the concept above to interpret the issue of our interest. With standard preferences, the firm and the consumer shares a common time discounting rate, which is usually determined by the market interest rate and can be ignored in some cases, without losing generality. With an extra time discounting rate between the current period and the next, the consumer puts more weight on her present gains while the firm puts equal weight on present and future profits. In this way, an introductory offer, which is offset by a rise in the price in the future, can be preferable to the consumer under certain conditions.

However, as we will show hereafter, the results of our model is not quite so simple. If we were satisfied with the effect of time-inconsistency on only one pair of two-part tariff, the result would be exactly in the way we have expected. Actually, in our model there is one firm facing contract designing problem with respect to two types of time-inconsistent consumers with different demands. Consumers pay a fixed fee in the first period for the access to repurchasable goods in the future. We show that with some restrictions on certain variables, the model can also be specialized into one with time-consistent or only one type of consumers. Our findings are different from those in the studies of DellaVigna and alike in the form that instead of exploiting the consumers' time-inconsistency, the firm is obliged to adjust its prices so as to guarantee the consumers' non-negative expected surplus, which *ex post* grants the consumers more surplus at the price of a loss of the firm's profit and total surplus of the society. And the most interesting part of our result might be that the interaction between time-inconsistency and contract design generates unique patterns in the firm's pricing strategy. We will show that the effect of time-inconsistency dominates when the difference between the demands of the two types is relatively small; when the difference becomes bigger, however, the mechanism of contract design begins to dominate, and the firm will have to adjust its prices in order to guarantee the IR and IC constraints.

As will be discussed afterwards, the loss of profit in the presence of time-inconsistency drives a perfectly competitive market into an oligopolistic one, in which an introdutory offer could result from either time-inconsistency of the consumers or the competition for consumers between firms, or both of them. That also a reason why we choose to assume monopoly in the model. Two other assumptions will also be discussed about: the maintenance of exclusive compatibility, which is critical in charging a two-part tariff that is incurred in the present and future periods; and the commitment to a per-unit price, the absence of which might also result in an introductory offer although for a different reason, as we will demonstrate in Subsection 2.6.

Besides the issues of time-inconsistency, a number of studies have been made regarding introductory offers. In some of them, introductory offers are related to the issue of entry via endogenous quality choice by an incumbent firm (Farrel, 1984, Allen, 1984, both cited by Doyle, 1986). But quality is not considered to be a choice variable by either our model or Doyle (1986). Doyle (1986) demonstrated that introductory offers may arise out of a market structure characterized by uncertainty, enabling stores to undertake ex post monopoly pricing. Meanwhile the expected discounted profits of them is zero, as cost in introductory offers and advertisements arising from competition would absorb potential monopoly rents. On the other hand, we can infer that when the market is not under perfect competition, firms can exploit the market structure.

Theories also show that the difference between introductory offers in the setup cost and conventional two-part tariff can be attributed to the consumers' preference towards quality. Suppose that the consumer only cares about the quality instead of everything else (Tirole, 1988). If we let the quality multiplied by a taste factor represent the consumer's valuation for one product, the consumer would only purchase when the price is not greater than the valuation. This can give rise to interesting features, for example, under certain circumstances the optimal pricing may merely involve the joint price of the fixed and variable part. Meanwhile, as Tirole only discusses about introductory offers in repeat purchases in his book, we cannot directly apply his theory to explain introductory offers made in two-part tariffs. Farrell and Shapiro (1989) analyzed the relation between setup costs and quality offered by the seller. Their findings assert that when setup costs are observable, the first best can be achieved without requiring long-term contracts that specify the quality. On our end, we would expect to find that when setup costs signify the quality when they are observable to the consumers.

Shapiro (1983) studied the optimal pricing strategy of experience goods when consumers initially overestimate and underestimate the quality. The optimal way to build a reputation when consumers underestimate the quality is to use a low introductory price followed by a higher regular price. Seeing from a different angle, introductory offers can serve as signals of quality. The signaling effect of introductory offers has been studied through time. As in the literature of Ellison (2009), firms will disclose all relevant information to consumers if the information is costless to disclose and disclosures are credible (Grossman, 1981; Milgrom, 1981). However, in many situations the disclosure of information is costly. Milgrom and Roberts (1986) tried to formalize the studies by Nelson (1970, 1974, 1978) on the quality signaling of experience goods through advertising, which sometimes need not offer direct information on the product, together with the role of pricing. They discussed the effect of advertising and introductory offers as dissipative signals on separating equilibriums of firms with high and low quality. They assume that initial sales be increasing in the perceived quality, but without stating the mechanism why higher quality leads to more sales. On the other hand, Bagwell (1987) demonstrated that low introductory prices can be used to signal low production costs, and hence low prices in the second period. The logic behind this might be that an introductory offer reflects the firm's confidence of earning profit in the future, by low marginal cost in Bagwell's study, and by high quality in ours. We try to formalize on this signaling effect with our second approach.

Our second model wants to address the signaling effect of introductory offers in setup costs. We'll show that when consumers care only about the quality of the consumable products, the optimal pricing strategy involves only the joint price: the total weighted price of the fixed and variable parts. Under such a circumstance the high-type firm[1] can adjust both parts of the price simultaneously so as to lower the expected profit from imitating behaviors of the low type, without deviating from its optimal pricing. This effect facilitates separating strategies by undermining the incentive of imitation of the low type.

It might be noteworthy that, the cases in which the purchase includes a setup price that an introductory offer is made was not mentioned in any of these studies discussed above, regarding the two effects we have discussed. Although the principles behind making introductory offers are similar, some features distinguish the two-part tariff case from common cases. Under most circumstances the setup price brings little direct utility to the consumer, but only enables the consumer to enjoy utility from the consumable products. Often, similar to the cases of printers and capsule coffee machine, a small amount of consumable products come together in the initial package, the price of which is included in the setup price, so that the consumer learns the quality on paying the setup cost. Of course, the setup cost can vary within a wide range, as the firm can price discriminate through machines with different characters. Ellison (2005) notes that in this case the consumers who are more price-sensitive have a lower willingness to pay for extra improvements. We can infer that this is also the reason why introductory offers are mainly made in the basic (or cheapest) version of the setup purchase. Although we still assume that the consumers enjoy no direct utility from paying the setup cost, setup equipments with different prices help contract designing which provides different two-part tariffs.

The rest of this paper is organized as the following. In Section 2 we build a model with one firm solving the contract designing problem with respect to time-inconsistent consumers, with different levels of demand. In Section 3 we model the signaling effect of introductory offer in the setup cost. We briefly discuss about the robustness of the models in Section 4. Then we conclude with Section 5.

2 The Time-Inconsistency Approach

In this section we set up a two-period model with one firm and two types of consumers. We show how time-inconsistency of the consumers generates discrepancy in the firm's contract design of two-part tariffs in the frame of second-degree price discrimination, comparing with the standard case. As the concept of time-inconsistency is taken from behavioral IO, we may also designate this model as the behavioral model hereafter.

[1] Naturally, we denote firms which provide high quality as the high type, firms providing low quality as the low type.

First we describe the basic settings of the model, and then we're going to find out the effect resulting from time-inconsistency of the consumers in the scenario of one firm and two types of consumers, as well as the impact on welfare and profit. We then show that the basic model can be transformed into a variant with one type of firm, in which our main results still stand. At the end of this section we examine the validity of a few assumptions that are critical to this model.

2.1 Basic Settings

The monopolistic firm produces one product with a per-unit marginal cost c, requiring the consumers to pay a setup cost in order to be able to consume it. There exist two types of consumers whose numbers are both normalized to 1, with different demand functions: $D_H(p) = \theta_H - p$ and $D_L(p) = \theta_L - p$, where $\theta_H \geq \theta_L$. We denominate the consumer with higher demand as the high type, and that with lower demand as the low type. In our case it doesn't affect the results whether they are sophisticated consumers that are aware of the time-inconsistency of themselves or naive consumers that are unaware of it. As we will show, the time-discounting factor affects only the consumers' evaluation of their surplus, but not their future actions.

Knowing the demand functions, the firm wants to design two sets of two-part tariffs (L_H, p_H) and (L_L, p_L) for each type of consumer. For simplicity, we assume that the firm doesn't have any marginal cost by providing the setup instalation, which doesn't affect the result of the model.

In the first period, each consumer faces the decision of whether to purchase the product and which tariff to choose. A comsumer buys the product as long as she enjoys non-negative surplus, and chooses the tariff that grants her more surplus. In the second period each consumer decides how many units of products to buy according to her demand function.

The firm wants to garantee that each type of consumer will buy and that each type chooses her corresponding tariff, i.e. the high type chooses (L_H, p_H) and the low type chooses (L_L, p_L). Based on this the firm designs tariffs that maximize its expected profit from both types. Conventionally, we expect that consumers with higher willingness to pay would prefer the set of tariff with a higher fixed fee and a lower per-unit price, while consumers with lower willingness to pay would prefer the set of tariff with a lower fixed fee and a higher per-unit price.

In the next subsections we solve the model with the assumption that consumers are time-inconsistent, i.e. they discount future prices and utilities in their perception by a factor β while the firm doesn't. We expect to discover a fall in the fixed fee as the result of a distortion in the consumers' perceived surplus, and a raise in the per-unit price as the firms' compensation for their profit loss in the fixed fee. In this model we define a fall in the fixed fee in the time-inconsistent case compared with the time-consistent case as an introductory offer. We seek the existence of such introductory offer in both L_H and L_L.

2.2 Optimal Pricing with Time-Inconsistency

In this subsection we assume that when consumers evaluate in period 1 whether to purchase and which tariff to choose, their surplus and the prices they'll pay in period 2 are discounted by a factor $\beta \in (0, 1]$. It might be useful to have in mind that when $\beta = 1$, the consumers are time-consistent.

Given the prices for each type, the gross utility that a consumer enjoys is $\int_0^{Q_j}(\theta_j - Q_j)dQ_j$, where $Q_j = \theta_j - p_j$; $j \in (H, L)$. When the consumer evaluates her surplus, she multiplies the gross utility with a time discounting factor, since she only enjoys the utility in the second period; she then substracts the prices she has to pay, also discounting the per-unit price paid in the second period. Thus the net surplus that a time-inconsistent consumer anticipates is $E(CS_j) = \int_0^{Q_j}(\theta_j - Q_j)dQ_j - Lj - \beta p_j(\theta_j - p_j)$, $j \in (H, L)$.

However, when the time comes to period 2, the consumers no longer take β into account when they decide how many products to buy. The firm, on the other hand, does not suffer from time-inconsistency, hence has the incentive to exploit the consumers with tariffs that are different from the standard case. With this assumption we solve the problem about the firm's contract design and see what difference does time-inconsistency make to the result and whether, or by how much it is as we have expected.

With the consumers' extra time discounting factor, β, the firm maximizes its expected profit from both types:

$$\max_{L_H, L_L, p_H, p_L} \pi = L_H + (p_H - c) \cdot (\theta_H - p_H) + L_L + (p_L - c) \cdot (\theta_L - p_L) \quad (2.1)$$

subject to

$$\beta \int_0^{Q_L} (\theta_L - Q_L)dQ_L - L_L - \beta p_L(\theta_L - p_L) \geq 0(IRL); \quad (2.2)$$

$$\beta \int_0^{Q_H} (\theta_H - Q_H)dQ_H - L_H - \beta p_H(\theta_H - p_H) \geq 0(IRH); \quad (2.3)$$

$$\beta \int_0^{Q_L} (\theta_L - Q_L) dQ_L - L_L - \beta p_L(\theta_L - p_L) \geq \beta \int_0^{Q'_L} (\theta_L - Q'_L) dQ'_L - L_H - \beta p_H(\theta_L - p_H) (ICL), \tag{2.4}$$

where $Q'_L = \theta_L - p_H$;

$$\beta \int_0^{Q_H} (\theta_H - Q_H) dQ_H - L_H - \beta p_H(\theta_H - p_H) \geq \beta \int_0^{Q'_H} (\theta_H - Q'_H) dQ'_H - L_L - \beta p_L(\theta_H - p_L) (ICH), \tag{2.5}$$

where $Q'_H = \theta_H - p_L$.

It is worth noticing that in the firm's maximization problem there appears no time discounting factor. β only exerts its effect on the consumers' expected surplus in the constraints.

IRL is binding and thus equivalent to

$$L_L = \frac{\beta}{2}(\theta_L - p_L)^2. \tag{2.6}$$

Simplifying (2.3) yields

$$L_H \leq \frac{\beta}{2}(\theta_H - p_H)^2. \tag{2.7}$$

As we know that IRL is binding, i.e.

$$\beta \int_0^{Q_L} (\theta_L - Q_L) dQ_L - L_L - \beta p_L(\theta_L - p_L) = 0,$$

Constraint ICL is equivalent to

$$L_H \geq \frac{\beta}{2}(\theta_L - p_H)^2. \tag{2.8}$$

And rearranging (2.5) we have

$$L_H \leq L_L + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2.$$

Substituting L_L to get

$$L_H \leq \frac{\beta}{2}(\theta_L - p_L)^2 + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2. \tag{2.9}$$

It is obvious that $\frac{\beta}{2}(\theta_L - p_L)^2 - \frac{\beta}{2}(\theta_H - p_L)^2 < 0$, so we know that

$$\frac{\beta}{2}(\theta_L - p_L)^2 + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2 < \frac{\beta}{2}(\theta_H - p_H)^2.$$

Also, we have

$$\frac{\beta}{2}(\theta_L - p_L)^2 + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2 \geq \frac{\beta}{2}(\theta_L - p_H)^2. \tag{2.10}$$

Proof: The inequality above is equivalent to

$$(\theta_L - p_L)^2 + (\theta_H - p_H)^2 - (\theta_H - p_L)^2 - (\theta_L - p_H)^2 \geq 0.$$

Rearrange to get:

$$(\theta_L + \theta_H - 2p_L)(\theta_L - \theta_H) + (\theta_H + \theta_L - 2p_H)(\theta_H - \theta_L) \geq 0,$$

Which can be further reduced to

$$2(\theta_H - \theta_L)(p_L - p_H) \geq 0.$$

The inequality holds for $p_L > p_H$.

Q.E.D.

Thus if we let (2.9) be binding, constraints (2.7) and (2.8) will be satisfied as well. Now constraint (2.9) becomes:

$$L_H = \frac{\beta}{2}(\theta_L - p_L)^2 + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2. \tag{2.11}$$

Substitute (2.6) and (2.11) into (2.1) and the maximization problem becomes:

$$\max_{p_H, p_L} \pi = \beta(\theta_L - p_L)^2 + \frac{\beta}{2}(\theta_H - p_H)^2 - \frac{\beta}{2}(\theta_H - p_L)^2 + (p_H - c)(\theta_H - p_H) + (p_L - c)(\theta_L - p_L) \tag{2.12}$$

F.O.C.:

$$\partial\pi/\partial p_H = -\beta\theta_H + \beta p_H + \theta_H - 2p_H + c = 0;$$

$$\partial\pi/\partial p_L = -2\beta\theta_L + 2\beta p_L + \beta\theta_H - \beta p_L + \theta_L - 2p_L + c = 0;$$

S.O.C.:

$$\partial^2\pi/\partial p_H^2 = \beta - 2 < 0;$$

$$\partial^2\pi/\partial p_L^2 = \beta - 2 < 0.$$

Solve to get:

$$p_H^* = \frac{1}{2-\beta}c + \frac{1-\beta}{2-\beta}\theta_H; \tag{2.13}$$

$$p_L^* = \frac{1}{2-\beta}c + \frac{\beta}{2-\beta}\theta_H + \frac{1-2\beta}{2-\beta}\theta_L. \tag{2.14}$$

Substitute (23) and (24) into (20) and (21) we have

$$L_H^* = \frac{\beta}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 + \frac{\beta}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 - \frac{\beta}{2}(\frac{2-2\beta}{2-\beta}\theta_H - \frac{1}{2-\beta}c + \frac{2\beta-1}{2-\beta}\theta_L)^2; \tag{2.15}$$

$$L_L^* = \frac{\beta}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2. \tag{2.16}$$

If we let $\beta = 1$, the results are equivalent to those we can get with time-consistent consumers:

$$p_H^{C*} = c; \tag{2.17}$$

$$p_L^{C*} = c + \theta_H - \theta_L; \tag{2.18}$$

$$L_H^{C*} = \frac{1}{2}(2\theta_L - \theta_H - c)^2 + \frac{1}{2}(\theta_H - c)^2 - \frac{1}{2}(\theta_L - c)^2; \tag{2.19}$$

$$L_L^{C*} = \frac{1}{2}(2\theta_L - \theta_H - c)^2. \tag{2.20}$$

2.3 The Effect of Time-Inconsistency on Pricing Strategy

In this subsection we evaluate the effect of β on the firm's contract design, in order to discover whether they exert the effects in the way we have expected.

Proposition 1 (Monopolistic firm, 2 types of consumers, time-consistent vs. time-inconsistent). With the consumers' time-inconsistent preferences, the firm makes an introductory offer accompanied by a raise in the per-unit prices to the consumers with higher willingness to pay, while it does the same to the consumers with lower willingness to pay as long as the difference between the demands of the two types is small enough($\theta_H - \theta_L < \frac{1}{2}(\theta_L - c)$). In these cases, each part of the tariffs deviates further from their ordinary values as β gets smaller, i.e. the time-inconsistency becomes more severe.

Proof of Proposition 1. To see the effect of β on the firm's pricing, we take derivatives of each part of the tariffs with respect to β:

$$dp_H^*/d\beta = \frac{c-\theta_H}{(2-\beta)^2} < 0, \tag{2.21}$$

Which implies that the smaller β is, i.e. the more severe the consumers' time-inconsistency is, the larger p_H grows.

$$dp_L^*/d\beta = \frac{c + 2\theta_H - 3\theta_L}{(2-\beta)^2} \tag{2.22}$$

$dp_L^*/d\beta < 0$ if $3\theta_L > c + 2\theta_H$, or after rearrangement,

$$\theta_H - \theta_L < \frac{1}{2}(\theta_L - c). \tag{2.23}$$

This is an assumption that we would like to maintain throughout this model, in the form that facilitates our discussion about the issues of welfare and profit:

Assumption 1 The difference between the demands of the two types is moderate that $\theta_H - \theta_L < \frac{1}{2}(\theta_L - c)$: the difference between θ_H and θ_L is smaller than half the difference between θ_L and c.

Under this assumption, the smaller β is, the higher p_L becomes.

Then, about L_H we have

$$dL_H^*/d\beta = \frac{1}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 + \beta(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)\frac{3\theta_L - c - 2\theta_H}{(2-\beta)^2}$$

$$+\frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 + \beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2} - \frac{1}{2}(\frac{2-2\beta}{2-\beta}\theta_H - \frac{1}{2-\beta}c + \frac{2\beta-1}{2-\beta}\theta_L)^2$$

$$-\beta(\frac{2-2\beta}{2-\beta}\theta_H - \frac{1}{2-\beta}c + \frac{2\beta-1}{2-\beta}\theta_L)\frac{-2\theta_H - c + 3\theta_L}{(2-\beta)^2}$$

$$= \frac{\beta}{(2-\beta)^2}(3\theta_L - c - 2\theta_H)(\theta_L - \theta_H) + \frac{1}{2}\cdot\frac{3\beta\theta_L - 2c + (2-3\beta)\theta_H}{2-\beta}(\theta_L - \theta_H)$$

$$+\frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 + \beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2}$$

$$= \frac{3\beta(4-\beta)(\theta_L - \theta_H) - 4c + 4\theta_H}{2(2-\beta)^2}(\theta_L - \theta_H) + \frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 + \beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2}$$

$$= \frac{3\beta(4-\beta)(\theta_L - \theta_H)^2}{2(2-\beta)^2} + \frac{2(\theta_H - c)(\theta_H - \theta_L)}{(2-\beta)^2} + \frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 + \beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2} > 0 \tag{2.24}$$

From which we infer that, as β gets smaller, L_H falls together with it. It means that as the high-type consumers become more time-inconsistent, the firm makes introductory offers to them with a greater degree.

The effect of β on L_L^* is clear:

$$dL_L^*/d\beta = \frac{1}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 + \beta(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)\frac{3\theta_L - c - 2\theta_H}{(2-\beta)^2} \quad (2.25)$$

As we know that $\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H = \theta_L - p_L > 0$, $dL_L/d\beta > 0$ if $3\theta_L > c + 2\theta_H$, i.e. *Assumption 1* holds.

Thus we infer that as β gets smaller, L_L^* also becomes smaller.

Q.E.D.

We can further reduce (2.25) as

$$dL_L^*/d\beta = \frac{(1+\beta)\theta_L - c - \beta\theta_H}{2(2-\beta)^3}[(2-\beta)(1+\beta)\theta_L - (2-\beta)c - \beta(2-\beta)\theta_H + 6\beta\theta_L - 2\beta c - 4\beta\theta_H] \quad (2.26)$$

As we know that $\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H = \theta_L - p_L > 0$, $dL_L^*/d\beta > 0$ if

$$(-\beta^2 + 7\beta + 2)\theta_L - (\beta + 2)c + (\beta^2 - 6\beta)\theta_H > 0,$$

Which can be transformed into

$$(\beta^2 - 6\beta)(\theta_H - \theta_L) + (\beta + 2)(\theta_L - c) > 0.$$

Solve to get

$$\theta_H - \theta_L < \frac{\beta + 2}{6\beta - \beta^2}(\theta_L - c).$$

When this condition holds, we can infer that as β gets smaller, L_L^* also becomes smaller.

Further, it's easy to discover that

$$\frac{\beta + 2}{6\beta - \beta^2}(\theta_L - c) > \frac{1}{2}(\theta_L - c),$$

Which implies that when $\frac{1}{2}(\theta_L - c) < \theta_H - \theta_L < \frac{\beta+2}{6\beta-\beta^2}(\theta_L - c)$, the firm lowers both L_L and p_L at the same time.

Now we have a corollary that describes the firm's pricing strategy in more detail based upon *Proposition 1*:

Corrollary 1-1 (Monopolistic firm, low-type consumers, time-consistent vs. time-inconsistent). With the low-type consumers' time-inconsistent preferences, the firm makes an introductory offer accompanied by a raise in the per-unit prices as long as the difference between the demands of the two types is small($\theta_H - \theta_L < \frac{1}{2}(\theta_L - c)$); it lowers both part of the tariff when the difference is moderate($\frac{1}{2}(\theta_L -$

$c) < \theta_H - \theta_L < \frac{\beta+2}{6\beta-\beta^2}(\theta_L - c)$); and raises the fixed fee while lowering the per-unit price if the difference is large($\theta_H - \theta_L > \frac{\beta+2}{6\beta-\beta^2}(\theta_L - c)$). In any of these cases, as time-inconsistency becomes more severe, prices deviate further from their values with time-consistent consumers.

With the consumers' time-inconsistent preferences, the firm responds with two-part tariffs different from those when the consumers are time-consistent. To the consumers with higher willingness to pay, the firm makes an introductory offer accompanied by a raise in the per-unit prices. The firm does the same to the consumers with lower willingness to pay as long as the difference between the demands of the two types is small enough($\theta_H - \theta_L < \frac{1}{2}(\theta_L - c)$). In any of these cases, as β gets smaller, i.e. the time-inconsistency becomes more severe, the firm makes a greater degree of introductory offers in the setup costs L_H and L_L and a higher raise in the per-unit prices p_H and p_L.

The interpretation for our results can be quite intuitive. Let's first look at the case when *Proposition 1* holds. When the consumers are time-inconsistent, they consider both their utility and the price they'll pay with the same time discounting factor β. Thus their expected surplus shrinks proportionally which gives the firm an incentive to make introductory offers, in the form of lowering the setup cost so that the constraints IRL, IRH, ICL and ICH would maintain. Meanwhile, in order to compensate for the loss caused by introductory offers, the firm raises the per-unit prices to compensate itself, which seems to be tolerable by the consumers as they are time-inconsistent.

In addition, the difference between the various parts of tariffs in time-inconsistent cases and themselves in the time-consistent case monotonically increases as β gets smaller, which represents a more severe time-inconsistency. For either the two-type model or the one-type variation, if β is sufficiently small, the setup cost may even be smaller than the marginal cost of producing it, with the per-unit price high above its marginal cost. This is conformable with what we can observe in some markets, which challenges the conventional price discrimination theory.

Yet the above is only the ideal situation when the demands of the two types of consumers have a relatively small difference from each other. The interaction between time-inconsistent preferences and contract designing shows some interesting features on the tariff for the low-type consumers. The original two-part tariff as a whole becomes less attractive when the consumers are time-inconsistent. This effect does not dominate until the demand of the low type becomes small enough compared to that of the high type: $\frac{1}{2}(\theta_L - c) < \theta_H - \theta_L < \frac{\beta+2}{6\beta-\beta^2}(\theta_L - c)$. In this situation, the demand of the low type is so low that the firm can no longer compensate itself by exploiting the consumers' time-inconsistency and raising the per-unit price: it can only guarantee the low type's participation (maintain IRL constraint) by lowering both parts of the tariff at the same time.

Another impact of time-inconsistency on the consumers' preferences to the tariffs is that the fixed fee paid in the present is weighted more than the per-unit price paid in the future. From (2.17)~(2.20) we can see that, for time-consistent consumers, $p_H^C < p_L^C$, while $L_H^C > L_L^C$. The high-type consumer pays a higher fixed fee in the present for a lower per-unit price in the future; the low-type consumer pays a lower fixed fee in the present at the cost of a higher per-unit price. When the consumers become time-inconsistent, they attribute more weight to the fixed fee than the per-unit price, hence (L_H, p_H) becomes less attractive compared with (L_L, p_L). Meanwhile, as the difference between the demand of the two types becomes bigger, it becomes more difficult to reconciliate this difference in contract design. It might appeal to the firm that the high-type consumers contribute to a greater part in the firm's profit function than the low type, and thus should be attributed higher priority. When the demand of the high-type consumer is high enough compared with that of the low-type consumer, i.e. $\theta_H - \theta_L > \frac{\beta+2}{6\beta-\beta^2}(\theta_L - c)$, the firm raises L_L and lowers p_L. This adjustment serves to maintain constraint ICH, or in other words, to prevent the high type from choosing the tariff for the low type. Adjusting the tariff in such a direction is ineffective in maintaining the low type's participation; yet this is worthwhile for the firm since the high type contributes a much more important part of profit.

As we are going to show in Subsection 2.5, when there exists one firm and one type of consumers, the effect of contract design disappears and there is left only the effect of time-inconsistency, which we can then observe more clearly. Nevertheless, the above-mentioned interaction between time-inconsistency and contract design is more fascinating for deeper thinkings and discussions.

2.4 Welfare and Profit

In this subsection we discuss about the effect time-inconsistency exerts on the consumers' surplus and the firm's profit, with the premise that we maintain Assumption 1, as we are more interested in the general situation that the firm makes introductory offers to both types of consumers accompanied by a raise in per-unit price. It is worth noticing that instead of the surplus perceived by the consumers in the first period, we should look at the factual surplus after the two periods.

With time-consistent preferences, IRL is binding, thus the low-type consumer enjoys the net surplus

$$CS_L^C = 0;$$

While the high-type consumer enjoys the net surplus

$$CS_H^C = \int_0^{Q_H} (\theta_H - Q_H)dQ_H - L_H^C - p_H^C(\theta_H - p_H^C) = \frac{1}{2}(\theta_H - p_H^C)^2 - L_H^C. \quad (2.27)$$

With generalized (time-consistent and time-inconsistent) preferences, the low-type consumer enjoys the net surplus

$$CS_L = \int_0^{Q_L} (\theta_L - Q_L)dQ_L - L_L - p_L(\theta_L - p_L) = \frac{1}{2}(\theta_L - p_L)^2 - L_L$$

$$= \frac{1-\beta}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 > 0, \quad (2.28)$$

Which is the difference between the values of L_L with time-consistent and time-inconsistent consumers. As we know from Proposition 1, L_L becomes smaller as β gets smaller, so CS_L becomes larger.

The high-type consumer enjoys the net surplus

$$CS_H = \int_0^{Q_H} (\theta_H - Q_H)dQ_H - L_H - p_H(\theta_H - p_H) = \frac{1}{2}(\theta_H - p_H)^2 - L_H. \quad (2.29)$$

Take derivative with respect to β:

$$dCS_H/d\beta = \frac{\partial CS_H}{\partial p_H}\frac{dp_H}{d\beta} - \frac{dL_H}{d\beta} = -(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{c-\theta_H}{(2-\beta)^2} - \frac{3\beta(4-\beta)(\theta_L-\theta_H)^2}{2(2-\beta)^2}$$

$$-\frac{2(\theta_H-c)(\theta_H-\theta_L)}{(2-\beta)^2} - \frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 - \beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2}$$

$$= (\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2(\frac{1}{2-\beta} - \frac{1}{2} - \frac{\beta}{2-\beta}) - \frac{3\beta(4-\beta)(\theta_L-\theta_H)^2}{2(2-\beta)^2} - \frac{2(\theta_H-c)(\theta_H-\theta_L)}{(2-\beta)^2}$$

$$= -\frac{\beta}{2(2-\beta)}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 - \frac{3\beta(4-\beta)(\theta_L-\theta_H)^2}{2(2-\beta)^2} - \frac{2(\theta_H-c)(\theta_H-\theta_L)}{(2-\beta)^2} < 0 \quad (2.30)$$

Which implies that the high-type consumer enjoys more net surplus as β gets smaller.

So we know that with time-inconsistency, the net surplus of both high-type and low-type consumers increases. Now we would like to see what happens to the firm's profit from these two types. The firm earns from the high-type consumer the profit

$$\pi_H = L_H + (p_H - c) \cdot (\theta_H - p_H). \tag{2.31}$$

The effect of change in β on π_H is

$$d\pi_H/d\beta = \frac{dL_H}{d\beta} + \frac{\partial \pi}{\partial p_H}\frac{dp_H}{d\beta}$$

$$= \frac{3\beta(4-\beta)(\theta_L - \theta_H)^2}{2(2-\beta)^2} + \frac{2(\theta_H - c)(\theta_H - \theta_L)}{(2-\beta)^2} + \frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2$$

$$+\beta(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)\frac{\theta_H - c}{(2-\beta)^2} + \frac{\beta}{2-\beta}(\theta_H - c)\frac{c - \theta_H}{(2-\beta)^2}$$

$$= \frac{3\beta(4-\beta)(\theta_L - \theta_H)^2}{2(2-\beta)^2} + \frac{2(\theta_H - c)(\theta_H - \theta_L)}{(2-\beta)^2} + \frac{1}{2}(\frac{1}{2-\beta}\theta_H - \frac{1}{2-\beta}c)^2 > 0 \tag{2.32}$$

Which means that as β gets smaller, the firm earns less profit from the high-type consumer.

The firm earns from the low-type consumer the profit

$$\pi_L = L_L + (p_L - c) \cdot (\theta_L - p_L). \tag{2.33}$$

Take derivative with respect to β:

$$d\pi_L/d\beta = \frac{dL_L}{d\beta} + \frac{\partial \pi}{\partial p_L}\frac{dp_L}{d\beta}$$

$$= \frac{1}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 + \beta(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)\frac{3\theta_L - 2\theta_H}{(2-\beta)^2}$$

$$+\frac{\beta}{2-\beta}(3\theta_L - 2\theta_H - c)\frac{c + 2\theta_H - 3\theta_L}{(2-\beta)^2}$$

$$= \frac{1}{2}(\frac{1+\beta}{2-\beta}\theta_L - \frac{1}{2-\beta}c - \frac{\beta}{2-\beta}\theta_H)^2 + \frac{\beta(3\theta_L - 2\theta_H - c)}{(2-\beta)^2}(\theta_H - \theta_L) > 0, \tag{2.34}$$

Given that Assumption 1 holds, i.e. $3\theta_L > c + 2\theta_H$. So we know that as β gets smaller, the firm earns less profit from the low-type consumer.

Generalizing what has been discussed about in this subsection, we have:

Corollary 1-2 (Monopolistic firm, 2 types of consumers, time-consistent vs. time-inconsistent). Time-inconsistency causes distortion to the consumers' surplus and the firm's profit. For the high-type consumers and for low-type consumers that satisfy *Assumption 1*, as time-inconsistency gets more severe, the consumers *ex post* enjoy more net surplus and the firm earns less profit from them.

We would still want to know the effect of time-inconsistency on total surplus. Suppose there is no setup cost and time-inconsistency, and a social planner chooses (p_H, p_L) in order to maximize the total surplus with respect to both types of consumers:

$$\max_{p_H} TS_H = \int_0^{Q_H} (\theta_H - Q_H - c)dQ_H, Q_H = \theta_H - p_H, \tag{2.35}$$

i.e.

$$\max_{p_H} TS_H = (\theta_H - c)(\theta_H - p_H) - \frac{1}{2}(\theta_H - p_H)^2$$

F.O.C.:

$$\partial TS_H / \partial p_H = c - \theta_H + \theta_H - p_H = 0;$$

S.O.C.:

$$\partial^2 TS_H / \partial p_H^2 = -1 < 0,$$

So we have the maximizing solution

$$p_H^* = c. \tag{2.36}$$

The social planner also solves the problem:

$$\max_{p_L} TS_L = \int_0^{Q_L} (\theta_L - Q_L - c)dQ_L, Q_L = \theta_L - p_L, \tag{2.37}$$

Which similarly leads to

$$p_L^* = c. \tag{2.38}$$

From Proposition 1 we know that, with the consumers' time-inconsistency, both p_H and p_L increases, deviating further from their optimal levels that maximize the total surplus. The setup costs serve only as a transfer of consumers surplus to the firm's profit, without affecting total surplus. Hence we have:

Corollary 1-3 (Monopolistic firm, 2 types of consumers, time-consistent vs. time-inconsistent). Under Assumption 1, total surplus of the society is undermined by time-inconsistency, deviating further from social optimal than in the time-consistent case.

When consumers evaluate their decisions at the beginning, time-inconsistency let them perceive less surplus than they actually will enjoy. So the firm has to sacrifice a part of its profit to retain the consumers, which *ex post* grants the consumers more surplus than in the time-consistent case. Generally, time-inconsistency of the consumers causes distortion to the total surplus of the society, which suffers from a loss as a result of deviating further from social optimal. The most interesting aspect is that, it is the time-consistent firm suffering from a loss instead of the time-inconsistent consumer. On the contrary, the time-inconsistent consumer enjoys more net surplus because in the first period, when she evaluates her purchase, she perceives less surplus than that she is going to enjoy in fact. In our case, when there are two players in a game, one rational and one irrational, the rational player is at disadvantage instead of the irrational player. But whether this is true to other applications remains to be explored.

Throught these subsections we have assumed 2 types of consumers in order to discuss about the interaction between time-inconsistency and contract design. If we want to address only to the effect that time-inconsistency exerts on a two-part tariff, we can suppose that the two types have equal demands, i.e. $\theta_H = \theta_L = \theta$, which ends up in the same situation: one firm, one type of consumers. In the next subsection we are going to show that our model can be simplified to one with one monopolistic firm and one consumer, and we shall also find out whether our conclusions still hold in the simplified variation of the model.

2.5 One Monopolistic Firm, One Consumer

In this subsection we discuss about a variation of the time-inconsistency model, assuming that there is one monopolistic firm and one type of consumers whose number is normalized to 1, with the demand function $D(p) = \theta - p$, which is equivalent to assuming $\theta_H = \theta_L = \theta$. The other setups are the same with our two-type model.

With time-inconsistent consumers, the firm maximizes its expected profit:

$$\max_{L,p} \pi = L + (p - c)(\theta - p) \tag{2.39}$$

subject to

$$\beta \int_0^Q (\theta - Q)dQ - L - \beta p(\theta - p) \geq 0, \tag{2.40}$$

Where $Q = \theta - p$.

We know that constraint (2.40) must be binding, hence

$$L = \frac{\beta}{2}(\theta - p)^2. \tag{2.41}$$

Plug (2.41) back into (2.39) and the maximization problem becomes:

$$\max_p \pi = \frac{\beta}{2}(\theta - p)^2 + (p - c)(\theta - p) \tag{2.42}$$

F.O.C.:

$$\partial \pi / \partial p = -\beta\theta + \beta p + \theta - 2p + c = 0;$$

S.O.C.:

$$\partial^2 \pi / \partial p^2 = \beta - 2 < 0.$$

Solve to get:

$$p^* = \frac{1}{2 - \beta}c + \frac{1 - \beta}{2 - \beta}\theta. \tag{2.43}$$

Substitute into (2.41) to get:

$$L^* = \frac{\beta}{2}(\frac{1}{2 - \beta}\theta - \frac{1}{2 - \beta}c)^2. \tag{2.44}$$

In fact, the results of the two-type model also apply for the one-type variation. If we directly substitute using $\theta_H = \theta_L = \theta$ into (2.13) and (2.14), we get:

$$p^* = \frac{1}{2 - \beta}c + \frac{1 - \beta}{2 - \beta}\theta.$$

It is easy to discover that by substituting into either equation, we can get the identical result. Similarly, by substituting $\theta_H = \theta_L = \theta$ into (2.15) and (2.16) we have

$$L^* = \frac{\beta}{2}(\frac{1}{2 - \beta}\theta - \frac{1}{2 - \beta}c)^2.$$

Again, the result is the same by substituting into either equation.

When $\beta = 1$, we have the result with time-consistent consumers. Plug $\beta = 1$

into (2.45) and (2.46) so that we have

$$p^* = c; \tag{2.45}$$

$$L^* = \frac{1}{2}(\theta - c)^2. \tag{2.46}$$

The result is consistent with the standard price discrimination theory, i.e. the per-unit price is equal to marginal cost and the fixed fee is equal to the consumer's gross surplus.

In order to find out the effect of β on the two-part tariff, we take derivatives of p and L with respect to β:

$$\partial p^*/\partial\beta = \frac{1}{(2-\beta)^2}c + \frac{-\theta(2-\beta)+\theta(1-\beta)}{(2-\beta)^2} = \frac{c-\theta}{(2-\beta)^2} < 0 \tag{2.47}$$

$$\begin{aligned}\partial L^*/\partial\beta &= \frac{1}{2}(\frac{1}{2-\beta}\theta - \frac{1}{2-\beta}c)^2 + \frac{\beta}{2}\cdot 2(\frac{1}{2-\beta}\theta - \frac{1}{2-\beta}c)[\frac{\theta}{(2-\beta)^2} - \frac{c}{(2-\beta)^2}] \\ &= \frac{1}{2}(\frac{\theta - c}{2-\beta})^2 + \frac{\beta(\theta-c)^2}{(2-\beta)^3} > 0 \end{aligned} \tag{2.48}$$

Which implies that the smaller β is, i.e. the more time-inconsistent the consumer is, the more the setup cost falls and the more the per-unit price increases.

Generalizing the above discussed we have:

Proposition 2 (Monopolistic firm, 1 type of consumer, time-consistent vs. time-inconsistent). With the consumer's time-inconsistent preferences, the firm's optimal strategy varies from that when the consumer is time-consistent. The firm makes an introductory offer by lowering the setup cost, and raises the per-unit price at the same time. The more time-inconsistent the consumer is, i.e. the smaller β is, the more the setup cost falls and the per-unit price raises.

We would also like to discuss about the issue of welfare and profit. When the consumer is time-consistent, she enjoys the net surplus

$$CS^C = \int_0^Q (\theta - Q)dQ - L - p(\theta - p) = 0, \tag{2.49}$$

As we know that constraint (2.40) is binding.

In this case the firm earns the profit

$$\pi^C = L + (p-c)(\theta - p) = \frac{1}{2}(\theta - c)^2, \tag{2.50}$$

And the total surplus is

$$TS^C = \frac{1}{2}(\theta - c)^2. \tag{2.51}$$

When the consumer is time-inconsistent, instead of the net surplus perceived in the first period, in fact she enjoys

$$CS = \int_0^{Q'} (\theta - Q')dQ' - L' - p'(\theta - p') = \frac{1-\beta}{2(2-\beta)^2}(\theta - c)^2 > 0. \tag{2.52}$$

And the firm earns the profit

$$\pi = L + (p - c)(\theta - p) = \frac{1}{4 - 2\beta}(\theta - c)^2; \tag{2.53}$$

In this case the total surplus is

$$TS = \frac{3 - 2\beta}{2(2-\beta)^2}(\theta - c)^2. \tag{2.54}$$

It is easy to discover that $\pi < \pi^C$, $TS < TS^C$.

If we take derivatives of CS, π and TS with respect to β, we have:

$$dCS/d\beta = -\frac{\beta}{2(2-\beta)^3}(\theta - c)^2 < 0, \tag{2.55}$$

Which implies that the smaller β is, i.e. the more time-inconsistent the consumer is, the more net surplus she enjoys *ex post*;

$$d\pi/d\beta = \frac{2}{(4 - 2\beta)^2}(\theta - c)^2 > 0, \tag{2.56}$$

Which means that as β gets smaller, the firm earns less profit;

$$dTS/d\beta = \frac{1-\beta}{(2-\beta)^3}(\theta - c)^2 > 0, \tag{2.57}$$

Which means that as β becomes smaller, the total surplus of the society shrinks.

Therefore we have:

Corollary 2-1 (Monopolistic firm, 1 type of consumer, time-consistent vs. time-inconsistent). With the consumer's time-inconsistent preferences, the consumer enjoys more net surplus while the firm earns less profit, and the society suffers from a loss of total surplus compared with the time-consistent case. The difference of these values compared with the time-consistent case gets bigger as the consumer becomes more time-inconsistent, which is represented by a smaller

β.

When there is only one type of consumers, the problem caused by contract design disappears. The conclusions and the mechanism behind them are more obvious. When there exists time-inconsistency, in order to guarantee that the consumer buys, the firm has to make an introductory offer to grant the consumer non-negative perceived surplus. Then the firm compensates itself by raising the per-unit price, which is discounted by the time-inconsistent consumer and seems not as severe as it factually is. Also, as the consumers *ex ante* expect less surplus than they actually get, time-inconsistency causes distortion so that the consumers *ex post* enjoy more surplus at the price of less profit for the firm and a loss of total surplus of the society.

2.6 Competition, Compatibility and Commitment to Price

There are three assumptions playing critical roles in this model. So far we have assumed monopoly of the firm, as most of the studies mentioned in the introduction do. The second of our important assumptions is that the firm provides the equipments with special designs so that the products of one firm is only compatible with the equipement of the same firm. And the third one is that the firm commits to a per-unit price it announces at the beginning.

From *Proposition 1* and *Proposition 2*, we see that the firm is expected to make an introductory offer accompanied by a raise in the per-unit price in response to the consumers' time-inconsistency (with some restrictions when in *Proposition 1*). In this subsection, first we are going to see how would a firm choose its pricing strategy if it doesn't have commitment to a per-unit price. Next, we would like to discover whether the firm carries on similar strategies and whether maintaining exclusive compatibility is the firm's priority in markets with different levels of competition.

2.6.1 Commitment to Per-unit Price

In Subsection 2.5 we have shown that for a market with one monopolistic firm and one type of consumer, the firm makes an introductory offer together with a raise in per-unit price when the consumer is time-inconsistent. Now, with other assumptions remaining the same, we would like to change the assumption about price commitment and see what would be the firm's pricing strategy. Here we assume that despite that the firm has announced a per-unit price p in the first period, it can set another per-unit price p_M in the second period, which is equal to the monopolistic price since the consumers are locked-in and face no other choices.

In the second period the firm maximizes its profit from selling the consumable goods:

$$\max_{p_M} \pi_2 = (p_M - c)(\theta - p_M) \tag{2.58}$$

F.O.C.:

$$\partial \pi_2 / \partial p_M = \theta - 2p_M + c = 0 \tag{2.59}$$

Solve to get

$$p_M^* = \frac{c + \theta}{2} \tag{2.60}$$

The consumers are aware of this fact and won't believe any price announced in the first period, and they expect to enjoy the surplus $E(CS) = \beta \int_0^Q (\theta - Q)dQ - L - \beta p_M(\theta - p_M)$, where $Q = \theta - p_M$.

The firm solves the problem:

$$\max_{L, p_M} \pi = L + (p_M - c)(\theta - p_M) \tag{2.61}$$

subject to

$$\beta \int_0^Q (\theta - Q)dQ - L - \beta p_M(\theta - p_M) \geq 0, \tag{2.62}$$

Where $Q = \theta - p_M$.

Substitute 2.60 into both and the problem becomes

$$\max_{L} \pi = L + \frac{(\theta - c)^2}{4} \tag{2.63}$$

subject to

$$\frac{\beta(\theta - c)^2}{8} - L \geq 0 \tag{2.64}$$

Which is equivalent to

$$L \leq \frac{\beta(\theta - c)^2}{8} \tag{2.65}$$

Because of monotonicity, we know that the constraint is binding and thus

$$L^* = \frac{\beta(\theta - c)^2}{8} \tag{2.66}$$

Now if we compare 2.60 and 2.66 with 2.43 and 2.44, we can find that

$$\frac{c + \theta}{2} > \frac{1}{2 - \beta} c + \frac{1 - \beta}{2 - \beta} \theta \tag{2.67}$$

And

$$\frac{\beta(\theta-c)^2}{8} < \frac{\beta}{2}(\frac{1}{2-\beta}\theta - \frac{1}{2-\beta}c)^2. \tag{2.68}$$

Which means that compared the case with price commitment, the firm charges a lower fixed fee and a higer per-unit price for the same β. As we have already discussed, it deviating further from the ordinary price means a greater loss in profit.

Generalizing the above we have:

Corollary 2-2 (Monopolistic firm, 1 type of consumer, time-consistent vs. time-inconsistent, no price commitment). When the firm doesn't commit to a per-unit price, it also makes introductory offers together with a raise in the per-unit price to time-inconsistent consumers, and these prices deviate further from their ordinary values than with price commitment. The firm loses profit when it doesn't commit to a per-unit price.

In reality, the firm might have some gains the first time it breaks its price commitment; but once the consumers realize that the firm is not playing by the rules, non-commitment no longer works and causes loss instead.

2.6.2 Perfect Competition

When the market is perfectly competitive and does not suffer from the time-inconsistency problem, products of different firms are homogeneous that the consumers are indifferent in choosing which product to buy. However, once a consumer chooses a firm and pays the setup cost, she may face the lock-in problem: she can only continue to buy the products of the same firm unless she is willing to pay a switching cost to turn to another firm, if the firm chooses to maintain exclusive compatibility. But if the firms commit to their per-unit prices, there is weaker incentive[2] for exclusive compatibility. The consumers *ex ante* evaluate their surplus and chooses the firm that maximizes their expected surplus, which is equal over each firm under perfect competition. Once a consumer chooses a firm, she has little incentive to turn to another firm even if she doesn't have to buy another equipment, because products of other firms won't grant her any more surplus as the goods are homogeneous. Without exclusive compatibility, each firm sets its fixed fee/setup cost and per-unit price equal to its marginal cost, which is equal to all firms in the market, because when it comes to the second period, if a firm raises its per-unit price, it will lose all its consumers. Since in the end each firm earns zero expected profit, why bother

[2] A firm with weak incentive for exclusive compatibility may or may not choose to maintain it, while a firm with strong incentive will almost certainly pay the cost.

maintaining exclusive compatibility at the risk of driving away consumers who hate losing the opportunity of trying other brands? Additionally, it is natural to believe that designing and producing a new type of equipment incurs a remarkable cost, which also weakens the incentive for exclusive compatibility.

On the other hand, if the firms don't have to commit to a certain per-unit price, they will have the incentive to attract the consumers with an introductory offer and then exploit them with a high per-unit price as long as the consumers don't turn to another firm. Hence it is necessary for the firms to maintain exclusive compatibility to prevent the consumers from switching. Each firm charges a monopolistic per-unit price to extract profit as the consumers are locked-in; however, if consumers can expect the non-commitment pricing behavior, the will expect monopolistic per-unit prices and take them into account in their evaluation *ex ante* of whether or which to buy. In a perfectly competitive market, the firms has to lower their fixed fee until each firm earns zero expected profit, i.e. they have to sell the setup equipments at a price lower than its marginal cost. So we have:

$$L < MC_L;$$

$$p > c.$$

If we introduce the consumers' time-inconsistent preferences to the market, even when the firms commit to their per-unit prices, the firms make introductory offers in the fixed fee and charges a higher per-unit price, which requires exclusive compatibility for the lock-in effect. However, as we have discovered in Section 2, with time-inconsistent consumers, the firm loses profit compared with the case with time-consistent consumers. As each firm earns zero profit in a perfectly competitive market, time-inconsistent consumers eventually lead to negative profit of the firms. Some firms must exit the market until each firm's profit becomes non-negative with time-inconsistent consumers. This implies that each firm would have positive profit if the consumers were time-consistent, meaning that the market is oligopolistic. We infer that time-inconsistent consumers don't exist in perfectly competitive markets; even if they do, the market will be forced to evolve to an oligopolistic one. We will also discuss about the firms' pricing strategy in an oligopolistic market.

2.6.3 Monopolistic Competition

In a market with monopolistic competition, each firm sets its price above the marginal cost while still being able to retain some demand, although the expected profit is still zero. As to the two-part tariff, we can infer that both fixed fee and per-unit price are above their marginal cost when the consumers are time-consistent. In this case each firm has weaker incentive to maintain exclusive compatibility, since the demand is mostly retained by its product which is differentiated from that of other firms. When the firm doesn't commit to a per-unit price, or when the consumers are time-inconsistent, the firms make introductory offers in the fixed fee, which is no longer guaranteed to be lower than its marginal cost and is accompanied by a raise in per-unit price as well. Similar with the perfectly competitive market, the consumers' time-inconsistency will drive monopolistic competition into oligopoly.

2.6.4 Oligopoly

In an oligopolistic market, each firm earns a positive profit, while the equilibrium may not necessarily be symmetric, depending on various factors of each firm: product differentiation, cost, location, etc. It is very natural that firms would want to differentiate their products in an oligopolisitic market; but it is when the products are homogeneous that each firm has more incentive to maintain exclusive compatibility, because consumers can easily turn to other homogenous products. When products are differentiated, on the other hand, some consumers will remain loyal even without exclusive compatibility; nevertheless, exclusive compatibility may still be an effective device to retain consumers, which is even more so when firms don't have to commit to a per-unit price. In our case, if the goods are differentiated by quality, we can infer that they're nearly homogeneous from aspects other than the quality, which arouses the incentive for maintaining exclusive compatibility. When consumers become time-inconsistent, the response of each firm is pretty much like that of a monopoly: an introductory offer in the setup cost and a raise in the per-unit price. Also, just like in the monopolistic case, consumers *ex post* enjoy more surplus while the firms lose some profit. Those firms whose profit becomes negative after taking this loss will exit the market (if the equilibrium is asymmetric); or some firms will exit until the profit for the remaining firms becomes non-negative (if the equilibrium is symmetric). With the consumers' time-inconsistency, firms have stronger incentive to maintain exclusive compatibility, since they're losing money in the setup cost and they expect to make up with a higher per-unit price. The loss will be even greater if they can't retain their consumers for the second period.

Even without the effect of time-inconsistency, the firms still have the incentive for introductory offers, as they compete for consumers with each other. Time-inconsistency, on the other hand, acts to exacerbate this effect. This is one of the reasons why we choose to address the effect of time-inconsistency using the assumption of monopoly: in an oligopolistic market, various features of the market and of each firm eventually lead to various levels of introductory offer, which can be mixed up with the effect of time-inconsistency. Hence, it would be difficult to draw a clear conclusion for an oligopolistic assumption.

2.6.5 Hitchhiking Products

Apart from the conditions of an oligopolistic market described above, there might be another form of market structure: some firms may only provide products that are compatible with equipments of a certain brand, without having to sell the setup equipment itself. After one of the oligopolistic firms has decided with a design of equipment and its compatible product, it can no longer change the design, which opens the chance of "hitchhiking". Another firm can sell products compatible with this design at a lower price, benefiting from the existing consumers of this brand, which is more likely to happen to a brand with higher quality or higher fame. There are two sides of effects of "hitchhiking" on the sales of the incumbent firm. In the short run, hitchhiking products may steal a part of consumers from the incumbent firm with their low price. If the firm is confident with the quality of its product, this is not much of concern as the consumers who choose the firm has a higher preference for quality and are not likely to be very interested in products of low quality. But still, the consumers might feel difficult to resist the temptation to try out a new product. Of course, the incumbent firm can prevent hitchhiking with patent right, membership or other methods. However, in the long run, if more and more firms are hitchhiking the same incumbent firm, they may create a network effect which makes the setup equipment of the incumbent firm more attractive to consumers. Introductory offers in the setup goods are helpful in building up a network effect, while a strictly maintained exclusive compatibility may cause a loss of potential market share from this sense. But first of all, the existence exclusive compatibility is the premise of hitchhiking products, for there would be no such thing as hitchhiking if the setup goods are compatible with any consumable good from the beginning.

In this section we have worked on a model with only one firm that provides a single product. The firm's pricing strategy towards different types of consumers demonstrates patterns that vary from each other: the firm always makes an introductory offer in the fixed fee to the high-type consumers, while it does not carry on the same strategy towards the low-type consumers. It is natural to suppose that the consumers with a higher willingness to pay would also appreciate products with higher quality. Hence, when the quality of a product is differentiated, can we infer that the quality is more likely to be higher when we observe an introductory offer in the fixed fee? Or, in other words, does an introductory offer in the fixed fee serve as a signal of quality? Suppose there are two firms producing goods with differentiated qualities, and the consumers form their expectations for the quality according to the prices. Then an introductory offer might give a signal of high quality, which makes it less profitable for the low-quality firm to imitate. That's what we're going to model in the next secton.

3 The Signaling Approach

In this section we set up a two-period model with two types of firms with different levels of quality and continuous types of consumers. We show that when the consumers only care about the joint price they pay, the optimal pricing for the firm consists of an optimal joint price which allows the high-type firm to adjust either part of the price, in the form that gives the signal of high quality and prevents the low-type firm from imitating.

3.1 Basic Settings

There are two types of firms in the industry of a certain kind of product with different qualities. The high-type firms provide products with high quality S_H at the marginal cost c; the low-type firms provide products with low quality S_L at 0 marginal cost. Each firm has monopolistic power within its territory. At the beginning, each type of firms simultaneously announces their two-part tariff to all the consumers, according to their optimal pricing strategies without the signaling problem: (L_H, p_H) for the high type and (L_L, p_L) for the low type, respectively. The setup price is paid in period 1 and the consumers can decide whether to buy one unit of product for the per-unit price or not to buy at all. Both types of firms provide the setup equipment at the marginal cost C_M, as we assume that the basic function of the equipment doesn't vary much; but each type of equipment is specially designed so that it is only compatible with products of the same type.

The consumers, however, have incomplete information of the quality provided by the firm, and don't know whether they are located in the territory of a high-type or a low-type firm. Thus they expect the quality of the product on observing its setup cost and the per-unit price, i.e. if they observe (L_H, p_H), they expect the quality to be S_H; if they observe (L_L, p_L), they expect the quality to be S_L. Each consumer has a factor of taste, θ, which grants her the utility θS for a product with quality S. The number of consumers in each territory is normalized to 1, and their factor of taste, θ, is uniformly distributed between 0 and 1. Therefore the consumers would buy if

$$\theta S - p \geq 0$$

Hence only the consumers whose θ is high enough would choose to buy:

$$\theta \geq \frac{p}{S}$$

From which we get the demand function

$$D(p) = 1 - \frac{p}{S} \tag{3.1}$$

This gives the incentive to a low-type firm to sell at the tariff (L_H, p_H) in order to disguise itself as a high-type firm. The consumers don't learn the true quality until buying the setup equipment, and decide whether to buy in the second period according to the new information. For example, a capsule coffee machine is usually packed with several units of coffee capsules; a new printer can print a number of files before running out of ink, etc., which permits the consumers to utilize the product and learn about the quality right after buying the equipment. In the second period the low-type firm can choose to maintain the price p_H as it has announced before, or change it to p_L to be compatible with its revealed type.

3.2 Optimal Pricing without the Signaling Problem

When the consumer evaluates her purchase in the first period, she would buy if

$$E(CS) \geq 0$$

i.e.

$$\theta E(S) - L_H - p_H \geq 0$$

If the consumers expect that $S = S_H$:

$$\theta S_H - L_H - p_H \geq 0,$$

i.e.

$$\theta \geq \frac{L_H + p_H}{S_H}.$$

Therefore we have the demand function

$$D(L_H, p_H) = 1 - \frac{L_H + p_H}{S_H} \tag{3.2}$$

Define $P_H^T = L_H + p_H$,
Then the demand function is equivalent to

$$D(P_H^T) = 1 - \frac{P_H^T}{S_H}. \tag{3.3}$$

The high-type company then solves the problem:

$$\max_{P_H^T} \pi^H = (P_H^T - C_M - c)(1 - \frac{P_H^T}{S_H}) \tag{3.4}$$

F.O.C.:

$$\partial\pi/\partial P_H^T = 1 - \frac{2}{S_H} P_H^T + \frac{C_M + c}{S_H} = 0,$$

So the optimal joint price is

$$P_H^{T*} = \frac{S_H + C_M + c}{2}. \tag{3.5}$$

Which is the optimal joint price for the high type. It implies that as long as the high-type firms come up with a two-part tariff which sums up to the optimal joint price, their profits are at the optimal value. Nevertheless, if the low-type firms imitate the pricing of the high-type firms, in the long run the high type wouldn't be able to give a credible signal of its quality, hence undermine the profiting ability. So the high-type firms can adjust either part of the tariff without changing the joint price, in the form that deters the low type's imitation. We are going to show that by making an introductory offer, i.e. reducing L_H to the degree that it is below its marginal cost, the low type would be deterred from imitating.

Similarly, for the low type, the optimal joint price is

$$P_L^{T*} = \frac{S_L + C_M}{2}, \tag{3.6}$$

As we assume that providing low quality incurs zero marginal cost.
It is obvious that

$$P_H^{T*} > P_L^{T*}$$

When the low type sets its joint price to optimal value, its profit is

$$\pi_0^L = (1 - \frac{P_L^T}{S_L})(P_L^T - C_M) = \frac{1}{4S_L}(S_L - C_M)^2. \tag{3.7}$$

However, this profit is based upon the fact that the consumers correctly expect the low quality and make their purchase decisions according to it. If the consumers' expectation is misled by the imitation of the low type, there is the possibility for a pricing that brings a higher (although not necessarily) profit. In the next subsection we're going to discuss about this possibility.

3.3 Imitation of the Low-type Firm

If the low type imitates the high type, in the first period, it has to announce (L_H, p_H) so that the consumers will infer from the price that the quality is S_H.

On observing the two-part tariff, the consumers expect that $S = S_H$. The demand in the first period is

$$D_1(P_H^T) = 1 - \frac{P_H^T}{S_H}. \tag{3.8}$$

Then in the second period, the low type faces 2 choices:

Choice 1 Maintain p_H as it has announced in the first period.

Knowing the true quality of the product, consumers will purchase if

$$\theta S_L - p_H \geq 0$$

So that

$$D_2(p_H) = 1 - \frac{p_H}{S_L} \tag{3.9}$$

In this case the low type's total profit in both periods is

$$\pi_1^L = (1 - \frac{L_H + p_H}{S_H})(L_H - C_M) + (1 - \frac{p_H}{S_L})p_H \tag{3.10}$$

Directly taking partial derivative to get

$$\partial \pi_1^L / \partial L_H = -\frac{2}{S_H} L_H + \frac{C_M}{S_H} + 1 - \frac{p_H}{S_H} = (1 - \frac{P_H^T}{S_H}) + \frac{C_M - L_H}{S_H}$$

$$\partial \pi_1^L / \partial p_H = \frac{C_M - L_H}{S_H} - \frac{2p_H}{S_L} + 1$$

As $p_H = P_H^{T*} - L_H$, taking P_H^{T*} as determined, we have

$$\frac{\partial p_H}{\partial L_H} = -1,$$

So that

$$d\pi_1^L / dL_H = \partial \pi_1^L / \partial L_H + \frac{\partial \pi_1^L}{\partial p_H} \cdot \frac{\partial p_H}{\partial L_H} = \partial \pi_1^L / \partial L_H - \partial \pi_1^L / \partial p_H$$

$$= (1 - \frac{P_H^{T*}}{S_H}) + \frac{C_M - L_H}{S_H} - (\frac{C_M - L_H}{S_H} - \frac{2p_H}{S_L} + 1)$$

$$= 1 - \frac{P_H^{T*}}{S_H} + \frac{2p_H}{S_L} - 1 = \frac{2p_H}{S_L} - \frac{P_H^{T*}}{S_H} \tag{3.11}$$

So that we have

$$d\pi_1^L / dL_H \geq 0$$

If

$$L_H \leq \frac{2S_H - S_L}{S_L} p_H;$$

Or

$$d\pi_1^L / dL_H < 0$$

If

$$L_H > \frac{2S_H - S_L}{S_L} p_H.$$

In order to reduce the low type's expected profit from imitating, the high type either reduce L_H as much as possible when $d\pi_1^L/dL_H \geq 0$, or increase L_H as much as possible when $d\pi_1^L/dL_H < 0$, keeping P_H^{T*} determined. Then two extreme cases are $L_H = 0$ or $p_H = 0$. Now we discuss what is the expected profit of the low type in these two cases.

When L_H approaches zero,

$$\lim_{L_H \to 0} \pi_1^L = (1 - \frac{p_H}{S_H})(-C_M) + (1 - \frac{p_H}{S_L})p_H = C_M(\frac{P_H^{T*}}{S_H} - 1) + (1 - \frac{P_H^{T*}}{S_L})P_H^{T*} \tag{3.12}$$

As in this case $P_H^{T*} = p_H$.

When p_H approaches zero,

$$\lim_{p_H \to 0} \pi_1^L = (1 - \frac{L_H}{S_H})(L_H - C_M) + 0 = (1 - \frac{P_H^{T*}}{S_H})(P_H^{T*} - C_M)$$

$$= C_M(\frac{P_H^{T*}}{S_H} - 1) + (1 - \frac{P_H^{T*}}{S_H})P_H^{T*} \tag{3.13}$$

As $S_H > S_L$, we know that

$$C_M(\frac{P_H^{T*}}{S_H} - 1) + (1 - \frac{P_H^{T*}}{S_L})P_H^{T*} < C_M(\frac{P_H^{T*}}{S_H} - 1) + (1 - \frac{P_H^{T*}}{S_H})P_H^{T*},$$

i.e.

$$\lim_{L_H \to 0} \pi_1^L < \lim_{p_H \to 0} \pi_1^L, \tag{3.14}$$

Which implies that, by choosing $L_H = 0$, the high type can reduce the low type's expected profit by imitation more than by choosing $p_H = 0$. Hence reducing L_H while keeping the joint price P_H^{T*} at its optimal level is an effective strategy to prevent the low type from imitating.

Choice 2 Return to the low type's price p_L.

Consumers will purchase if

$$\theta S_L - p_L \geq 0$$

So that

$$D_2^L(p_L) = 1 - \frac{p_L}{S_L}$$

In this case the total profit in both periods of the low type is

$$\pi_2^L = (1 - \frac{L_H + p_H}{S_H})(L_H - C_M) + (1 - \frac{p_L}{S_L})p_L \tag{3.15}$$

Take partial derivative with respect to L_H and p_H to have

$$\partial \pi_2^L / \partial L_H = (1 - \frac{P_H^{T*}}{S_H}) + \frac{C_M - L_H}{S_H}; \tag{3.16}$$

$$\partial \pi_2^L / \partial p_H = \frac{C_M - L_H}{S_H}. \tag{3.17}$$

As $p_H = P_H^{T*} - L_H$, we have

$$d\pi_2^L / dL_H = \partial \pi_2^L / \partial L_H + \frac{\partial \pi_2^L}{\partial p_H} \cdot \frac{\partial p_H}{\partial L_H} = \partial \pi_2^L / \partial L_H - \partial \pi_2^L / \partial p_H$$

$$= (1 - \frac{P_H^{T*}}{S_H}) + \frac{C_M - L_H}{S_H} - \frac{C_M - L_H}{S_H} = 1 - \frac{P_H^{T*}}{S_H} \geq 0 \tag{3.18}$$

Which implies that the high type can reduce the expected profit of the low type if it imitates, by keeping the joint price fixed and reducing the setup cost.

Generalizing the response of the high type with respect to the above two choices of the low type, we have

Proposition 3 (two types of firms, continuous type of consumer, signaling *effect).* Keeping its joint price $P_H^{T*} = L_H + p_H$ at the optimal level, the high-type firms can reduce the expected profit of the low type from imitating their two-part tariff by making introductory offers in the setup cost, thus weakening the low type's incentive to imitate and maintain their pricing as a signal for high quality.

3.4 Interpretation and Discussion

Intuitively, the high type is losing money by making an introductory in the setup cost to the extent that the price is even below the marginal cost of the equipment. But the high type is confident with the quality it is providing that the consumers will return to buy the product, which will generate more profit that more than compensate the loss by making the introductory offer. If the low type imitates, without doubt, it also loses money in the first period. When it comes to the second period, it either maintains a high per-unit price that is well above the factual value of its low quality, so that very few consumers would continue to buy; or return to the low price that is not enough to compensate for the loss in the first period.

Our finding, however, does not completely eliminate the possibility for imitation, as it is difficult to decide whether the extreme values of π_1^L and π_2^L is strictly below π_0^L without further restrictions on the relations between S_H, S_L, C_M and c. There is the possibility for the low type to enjoy a higher profit from imitating even if this profit has already been reduced by the high type's pricing strategy. For instance, in the second period the low type might still be able to retain more demand compared with the situation without imitation, as for the consumers the setup cost is already a sunken cost and making the purchase decision in the second period only involves the per-unit price.

It might seem that in the second period, once the consumers learn the ture quality, it is a better option returning to the original low price in order to retain more demand. But if the game is repeated, this may not be quite feasible for the low type, as it cannot frequently change its price as it wills. And by returning to the low price, it exposes to future potential consumers of its imitation.

4 Robustness

4.1 Repeated Periods

Our study focuses on introductory offers made upon the fixed fee in a two-part tariff, which constitutes a specialized case for introductory offers in repeat purchases. Despite that the mechanism of our model might differ from that in repeat purchases with a single price, the reasoning may also apply. Introductory offers in repeat purchases can be more attractive to time-inconsistent consumers, especially when the product is new to them, as they are taking a risk of buying an unfamiliar product at present for the possibility of buying a product that brings positive (or at least non-negative) surplus in the future. As well, consumers may infer from an introductory offer that the firm is providing high quality as it is confident of its quality that is willing to sacrifice present profit for future sales.

Although our model consists of merely two periods, it can be seen as an abbreviated form of multiple periods. Imagine if we define

$$p_H = \sum_{t=1}^{+\infty} \delta^t p = \frac{\delta}{1-\delta} p_h, \tag{4.1}$$

Where p_h represents the price charged by the high-type firms in each subdivided period. The same applies to p_L in both models. Similarly, we can also define

$$S_H = \sum_{t=1}^{+\infty} \delta^t s_h = \frac{\delta}{1-\delta} s_h; \tag{4.2}$$

$$S_L = \sum_{t=1}^{+\infty} \delta^t s_l = \frac{\delta}{1-\delta} s_l; \tag{4.3}$$

Meaning that we can use a single variable to represent the quality of the product enjoyed in the many periods in the future, since we get the utility directly by multiplying the quality by θ.

We can define other variables in the same way, with all the variables sharing the same time discounting factor δ. Without losing generality, we let one period represent these endless periods, as what matters is the decision made by the consumers at the beginning of the second period. Using one period as a generalization of the future periods does not affect the final results. Let's take the model in Subsection 2.5 with one firm and one consumer as an example. If we assume that there are endless periods in the future, the firm still charges the fixed fee only once and its time-discounted profit would be:

$$\pi_T = L + \sum_{t=1}^{+\infty} \delta^t (p-c)(\theta - p) = L + \frac{\delta}{1-\delta}(p-c)(\theta-p). \tag{4.4}$$

The consumers' expected surplus has to be non-negative, with an extra time-discounting rate β between the present and the future periods:

$$\sum_{t=1}^{+\infty}\beta\delta^t \int_0^Q (\theta - Q)dQ - L - \sum_{t=1}^{+\infty}\beta\delta^t p(\theta - p) \geq 0, \tag{4.5}$$

i.e.

$$\frac{\delta}{1-\delta}\beta \int_0^Q (\theta - Q)dQ - L - \frac{\delta}{1-\delta}\beta p(\theta - p) \geq 0. \tag{4.6}$$

Where $Q = \theta - p$.

The firm's problem becomes

$$\max_{L,p} \pi_T = L + \frac{\delta}{1-\delta}(p-c)(\theta - p) \tag{4.7}$$

subject to

$$\frac{\delta}{1-\delta}\beta \int_0^Q (\theta - Q)dQ - L - \frac{\delta}{1-\delta}\beta p(\theta - p) \geq 0.$$

The constraint must be binding and leads to

$$L = \frac{\delta}{1-\delta}\frac{\beta}{2}(\theta - p)^2. \tag{4.8}$$

Plug (4.8) back into (4.7) and the maximization problem becomes:

$$\max_{p} \pi = \frac{\delta}{1-\delta}\frac{\beta}{2}(\theta - p)^2 + \frac{\delta}{1-\delta}(p-c)(\theta - p). \tag{4.9}$$

Now if we compare (4.9) with (2.42), we can discover that there is no difference except a common multiplyer $\frac{\delta}{1-\delta}$, which does not affect the optimal value of p:

$$p^* = \frac{1}{2-\beta}c + \frac{1-\beta}{2-\beta}\theta. \tag{4.10}$$

The optimal value for L is

$$L^* = \frac{\delta}{1-\delta}\frac{\beta}{2}(\frac{1}{2-\beta}\theta - \frac{1}{2-\beta}c)^2. \tag{4.11}$$

Which is only a propotional change and does not affect the result we get from the derivative:

$$\partial L^*/\partial\beta = \frac{1}{2}\frac{\delta}{1-\delta}(\frac{\theta - c}{2-\beta})^2 + \frac{\delta}{1-\delta}\frac{\beta(\theta - c)^2}{(2-\beta)^3} > 0, \tag{4.12}$$

Which shares a very similar result with (2.48).

The same reasoning applies to both the behavioral model and the signaling model. The firm and the consumers share a common time-discounting factor δ through time, and the only difference that affects the decisions of both parts is the consumers' time-inconsistency, represented by an extra time-discounting factor β between the present and the future periods. A δ that proportionally multiplies each variable is unnecessary and only complicates the calculation. Since using one period to represent the future periods does not essentially affect the results, a two-period model would be better in its simplicity and intuitiveness which facilitates discussion and interpretation.

Meanwhile, the fact that the product is sold for many periods after the purchase of the setup equipment helps interpret some of our findings. For example, in the behavioral model, in repeated sales, the firm is more willing to sacrifice its profit from the fixed fee for a raise in the per-unit price. In the signaling model, in *Choice 1* made by the low-type firm that imitates, the reason why the high type choose to lower L_H instead of p_H can be very well explained if we know that the product will be sold for many periods in the future. The multi-period scenario also provides an explanation for the limitation mentioned in Section 3.4.

4.2 Competition

In our model(s) we assume monopoly of the firm(s), as most of the studies mentioned in the introduction do. Notwithstanding, Ellison (2009) points out that competition does not qualitatively eliminate exploitations of firms when consumers are homogeneous, or when they're heterogeneous with the firm providing only a single quality level. Grubb (2015) suggests that competition only partially protect naive consumers. For our signaling model, formalizations with competitive or oligopolistic assumptions would be desirable in order to discover how signaling effect works when decisions of different firms interact with each other, as is intended for our future study. But one should be aware that a perfect competition is not possible since products are differentiated by quality. In the signaling model, maintaining exclusive compatibility of its own product with the setup equipment is pivotal, especially to the firm providing high quality.

In our behavioral model, however, with the consumers' time-inconsistent preferences, the firm loses profit instead of exploiting the consumers. As already shown in Subsection 2.6, this feature leaves there little chance for a perfectly competitive market in the presence of the consumers' time-inconsistency. As the market evolves towards an oligopolistic one, competing for consumers would become another incentive for introductory offers, which would mix up with the effect of time-inconsistency. So it is best to use the assumption of monopoly in our model.

5 Conclusion

In this paper we have taken two approaches modeling the pricing strategy of firm(s), in order to address to introductory offers made upon the fixed fee of the two-part tariff.

Each model consists of two periods. In the first period the consumers pay the fixed fee in exchange for the setup equipment; and we let the second period represent the many periods in the future in which the consumers continue to purchase consumable products. We show that compared with multi-period models, two-period models enjoy mathematical simplicity and intuitiveness, while leading to results essentially the same with those we can get from multi-period models.

The first model tries to find out the effect of the consumers' time-inconsistency on the firm's contract design. We find that for the high-type consumers and for low-type consumers whose demand has a relatively small difference from the high type, as the consumers become more time-inconsistent, the more the firm raise the per-unit price and lower the fixed fee, i.e. makes a greater introductory offer; meanwhile, the consumers *ex post* enjoy more surplus and the firm loses more profit; the total surplus of the society also suffers from a loss. When the firm only faces one type of consumers, our conclusions also stand. If the firm doesn't commit to a per-unit price, it still has a similar pricing strategy, although there is a different mechanism that leads to this result.

Our findings also imply that when there are two types of consumers whose demands have too big a difference, the effect of contract design may prevail that of time-inconsistency in the firm's pricing strategy: in the tariff for the low-type consumers, both part may fall with time-inconsistency when the difference of demands is moderate; and the fixed fee may rise while the per-unit price falls when the difference is big enough. We uphold that this is because of the firm's incentive to maintain the IR and IC constraints, especially when the low-type consumers contribute a relatively small part of profit to the firm.

In the examination of robustness for the behavioral model, we find that time-inconsistent consumers eventually push a competitive market into oligopoly since they reduce the expected profit of each firm. In oligopoly, firms make introductory offers in response to time-inconsistency. We also uphold that a firm has stronger incentive to maintain exclusive compatibility of its setup equipment and product when the market is less competitive, or when consumers are time-inconsistent, or when it doesn't have commitment to a per-unit price; a firm has weaker incentive for exclusive compatibility when the market is more competitive or when the firms provide differentiated products.

Future studies with respect to the time-inconsistency model (or the behavioral model) may want to explore how time-inconsistency works with more generalized forms of demand and profit functions.

In the second model, the signaling model, we show that if consumers make their expectations about quality based upon the prices they observe, the firm providing high quality may utilize an introductory offer in the setup cost to reduce the expected profit of the low-type firm from imitation, thus guarantee the validity of the signaling effect of its prices. Future studies might introduce the situation where two firms compete for demand in the same market, and try to find the existence of a separating equilibrium.

This paper discusses about the two above-mentioned mechanisms that induce a firm to make introductory offers in the fixed fee of two-part tariff. There may still be other mechanisms behind this pricing pattern; or it may be the synthesis of various mechanisms that actually leading to the result. We shall leave that for future explorations.

References

Allen, F. 1984. "Reputation and Product Quality." *Rand Journal of Economics*, Vol. 15, pp. 311-327.

Bagwell, Kyle. 1987. "Introductory Price as a Signal of Cost in a Model of Repeat Business." *Review of Economic Studies*, LIV, 365-384.

DellaVigna, Stefano and Malmendier, Ulrike. 2004. "Contract Design and Self-Control: Theory and Evidence." *Quarterly Journal of Economics*, 119(2), pp. 353-402.

DellaVigna, Stefano and Malmendier, Ulrike. 2006. "Paying Not to Go to the Gym." *The American Economic Review*, 96(3), pp. 694-719.

DellaVigna, Stefano. 2009. "Psychology and Economics: Evidence from the Field." *Journal of Economic Literature*, 47(2), pp. 315-372.

Doyle, Chris. 1986. "Intertemporal Price Discrimination, Uncertainty and Introductory Offers." *Economic Journal*, 96, pp. 71-82.

Ellison, Glenn. 2005. "A Model of Add-on Pricing." *Quarterly Journal of Economics*, 120, 585-637.

Ellison, Glenn. 2009. "Bounded Rationality in Industrial Organization." In *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, Volume II:142-174.

Farrell, Joseph. 1984. "Moral Hazard in Quality, Entry Barriers, and Introductory Offers." *M.I.T. working paper.*

Farrell, Joseph and Shapiro, Carl. 1989. "Optimal Contracts with Lock-In." *The American Economic Review*, 79(1), pp. 51-68.

Frederick, Shane; Loewenstein, George and O'Donoghue, Ted. 2002. " Time Discounting and Time Preference: A Critical Review." *Journal of Economic*

Literature, 40(2): 351-401.

Grossman, Sanford J. 1981. "The Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics*, 24, 461-483.

Grubb, Michael D. 2015. "Overconfident Consumers in the Marketplace." *Journal of Economic Perspectives*, 29(4), pp. 9-36.

Loewenstein, George, and Prelec, Drazen. 1992. "Anomalies in Intertemporal Choice: Evidence and an Interpretation." *Quarterly Journal of Economics*, 107(2): 573-97.

Milgrom, Paul. 1981. "Good News and Bad News: Representation Theorems and Applications." Bell Journal of Economics, 12, 380-391.

Milgrom, Paul, and Roberts, John. 1986. "Price and Advertising Signals of Product Quality." *Journal of Political Economy*, 94(4), 796-821.

Nelson, Phillip. 1970. "Information and Consumer Behavior." *Journal of Political Economy*, 78, 311-329.

Nelson, Phillip. 1974. "Advertising as Information." *Journal of Political Economy*, 81, 729-754.

Nelson, Phillip. 1978. "Advertising as Information Once More." In *Issues in Advertising: The economics of Persuation*, edited by David G. Tuerck. Washington: American Enterprise Inst.

Shapiro, Carl. 1983. "Optimal Pricing of Experience Goods." *The Bell Journal of Economics*, 14(2), pp. 497-507.

Thaler, Richard H. 1981. "Some Empirical Evidence on Dynamic Inconsistency." *Economics Letters*, 8(3): 201-07.

Tirole, Jean. 1988. "*The Theory of Industrial Organization.*" The MIT Press.