

Stochastic Evaluation of Deepwater oil prospects in Portugal
using Monte Carlo Simulation

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Project submitted as partial requirement for the conferral of
Master in Finance

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April 2013

Acknowledgements

I owe my sincere gratitude to Professor António Barbosa for the expert guidance, the confidence and the persistent help that provided me an opportunity to grow both personally and professionally in a unique work environment.

Dra. Catarina Ceitil deserves a word of acknowledgment for her valuable and expert advices and review which significantly improved the quality of the dissertation.

I am equally indebted to Dr. Roland Muglli, Eng. Walter Waes, Eng. Miguel Sinval and Geoscientist Carolina Libório, for their priceless assistance when making available their support in a number of ways.

My friends, Joana Vieira, Daniel Valbom, Pedro Gancho and João Jeremias deserve my sincere appreciation for all the support and attention in the last months.

Special words go to my family: my parents, Manuel and Elsa and my brother, Miguel. A special thought is devoted to them for a never-ending support and encouragement, who raised me with constant love and whose guidance is with me in all my pursuits. I owe them my eternal gratitude.

Finally, I save my last acknowledgement, for my girl and my best friend, Paula Pereira da Silva, for every word, for every love push and even for her silent look that always kept me in the way of accomplishing this dissertation. I would never had done it if she wasn't present in my life in such a deeply way.

Abstract

Portugal, like many other countries, is an importer of oil resources, however is one of the few countries, with potential for large oil discoveries along its unexplored exclusive economic zone. The potential is justified from the geological similarity with the recent deepwater discoveries, on the other side of the Atlantic Ocean (Brazil, Gulf of Mexico and Eastern Canada). This study performs a Stochastic Evaluation methodology, for assessing the oil potential of the deepwater concessions located in the Peniche and Alentejo Basins.

A project life cycle in the Oil&Gas sector, considers important uncertainties related to yields and costs. The main uncertainties are related to Recoverable Oil Volumes, Production rates, the Brent Price and the Capex&Opex structure. Recoverable Oil Volumes are calculated and framed according to the geological software, GeoEx, which discloses a statistically relevant LogNormal Distribution. The yearly Production rates are randomly selected from historical distributions derived from similar reservoirs. The Brent price is forecasted in a Mean Reversing process with Jumps diffusion from the Geometric Brownian model. Finally, the Capex&Opex structure follows a Triangular Distribution estimated by experienced project managers.

The uncertainty adjustment is prior to the modeling of the main variables that distress Cash Flows from the Monte Carlo Simulation. The simulation preserves the stochastic nature to the Cash Flows, since it is a sum of random variables. At the end of the study, all possible NPV's are given as Probability Density Distribution that expresses the probability of the economic value for the Peniche and Alentejo Basin.

Key words: Uncertainty, Stochastic Modeling, Investment Valuation, Monte Carlo Simulation

JEL Classification: Q40; C63; O22

Resumo

Portugal, como muitos outros países, é um importador de petróleo, porém, é um dos poucos países com potencial para grandes descobertas de óleo, ao longo de sua zona exclusiva econômica inexplorada. O potencial é justificado a partir da semelhança geológica, com as recentes descobertas em águas profundas, no outro lado do Oceano Atlântico (Brasil, Golfo do México e do Canadá Oriental). Este estudo executa a metodologia de avaliação estocástica, para estimar o potencial petrolífero das concessões de águas profundas, na Bacia de Peniche e Alentejo.

O ciclo de vida de um projeto no sector de *Oil&Gas*, incorpora incertezas importantes relacionadas com rendimentos e custos. As principais incertezas prendem-se com os Volumes Recuperáveis de óleo, com as Taxas de Produção, o preço do *Brent* e com a estrutura de *Capex&Opex*. Os Volumes de óleo recuperáveis são calculados e enquadrados de acordo com o *software* geológico, *GeoEx*, que expõe uma Distribuição LogNormal estatisticamente relevante. As Taxas de Produção anuais são aleatoriamente selecionadas a partir de distribuições históricas de reservatórios semelhantes. O preço do *Brent*, é estimado num processo de Reversão à Média com difusão de Saltos, a partir do modelo Browniano Geométrico. Finalmente, a estrutura de *Capex&Opex* segue uma Distribuição Triangular, estimada por gestores de projeto experientes.

O ajuste das incertezas é anterior à modelagem subjacente das principais variáveis que afectam os *Cash Flows* na simulação de *Monte Carlo*. A simulação preserva a natureza estocástica para os *Cash Flows*, uma vez que é uma soma de variáveis aleatórias. No final do estudo, todos os *NPV's* simulados são dados numa Distribuição Densidade de Probabilidade que expressa a probabilidade do valor económico da Bacia de Peniche e Alentejo.

Palavras-chave: Incerteza, Modelização Estocástica, Avaliação de Investimentos, Simulação de Monte Carlo

Classificação JEL: Q40; C63; O22

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List of Abbreviations

A-D	Anderson-Darling
AIC	Akaike Information Criterion
BIC	Bayesian Information Criterion
BMMRJD	Brownian Motion Mean Reversing with Jumps Diffusion
BOE	Barrel of oil and equivalent
CDF	Cumulative Distribution Function
CF	Cash Flow
DCF	Discounted Cash Flows
DGGE	Direção Geral de Geologia e Energia
DPEP	Divisão para a Pesquisa e Exploração de Petróleo
E&P	Exploration and Production
EIA	Energy Information Administration
FPSO	Floating production storage and offloading
GBM	Geometric Brownian Motion
K-S	Kolmogorov-Smirnov
MLE	Maximum Likelihood Estimators
NPV	Net Present Value
O&G	Oil and Gas
OPEC	Organization of the Petroleum Exporting Countries
PDF	Probability Density Function
POS	Probability of success
PP&E	Power Plant and Equipment
PSC	Production Sharing Contract
SPE	Society of Petroleum Engineers
STOIP	Stock Tank Oil Initially in Place
USD/BBL	United States Dollars per barrel

1. Introduction

1.1. Scope

Offshore oil is formed from organic matter that has been buried, namely at the bottom of the ocean, and encased in clay, sand and crumbled rock. Thereby, different strata are formed, which are placed under high pressure and heat, forming Crude oil and natural gas. Through the porous layers of rock, the mixture of Crude oil, natural gas and saltwater slowly migrates, taking place over thousands of years. Afterwards, this mixture accumulates in the porous sandstone and limestone layers in the bedrock, where overlying impermeable layers prevent it from getting any further.

The continued growth of energy demand had motivated Oil&Gas companies to find and produce increasing quantities of hydrocarbons, which requires more than merely ramping up production from traditional sources (e.g. injector wells). As time goes by, it is relatively, easy to produce resources in new locations (e.g., the deepwaters), that were inaccessible some years ago.

White (2007) explains that within the past 15 years, deepwaters has evolved from being a “technological frontier to a strategically important component of the world’s oil industry”. In the past, Crude oil that was identified at more than 2000m of depth, was given as impossible or very difficult to extract. New drilling technologies, unlocked those worldwide recoverable oil volumes of the deepwaters regions that were still underexplored and holding considerable potential. Chakhmakhchev & Rushworth’s (2010) research points that the increasing interest began between 2005-2010, after the firsts world class discoveries of heavy oil in deepwaters, namely in Libya, Ghana, Brazil, Russia, México, China and Mozambique. These discoveries have 1P reserves estimates of around 400 billion of BOE.

Portugal is one the countries with the largest exclusively economic zones in the world and with a comparable basin within the most recent oil discoveries. In these areas relies potential that many see but few are exploring it (Petrobras, Galp Energia and Partex). Portugal’s potential in deepwater oil, is going to be the basis of this dissertation, with particularly interest in the Stochastic Evaluation as a function of the main uncertainties that an oil field are exposed.

1.2. Objectives

This study aims to propose Stochastic Evaluation for an upstream Oil&Gas project investment decision-making. The overall goal of the research is to address a dynamic evaluation applied to the identified oil prospects in Portugal. The stochastic form is assumed in the Monte Carlo Simulation to model the major uncertainties such as Recoverable Oil Volumes, Production rate, Oil prices and the Capex&Opex cost structure.

The output of this simulation will return an empirical NPV Distribution associated to a Probability Density Function.

The main activities that supported this study can be systematized as follows:

1. Review of the background knowledge concerning the Oil&Gas sector such as Major Companies and Hydrocarbon Reserves classification.
2. Review of the foundations behind the project life cycle of an Oil&Gas field with the existing Upstream Fiscal Policies.
3. Characterization of the Stochastic Project Evaluation concept.
4. Defining the Uncertainties and fitting into empirical distributions in a generic Oil&Gas evaluation model.
5. Perform the oil price forecast with the model Mean Reversing with Jumps.
6. Computing the project discount rate.
7. Establishment of the Monte Carlo Simulation.
8. Aggregate and discuss of the Empirical NPV Distribution.

1.3. Organization of the Dissertation

This dissertation is organized in 6 chapters. In Chapter 1 a brief introduction is made focusing on the scope and objectives of the presented study.

In Chapter 2 a state-of-the-art review is made on the Oil&Gas sector and it is presented some of the technical background knowledge useful in an oil project evaluation.

Chapter 3 describes the Stochastic Project Evaluation using Monte Carlo. Concerning the uncertainties of the simulation, there is also a description on the used Distributions, the Fitness measures and sensibility analysis performed.

Chapter 4 stands by the collected data and the methodology used on this research. It is subdivided into Technical and Financial data.

The detailed Stochastic Evaluation of the Alentejo and Peniche Basin is exhibit in Chapter 5. The use of the aforementioned methodology framed in the Monte Carlo Simulation discloses the NPV distribution for the oil prospects in Portugal.

Finally, Chapter 6 presents the main conclusions drawn by this dissertation, highlighting what is thought to be its main contributions. Some ideas about the possible subsequent research related to the incorporation of the stochastic form of discounting Cash Flows are also suggested.

Due to the amount of information that supports this research, some of the figures and tables were not included in the main text and were organized in Appendix. In each one, a sequential numbering was applied from Appendix 1 to Appendix 13.

2. Oil&Gas Contextualization

2.1. Industry & Fiscal Policies

The Oil&Gas industry is a very intensive capital investment sector with projects having long life cycles (e.g. 30 years), which determines the direction and course of billions of dollars every year. According to Brown (2011), the upstream industry spending in 2011, reached to a new high record with an annual investment around US\$450 billion which represented a 7% growth compared to 2010. It is also stated that 2013 can reach to a new record if this growth is maintained, reaching US\$500 billion.

With a US\$22billion spent in 2011, ExxonMobil has the largest capital spend of all Oil&Gas companies. Chevron and Petrobras will be running close in 2012 and 2013. Chevron total upstream spending could exceed US\$25billion by 2013. Several of Petrobras' giant oil discoveries in the Santos Basin should be progressing through development in a similar timeframe and its annual capital expenditure could reach US\$28billion by 2014.

The abovementioned underlying investment has its genesis in the interaction of the companies that compose Oil&Gas value chain (Figure 1). The upstream focus is finding and extracting minerals and resources from the soil. Midstream companies are determinant in transporting Oil&Gas resources via ships or pipelines that carry these commodities over great distances. Downstream companies are in charge for the refining phase according to market commodities and turning them into final products (e.g. jet fuel, automobile gasoline, diesel, naphtha, etc.). Oil Field Services are companies specialized on providing services to companies that explore, produce, refine or sell hydrocarbon products. They might maintain their oil fields, repair broken spots, upgrade the technology, or even provide security for workers. Finally, there are Integrated Majors that do everything described in above but in different proportions. The most common combination is to focus on upstream and downstream, and then leave the rest to dedicated specialists.



Figure 1 - Upstream (left), Midstream (middle) and Downstream (right).

The sector can be very distinct of the regular business in many practices. One of the main differences is in Prices, that can't be controlled. Revenue can be quite an uncertainty in a project evaluation with such long Life Cycles because Oil&Gas companies follow the commodity price cycles. Investing in energy or mining company

is almost like investing in the underlying commodity, which means being comfortable with giant price swings.

The large spending in an Oil&Gas project has also a significant full stop regarding the oil probability of success (POS). The POS signals the probability of finding and extracting commercially and technically the oil resources. In deepwaters, the range of the POS, may diverge between 5% and 30%, when an exploratory well is being drilled, which means that the remaining probability may conduct to a dry well or the oil may not be economically and/or technically produced.

In the scope of the petroleum industry, the Political aspects have rather significant relevance. Countries around the world have a great impact on the industry's players, not only for being the owners of hydrocarbon (i.e. oil and gas) resources but also for controlling the hydrocarbon reserves. Thereby, the governments have the right to sell concessions to different companies, which grant an exclusive right for exploration and production of oil within a specified geographical area at a given time horizon.

Moreover, and according to EIA¹ (2008), the governments are also allowed to favor national oil companies and exclude foreign ones from the process. In this context, and in addition to individual countries, there is an intergovernmental organization consisting of the twelve world's major oil-exporting nations: the Organization of the Petroleum Exporting Countries (OPEC) which works to coordinate and unify the petroleum policies of the member countries. Today's OPEC nations were the ones to actually nationalize oil production in their countries and take over most of the business from big oil corporations.

Contributing with the available data of the BP Statistical Review, 2008 which attributes 75.5% of the world's oil resources to OPEC's control. Taking into account the oil depletion elsewhere on the globe, its geopolitical influence is only likely to enhance considering that these oil producing countries own significant bargaining power. In fact, Kvalegag (2009) states that OPEC nations own two thirds of the world's oil reserves and represents about 35 % of the world's oil production, with oil that is one of the cheapest to produce. OPEC has been known to possess great control over oil price levels, which corroborates the meaningful bargaining power of this organization when it comes to granting oil-fields-concession rights to international companies. An evident factor of uncertainty is the oil price which is highly dependent of OPEC supply and control decisions.

Nowadays, Oil&Gas exploration and production, require the concomitant consideration of a variety of concerns namely, political, economic, social, technical, environmental and legal (Figure 2).

¹ Energy Information Administration publishes Annual Energy Outlook 2008

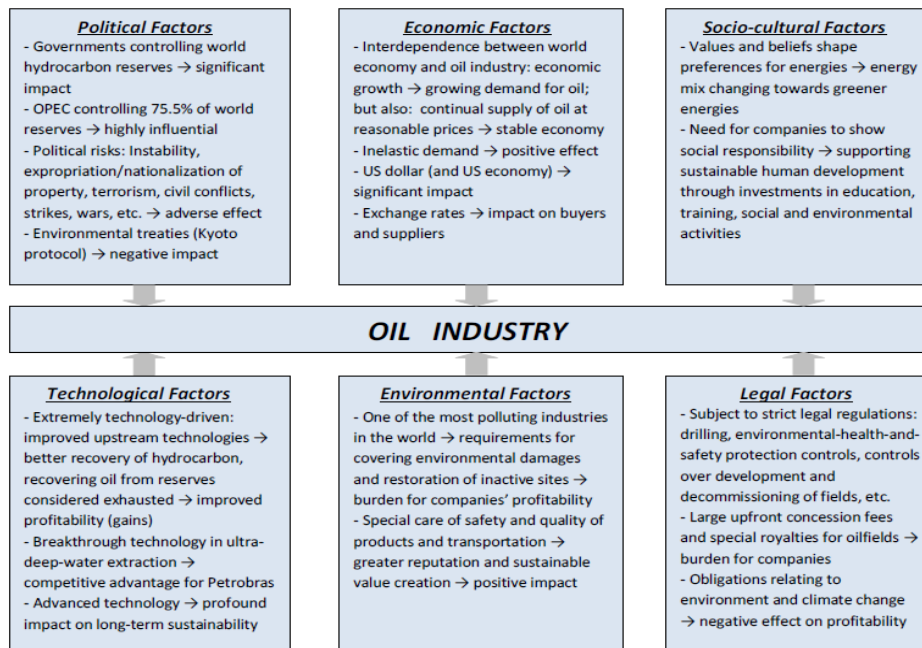


Figure 2 - Summary of macro-environment-PESTEL factors (adapted from Talevski & Lima, 2009).

Despite the abovementioned characteristic, another important relies on the Balance Sheet of such companies. Banks, insurance firms and E&P companies are very dependent on its balance sheet. The reserves contain its most important asset since it's the source of future revenues and profits. For a regular company, assets tend to move up and to the right. Items such as PP&E, inventory or accounts receivable, increase as the company grows. However, as an Oil&Gas company produces more and more and earns revenue, its assets decline because they use up all those valuable resources found in the ground. Finally, Oil&Gas companies have specific accounting with different line items. There are two different accounting methodologies: full cost and successful efforts.

Fiscal Policies

According to Tordo (2007), from World Bank, the Global Market for Oil&Gas Exploration gathers attention from Governments and Companies. Governments intend to attract capital and technology to develop their hydrocarbon sector. Several countries have used favorable taxation in Oil&Gas, to support the development of the sector in addition to relevant sector reforms and for a government policy signal to the market, intended to increase investment decisions.

The challenge of an efficient fiscal system is to induce maximum effort from the Oil&Gas companies while ensuring that the host government is adequately compensated. In order to devise and apply the appropriate policies, strategies and tactics, each must assess its position in the global marketplace and evaluate its particular situation, boundary conditions, concerns and objectives. Companies look for investment opportunities that suit their corporate strategies and risk-reward profiles. The initial decision to invest and the resulting allocation of revenue and benefits are greatly influenced by the content of existing legal arrangements and fiscal policies.

The author Ibrahim (2005), emphasizes the two big fiscal systems in the upstream industry. Despite the inherent differences of some taxes, the fundamental change between them comes from different attitudes towards the ownership of mineral resources:

- **A Concessionary Contract** allows a private ownership of mineral resources, like in Brazil. In this case, there is a title transfer of the minerals to a company or consortium. Afterwards, the company is subject of Royalties and others tax payments. All contracts established are negotiable and include different tax rates and different underlying calculation, but, usually the taxes are the ones present in the bellow Table 1.

Table 1 – Fiscal Description of the Concessionary Contract

Concessionary Contract	Description
Signature Bonus	Amount paid by the company that won the auction of the concession. Size of payment reflects expected return from investment with risks.
Acreage Rentals	Annual tax due by the company for the exploration and production depending on the field size and geological characteristics.
Royalties	Value owed to the government upon extraction of the mineral despite taking the ownership of all minerals found on the acreage.
Special Participation Tax	Varies according to well location (offshore/onshore), years of production taking into account the profitability of the oil field.
Income Tax	Tax due to the Net Income generated
= Government Take	Is the sum of the total taxes paid to the Government

- **A Contractual Contract** retains the Government the ownership of minerals. Oil&Gas companies have the right to receive a share of the production or revenues from the sale of hydrocarbons (Table 2). This is outlined in the so-called Production Sharing Contract (PSC), like in Angola. Host Governments are usually represented by national companies, an oil ministry or both. The term contractor is used to denote the international company operating the oil or gas field. The contractor then funds the required activities and is eventually reimbursed out of a dedicated share of the production.

Table 2 - Fiscal Description of the Contractual Contract

Contractual Contract	Description
Signature Bonus	Amount paid by the company that won the auction of the concession. Size of payment reflects expected return from investment with risks.
Profit Oil	Profit oil is production shared between the state and the contractor depending on the ROR (Rate of Return).
Petroleum Revenue Tax	Varies according to well location (offshore/onshore), years of production and is a tax that takes into account the profitability of the oil field after initiating the production.
Income Tax	Tax due to the Net Income generated
= Government Take	Is the sum of the total taxes paid to the Government

2.2. Life Cycle of an Field Project

Quoting Tordo (2007) from the World Bank, the stages of a typical Oil&Gas project can be defined by the stepwise description and summary depicted in Figure 3.

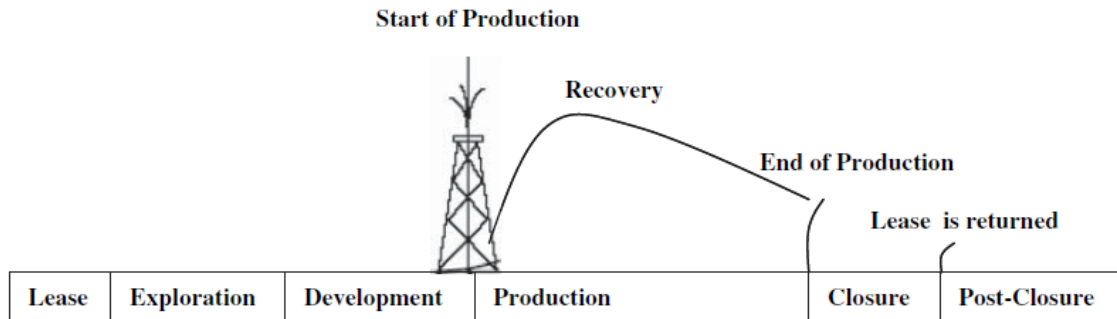


Figure 3 - The Project Cycle (adapted from Tordo, 2007)

- **Licensing:** the government grants a license (lease, or block area) or enters into a contractual arrangement with an oil company or group of oil companies to explore and develop a field with or without transferring the ownership of the mineral resources. This is the period where auctions for blocks and signature bonus occur.
- **Exploration:** after acquiring the rights, the oil company carries out geological and geophysical surveys such as seismic surveys and core borings. The data so acquired are processed and interpreted and, if a play appears promising, exploratory drilling is carried out. Depending on the contracts celebrated, the consortium may be obliged to drill a minimum work commitment of wells. The location of the well induces Rig, Drill ship, Semisubmersible, Jack-up, or a FPSO that will be used.
- **Appraisal:** if hydrocarbons are discovered, further delineation wells are drilled to establish the amount of recoverable oil, production mechanism, and structure type. Development planning and feasibility studies are performed, and the preliminary development plan is used to estimate the development costs.
- **Development:** if the appraisal wells are favorable and the decision is made to proceed, then the next stage of development planning commences using site-specific geotechnical and environmental data. Once the design plan has been selected and approved, contractors are invited to bid for tender. Normally, after approval of the environmental impact assessment by the relevant government entity, development drilling is carried out and the necessary production and transportation facilities are built.
- **Production:** once the wells are completed and the facilities are commissioned, production starts. Workovers must be carried out periodically to ensure the continued productivity of the wells, and secondary and/or tertiary recovery may be used to enhance productivity at a later time.

- **Abandonment:** at the end of the useful life of the field, which for most structures occurs when the production cost of the facility is equal to the production revenue (the so-called “economic limit”), a decision is made to abandon. For a successful removal, operators generally begin planning one or two years prior to the planned date of decommissioning (or earlier depending on the complexity of the operation).

2.3.Global Hydrocarbon Reserves

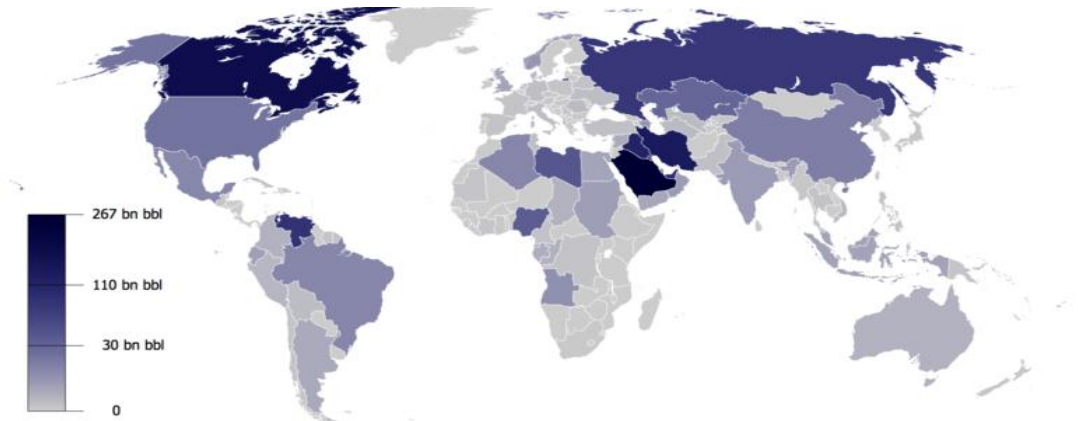


Figure 4 – Proven Oil Reserves as published by the CIA Factbook (2009)

The available data from OPEC at the beginning of 2011, the highest proven oil reserves (including non-conventional oil deposits) are in Venezuela (20% of global reserves), Saudi Arabia (18% of global reserves), Canada (13% of global reserves) and Iran (9%) – Figure 4.

Predictions and calculations about total amount of extractable reserves and Production rates are important inputs in valuations of Oil&Gas field developments. It should be stressed that wrong inputs may lead to bad investment decisions and unprofitable projects (Kvalegag, 2009). Nevertheless, an evident factor of uncertainty is the exact amount of Recoverable Oil Volumes of the petroleum reserves in Oil&Gas fields.

Petroleum reserves are firstly forecasted through seismic surveys and exploration drilling, being the calculations and predictions made on the basis of the results. However, errors and uncertainties may be associated to these calculations and thus the actual amount of extractable volumes may turn out to be more or less than predicted. Moreover, predictions about Production rates are subject to the same uncertainty as the characteristics of petroleum Recoverable Oil Volumes. This topic will be further discussed in Section 2.4.

In this context, and according to the definitions approved by the Society of Petroleum Engineers (SPE) and the World Petroleum Council (WPC), Reserves are classified as Proven, Probable and Possible, depending upon the likelihood of their recovery (Menabde, 2008). Contingent and Prospective resources are respectively two more gross categories estimates for oil resources (Figure 5).

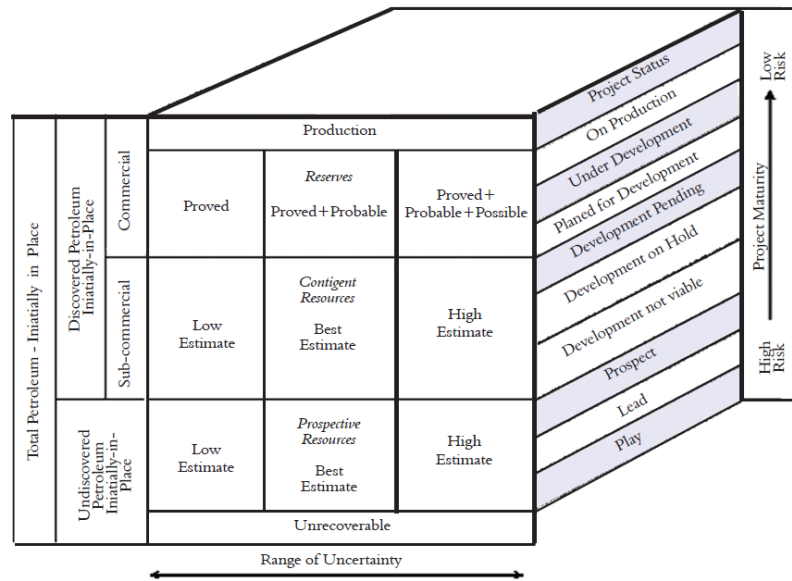


Figure 5 - Petroleum Resource Classification Scheme (adapted from Suslick, 2009)

- **Proven (1P) reserves:** by analysis of geologic and engineering data, these reserves can be estimated with reasonable certainty to be from a present time, commercially recoverable from known Recoverable Oil Volumes and under current economic conditions, operating methods and government regulations. If deterministic methods are used, the term "reasonable certainty" is intended to express a high degree of confidence that the quantities will be recovered. In the case of probabilistic methods are used, there is at least a 90% probability that reserves recovered will be equal or exceed the 1P estimate (90% confidence or P90). In this context, the term "proven" refers to the actual quantities of oil and not only to the productivity of the well or the reservoir. Reserves can be classified as proven if the facilities to process and transport it to the market are operational at the time of the estimate, or if there is a reasonable expectation that these facilities will be created.
- **Proven plus Probable (2P) reserves:** these are the sum of proven (1P) and probable reserves and are a category of unproved reserves. The unproved reserves are based on geologic or engineering data, similar to that used in estimates of proven reserves. However, technical, contractual, economic, or regulatory uncertainties preclude such reserves being classified as proven. Probable reserves are those quantities of hydrocarbons which, by analysis of geological and engineering data, are less likely to be recovered than proven reserves but more likely than possible reserves. In the case of probabilistic methods are used, there is at least a 50% probability (or P50) that reserves recovered will exceed the estimate of 2P.
- **Proven, Probable plus Possible (3P) reserves:** these are the sum of proven, probable (2P) and possible reserves and are a category of unproved reserves, similar to the probable reserves. Possible reserves have a lower probability of recovery than probable reserves. These are those reserves that, to a low degree of certainty (10% confidence or P10), are recoverable. Thus, there is relatively high risk associated with these reserves.

In addition to the abovementioned definitions of reserves, a further category exists on which for whatever reason are not, deemed commercially recoverable at the present time, namely **Contingent resources (3C)**. Contingent resources may be of a significant size, but still have constraints to development. This may occur for several reasons, such as those related to the maturity of the project, to the technology (it is necessary to develop and test new technology that allows commercially exploit of the estimated quantities), or the market (sales contracts are not yet in place or need to install infrastructure to bring the product to the customer). The quantities in this class are not considered as reserves.

Finally, **Prospective resources** refer to quantities of hydrocarbons estimates, at a present time, to be potentially recoverable from unknown Recoverable Oil Volumes by the application of future development projects. The estimate of the volumes of a certain prospect is subject to commercial and technological uncertainties. The quantities in this class are not classified reserves or contingent resources.

2.4.Recoverable Oil Volumes & Production Rates

Several uncertainties are always associated to the calculation of Recoverable Oil Volumes estimates of oil and gas. In this context, higher uncertainties conduce to a wider range of input values in the distribution. The reliability depends on the quality and amount of the available data on which the technical studies are based on. As seen in the previously section, the Reserve, Contingent and Prospect estimates can be framed in one of the stage of the project's life cycle, presented in Section 2.3.

The technical knowledge behind those estimates has the same input assumptions:

- **Stock Tank Oil Initially In Place (STOIIP)** is the product of Gross Rock Volume (GRV) with the net/gross ratio, porosity (ϕ), oil saturation (S_o) and oil expansion factor ($FVF = 1/B_o$). The same equation applies to the gas scenario, replacing oil saturation and expansion factor with gas saturation (S_g) and formation factor (B_g). Thus, the expression (1) used for this calculation is given by:

$$STOIIP = GRV \cdot \frac{Net}{Gross} \cdot \phi \cdot S_o \cdot FVF \quad (1)$$

- **Recoverable Oil Volumes** is the product of STOIIP by the recovery factor (R_f), Equation (2). Based on this calculation it is possible to determine the expectable amount of extractable volumes that actually can be recovered through production. When all the different segments have their own volumes calculated, with or without dependencies between them, they are all combined into one prospect.

$$Recoverable\ Volumes = STOIIP \cdot R_f \quad (2)$$

The Recoverable Oil Volumes estimates are described in Section 4.1 and 4.2.

Production Rate

According to Dake (1998), the Oil Production rate occurs in a predictable manner based on geological circumstances, governmental policies, and engineering practices. The shape of the decline curve is conditional whether it is being considered to a certain Basin, to a set of fields, to certain the number of wells, etc.

This shape will determine the production occurrence and the subsequently the potential Revenue in a producing field. Therefore, there are several yearly uncertainties associated with the establishment of production's forecast and thus the estimates of reserves mentioned in Section 2.3.

The commons process of estimating the quantities of recoverable oil per annum/day relies on the use of certain generally accepted analytical procedures.

These analytical procedures fall into four broad categories or methods to estimate reserves:

1. Volumetric-based methods;
2. Performance history-based methods;
3. Mathematical models;
4. Analogy to other Recoverable Oil Volumes.

The abovementioned categories may be used singularly or in combination by the reserve evaluator, who must select the method or combination of methods which in their professional judgment is most appropriate. This decision is based on the nature and amount of reliable geoscience and engineering data available at the time, the established or anticipated performance characteristics of the reservoir being evaluated and the stage of development or producing maturity of the property.

In this context, it is assumed that factors which influenced the performance trends of wells or fields in the past will continue to govern their performance in the same manner. In other words, it is expected that what has occurred will continue to occur. This assumption allows extrapolating future trends from past performance.

Decline curve analysis is a traditional means of predicting oil well production behavior at different points in time based on its past production history, being the most common method of forecasting Oil&Gas production. The production rate depends upon many factors, such as, reservoir drive mechanism, pressure behavior, rock and fluid properties, production problems (e.g. existence of sand, scale or asphaltenes) and work overs. Moreover, contractual issues regarding the well duration or well performance can be also decisive in the establishment of a production limit.

Therefore, despite being an important input for petroleum economics analysis, it should be stressed that the plot production decline curves involves several uncertainties that directly affect the expected estimates of annual Recoverable Oil Volumes. Additionally in Section 4.1 and 4.2 there is the description of the methodology used in this dissertation.

2.5. Mean Reversing with Jumps Model

The Mean Reversion Model with Jumps is a model generated by Geometric Brownian Motion, GBM, from historical time series (i.e. Oil Prices) with a jump diffusion process. The GBM component of the model, declares that changes in the Oil Price series in one unit of time by a certain amount is Normally Distributed with mean, μ , and variance, σ^2 .

The Normal Distribution can be a choice for a lot of variables because a variable Y, is additively affected by many independent random variables accordingly to the Central Limit Theorem.

The Poisson-Gaussian model of Mean Reversion, also known as Mean Reversion with Jumps diffusion, was proposed Vasicek (1977) by the formula of Eq.(3) and Eq. (4):

$$Y_t = \left(\mu - \frac{\sigma^2}{2} \right) t + K_t \mu_J + N_t \sqrt{\sigma^2 t + \sigma_J^2 K_t} \quad (3)$$

$$r_t = \left(\mu - \frac{\sigma^2}{2} \right) t + N_t \sigma \sqrt{t} \quad (4)$$

Where:

μ – Mean

σ - Standard Deviation

μ_J - Mean jump size

σ_J - Volatility jump size

r_t – Return in time t

N_t - sample from the standard Normal Distribution $\sim N(\mu, \sigma^2)$ with $t \geq 0$,

K_t - sample from the Poisson (λt) Distribution with $t \geq 0$,

The researcher Vose (2010) proved that a regular commodity price follows a Mean Reversing Process if it has a tendency to return to some average value over time. This means that investors may be able to forecast future returns better by using information on past returns to determine the level of reversion to the long-term trend path.

The Jump diffusion component is a refinement intended to denote for sudden shocks to the variable that occur randomly in time reflecting arrival of information. There are two types of information; the normal news with smooth variation in the oil prices captured in the GBM and the abnormal news held with jumps in the prices. The abovementioned model intends to recognize that beyond the usual background randomness of a time series variable there will be events or abnormal news that have a much larger impact on the variable.

The frequency of the jumps, is usually modeled as a Poisson Distribution with intensity λ , in some time increment, t, hence it will be modeled as a Poisson ($\lambda ; t$) jumps. The jump size for r_t is usually modeled as Normal (μ_J, σ_J) for mathematical convenience and ease of estimating the parameters. The parameter, λ , estimation

cannot be too high (i.e. $\lambda > 0.2$), because the Poisson jumps are meant to be rare events, not form part of each period's volatility.

The jump process is independent from the Wiener increment, from the continuous process. In this context, the Poisson process is independent of the Mean Reversing process. The process assumes that there is the same probability of jump-up and jump-down, that is, with a frequency 1/2 occur a jump up or with a frequency 1/2 occurs jump down (Figure 6). The equal probability for jump up or jump down, relates to the expected jump size as being zero, since the expected value is independent of the jump. It should be stressed that the jump diffusion adds fatter tails to the Normal Distribution from the mean-reversion component.

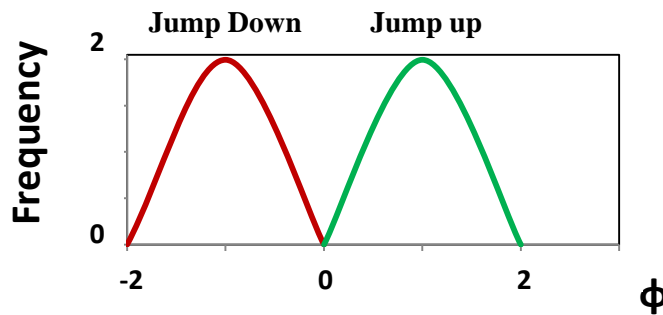


Figure 6 - Frequency of Jumps in Poisson Process

In concrete terms, the smooth variation is modeled by a mean-reversion process in a continuous process, whereas the jumps are modeled with a Poisson process in a discrete time process. Mean Reversion with Jumps reflects a revert force over the variable pulling towards to an equilibrium level according to μ , the drift parameter and σ , volatility parameter.

It is assumed that shocks occur according to a Poisson process with parameter λ , and that at such times, there is a jump in the process that is normally distributed with parameters μ_j and σ_j . This model has more economic logic and considers large variation of the underlying variable due the arrival of abnormal rare news

The velocity of the reversion process is given by the parameter λ . Jumps are represented by the term K_t , which most of time is zero and sometimes occur jumps of uncertain size λ and with arrival rate t .

3. Stochastic Project Evaluation Background

3.1. Definition

A deterministic evaluation through DCF can be easy and straightforward. However, deterministic variables as inputs contribute to a static evaluation that might underestimate uncertainties and being one of the main causes of a project failure (Erdem, 2008). Risk assessment assumes a vital role for economical sustainability of one project.

Wolf & Belaid (2009) studies classify Stochastic Project Evaluation as an improved estimation of Cash Flows that can be obtained by Monte Carlo Simulation. It is classified as an economic method in which one or more variables within the model are treated as stochastic or random. The random variables are usually constrained by distributions assumptions that were subject of historical data and/or fitting tests. The stochastic nature is also attributed to Cash Flows since it is a sum of random variables. The NPV estimate, will be associated to a probability distribution function as a result of Monte Carlo Simulation.

Figure 7, addresses an illustration of the empirical NPV distribution as a function of stochastic variables. In this case the $NPV = f(X, Y, Z)$. In this dissertation, the assumption of this distribution of the underlying variable is was justified base on fitting tests or subsequent work prof. By the end of the day, it is possible to address an NPV estimate and its probability of occurrence.

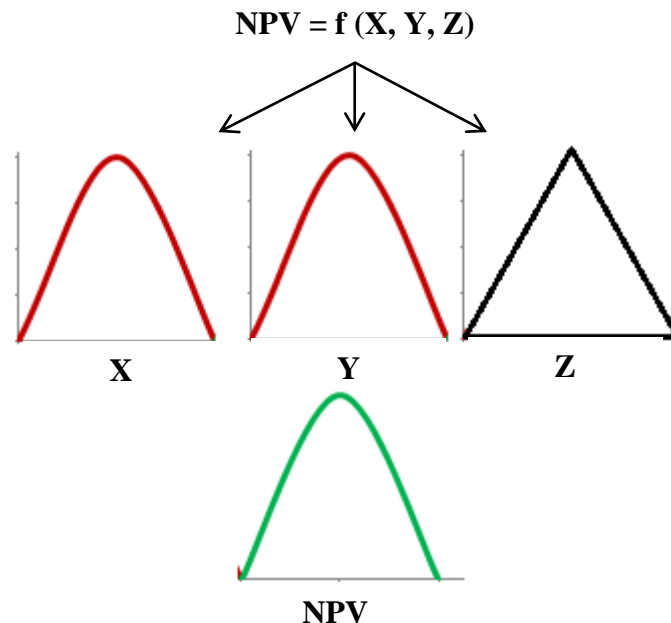


Figure 7 - Stochastic Project Evaluation (adapted from Belaid and Wolf (2009))

3.2. Monte Carlo Simulation

Talevski and Lima (2009) defines Monte Carlo Simulation as a problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables.

According to Holtan (2002), Monte Carlo simulations fast become the technology of choice for evaluating and analyzing assets. Two of the main virtues of simulation are flexibility and simplicity. Firstly it's flexible, because there isn't no uncertainty that cannot be modeled and simple for the easy implementation in a *MS Excel* spreadsheet. During the simulation, many samples paths of the uncertain Cash Flows components are generated.

A single sample path represents one possible set of values for a particular component for a determined time period. It is typical to name iteration, for a single set of sample paths. Calculating NPV using Monte Carlo Simulation is the same as generating a number of iterations which each one represents a different set of sample paths for uncertainties which originate an associated NPV.

Monte Carlo Simulation tends to follow a particular pattern:

1. Defining a domain of possible inputs variables;
2. Generate inputs randomly from a probability distribution over the domain;
3. Perform deterministic computation on the inputs;
4. Aggregate the results in a stochastic form.

Galli (1999), stated that Monte Carlo simulation is a natural extension of the standard NPV base case by allowing for that fact that variables are not known with certainty. In fact, any evaluation or risk assessment is uncertain. Standard statistical distributions such as the Normal, LogNormal, Poisson and the Triangular Distributions may be used to describe the input parameters.

Collecting historical representative data from the established key uncertainties and fitting them, into a distribution function is one of the primordial steps behind Monte Carlo Simulation. The principle behind fitting distributions to data, is to achieve the type of distribution (e.g. Normal, LogNormal, Triangular, Poisson) and the associated value of the parameters (e.g. mean, variance, minimum, maximum, mode and lambda) that leads to the highest statistical significance probability of producing forecasts from the observed data.

In probability theory, a Probability Density Function (PDF) characterizes the probability of each value occurring from a range of possible values. The density at a point refers to the probability that the variable will have a value in a narrow range about that point. The Cumulative Distribution Function (CDF) describes the probability that a real valued random variable X with a given probability distribution will be found at a value less than or equal to a certain value. Intuitively, it is the "area so far" function of the probability distribution.

The parameters of a probability density function can be computed according to Maximum Likelihood method proposed by Law and Kelton in (1982). For any Probability Density Distribution $f(x)$ with a parameter α and with a correspondent observation data (X_i) an expression called the Likelihood can be defined as:

$$L = \prod_{i=1}^n f(X_i ; \alpha) \tag{5}$$

$$\frac{dL}{d\alpha} \tag{6}$$

To find the Maximum Likelihood Estimator (MLE), use (X) to maximize L with respect to α . Then solve equation 3 for α .

Distributions

i) Normal Distribution

The Normal (or Gaussian) Distribution was introduced by the mathematic Karl Friedrich Gauss (1809) and is the most used distribution in statistics.

Normal Distribution - Probability Density Function (PDF) that defines as:

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < +\infty \tag{7}$$

Normal Distribution - Cumulative Distribution Function (CDF) defined as:

$$\frac{1}{2} \left[1 + e \left(\frac{(x - \mu)}{\sqrt{2\sigma^2}} \right) \right] \tag{8}$$

Where:

- μ : Mean
- δ : Standard deviation
- e : Base of the natural logarithm
- π : Constant pi

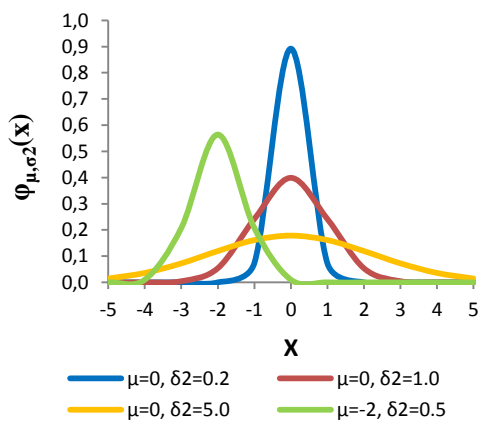


Figure 8 - Normal Probability Density Function (PDF)

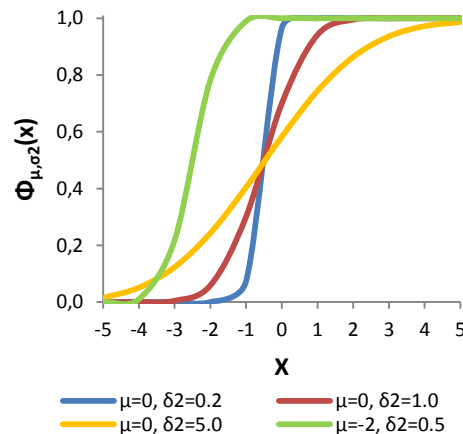


Figure 9 - Normal Cumulative Distribution Function (CDF)

ii) LogNormal Distribution

The LogNormal Distribution, introduced by Hazen (1930), is obtained by applying the Normal Distribution to the logarithmic transformed sample. The logarithmic transformation reduces the skewness of the sample, thus making reasonable the application of the Normal law in cases where the skewness of the sample is too high.

The Probability Density Function (PDF) defined as:

$$\frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \tag{9}$$

The Cumulative Distribution Function (CDF) defined as:

$$\frac{1}{2} + \frac{1}{2} \left[e \left(\frac{\ln x - \mu}{\sqrt{2\sigma^2}} \right) \right] \tag{10}$$

Where:

- μ : Mean
- δ : Standard deviation
- e : Base of the natural logarithm
- π : Constant pi

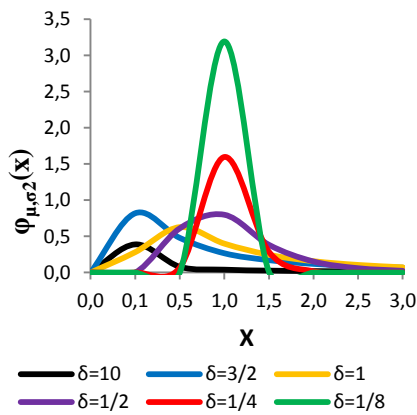


Figure 10 - LogNormal Probability Density Function (PDF)

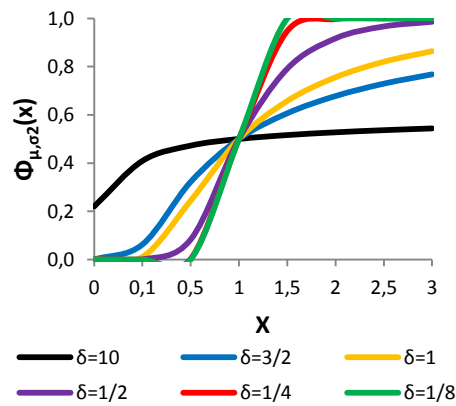


Figure 11 - LogNormal Cumulative Distribution Function (CDF)

iii) Triangular Distribution

The Triangular Distribution proposed by Achenwall (1757) is Continuous Probability Distribution described by a minimum value (a) to a maximum value (b) and most likely (c) value (mode) where $a < b$ and $a \leq c \leq b$. The distribution is used to describe what it is believed about random variables and is typically used when the distribution is only vaguely known through its extremes and its mode.

Triangular Distribution – Probability Density Function (PDF) defined as:

$$f(x) = \begin{cases} 0, & \text{for } x < a, \\ \frac{2(x-a)}{(b-a)(c-a)}, & \text{for } a < x \leq c, \\ \frac{2(b-x)}{(b-a)(b-c)}, & \text{for } c < x \leq b, \\ 0, & \text{for } b < x. \end{cases} \quad (11)$$

Triangular Distribution – Cumulative Distribution Function (CDF) defined as:

$$F(x) = \begin{cases} 0, & \text{for } x < a, \\ \frac{(x-a)^2}{(b-a)(c-a)}, & \text{for } a < x \leq c, \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, & \text{for } c < x \leq b, \\ 1, & \text{for } b < x. \end{cases} \quad (12)$$

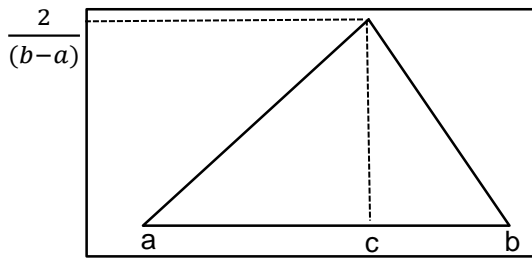


Figure 12 - Triangular Probability Density Function (PDF)

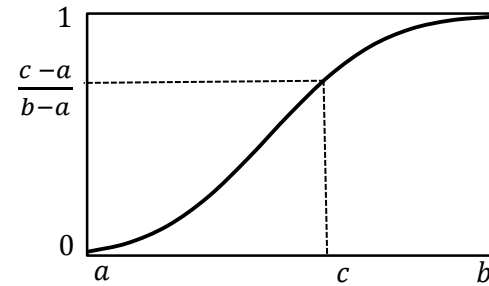


Figure 13 – Triangular Cumulative Distribution Function (CDF)

iv) Poisson Distribution

Finally, the Poisson Distribution was introduced by Poisson (1837) as binominal distribution limit. It is a common distribution for modeling the frequency of events during a fixed time interval. When the Poisson Distribution are related to a time interval t , then the expected number of events during this time interval is $\mu = \lambda t$. This distribution has a single parameter μ which is the expected number of events during the interval.

Poisson Distribution – Probability Density Function (PDF) defined as:

$$\frac{e^{-\mu} \lambda t^x}{x!}, x = 0, 1, 2, \dots \quad (13)$$

A Poisson process is a time series associated with Bernoulli trials. Trials with two possible outcomes: the event occurs (success) or the events does not occur (failure). This process is related to parameter $\lambda > 0$, the expected number of events per unit time which is called intensity of a Poisson process.

$$\lambda e^{-\lambda t}, \lambda > 0, t > 0$$

Observing Figure 14, the horizontal axis is the index x , the number of occurrences. The function is only defined at integer values of x .

Poisson Distribution – Cumulative Distribution Function (CDF) defined as:

$$e^{-\lambda} \sum_{i=0}^x \frac{\lambda^i}{i!} \tag{14}$$

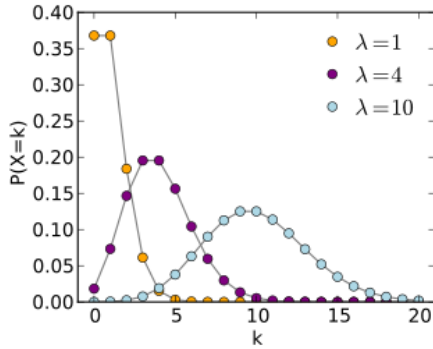


Figure 14- Poisson Probability Density Function (PDF)

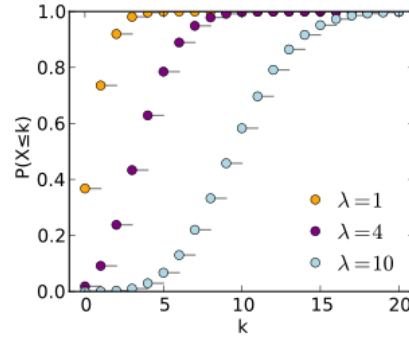


Figure 15 - Poisson Cumulative Distribution Function (CDF)

Noting Figure 15, the horizontal axis is the index x , the number of occurrences. The CDF is discontinuous at the integers of k and flat everywhere else because a variable that is Poisson distributed only takes on integer values.

Distribution Fitness

There are many types of distributions and there are a few of them that can be quite suitable to a certain data but having useful differences in the models underlying them. A visual comparison may be a good indicator for a start, though one should appreciate that the data pattern, particularly for small data sets, will not usually look like the same pattern one would see if the dataset was large. Thus, one usually tries to fit several types of distributions to the data set and then compare how well they fit the data.

It is important to consider whether the properties of the fitted distribution, particularly if the range and any skewness are appropriate. Jankauskas and McLafferty (1995) formally states that the goodness-of-fit is defined as the probability of the data given the parameters previously estimated in processes as Maximum Likelihood estimators. There are two statistics tests proposed by the authors used to the fitness accuracy:

- i) Kolmogorov-Smirnov Test
- ii) Anderson-Darling Test

The information criteria are used rank the goodness-of-fit that are statistically significant by the abovementioned tests. Vose (2010) uses the following information criteria's:

- iii) AIC - Akaike information criterion
- iv) BIC - Bayesian information criterion

The fitness accuracy proposed by the Kolmogorov-Smirnov test does not depend on the number of intervals, which makes it powerful than the Chi-Square test. A weakness of the Kolmogorov-Smirnov test is that it does not detect tail discrepancies.

The Kolmogorov-Smirnov statistic defined in Law and Kelton (1982), is:

$$D_n = \sup[|F_n(x) - \hat{F}(x)|] \quad (15)$$

Where:

- n – total number of data points;
- $\hat{F}(x)$ – the hypothesized distribution;
- $F_n(x) = \frac{N_x}{n}$;
- N_x – the number of X_i 's less than x .

The Kolmogorov-Smirnov interpretation:

$$\begin{cases} H_0: \text{The Distribution is a good fit} \\ H_1: \text{The Distribution is not a good fit} \end{cases}$$

The Anderson-Darling Test is very similar to the Kolmogorov-Smirnov Test, but it places more emphasis on extreme tail values. It does not depend on the number of intervals. In other words, The A-D test value is simply the average squared difference between the empirical cumulative function and the fitted cumulative function, with a special weighting designed to accentuate the tails of the distribution.

The Anderson-Darling Statistic used by BesFit, as defined in Anderson and Darling (1954) is:

$$A_n^2 = n \int_{-\infty}^{+\infty} [F_n(x) - \hat{F}(x)]^2 \Psi(x) \hat{f}(x) dx \quad (16)$$

Where:

- $\Psi = \frac{1}{\hat{F}(x)[1-\hat{F}(x)]}$;
- $\hat{f}(x)$ – the hypothesized density function;
- $\hat{F}(x)$ – the hypothesized distribution function;
- $F_n(x) = \frac{N_x}{n}$;
- N_x – the number of X_i 's less than x .

The Anderson-Darling interpretation:

$$\begin{cases} H_0: \text{The Distribution is a good fit} \\ H_1: \text{The Distribution is not a good fit} \end{cases}$$

Ranking of Distributions

After having statistically significant parameters of an underlying distribution, there is the need of ranking the goodness-of-fit. Vose (2010) recent publication, mentions the Kolmogorov-Smirnov and Anderson-Darling fit statistics, as technically inappropriate method for distribution ranking of fitting data. They are limited if there is the need to precise observations and incorporate censored, truncated or binned data.

It is possible to rank each fitted distribution by the information criterion of choice. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are the most used to compare different fitted models against each other. The analysis is the following, as the lower the value of the information criteria, the better for the fit. The reasoning behind both information criteria statistics is that, the better model is the one that explains the data well with a minimum number of free parameters.

The @Risk add-in software uses the statistics based in the Log-Likelihood information criteria in truncated, censored and binned data. Realistically, most of the time, fitting a continuous distribution to a set of precise observations and use statistical measures of fit called information criteria. These statistics are based on the computation of the log likelihood of the fitted distribution when producing the set of observations. This means that one can use maximum likelihood, as the fitting method and be consistent with the goodness of fit statistic. If the information criteria penalize distributions with greater number of parameters, and thus help avoid the over-fitting problem.

Akaike information criterion - AIC

$$AIC = \left(\frac{n-2k+2}{n-k+1} \right) - 2\ln[L_{max}] \quad (17)$$

The AIC (Akaike 1974) is the less strict of the two proposed, in penalizing the loss of degrees of freedom.

Bayesian information criterion - BIC

$$BIC = k\ln[n] - 2\ln[L_{max}] \quad (18)$$

The BIC (Schwarz, 1978) is the strictest in penalizing loss of degree of freedom by having more parameters.

Where:

- n - number of observations (e.g. data values, frequencies);
- k - number of parameters to be estimated;
- L_{max} - the maximized value of the log-Likelihood for the estimated model.

Uncertainties Sensibility

When creating a stochastic economic model, it is important to focus on the Uncertainties variables that most impact CF's. Uncertainties with minor impact on profit, will invariably have minor impact on the project value. Trying to model all uncertainties, will inevitably bias the model and becoming it unrealistic. The main focus, should be addressing a stochastic nature to uncertainties that most affect the project value and assume a deterministic form for minor variables which impact can be negligible when comparing to the first ones. These might involve the use of the methodology behind the model structure or could be related to the actual values that have been used to populate the model. Knull (2007), suggests the Tornado Sensibility Analysis for Uncertainties, in order to determine the variables with the greatest influence in a project evaluation.

Once more, this methodology consists on testing which variables have the greatest or lowest influence on the model's results. Each variable in the model is assumed to occur by its extreme values in *ceteris paribus* assumption, regarding other variables. Subsequently, all other input values are set fixed in a base case values.

In this context, the extreme input value simulated is used in the input variable cell recalculating the NPV estimate. The same steps are repeated for other extreme input value for each input variable. As a result, Tornado Chart shows a ranking of the single factor sensitivity analysis (Figure 16).

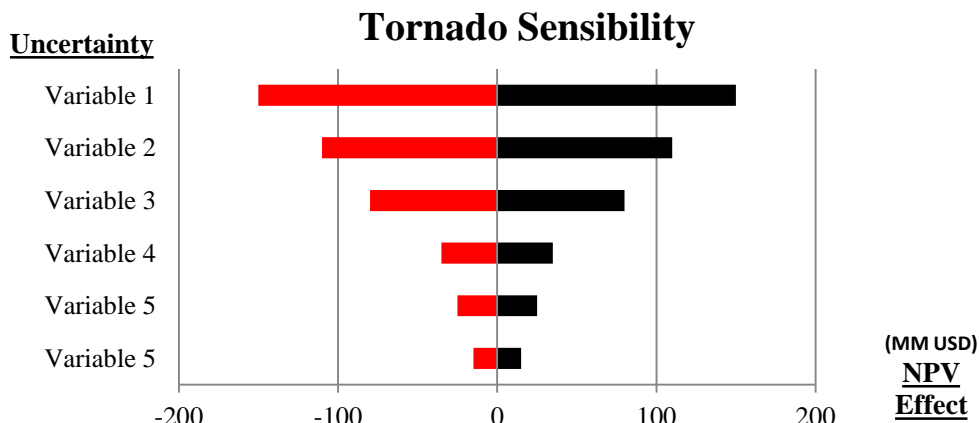


Figure 16 - Tornado Analysis chart example

The NPV ranking effect by variable is achieved after N simulations being quantitatively visible as the most influent variables presenting the largest bars. In this example, extreme values from variables that represent a negative effect on NPV are represented in red versus extreme values that represent a positive effect, in black. For instance, a lower extreme value in the variable “Sales”, may be represented in the red bar, since it decreases NPV estimate.

3.3. Discount Rate

Back in the 60's, Sharpe (1964), Lintner (1965) and Mossin (1966) were the developers of one of the most relevant achievements in the financial world, the Capital Asset Pricing Model (CAPM). These authors examined the relationship between risk and return of a financial asset.

Sharpe (1964) formula assumes a rational investor that should not take on any diversifiable risk. As so, only systematic risk is rewarded within the scope of this model. Therefore, the required return on an asset, that is, the return that compensates for risk taken, must be linked to its contribution to overall portfolio riskiness opposing to its "stand alone risk". This financial theory, accepts the notion that an asset return should be proportional to the risk incurred by its holder. The following equation is the basis of this model:

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f] \quad (19)$$

Where:

- $E(r_i)$: Expected return
- r_f : Return of the risk-free asset
- β_i : Sensitivity of the Project to movements in the market return
- r_m : Return of the benchmark asset

A project discount rate can't be directly observed in the market. In previous studies, Pindyck (2010), enhances the use of CAPM, when computing a project discount rate. It should be stressed, that the risk perception must be adjusted to the competitive expected rate of return on the project. The author identifies the project beta estimate assuming a selected and representative sample of companies within the project industry. Therefore, asset betas will represent the business risk of the project business operations. Supplementing with Head (2008) research, the usage of proxy betas to represent the business risk of the investment project and the return of the index where these companies rely, as the Market return benchmark.

Asset Betas might have slightly different values after ungeared reflecting small differences in business operations and business. One refinement suggested by Pindyck (2010) is to perform an arithmetic average to Asset Beta, proportionally of its market capitalization within the collected proxy companies. Average Asset Beta can be computed according:

$$f(\bar{\beta}) = \sum_{n=1}^{\infty} (X_n \cdot \beta_n) \quad (20)$$

Where:

- X_n – Proportional Market Capitalization of the collected proxy Company n ;
- β_n – Asset Beta of the Company n .

4. Data and Methodology

In this chapter, it will be presented the data and methodology used to compute the Stochastic Project Evaluation, through the statically methods, described in Chapter 3.

For better understanding, this chapter appears divided by Technical and Financial segments. The Technical segment, identifies the process behind an estimate for Recoverable Oil Volumes through GeoEx software. The next topic, defines the random process for computing the annual Production rates that originate Cash Flow.

Pointing out to the Financial segment, it starts by recognizing the Oil&Gas players present in deepwater projects. Financial returns are used for benchmarking purposes, in order to compute the discount rate from CAPM. Aiming at the NPV distribution for the potential Oil Prospects in Portugal, this chapter presents some of those assumptions.

4.1. Technical Data

i) Recoverable Oil Volumes

GeoEx software, supplied one of the main inputs of this Project Evaluation, the Recoverable Oil Volumes. The estimate of the volumes can be framed in the Prospective Resources of reserves mentioned in Section 2.4 since is still subject to commercial and technological uncertainties. The parameters estimates were based on the available geological data, interpreted by geophysics (Appendix 4 and Appendix 5). The software required the following data inputs:

- HC water contact [m] –boundary between the Hydrocarbons and the water; it can be Oil-Water, Gas-Water or Gas-Oil-Water.
- Spill point depth [m] – The spill point marks the maximum area to hold hydrocarbons; generally, is given by the last closing contour of the trap, and is a constant number.
- Net/gross ratio [decimal] – This ratio represents the portion of the reservoir that is actually effective; for instance, in a package of sandstones (gross rock), there are levels with more or less clay and/or silt content – the levels with cleaner sands and better porosity are the net.
- Porosity [decimal] – Porosity represents the volume of empty space (pores) within the rock, and it can be primary or secondary.
- Oil/Gas saturation [decimal] – Depends on the porosity and wettability of a rock.
- Oil/Gas expansion factor (1/Bg) [scf/cf] – related to the overburden, and type of oil/gas. It refers to the increase in volume of the hydrocarbon at atmospheric conditions of pressure and temperature. Heavy oils will have a low expansion factor, whereas gas will have the highest. The higher the overburden rock, the higher the expansion will be, when the hydrocarbons reach the surface.
- Recovery factor [decimal] – This factor is related with the type of oil, and the porosity and permeability of a rock.

ii) Production Rates

In order to assess annual Cash Flows, there is the need of knowing the incoming production per year from the total Recoverable Oil Volumes estimate. In this context, the Production rates were collected from historical oil projects supplied by Wood Mackenzie¹ data base. The Production Profile from a reservoir can be the sum of Annual Production rates which can be defined as the annual production percentage of the Total Recoverable Oil Volumes. The proposed formula is:

$$\text{Annual Production Rate}_n = \frac{\text{Annual Production}_n}{\text{Recoverable Volumes}} \quad (21)$$

4.2. Technical Methodology

All non-financial data, reflect Geophysic Carolina Libório, internal view about the performed geological processing for the Recoverable Oil Volumes and Production rates. The parameters of those estimates are used to roll out Monte Carlo simulation. The path estimate is standing in this chapter, for each uncertainty.

i) Recoverable Oil Volumes

Using GeoX (GeoKnowledge) Software, it is possible to load the major inputs, described in Section 4.1, that reflect the reservoir main characteristics being able to reach to an estimate for the Recoverable Oil Volumes. All geological inputs, described, have a well fitted LogNormal Distribution, according to sector practice. Thus, the Recoverable Oil Volumes, is given in a statistical confidence interval framed in a LogNormal Distribution with $X \sim \text{LogN}(\mu, \sigma)$. (Appendix 3 - GeoEx output application for Portugal Prospects). The results obtained for Recoverable Oil Volume estimate for parameters ($\mu = 354,5$ and $\sigma = 200,2$), are used to perform the simulation in the stochastic economic model as described furthermore in Section 5.3.

i) Production Rates

There were collected more than 300 historical oil fields projects all over the world, from Wood Mackenzie. To assess the possibility of having several comparable oil fields, projects had to comply benchmarked criteria similar with Portugal Oil Prospects.

Firstly, this research started to range the collected oil fields, according with 5 intervals of Recoverable Oil Volumes estimates:

- 150-300 MM BOE (30 Oil fields)
- 300-400 MM BOE (122 Oil fields)
- 400-700 MM BOE (76 Oil fields)
- 700-1000 MM BOE (66 Oil fields)
- 1000-1500 MMBOE (41 Oil fields)

Subsequently, oilfields can have quite different features despite its Recoverable Oil Volumes. A selection from those production profiles was needed to take in account other relevant characteristics. To become comparable to the project subject of this dissertation, our data sample was filtered to the oil field projects that had similar reservoir sediments and salt mobility within Portugal basin, only projects with maximum 25 years of production , water depth between 200 to 3500m and a number of wells drilled in the prospects between 30 to 50 wells.

As a result of recent deepwater discoveries, this additional filter resulted in a “shrink” of the collected data sample for 23 oil fields projects comparable with Portugal prospects:

- 150-300 MM BOE (3 Oil fields)
- 300-400 MM BOE (9 Oil fields)
- 400-700 MM BOE (6 Oil fields)
- 700-1000 MM BOE (3 Oil fields)
- 1000-1500 MMBOE (2 Oil fields)

Finally, the Productions Rate, proposed in Equation 21, was selected according to the Mean Recoverable Oil Volumes estimate produced in the Monte Carlo iterations. In this context, an estimate from GeoEx for the Recoverable Oil Volumes, defines a random process for the selection of one of the 23 oil field projects, proposing the underlying Production profile as the one to reflect the annual Revenues.

4.3.Financial Data

The data set comprises representative daily returns of companies, for the Crude oil commodity, 10 year yield bond from the Federal Government of Germany and the Dow Jones U.S. Integrated Oil&Gas Index (DJUSOL) from January 2, 2002 to December 31, 2012. Note that the sample encompasses the period where Deepwater discoveries started to be announced, the period where oil prices ranged between 30 and 140 USD per barrel and also the most recent crisis in the financial market. During this period many discoveries that were thought to be impossible to recovery revealed a specific new oil era. The subprime crisis began in the summer of 2007 and which consequences are still reflected nowadays. Many financial institutions suffered large losses all around the world making it reasonable to consider an estimation of the accuracy when predicting these losses. The data were obtained through Bloomberg and embodies the following financial assets:

- Brent (OILB)
- 10y yield bond from Federal Government of Germany
- Dow Jones U.S. Integrated Oil&Gas Index (DJUSOL)

Selection of comparable companies:

- BG Group PLC (BRGYY)
- BP PLC (BP)
- Chevron Corp (CVX)
- China Petroleum & Chem (SNP)
- ConocoPhillips (COP)
- Ecopetrol SA ADR (EC)
- ENI SpA (E)
- Exxon Mobil Corp (XOM)
- Petróleo Brasileiro SA (PBR)
- Marathon Oil Corp (MRO)
- Petrochina Co Ltd (PTR)
- Royal Dutch Shell PLC (RDSB)
- YPF Sociedad Anonima (YPF)
- Statoil (STO)
- Total SA (TOT)

4.4.Financial Methodology

i) Oil Prices

The Brent prices will be used for forecasting purposes in Section 5.3, framed in a Mean Reversion Model with Jumps. The Bloomberg data sample provided the Brent and subsequently the log daily returns were computed from 2002 to 2012 as follows:

$$r_t = \log \left(\frac{P_t}{P_{t-1}} \right) \tag{22}$$

Where:

- r_t : Return of the financial asset
- P_t : Last Price of day t
- P_{t-1} : Last Price of day t-1

ii) Mean Reversing with Jumps

The Mean Reversing with Jumps is a Geometric Brownian Motion, from historical time series with a jump diffusion process. Previously in Section 2.5, it is stated the importance of the selected Brent time series range. This interval will conduct to a certain drift equilibrium level, as being one important assumption in the model. From the series it is possible to declare a mean of 70 usd/bbl and a standard deviation of 31 usd/bbl, between 2002 and 2012. This range was selected according to the time frame where deepwater discoveries were announced to the financial markets comprising the assumptions and relevance of this dissertation (Appendix 13).

Analyzing Brent prices, it is possible to observe the stochastic behavior of the commodities price pictured in Figure 17. The Brent series reveals a visible stochastic behavior whose state is non-deterministic. It is not possible to trace an horizontal line without making a huge deviation between the actual behavior and that line.

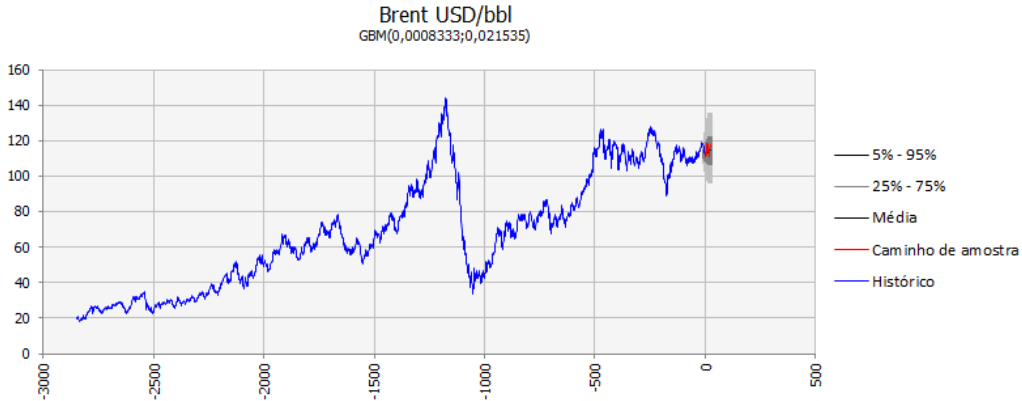


Figure 17 - Brent closing Price between 2002 and 2012 –Bloomberg source

The @Risk add-in, is able to perform the transformations that are need when the mean, variance, and autocorrelations of the original series are not constant in time. It is possible to detrend and perform first or second order differences in the series. The original series (Figure 17) was automatically transformed in order to achieve stationary (Figure 18 and Figure 19).

In this dissertation a first order difference was enough to achieve the stationary assumption respected and such a series is said to be difference-stationary. Sometimes it can be hard to tell the difference between a series that is trend-stationary and one that is difference-stationary.

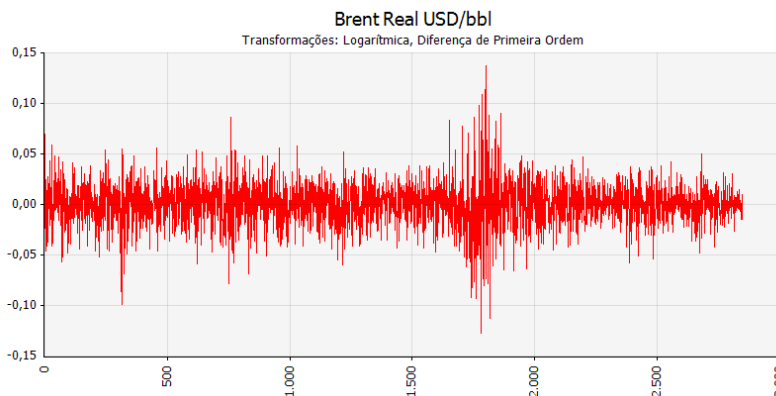


Figure 18 - First Difference of the Brent between 2002 and 2012 - Bloomberg source

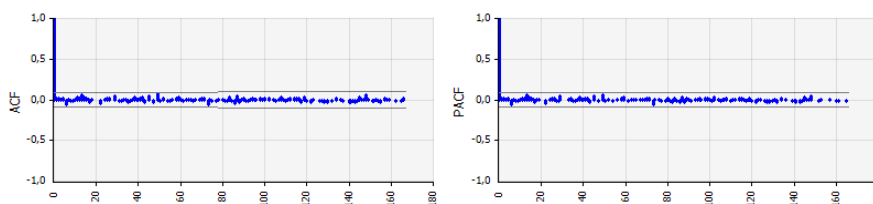


Figure 19 - Autocorrelation and Partial Autocorrelation Charts

iii) Asset Beta and Average Beta

The beta estimate for a single asset, were also collected through Bloomberg. The Levered Beta from the observed stock returns, are given by:

$$\beta_i = \frac{Cov(r_t, r_m)}{\sigma_m^2} \quad (23)$$

Where:

β_i : Sensitivity of the particular share to movements in the market return, also known by Equity or Levered Beta

r_t : Return of the financial asset

r_m : Return of the benchmark asset, assumed to be Dow Jones U.S. Integrated Oil&Gas Index (DJUSOL)

σ_m^2 : Variance of the benchmark asset, assumed to be the Dow Jones U.S. Integrated Oil&Gas Index (DJUSOL)

As mentioned, all Financial Returns were collected from Bloomberg. Using this type of returns, the rise of Equity betas, is inevitable in the sense that this measure of risk sensibility aggregates, both financial and business risk. When evaluating a project, the main interest, is to isolate the Asset Beta, reflecting only its business risk. It is possible to get Asset Beta from the observed Equity Beta using the following derivation:

$$\beta_e = \beta_a \frac{V_e + V_d(1 - T)}{V_e} \quad (24)$$

Where,

β_e : Equity beta;

β_a : Asset beta;

V_e : Market value of company's shares;

V_d : Market value of company's debt;

$V_e + V_d(1 - T)$: Market Value after tax;

T : Company profit tax rate.

The Average Asset Beta can be computed according:

$$f(\bar{\beta}) = \sum_{n=1}^{\infty} (X_n \cdot \beta_n) \quad (25)$$

Where:

X_n – Is the proportional Market Capitalization of the Company n from the overall collected sample proxies;

β_n – Is the Asset Beta of the Company n .

iv) NPV Distribution

In this Oil&Gas project evaluation, Net Present Value is a function of several variables, being identified 4 main uncertainties: Recoverable Oil Volumes, Oil Price, Production Rate and Capex&Opex costs.

The Stochastic form, previously presented for that 4 main uncertainties results in a stochastic form for the Output, namely the NPV Distribution.

Other effects such as Depreciation & Amortization and Fiscal Taxes are the best examples of consequences of these uncertainties being treated in Section 5.2.

This dissertation, proposes the relation between these variables described by the following equations:

$$NPV = \sum_{t=1}^n \frac{EBIT \text{ post tax}_t - Exp. CAPEX_t - Dev. CAPEX_t + D\&A_t}{[1 + r]^t} \quad (26)$$

$$EBIT \text{ post tax}_t = (EBITDA_t - D\&A_t) \times (1 - Income \text{ tax}_t) \quad (27)$$

$$EBITDA_t = Production_t \times Price_t - Opex_t - Royalties_t - Gov. Taxes_t \quad (28)$$

Where:

$Price_t$: Price in year t of Crude forecasted with Mean Reverting with Jumps

P_t : Production of year t estimated according to production rate estimate

$Opex_t$ – operating costs of year t ;

$D\&A_t$ – Depreciation & Amortization of year t ;

$Exp. Capex_t$: Exploration Investments of year t ;

$Dev. Capex_t$: Development Investments of year t ;

$Royalties_t$ – Production tax paid to host government the year t ;

$Gov. Tax_t$ – Acreage tax + Signature Bonus to host government the year t ;

$Income Tax_t$ – Income tax paid to host government the year t ;

r – Project adjusted discount rate in year t ;

5. Stochastic Evaluation of Portugal Oil Prospects

In this chapter there will be presented the Stochastic Project Evaluation applied to the Portugal Oil Prospects.

Portugal is seen by upstream sector, as a country with high potential in terms of natural resources that can be explored in its exclusive economic zone. Recent deepwater technology enabled the study of Portugal Basins in a much more detailed form. Boosted by the analogy involving the cross Atlantic Ocean Basins of Eastern Canada where recent deepwater discoveries have been delineated.

Up to date, there were drilled more than 50 wells onshore and 12 offshore wells in Portugal. All offshore wells were drilled in the 70s, at water depths of 200 meters, yielding several oil and gas shows, but no commercial discoveries were announced.

5.1. Location and Potential Oil Reserves in Portugal

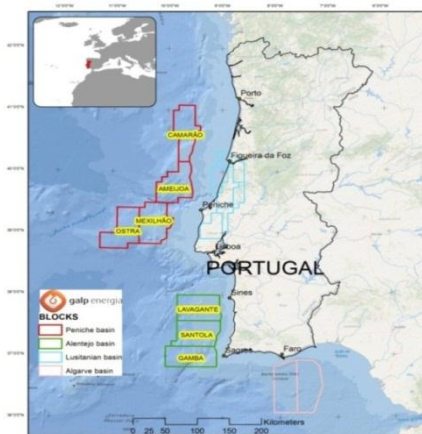


Figure 20 - Portugal offshore blocks

Currently, Portugal has two offshore areas with prospective oil blocks, under legal concession (Table 3). Both areas are in exploration phase and the underlying blocks are deepwater projects at depths of over 400m and placed at more than 50 km from shore.

In Alentejo Basin, there are 3 blocks concessioned (Lavagante, Santola and Gamba) with a total acreage of 12.169 Km. In Peniche Basin, the underlying consortium has 4 blocks (Ostra, Camarão, Amêijoia, Mexilhão) under evaluation with a total acreage of 9.100 Km (Figure 20).

Table 3 - Blocks Information

Basin	Depth (m)	Area (Km ²)	Consortium
Alentejo	400 to 3000	12.169	Petrobras (50%, Op.) Galp Energia (50%)
Peniche	400 to 3500	9.100	Petrobras (50%, Op.) Galp Energia (30%) Partex (20%)

According to consultant DeGolyer and MacNaughton² 2011 report, in case of a discovery, the potential prospects in Portugal, might have a substantial quantity of recoverable oil of 863 Million of BOE, subdivided as following:

The Alentejo Basin blocks have a mean recoverable volume estimate of 679 Million of BOE with a standard deviation of 321 Million of BOE. The Peniche Basin blocks have a lower mean recoverable volume of 184 Million of BOE standard deviation of 104 Million of BOE.

As stated previously, in Section 4.2, this research has a current estimate for Portugal Blocks of 355 Million of BOE with a standard deviation of 202 Million of BOE.

² DeGolyer and MacNaughton is an international consultant for reservoir, appraisal, field development planning services.

5.2. Portugal Fiscal System in Oil&Gas

The Portuguese Petroleum Legislation (Decree-Law nr. 109/94), opened to the world its resources for potential investment interest. Portugal Government has the upstream regulator in the DGGE³, which has the responsibility of managing, organizing and integrate all technical information of the national upstream sector. There is a specific division, DPEP⁴, for the exploration and production of Oil&Gas where the petroleum legislation and fiscal terms in Portugal's territory is regulated.

As said previously, until today, there isn't any commercial oil discovery on deep offshore in Portugal, thus the governments intend to attract investment simplifying with competitive fiscal policies for Oil&Gas investments. The DPEP framed the portuguese fiscal system as a Concessionary Contract (Section 2.1). Thus, a consortium or a company from upstream that operates in the Portuguese offshore, are subject of specific fiscal terms under the celebrated in Table 4.

Table 4 - Fiscal Description of the Concessionary Contract

Concessionary Contract	Description
Acreage Rentals	Varies according to the Project Cycle phase. Between 131 \$/Km, 261 \$/Km2 or 1.307 \$/Km2.
Royalties	Varies according to Production. Between 2%, 5% or 7% of the Gross Revenue.
Incremental income tax.	Varies according to EBT. Between 3% or 5% of the taxable income.
Income Tax	IRC of 25% and Municipal Tax Corporate income tax of 1,5% of the taxable income.
= Government Take	Sum of the total taxes

- Acreage Rentals: Annual payment/km2, varies in the contract vary according to the phase of the project:
 - 131 \$/Km², if the project is in Exploration phase
 - 261 \$/Km², if the project is in Development phase
 - 1.307 \$/Km², if the project is in Production phase
- Royalties is an annual production tax payment triggered by the accumulative production being paid:
 - 2% of the total Revenue for the first 5 Millions of BOE.
 - 5% of the total Revenue for production between 5 to 10 Millions of BOE.
 - 7% of the total Revenue for production above 10 Millions of BOE.
- Incremental income tax to the taxable income:
 - An additional 3% is applicable to companies with EBT for taxable amounts above 1.5M€.
 - An additional 5% is applicable to companies with EBT for taxable amounts above 10M€.

³ DGGE - Direção Geral de Geologia e Energia

⁴ DPEP - Divisão para a Pesquisa e Exploração de Petróleo

- Two corporate income taxes are owed to the host Government:
 - The Corporate Income Tax (IRC) of 25%
 - The Municipal Tax Corporate Income tax of 1.5%

The creation of tax-free provisions is allowed and must be invested in oil exploration or research in national territory within three years, without exceeding: 30% of the gross sales value of the oil produced in the concession area during the provision year; 45% of the taxable collectable oil that would be calculated without the provision.

Investments in prospecting and oil exploration may be amortized as provided in the IRC code tax, from the beginning of production. However, the investments attributable to a discovery and assessment are deductible up to 100% in the first year of full production.

5.3. Monte Carlo Uncertainties

Using the stochastic project simulation for evaluating upstream oil projects imposes stochastic behavior in uncertainties that most arm the project value. The next step of this research, relates to the explanation of the fitted distributions and models for the uncertainties considered in the Monte Carlo simulation.

In Table 5, it is possible to observe the summary of the uncertainty assumption. The subsequent explanations for each one of them are presented in bellow.

Table 5 - Summary of uncertainties treatment

Uncertainty	Assumption
Recovery Oil Volume	LogNormal Distribution
Production Rate	Empirical Distribution
Oil Price	Mean Reverting with Jumps
Seismic & Data Acquisition	Triangular Distribution – Capex
Exploration Well unit cost	Triangular Distribution – Capex
Appraisal Well unit cost	Triangular Distribution – Capex
Injector Well unit cost	Triangular Distribution – Capex
Production Well unit cost	Triangular Distribution – Capex
Other Costs	Triangular Distribution – Capex
FPSO unit cost	Triangular Distribution – Opex
Abandonment Cost	Triangular Distribution – Opex
Variable Opex per well	Triangular Distribution – Opex

Reservoir Production Estimation – LogNormal Distribution

The Oil&Gas is an industry with more than one century and much evidence has been collected through the years. Accordingly to the 1991 annual outlook for Oil & Gas from EIA⁵ demonstrated preponderance evidence for Recoverable Oil Volumes when associated its behavior to the LogNormal Distribution. Sharing with the previous statement is Demirmen (2007) from SPE⁶, stating it as the LogNormal as the most common distribution for fitting the Recoverable Oil Volumes.

The main reasons for this assumption, relies to LogNormal Distribution range, which takes on values from 0 to +∞. Subsequently, being rightward skewed implies that ranges of higher Recoverable Oil Volumes are less likely to occur than equivalent ranges of smaller Recoverable Oil Volumes. This assumption is frequent, in this type of detailed geological analysis, being a more conservative approach than within Normal Distribution. In fact, the LogNormal Distribution differs, significantly from the Normal Distribution wherein, the mean, median and mode are more identically valued and the tails of the distribution are symmetrically distributed about that value. The relative frequency of outcomes in a LogNormal Distribution is greatest for estimates that are smaller than the mean value. Log Normal reflects a long right tail of the distribution where possible but increasingly improbable high valued outcomes are located.

In this dissertation and as stated previously in Section 4.1, the size of the distribution of hydrocarbon prospects is derived from distinct distributions of the relevant geological variables. The application of these random variables in a multiplicative computation results in a Recoverable Oil Volumes distribution that is approximately LogNormal, with the following results:

Table 6 - Rec.Volumes Statistics

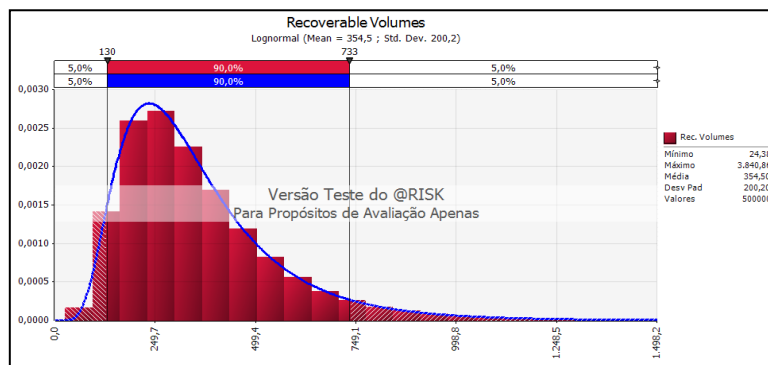


Figure 21 – Probability Density Function for Rec. Volumes

Statistic Parameters	
Minimum	24
Maximum	3.841
Mean	354,5
Mode	232,5
Median	308,7
Stand. Dev.	200,2
Skewness	1,8750
Kurtosis	9,8260
Percentiles	
1%	91
5%	130
25%	216
50%	309
75%	440
95%	733
99%	1.050

In view of **Figure 21** and **Table 6**, disclose the results obtained for the Recoverable Oil Volumes for Portugal oil prospects. The simulation was performed 5.000 iterations and described the above probability density function.

⁵ EIA - Energy Information Administration

⁶ SPE - Society of Petroleum Engineers

It should be stressed that previously in Section 4.2, the GeoEx software computed a Mean estimate of 354,5 million BOE with a standard deviation of 200,2 million of BOE which is the major input on iterations. In this simulation, the minimum estimate for Recoverable Oil Volumes is 24 million of BOE's and maximum of 3.841 million of BOE.

Table 7 - Statistics for Rec. Oil Volumes

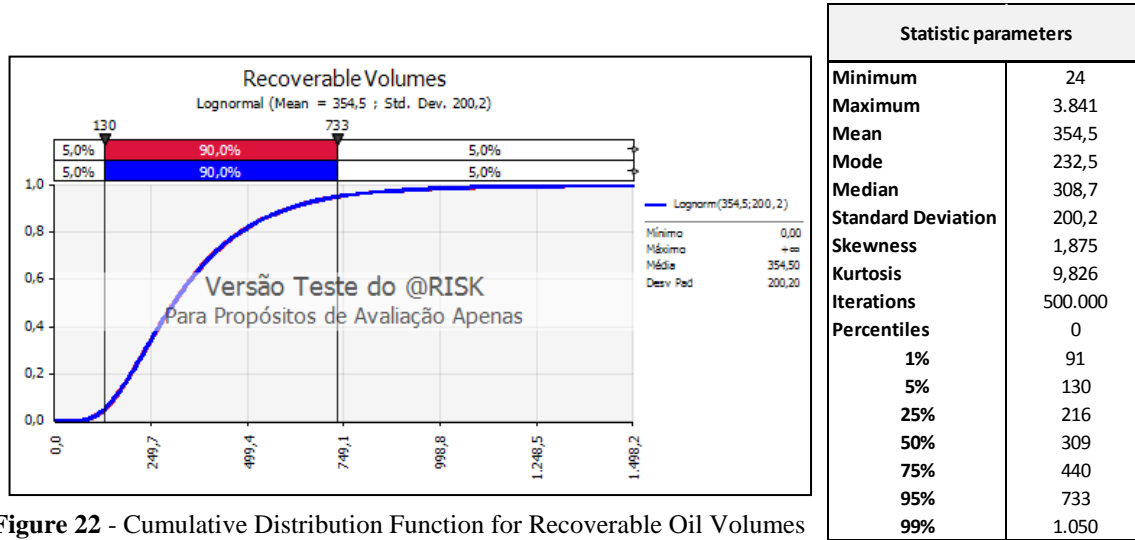


Figure 22 - Cumulative Distribution Function for Recoverable Oil Volumes

Based on **Figure 22**, it is possible to state with a 90% of confidence, the depicted simulation, shows a cumulative probability for the estimate for Recoverable Oil Volumes being around 130 and 733 million of BOE.

Observing **Table 7** and focusing in the extreme values, the abovementioned distribution, disclose a positive skew of 1,875 which indicates that the tail on the right side is longer than the left side and the bulk of the values lie to the left of the mean. As so there is 5% of probability that Recoverable Oil Volumes, range between 0 and 130 million BOE. Pointing out, the upper bound percentile, there is a 5% of probability that volumes range between 733 and 3.841 million of BOE.

Production Rate

After having an estimate for the Recoverable Oil Volumes, there was the need of having the behavior of the underlying production. Assessing to the annual production rate, from the Recoverable Oil Volumes estimate, is required for the annual Cash Flow of the project and for the taxation that might trigger.

In the Oil&Gas industry it is common practice to use historical production rates as empirical distributions, using ensemble data assimilation from historical reservoirs of oil. In fact, Dake (1998), proposes this methodology as being one of the most appropriate procedures (Section 2.4).

The shape of this uncertainty can be very peculiar since every reservoir behaves as a singular one, mainly due to the underlying development of the project. The production

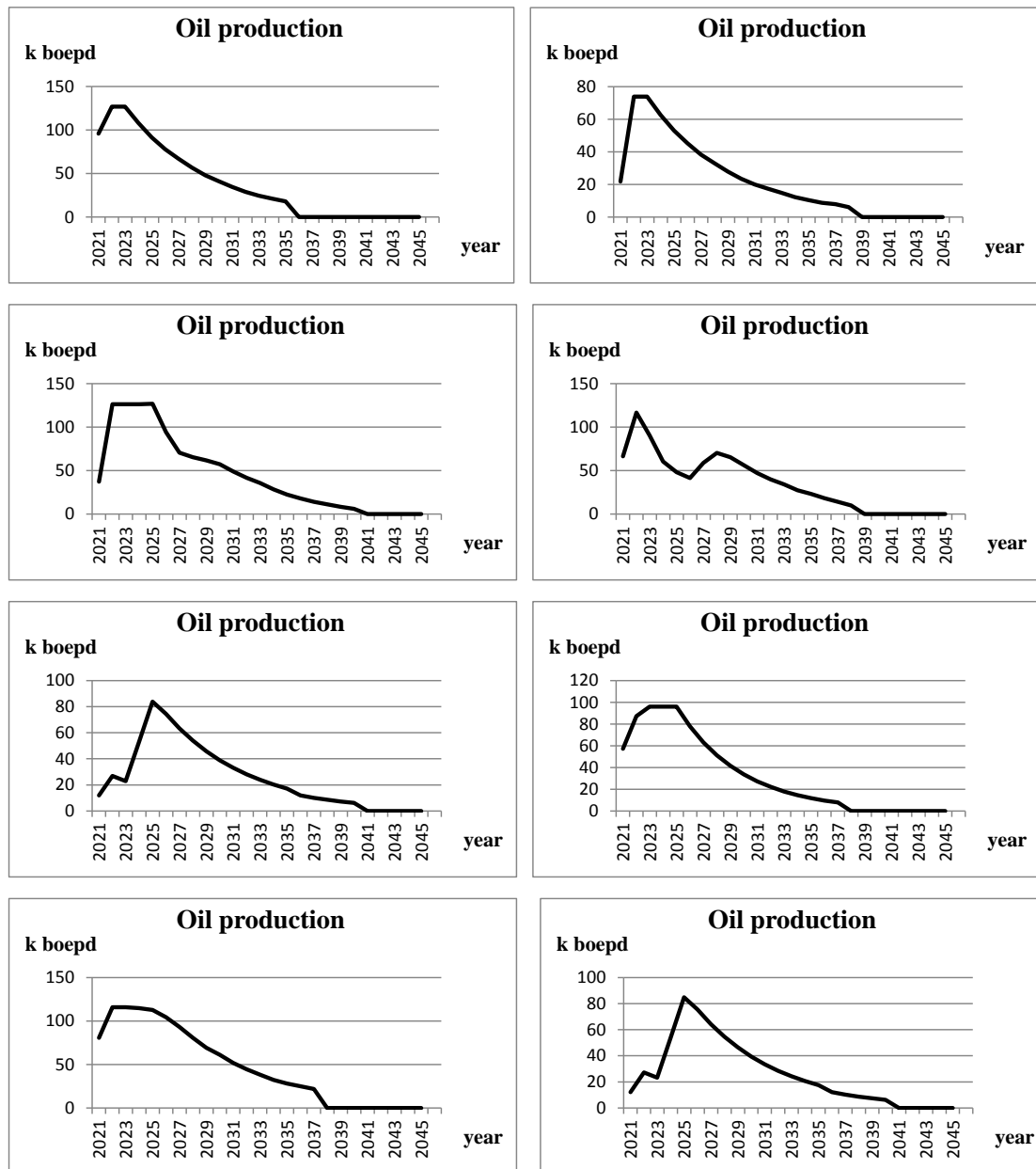
rate and the peak oil⁷ level is conditional whether it is being considered to a certain Basin, to a Reservoir, to a set of fields, to the Recoverable volume, to the number of wells, to the work over timing or if experience technical problems, etc.

After the peak oil, it is expected to enter in a terminal decline as a consequence of the outflow pressure from the reservoir. Same uncertainty is addressed to the peak oil level and timing on which point the barrels of oil and equivalent will enter in decline.

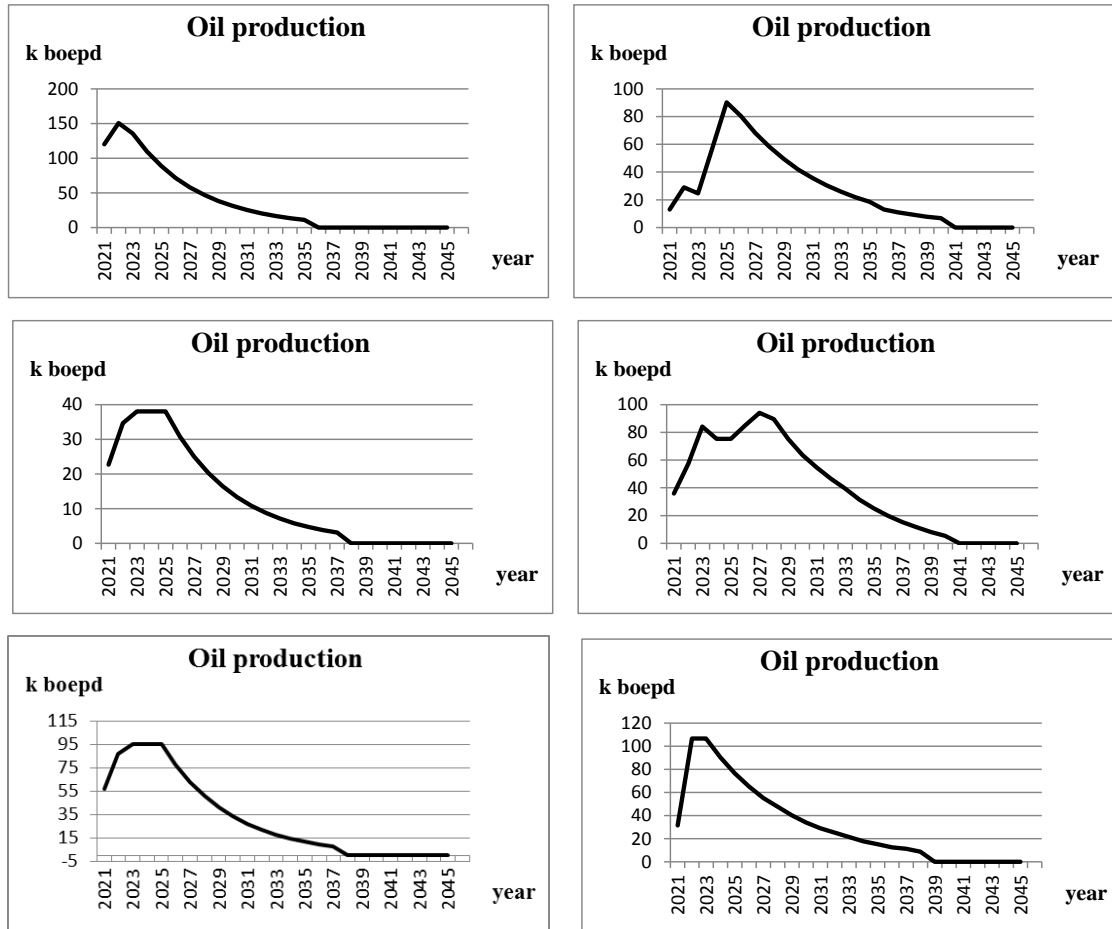
In this dissertation, it is modeled a technical selection of historical production rates (Section 4.2) with a random mimic as a function of the Recoverable Oil Volume estimate, shaping an empirical distribution in each simulation..

Beholding to Figure 23, are some examples of the Empirical Distributions used in the depicted simulation.

Figure 23 - Production Rate Shapes collected from 14 random historical reservoir data



⁷ Peak oil is the point in time when the maximum rate of petroleum extraction is reached



Oil Price – Mean Reversion with Jumps

Accordingly to Dias (2012), for forecasting purposes, the decision between a deterministic model and any of the known stochastic models, the second pick will be for sure more realistic than the first one. In fact a stochastic series is one whose state is non-deterministic.

This author augments, that the most appropriate method for estimating commodities prices, like WTI or Brent, are stochastic processes, in particular the Poisson-Gaussian model of Mean Reversion, also known as Mean Reversion with Jumps.

This dissertation performed fitting tests in order to assess the most appropriate stochastic model that described the Brent series between 2002 and 2012 (Section 2.5 and Section 4.4).

Analyzing Table 8, which reflects the significant statistical models computed by @Risk, the Mean Reverting with Jumps (BMMRJD⁸), is ranked 1st in both AIC and BIC statistics. By this sharing with the above mentioned author.

⁸ BMMRJD – Brownian Motion Mean Reversing with Jumps Diffusion

Table 8 - Information Criteria for Brent Series between 2002 and 2012

Ajuste	AIC	BIC
BMMRJD	-13765,614	-13753,71
BMMR	-13765,13..	-13747,27..
ARCH (1)	-13763,71..	-13739,91..
GBM	-13762,08..	-13738,27..
GARCH(1,1)	-13751,16..	-13727,36..
ARMA(1,1)	-13567,63..	-13543,82..
GBMJD	---	---
BMMRJD	---	---
AR(1)	N/A	N/A
AR(2)	N/A	N/A
MA(1)	N/A	N/A

After framing the Mean Reversing with Jumps as the most appropriate model, the @Risk add-in, proposed the MLE parameters and computed a simulation depicted in the Figure 24.

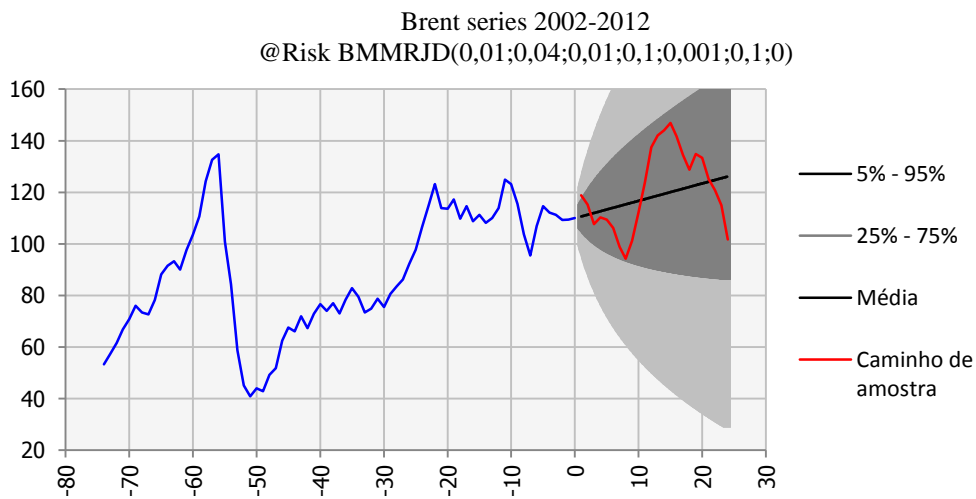


Figure 24 - Oil Price framed in the Mean Reversing with Jumps

Parameters:

- RiskBMMRJD ($\mu; \sigma; \alpha; \lambda; \mu_j; \sigma_j; Y_0$)
- RiskBMMRJD (0,01 ; 0,04 ; 0,01 ; 0,1 ; 0,001 ; 0,1 ; 0)

In concrete terms the parameters interpretation from the generated Brownian motion process, can be read as a proposed drift of 1%, volatility of 4%, speed of reversion of 0,01, jump rate of 0.1, jump size mean of 0,1%, jump size standard deviation of 10%, and value of 0% at time 0. The proposed formula results in a random variation that is multiplied by the equilibrium Brent level of 70 usd/bbl.

It is assumed that shocks occur according to a Poisson process with parameter λ , and that at such times, there is a jump, in the process that is normally distributed with parameters $\mu_j=0,1\%$ and $\sigma_j=10\%$. This model has more economic logic and considers large variation of the underlying variable due the arrival of abnormal rare news.

Capex&Opex Cost Structure – Triangular Distribution

Several authors use Triangular Distributions, in order to assess to a break down cost approach for a project outflow structure. In fact, Larham (2012) proposes the Triangular Distribution as the most accurate and simple conduct for estimating the cost as a result of an activity. The total project cost, is the sum of the random variables representing the costs of the individual activities, which are assumed to be independent. The researcher also emphasises the use of the @Risk add-in for *MS Excel*, as one of the more common tools for the Monte Carlo Simulation.

In this dissertation the complex structure cost of an oil field project is simplified as estimated in function of the number of wells and the total recoverable oil volumes. All assessments for the costs of an activity are specified in the form of a three point estimate comprising: Maximum, Average and Minimum cost. The three point estimates are then turned into a convenient probability distribution with @Risk-add in. The Appendix 1, shows the economic modeling behind all assumptions.

It should be pointed out that Table 9 and Table 10 embody these important estimates for variables like: Seismic & Data acquisition cost, the well unit cost (Exploration, Injectors, Appraisal, and Production), Others costs, the Abandonment cost, the FPSO purchase and Variable Opex⁹.

Table 9 - Capex Structure Assumptions

CAPEX					
Seismic & Data acquisition			Exp Well Unit cost (MM\$/well)		
Max	Average	Min	Max	Average	Min
15	10	8	130	100	90
Inj. Well Unit cost (MM\$/well)			App. Well Unit cost (MM\$/well)		
Max	Average	Min	Max	Average	Min
130	100	90	140	120	90
Prod. Well Unit cost (MM\$/well)			Other Costs		
Max	Average	Min	Max	Average	Min
140	120	90	15	10	8

Table 10 - Opex Structure Assumptions

OPEX					
Abandonment Cost (MM\$)			FPSO Cost (MM\$)		
Max	Average	Min	Max	Average	Min
2.000	2.300	2.500	1.800	1.500	1.300
Variable OPEX (MM\$/well)					
Max	Average	Min			
15	10	8			

It is important to stress that there are some sunk cost from the Exploration phase that are not being considered in this project evaluation. Namely, the Signature Bonus and

⁹ The costs estimates were intermediated with interviews at experienced managers in Deepwater projects in Galp Energia. Dr. Roland Muggli (Exploration Director), Walter Waes (Reservoir Eng) and Eng. Miguel Sinval (Senior Drilling Engineer) supplied important estimates, regarding the development plan for the prospects in Portugal.

Seismic Acquisition & Processing that has been performed until the end of 2012. These costs had past disbursements of cash as part of the required process to enter the block by the underlying consortiums and part of the current work commitments described in Table 11.

Recalling for the variable number of production years, as a result of the random empirical production rate distribution considered in each iteration (Section 4.2).

Table 11 - Phase of the Project and Minimum Work Commitments

Phase of the Project	Starts	Ends	Min. Work Commitments
Licensing	2010	2013	Signature Bonus
Exploration	2013	2016	1 Exploration well + Seismic
Appraisal	2017	2019	2 Appraisal wells
Development	2020	2020	-
Production	2021	variable	-
Abandonment	variable +1 year	-	-

The Exploration and Appraisal phase have minimum work commitment which is assumed to be accomplished for both blocks. In those years, it is forecasted that 1 Exploration and 2 Appraisal wells should be accomplished, thus, the only uncertainty is their cost but not the number of wells.

The simulation starts with the number of production and injectors wells as a function of the Recoverable Oil Volumes. Again, for a single iteration in the Monte Carlo for the Recoverable Oil Volumes it is creating an underlying Development Plan scenario.

The results obtained for the Recoverable Oil Volume constructs a Development Plan scenario predicting the number of wells and the number of wells need per year for extracting those volumes (Table 12). For instance, if the Recoverable Oil Volume estimate is within 200 and 500 million of BOE, there will be forecasted 13 Production wells, 10 Injectors wells. The frequency of the well drilling will be 4 wells and 2 wells, per year, for production and injector wells respectively. Both of them starting in the first year of the first oil (2021) until the last year of Production.

Table 12 - Development Plan Assumptions as function of Recoverable Oil Volumes

Development Plan Assumptions				
Rec. Oil Volume	# Wells	# Wells	# Wells/year	# Wells/year
MM BOE	Prod.	Inj.	Prod.	Inj.
100	3	2	1	1
200	5	4	2	2
500	13	10	4	2
1.000	25	20	8	4
2.000	50	40	14	7
5.000	125	100	25	15
10.000	250	200	25	15

Finally, the cost of the Abandonment year, is allocated in the immediate year after the final year of production, in order to minimize the legal environmental impact process of this Oil&Gas activity.

5.4. Discount Rate

This dissertation, selected a representative number of companies that should be comparable to the underlying project. Namely companies present in the Dow Jones US Integrated Oil&Gas Index with past and current deepwater offshore investments presented in Section 4.3. The Unlevered Beta and Market Capitalization estimate are accessible in Table 13.

Table 13 - Detailed Market Capitalization and Beta estimates for comparable companies

	Petrochina Co Ltd	China Petroleum	Statoil	Exxon Mobil Corp	Chevron Corp
Market Cap. (MM USD)	1.998.748	654.087	443.222	389.648	210.516
Market Cap. (%)	44,1%	14,4%	9,8%	8,6%	4,6%
Unlevered Beta	0,930	0,900	0,800	0,900	1,000

	Royal Dutch Shell PLC	Petrobrás SA	Ecopetrol SA ADR	Total SA	BP PLC
Market Cap. (MM USD)	164.686	124.300	116.670	88.067	81.221
Market Cap. (%)	3,6%	2,7%	2,6%	1,9%	1,8%
Unlevered Beta	0,530	1,140	0,760	0,690	0,870

	Conoco Phillips	Eni SpA	BG Group PLC	Marathon Oil Corp	YPF Sociedad Anonima
Market Cap. (MM USD)	70.749	66.651	55.946	21.677	22.620
Market Cap. (%)	1,6%	1,5%	1,2%	0,5%	0,5%
Unlevered Beta	0,800	1,650	1,200	1,420	1,094

The abovementioned table enabled to compute an equivalent unlevered beta for the underlying project subject of this dissertation.

The unlevered Betas are multiplied by the weight average of the market capitalization within the collected sample. As a result the average beta from the 15 companies is $\bar{\beta}=0,910$.

Table 14 - Average unlevered Asset Beta

Total Market Cap.	4.528.473
Avg. Project Beta	0,910

The risk-free rate can typically be taken as the interest rate on a bond whose maturity matches with the project life. An oil field project, usually have more than 10 years of economic evaluation, therefore, this research choose to use the yield of the Federal Government of Germany as a benchmark for the risk free rate, with respected to the longer maturity available, the 10-year government note. The Government of Germany holds an historic yield of 3,83%, between 2002 and 2012. Subsequently, to the market return, the benchmark used was the return from the Dow Jones US Integrated Oil&Gas Index (10,35%) in the same period referred in above.

The project discount rate accomplished a 9,76% , computed as follows:

$$r = r_f + \beta_i [E(r_m) - r_f]$$

$$r = 3,83\% + 0,910 [10,35\% - 3,83\%] = 9,76\%$$

This discount rate is quite similar to the discount rate required by the U.S. Securities and Exchange Commission (SEC) for the Oil&Gas E&P companies with publicly issued securities to disclose their reserve levels and values on an annual basis using a specified set of assumptions, including a 10% discount rate.

5.5. Investment Analysis

NPV Distribution

The Monte Carlo simulation run over the entire *MS Excel* Spreadsheet, including all the input assumptions discussed previously in Section 5.3. The simulation performed a total of 5.000 trials, in the @Risk add-in, meaning that 5.000 NPV's were derived from multiple combinations of the input assumptions. The application of at least 5.000 of iterations seems to be recommendable. The step from 5.000 to 10.000 iterations does not increase the smoothness of the function significantly, but rises computation time considerably.

As mentioned in Section 3.2., the @Risk add-in simulator, is automatically fitting the distributions, using maximum likelihood estimators to continuous data and providing goodness-of-fit statistics based on:

- Anderson-Darling test,
- Kolmogorov-Smirnov test
- Akaike information criteria
- Bayesian information criteria

The Fitting results are exposed in Table 15, by displaying the list of fitted distributions according to the goodness-of fit that generated valid fit results. The goodness-of-fit statistic provides a quantitative measure of how closely the distribution of the data being fit resembles the fitted distribution. A lower statistic indicates a better the fit. The Figure 25 graphically points out the Normal Distribution and the PDF of the simulation performed.

Once more, Table 15, is displaying quantitatively the Normal Distribution, as the most accurate candidate from the ranking of all statistical tests. Thus, the Normal Distribution describes the behavior of the uncertainties combination. The A-D and K-S tests are statistically relevant and the fitted data from the simulation can be seen as normal continuous data. After having this certainty it is possible to conclude that AIC and BIC provide and rank the Normal Distribution selection since this statistic is the lowest among the remaining distributions.

Table 15 - Ranking of Distribution Fitness

Ajuste	A-D	K-S	AIC	BIC
Normal	4,4578	0,0235	94283,5294	9463,9826
Lognorm	5,2336	0,0237	94286,5054	94266,9586
Lognorm2	5,2336	0,0237	94286,5054	94266,9586
Erlang	5,4158	0,0264	94289,0251	94272,5061
Pearson 5	6,2255	0,0270	94292,0529	94275,9931
Gamma	6,4719	0,0283	94322,6735	94303,1267
ExtValue	7,4733	0,0333	94328,3735	94308,8267
LogLogistic	13,4368	0,0367	94445,9317	94426,3849
Weibull	27,0098	0,0481	94660,7438	94641,1970
Logistic	46,5663	0,0620	94806,7411	94793,7091
Rayleigh	59,8632	0,0791	95036,6774	95023,6454
InvGauss	60,0764	0,0809	95180,5928	95167,5608
Laplace	77,9841	0,0847	95448,6362	95435,6042
Pert	264,2026	0,1346	96284,6845	96265,1378
Expon	709,5253	0,1882	97942,6861	97929,6541
Triang	1112,6019	0,2927	98226,0289	98206,4822
Levy	1302,3639	0,2929	98717,4255	98704,3936
Uniform	2460,5659	0,3422	98723,9445	98704,3977
Student	25940,4408	0,4397	102710,0964	102697,0644
ExtValueMin	---	0,5308	103287,0365	103274,0045
Pareto2	---	0,8348	163821,4676	163814,9512
Beta General	N/A	N/A	N/A	N/A
ChiSq	N/A	N/A	N/A	N/A
Pareto	N/A	N/A	N/A	N/A
Pearson 6	N/A	N/A	N/A	N/A

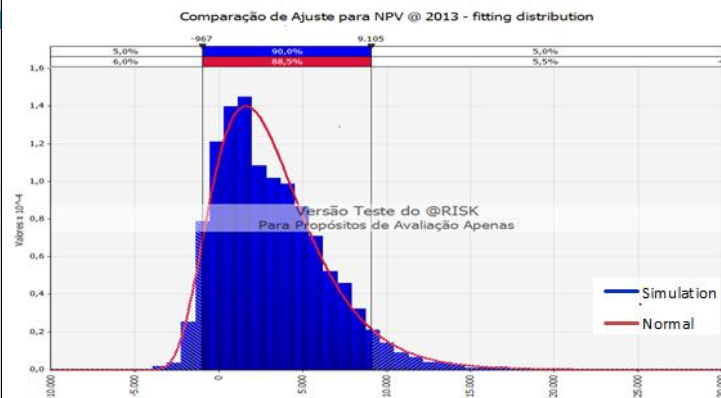


Figure 25 - NPV Probability Density Function

Rendering to the Normal Distribution, Table 16 and Figure 26 display the simulation’s behavior over the results obtained. It is exposed the Normal Distribution as the most fitted distribution of the underlying NPV simulation, with MLE parameters: **(Mean = 3.179 Million USD ; Standard Deviation = 3.282 Million USD)**

The Mean of the Normal Distribution (3.179 Million USD), is identically close to the average NPV (3.180 Million USD) which indicates that the input assumption distribution and parameters were set appropriately.

The Standard Deviation of the Normal Distribution (3.282 Million USD) and the Standard Deviation from the simulation (3.286 Million USD) are two of the most important pieces of information from this process. Both being larger than the Mean NPV, indicating a greater the range of possible NPV’s and addressing an huge uncertainty and turning the Mean NPV less representative.

With such a large Standard deviation, the extreme values from the advising Probability Density Function should be looked in detail in order to assess the real perception of risk. Accordingly to the CDF, from the obtained simulation, there is a 5% probability that NPV is equal to or less than -967 Million USD. It should also be stressed that the minimum accomplished in this simulation is -3.934 Million USD.

If the analyst is interested in a 90% level of confidence from the CDF, the NPV interval is between the lower endpoint -967 Million USD and the upper endpoint 9.105 Million USD.

Lastly, the upper limit has a 5% of probability that upper bound of the outcome NPV is equal or greater to 9.105 Million USD. Moreover, the maximum uplift accomplished in this simulation is 26.600 Million USD evidencing the positive skewness.

Table 16 - NPV Simulation

	Input	Normal
Minimum	-3.934	-6.067
Maximum	26.600	∞
Mean	3.180	3.179
Mode	1.383	1.596
Median	2.594	2.637
Stand. Dev.	3.286	3.282
Skewness	1,0602	1,0647
Kurtosis	3,8231	3,0000
Percentiles		
1%	-1.860	-2.106
5%	-967	-1.117
25%	700	817
50%	2.594	2.637
75%	5.114	4.950
95%	9.105	9.329
99%	13.106	13.274

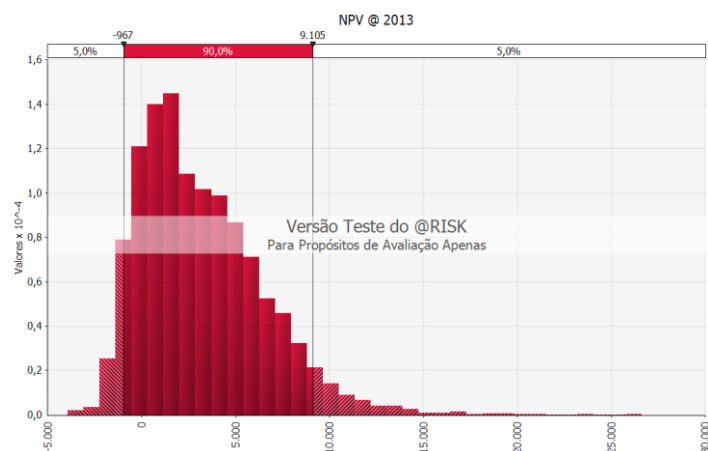


Figure 26 - NPV Probability Density Function

When benchmarking the excess Kurtosis measure of 3,8231 with the coefficient associated with a Normal Distribution, which is around 3, signals that the probability

of obtaining an extreme value is higher. Furthermore, the higher the Kurtosis coefficient is above 3, the more likely that NPV's will be either extremely large or extremely small.

However, the positive skewness of 1,0602, indicates that the distribution is positively skewed. Applying this to underlying NPV Probability Density Function, a positive skew, means occurrence of frequent small negative outcomes and a few extreme gains. The positively skewed distribution can also be confirmed by the mode (1.383 Million USD) being less than the median (2.594 Million USD) which is less than the arithmetic mean (3.180 Million USD).

The Table 17 and Figure 27, exalt the probability of NPV is equal or less than zero USD, with reference to the advising PDF ¹⁰. There is a 15,5% probability that investors are rewarded to the discount rate of 9,76%.

Furthermore, there is a 79,5% of confidence that the NPV interval is between the lower endpoint 0 USD and the upper endpoint 9.105 Million USD.

Finally, the upper limit has a 5% of probability that upper bound of the outcome NPV is equal or greater to 9.105 Million USD.

Table 17 - NPV Simulation

	Input	Normal
Minimum	-3.934	-6.067
Maximum	26.600	∞
Mean	3.180	3.179
Mode	1.383	1.596
Median	2.594	2.637
Stand. Dev.	3.286	3.282
Skewness	1,0602	1,0647
Kurtosis	3,8231	3,0000
Percentiles		
1%	-1.860	-2.106
5%	-967	-1.117
25%	700	817
50%	2.594	2.637
75%	5.114	4.950
95%	9.105	9.329
99%	13.106	13.274

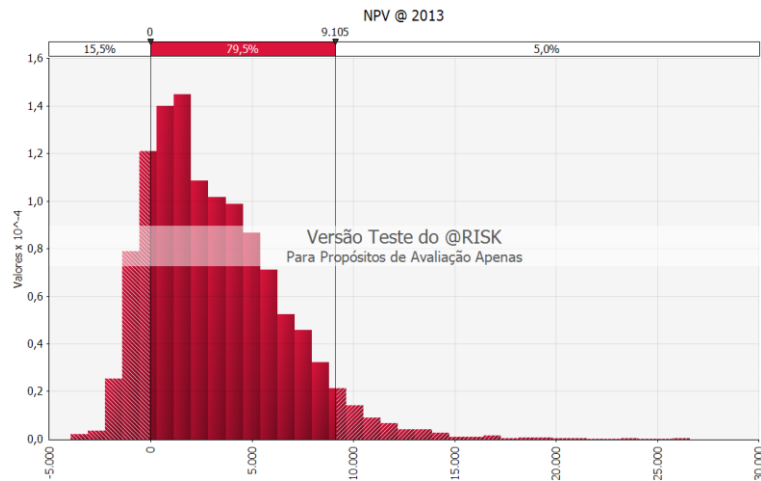


Figure 27 - NPV Probability Density Function

Sensibility Analysis

The established uncertainties used in Monte Carlo, raises the need of knowing, which were the most relevant variables.

Ignoring stochastic behavior for uncertainties such as oil prices, Recoverable Oil Volumes and cost structures (Capex&Opex) are not very recommended in an Oil Field Project Evaluation. In a natural resource extraction project, like upstream oil projects, the resource price and the resource amount are usually the two main uncertain variables.

¹⁰ PDF – Probability Density Function

This research started by performing a Tornado Analysis, in order to assess the importance of the variables that shape the Business Model for the Portugal Prospects. Using @Risk add-in, the Tornado Sensibility of the inputs, is pictured by displaying a quantitative ranking of the inputs that most impact the output, in this case the NPV Distribution. Inputs that have the largest impact on the output distribution have the longest bars (Figure 28).

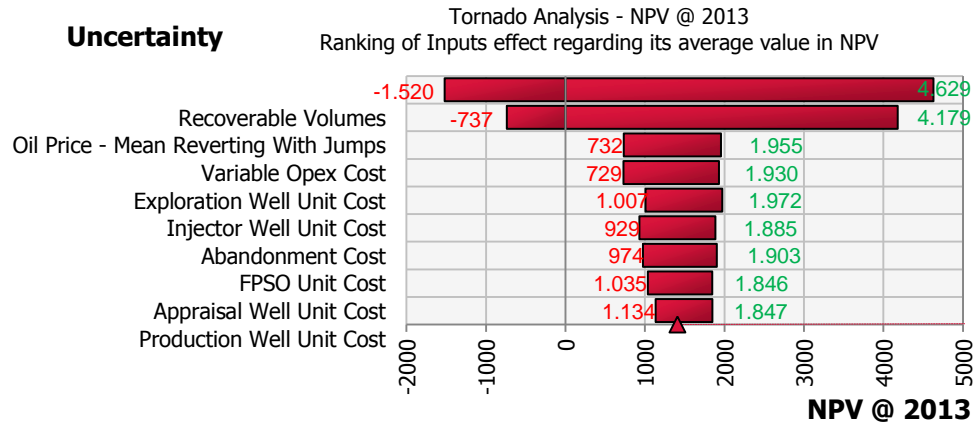


Figure 28 - Tornado Analysis for Empirical NPV Distribution

Analyzing, the Tornado Sensibility, it is possible to depart the ranking of the variables that have bigger impact in the NPV Distribution. The Uncertainties regarding the Recoverable Oil Volumes and the Oil price assume the most important impacts in the Project Evaluation. Observing Figure 27, it is possible to conclude that both variables are critical, since they are individually capable of destroying the Project value but also turning it into a very profitability investment. The effect of Recoverable Oil Volume towards the Mean of the NPV can range between -1.520 Million USD and 4.629 Million USD. Regarding the Oil price effect can range between -737 Million USD and 4.179 Million USD.

Noticing for the Figure 29 and Figure 30, reveal the concentration of results during the simulation performed.

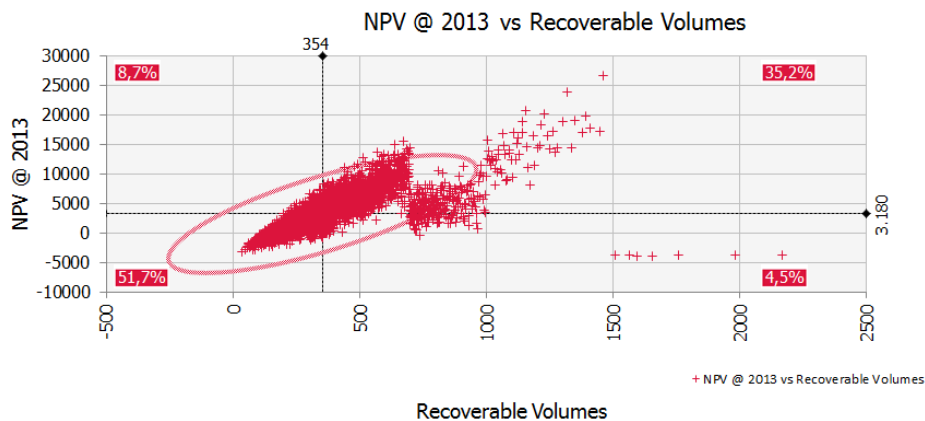


Figure 29 - Sensibility between Recoverable Oil Volumes and NPV

In Figure 29 the vertical axes are defined by the Mean estimate for Recoverable Oil Volumes (354 Million of BOE) and the horizontal axes are molded by the Mean estimate for the NPV (3.180 Million USD).

It is clear that Recoverable Oil Volumes estimate is being very conservative since 51,7% of the simulations were below both Mean estimates. The reason for this is subsequent, to defined rightward skewed Log Normal Distribution, which implies that ranges of higher Recoverable Oil Volumes are less likely to occur than equivalent ranges of smaller Recoverable Oil Volumes.

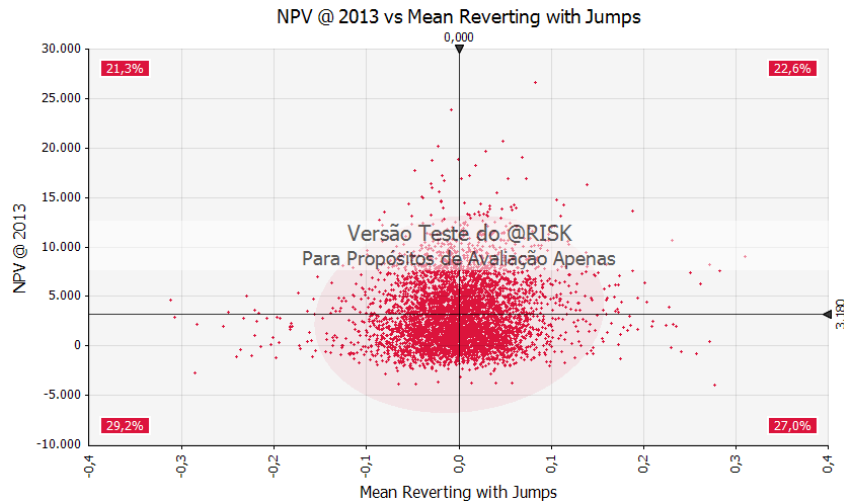


Figure 30 - Sensibility between Oil Price and NPV

Perceiving for the Figure 30, the vertical axes are defined by the variation towards the equilibrium level of the model (90 usd/bbl) and the horizontal axes are molded by the Mean estimate for the NPV (3.180 Million USD). It is noticeable that the biggest cloud is around the equilibrium level of brent since the 4 quadrants are similar equally distributed around 25%. The main conclusion is that price affects the project value, however the first jump determines the positive or negative effect on it. If the jump are pointing to a high levels of brent, the project values increase, if the jump negatively departs, the project value will inevitably decrease.

6. Conclusion and Future Developments

An Oil&Gas project with particular interest in deepwater, is designed for several years with billionaire disbursements of cash without any guarantee of having the desired remuneration. During the early stages of the project, the geological information of its potential is nothing more than, a quick look through the neck of a dark bottle, that can be quite full but also empty.

This dissertation carried out a Stochastic Project Evaluation applied to the potential oil prospects in the deep portuguese waters, namely, in the Alentejo and Peniche Basin. Both projects are in the exploration phase, and there are many unknowns that are classified and treated as uncertainties. The Recoverable Oil Volumes and the Oil price are the two biggest uncertainties that most threat the project value. Each one of them is able to turn the project negative by its own behavior. In addition, both uncertainties are hampered by the improbability of the required Capex&Opex investment for developing those fields.

The Monte Carlo simulation has the advantage of being based on the estimated Cash Flows and therefore fits perfectly into the uncertainties abovementioned, assessing a Probability Density Function for the NPV. The Stochastic Evaluation of this project was simulated by @Risk add-in, for 5.000 trials and derived a PDF with Mean equal to 3.179 Million USD and a Standard Deviation of 3.180 Million USD. There is a larger dispersion of the results, evidencing the uncertainty environment of the Oil&Gas industry. The risk and reward concept, is expressed in the simulation by addressing a probability of 84,5% for the NPV turn higher than zero, enhancing a potential attractive upside for this amount of dispersion.

Finally, this dissertation highlights suggestions for future developments related to the Stochastic Evaluation Project for deepwater projects. A first improvement of this work stands for the independency of the uncertainties. As years go by, huge amounts of data is appearing and may be relevant to a future study incorporate different types of correlation between the uncertainties. Secondly, the production rates that are associated to Recoverable Oil Volumes, should ideally be done by well, instead of assessing to the whole reservoir. Thirdly, the Capex&Opex cost structure, it is suggested that should be thought to be as function of the activities durations. For instance, within the drilling activity and the rigs which are performing it. A final recommendation relies for the hypothesis of including the financial Markowitz method, in order to assist the decision of accepting the underlying risk of a project. This could be realized by crossing the NPV probability and the investor utility curve.

Bibliography:

Akaike, H. (1974): "A New Look at the Statistical Model Identification," *I.E.E.E. Transactions on Automatic Control*, AC 19, 716-723.

Anderson T. W. and Darling D. A. (1954). A Test of Goodness of Fit. *Journal of the American Statistical Association*, 49(268): 765-769.

Belaid F. and Wolf D. (2009). *Stochastic evaluation of petroleum investment projects using Monte Carlo Simulation*. Universite du Littoral Côte d'Opale. In 20th May of 2009.

BP Statistical Review of World Energy (2008). June 2008: 03-94.

Chakhmakhchev A. and Rushworth P. (2010). *Global Overview of Recent Exploration Investment in Deepwater - New Discoveries, Plays and Exploration Potential*. In Search and Discovery Article #40656. 17th December 2010.

Chance D. M. (2008). *Monte Carlo Simulation*. Louisiana State.

De Golywer and Maughton (2011). *Prospective Resources in 2011 for Galp Energia*.

Dake, L.P. (1998) *Fundamentals of reservoir engineering*. Netherlands: Elsevier.

Demirmen, F. (2007). *Reserves Estimation: The Challenge for the Industry*. In May 2007. Society Petroleum Science & Engineering (SPE) 81-89.

Dias (2012), *Monte Carlo Simulation of Mean Reversion with Jumps*. São Paulo: Wiley

Dias, M. (2004). *Valuation of Exploration and Production Assets: an overview of real option models*. Society Petroleum Science & Engineering (SPE) 93-114.

Energy Information Administration. (2008). *Annual Energy Outlook 2008, with Projections to 2030*. June 2008. 13-88

Erdem, Ô. (2008). *Uncertainty Assessment for the evaluation of Net Present Value of a mineral deposit*. Middle East Technical University. In December 2008

Evans, M., Hastings, N., and Peacock, B. (2000). *Triangular Distribution*. Ch. 40 in *Statistical Distributions*, 3rd ed. New York: Wiley, pp. 187-188.

ExxonMobile. (1991). *Outlook for Oil & Gas*. Corporate Files.

Galli A., Armstrong M. and Jehl B. (1999). *Comparing Three Methods for Evaluating Oil Projects: Option Pricing, Decision Trees, and Monte Carlo Simulations*. Society of Petroleum Engineers (SPE) 529: 93-114

Gauss, C. F. (1809). *Theoria motvs corporvm coelestivm in sectionibvs conicis Solem ambientivm*. (in Latin). *Theory of the Motion of the Heavenly Bodies Moving about the Sun in Conic Sections*] English translation. Boston: Little, Brown and Company

Hazewinkel, M. (2001), Normal Distribution, *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4.

Head (2008). *Project specific discount rates*. The capital asset pricing model – Part 2. In April 2008. 45-48.

Holtan, M. (2002). *Using simulation to calculate the NPV of a project*. Onward Inc.

Ibrahim, O. (2005). *Uncertainty Analysis in Economic Evaluations*. *Business Mathematics and Informatics*. Faculty of Science, Amesterdam.

Jankauskas L. and McLafferty S. (1995). *BestFit, Distribution Fitting Software by Palisade Corporation*. Winter Simulation Conference 1995 pp: 457-461.

Knull (2007). *Accounting for Uncertainty in Discounted Cash Flow Valuation of Upstream Oil and Gas Investments*. Article. August 2007.

Kvalevåg, T. (2009). *How do discounted cash flow analysis and real options differ as basis for decision making about oil and gas field developments?* Copenhagen Business School. Master in Finance and Strategic Management.

Larham R. (2012). *Project Costing with the Triangular Distribution and Moment Matchingmore*. MHF Journal, Issue 3.

Law, A. M. and Kelton, W. D. (1982). *Simulation Modelling and Analysis*. McGraw-Hill Publisher.

Litner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *The Review of Economics and Statistics*, 47(1): 13-37.

- Menabde, I. (2008). *Valuation of Integrated Oil & Gas Companies*. Copenhagen Business School. MSc in International Business and Economics: Cand. Merc Finance and Strategic Management (FSM).. In 9th October 2008.
- Mossin, J. (1966). Equilibrium in Capital Asset Market, *Econometria*, 34(4): 768-783.
- Pindyck, R. (2010). *Investments in Offshore oil and Natural Gas deposits in Israel: Basic Principles*. Massachusetts Institute of Technology. In 13th November 2010.
- Poisson S.D. (1837). *Probabilité des jugements en matière criminelle et en matière civile, précédées des règles générales du calcul des probabilités*. Paris, France: Bachelier.
- Schwarz, G. (1978). Estimating the dimension model. *Ann. Statist*, 6(2): 461-464.
- Sharpe, W. (1964). Capital Asset Prices; A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3): 425-442.
- Talevski, D. and Lima, A. (2009). *Strategic and financial analysis in the oil industry: Petrobras shareholders value potential and fair value of stock*. AARHUS School of Business. Master in Finance and International Business.
- Tordo, S. (2007). *Fiscal Systems for Hydrocarbons*. World Bank, Washington DC, working paper no.123.
- Vasicek (1977). *An Equilibrium Characterisation of the Term Structure*. Journal of Financial Economics 5: 177–188.
- Vose, D. (2010). *Fitting distributions to data and why you are probably doing it wrong*. Paper in 15th February 2010.
- White, G. (2007, November 21), *Does Deepwater Oil Discovery Disprove the Peak Oil Theory?* Fleet Street Invest.

Appendix

Appendix 1: Economic Modeling for the Peniche and Alentejo Basin

The screenshot shows a financial model spreadsheet with columns for years from 2013 to 2028. Rows include production volumes (Oil: 26.13 to 46.00 kboepd; Gas: 9.54 to 16.79 kboepd), cash flow components (Sales: 1,136 to 2,864 MMUSD; OPEX: 131 to 201 MMUSD; EBITDA: 1,946 to 3,488 MMUSD), and summary metrics (NPV @ 2013: 1.715; NPV / BBI: 7.098664; CAPEX / BBI: 26).

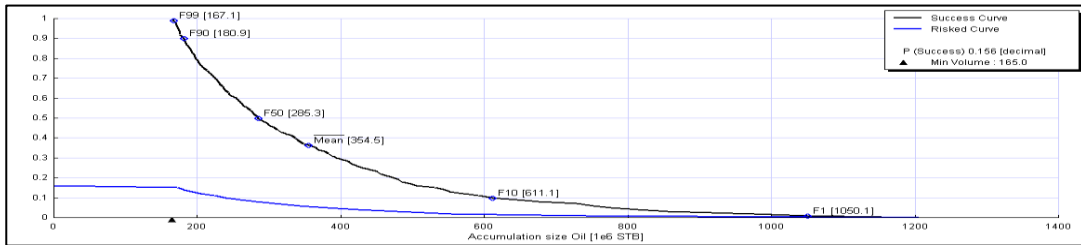
Appendix 2: Running the Monte Carlo Simulation with @Risk

The screenshot displays the same spreadsheet as Appendix 1, but with the @RISK simulation interface overlaid. The 'Simulando' window shows a progress bar at 10%, 49,600 iterations of 50,000, 1 of 1 simulation, and a remaining execution time of 00:02:28. The background spreadsheet data is visible through the window.

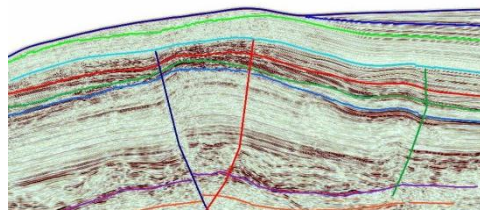
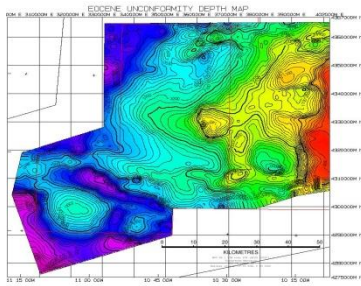
Appendix 3: GeoEx application for the Recoverable Oil Volumes

After all GeoEx inputs loading (Section 4.2.), the software returns several confidence interval values ranging from P99, P90, P50, P10, P1 and also a mean and volatility estimate for Recoverable Volumes. The P99 is the minimum value, with a 99% chance of being found; the P1 value stands for a 1% probability of finding the volume associated with it. The P50 is a volume that corresponds to a probability of 50% and the mean is the arithmetic average; P50 and mean are not the same values, although they can be approximate – it all depends on the type of distributions one applies on the input values.

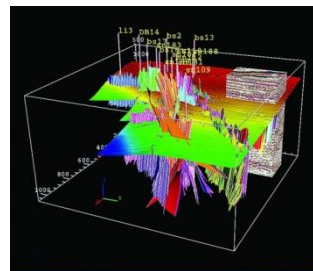
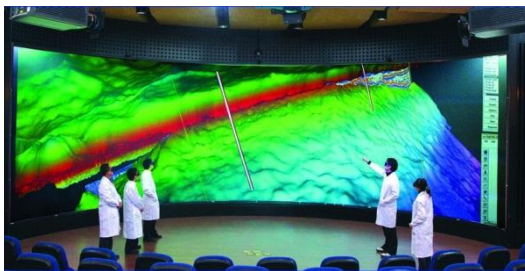
Portugal Recoverable Volumes Scenarios	Dist. type	Mode	Mean	Std. dev.	F99	F90	F50	F10	F10	Overall probability
Failure										0,843945
Success volume	MC(500)	173,2	354,5	200,2	167,1	180,9	285,3	611,1	1050,1	0,156055
Unconditional prospect	MC(500)-r		55,3	151	0	0	0	239,9	742,7	1



Appendix 4: 2D Seismic processing



Appendix 5: 3D Seismic processing



Appendix 6: FPSO and Semi-submersible for producing oil in deepwaters



Appendix 7: Production Profile Simulation for Production rate

<u>GeoEx Simulation</u>						
<u>Recoverable Volume</u>						
μ	354,50	Random Production Profile (MM BOE)				
σ	200,20					
<u>Year</u>	<u>Selected</u>	<u>1.500 to 1.000</u>	<u>1.000 to 700</u>	<u>700 to 400</u>	<u>400 to 300</u>	<u>300 to 150</u>
1	1%	2%	1%	4%	1%	2%
2	2%	6%	3%	6%	2%	4%
3	3%	7%	4%	10%	3%	4%
4	4%	8%	4%	12%	4%	8%
5	4%	7%	4%	12%	4%	13%
6	4%	7%	4%	11%	4%	12%
7	4%	6%	4%	10%	4%	10%
8	4%	6%	4%	8%	4%	8%
9	4%	5%	4%	6%	4%	7%
10	4%	5%	4%	5%	4%	6%
11	4%	5%	4%	4%	4%	5%
12	4%	4%	4%	3%	4%	4%
13	4%	4%	4%	3%	4%	4%
14	4%	4%	4%	2%	4%	3%
15	4%	4%	4%	2%	4%	3%
16	4%	3%	4%	1%	4%	2%
17	4%	3%	4%	1%	4%	2%
18	4%	3%	4%	0%	4%	1%
19	4%	3%	4%	0%	4%	1%
20	4%	2%	4%	0%	4%	1%
21	4%	2%	4%	0%	4%	0%
22	4%	2%	4%	0%	4%	0%
23	4%	1%	4%	0%	4%	0%
24	4%	0%	4%	0%	4%	0%
25	4%	0%	4%	0%	4%	0%
TOTAL	100%	100%	100%	100%	100%	100%

Appendix 8: “Potential Oil in Portugal? Alentejo more probable than Peniche”

Galp Petróleo em Portugal? Alentejo mais provável que Peniche DN – 06-03-2013

É mais provável encontrar petróleo em Portugal na bacia do Alentejo do que na bacia de Peniche, garantiu na terça-feira o presidente executivo da Galp, Ferreira de Oliveira, no final de uma apresentação da estratégia de negócio da petrolífera em Londres, escreve o Diário de Notícias (DN).



Há Petróleo em Portugal... Onde? O presidente da Galp diz que é mais provável encontrar o ouro negro no Alentejo do que em Peniche, dois anos depois de a petrolífera ter começado a estudar as bacias das duas zonas à procura de reservas de crude. “Neste momento, nesta fase dos estudos geológicos, a probabilidade de investimento seria mais na bacia do Alentejo do que na bacia de Peniche”, disse ontem Ferreira de Oliveira, citado pelo DN, no final da apresentação da estratégia de negócio da petrolífera até 2017, em Londres. Contudo, o presidente da Galp acrescentou que à medida que a prospecção for avançando estas estimativas podem mudar de rumo. “Portugal é um investimento de risco”.

Appendix 9: “Brent can reach to 190\$ in the final of the decade”

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		Âmbit: Economia, Negócios e...	Corte: 1 de 1

Brent pode chegar aos \$190 no final da década

ESTUDO DA OCDE APONTA PARA VALORIZAÇÃO DE 76% DO PETRÓLEO ATÉ 2020



Petróleo pode quase duplicar de preço com recuperação do crescimento mundial e aumento da procura por parte dos países emergentes

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 A pressão da procura dos mercados emergentes deverá elevar o preço do Brent, o petróleo que serve de referência às importações portuguesas, para os 190 dólares em 2020. Este é o cenário base assumido num estudo da Organização para a Cooperação e Desenvolvimento Económico (OCDE), que tem em conta uma recuperação do crescimento da economia mundial para níveis anteriores à crise. Caso o PIB registe um crescimento mais robusto, a cotação pode superar em muito os 200 dólares.

O estudo de quatro economistas do departamento de Economia da OCDE constrói um modelo de previsão para o preço do Brent. O cenário base assume o regresso a um crescimento económico mundial ligeiramente inferior ao registado entre 1998 e 2007 para o período entre 2011 e 2020. E um aumento da procura por petróleo, concentrada nos países emergentes, em particular na China (4,7% ao ano), na Índia (5,2%) e na Indonésia (4%). O modelo incorpora também uma menor intensidade na utilização de energia obtida a partir do petróleo, em resposta ao aumento do preço, e o impacto negativo da alta das cotações no PIB.

“A análise sugere que uma retomada gradual do crescimento da economia mundial estará associada a uma subida do preço do petróleo para 190 dólares por barril em 2020”, diz o estudo. O que implica uma valorização de 76%

face à cotação actual de 108 dólares.

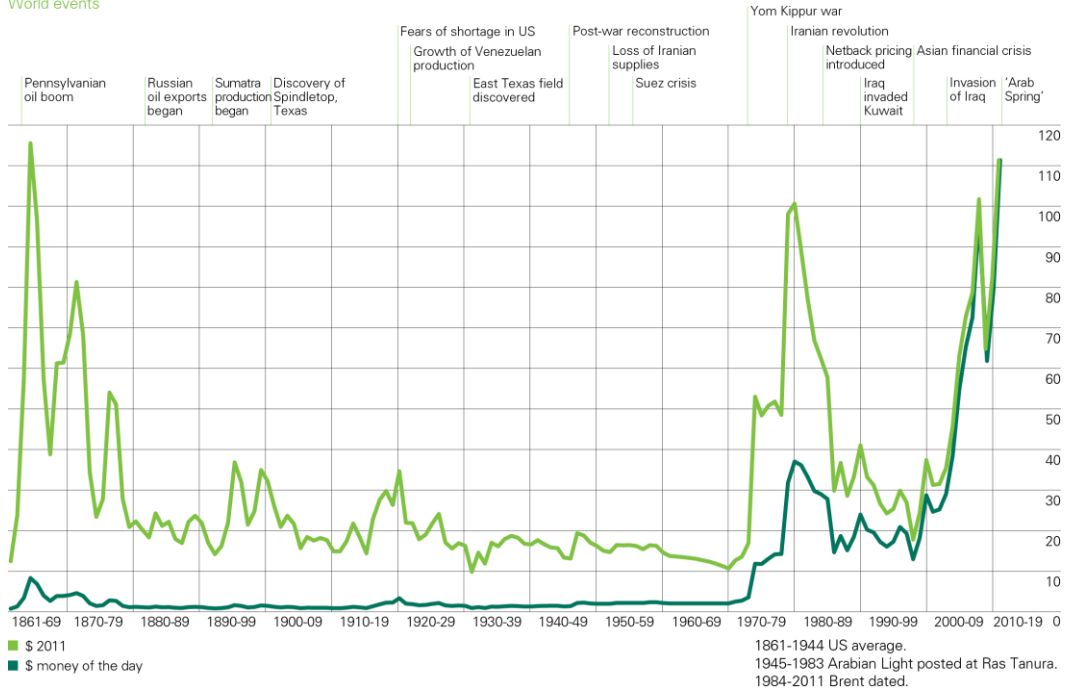
A previsão assenta num cenário base. Os autores salientam que “tendo em conta perspectivas plausíveis para a procura e a oferta, existe o risco de os preços subirem para entre 150 a 270 dólares por barril em termos reais [descontando a inflação] até 2020, em função da resposta da procura e oferta, e do prémio de risco que está incorporado nos preços actuais devido ao receio de escassez no futuro”. Um desses cenários alternativos estima que o preço chegue aos 240 dólares, caso a economia mundial cresça a um ritmo 1% superior.

O Japão e a União Europeia são as regiões cujas economias serão menos penalizadas pela subida da cotação do petróleo. Índia e Indonésia vão sentir um impacto maior. O estudo sugere acabar com os subsídios à utilização de combustíveis fósseis, em particular nos países emergentes.

Appendix 10: BP Energy Outlook 2012 – World events and Crude oil prices 1861-2011

Crude oil prices 1861-2011

US dollars per barrel
World events

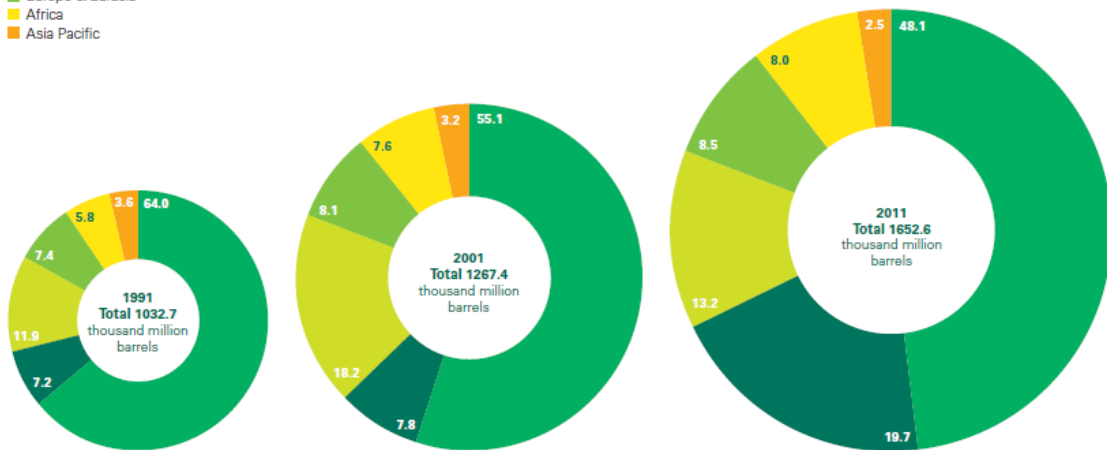


Appendix 11: BP Energy Outlook 2012 – Distribution of Proved Reserves

Distribution of proved reserves in 1991, 2001 and 2011

Percentage

- Middle East
- S. & Cent. America
- North America
- Europe & Eurasia
- Africa
- Asia Pacific



Appendix 12: ExxonMobil Energy Outlook 2012 – Turning Crude oil into usable products



Appendix 13: Possible Brent prices accordingly to the type of Energy and Proved Reserves in 2008

