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# Equivalent Dielectric Description of Transmit-arrays as an efficient and accurate method of analysis

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**Abstract**— Transmit-arrays (TAs) provide cost-effective solutions for various antenna applications, including satellite and terrestrial communications. Usually, these antennas have electrically large apertures, comprising thousands of fine-tuned subwavelength unit-cells. This makes full-wave simulations demanding in terms of computational resources, constraining the antenna design and optimization. Herein, we present an efficient method for the reduction of the TA's computational complexity that still provides accurate results for the main figures of merit of the antenna. For the chosen example, the simulation was 3 times faster and required 50% less memory. Yet, as the complexity of the problem is further scaled, this method is expected to become even more effective.

**Index Terms**— Effective media, transmit-arrays, flat-lens, non-periodic array, computational efficiency.

## I. INTRODUCTION

Transmit-arrays (TAs) have been an attractive solution to realize low-cost, low-profile, high-gain steerable pencil beam in many point-to-point and satellite communication applications [1]. These arrays comprise a collection of phase shifting unit-cells, which can provide beam collimation for the transmitted radiation coming from a primary feed. The phase shifts of the unit-cells are tuned by a careful design of sub-wavelength metallic features in each layer. Performing a full-wave analysis of these electrically large antennas, properly accounting the unit-cell fine sub-wavelength details is computationally challenging. To tackle this problem, various approaches have been proposed and employed [2]-[4]. PO/GO based methods, although extremely fast, have limitations regarding the estimation of gain, scan loss, and side lobe, all essential parameters to assess the performance of the TA.

In this communication, we show that scattering-based homogenization methods can be applied to a wide class of non-periodic planar structures. We focus on passive TAs, although the theory can be easily applied to reflect-arrays. We evaluate the efficiency of the proposed method in terms of accuracy and resource saving. The equivalent TA is assembled by replacing each unit-cell by a homogenous lossless isotropic dielectric material with proper permittivity

and permeability values. Although homogenization theory is a mature topic [5],[6], it is usually employed for the analysis of periodic structures. Reference [3] is one exception, but it still does not answer the following questions: is it possible to have a homogenization process that accurately evaluates a vast class of TAs? If so, what computational advantages can be achieved?

We applied the developed method for the analysis of a wide-angle beam scanning TA [1]. The shape of the main beam was kept when compared to the full simulation model, with antenna gain differences less than 1 dB. With the proposed method, the simulation ran more than 3 times faster and the memory was reduced by half.

## II. HOMOGENIZATION OF UNIT CELLS

By applying suitable homogenization techniques to the real unit-cells, we can get an equivalent, and yet simpler description of the original TA. Some assumptions are needed. First, the homogenization theory is valid under the assumption that the unit-cell is sub-wavelength, which fits into the design requirements of TAs. In fact, the size of the TA unit-cell is usually smaller half of the free-space wavelength to avoid grating lobes. Another assumption is that the scattering responses of the real and equivalent cells are relatively independent of the angle of incidence. This is also a usual requirement when designing TA, thus making this prerequisite valid for most unit cell designs. Moreover, an isotropic material description of the cells is considered. This restriction applies to cells that are polarization insensitive which is also a usual specification, especially for satellite applications. Finally, we are assuming the lossless case which is a reasonable approximation for typical TA unit-cells.

We adopt a homogenization method based on the unit-cell scattering parameters. Each TA unit-cell is excited by a linearly polarized plane wave for normal incidence. By analyzing the unit cell using periodic boundary conditions (i.e. as a frequency selective surface) the transmission,  $T$ , and reflection coefficients,  $\Gamma$  are known *a priori*. One possible option to define the material properties is to match these

coefficients to the ones of a homogeneous dielectric slab with relative permittivity  $\epsilon_r$  and permeability  $\mu_r$ . However, the corresponding inversion may lead to some problems such as excessive phase sensitivity of the model near Fabry-Perot resonances and multiplicity of the solutions [6]. This inversion can be greatly simplified if the multi-reflections inside the cell are neglected, which can be reasonable to do in the context of quasi-transparent unit-cells. In this case we can simply consider that

$$\begin{aligned} T_{eq} &= \frac{2\sqrt{\mu_{eq}}}{\sqrt{\epsilon_{eq}} + \sqrt{\mu_{eq}}} e^{-j\sqrt{\epsilon_{eq}\mu_{eq}}H_{cell}}, \\ \Gamma_{eq} &= \frac{\sqrt{\mu_{eq}} - \sqrt{\epsilon_{eq}}}{\sqrt{\mu_{eq}} + \sqrt{\epsilon_{eq}}} \end{aligned} \quad (1)$$

where  $H_{cell}$  is the height of the unit cell. The equivalent model is driven by matching the transmission ( $T_{eq}$ ) and reflection ( $\Gamma_{eq}$ ) coefficients of the equivalent material with the transmission ( $T_r$ ) and reflection ( $\Gamma_r$ ) coefficients of the real cell, respectively. Because the working principle of the TA only depends on the relative phase difference between the unit-cells, an arbitrary constant phase  $\varphi_0$  can be defined to provide an additional degree of freedom. Therefore, it follows

$$\begin{cases} |T_{eq}| = |T_r| & |\Gamma_{eq}| = |\Gamma_r| \\ \arg(T_{eq}) = \arg(T_r) + \varphi_0 \end{cases} \quad (2)$$

The value of  $\varphi_0$  allows additional control of the permittivity and permeability equivalent parameters  $\varphi_0$ .

Let us consider as an example the unit-cell family developed in [1]. This PD set is composed of  $N = 63$  cells designed to operate at 30 GHz. Each cell has five layers of stacked square patches with various dimensions ( $L_1, L_2, L_3, L_2, L_1$ ), see Fig. 1. The square shape of the patches ensures the cell response is the same to  $x'$ - and  $y'$ -polarized incident waves. This allows to consider an equivalent isotropic medium description for each cell, i.e.,  $\epsilon_{eq}^k$  and  $\mu_{eq}^k$ . The superscript  $k$  discriminates the different unit-cells, with  $1 \leq k \leq N$ . Inverting this problem gives the set of equations

$$\epsilon_{eq}^k = -\frac{\arg(T_r^k) + \varphi_0}{k_0 H_{cell}} \frac{1 + |\Gamma_r^k|}{1 - |\Gamma_r^k|} \quad (3)$$

$$\mu_{eq}^k = -\frac{\arg(T_r^k) + \varphi_0}{k_0 H_{cell}} \frac{1 - |\Gamma_{PD}^k|}{1 + |\Gamma_{PD}^k|} \quad (4)$$

One should stress, that there is an equally valid model where the permittivity and permeability values are interchanged. Moreover, the value of  $\varphi_0$  is chosen such that, for every cell,  $\mu_{PD}^k \geq 1$  and  $\epsilon_{PD}^k \geq 1$ . This condition restricts the analysis to more conventional parameters values where  $n_{PD}^k = \sqrt{\epsilon_{eq}^k \mu_{eq}^k} \geq 1$ . Thus, for the lossless and dispersive regime that we assume we get a causal description of all cells,

otherwise we would get superluminal energy velocities. The refractive index of the equivalent material is given by

$$n_{eff}^k = \frac{|\arg(T_{eq}^k) + \varphi_0|}{k_0 H_{cell}} \sqrt{\frac{1 + |\Gamma_{eq}^k|}{1 - |\Gamma_{eq}^k|}} \quad (5)$$

Therefore,  $\varphi_0$  can also be adjusted to limit the maximum value of  $n_{eff}^k$  which defines the electrical size of the problem that we intend to keep as low as possible to improve the simulation efficiency. We should stress that the required number of meshing cells for the full-wave simulations depends on the wavelength inside the structure, meaning that higher refractive indices implies a denser discretization. The scattering parameters for the PD unit cells are represented in Fig. 2. These cells have high transmission (above -0.3 dB) and low reflections (below -11 dB). The equivalent material parameters are represented in Fig. 3 as the corresponding refractive indices  $n_{PD}^k$  and impedances  $Z_{eq}^k = \sqrt{\mu_{eq}^k / \epsilon_{eq}^k}$ .

According to the aforementioned criteria, we get  $\varphi_0 = -333.9^\circ$  which guarantees the lowest refractive index values ( $1.3 \leq n_{eq}^k \leq 4.4$ ) with  $\epsilon_{eq}^k \geq \mu_{eq}^k \geq 1$  for every cell. In Fig. 4, we compare the magnitude of the reflection and transmission coefficients for PD and equivalent unit cells. We can observe that the transmission coefficients of the PD unit-cell and the equivalent unit-cell are very similar, whereas some differences may occur in the reflection coefficients when there is higher impedance mismatch with free-space ( $Z_{eq}^k / \eta_0 < 1$ ).

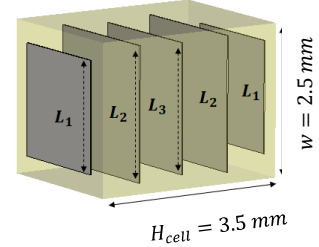


Fig. 1 structure of the PD cell introduced in [1].

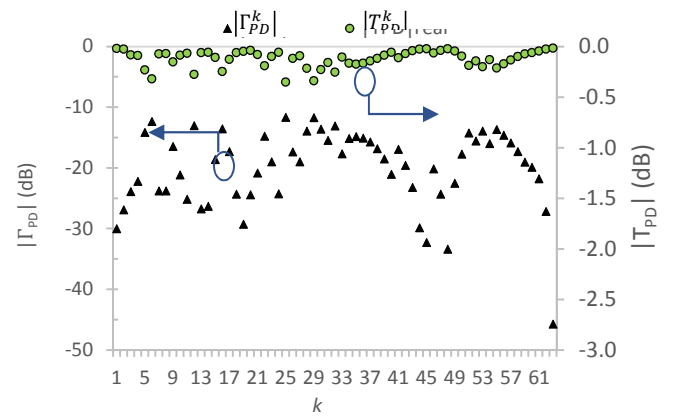


Fig. 2 – Scattering parameters of the PD unit-cells [1] for 30 GHz.

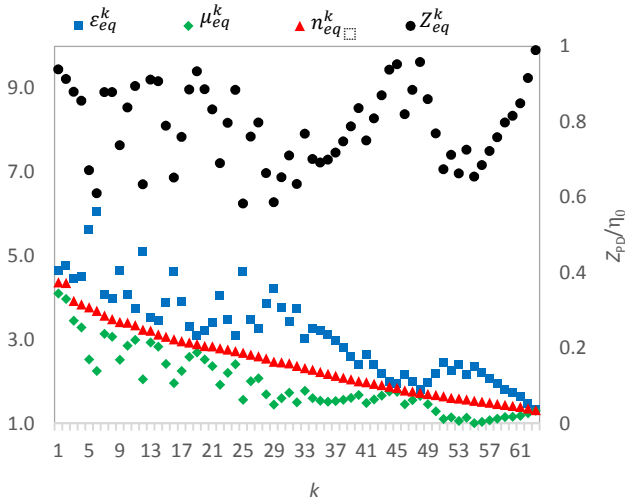


Fig. 3 –Equivalent constitutive parameters of the PD unit-cells for 30 GHz.

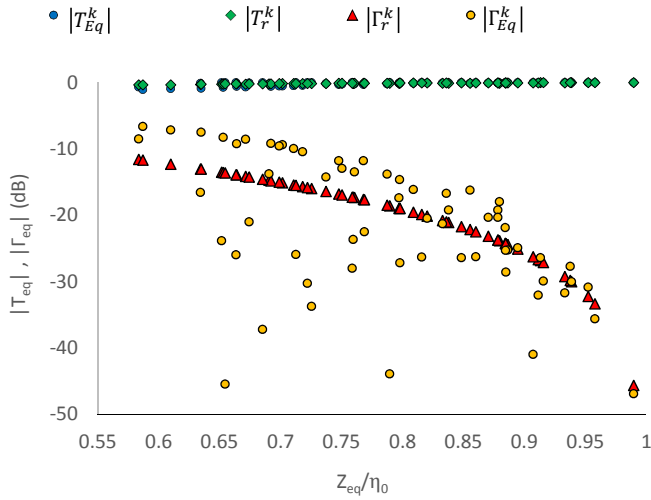


Fig. 4 Transmission and reflection coefficients for PD and equivalent cells as a function of the medium impedance.

### III. EQUIVALENT DIELECTRIC TA

The TA has a rectangular  $140\text{mm} \times 190\text{mm}$  aperture and focal distance of  $100\text{mm}$  [1]. The maximum gain is 30 dBi at 30 GHz and beam-steering is achieved by translating the feed along the longer axis of the TA with constant  $z$ . The feed is a 14.5 dBi standard gain Ka-band horn. The Hybrid TLM-FDTD approach, available in the  $\text{\textcircled{R}}$  CST Microwave Studio, is used for the analysis of the TA. The simulations were performed with a high-end workstation (Dell T7610, 128 GBytes RAM, 2 Processors at 2.5 GHz) using Tesla K20c Graphic process units (GPUs) to accelerate to computing time of this electric large problem. The simulated radiation patterns of the TA and the equivalent TA are presented in Fig. 5. The radiation patterns are obtained at 30 GHz for different  $x$ -positions of the horn:  $a=+30\text{mm}$ ,  $0\text{mm}$ , and  $-27\text{mm}$ . It can be seen from this figure that the TA and

the equivalent TA direct the beam in the same direction for all the positions of the feed. Moreover, the gain of the two TAs in the beam maximum direction is in great agreement with a maximum difference of only 0.5 dB for  $a=0\text{mm}$ . Furthermore, the equivalent TA also presents a good estimation of the side lobe levels and even of the back lobe. The equivalent TA full-wave simulation required about 43 minutes and 3.84 Gbytes of GPU memory, whereas using the real TA takes more than three times that amount of time and almost twice the memory

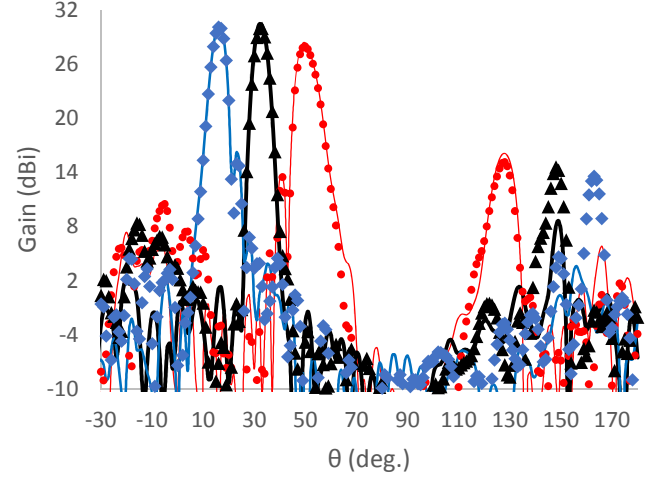


Fig. 5 Full-wave simulated TA (points) and equivalent TA (solid lines) at 30 GHz.

Table I summarizes the required memory and time to simulate both TAs. It is shown that the equivalent TA provides the presented results in Fig. 5 in only about 43 minutes and required memory of 3.84 Gbytes, whereas the PD TA takes more than three times that amount of time and almost twice the memory. As mentioned, as the complexity of the problem is further scaled, the gain of this method is expected to become even higher.

TABLE I  
COMPARISON BETWEEN REQUIRED RESOURCES FOR FULL-WAVE SIMULATION OF THE PD TA AND THE EQUIVALENT TA IN CST

Machine: Dell T7610, 128 GBytes RAM, 2 Processors at 2.5 GHz		
TLM – mesh (CST)	PD TA	Equivalent TA
Time step ( $10^{-14}$ s)	3.9874886	10.00374
Number of time Steps	182 764	72 850
Simulation time	148m 54 s	42m 56 s
Number of used GPUs	2 GPU Tesla K20c	2 GPU Tesla K20c
Allocated memory (Gbytes)	7.429	3.843

#### IV. CONCLUSIONS

The paper presents a simple and yet effective homogenization method to replace complex-shaped TA unit-cells, useful for highly resource-demanding simulations. The method was tested on a previously published wide-angle beam steering TAs working in the Ka-band. The equivalent cells performed very close to the actual ones, providing very good estimates of the TA beam directions, beam gain, gain scan loss and side-lobe level. The simulation time could be reduced by a factor of 3, confirming the effectiveness of the proposed method. The method can be adapted to reflect-array cells as well.

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#### REFERENCES

- [1] E. B. Lima, S. A. Matos, J. R. Costa, C. A. Fernandes, and N. J. G. Fonseca, “Circular polarization wide-angle beam steering at Ka-band by in-plane translation of a plane lens antenna,” *IEEE Trans. Antennas Propag.*, vol. 63, no. 12, pp. 5443-5455, Dec. 2015.
- [2] P. Naseri, S. A. Matos, J. R. Costa, and C. A. Fernandes, “A fast computational algorithm to evaluate large transmit-arrays,” in Proc. 12th Eur. Conf. Antennas Propag. (EuCAP), London, UK, Apr. 2018, pp. 1–4.
- [3] Seyed Mohamad Amin Momeni Hasan Abadi and Nader Behdad, “Design of wideband, FSS-based multibeam antennas using the effective medium approach,” *IEEE Trans. Antennas Propag.*, vol. 62, no. 11, pp. 5557-5564, Nov. 2014.
- [4] H. Kaouach, L. Dussopt, J. Lantéri, T. Koleck, and R. Sauleau, “Wideband low-loss linear and circular polarization transmit-arrays in V-band,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 7, Jul. 2011.
- [5] D. J. Bergman, “The dielectric constant of a composite material — a problem in classical physics,” *Phys. Rep.* 9, pp. 377 – 407, 1978.
- [6] N. G. Alexopoulos, C. A. Kyriazidou, and H. F. Contopanagos, “Effective Parameters for Metamorphic Materials and Metamaterials Through a Resonant Inverse Scattering Approach” *IEEE transactions on microwave theory and techniques*, vol. 55, no. 2, 2007