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COSTLY REVERSIBLE DISINVESTMENT OPTION IN A VALUATION OF RENEWABLE ENERGY CASE

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Abstract

The necessity to improve the company valuation techniques considering the relevant factors of the market and the flexibility of change investment positions during the period lead academics search for new valuation techniques. Also, investors interested in analyzing the financial information to know before invest at the project when he will reimbursed for profits and if the project goes wrong how much is the value of the investment. In this context real options valuation raises as a concept imported from financial markets to evaluate projects and companies as can incorporate the fluctuations of the market and decisions defined by the investor.

Santos et al. (2014) presented a case of renewable energy on a mini-hydro plant where evaluates the project both with traditional methods and real options studding the ability to defer the entry in the project until 5 years. Despite being a surplus to present the two valuation methods, the real options valuation analyzed is quite limited.

This project intends to demonstrate that the application of real options with flexibility at disinvestment decision leads to a more accurate decision. To accomplish this purpose, we will revisit the same base case but including dividends and an option to disinvest during the life of the project in a costly reversible situation with the binomial model.

Key-words: Real Options, Flexibility, Costly Reversibility, Binomial Model

JEL Classification System: G30; G31; D81.

Resumo

A necessidade de melhoria das técnicas de avaliação nas empresas adaptadas aos factores relevantes do mercado, bem como a flexibilidade de alteração de posições de investimento durante um período levaram a que os académicos investigassem novas formas de avaliação. Além disso, também é do interesse do investidor analisar indicadores financeiros como quando é que será reembolsado e qual é o momento em que recupera o seu investimento caso as condições se apresentem como desfavoráveis. Neste contexto, a avaliação de projetos pelo método das opções reais apareceu como um método importado dos mercados financeiros para avaliar projetos de investimento e empresas onde são refletidas as flutuações do mercado e que decisões o investidor pode tomar decorrente do mesmo.

Santos et al. (2014) apresentaram um caso no sector das energias renováveis de uma instalação hidráulica, que procura avaliar o projeto pelos métodos tradicionais e pelo método das opções reais, estudando a possibilidade de adiar a entrada do projeto até ao quinto ano. Embora as diferenças apresentadas sejam uma mais-valia para a apresentação dos diferentes métodos, a sua avaliação efetuada nas opções reais apresentou-se bastante limitada.

Este projeto demonstra que a aplicação do método das opções reais com a introdução de flexibilidade na decisão de desinvestimento leva a que a decisão do mesmo se torne mais correta. Assim, será estudado num modelo binomial o mesmo caso base onde é analisado o incremento de dividendos no projeto e uma opção de abandono ou desinvestimento durante a sua vida útil num contexto de não recuperação total do seu investimento.

Palavras-chave: Opções Reais, Flexibilidade, Custos com Reversão, Modelo Binomial

JEL Classification System: G30; G31; D81.

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Abbreviations

NPV – Net Present Value

CAGR – Compound Annual Growth Rate

NBV- Net Book Value

DY- Dividend Yield

1. Introduction

The theme of real options is relatively recent. The concept appears in consequence of the necessity of several academics and practitioners trying to achieve more reliable project valuations. They conclude that, as in the financial markets, the flexibility of the fluctuations of the project value and other market factors can lead to strategy management's misstatements. The academics absorbed the theory of financial options valuation to develop a corporate finance model to value project's investment.

Financial options contracts started being traded in 1973 in the Chicago Board Options Exchange (CBOE). In the same year, Black and Scholes (1973) provided an innovative method to value stock options. They associated warrants to options to compute the price of financial options assuming that the price of the underlying stock followed the Geometric Brownian Motion assumption.

On the other side, several academics and practitioners were dissatisfied with the traditional methods applied to value investment projects. They considered that the inflexibility to value a project tended to overvalue it and not to describe with reliability the future cash-flow. Myers (1977) was one of the first academics that presented a new approach applying the concept of financial options in the corporate finance field, by perceiving discretionary investment opportunities as growth options. Many articles were published afterwards but only in 1994 and 1996 were published two books that nowadays still continue to be references, about Real Options, namely Dixit and Pindyck (1994) and Trigeorgis (1996).

For example, Trigeorgis (1996) explained that traditional methods cannot properly capture management's flexibility to adapt and revise later decisions in response to unexpected market developments. The flow of new information can influence the development of the project and defraud the management's cash-flow expectations.

Nowadays, Real Options are more used in the academic side, the market continuing to prefer the use of traditional methods. Usually the managers develop the traditional discounted cash-flow with several scenarios analysis keeping up with a reassessment during the life of the project.

Santos et al. (2014) presented the advantages of applying real options rather than traditional methods in a mini-hydro plant. They considered that investment is irreversible, i.e., it cannot be reverted. This does not reflect exactly the reality, as if the investor wanted to stop these project, he would still sell the initial investment. The project is also limited, as they did not include dividends. This is an essential factor for every investor to get his money back and reduce risk.

The aim of this thesis is to explain the importance of real options, i.e., how using there in an investment can add value to investor analysis at the moment of spending his money. The thesis' main goal is not to apply complex methods of real option valuation, but explain that combining more options to a project valuation provides a more reliable analysis to the investor, when comparing with and without a disinvestment option in the renewable energy sector.

The thesis is separated in chapters. Chapter 2 presents the literature review and is organized in several sections. The section 2.1 studies Real Options Concept, Payoffs, and the differences between real options and traditional methods. In section 2.2 it is described the stochastic processes and section 2.3 discusses the Black-Scholes formula as a valuation method to real options. Posteriorly in sections 2.4. and 2.5. it is described the binomial model and studied the investments and disinvestments decisions in a context of costly reversibility option respectively. In chapter 3 the case study analysis is performed where it is done a case base description, and discussed the limitations and project assumptions. In chapter 4 it is described the development of the case and chapter 5 describes the results. Finally, chapter 6 presents the conclusion of thesis.

2. Literature Review

2.1. Real Options Valuation

2.1.1. The Concept of a Real Option

The concept of an option is simple as an individual that has the right over something that a counterpart is obliged to satisfy. Options are used since a long time ago in the quotidian life. In the decade of 80's appeared options associated to financial markets as commonly known as financial options. It is a contract over some kind of financial instrument that was formally defined and traded in the market. A financial option contract gives the right but not the obligation to buy (call option) or sell (put option) the content of a contract that defines the transaction amount (contract size), the underlying asset transacted, the determined future price to sell or buy (Strike), at a defined time to exercise that can be only at the maturity date (European-Style Option) or until the maturity date (American-Style Option).

As usual in a trade, there are two parties involved. The buyer, the player that buys the option, takes a long position and has the right over the exercise of the option; the seller, player that sells the option, takes a short position in the option. The seller has the obligation to correspond the decision of the buyer. But this is not an unfair deal. The buyer of the option pays a premium, the price of buying the financial option contract, to the seller, and gives the opportunity to make a profit if the option is not exercised. This gives a final profit or loss at the maturity date. The payoffs associated with the exercise or not of the buyer and the seller are defined in next section.

As described above, the concepts of real options valuation described above are the same used in financial options contract. The difference between them is the inputs associated, that in the financial options are market's inputs and in the real options valuation are corporate finance's inputs, as shown in the following table.

Concepts	*Explanations*
Call Option	Present value of a future project investment
Put Option	Present value of a future project disinvestment, i.e., reduce or shut down option to sell the Asset.

Strike	Value at which the player may invest or disinvest in determined project
Maturity Date	Period for which the option can be exercised
Underlying Asset	The project value

Table 1 - Explanation of Real Options Inputs

Source: *Trigeorgis - Managerial Flexibility and Strategy in Resource*

Trigeorgis (1996) describes that regarding new information management is willing to take new decisions that cannot be incorporated using the traditional methods. There decisions are options that could be incorporated in the project to improve its reliability:

- Option to defer: Management holds the initial defined investment regarding the new information of the market.

- Time to build option: The investment is done as the initial defined stages are completed.

- Option to expand: If the market condition goes favorable, management can decide to expand the scale of the project and accelerate the production.

- Option to contract: If the market condition goes bad, management can reduce the project regarding the associated loss.

- Option to abandon: If the market condition goes unfavorable management can close the project.

- Option to switch: If the market condition change, the project can also be adapted to the demand by switching defined inputs of the project.

- Growth options: An option of early investment that can be strategic to develop new projects.

2.1.2 Options Positions, Payoff and Profit

A call or put option is a contract giving the right to buy or sell determined settlement negotiated in the moment t_0, with certain conditions, by two parties. The buyer takes the right to exercise the call or put option (Long position) and the seller has the obligation to sell or buy if the option is exercised (Short position).

When the investor takes a long position, must pay the price of the option, which is the risk premium that the counterpart is willing to take to receive the money in for the option traded.

By taking a long position the buyer pays the money and only has potential gains in the future, instead of the seller, that receives initially the money and has potential losses in the future. The buyer is not exposed to downside risk. In the case of a naked position the premium to be received by the seller is not immediately cashed in, but instead is included in the margin account.

At the exercise or expiration date, the buyer exercises or not the option and the following payoffs can be achieved:

	Long Call	*Long Put*
Exercise	$S_t - K$	$K - S_t$
No exercise	0	0

Table 2 - Payoffs of a Call and Put Option

where S_t is the value of the settlement at exercise date and K is the strike price of the option.

For a call option, the buyer pays the strike and receives the settlement. The payoff corresponds to the difference between the value of the settlement and the strike. In the case of a put option, the buyer of the option, the one that has the long position, sells the settlement and receives the strike price of the option.

Although the transaction is completed, the payoff is not the profit or loss of the operation. It is necessary to take into account the premium paid by the buyer. At the inception of the contract the final profit or loss of the operation is the premium paid plus the payoff in the exercise moment (if we ignore, for simplicity, the time value of money).

2.1.3 Factors that Affect the Option Pricing

Options are financial instruments that are influenced by several factors inherent of a project. So, as the factors change, the price of the option is adjusted regarding the expected payoffs.

This price is influenced by the following factors:

Current Price, S_0: The fluctuations of the current price will impact in the payoffs of execution of the call option. If the current price increases, the payoff of a call option will also increase, becoming more valuable. On the other hand, the value of the put option will increase if the stock price falls, becoming more valuable.

Strike Price, *K*: In the case of a call option, the option will become more valuable as the strike price is lower, increasing the payoffs of the operation. Considering two call options, c_x and c_y, with a strike: 20€ K_x and 10€ K_y, and the same market conditions for each one. The payoff at maturity of C_y is always 10€ higher than C_x, so the call option C_y is more valuable than the call option C_x. Using the same example, put option will become less valuable as the strike price falls.

Time to expiration, *T*: The investor is willing to pay a higher price as there are more possibilities to execute the option until maturity date. For example, two call options, c_x and c_y, with a maturity date: 12m (T_x) and 24m (T_y), and the same market conditions for each one. The option *"y"* gives the possibility to execute the option in the same time of option *"x"* plus 12 months. This increase of time needs to be rewarded in the premium of the option. So it is expected that as the maturity date is extended, the option is more valuable. This is applied to American-style options. In the case of European-style options that is not always true. Despite the fact that as maturity may arise, the stock price has more possibilities to swing, if it is in a situation of payment of dividends in a short time, a call option with shorter time to expiration (before the dividends' payment) could be worth more than one with a longer time (after dividends' payment).

Volatility, σ: The volatility of an option is the value that measures the extent of the swing of the settlement price. In long positions downside risk is limited and the gains are potentially unlimited, so the probability of the investor having deep in the money positions is higher, as volatility increases. The investor can also increase his payoffs if the volatility of the price is higher.

Risk Free Interest Rate, R_f: The risk free interest rate corresponds to the interest rate that is rewarded an investment in a situation of risk neutral world. If the risk free interest rate increases, the required return of the investor will be higher, lowering the present value of the strike price. Thus, this situation let call options' price increase. In the case of put option, the price falls, regarding the decline of the current value of the strike.

Dividends: Distribution of dividends by the companies tends to decapitalize the companies and decrease their market value. This influence the options prices because the stock price decreases, increasing the value of put options and decreasing the value of call options.

The next table summarizes the comparative statics analysis described above:

Factors	c_t	C_t	p_t	P_t
Current Price, S_0	+	+	-	-
Strike Price, K	-	-	+	+
Time to Expiration, T	+/-	+	+/-	+
Volatility, σ	+	+	+	+
Risk-Free Interest Rate, R_f	+	+	-	-
Dividends	-	-	+	+

Table 3 - Factors affecting Real Options Pricing

2.1.4. Real Options vs Traditional Valuation Methods

Nowadays the traditional valuations methods are still used to value projects and make decisions. Moreover, it is still used by top management of international companies to make strategic decisions. In fact, traditional valuation methods continued to be easy to compute, analyze and gather conclusions. However, they can lead to irreversible misstatements, in consequence of their own limitations.

The traditional net present value (NPV) is one of the traditional valuation methods. This method is applied to discount the future cash-flows of a project to the present. Despite the model being simpler to apply, the inherent limitations can lead to distorting the real value of the project. The main problem is that the project only incorporates the present information to estimate the future cash-flow. Therefore, if there is any change in the market in the future, it cannot incorporate this new information when computing the net present value. Additionally, when the discounted cash-flows are computed, it is considered only the most likely scenario. If the firm has ability to stop losses or increase the profit, it cannot be incorporated in the discounted cash-flows. This model, generally, underestimates the real value of the project because it cannot incorporate flexibility. The ability to take only a decision of getting in the project or leave it gives a time range limitation. The investor may not invest in a project in the present, but at some future period he may get in regarding the changes in the market. Trigeorgis (1996) explained that the discounted cash-flow valuation underestimates the investment opportunities, creating myopic decisions, and inefficiency in competitiveness because it does not incorporate key factors as mentioned above.

The real options approach incorporates characteristics as irreversibility, uncertainty and timing. It is possible for the investor to choose to invest, contract, temporarily shut down, switch, defer, expand, choosing the optimal time to act, considering the uncertainty of the market.

2.2. Stochastic Price Process

To compute the price of options with Black and Scholes (1973) and Merton (1973) models it is also mandatory to compute the expected future price of the underlying asset. This price of the underlying asset has a price process associated. Hull (2012) defined a stochastic process as any variable whose value changes over time in an uncertain way.

There are two types of stochastic processes: the discrete-time and the continuous-time. Hull (2012) defined the primer as the one where the value of the variable can change only at a certain fixed points in time and the latter as the one that can assume variables in a defined continuous period, i.e. , with no gaps between two points in time.

Besides the two types of time, there are also two types of variables: the discrete values that can only assume certain values in a range of time and the continuous variables that can be any value considering a period of time.

2.2.1. Markov Stochastic Process

Regarding the Markov Process, it is a particular case of the stochastic process developed by Andrey Markov. Hull (2012) explained that the Markov Process is a continuous price process where the current value is the only relevant factor for predicting the future, since it incorporates all past information and the future prices are not dependent of the price in the past. Also, consider that the market has a weak form pattern, so it is impossible to make above-average returns analyzing past information.

To compute the future price, the Markov process considers a probability that follows a normal distribution N $(0,T^2)$, where T is a period of time. So, the standard deviation of the process is the time of a defined period.

2.2.2. Wiener Process

Wiener Process, or more known as Brownian Motion, is a particular case of a Markov Stochastic Process. This was initially applied in physics to describe the unpredictable molecular moves, being posteriorly a finance concept, regarding the fluctuations of stock prices.

The Brownian Motion process considers two properties to describe the unpredictable movements of a variable stock Z:

1. The change of ΔZ during a small period of time Δt is:

$$\Delta Z = \epsilon\sqrt{\Delta t} \tag{1}$$

where ϵ ~ N (0,1).

2. The values of ΔZ for two different interval of time are independent.

The first property implies ΔZ follows a normal distribution. The second property defines that the price is independent so it follows the Markov Process described above.

2.2.3. Generalized Wiener Process

The Generalized Wiener Process is an improvement made over Brownian Motion. Despite the unpredictable fluctuations of the price in the Brownian Motion, the expected future value of the stock follows a normal distribution with mean 0. So, to mitigate the problem, the Generalized Wiener Process incorporates a drift factor in the model:

$$dx = ad_t + bd_z, \tag{2}$$

where a is the expected drift rate per unit of time and b is the noise of the variance.

2.2.4. Geometric Brownian Motion

Considering the price process described above, no one can solve the main problem that a price process of a stock has: the required return of investors on a stock is independent of the stock prices. If it is considered the constant expected drift rate and variance as in the Wiener process, its variables swing in an absolute value, while the return is considered in percentage. Geometric Brownian Motion is an improvement of a Wiener Process in the way that was projected so that fluctuations are relative and not in an absolute manner:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t \sim N(\mu dt, \sigma^2 dt), \tag{3}$$

S_t is the value of the project, dS_t is the absolute variation in the project, μ is the expected rate of return, dt is the variation in a continuous time, σ is the volatility, and $dZ_t = \epsilon\sqrt{dt}$, where d is the instantaneous change in time.

Delving the formula, it is possible to achieve that:

$$S_t = S_0 e^{\left(r-\frac{\sigma^2}{2}\right)t+\sigma\sqrt{t}\epsilon} \tag{4}$$

It is important to explain that regarding the different risk in the market, it is essential to consider that we are in a risk neutral world. This implies that μ was substituted by r_f that is the risk free interest rate. The formula above also demonstrates that the value of a future date of a project is defined by the current value capitalized with a risk-free rate plus the volatility input that is lognormal distributed. So, if the volatility input is lognormal distributed and as the value of the project depends on it, is possible to conclude that the present value of the project follows a lognormal distribution as well as the volatility.

2.3 The Black –Scholes-Merton Formula

Until the 70's, most of the research on pricing options was in terms of pricing warrants, which is the right to buy a share of the firm in a certain price during a given period of time (Galai and Schneller, 1978). Despite the fact that the concept of the warrant is similar to an option, the main difference is that the option is issued by an individual and the warrant is issued by a company when it issues new debt to attract the investor, increasing the outstanding shares when it is exercised. In the principle of the decade of 70, Fisher Black, Robert Merton and Myron Scholes developed a new formula to valuate options that nowadays is still in use. It is called the Black-Scholes-Merton Model.

Black and Scholes approach the valuation of the warrants to the valuation of the options, deriving a formula to price it considering several assumptions to be in a market with "ideal conditions". They considered as assumptions to apply the model the following:

- The short-term interest rate is known and constant through time;
- The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price;
- The option can only be exercised at maturity;
- There are no transactions costs in the market;
- It is possible to borrow any fraction of a stock, buy, or hold;
- There is no penalty for short selling.

Considering the assumptions described, Black-Scholes defined that the price follows a Geometric Brownian Motion. So to compute the price of the option, the Black-Scholes Model assumes risk neutral world. The option's price is influenced by constant variables such as time, volatility and the ratio of the stock price and the strike of the option and risk free rate. They developed a formula known as Black-Scholes formula:

$$c_0 = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{5}$$

$$p_0 = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \tag{6}$$

where $N(d_1)$ and $N(d_2)$ are the cumulative probability that follows a normal distribution function. Regarding the second one, $N(d_2)$, is the probability of the option being exercised.

S_0 is the current value of the project and Ke^{-rT} is the current value at which the project could be sold. The variables "r" and "T" are the risk-free interest rate and the time, respectively.

The parameters d_1 and d_2 are defined as follows:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{7}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{8}$$

During the life of the project, it could have any type of dividends that were mandatory to pay regarding the evolution of the project as, for example, dividends to shareholders or royalties. It pushes down the value of project and needs to be incorporated when a valuation of a project is done. Until this point, it was not considered the hypotheses of the project could paying dividends but it can also be included in the Black-Scholes-Merton Model.

Considering including this, the price process is changed to:

$$dS_t = (r-q)S_t dt + \sigma S_t dZ_t \tag{9}$$

and the Black-Scholes formula is modified to:

$$c_0 = e^{-rT}\left[S_0 e^{(r-q)T} N(d_2) - KN(d_1)\right] \tag{10}$$

$$c_0 = e^{-rT}\left[e^{(r-q)T} N(d_2) - KN(d_1)\right], \tag{11}$$

where

$$d_1 = \frac{\ln(\frac{S_0}{K})+\left(r-q+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{12}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{13}$$

The dividends above were present as continuing dividends. There are also discrete dividends where to achieve the optimal point of exercise of the option it is mandatory to compute the present value of the dividends and compare it to the intrinsic value of exercise the option at the moment.

2.4 Binomial Model

2.4.1. One Step Binomial Model

There are several types of models for pricing an option. One of the most popular, and applied in this thesis, is the binomial tree model. The model follows a simple concept: in each step the price of the options moves up or down, considering a defined probability. Hull (2012) defined the binomial tree as a diagram representing different possible paths that might be followed by the project over the life of an option. The underlying assumption is that the stock price follows a random walk.

To turn the model reliable, the first assumption that is needed to value an option is the non-arbitrage principle. Using the model, the risk of the investor having a free lunch (profit with no risk) is eliminated.

Using a simple analysis described in Hull, (2012), it will be considered only one step in the model, i.e., there will be only two moments of pricing the project, moment t_0 and t_1, where t_0 is the initial moment and t_1 is 3 months after. The option is a European-style option, being only possible to be exercise at the maturity. At month three, the project can reach value 22 or 18 and the investor will exercise if the value of the project achieves 21 (strike of the option). The probability of outcome each of the scenarios is equal. As it is possible to disclose the value of the project at 1 month, there is no uncertainty, being the required return of the investor equal to the risk free interest rate (10%). Therefore, the project is in a context of risk neutral world.

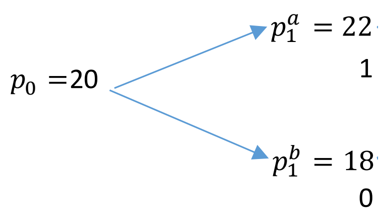

Figure 1 - One Step Binomial Model

The value of the option at maturity is as follows:

$$c^a = \max(22 - 21; 0) = 1 \tag{14}$$

$$c^b = max(18 - 21; 0) = 0 \tag{15}$$

Despite, the value of the option at maturity being clearly achieved, the investor can also invest initially in the project, reducing the risk of getting losses in the investment.

To get a riskless outcome of the investment, it is necessary to compute the riskless portfolio.

$$22\Delta - 1 = 18\Delta \tag{16}$$

$$\Delta = 0.25, \tag{17}$$

where Δ is the ratio of change in the price to the change in the price of the project. So, to get a riskless investment of the project, the investor can invest initially 25% in the project and short the investment selling a put option of the project.

The outcomes of the operation are:

$$Scenario\ A: 22 \times 0.25 + (21 - 22) = 5.5 - 1 = 4.5 \tag{18}$$

$$Scenario\ B: 18 \times 0.25 = 4.5 \tag{19}$$

No matter the swings of the price value, the output of the position is always 4.5. As mentioned above, the project is in a context of risk neutral world, so is rewarded at the risk free interest rate, 10%. The present value of the payoff is:

$$4{,}5e^{-0.10\times 3/12} = 4.3889 \tag{20}$$

it follows that the value of the option is:

$$20 \times 0.25 - c = 4.3889 \tag{21}$$

$$c = 0.6111 \tag{22}$$

The present value of the call is 0.6111. The call is in equilibrium at a price of 0.6111, satisfying the non-arbitrage condition. It is important to emphasize that the project is in context of risk neutral world. In real world, each project has its associated risk and expected rate of return.

Gathering the generic concepts it is possible to compute the price of an option with:

$$c_{0,0} = e^{-r\Delta T}[pc_{1,1} + (1 - p)c_{1,0}] \tag{23}$$

where $p = \frac{e^{r\Delta t} - d}{u - d}$ is the probability of an up movement in the value of the project. The parameters *u* and *d* are the down and up factor respectively:

$$u = e^{\sigma\sqrt{\Delta t}} \quad (24)$$

$$d = e^{-\sigma\sqrt{\Delta t}} \quad (25)$$

Note that Δt is the difference between two periods. Applying the concepts to the example above:

$$22p + 18(1 - p) = 20e^{0.10\times3/12} \quad (26)$$

$$p = 0.6266 \quad (27)$$

So, the probability of the call moving up is 0.6266.

2.4.2. Two Step Binomial Model

The binomial model can also be expanded for multiple periods of time. Although the option could be exercised at the maturity, the option may have several steps until it. It will be assumed the same case described above but with a maturity of 6 months. So, the two steps in the binomial model, at month 3 and 6, respectively, reflect these situations. Note that the initial idea is the same: compute the price of the option in the beginning of the tree.

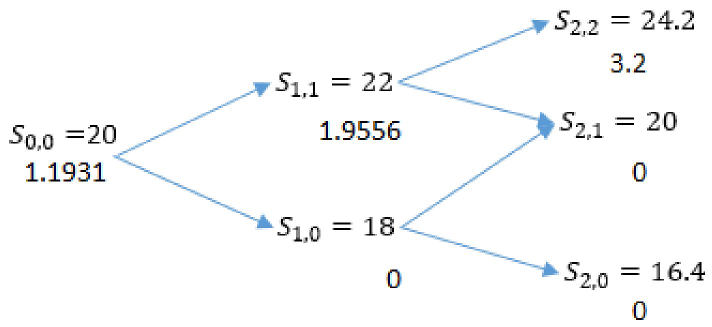

Figure 2 - Two Step Binomial Model

On the diagram it is described the value of the project in each step, and below the value is described the respective payoff of each node. In this case, there is only one node where the investor can execute the option. It is necessary to compute:

$$C_{1,1} = e^{-0.10\times3/12}(3.2 \times 0.6266 + \; 0 \times 0.3734) = 1.9556 \quad (28)$$

Finally, the first node of the tree:

$$C_{0,0} = e^{-0.10\times3/12}(1.9556 \times 0.6266 + \ 0 \times 0.3734) = 1.1931 \quad (29)$$

The price of the option is 1.1931.

2.4.3. American Style Option in Binomial Tree

Until this point we only considered one step of exercise, as in the case of European-style options. However, it is possible also to consider the situation of American-style options. This is the type of option that will be analyzed in the thesis.

The process is also exactly the same, but at each node the investor should decide if is in the optimal point to exercise the option, i.e., choose between exercising and receiving the intrinsic value or moving on more step forward.

To take a clear example of that, a dividend will be added in the case described in the previous section. The dividend has a value of 2.5€ immediately after month 3. The payoffs are described below the price in each node:

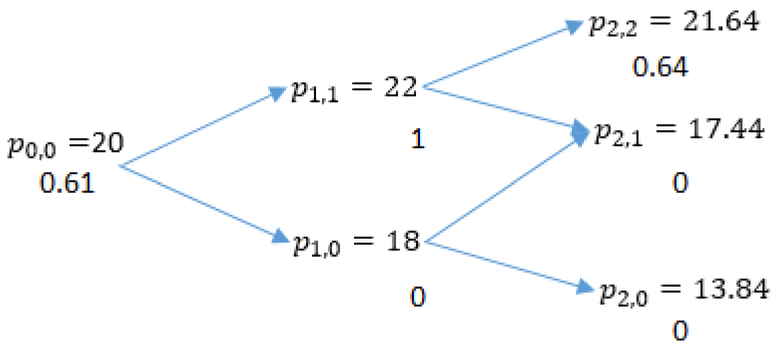

Figure 3 - American Style Option in Binomial Tree

Computing the payoffs in each node:

$$c_{1,1} = \max\ (\ e^{(-0.1\times3/12)} \times 0.6266\ \times (3.2 - \ e^{(0.1\times3/12)} \times 2.5)\,; 22 - 21) = 1 \quad (30)$$

$$c_{0,0} = \max\ (\ e^{(-0.1\times3/12)} \times 1 \times 0.6266\,; 20 - 21) = 0.6111 \quad (31)$$

At node (1,1) the better payoff is to execute at the node. Therefore, it is optimal to early exercise the option.

2.4.4. American Style Option paying Continuous Dividend-Yield

Besides payments of discrete dividends, as described in previous section, it is also possible to add continuous payments with a dividend yield.

Hull (2012) explains that considering a dividend yield q, and a risk free interest rate of r (as it is in risk neutral world), the capital gains must be compute with $r - q$. For example, if an investor has stock at S_0, the expected return of the project must be $r - q$ after one time step t. So:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} \tag{32}$$

The remaining formulas described at section 2.4.1. do not change.

2.5 Investment Decision in a Costly Reversibility Context

2.5.1. Investment Decision

As mentioned in section 2.1.4 real options can give flexibility to make decisions considering the new information of the market, approaching the valuation of the project to the real world. One of the options that the investor can take is the ability to wait until favorable conditions in the project occur. Once the conditions to invest are favorable the investor enters in the market. . The case that will be applied in this thesis studied this situation, where the investor waits to invest until favorable conditions appear.

The ability to delay an investment until getting favorable conditions can be a surplus of an investor, when analyzing an investment project. In fact, he could decide to continue in the idle state or enter in the market, i.e., exercise the option to invest in the project. The value of the project is a combination of:

$$\bar{X} + F(V) \rightarrow V \tag{33}$$

where the value of the project (V) corresponds to the amount invested $(\bar{X})$ plus the future payoffs $F(V)$. Moreover, $F(V)$ is our value of the option to invest and must obey the following boundary conditions:

$$F(0) = 0 \tag{34}$$

$$F(\bar{V}) = \bar{V} - \bar{X} \tag{35}$$

$$F'(V - \bar{V}) = 1 \tag{36}$$

Regarding the first assumption, if follows that if the future payoffs are zero, the value of the project is also zero. The second condition refers that by investing in a project the firm receives the net payoff of $\bar{V} - \bar{X}$. $\bar{V}$ is the critical asset pricing which is the optimal point to invest in the project. The third condition is the high contact condition, i.e., the opportunity cost of waiting is equal to the value of the option to invest.

In the context of American perpetual options, and using the Geometric Brownian Motion (see chapter 2.4), $F(V)$ can be achieved with:

$$F(V) = \begin{cases} (\bar{V} - \bar{X}) \left(\frac{V}{\bar{V}}\right)^{a} \rightarrow V < \bar{V} \\ \left(V - \bar{\bar{X}}\right) \rightarrow V > \bar{V} \end{cases} \tag{37}$$

where

$$\bar{V} = \frac{a}{a-1}\,\bar{X} \tag{38}$$

and

$$a = \frac{1}{2} - \frac{r-q}{\sigma^2} + \sqrt{\left(\frac{r-q}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{39}$$

Note that $\frac{a}{a-1}$ is the increment in the project value, if we invest one more coin and a is the elasticity.

2.5.2 Costly reversibility in a Disinvestment Option

The market conditions are not always in a favorable context. Once the investor enters in the project, the conditions can deteriorate and the investor may leave the project. One of the options that the investor can take is the ability to disinvest in a project, that is, an option that if the conditions go unfavorable, the investor can contract the investment or abandon the project. In this thesis, the option that will be analyzed is the possibility of the investor abandoning the project.

In fact, Trigeorgis (1996) considered an option to abandon or disinvest as a resale value of capital equipment and other assets if the market conditions go down and the investors do not want to continue the project. This option can be valued as an American-style put option, where the exercise price is the resale value or the best alternative value. Also, several investigators already studied the ability to recover the investment done, and increment this added value to the analysis of the investment. Dixit (1989) consider entry and exit decisions under uncertainty with hysteresis. So, as the decision of disinvestment is taken the investor could not recover all the initial investment. Dias and Shackleton (2011) considered that most capital expenditure in a firm or specific industry should be considered as sunk cost. Even if the capital expenditures are not from a specific industry, they could not be totally recovered. So a loss option should be included, that increments the value of the initial investment that could not be recovered. Keswani and Shackleton (2006) also analyzed a disinvestment option in the context of costly reversibility and concluded that the investor do not run in additional cost if when stopping the project, he cannot recover all the investment done.

Actually, considering an option to abandon in a project (a perpetual American put option), gives more flexibility to the investor to take accurate decisions, even if he cannot recover all the investment. So, it is possible to achieve the value of the disinvestment option considering the following:

$$\underline{X} \leftarrow V + F(V) \tag{40}$$

where $\underline{X}$ represent the divestment that can be recovered, V represents the assets value of the project and $F(V)$ is the option to disinvest. As the value V is higher, the value of the option turns to 0. The investor has less incentive to sell the project, if it is profitable. However, to, $F(V)$ must obey the following boundary conditions:

$$F(\infty) \rightarrow 0 \tag{41}$$

$$F(\underline{V}) = \underline{X} - \underline{V} \tag{42}$$

$$F'\left(V = \underline{V}\right) = -1 \tag{43}$$

Considering the first condition is expected that once V tends to infinity the disinvestment options tends to zero, as the firm does not have incentive to disinvest. Regarding the second boundary condition $\underline{V}$ defines the optimal position to disinvest where splits the stop and ongoing project, i.e. exercise the optimal disinvestment option. Exercising the option, the firm gets the payoff of $\underline{X} - \underline{V}$. The third boundary condition explains that disinvestment is optimal if the marginal value of investment in a project is inversely proportional to invest, so that the benefits of investing one more coin in the project are the same as the benefits to disinvest.

In the context of American perpetual options, and using the Geometric Brownian Motion (see chapter 2.4), $F(V)$ the disinvestment option can be achieved with:

$$F(V) = \begin{cases} \left(\underline{X} - \underline{V}\right)\left(\frac{V}{\underline{V}}\right)^{b} \rightarrow V > \underline{V} \\ \left(\underline{X} - V\right) \rightarrow V < \underline{V} \end{cases} \tag{44}$$

where

$$\underline{V} = \frac{b}{b-1}\underline{X} \tag{45}$$

and

$$b = \frac{1}{2} - \frac{r-q}{\sigma^2} - \sqrt{\left(\frac{r-q}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{46}$$

$\underline{V}$ is the value of the project at which it is optimal to disinvest, $\underline{X}$ is the disinvestment proceeds that were initially incurred, with $\alpha = \frac{\underline{X}}{\overline{X}}$ being the percentage recovered of the initial costs, and b is the elasticity.

2.5.3. Entry and Exit Decisions

Several valuation projects may not capture all the flexibility values of the project. Most projects in the real word are not characterized by one decision of entry and exit, but with both decisions simultaneously. In order to capture the reliable value of the project, when the investor takes the decision of entry in the project, he should already consider in the project's valuation the value of taking the decision of disinvesting in the future.

Keswani and Shackleton (2006) consider this situation and conclude that if the investor has the ability to disinvest in the project he should compare the value of the project plus the value of the option to close to the disinvestment value. Additionally, if the investor has always the ability to reverse the decision at any times, then the manager should compare the value of the project close plus the option to open to the value of the project open plus the option to close. So, to get in the project the following assumption must be satisfied:

$$\overline{X} + Open\ Option\ at\ \overline{V} \rightarrow \overline{V} + Closing\ Option\ at\ \overline{V} \quad (47)$$

and to be in a disinvestment point it must satisfied:

$$\underline{X} + Open\ Option\ at\ \underline{V} \leftarrow \underline{V} + Closing\ Option\ at\ \underline{V} \quad (48)$$

When the project value reaches the optimal point to invest, the investor wants to change the situation of idle project, where has the amount to invest and the option of invest to a context of having the project running and the option to disinvest, to an active project.

To disinvest the project needs to reach an optimal value where the investor tends to close the project, sell the assets and has the option to invest despite continuing with the project.

3. Case Study Analysis

3.1. About the Case

The renewable energy sector was described by Santos et al. (2014) as a sector with a high level of uncertainty regarding the liberalization of the market. So, the traditional methods applied to value the project cannot capture the risk and decrease the expected value of the project. Considering the high investment and the specificity of the project, the authors considered that the project is in a situation of total irreversibility. Also, the authors included an option to delay the project that gives more flexibility because the investor can delay the project until having a certain decision. In fact, the presence of irreversibility made the authors to assume that the project was completely irreversible, only studying the option to defer the project, i.e., wait until changes in the market.

The article Santos et al. (2014) will be used as the basis to the case study analysis of the thesis. It demonstrates that applying real options valuation, more specifically an option to defer, can augment the project value when compared with the valuation via traditional methods.

Regarding the case, the authors evaluate a mini hydro plant project with an installed capacity of 500kW. The project starts at 2006 and is expected to operate over 50 years (turbines and generator), with a reinvestment of a new transformer at the year 25. The company has an incentive that covers 40% of the investment up to 1000€/kW, 25% of the investment comes from the equity and the remaining 35% are financed through a loan. The loan must be paid in 9 years with a grace period of 3 years. The annual payments are subject to an interest rate of 6.5% and annual constant payments over 10 years. There is also an inflation rate of 3%[1] and the risk-free interest rate is 7%. To compute the value of the option it was considered as up-factor the value of 1.49 and for p, probability of an up movement, the value of 0.49. The project has an initial investment of 830,000€ essentially in fixed assets. Despite the assumptions of the project valuation seeming unrealistic, they

[1] Inflation rate will be considered as our Compound Annual Growth Rate.

$$CAGR = \frac{Ending\ Value}{Beginning\ Value}^{(1/years)} - 1$$

were maintained to be possible to compare with the real options valuation. Also it was not considered changes in technological, environment policies, and in fuel costs.

The authors only considered as variable the price of the electricity in a stochastic process, more specifically the Geometric Brownian Motion. They used the software Crystal Ball and Monte-Carlo simulation to compute the probability factors and the volatility, i.e., the standard deviation of 40%.

To evaluate the option, the authors considered an American-Style option with no dividends with 5 times of fluctuation of the project value and also 5 steps of entry decision nodes. They defined the investment decision comparing the static NPV, that is the net present value the project start at this point, and the project value of the option to delay:

$$\begin{cases} Option\ value\ of\ Delay > Static\ NPV \rightarrow Delay \\ Option\ value\ of\ Delay < Static\ NPV \rightarrow Invest \end{cases}, \tag{49}$$

where

$$Option\ value\ of\ Delay = Project\ Value\ (\ with\ option\ to\ delay) - \ Static\ NPV. \tag{50}$$

The authors developed a binomial tree and reached the following conclusions:

Years

0	1	2	3	4	5
					Invest
				Invest	
			Invest		Invest
		Invest		Invest	
	Invest		Delay		Invest
Delay		Delay		Delay	
	Delay		Delay		Do not invest
		Delay		Delay	
			Delay		Do not invest
				Delay	
					Do not invest

Figure 4 - Base Case Conclusions Analysis

Source: *Lucia Santos – Real Options versus Traditional Methods to assess Renewable Energy*

They concluded that despite the NPV in the first node, time 0, being positive, it is better to wait in order to lower the uncertainty in the project. They also concluded that using real options gives more flexibility to the investor and emphasizes the limitations of the discounted cash-flow method. Therefore, the valuation increases as is incremented by the real options valuation.

3.2. Case Study Limitations and Project Goals

As in other corporate finance projects, it is impossible to capture all the variables' effects that surround projects. Moreover, some of them do not materially affect the results and could be hard to determine.

The selection of this article as base case to this project is essentially related to considering that there are relevant effects not included in the case that can, potentially, modify the investor decision. As discussed over the thesis, the surplus of real options valuation is the flexibility of taking decisions regarding the scenario. Considering that we are an investor and we can decide between two projects in the same conditions, where one can be reverted and the other cannot, it is natural that the choice falls over the project with flexibility. But, how much does it represent for the project value in the present? It can make the difference between entering in the market or keeping the money in the pocket. In the article the authors do not take in consideration the disinvestment decision, considering only the entry scenario. This can lead to an incorrect decision as the information is not complete. Adding the disinvestment possibility, the investor could invest with worse conditions as he has the possibility to leave the project and stop losses and sell the assets. To develop this goal it will be necessary to make an assumption not taken in the case that is the strike of the disinvestment option. See chapter 3.3.1. where this thematic is discussed.

Other limitation is related to the dividends of the project. When the authors assume that there are no dividends in the project, this is not exactly true[2]. Besides the flexibility, each investor tries to get a required return of the project. The principal reason is to reduce the investment risk exposure, reinvest or invest in another project. So, the dividends paid during the project should be considered in the project as they decrease its value.

The approach taken will be with non-complex development to enhance the real world approach and compare our results to the case. To start at the same point, the case will be replicated and subsequently it will be added the disinvestment option and dividends to analyze if the entry decision is modified, what is the gain or loss of adding these variables

[2] Even though this issue is not discussed by them, it is important to note that in the absence of dividends the value of the American-Style call is actually equal to corresponding European-Style call.

and when should it be optimal to decide to invest or disinvest. Also, a binomial tree will be constructed so that it can be possible to analyze the decision that should be taken by the investor, i.e., when it will be optimal to entry, maintain the project and disinvest in a situation of costly reversibility.

3.3 Project Assumptions

3.3.1. Strike

Considering that it will be applied a disinvestment option, it is critical to define the disinvestment strike, i.e., the value that will be recovered when the option is executed. Unlike financial assets, a project finite life makes it lose value over its life, thus it is not possible to apply the theoretical framework presented in section 2.5.2, where it is exposed a disinvestment context in a costly reversible situation with an infinite life. So, to achieve a realistic assumption for the project, a non-constant strike during the project life will be used.

To define the strike it will be assumed the NBV[3] of the fixed assets at each period. Sometimes the amount invested may not be totally spent in fixed assets as it is necessary to do studies and incur in other costs not directly related to fixed assets, but to simplify it will be assumed that the amount initially invested is represented by all capitalized costs and will be depreciated over the life of the project. The NBV will depreciate in 50 years, which corresponds to the life of the project. As described in the base case analysis, a new transformer is necessary in year 25, so the strike will increase in that amount.

The present assumption assumes some uncertainty on the recoverable value, thus a scenario analysis with total irreversibility[4], with a recoverable value of 33% and 66% of the NBV and totally recoverable NBV will be developed. For this purpose, it will be applied the following function:

$$\underline{X} = \alpha \times |\bar{X}| \times \left(1 - \frac{T}{50}\right) \quad , \tag{51}$$

[3] NBV = Net Book Value. Net Book Value is defined as the amount invested in fixed assets less the accumulated depreciation.
[4] Scenario developed in the base case.

where $|\bar{X}|$ is the initially amount invested in absolute terms, $\underline{X}$ is the recovered value that will be recovered at T time and α is the percentage recovered of the NBV. In the graph bellow it is presented the Strikes that will be used in the scenarios analysis:

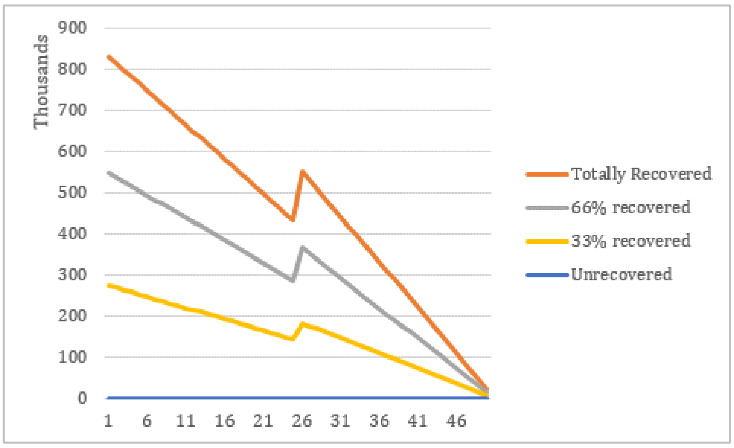

Figure 5 - Strike Evolution during the Project Life

In Section 5 the results will be compared the results and it will be assessed which is the best investment decision and if the change in the strike significantly changes the investment decision.

3.3.2 Dividends

As described in section 3.1. the analysis of real options valuation of Santos et al (2014) does not include dividends as variable. Usually dividends are one of the decision factors that influence the investment decision as the investor wants to be remunerated according to the risk of the project. Moreover, he wants to redeem cash-flows to reduce the initial investment risk exposure in the project, invest in another project or reinvest.

For the reason presented above, our case will include dividends. For this purpose, it will be applied a payment with continuous dividend yield, q. As benchmark of a relevant company in the sector of renewable energy EDP SA will be used, a company listed in Euronext Stock Exchange in Lisbon. This company is the only listed in Portugal that operates renewable energy in Portugal. Regarding the dividends distribution, it will be assumed the average of dividend yield[5] of the last 3 years, 6.16% (EDP, 2017).

Years	*DY*
2013	7.00%

[5] To compute the Dividend Yield was considered the amount distributed over the share value at the end of the year. To simplify, was considered the dividend yield as a continuous dividend yield.

2014	5.67%
2015	5.81%
Average	6.16%

Table 4 - EDP Dividend Yield

3.3.3. Project Value

At a project valuation context, the valuation of the settlement is defined by the sum of the discounted cash-flows of each period. This value of the project has constant cash-flows over a perpetuity life. Considering the case that will be developed, due to the investment in the transformer at year 25, financing activity and other non-constant costs, the cash-flows are not constant over the 50 years useful life. Figure 6 and 7 presents the fluctuation of the current cash-flows and discounted cash-flows where the current cash-flows follows the left axis and discounted cash-flows follows the right axis:

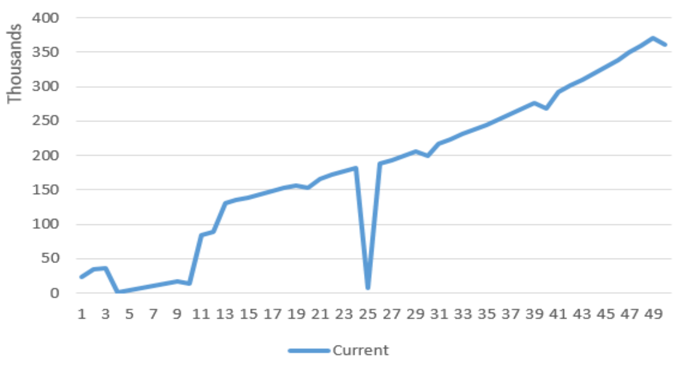

Figure 6 – Base Case Current Cash-Flows

Figure 7 Base Case Discounted Cash-Flows

Therefore, in order to get the project value more close to a reliable value and not to influence the investment decision, we will compute the project value summing the discounted cash-flows between T[6] and 50 years at node T. Until this point, the cash-flows before T were excluded but this project value is only correct if its value does neither appreciate nor depreciate. For this purpose, the project value will be multiplied by the up or down factor elevated by the number of times[7] that goes up or down since $T = 0$, ie the beginning of the project.

Other thematic in the base case is the possibility of the transformer investment on year 25. Considering that the project value may be so low that the investor does not want to buy the new transformer, as it can lead to certain points where the best option is not to invest in the transformer and sell the assets at year 25.

So, to evaluate the project value until the year it will be considered these two scenarios of valuation of the project and the scenario with better valuation will predominate. The following describes the valuation of the project value.

The following formula explains the value of S_t at node (t,y):

$$S_t = \begin{cases} \max(\sum\left(CF_{t-t_{50}}\right)^{u\times y} - Transformer \times e^{(t-t_{25})\times r_f}; \sum\left(CF_{t-t_{25}}\right)^{u\times y}) & if\ t < 25 \\ \sum\left(CF_{t-t_{50}}\right)^{u\times y} & if\ 25 \leq t < 50 \end{cases} \quad (52)$$

where u is the up factor and y is the number of times that the project values goes up or down and t is the current operation years and r_f is the risk free interest rate.

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[6] T= Period ran since the project begins.
[7] For this analysis will be considered the net of ups or downs that project value suffered. For example, if the project value goes up 3 times and goes down 1 times, will be considered only 2 up times.

4. Case Development

The analysis of the case was developed in phases. Before starting to develop our aims, it was replicated the base case to get the accuracy of the data and conclusions. The development of the case was done with the inputs described in 3.1. where the traditional methods valuation and the binomial tree was constructed.

Posteriorly it was developed the disinvestment American-Style option in a situation of costly reversibility with a binomial tree with 50 times. Subsequently it was developed the investment American-Style option considering the prior disinvestment option and finally it was defined the investment/disinvest decisions in each node. During these phases it was considered the assumptions taken on dividends, project value and the recovered value, i.e., strike. The case was developed using Microsoft Office Excel Tool.

4.1. Base Case

As mentioned above, in order to get consistent results and compare the conclusions it is critical to have the same base case as the Santos et al (2014). To create that it was replicated the traditional methods described and constructed a binomial tree to achieve the results revealed in the article. The results are presented below:

Years: 0	1	2	3	4	5
					6,512,499
					5,682,499
				4,365,459	
				3,591,572	
			2,926,255		2,926,251
			2,204,687		2,096,251
		1,961,527		1,961,525	
		1,314,527		1,187,638	
	1,314,851		1,314,850		1,314,848
	763,592		647,461		484,848
881,371		881,370		881,369	
433,660		343,632		221,322	
	590,800		590,799		590,799
	178,808		101,028		0
		396,025		396,024	
		46,117		0	
			265,463		265,463
			0		0
				177,945	
				0	
					119,280
					0

Value of the project
Project value of delay

Figure 8 – Base Case Project Value of Delay

Source: *Lucia Santos – Real Options versus Traditional Methods to assess Renewable Energy*

Regarding the figure above and comparing it to the tree presented in the article it is possible to verify that there are only differences of no more than one euro. Santos et al (2014) presented the option applied as an American-Style option, but, in fact, it is equivalent to a European Style option as there are no dividends in the base case. This means that the true value is the one that is obtained by the Black-Scholes Model (1973).

4.2. Developing the Disinvestment Option

Considering that the disinvestment option will influence the initial decision of investment, we start the analysis developing the disinvestment scenario. A binomial tree of 55 nodes was constructed. Note that the first 5 periods are developed for the scenario of entry where the investor has the investment option and the subsequent 50 year correspond to operation of the project. In the case that the option is executed early, it will be considered that the project is node 5, where starts the operation of the project. After node 5, the project is in a situation of disinvestment so it will be analyzed this scenario. Figure 9 presents the Binomial Tree with 55 nodes where the gray shows where it will be analyzed the option of delaying the project and the orange zone is the second scenario where it is analyzed the disinvestment scenario.

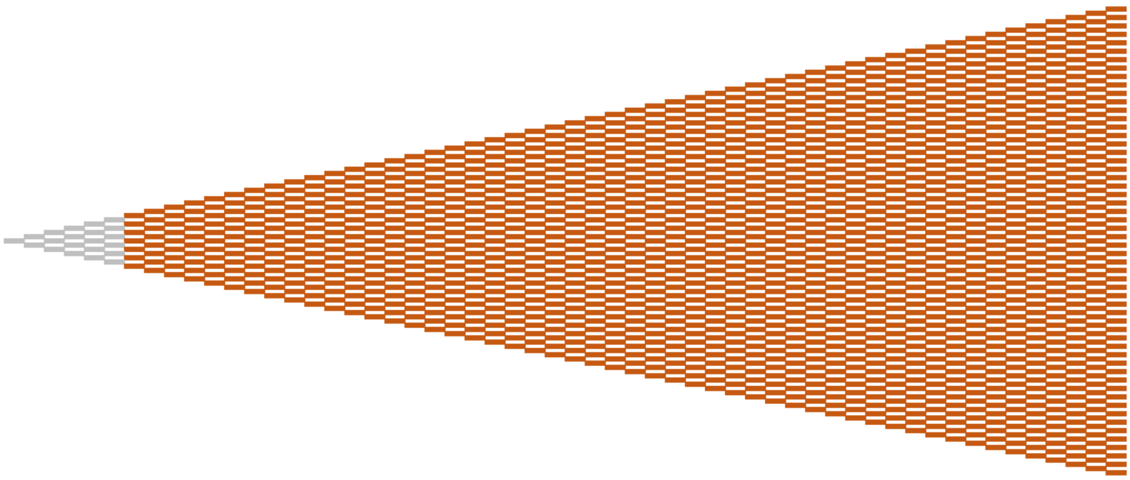

Figure 9 - Binomial Tree to evaluate the project

After constructing the Binomial Tree, it was computed the project value and strike at each node as defined in section 3.3.4 and 3.3.3 respectively. Then, we started the disinvestment option analysis, being necessary to compute the probability, p, of an up movement as defined on chapter 2.4.1. to include dividends. A new p was computed:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} = \frac{e^{(0.07-0.0616)} - 0.67}{1.49 - 0.67} = 0.4116 \quad (53)$$

Posteriorly the payoffs at all nodes were computed, as defined at section 2.1.2, and to achieve the value the disinvestment option, it was applied the thematic presented at section 2.4.3 and developed the following formula:

$$DO_{t-1}^{y} = e^{-0.07} \times \left[p \times Max\left(K_t^{y+1} - S_t^{y+1}; DO_t^{y+1}\right) + (1-p) \times \ Max\left(K_t^{y-1} - S_t^{y-1}; DO_t^{y-1}\right)\right] \quad (54)$$

where DO_{t-1}^{y} is the disinvestment option that goes up y times and is at period $t-1$ and S is the settlement, ie, the project and K is the strike, p was defined and computed at formula 48. Note that at node 55 (50th year of operation) the option has no value as we can only execute or run the project until the end, i.e. $DO_t^{y+1} = 0$. The option valuation was computed from the end to the beginning.

After concluding the option valuation, the disinvestment decision was developed. For this purpose was defined the following conditions:

$$\begin{cases} S_t + DO_t < K_t \rightarrow Disinvest \\ S_t + DO_t > K_t \rightarrow Keep\ Run\ the\ Project \end{cases} \quad (55)$$

The investment decision is intuitive as the investor wants to continue running the project until the recoverable value is higher than the project value plus the disinvestment option, and when this point is reached the investor leaves the project getting the strike. Note that despite section 2.5.3. is related to perpetuity projects, the intuition of changing an active project to an idle is the same as applied in the case.

Figure 10 presents a short summary of the tree nodes of the binomial tree developed in the numerical computations.

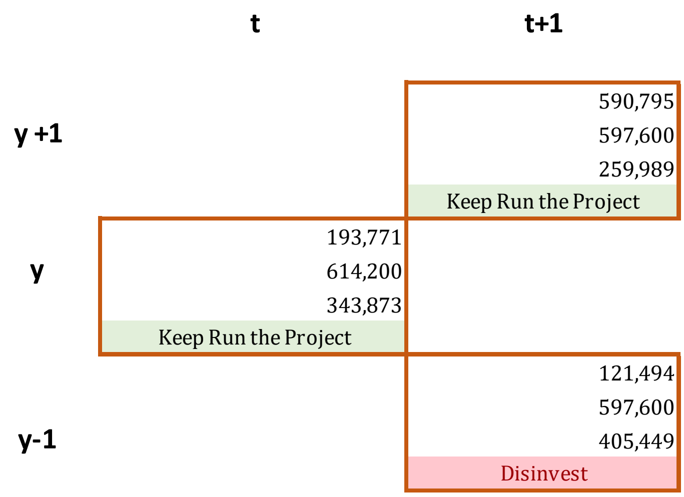

Figure 10 - Disinvestment Option Binomial Tree

The value presented on the top in each node is the project value, where it was applied the formula 52 and the value below corresponds to the recoverable value, i.e. strike K at the node. The third value is the disinvestment option that was defined with the formula 54 and on down is the investment decision at the node. As can been see the node (t+1, y-1) is in a situation of disinvestment as:

$$121{,}494 + 405{,}449 < 597{,}600 \quad (56)$$

where $S_{t+1} = 121{,}494$, $DO_{t+1} = 405{,}449$ and $K = 597{,}600$. So, at this node it is optimal to disinvest.

4.3. Developing the Investment Option

In the context of the development of the investment option it will be used the binomial tree priorly constructed and developed the first 5 periods of the tree.

At this scenario, the project value does not lose value during the life depreciation as it does in an idle situation. Once the call is executed, the project enters in a disinvest scenario and the project starts to lose value over its life.

To get the investment option value it was necessary to recompute the call options of the base case. To do that it was applied the following formula:

$$IO_{t-1}^{y} = e^{-0.07} \times \left[p \times Max\left(S_t^{y+1} - K_t^{y+1}; IO_t^{y+1}\right) + (1-p) \times Max\left(S_t^{y-1} - K_t^{y+1}; IO_t^{y-1}\right)\right] \quad (57)$$

where IO corresponds to the investment option. The remaining variables were explained in the prior section.

After concluding the option valuation, the investment decision was developed. For this purpose, the following conditions were defined:

$$\begin{cases} IO_t < S_t - K_t + DO_t \rightarrow Invest \\ IO_t > S_t - K_t + DO_t \rightarrow Delay \end{cases} \tag{58}$$

As in the prior section, the investment is intuitive as the investor wants to enter in the project when the cash-flow that will be generated plus the disinvestment option is higher than the investment option. This investment decision is also related to section 2.5.3. as the investor wants to change the scenario when getting in the project gives more incentive than remaining in an idle state.

Figure 11 shows an example of the binomial tree developed to present the practical execution:

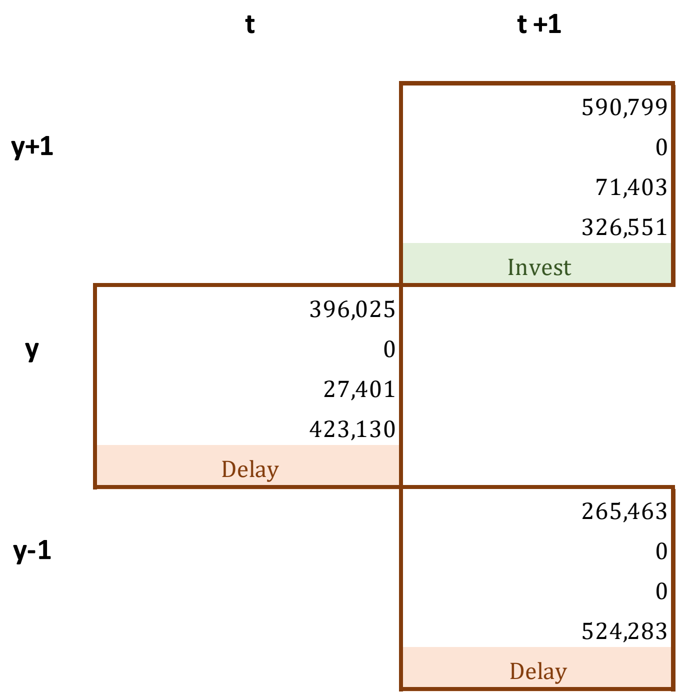

Figure 11 - Investment Option Binomial Tree

The value presented at the top of each node is the project value that flows with the up or down factor. The value below corresponds to the maximum between the payoff and zero if the investor exercises the option at this point. The third value is the value of delaying the project's option, i.e., IO as defined at formula 57. As can been see the node (t, y) is in an investment situation as:

$$27{,}401 > 396{,}025 - 830{,}000 + 423{,}130 \qquad (59)$$

where $S = 396{,}025$, $K = 830{,}000$, $DO_{t+1} = 423{,}130$ and $IO = 27{,}401$. So at this point the optimal decision is to delay the investment on more step.

5. Results

In this chapter it will be presented the project valuation that has been developed. We will gather the conclusions initially with the analysis of the increment of dividends in the project and compare the investment decisions with the base case and then it will be analyzed the disinvestment option during the operation cycle. Posteriorly the investment decisions will be analyzed with an increment of a disinvestment option at a costly reversibility situation with several scenarios as described in section 3.3.1. compared with the base case and subsequent investment decisions.

5.1. Base Case with Dividends

As discussed at section 3.3.2., to evaluate the project, we first include a payment of a continuous dividend yield of 6.16%. To achieve the impact of dividends it will be assumed that we are in an irreversibility context. The decision tree is presented in figure 12 where each node (from the top to the bottom) presents the project value, payoff of exercising the option and the option value of delaying. Below it we also show my investment decisions defined with and without dividends comparing with the base case. Regarding the investment decisions presented it is possible to verify that although all nodes under $y = 0$ have the same investment decision, when the project increases the value ($y > 0$) the investment decision becomes different. Comparing initially the investment decision without dividends, it should not have different investment decisions. In fact the investment decisions defined are different. The base case defines that investment decisions should follow as defined in section 3.1. arguing that the opportunity cost of differing one period should be compared to the NPV at the node. It seems not to be the most correct decision as the investor will only change from an idle situation to an active one when the operation value will be higher as defined at section 4.2. and 2.5.3. Approaching to investment scenario with and without dividends we verify that naturally it is optimal to early exercise the investment. As can be seen in Figure 12, at node (2,2) it will only be necessary that the project value grows two time steps to be at in a favorable investment context. This situation is explained by the probability of capital gains that were significantly low as compared without dividends and consequently the value to delay the project is lower. Therefore the investor will only invest when he considers that project at a moment that is better to become active than at an idle situation. As in the base case, the investment decision at moment $t = 0$ continues to be better to not get in the project.

Project value
Payoff at the node
Call value
Investment decision with dividends
Investment decision without dividends
Investment decision base case

Y \ T	0	1	2	3	4	5
5						6,512,499 5,682,499 - Invest Invest Invest
4					4,365,459 3,535,459 3,330,775 Invest Delay Invest	
3				2,926,255 2,096,255 1,977,550 Invest Delay Invest		2,926,251 2,096,251 - Invest Invest Invest
2			1,961,527 1,131,527 1,098,690 Invest Delay Invest		1,961,525 1,131,525 1,070,454 Invest Delay Invest	
1		1,314,851 484,851 568,639 Delay Delay Invest		1,314,850 484,850 536,310 Delay Delay Delay		1,314,848 484,848 - Invest Invest Invest
0	881,371 51,371 278,046 Delay Delay Delay		881,370 51,370 244,986 Delay Delay Delay		881,369 51,369 186,063 Delay Delay Delay	
-1		590,800 0 109,048 Delay Delay Delay		590,799 0 71,403 Delay Delay Delay		590,799 0 - Not Invest Not Invest Not Invest
-2			396,025 0 27,401 Delay Delay Delay		396,024 0 0 Delay Delay Delay	
-3				265,463 0 0 Delay Delay Delay		265,463 0 - Not Invest Not Invest Not Invest
-4					177,945 0 0 Delay Delay Delay	
-5						119,280 0 - Not Invest Not Invest Not Invest

Figure 12 - Binomial Tree Dividends Analysis

5.2. Disinvestment Option

Before starting the analysis of investment decision with a disinvestment option it is important to analyze the behavior of the disinvestment scenario during the project life. For this purpose, it was developed the binomial tree displayed in Figure 13 with scenarios of full net book value recovery and a recovery of 66% and 33% of the net book value, as mentioned at section 3.3.1. The vertical axis corresponds to the up or down movements done by the project and the horizontal axis corresponds to operation years.

As it is possible to observe in figure 13, we are in the presence of a classical situation of incrementing flexibility in a project, where when the project value becomes worth it is better to disinvest than continues the project thus the project do not operates at adverse production conditions. Note also that as the recovery of the project is higher the optimal disinvestment decisions tends to get favorable nodes as the investor will recover more initial investment. Other particular situation is the year 25, where is the reinvestment of a transformer or the leave of the project. At this time, it is noted that optimal disinvestment decision has a sharp increase, which indicates that it is more valuable to operate until year 25 than reinvest and continue project, i.e. the discounted future cash flows plus the disinvestment options do not cover the investment in the transformer. Regarding the years following the year 25, the disinvestment option decision tends to increase once the project reaches the end. Moreover, as the project value depreciates, the option value of disinvesting tends to zero as the uncertainty of the project value is decreasing, doing that optimal disinvestment decisions reach to more favorable condition nodes.

Figure 13 - Disinvestment Option

5.3. Investment Decision under Dividends and Disinvestment Option

Until this point, it was only developed an individual analysis and evaluated the impact in the project. In this section, it will be entered in an aggregated thematic to gather the final conclusions of the thesis.

To evaluate the investment decisions two output results were created, table 5 and figure 14, where are presented the disinvestment option values at each node y over different recovery investment scenarios and investment decisions, respectively. As can be seen in figure 14, it is possible to verify that nodes with higher and lower (y < -1 and y > 1) project values present consistent decisions, as the project lost or gained sufficient value that incrementing a disinvestment option will not change the investment decision. On the other hand, the intermediate nodes (1 > y > -1) presents diversified investment decisions, being essentially related to disinvestment option. Approaching initially to 100% NBV recovery investment scenario, it is possible to verify that it is the more favorable investment scenario, thus it recovers more investment if the project loss value. Considering table 5, if the project is at node y=0, the fact of existing a disinvestment option increments the valuation at 241.281€ comparing with 38.318€ if disinvestment covers only 33%. Therefore the impact between the scenarios is so high that it materially affects the investment decision, i.e., incrementing a disinvestment option tends to make investors consider to invest at less healthier nodes, where if it did not exist the favorable decision would have been to delay or disinvest.

Y	-5	-4	-3	-2	-1	0	1	2	3	4	5
K=100%	659,456	592,005	519,215	423,130	326,551	241,994	185,281	136,843	106,237	81,198	63,932
K=66%	396,334	334,280	281,338	209,518	160,656	119,104	91,826	69,006	54,215	42,619	33,778
K=33%	149,596	114,869	88,129	65,426	50,600	38,318	30,298	24,170	19,342	15,847	12,633

Table 5 - Disinvestment Option Values

Additionally, the only node where it is favorable to invest and, even if project loses one time step value continues to be favorable is the $K = 100\%$ scenario. Considering the remaining scenarios it is possible to verify that $K = 66\%$ is favorable to invest when the project grows one time step. For $K = 33\%$ and irreversibility scenario, if the project is at maturity, where it is only necessary to grow one time step it, is always necessary to the project to grows two time steps for the investment in the project be favorable.

Y \ T	0	1	2	3	4	5
5	*K =100%* *K=66%* *K=33%* *K= 0* *Base Case*					Invest Invest Invest Invest Invest
4					Invest Invest Invest Invest Invest	
3				Invest Invest Invest Invest Invest		Invest Invest Invest Invest Invest
2			Invest Invest Invest Invest Invest		Invest Invest Invest Invest Invest	
1		Invest Invest Delay Delay Invest		Invest Invest Delay Delay Delay		Invest Invest Invest Invest Invest
0	Invest Delay Delay Delay Delay		Invest Delay Delay Delay Delay		Invest Delay Delay Delay Delay	
-1		Delay Delay Delay Delay Delay		Invest Delay Delay Delay Delay		Invest Not Invest Not Invest Not Invest Not Invest
-2			Delay Delay Delay Delay Delay		Delay Delay Delay Delay Delay	
-3				Delay Delay Delay Delay Delay		Not Invest Not Invest Not Invest Not Invest Not Invest
-4					Delay Delay Delay Delay Delay	
-5						Not Invest Not Invest Not Invest Not Invest Not Invest

Figure 14 - Investment Decisions under Dividends and Disinvestment Option

6. Conclusion

This section aims to present the major conclusions from the analysis of research and the case. The objective of this thesis was to develop a real options valuation focusing on flexibility.

Santos et al. (2014) developed the base case in a context of irreversibility and compared an option to delay with the traditional NPV. In both cases, it is assumed that once the investment is made, there is no reversion possibility. Considering that, it was studied the thematic and applied to the analysis the disinvestment possibility and dividends. In order to do that it was developed an analysis using a Binomial Tree and included an American-style option with dividends at renewable energy sector considering several scenarios of reversibility of the project.

Considering the valuation conclusions, Figures 12 and 13 present a comparison with and without dividends and disinvestment option respectively. Regarding Figure 12, it was interesting to verify that dividends inclusion forces the project to get in at an earlier context as the probability of capital gains were significantly low as compared without dividends and consequently the value to delay the project is lower. It is important to emphasize that the best investment decision in most of scenarios at $t = 0$ is to delay, having only one scenario, the one with the more reversible costs where is favorable to invest. Also, if the project is at maturity and depreciates only one time step is favorable to invest in the best scenario. Other important conclusion is that incrementing an option to disinvest makes the investment node range more broadly where is possible to have favorable investment nodes at lower project values.

Regarding the disinvestment context, if the investor has a disinvestment possibility, it gives a decision line to investor where he can stop losses, if the project loss value, disinvesting. Also, if the investor are reaching to a reinvestment situation the decision line tends to grows as to continue the project value need to be more valuable to support the investment. Other interesting conclusion is that this decision line tends to grow as the project comes to maturity, the main reason for that is the reduction of uncertainty.

Nowadays, discounted cash-flows method continues to be the method that management recognizes as more reliable to evaluate, as it is more easy to develop and analyze. To contradict it is essential that finance specialists, the ones to whom is left the responsibility to develop a valuation, demystify the complex applicability, start using the method and present it as the best method to evaluate a project. Moreover it is essential to achieve the real limitations of each project and how they can be solved applying flexibility through a real option approach, developing valuation with several options that management recognizes real value.

At last, we suggest that it should be interesting to analyze the project with other options in a context of infinity useful life where it can be analyzed the new entry decisions and re-entry decisions in the project.

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