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Department of Economics / Department of Political Economy

## **The effects of Quantitative Easing on U.S. Inflation and Output**

André Manuel Da Silva Ribeiro

Master in Economics

Supervisor:

Ph.D. Economics, Diptes Chandrakante Prabhudas Bhimjee, Invited Assistant Professor

ISCTE – IBS

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# Abstrato

A presente Dissertação tem como objetivo fornecer informação quanto ao impacto dos Programas de Q.E. (i.e., *Quantitative Easing*) na Produção e na Inflação, nomeadamente, como a compra de títulos de dívida pública americana de diferentes maturidades e M.B.S (i.e., *Mortgaged-backed securities*) impactam as referidas variáveis. Esta Dissertação foca-se no período pós-crise entre o segundo trimestre de 2009 e o último trimestre de 2019, usando como base de dados os recursos disponibilizados pela Reserva Federal dos Estados Unidos. Esta Dissertação usa dados com uma periodicidade mensal para seis séries temporais. O número total de observações é de 129 observações (N=129) para cada. Utilizando uma metodologia VAR (i.e., Vector Auto-Regressive) esta Dissertação conclui que as políticas de Q.E. produzem um impacto tanto na Produção Industrial como na Inflação. A natureza deste impacto (i.e., positivo ou negativo) não conseguiu ser definitivamente aferida devido ao seu comportamento intermitente. No entanto, para a Produção Industrial, o sinal do impacto acabou em valores positivos para as M.B.S e para os títulos com uma maturidade entre 5 e 10 anos. Em relação à Inflação, quase todos os títulos de dívida considerados, bem como as M.B.S, acabaram com valores positivos, a exceção foi o título de dívida pública com uma maturidade de 10 anos. Em suma, a hipótese de que os programas de Q.E. não provocam qualquer tipo de reação por parte da Produção Industrial e da Inflação é rejeitada.

Códigos JEL: E52, E58

*Palavras-Chave:* Política Monetária, Quantitative Easing, Estados Unidos, Inflação, Produção Industrial

# Abstract

The present dissertation aims to provide insight on how Quantitative Easing Programs impacts both GDP and CPI, specifically, how the purchase of securities ranging from U.S. Treasury Securities to Mortgaged-backed Securities (i.e., MBS) impact said variables. It focuses on the post-crisis period between the second quarter of 2009 and the last quarter of 2019, using U.S. Federal Reserve data resources. It uses monthly data collected for six time-series, giving a total of 129 observations (N=129) for each. Using a Vector Auto-Regressive (i.e., VAR) approach, this dissertation concluded that Q.E. triggers a response in both Industrial Production and CPI, persistent for at least 15 lag periods. This response is stronger for Industrial Production, having a comparatively weak impact on CPI. The nature of this impact (i.e., positive or negative) could not be definitely inferred, as, across the 15-lag period, the behavior was mostly intermittent. However, for Industrial Production the sign of the response ended on a positive value for M.B.S and U.S. Treasury Securities maturing in 5 to 10 Years. For CPI all the U.S. Treasury securities and M.B.S ended on a positive value, with the exception of the U.S. Treasury Securities Maturing in 10 Years. Thus, the claim that Q.E. Programs do not trigger any type of response at either the Industrial Production or CPI level is safely rejected.

JEL codes: E52, E58

*Keywords:* Monetary Policy, Quantitative Easing, United States, Inflation, Industrial Production

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# 1. Introduction

In the last decades of the 20<sup>th</sup> century the United States struggled with the problematic issue of high inflation. In this fight, several traditional monetary policy measures were pursued to address not only this issue but also short-term fluctuations in employment and output. The outcome was a success, insofar as the U.S. returned to low and well-established inflation levels and expectations, as well as witnessing a steep rise in employment and output across the nation. Despite the positive result that followed shortly after the Monetary Policy measures, later years of chronic low inflation deemed problematic. Taking the case of Japan, which went through a similar situation, it was clear that low inflation could rapidly turn into a self-perpetuating trap, because in this case, the low levels of inflation posed a challenge for the traditional methods.

During the first decade of the 21<sup>st</sup> century, with the onset of the Global Financial Crisis of 2007-2009, most of the advanced western economies fell into a state of deep recession, beyond the scope and help of traditional containment measures. First the countries' central bank authorities cut the short-term interest rate to the unprecedented level of zero (or close to it). This, alone, was not capable of containing the crisis, propelling Monetary Policy Authorities to undertake extraordinary measures, later known as Quantitative Easing Programs.

Today, after little more than a decade from these events, the Federal Reserve's main macroeconomic models predict that the use of the pre-crisis methods would result in a constraint of short-term interest at the zero level, in as many as one third of the time (**Kiley and Roberts, 2017**).

These alternative measures, which essentially consist of large-scale purchases of financial assets, used in combination with public disclosure (i.e., detailed descriptions of the type of asset purchases), are seen by many economists as having contributed substantially to the economic recovery. As such, it is a major objective of this Dissertation to critically address and review the specific effects of the U.S. Quantitative Easing program in the United States' inflation and output, since these monetary programs were implemented. In order to better understand this important research topic, the following chapter introduces the theme, by first defining Q.E.

The present Dissertation uses a VAR Methodology in order to answer the empirical research question addressed in the present Dissertation. The dataset uses monthly variables, all directly retrieved from the Federal Reserve Bank of St. Louis.

The present Dissertation's findings suggest that Q.E. Programs that consist in the purchase of U.S. Treasury Securities and M.B.S have a significant impact both at the CPI and Output level. This impact is comparatively larger in Output than in CPI.

The present Dissertation is organized as follows: section 2 provides a contextualizing chapter to Q.E.; section 3 analyses the most relevant academic literature; section 4 describes the main methodology and data herein used, while section 5 discusses the motivation behind the empirical method; section 6 describes and critically discusses the empirical findings; and, lastly, section 7 concludes.



## **2. Defining Quantitative Easing - contextualization and evolution**

### **2.1. Defining Q.E.**

According to **Williamson, S. (2018)**, “*Q.E. consists of large-scale asset purchases by central banks, usually of long-maturity government debt but also of private assets, such as corporate debt or asset-backed securities. Typically, Q.E. occurs in unconventional circumstances, when short-term nominal interest rates are very low, zero or even negative.*”

**(Williamson, 2019: Introduction.)**

In sum, Quantitative Easing increases the money supply by purchasing assets with newly created bank reserves in order to provide banks with more liquidity. This, however, is not without a cause, so what are, in theory, the effects of Q.E. on the Output and Inflation (i.e., Gross Domestic Product and CPI)?

### **2.2. The theoretical effects of Q.E.**

If central banks simply increase the money supply, it can cause widespread inflation. In a worst-case scenario, the central bank may cause inflation through Q.E. without economic growth, causing a period of so-called stagflation. Although most central banks are created by their countries’ governments and are involved in some regulatory oversight, central banks can't force the banks under its oversight to increase lending or force borrowers to seek loans and invest. If the increased money supply does not work its way through the banks’ balance sheet and into the economy, Quantitative Easing may not be effective, except as a tool to facilitate deficit spending.

Another potentially negative consequence is that Quantitative Easing can devalue the domestic currency. For manufacturers, this may help stimulate growth because exported goods would be cheaper in the global market. However, a falling currency makes imports more expensive, which can increase the cost of production and consumer price levels.

### **2.3. Federal Reserve balance sheet evolution in recent times**

In order to better grasp the implications and magnitude of these programs it is relevant to analyze the Federal Reserve Balance Sheets for the period between 2007 and 2020 (**Figure 1**). The starting point to Q.E. is clearly visible, as the program was initialized in the midst of the 2008

financial crisis. Afterwards, the total assets held by the Federal Reserve surged quite significantly, capping at an unprecedented level of 4.516.077 Million US Dollars, a value 5.3 times greater than its pre-crisis maximum for the period (equal to 851.755 Million US Dollars).

The upward spikes in **Figure 1** are, of course, coincident with the Federal Reserve release of its Quantitative Easing programs, denominated Q.E1- Q.E3 and *Operation Twist*, between December 2008 and October 2014 respectively.

Moreover, the size of the programs is of great relevance, as well as the type of assets purchased in each program. Both are of great concern to central bankers, as according to **Williamson, S. (2018)**, “*In essentially all of the Q.E. programs conducted in the world during and after the financial crisis, central banks seemed primarily interested in how the type and quantity of asset purchases would affect financial market conditions and, ultimately, inflation and aggregate economic activity*” (**Williamson, 2019: What is Quantitative Easing?**). It is essential to note, after a first analysis on the nature of these programs, the switch to assets of higher average maturity, from Short-term Maturity T-Bills to Long-term T-Bills. This shift was so abrupt that between 2007 and late 2014 the average duration of the Federal Reserve portfolio increased from 1.6 years to 6.9 years (**Engen, Laubach, and Reifschneider, 2015**). This is also noteworthy if we analyze the chronogram of the Time Series in **Figures 2 to 5**. In addition, the following table provides a summary analysis regarding the increase or decrease in the Holdings Value of each Security type.

<b>Holdings of Short-Term and Long-Term Securities</b>			
Short Term Securities (less than a year)	91 days to 1 year	N/A	N/A
Holdings after 2008 crisis	Decrease	-	-
Long Term Securities (more than a year)	1-5 years	5-10 years	>10 years
Holdings after 2008 crisis	Increase	Increase	Increase

**Figure 6 – Transition from Short to Long-term Securities by the Federal Reserve**

## 2.4. The theory behind the switch in the asset's average maturity

Quantitative Easing works through two main channels: 1) by reducing the net supply of long-term assets, which increases prices and lower their yields; 2) by signaling policymakers' intention to keep short-term interest rates low for an extended period. Both channels helped ease financial conditions in the post-crisis era, so if the Q.E. successfully manages to reduce the long-term interest rates the economy will respond much in the same way that it does to conventional monetary easing, as a lower cost of capital, higher wealth, a weaker currency, and stronger balance sheets increase spending on domestic goods and services. In sum, this can be seen in Eq. (1), as a bond yield is given by:

$$\text{Current Yield} = \frac{\text{Annual Coupon Payment}}{\text{Bond Price}} \quad (1)$$

By reducing the net supply, the denominator (i.e., the Bond Price) increases, which will lower the current yield value. This in turn translates into lower interest rates offered on loans (i.e., mortgages or business loans) as rates on government bonds tend to affect other interest rates in the economy, leading to increased spending. Additionally, following a similar line of reasoning, as mortgage rates decline, the demand for housing is likely to increase as well. All this aggregate spending is bound to stimulate the economy. However, one must always take into consideration that this type of market operations should not be viewed simply as mechanism to provide increased liquidity in the midst of a crisis. In fact, the said mechanism is more complex, as the central bankers must consider the size, type, and timing of these asset purchases to mitigate potentially negative scenarios as mentioned before.

The following section presents the conducted literature review, detailing the most relevant research on Quantitative Easing and its effects on yield prices, and thus, on Output and Inflation.



### 3. Literature Review

The effects of Quantitative Easing (Q.E.) have been extensively researched, first starting by how it effects the pricing of different classes of assets. **Gagnon et al. (2011)** observes that over a horizon of 5 days after press release, yields on 10-year U.S. treasuries dropped a full percentage point, and its MBS 10-year counterpart far more than this value. This result holds substantially well even for shorter or longer event windows (**Joyce et al, 2011**).

However, the same event study methodology, when applied to later rounds of Q.E. has shown to have fewer substantial results, even when adjusting for the difference in program size (**Krishnamurthy and Vissing-Jorgenson, 2011**). A possible reason for this loss in impact is that the first round of Q.E. provided critical liquidity to the financial markets in a period of dire need. This hypothesis suggests that, as a policy tool, Q.E. is only to be used in periods of high financial distress, limiting its use in regular, more controlled periods.

The second biggest critique is that these studies, based on event windows, capture asset price changes over a short time, revealing only short-term liquidity effects, which are expected to dissipate within a short time span. If, for instance, we consider the 10-year yield after the first Q.E. program (i.e., Q.E1) was implemented, between December 2008 and December 2015, its value was, surprisingly, higher than before the intervention, thus supporting the work of **Wright (2011)** which argues that the effects of Q.E. are mitigated fairly quickly.

A counter-argument for the perceived loss in effectiveness of subsequent rounds of Q.E. is, as reported by **Gagnon (2018)**, that these same effects were already anticipated by the financial markets, and thus, incorporated in the first round of Q.E. This, in fact, should be considered a robust counter-argument as primary market dealers had higher expectations regarding the size of the second round, prior to its announcement, thus, when the official details were released, the price of 10-year yields rose slightly, reflecting the disappointment of the investors. A solution to this type of issues, that emerge from the event study methodology, is to extend the event day horizon, thus capturing multiple announcements, as explained by **Gagnon (2018)**. This, however, adds additional noise from non-monetary policy related news, further affecting asset prices.

Other authors focus on how the asset purchases were handled. Looking at the composition of the purchase's portfolio, namely the M.B.S. to Treasuries ratio, Q.E1 had a substantially higher ratio than Q.E2, which included more U.S. Treasuries in relative terms. If the type of assets is a significant matter, then an unexpected change in the mix would result in significant impacts on the

yields of these assets. This, indeed, seems to be the case, when considering the evidence from **Krishnamurthy and Vissing-Jorgenson (2011)**. This research points that, when comparing Q.E1 and Q.E2, the program with higher M.B.S. to U.S. Treasury ratio reported having seen a drop in yields of M.B.S. greater than its counterpart. **Neely (2010) and Gagnon et al. (2011)** also build on this reasoning that Q.E. is an effective tool, as both researchers argue that even assets not subject to purchases (e.g., corporate bonds, equities, and foreign assets) by the U.S. Federal Reserve moved substantially much in the same way as it would be expected from a conventional monetary policy approach. Moreover, Q.E. stimulates the issuance of corporate bonds (**Lo Duca, Nicoletti, and Martinez, 2016**) and triggers a lower cost of insuring against corporate credit risk via Credit Default Swaps (**Gilchrist and Zakrajsek, 2013**).

Overall, the great critique against Q.E. effectiveness is that it is non-persistent over time, having only transitory effects on long-term yields, due to pure liquidity effects. This argument does not hold robust once we control for the anticipation of future rounds of Q.E. by market participants. However, if this was true, then, ultimately, proficient investors would profit from betting on the mitigation of the effects on asset pricing. This is a difficult assumption to hold, as **Neely (2016)** shows that time-series models implying mitigation effects did not predict as well as other models, that relied on the simple assumption that prices today will be the same as tomorrow. Continuing to support the robustness of the hypothesis that Q.E. does have persisting effects on yield prices, **Ihrig et al. (2018)**, estimate an arbitrage-free model which allows current and expected holdings of securities by the U.S. Federal Reserve to influence yields, finding significant long-term effects from the implementation of these programs. An estimation of cumulative effects of the purchases on the 10-year U.S. Treasury yields exceeded 120 basis points when net purchases ended in October 2014, and was still about 100 basis points as of the end of 2015; thus, the supposed argument for loss in effectiveness of later rounds of Q.E. does not hold well. Also supporting the long-term effectiveness of Q.E., **Wu (2014)**, credits the U.S. Federal Reserve policy for the 217 base point decline in 10-year U.S. Treasury yields between 2008 and 2013. In sum, research seems to support, with substantial results, that Q.E. is indeed an effective monetary policy tool, having real and persistent effects on asset prices and thus in the economy (i.e., Gross Domestic Product).

Another approach that is reinforced with the new Q.E. policies is the idea of forward guidance, which is, in simple terms, presenting to the general public a description of the measures Central Banking authorities will implement, how they will be implemented, and when they will be

implemented. The idea is to provide insight on how monetary policy makers expect the economy and policy to evolve over time. Although it is not a novel approach, having been already used during the pre-crisis period on several policy announcements, if we are to research how Q.E. policies impact Inflation and Output, it is relevant to analyze every component of such policies, including the ones which are used simultaneously, in support of Q.E. programs, as is the case with forward guidance.

Pre-crisis research by **Gürkaynak, Sack, and Swanson (2005)**, who use an event study methodology with high-frequency data, indicates that the effects on asset pricing can be encapsulated in two factors: 1) reactions to unexpected changes in the Federal Reserve Funds Rate; and 2) expectations on the future path that the Federal Reserve Funds Rate may take. These are also called implicit and explicit factors of policy announcements (**Gürkaynak, Sack, and Swanson, 2005**). Forward guidance point 2), associated with expectations on the future path of the Federal Reserve Funds Rate, seems to be more influential in determining U.S. long-term yields. Providing remarkable insight on the effectiveness of such measures, **Woodford (2013)** states that exposing insight on future outlook is a valuable approach at the zero-lower bound, as optimal monetary policy in those circumstances may be relatively time-inconsistent, meaning that the commitment made today to achieve a set interest-rate, or other related actions, will ultimately be inconsistent as the situation changes and incentives to keep those settings differ. As an example, the author observes that when short term rates cannot be lowered any more, institutions may want to reduce long term rates by communicating that they intend to keep short term rates lower for an extended period of time; nevertheless, this might constitute a risky assessment, as it involves a potential overshoot of the inflation target. Communicating with clarity also creates a reputational stake for the central bank to follow through, providing, perhaps, more motivation to strive for effectively implementing the policies announced. Reinforcing the role for clarity, **Campbell et al. (2017)** points that the U.S. Federal Reserve forward guidance policy greatly benefits from clarity in expositions, which lead to improved macroeconomic effects, as the probability of misinterpretation by the target audience is vastly reduced.

Corroborating these findings, **Gust et al. (2017)**, using a D.S.G.E. model, find that market agents gradually assimilate the message: lower for longer Federal Funds rates, so the effects on the 2-year U.S. treasury yields only became apparent after a few months. Thus, the clarity of exposition poses as a factor which contributes to the lowering of this lag period for effectiveness.

Until this point, we discussed evidence showing Q.E. Programs to be effective in lowering the yields of specific securities, but this also works as a channel to stimulate economic activity by incentivizing spending. However, it is also quite relevant to analyze literature that studies the direct effects of Q.E. Programs on Price levels (i.e., CPI) and Output (i.e., GDP, Industrial Production). Research by **Chenkelberg and Watzka (2013)** enlightens this question, as they address Q.E. conducted by the Bank of Japan; nonetheless its findings are relevant across other developed nations such as the United States. Their main findings argue that unconventional policy actions can positively impact Price levels and Industrial Production, even when the economy is at the Zero-Lower Bound. However, the Q.E. shock reported was rather weak and of temporary nature, taking time to show its reach, namely about 2 periods.

On the other hand, **Girardin and Moussa (2011)** argue that quantitative easing was able to provide considerable stimulation to both output and prices. Supporting the line of argumentation of the latter, **Matsuki and Satoma (2015)** also mention that the purchase of long-term securities increases economic activity; in addition, it lowers rates, positively impacting inflation rates. A common point across the two previously mentioned researches is that Q.E. does have real and measurable effects on the economy, they differ in regards to the size and persistence of such effects.

Even considering this fact, the majority of the literature seem to point towards a positive and persistent effect both for CPI and Output, either considering the direct researches or the researches measuring the effective lowering of the securities yield (which, in this case, subsequently translate into economic effects). Additionally, it should be observed that forward guidance and Q.E. are quite closely related and should be combined for increased effectiveness. Accordingly, it is of the utmost important that Central Banks continue to improve communication frameworks, incorporating a more systematic approach to forward guidance, as well as promoting further clarity of exposition. Empirical evidence on Q.E. shows that it continues to be helpful even beyond crisis periods, once accounting for market participants' expectations.



## 4. Data

### 4.1. Purpose of the Study

The purpose of this Dissertation is to assess the impact of Q.E. Programs, which started in December 2008 and lasted until October 2014, on two key Macroeconomic Variables:

- 1) **GDP - US Output.**
- 2) **CPI - Consumer Price Index.**

### 4.2. Defining the time window

The U.S. N.B.E.R. (i.e., National Bureau of Economic Research)<sup>1</sup> dates the turning point from the last trough as of the second quarter 2009. The recovery and expansionary period following the U.S. ‘Subprime’ Crisis lasted until early 2020, before the COVID19 Outbreaks.

During this research, the outbreak related events and its corresponding time period will not be included, as this inclusion might introduce “noise” in the statistics, as it describes an extraordinary circumstance, and, also, to prevent mixing an expansionary period with the beginning of a recessionary one.

**Defining the Expansionary Period (~10 Years):** First of April 2009 (Second Quarter of 2009) – First of December 2019 (Last Quarter of 2019).

### 4.3. Data Overview

Since the time window is defined for this Dissertation, it is now relevant to specify which time-series are to be included. First, an overview will be provided consisting of an economic theory analysis to support the reason why each variable is to be included. This is followed by the corresponding descriptive statistics and a short summary of the characteristics of the data. The dataset’s periodicity is monthly and seasonally adjusted. The dataset was retrieved from the Federal Reserve Bank of St. Louis. The focus of the study is to assess the impact of Q.E. on key

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<sup>1</sup> NBER (2020). *US Business Cycle Expansions and Contractions*. Available at: <https://www.nber.org/cycles.html>.

economic variables: GDP and CPI since its implementation in 2009 until the end of 2019. The motivation regarding this time window choice is to fully address the impact of the Q.E. programs.

Accordingly, the following variables were selected, given the actual composition of the Quantitative Easing programs: (i) Securities Held Outright: U.S. Treasury Securities: Maturing in Over 1 Year to 5 Years (Wednesday Level), (ii) Securities Held Outright: U.S. Treasury Securities: Maturing in Over 5 Years to 10 Years (Wednesday Level), (iii) Securities Held Outright: U.S. Treasury Securities: Maturing in Over 10 Years (Wednesday Level), (iv) Securities Held Outright: Mortgage-Backed Securities (Wednesday Level).

However, it is not possible to directly retrieve monthly data from official sources for GDP, as this variable is subjected to Quarterly measurements. As such, an alternative would be to take into consideration the Industrial Production as a proxy for GDP. Industrial Production has monthly data readily available from the official sources. Given this fact, the remaining variables are: (i) Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (CPI) and (ii) Industrial Production: Total Index.

The reason for choosing Industrial Production is that can be a good proxy for GDP if a very high correlation is present between both time-series (i.e., GDP and Industrial Production), as is the case (**Figure 7 – RStudio Output**).

A summary table of the RStudio output can be consulted below.

<b>Correlation between Gross Domestic Product and Industrial Production for the period: 2009-2019</b>	
Method	Correlation Coefficient
Spearman	0.9216249
Pearson	0.9097852

**Figure 8 – Correlation between G.D.P. and I.P. (Summary)**

## 4.4. Dependent Variables

- **Industrial Production**

A measure of the Output of the Industrial Sector of the Economy. For the United States this sector represented 11,154% of the U.S. GDP, as of 2017, and includes the following: manufacturing, mining, electric, and gas utilities.<sup>2</sup>

- **CPI**

The purchasing of financial assets leads to an increase of these assets held in the Federal Reserve Balance sheet. This market operation injects money into the economy, increasing the money supply, usually the higher the supply the lower the cost. Thus, this leads to a lower cost of money, which, in turn, is translated into higher Prices across Items in the U.S, discouraging saving as the purchasing power of deposits decreases and leading to an incentive for business and consumers to invest and spend.

## 4.5. Independent Variables (Q.E. Variables)

Q.E. programs focus on the purchase of specific types of financial assets. The bulk of these purchases are related to Long Term U.S. Treasury Securities maturing in over a Year and M.B.S (i.e., Mortgaged-Backed Securities). As such the Q.E. variables to include are:

- 1) **Securities Held Outright: U.S. Treasury Securities Maturing in Over 1 to 5 Years**
- 2) **Securities Held Outright: U.S. Treasury Securities Maturing in Over 5 to 10 Years**
- 3) **Securities Held Outright: U.S. Treasury Securities Maturing in Over 10 Years**
- 4) **Securities Held Outright: Mortgage-Backed Securities**

By purchasing these types of assets, the Federal Reserve lowers its net supply, increasing market prices and lowering yields. As a final result, this process ends up lowering the cost of money because the Federal Reserve injects money into the financial system.

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<sup>2</sup> World Bank (2020). *Manufacturing, Value Added (% Of GDP) - United States*. Available at: [https://data.worldbank.org/indicator/NV.IND.MANF.ZS?name\\_desc=false&locations=US](https://data.worldbank.org/indicator/NV.IND.MANF.ZS?name_desc=false&locations=US)

## **4.6. Descriptive Statistics**

In this section the descriptive statistics for each time-series will be presented. As such the following table contains: Maximum and Minimum values, Median, Mean, Standard Deviation, Skewness, Kurtosis and the result of the JB-Normality Tests for checking the Null Hypothesis of a normal distribution across the sample data. Additionally, the list of variables analysed: (i) Industrial Production, (ii) CPI, (iii) Securities Held Outright: U.S. Treasury Securities Maturing in Over 1-5 Years, (iv) 5-10 Years and (v) in 10 years as well as (vi) Mortgage-Backed Securities, for a total of six variables. The number of observations is 129 across the board, and this corresponds to a monthly periodicity for the period between 01/04/2009 and 01/12/2019. A short analysis of the summary Figure will be conducted on the following pages.

<b>Summary Statistics: All Variables</b>						
Descriptive Statistic	Industrial Production	CPI	MBS	U.S. Treasury 1-5 years	U.S. Treasury 5-10 years	U.S. Treasury 10+ years
Min	87.07	212.7	312534	197285	113279	104715
Median	102.54	236.3	154808 3	859839	449266	615596
Mean	101.75	235.6	136277 6	800853	491430	469865
Max.	110.55	258.4	177623 5	1241840	889104	662415
Std. Dev.	5.71	12.05	407923 .3	319124.4	236418.6	205487.4
Skewness	-0.65	-0.015	-0.5620 402	-0.277005 8	0.3520849	-0.6888461
Kurtosis	-0.089	-0.828	-1.0526 01	-1.37739	-1.270533	-1.295519
JB-Test (Normality)	p-value = 0.0095	p-value = 0.1578	p-value = 0.0017	p-value = 0.0027	p-value = 0.0035	p-value = 0.0407

**Figure 9 – Summary Statistics**

Analysis: In **Figure 9** it is important to notice that skewness values are negative across the board with the exception of U.S. Treasury Securities maturing in 5 to 10 years. This points to a distribution where the tail of the left side is longer than the tail on the right side, thus, the mean and median will be less than the mode.

In addition, in absolute terms, the skewness for CPI is very low (i.e., nearly zero) pointing for an almost symmetrical distribution. The rest of the values do not exceed 1 in absolute terms, pointing for moderate skewness.

Regarding Kurtosis, this statistic is also negative across the board, translating into a Platykurtic distribution. This is very good as it means the distribution is shorter and tails are thinner than in a normal distribution (i.e., the peak is lower and broader than in a Mesokurtic or normal distribution), thus leading to a fairly low number of outliers.

Finally, the Null Hypothesis for a Normal Distribution,  $H_0$ , is rejected for all time-series as the p-value is less than 0.05 (i.e., the 5% significance level) with the exception of the CPI which present a fairly high p-value of 0.1578.

A more graphical approach can be consulted in **Figures 10 and 11**, which show the evolution of all the variables for the period considered in this Dissertation. The most interesting fact to notice is how the Holdings Value across all the Q.E. Variables surged after 2009 as a response to the purchases by the Federal Reserve during the Q.E. programs. This is, of course, an unsurprising result, but it is important to notice how after some time the holdings of each type of security decreased with the exception of the longest-term securities held by the Federal Reserve, the securities maturing in 10 Years, which mostly maintained its value across time. Furthermore, it is also important to observe how the decrease in the holdings value of the remaining securities occurred during different stages, the first to be dropped was the Mortgaged-Backed Securities in 2011, followed by the Securities Maturing in 1 to 5 Years in 2012, while the securities considered fairly long-term (i.e., those maturing in 5 to 10 years) were only dropped in 2015.

The holdings value started to increase again between the end of 2012 and 2013, but not for the ones maturing in the 5 to 10 years range. As a consequence, the holdings value of the Mortgaged-Backed Securities and U.S. Treasury Securities maturing in 1 to 5 Years surged to unprecedented levels, reaching a high of 1.776.235 USD million for Mortgaged-Backed Securities and 1.241.840 USD million for the U.S. Treasury Securities maturing in 1 to 5 Years. After 2018, the holdings value decreased. Furthermore, it is still possible to observe how the holdings value across all

securities started to increase again in 2020, with an even more aggressive Q.E. approach, already confirmed by the Federal Reserve.

## **5. Methodology**

The present Dissertation uses a VAR regression. Unit root tests are used in order to reveal the stationarity of the time-series used in the model, and in order to determine the best model. This procedure is essential in order to provide an accurate data representation of the Impulse Response Functions.

### **5.1. Why the VAR model?**

When analysing the co-movement of several time series over time, the VAR model can be an efficient choice<sup>3</sup> as long as the data meets some basic requirements, the most important being the stationarity of the time-series. Since this Dissertation intends to capture the relationship between the 6 time-series considered, namely, the relationship between them as the quantities of each variable change over time, the VAR model seems to be a good fit, as this model generalizes an autoregressive model by allowing for multivariate time series. Additionally, on similar research mentioned previously in the literature section, a VAR based approach is often used, with significantly good results.

### **5.2. Advantages of the VAR model**

Choosing the VAR framework holds several, significant advantages, namely:

- 1) Choosing the Variables originates a problem in other models: which variables are exogenous and which variables are endogenous? The VAR model doesn't have this problem, as all variables are endogenous.
- 2) The VAR model allows the value of a variable to depend on its own lags and the lags of other variables, offering a robust structure which may be able to capture more characteristics of the data. Specifically, when lags are fitted, the relation between Q.E.

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<sup>3</sup> Christiane Bjørnland, H., 2000. *VAR Models In Macroeconomic Research*. Available at: [https://www.ssb.no/a/histstat/doc/doc\\_200014.pdf](https://www.ssb.no/a/histstat/doc/doc_200014.pdf)

variables and CPI or Output becomes more significant. The logic behind this assumption is related to the fact that: the markets need time to react to actions of the Federal Reserve.

### 5.3. Disadvantages of the VAR model

- 1) It is a linear regression model, so if the structure of database is not linear, it would be necessary to apply non-linear models.
- 2) It requires that all variables are stationary or integrated of the same order.

As a consequence of the requirements, the differencing process is used in order to remove a stochastic trend and to stabilize the mean (i.e., the sources of non-stationarity). The stationarity of the series will be tested using the following Unit Root testing procedures: (i) ADF; (ii) PP and (iii) KPSS. The ADF and PP unit root tests observe the null hypothesis that a time series is I (1), while the in KPSS, the null hypothesis assumes stationarity (or trend-stationarity).

Unit Root Tests			
Hypothesis	ADF	PP	KPSS
H0	Non-Stationary	Non-Stationary	Stationary
H1	Stationary	Stationary	Non-Stationary

**Figure 12 – Stationarity Tests Overview**

### 5.4. Why Log values?

The use of the log values has three main advantages<sup>4</sup>:

1. Many economic times series grow, on average, exponentially. The logarithm of the series grows approximately in a linear way and its absolute (constant) variation is approximately equal to the proportional change of the series.
2. The “variance-on-mean” relationship. The higher the mean the higher the variance. In this case of the logarithm of the series, it is approximately constant.
3. The predicted values of  $y_t$  from an untransformed linear regression may be negative. However, the predicted values from a log-transformed regression can never be negative.

<sup>4</sup> Dias Curto, J., 2019. *Time Series Models*. 1st ed. p.3.



## 5.5. The VAR Model

To better demonstrate the structure of a VAR model, it can be considered a simple, theoretical VAR model with two time-series variables,  $y_{1t}$  and  $y_{2t}$ , which consists of two equations, one for each of the variables, where the regressors in all equations are lagged values of all the variables. Assuming that the VAR contains two lagged values ( $p = 2$ ) of two endogenous variables ( $k = 2$ ) and let a constant be the only exogenous variable, the equations are:

$$\begin{aligned}y_{1t} &= a_{11}y_{1t-1} + a_{12}y_{2t-1} + b_{11}y_{1t-2} + b_{12}y_{2t-2} + c_1 + \varepsilon_{1t} \\y_{2t} &= a_{21}y_{1t-1} + a_{22}y_{2t-1} + b_{21}y_{1t-2} + b_{22}y_{2t-2} + c_2 + \varepsilon_{2t};\end{aligned}\tag{2}$$

where  $a_{ij}$ ;  $b_{ij}$ ;  $c_i$  are the parameters to be estimated.

Additionally, every error term has a zero mean and no serial correlation in individual error terms.

## 5.6. How to estimate the best fitting VAR model

For this purpose, knowing the optimal number of lags to consider in the VAR( $p$ ) model is crucial, and  $p$  represents the number of lags. The actual value of  $p$  can be determined according to a given information criterion: (i) BIC( $n$ ), (ii) AIC ( $n$ ) or the more conservative (iii) SC( $n$ ) and (iv) HQ( $n$ ) criteria.

Subsequently, a check on the stability for the system of difference equations is assessed. This is done by assuring the modulus of the eigenvalues of the matrix are less than one, if so, the system is stable. Additionally, other tests are recommended in order to check the goodness of fit of a VAR model. These include:

1. The multivariate ARCH-LM (Tests residuals for heteroskedasticity, meaning it tests squared residuals for autocorrelation, translating into a not constant variance, which breaks modelling assumptions);
2. The Jarque-Bera normality test for univariate and multivariate applied to the residuals of the VAR( $p$ ) (this validates the Normality of the residuals distribution);
3. The Breusch–Godfrey test (for testing the lack of serial correlation in the error terms of VAR).



## 6. Empirical Analysis

### 6.1. Stationarity Analysis

As we discussed previously, in order to apply the VAR model one of the main requirements is that the time-series are stationary (i.e., unit root free). To tests this requirement, an analysis of how the 6 time-series, without log differencing, behave is conducted, thus testing the Null Hypothesis for a Unit Root (ADF, PP) or a lack thereof (KPSS).

	ADF			PP			KPSS		
	Value of Test Statistic	Critical Value (5%)	Decision	Value of Test Statistic	Critical Value (5%)	Decision	Value of Test Statistic	Critical Value (5%)	Decision
<b>IP</b>	Tau3 = -1.55 Phi3 = 1.322	Tau3 = -3.43 Phi3 = 6.49	Don't Reject H0	-2.02	-3.45	Don't Reject H0	0.373	0.146	Reject H0
<b>MBS</b>	Tau3 = -1.55 Phi3 = 3.9	Tau3 = -3.43 Phi3 = 6.49	Don't Reject H0	-1.04	-3.45	Don't Reject H0	0.3701	0.146	Reject H0
<b>CPI</b>	Tau3 = -1.69 Phi3 = 1.49	Tau3 = -3.43 Phi3 = 6.49	Don't Reject H0	-1.76	-3.45	Don't Reject H0	0.2821	0.146	Reject H0
<b>U.S. Treasury 1-5 years</b>	Tau3 = -1.49 Phi3 = 1.41	Tau3 = -3.43 Phi3 = 6.49	Don't Reject H0	-0.54	-3.45	Don't Reject H0	0.3832	0.146	Reject H0
<b>U.S. Treasury 5-10 years</b>	Tau3 = -3.07 Phi3 = 4.95	Tau3 = -3.43 Phi3 = 6.49	Don't Reject H0	-1.95	-3.45	Don't Reject H0	0.5845	0.146	Reject H0

<b>U.S. Treasury 10 Years</b>	Tau3 = -1.86	Tau3 = -3.43	Don't Reject	-0.06	-3.45	Don't Reject H0	0.5959	0.146	Reject H0
	Phi3 = 2.77	Phi3 = 6.49	H0						

**Figure 13 – Stationarity Tests 1**

The results are clear on this case, none of the time series is Unit Root free, I(0). The ADF and PP tests that check for the null hypothesis of a unit root, I(1), are not rejected and the null hypothesis of the KPSS test, I(0) is rejected, thus it is important to proceed by removing the sources of non-stationarity, by considering the log values of each time-series as well as calculating the 1<sup>st</sup> differences. The following table represents the results when applying the same tests previously used on this transformed time-series.

	ADF			PP			KPSS		
	Value of Test Statistic	Critical Value (5%)	Decision	Value of Test Statistic	Critical Value (5%)	Decision	Value of Test Statistic	Critical Value (5%)	Decision
<b>IP</b>	Tau3 = -4.86	Tau3 = -3.43	Reject H0	-10.95	-3.45	Reject H0	0.1413	0.146	Don't Reject H0
	Phi3 = 11.8104	Phi3 = 6.49							
<b>MBS</b>	Tau3 = -3.7336	Tau3 = -3.43	Reject H0	-8.5428	-3.45	Reject H0	0.1486	0.146	Reject H0
	Phi3 = 4.8752	Phi3 = 6.49							
<b>CPI</b>	Tau3 = -6.6795	Tau3 = -3.43	Reject H0	-7.9596	-3.45	Reject H0	0.1346	0.146	Don't Reject H0
	Phi3 = 22.3593	Phi3 = 6.49							
<b>U.S. Treasury 1-5 years</b>	Tau3 = -3.7135	Tau3 = -3.43	Reject H0	-4.4351	-3.45	Reject H0	0.0639	0.146	Don't Reject H0
	Phi3 = 7.2841	Phi3 = 6.49							

<b>U.S. Treasury 5-10 years</b>	Tau3 = -4.1852	Tau3 = -3.43	Reject H0	-4.5283	-3.45	Reject H0	0.2751	0.146	Reject H0
	Phi3 = 6.9815	Phi3 = 6.49							
<b>U.S. Treasury 10 Years</b>	Tau3 = -1.4787	Tau3 = -3.43	Don't Reject H0	-4.3693	-3.45	Reject H0	0.1425	0.146	Don't Reject H0
	Phi3 = 1.2713	Phi3 = 6.49							

**Figure 14 – Stationarity Tests 2**

As can be observed from Figure 14, there are 3 time series in which all of the tests point for stationarity I (0). These are: (i) Industrial Production, (ii) CPI, (iii) U.S. Treasury 1-5 Years. While (i) U.S. Treasury maturing in 5 to 10 Years, (ii) U.S. Treasury Securities maturing 10 Years and (iii) M.B.S have 2 out of 3 tests pointing to stationarity. We conclude that these time-series are probably stationary given that the majority of the tests point to this conclusion.

## 6.2. The VAR Model – Estimation Results

It is now possible to proceed with the estimation of the VAR model. However, before estimating the model itself, the optimal number of lags for the VAR model has to be determined. This value can be determined according to an information criterion. A summary table of the results is presented below, additionally **Figure 15** refers to the RStudio Output.

<b>Information Criterion</b>				
AIC(n)	HQ(n)	SC(n)	FPE(n)	
17	17	17	17	

**Figure 16 – Lag Selection (Summary)**

The optimal number of lags is 17 according to all the Information Criterion (AIC, HQ, SC and FPE).

After estimating the VAR (17) model, it's overall stability should also be checked. Here stability does not refer to the coefficients' stability, but rather the stability of the system of the underlying difference equations. If the modulus of the eigenvalues of the companion matrix are less than one, the system is stable.

However, after estimating the VAR (17) model, the system is confirmed to be unstable as some of the eigenvalues of the companion matrix are greater than one (**Figure 17**). The same is

true for the VAR (16) model (**Figure 18**). The estimation of a VAR (15) model is therefore conducted, which is stable (**Figure 19 and Figure 20**).

In order to better understand the model, its condensed form is represented for both CPI and Industrial Production.

In condensed form the CPI equation is as follows (where C is a constant and  $\epsilon$  is the error term):

$$\begin{aligned} \text{CPI.l}d = & \sum_{i=1}^{15} \text{CPI.l}d.lag(i) + \sum_{i=1}^{15} \text{IP.l}d.lag(i) + \\ & \sum_{i=1}^{15} \text{MBS.l}d.lag(i) + \sum_{i=1}^{15} \text{U.S.Treasury1to5Y.l}d.lag(i) + \\ & \sum_{i=1}^{15} \text{U.S.Treasury5to10Y.l}d.lag(i) + \sum_{i=1}^{15} \text{U.S.Treasury10orMore.l}d.lag(i) + \\ & C + \epsilon \end{aligned} \quad (3)$$

For the detailed estimation results for the CPI equation please check **Figure 21**.

The estimation results for the Industrial Production equation is as follows (where C is a constant and  $\epsilon$  is the error term):

$$\begin{aligned} \text{IP.l}dd = & \sum_{i=1}^{15} \text{IP.l}d.lag(i) + \sum_{i=1}^{15} \text{CPI.l}d.lag(i) + \\ & \sum_{i=1}^{15} \text{MBS.l}d.lag(i) + \sum_{i=1}^{15} \text{U.S.Treasury1to5Y.l}d.lag(i) + \\ & \sum_{i=1}^{15} \text{U.S.Treasury5to10Y.l}d.lag(i) + \sum_{i=1}^{15} \text{U.S.Treasury10orMore.l}d.lag(i) + \\ & C + \epsilon \end{aligned} \quad (4)$$

For the detailed estimation results for the Industrial Production equation please check **Figure 22**.

Additionally, it is important to note that not all regressors from the last estimation (i.e., VAR (15)) are significant at the 5% level, as such, a re-estimation of the equation is conducted, considering only the significant regressors at the standard significance level of 5%.

The new estimation results (i.e., RStudio output) for the CPI and Industrial Production equation can be checked in **Figure 23 and 24 – RStudio Output**, respectively. Additionally, the following tables represent a summary of the regression Output.

<b>Estimation Results for the CPI Equation</b>				
Variable	Estimate	Standard Error	t value	Pr(> t )
Treasury5to10.ld.L2	0.0167347	0.0052320	3.199	0.00186 **
CPI.ld.L3	-0.2663362	0.0802954	-3.317	0.00127 **
INDPROD.ld.L3	0.0883382	0.0325719	2.712	0.00788 **
INDPROD.ld.L4	0.0718873	0.0332049	2.165	0.03279 *
Treasury1to5.ld.L4	-0.0118466	0.0049576	-2.390	0.01876 *
Treasury5to10.ld.L4	0.0329571	0.0063597	5.182	1.16e-06 ***
Treasury10.ld.L4	-0.0250821	0.0108478	2.312	0.02284 *
CPI.ld.L6	-0.2805747	0.0921596	-3.044	0.00299 **
MBSHoldings.ld.L6	0.0623683	0.0141351	4.412	2.61e-05 ***
INDPROD.ld.L8	0.0724685	0.0312183	2.321	0.02232 *
CPI.ld.L9	-0.2037927	0.0819086	-2.488	0.01451 *
Treasury10.ld.L13	-0.0569983	0.0098958	-5.760	9.51e-08 ***
MBSHoldings.ld.L14	-0.0208632	0.0065458	-3.187	0.00192 **
const	0.0033547	0.0003408	9.843	2.41e-16 ***
Significance codes: 0 '***', 0.001 '**', 0.01*', 0.05 '.', 0.1'', 1.				
Residual Std. Error 0.001483 on 99 degrees of freedom				
Multiple R-Squared: 0.6737		<b>Adjusted R-Squared: 0.6276</b>		

**Figure 25 – CPI Regression (Summary)**

The current CPI value is given by the sum of the lagged values of the above represented variables considering the estimated coefficient. The sign of the coefficient is of utmost importance in this economic analysis as it tells us what kind of impact (i.e., positive or negative) a specific variable or lagged terms of a variable have on the value of CPI.

It is clear how the lagged values of CPI have a negative impact on its current value, regardless of the lagged term considered. Industrial Production seems to have a positive impact on CPI levels across all lagged terms. The U.S. Treasury Securities maturing in 1 to 5 Years seem to have a negative impact, additionally the only lagged term of this variable significant to the model is the 4<sup>th</sup> term. In contrast, U.S. Treasury Securities Maturing in 5 to 10 Years seem to have a positive impact on CPI level, at the 4<sup>th</sup> lag. The U.S. Treasury Securities maturing in 10 Years have a negative impact at both lags, the 4<sup>th</sup> and 13<sup>th</sup>. Lastly, regarding M.B.S, the impact starts to be positive at the 6<sup>th</sup> lag, however, in the 14<sup>th</sup> lag it becomes negative.

In sum, increasing the holdings value of U.S. Treasury Securities maturing in 5 to 10 years seems to always show a positive impact on CPI levels, and the same is true for Industrial Production.

By contrast, U.S. Treasury Securities Maturing in 10 Years have a negative impact on CPI levels. The 3 remaining securities: (i) U.S. Treasury Securities Maturing in 1 to 5 Years; (ii) U.S. Treasury Securities Maturing in 5 to 10 Years; and (iii) M.B.S. seem to have an intermittent impact (i.e., ranging between positive and negative), as their corresponding impact is not clearly defined.

Looking at the R-Squared and Adjusted R-Squared values, these are 67,37% and 62,76% respectively. Throughout this Dissertation, the Adjusted R-Squared value will be considered over its counterpart. It indicates that the model's R.H.S. variables capture 62% of the variation of the L.H.S. Variable. The reason for this decision is bound by the nature of the model (multivariable regression) and the meaning of the two R-Squared Values. The non-adjusted R-Squared Value can be improved by adding new variables, regardless of how strongly correlated they are to the dependent variable, meaning this statistic can ultimately be increased even by pure chance. The adjusted R-squared provides an adjustment to the R-squared statistic such that an independent variable that has a correlation to the dependent variable increases adjusted R-squared and any variable without a strong correlation will make adjusted R-squared decrease (i.e., it penalizes the addition of random variables). As such, the latter metric is preferable to the former.



**Estimation Results for the Industrial Production Equation**

Variable	Estimate	Standard Error	t value	Pr(> t )
INDPROD.ld.L1	-0.2581858	0.0865982	-2.981	0.003653 **
CPI.ld.L2	0.5904010	0.2037092	2.898	0.004670 **
INDPROD.ld.L2	-0.2310744	0.0841456	-2.746	0.007227 **
CPI.ld.L3	0.9463219	0.2167299	4.366	3.25e-05 ***
Treasury5to10.ld.L3	0.0496799	0.0140601	3.533	0.000638 ***
CPI.ld.L4	0.6055912	0.2367441	2.558	0.012126 *
MBSHoldings.ld.L4	0.2791181	0.0534652	5.221	1.06e-06 ***
Treasury10.ld.L4	-0.1115649	0.0375858	-2.968	0.003799 **
MBSHoldings.ld.L5	-0.2151793	0.0602679	-3.570	0.000564 ***
Treasury1to5.ld.L5	-0.0357643	0.0122138	-2.928	0.004277 **
Treasury10.ld.L5	0.0861054	0.0380037	2.266	0.025764 *
CPI.ld.L7	0.4745130	0.2016774	2.353	0.020716 *
MBSHoldings.ld.L8	0.1105404	0.0334265	3.307	0.001337 **
CPI.ld.L10	0.8556733	0.2017551	4.241	5.20e-05 ***
CPI.ld.L11	0.5434864	0.2079139	2.614	0.010422 *
Treasury1to5.ld.L12	0.0349661	0.0116733	2.995	0.003504 **

CPI.l1.L13	0.5248093	0.1917897	2.736	0.007428 **
Treasury5to10.l1.L15	-0.0381255	0.0127222	-2.997	0.003490 **
const	-0.0049639	0.0008992	-5.521	2.98e-07 ***
Significance codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '.', 0.1 ' ', 1.				
Residual Std. Error: 0.003556 on 85 degrees of freedom				
Multiple R-Squared: 0.784		<b>Adjusted R-Squared: 0.7154</b>		

**Figure 26 - IP Regression (Summary)**

For Industrial Production, its lagged term 1 and 2 enter significantly and have a negative coefficient, meaning higher past values of Industrial Production have a negative contribution on its current value. For CPI lags 2,3,4,7,10,11 and 13 all enter significantly and with a positive coefficient, indicating that higher past CPI values have a positive impact in the current value of Industrial Production.

For U.S. Treasury Securities and M.B.S., the sign is not clearly defined, ranging between positive and negative. Specifically, Mortgage-Backed Securities enter significantly with the lagged terms: 4, 5 and 6 having an intermittent sign starting positive on the 4<sup>th</sup> lag. For U.S. Treasury Securities Maturing in 1 to 5 Years, the lagged terms are 5 and 12, starting with a positive coefficient at lag 5 and a negative at lag 12. Regarding U.S. Treasury Securities Maturing in 5 to 10 Years, the lagged terms are 3 and 15 with a positive coefficient in the 3<sup>rd</sup> and a negative on the 15<sup>th</sup>. Lastly, the U.S. Treasury Securities Maturing in 10 Years lagged terms 4 and 5 enter significantly with a negative coefficient at 4 and a positive one at 5.

Regarding the Adjusted R-Squared value, it is quite high, 71,54% of the dependent variable variation is being captured by the model, a rather good result.

### **6.3. Goodness of Fit tests**

After estimating the model, the most widely used goodness of fit tests are performed in order to detect any underlying VAR assumptions that might have been violated.

The standard procedure consists in the application of the following: (i) The multivariate ARCH-LM test, which test the Null Hypothesis of no ARCH effect, being the model

homoscedastic, (ii) the Jarque-Bera normality tests, which tests the null Hypothesis of a Normal Distribution, as well as separate tests for multivariate skewness and kurtosis; And, lastly, (iii) the Breusch and Godfrey LM test, for testing the lack of serial correlation in the residuals of a VAR(p) in the Null Hypothesis.

Detailed results of these tests can be consulted in **Figure 27 – RStudio Output**. Below is a summary of the results.

	<b>ARCH-LM</b>	<b>Jarque-Bera</b>	<b>Breusch-Godfrey LM</b>
P-Value	0.4178	JB = 0.5322 Skewness = 0.7298 Kurtosis = 0.2892	0.09092
Decision	Don't Reject H0	Don't Reject H0 (in any case)	Don't Reject H0

**Figure 28 - Stationarity and Normality Tests (Summary)**

The results are quite expressive, as the null hypothesis is not rejected in any of the cases, which means a lack of an ARCH effect, or in other words, the model does not contain conditional heteroscedasticity or autocorrelation in the squared series. Moreover, the Breusch–Godfrey test checks for autocorrelation in the errors in a regression model. It uses the residuals from the regression model and a test statistic is derived from it. The non-rejection of the null hypothesis means that there is no serial correlation of any order up to the 15<sup>th</sup> lag. Regarding the Jarque-Bera test, the results point to a non-rejection of the null hypothesis, which essentially constitutes a joint hypothesis of the skewness being zero and the excess kurtosis being zero, as such it is safe to assume the skewness and kurtosis do match that of a normal distribution. After estimating and testing the model for general goodness of fit it is relevant to present and then analyze the impact of a shock on both CPI and Industrial Production. Accordingly, the present Dissertation addresses the Impulse Response Functions associated with the adopted VAR specification (i.e., VAR (15)).

## 6.4. Impulse Response Functions

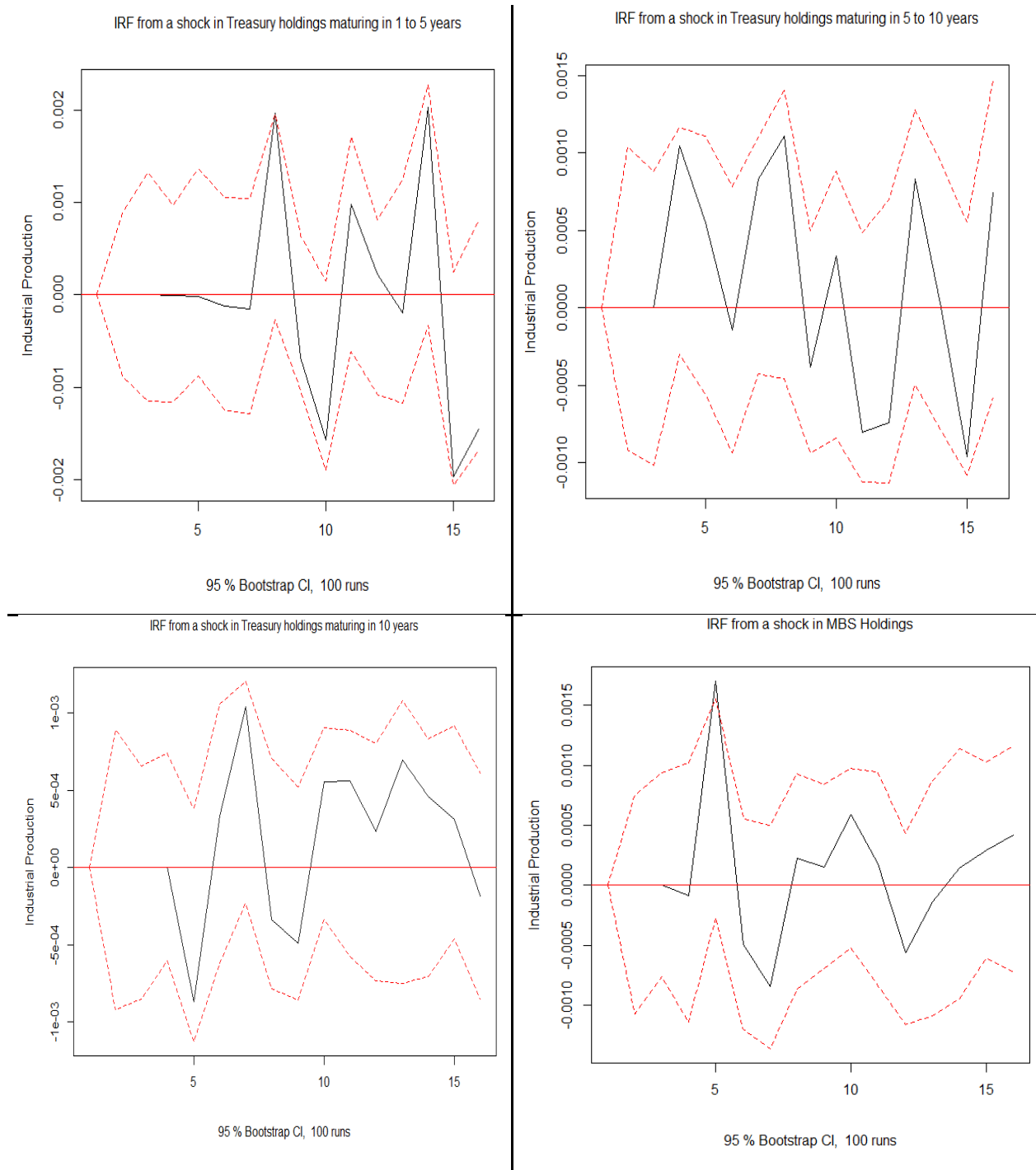
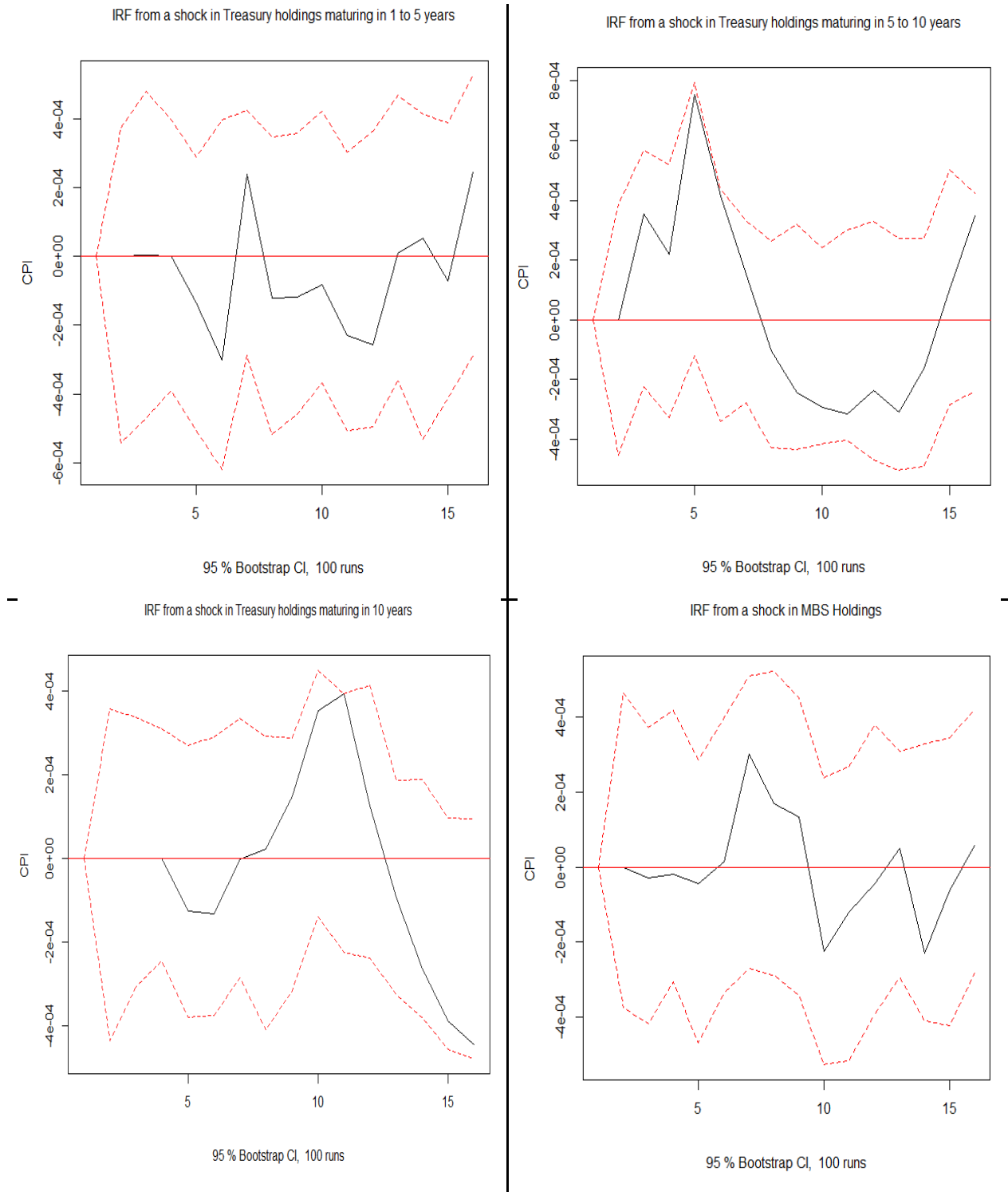


Figure 29 – Impulse Response Functions for IP

From the top-left side figure it can be observed that the effects of a shock in the given security holdings produce an early, very small and negative impact on Industrial Production, afterward it is possible to see positive spikes for lags 8, 12, 14 and negative spikes in lags 10, 13 and 15, ending on a negative note after lag 15. For the top-right side figure, a positive spike is registered in the lags 4, 7, 8, 10, 13 and a negative spike in lags 6, 9, 11, 12, 15, ending on a positive note. For the low-left side figure, a positive spike in lags 6 and 11 to 13 is observed, with negative spikes at lags 5, 8, 9 moving towards a negative result at the end. The last figure (i.e., low-right side figure starts by showing a small and negative spike at lag 4 and shows positive spike occurrences in lags 5, 7, 10 with negative spikes at lags 7 and 12, ending on a positive note.

A common occurrence across all the figures is that Industrial Production seems to respond negatively in the first periods to the shocks in the securities level with the exception of the reaction to the U.S. Treasury Securities Maturing in 5 to 10 Years which starts by triggering a positive response in Industrial Production. The type of security Industrial Production seems to take longer to respond to is the U.S. Treasury Securities Maturing in 1 to 5 Years. This response is then followed by an intermittent period of positive and negative spikes that ends on a positive value for U.S. Treasury Securities Maturing in 5 to 10 Years and M.B.S. The opposite is true for the remaining (i.e., U.S. Treasury Securities Maturing in 1 to 5 Years and U.S. Treasury Securities Maturing in 10 Years). For CPI, a similar analysis is conducted.



**Figure 30 – Impulse Response Functions for CPI**

For CPI, observing the top-left side figure it seems to register a positive spike in lags 7 and 14 and a negative spike at lag 6, 8 to 13, 15, ending on a positive note. For top-right side figure, positive spikes can be seen in lags 3 and 5 and negative spikes after lag 8 and until lag 14, ending positively. For the low-left side figure, positive spikes occur at lags 7 to 13 onwards, negative spikes at 5 to 7 and from 13 onwards, ending on a negative note. Lastly for the low-right side figure, positive spikes occur in lags 7 to 9 and 13, negative ones at lags 5, 10 and 14, ending with a positive result.

Overall, the behaviour can be described by a negative response early on across the board with the exception of the U.S. Treasury Securities Maturing in 5 to 10 Years. This is followed by a period of high volatility for (i) U.S. Securities Maturing in 1 to 5 Years and (ii) M.B.S, for (iii) U.S. Securities Maturing in 5 to 10 Years, after the 5<sup>th</sup> lag, a sharp decline with a recovery after the 10<sup>th</sup> lag returning at the 14<sup>th</sup> lag to a positive value. For the (iv) U.S. Securities Maturing in 10 Years the opposite is true, as it starts with a negative spike, followed by a steady increase until lag 10, and after this point it declines, ending with a negative value.

In sum, all securities end on positive values after the 15<sup>th</sup> lag with the exception of U.S. Securities Maturing in 5 to 10 Years.

## **6.5. Forecast Error Variance Decomposition**

Following this analysis a complementary one is conducted in order to study the difference between the real and the predicted value of the time series, that is the forecast error variance decomposition analysis (**Figure 31**). Both for CPI and Industrial Production the biggest contributor to forecast uncertainty are the lagged values of the variables themselves followed by Industrial Production for the CPI case and CPI for the Industrial Production case. Among the Q.E. variables the biggest contributor to the forecast uncertainty in CPI is the U.S. Treasury Securities Maturing in 5 to 10 Years and for the forecast uncertainty of Industrial Production, the M.B.S.





## 7. Conclusion

This Dissertation addresses a topic of increasing importance: the impact of Quantitative Easing programs on the real economy. The present Dissertation employs a VAR methodology and uses multiple time series data, namely: (i) U.S Treasury Securities maturing in 1 to 5 Years, (ii) U.S Treasury Securities maturing in 5 to 10 Years, (iii) U.S Treasury Securities maturing in 10 Years, (iv) M.B.S, (v) CPI and (vi) Industrial Production, which are extracted from the Federal Reserve Bank of Saint Luis data base.

Our findings suggest that the impact of these programs on Gross Domestic Product (using Industrial Production as a proxy) and CPI is mapped in an effort to contribute to its understanding and ultimately help fuel future research. A key finding, supported by the work of **Neely (2016)** and **Ihrig et al. (2018)**, is that Q.E. does have a significant and persistent impact in the economy both at the CPI level and at the Industrial Production level. Given that the Industrial Production is highly correlated with G.D.P, it can be used as a proxy for the latter, and, therefore, the Q.E. programs' impact on Industrial Production can be translated into a similar effect on G.D.P. Notwithstanding, **Neely (2016)** and **Ihrig et al. (2018)** also observe that the persistent effects on the economy is essentially felt through the effective lowering of the securities yield, which, in turns, incentivises spending, thus supporting the economy. This Dissertation addresses the said programs' final impact, directly measured in the economy. Accordingly, the nature of the impact (i.e., positive) resulting from Q.E. policies is evinced across the following securities: (i) MBS and (ii) U.S. Treasury Securities maturing in 5 to 10 years, both ending with a positive value after the 15<sup>th</sup> lag. However, the (iii) U.S. Treasury Securities maturing in 1 to 5 Years and (iv) U.S. Treasury Securities maturing in 10 Years point to a negative impact after the 15<sup>th</sup> lag. This Dissertation's findings also support the conclusions advanced by **Gust et al. (2017)**, who consider that market agents gradually assimilate the message for lower interest rates. In the present Dissertation's findings, this message can be observed in Industrial Production's lag in its response to Q.E. programs, only showing a response after 3 lag periods. This is a fact that is also defended by **Chenkelberg and Watzka (2013)**.

On the CPI side, results seem to indicate a positive impact on CPI (i.e., price levels), after the 15<sup>th</sup> lag, for all securities except for U.S. Treasury Securities maturing in 10 Years. The size of the impact however is smaller in absolute terms when compared to Industrial Production's impact. The response in the CPI is also not immediate, having a lag period of at least 3, similar to the case

for Industrial Production. This fact is also consistent with the previously mentioned researches. Here it can also be noticed that 3 out of the 4 securities, (i) U.S. Treasury Securities maturing in 1 to 5 Years, (ii) U.S. Treasury Securities maturing in 5 to 10 Years and (iii) M.B.S, seem to have a positive effect on CPI, a fact which is also supported by **Matsuki and Satoma (2015)**.

A very relevant finding, evident across Impulse Response Function graphs, both for CPI and Industrial Production is the considerable degree of volatility across the time period of 15 lags, exhibiting intermittent spikes of positive and negative responses. These results indicate that Q.E. does indeed have a measurable effect on the economy, but the definitive sign of this effect is hard to pinpoint, being relatively unstable at least until the 15<sup>th</sup> lag. Nonetheless, for CPI, 3 out of the 4 Impulse Response Functions expose a positive value after the 15<sup>th</sup> lag, and for Industrial Production, half of the Impulse Response Functions (i.e., 2 out of 4) show a similar result.

Additionally, it should be observed that the econometric approach used in the present Dissertation bears an intrinsic limitation bound to its relatively low time-window of 15 lag periods, as it is not sufficient to capture the Impulse Response Functions after this period. Likely, given a more extended time-window, the sign of the impact across the multiple IRFs for both Industrial Production and CPI could be assessed. In this sense, future researchers should consider a larger time-window, sufficiently sized to be able to capture the definitive impact that each of the Q.E. variables holdings value have on Output and CPI.

Lastly, in terms of the monetary policy implications associated with the impact of the resent Dissertation's findings, the results can be interpreted as a sign that policy makers are moving towards the right direction in finding novel solutions for stimulating the economy in the low interest rate environment under which we currently live in. Nevertheless, the way Q.E. programs are structured and released could need further calibration in order to identify the reason for the volatile response the current Dissertation has come across. Finding an answer to this phenomenon would be of great use for future policy makers and for the society as whole, helping mitigate the effect of a potential recession by only focusing on the instrument/procedures that lead to a positive response over both the short and the long term.

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# 9. Appendices

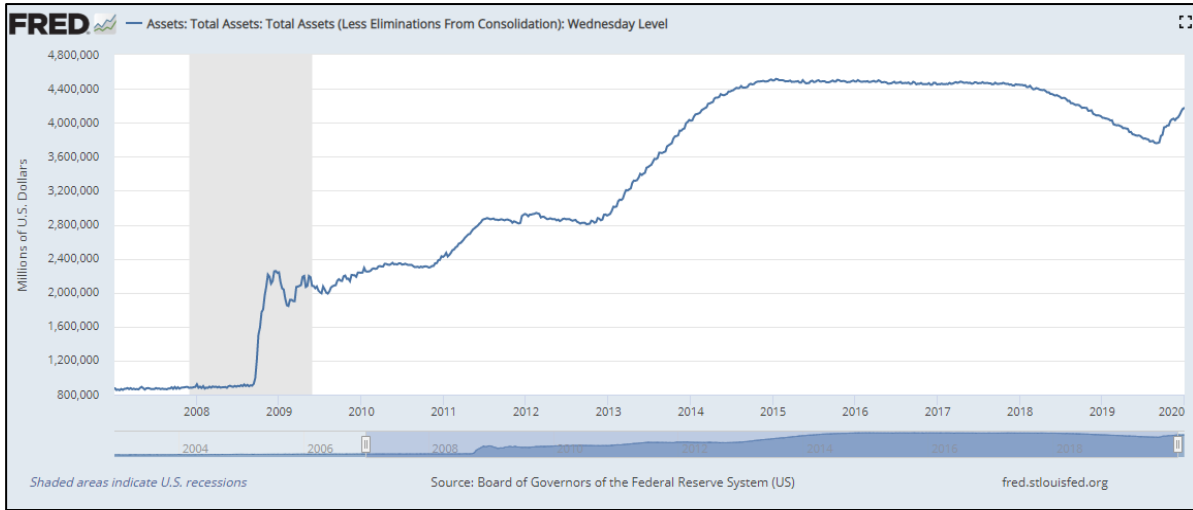


Figure 1 – Federal Reserve Total Assets (WALCL Timeseries)

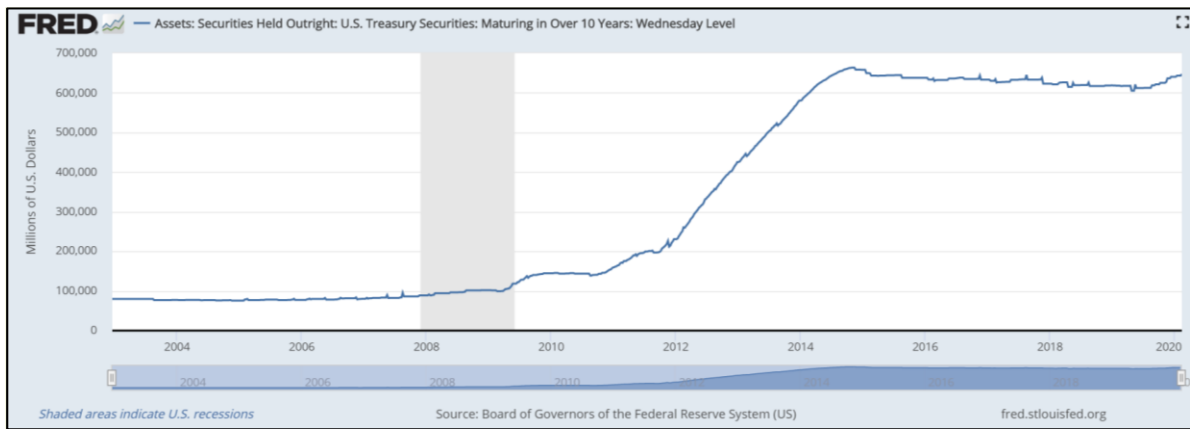


Figure 2 – Securities maturing > 10 years (TREAS10Y Timeseries)

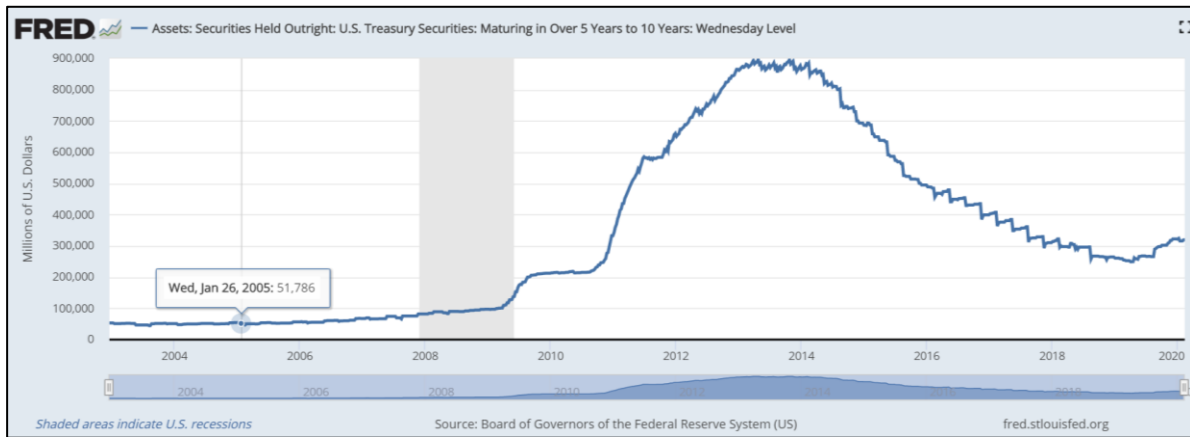


Figure 3 – Securities maturing 5 - 10 years (TREAS5T10 Timeseries)

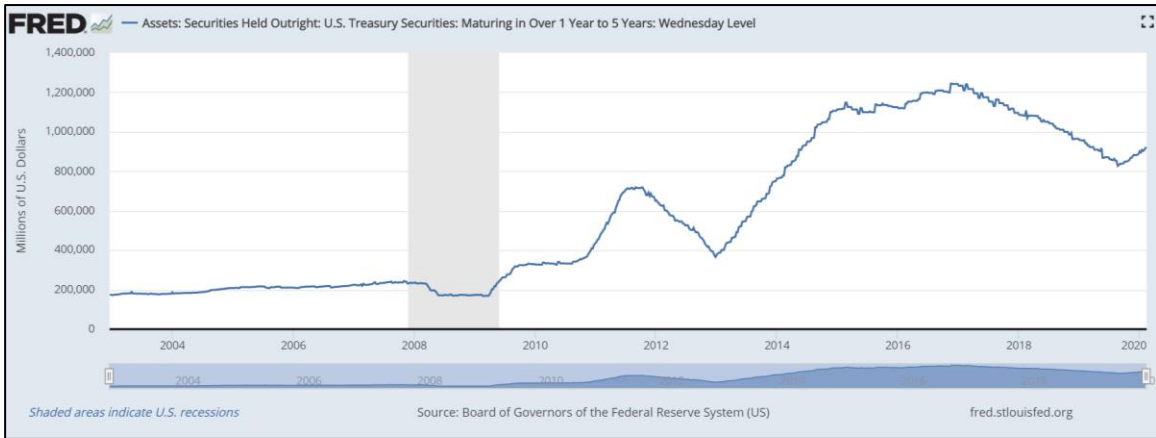


Figure 4 – Securities maturing 1 - 5 years (TREAS1T5 Timeseries)

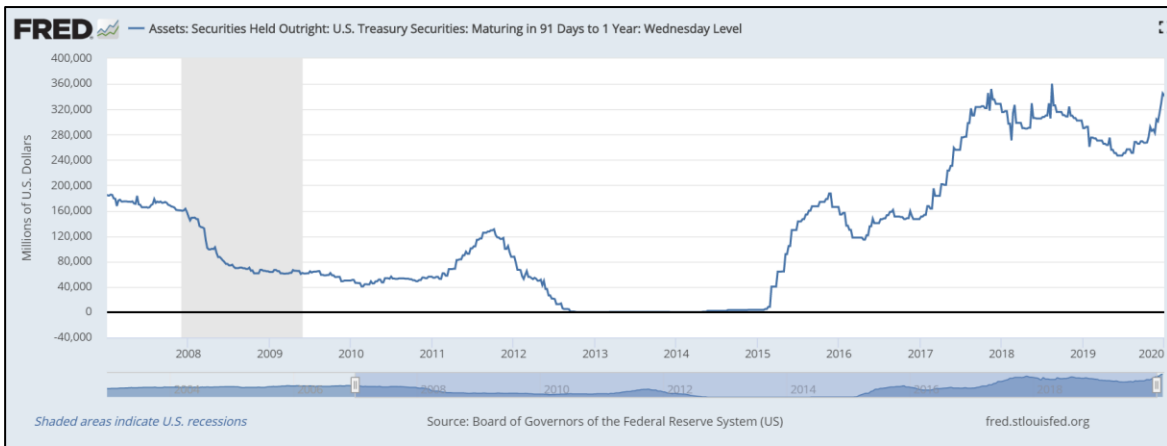


Figure 5 – Securities maturing 91 days – 1 year (TREAS911Y Timeseries)

```
> cor(INDPROD_Quarterly$INDPRO,GDP$GDP , method = "spearman")
[1] 0.9216249
> cor(INDPROD_Quarterly$INDPRO,GDP$GDP , method = "pearson")
[1] 0.9097852
```

Figure 7 – Correlation between Industrial Production and GDP



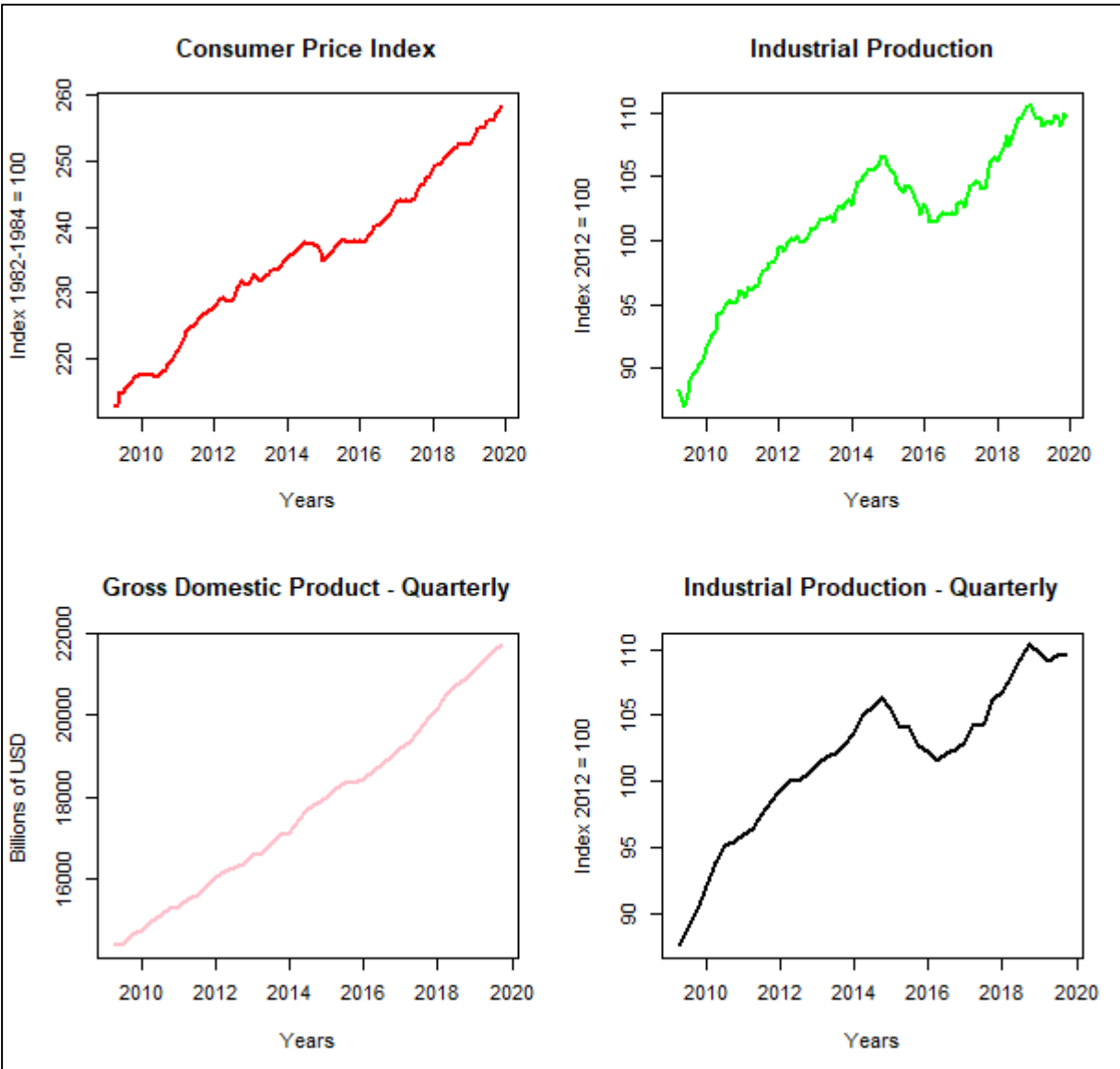


Figure 10 – Evolution of the CPI, GDP and Industrial Production between April 2009 and December 2019

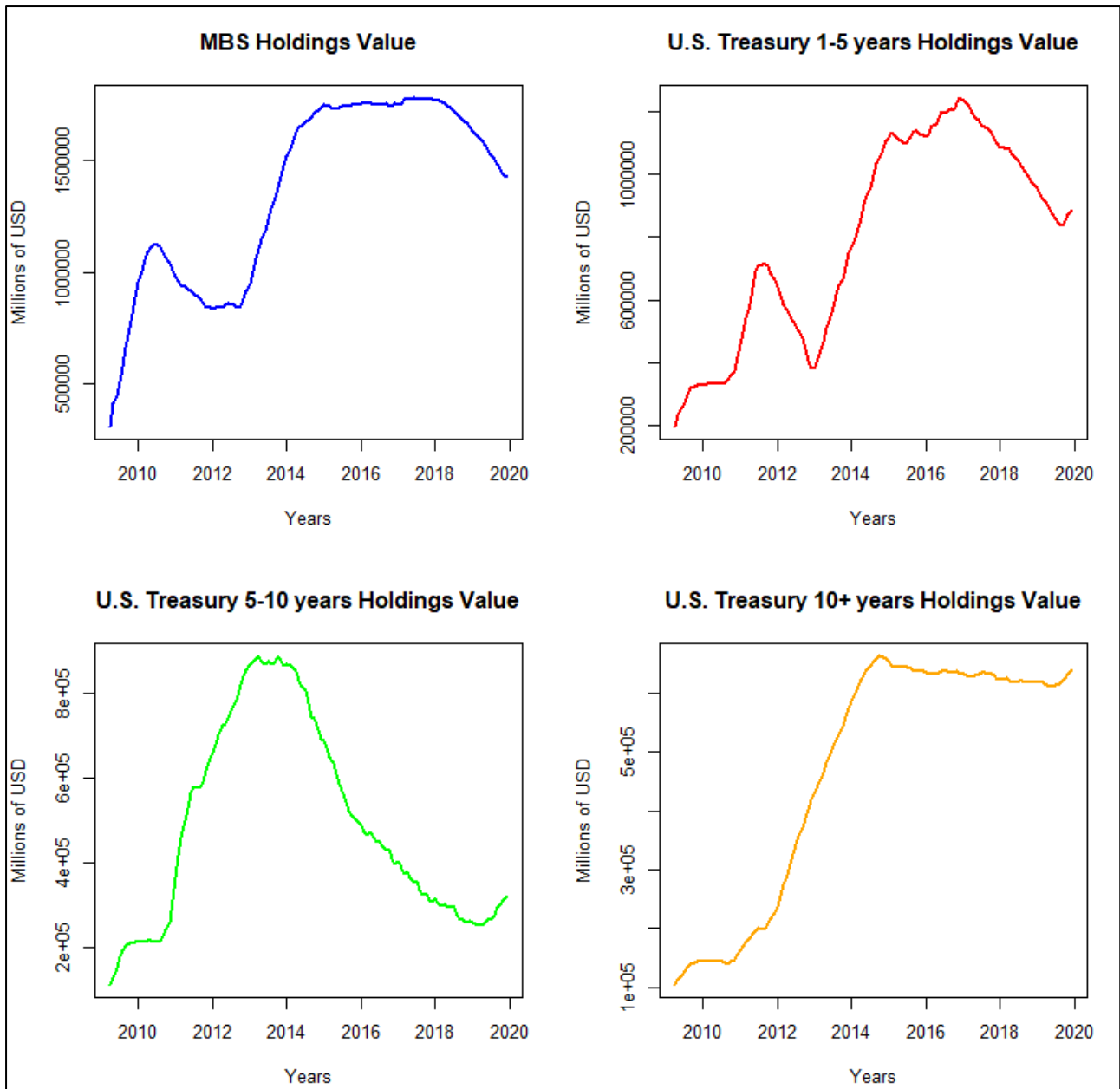


Figure 11 – Overview of all U.S. Treasury Securities and MBS Holdings Value between April 2009 and December 2019

```
> VARselect(dataseries1, lag.max=30, type="const")
$selection
AIC(n)  HQ(n)  SC(n)  FPE(n)
   17    17    17    17
```

Figure 15 – Lag selection by Information Criteria

```

> VAR17 <- VAR(dataseries1, p = 17, type="const")
> roots(VAR17)
[1] 1.05167277 1.02927716 1.02927716 1.01390453 1.01390453 1.00217794 1.00217794 1.00204530 1.00204530 0.99994577 0.99994577 0.99412332 0.99412332
[14] 0.99338532 0.99338532 0.99311483 0.99311483 0.99249615 0.99249615 0.99238520 0.99238520 0.99170604 0.99170604 0.98902699 0.98902699 0.98672626
[27] 0.98672626 0.98650539 0.98650539 0.98520554 0.98520554 0.98181138 0.98181138 0.97970795 0.97970795 0.97970399 0.97970399 0.97951450 0.97951450
[40] 0.97895120 0.97895120 0.97759512 0.97759512 0.97744298 0.97744298 0.97717735 0.97717735 0.97649396 0.97649396 0.97472012 0.97472012 0.97449048
[53] 0.97449048 0.97433688 0.97433688 0.97334911 0.97334911 0.97310349 0.97310349 0.97216096 0.97216096 0.97200328 0.97200328 0.97192004 0.97192004
[66] 0.96752122 0.96752122 0.96657901 0.96657901 0.96525355 0.96525355 0.96464394 0.96464394 0.96419939 0.96419939 0.96386387 0.96386387 0.96368005
[79] 0.96368005 0.96158001 0.96158001 0.95772304 0.95772304 0.94752944 0.94752944 0.94665798 0.94665798 0.94082515 0.94082515 0.94016290 0.94016290
[92] 0.91162640 0.91162640 0.89344349 0.89344349 0.72935334 0.72935334 0.72475042 0.72475042 0.61910122 0.61910122 0.50119170 0.50119170 0.07065394

```

Figure 17 – Eigen Values Matrix (VAR17)

```

> VAR16 <- VAR(dataseries1, p = 16, type="const")
> roots(VAR16)
[1] 1.00746207 1.00746207 1.00529784 1.00529784 1.00296645 1.00296645 1.00096804 1.00096804 0.98743498 0.98743498 0.98660426 0.98660426 0.98488005
[14] 0.98488005 0.98416984 0.98416984 0.98324714 0.98324714 0.98137402 0.98137402 0.98078904 0.98078904 0.97862486 0.97862486 0.97736503 0.97736503
[27] 0.97712587 0.97712587 0.97680351 0.97680351 0.97563731 0.97563731 0.97144198 0.97144198 0.97139072 0.97139072 0.97088311 0.97088311 0.97039336
[40] 0.97039336 0.96931795 0.96931795 0.96833794 0.96833794 0.96785913 0.96785913 0.96553285 0.96553285 0.96545921 0.96545921 0.96362269 0.96362269
[53] 0.96185404 0.96185404 0.96136565 0.96136565 0.96114307 0.96114307 0.95975171 0.95975171 0.95904323 0.95904323 0.95690632 0.95690632 0.95596817
[66] 0.95596817 0.95561926 0.95561926 0.94846245 0.94846245 0.93655910 0.93655910 0.92984256 0.92984256 0.92962876 0.92962876 0.92780621 0.92780621
[79] 0.91439641 0.91439641 0.90156409 0.90156409 0.89072786 0.89072786 0.88864360 0.88864360 0.88121284 0.88121284 0.84167057 0.84167057 0.78856695
[92] 0.62091439 0.49670428 0.49670428 0.20086140 0.20086140 0.09593769

```

Figure 18 – Eigen Values Matrix (VAR16)

```

> VAR15 <- VAR(dataseries1, p = 15, type="const")
> #TESTING IF THE SYSTEM IS STABLE
> roots(VAR15)
[1] 0.9971969 0.9971969 0.9955024 0.9955024 0.9953917 0.9953917 0.9910417 0.9910417 0.9851735 0.9851735 0.9819667 0.9819667 0.9804494 0.9804494
[15] 0.9803282 0.9803282 0.9776087 0.9776087 0.9774755 0.9774755 0.9760557 0.9760557 0.9730021 0.9730021 0.9713983 0.9713983 0.9713298 0.9713298
[29] 0.9706350 0.9706350 0.9706188 0.9688984 0.9688984 0.9688373 0.9688373 0.9687493 0.9687493 0.9662839 0.9662839 0.9661681 0.9661681 0.9661321
[43] 0.9661321 0.9649283 0.9649283 0.9575942 0.9575942 0.9562266 0.9562266 0.9555615 0.9555615 0.955479 0.955479 0.9522592 0.9522592 0.9487346
[57] 0.9487346 0.9450406 0.9450406 0.9387789 0.9387789 0.9334686 0.9334686 0.9333724 0.9333724 0.9320487 0.9320487 0.9187191 0.9187191 0.9184021
[71] 0.9184021 0.9156494 0.9156494 0.9091103 0.9091103 0.9082578 0.9082578 0.8960028 0.8960028 0.8720485 0.8720485 0.8598574 0.8598574 0.8281238
[85] 0.8281238 0.6635332 0.6635332 0.5927181 0.5927181 0.4034411 0.4034411 0.1259250

```

Figure 19 – Eigen Values Matrix (VAR15)

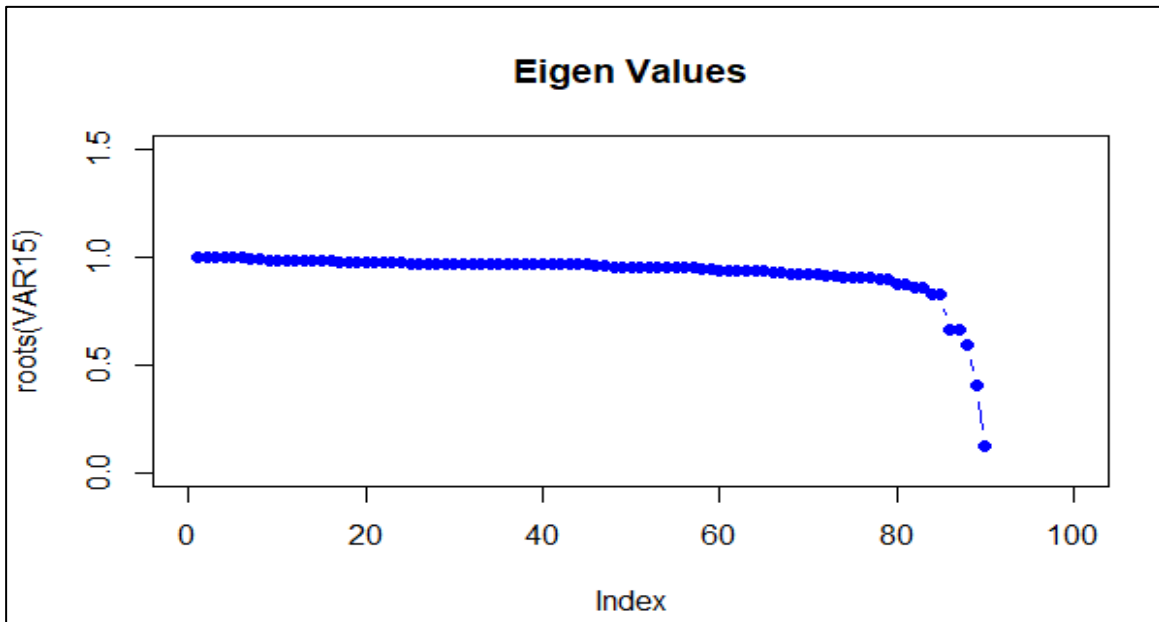


Figure 20 – Eigen Values Graphic Form (VAR15)

	Estimate	Std. Error	t value	Pr(> t )
CPI.1d.11	0.0124594	0.2159972	0.058	0.95452
INDPROD.1d.11	0.0138319	0.0854818	0.162	0.87293
MBSHoldings.1d.11	0.0744637	0.0745465	0.999	0.32871
Treasury1to5.1d.11	0.0054621	0.0284060	0.192	0.84928
Treasury5to10.1d.11	-0.0052241	0.0206553	-0.253	0.80268
Treasury10orMore.1d.11	-0.0232588	0.0555685	-0.419	0.67960
CPI.1d.12	-0.2291592	0.2090152	-1.096	0.28477
INDPROD.1d.12	0.0493815	0.0824017	0.599	0.55511
MBSHoldings.1d.12	-0.0186604	0.0798250	-0.234	0.81733
Treasury1to5.1d.12	-0.0309808	0.0332581	-0.932	0.36169
Treasury5to10.1d.12	0.0409819	0.0248103	1.652	0.11278
Treasury10orMore.1d.12	0.0004638	0.0552565	0.008	0.99338
CPI.1d.13	-0.3818737	0.2240862	-1.704	0.10244
INDPROD.1d.13	0.1039088	0.0839160	1.238	0.22868
MBSHoldings.1d.13	0.0127953	0.0826876	0.155	0.87844
Treasury1to5.1d.13	0.0569549	0.0350543	1.625	0.11846
Treasury5to10.1d.13	0.0042891	0.0271701	0.158	0.87601
Treasury10orMore.1d.13	-0.0396377	0.0617304	-0.642	0.52744
CPI.1d.14	-0.4069799	0.2282168	-1.783	0.08834
INDPROD.1d.14	0.2009633	0.0892966	2.251	0.03475 *
MBSHoldings.1d.14	-0.0271604	0.0760595	-0.357	0.72442
Treasury1to5.1d.14	-0.0507497	0.0390807	-1.299	0.20753
Treasury5to10.1d.14	0.0452237	0.0244956	1.846	0.07836
Treasury10orMore.1d.14	-0.0787563	0.0642054	-1.227	0.23293
CPI.1d.15	-0.0540825	0.2311007	-0.234	0.81713
INDPROD.1d.15	0.0733040	0.0941273	0.779	0.44441
MBSHoldings.1d.15	-0.0103420	0.0737445	-0.140	0.88975
Treasury1to5.1d.15	0.0124826	0.0380777	0.328	0.74615
Treasury5to10.1d.15	0.0270924	0.0273003	0.992	0.33180
Treasury10orMore.1d.15	-0.0475021	0.0645460	-0.736	0.46954
CPI.1d.16	-0.3272607	0.2004088	-1.633	0.11671
INDPROD.1d.16	0.1778384	0.0858239	2.072	0.05018
MBSHoldings.1d.16	0.0327067	0.0758650	0.431	0.67058
Treasury1to5.1d.16	-0.0361322	0.0292651	-1.235	0.22999
Treasury5to10.1d.16	0.0115516	0.0233586	0.495	0.62583
Treasury10orMore.1d.16	0.0185560	0.0497235	0.373	0.71258
CPI.1d.17	-0.3399715	0.2170982	-1.566	0.13163
INDPROD.1d.17	0.1574593	0.0943973	1.668	0.10948
MBSHoldings.1d.17	0.0846804	0.0837378	1.011	0.32289
Treasury1to5.1d.17	-0.0032084	0.0299473	-0.107	0.91565
Treasury5to10.1d.17	0.0353046	0.0225449	1.566	0.13163
Treasury10orMore.1d.17	0.0209250	0.0478408	0.437	0.66609
CPI.1d.18	-0.2899852	0.2283140	-1.270	0.21731
INDPROD.1d.18	0.2076998	0.0830991	2.499	0.02039 *
MBSHoldings.1d.18	0.0027196	0.0845720	0.032	0.97464
Treasury1to5.1d.18	0.0039498	0.0291407	0.136	0.89342
Treasury5to10.1d.18	-0.0118385	0.0246736	-0.480	0.63610
Treasury10orMore.1d.18	0.0390003	0.0508778	0.767	0.45150

CPI. 1d. 19	-0.5466935	0.2394067	-2.284	0.03242	*
INDPROD. 1d. 19	0.0734137	0.0889256	0.826	0.41791	
MBSHoldings. 1d. 19	0.0625736	0.0670323	0.933	0.36070	
Treasury1to5. 1d. 19	-0.0016631	0.0298492	-0.056	0.95607	
Treasury5to10. 1d. 19	0.0029412	0.0222981	0.132	0.89626	
Treasury10orMore. 1d. 19	-0.0402722	0.0513161	-0.785	0.44095	
CPI. 1d. 110	-0.4812648	0.2396277	-2.008	0.05703	.
INDPROD. 1d. 110	-0.0253211	0.0768162	-0.330	0.74480	
MBSHoldings. 1d. 110	-0.0369952	0.0795455	-0.465	0.64644	
Treasury1to5. 1d. 110	0.0166293	0.0340105	0.489	0.62972	
Treasury5to10. 1d. 110	-0.0098764	0.0227836	-0.433	0.66888	
Treasury10orMore. 1d. 110	0.0529635	0.0472700	1.120	0.27461	
CPI. 1d. 111	-0.2480730	0.2911124	-0.852	0.40331	
INDPROD. 1d. 111	-0.0179886	0.0736344	-0.244	0.80927	
MBSHoldings. 1d. 111	0.0339328	0.0631042	0.538	0.59616	
Treasury1to5. 1d. 111	0.0276813	0.0417342	0.663	0.51404	
Treasury5to10. 1d. 111	0.0083508	0.0246513	0.339	0.73800	
Treasury10orMore. 1d. 111	-0.0484146	0.0472571	-1.024	0.31674	
CPI. 1d. 112	-0.2226669	0.2322145	-0.959	0.34804	
INDPROD. 1d. 112	0.0625820	0.0798478	0.784	0.44153	
MBSHoldings. 1d. 112	-0.0517322	0.0598851	-0.864	0.39699	
Treasury1to5. 1d. 112	-0.0344797	0.0474009	-0.727	0.47465	
Treasury5to10. 1d. 112	-0.0059833	0.0250582	-0.239	0.81349	
Treasury10orMore. 1d. 112	0.0070734	0.0472410	0.150	0.88234	
CPI. 1d. 113	-0.2787531	0.2526632	-1.103	0.28184	
INDPROD. 1d. 113	-0.0927967	0.0748358	-1.240	0.22804	
MBSHoldings. 1d. 113	-0.0222890	0.0659824	-0.338	0.73871	
Treasury1to5. 1d. 113	0.0161541	0.0423707	0.381	0.70667	
Treasury5to10. 1d. 113	0.0041498	0.0242519	0.171	0.86570	
Treasury10orMore. 1d. 113	-0.1105936	0.0506738	-2.182	0.04004	*
CPI. 1d. 114	-0.1083860	0.2478624	-0.437	0.66617	
INDPROD. 1d. 114	-0.1455652	0.0798442	-1.823	0.08190	.
MBSHoldings. 1d. 114	-0.0347816	0.0418102	-0.832	0.41441	
Treasury1to5. 1d. 114	0.0343462	0.0387162	0.887	0.38461	
Treasury5to10. 1d. 114	0.0168814	0.0247148	0.683	0.50171	
Treasury10orMore. 1d. 114	-0.0299307	0.0545177	-0.549	0.58853	
CPI. 1d. 115	-0.1416230	0.2147321	-0.660	0.51640	
INDPROD. 1d. 115	0.0089330	0.0704683	0.127	0.90028	
MBSHoldings. 1d. 115	-0.0186098	0.0310314	-0.600	0.55483	
Treasury1to5. 1d. 115	-0.0256945	0.0243394	-1.056	0.30257	
Treasury5to10. 1d. 115	0.0100980	0.0249186	0.405	0.68922	
Treasury10orMore. 1d. 115	0.0182172	0.0423551	0.430	0.67130	
const	0.0096532	0.0031384	3.076	0.00553	**
---					
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					
Residual standard error: 0.001802 on 22 degrees of freedom					
Multiple R-Squared: 0.824, Adjusted R-squared: 0.1038					
F-statistic: 1.144 on 90 and 22 DF, p-value: 0.3729					

Figure 21 – Estimation results for the CPI equation

	Estimate	Std. Error	t value	Pr(> t )
CPI. 1d. 11	0.612059	0.531620	1.151	0.2620
INDPROD. 1d. 11	-0.268017	0.210391	-1.274	0.2160
MBSHoldings. 1d. 11	-0.040338	0.183477	-0.220	0.8280
Treasury1to5. 1d. 11	-0.049925	0.069914	-0.714	0.4827
Treasury5to10. 1d. 11	0.023694	0.050838	0.466	0.6457
Treasury10orMore. 1d. 11	0.056760	0.136767	0.415	0.6822
CPI. 1d. 12	0.881423	0.514436	1.713	0.1007
INDPROD. 1d. 12	-0.295882	0.202810	-1.459	0.1587
MBSHoldings. 1d. 12	0.165712	0.196468	0.843	0.4081
Treasury1to5. 1d. 12	-0.018819	0.081856	-0.230	0.8203
Treasury5to10. 1d. 12	-0.036578	0.061064	-0.599	0.5553
Treasury10orMore. 1d. 12	0.019965	0.135999	0.147	0.8846
CPI. 1d. 13	0.889054	0.551529	1.612	0.1212
INDPROD. 1d. 13	-0.236445	0.206537	-1.145	0.2646
MBSHoldings. 1d. 13	-0.133570	0.203514	-0.656	0.5184
Treasury1to5. 1d. 13	0.019514	0.086277	0.226	0.8232
Treasury5to10. 1d. 13	0.044889	0.066872	0.671	0.5090
Treasury10orMore. 1d. 13	0.096999	0.151933	0.638	0.5298
CPI. 1d. 14	0.556913	0.561696	0.991	0.3322
INDPROD. 1d. 14	-0.053402	0.219780	-0.243	0.8103
MBSHoldings. 1d. 14	0.280493	0.187200	1.498	0.1483
Treasury1to5. 1d. 14	-0.004309	0.096187	-0.045	0.9647
Treasury5to10. 1d. 14	-0.065264	0.060290	-1.083	0.2907
Treasury10orMore. 1d. 14	-0.098052	0.158025	-0.620	0.5413
CPI. 1d. 15	0.563123	0.568794	0.990	0.3329
INDPROD. 1d. 15	-0.112610	0.231670	-0.486	0.6317
MBSHoldings. 1d. 15	-0.356453	0.181503	-1.964	0.0623
Treasury1to5. 1d. 15	-0.055949	0.093718	-0.597	0.5566
Treasury5to10. 1d. 15	-0.058937	0.067192	-0.877	0.3899
Treasury10orMore. 1d. 15	0.113729	0.158863	0.716	0.4816
CPI. 1d. 16	0.378970	0.493254	0.768	0.4505
INDPROD. 1d. 16	0.079110	0.211233	0.375	0.7116
MBSHoldings. 1d. 16	0.177762	0.186722	0.952	0.3514
Treasury1to5. 1d. 16	0.027919	0.072028	0.388	0.7020
Treasury5to10. 1d. 16	0.053771	0.057491	0.935	0.3598
Treasury10orMore. 1d. 16	-0.091161	0.122381	-0.745	0.4642
CPI. 1d. 17	0.868923	0.534330	1.626	0.1181
INDPROD. 1d. 17	-0.201160	0.232334	-0.866	0.3959
MBSHoldings. 1d. 17	-0.057545	0.206099	-0.279	0.7827
Treasury1to5. 1d. 17	0.082175	0.073708	1.115	0.2769
Treasury5to10. 1d. 17	-0.019220	0.055488	-0.346	0.7323
Treasury10orMore. 1d. 17	0.057206	0.117747	0.486	0.6319
CPI. 1d. 18	0.512531	0.561935	0.912	0.3716

INDPROD.1d.19	-0.007555	0.218867	-0.035	0.9728
MBSHoldings.1d.19	-0.068216	0.164982	-0.413	0.6833
Treasury1to5.1d.19	-0.010519	0.073466	-0.143	0.8875
Treasury5to10.1d.19	-0.010553	0.054881	-0.192	0.8493
Treasury10orMore.1d.19	-0.068758	0.126301	-0.544	0.5916
CPI.1d.110	1.098180	0.589781	1.862	0.0760
INDPROD.1d.110	-0.184822	0.189063	-0.978	0.3389
MBSHoldings.1d.110	-0.097855	0.195780	-0.500	0.6222
Treasury1to5.1d.110	0.037085	0.083708	0.443	0.6621
Treasury5to10.1d.110	0.021188	0.056076	0.378	0.7092
Treasury10orMore.1d.110	0.078152	0.116343	0.672	0.5087
CPI.1d.111	0.824609	0.716497	1.151	0.2621
INDPROD.1d.111	0.173901	0.181232	0.960	0.3477
MBSHoldings.1d.111	0.042946	0.155314	0.277	0.7847
Treasury1to5.1d.111	-0.055695	0.102718	-0.542	0.5931
Treasury5to10.1d.111	-0.043556	0.060673	-0.718	0.4804
Treasury10orMore.1d.111	-0.045541	0.116311	-0.392	0.6992
CPI.1d.112	-0.164130	0.571535	-0.287	0.7767
INDPROD.1d.112	0.011718	0.196524	0.060	0.9530
MBSHoldings.1d.112	0.074461	0.147392	0.505	0.6184
Treasury1to5.1d.112	0.048207	0.116665	0.413	0.6835
Treasury5to10.1d.112	-0.020451	0.061674	-0.332	0.7433
Treasury10orMore.1d.112	0.062152	0.116271	0.535	0.5983
CPI.1d.113	0.754255	0.621864	1.213	0.2380
INDPROD.1d.113	-0.013649	0.184189	-0.074	0.9416
MBSHoldings.1d.113	-0.009170	0.162398	-0.056	0.9555
Treasury1to5.1d.113	-0.028651	0.104284	-0.275	0.7861
Treasury5to10.1d.113	0.009949	0.059690	0.167	0.8691
Treasury10orMore.1d.113	0.109400	0.124720	0.877	0.3899
CPI.1d.114	0.184644	0.610048	0.303	0.7650
INDPROD.1d.114	-0.149939	0.196516	-0.763	0.4536
MBSHoldings.1d.114	0.071343	0.102905	0.693	0.4954
Treasury1to5.1d.114	0.040910	0.095290	0.429	0.6719
Treasury5to10.1d.114	0.002115	0.060829	0.035	0.9726
Treasury10orMore.1d.114	-0.161201	0.134181	-1.201	0.2424
CPI.1d.115	0.178850	0.528507	0.338	0.7383
INDPROD.1d.115	-0.134819	0.173439	-0.777	0.4452
MBSHoldings.1d.115	0.017026	0.076376	0.223	0.8257
Treasury1to5.1d.115	-0.039509	0.059905	-0.660	0.5164
Treasury5to10.1d.115	-0.069807	0.061331	-1.138	0.2673
Treasury10orMore.1d.115	0.160659	0.104246	1.541	0.1375
const	-0.011736	0.007724	-1.519	0.1429
---				
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Residual standard error: 0.004435 on 22 degrees of freedom				
Multiple R-Squared: 0.8261, Adjusted R-squared: 0.1149				
F-statistic: 1.162 on 90 and 22 DF, p-value: 0.3566				

Figure 22 – Estimation results for the IP equation



	Estimate	Std. Error	t value	Pr(> t )	
Treasury5to10.l.d.12	0.0167347	0.0052320	3.199	0.00186	**
CPI.l.d.13	-0.2663362	0.0802954	-3.317	0.00127	**
INDPROD.l.d.13	0.0883382	0.0325719	2.712	0.00788	**
INDPROD.l.d.14	0.0718873	0.0332049	2.165	0.03279	*
Treasury1to5.l.d.14	-0.0118466	0.0049576	-2.390	0.01876	*
Treasury5to10.l.d.14	0.0329571	0.0063597	5.182	1.16e-06	***
Treasury10orMore.l.d.14	-0.0250821	0.0108478	-2.312	0.02284	*
CPI.l.d.16	-0.2805747	0.0921596	-3.044	0.00299	**
MBSHoldings.l.d.16	0.0623683	0.0141351	4.412	2.61e-05	***
INDPROD.l.d.18	0.0724685	0.0312183	2.321	0.02232	*
CPI.l.d.19	-0.2037927	0.0819086	-2.488	0.01451	*
Treasury10orMore.l.d.113	-0.0569983	0.0098958	-5.760	9.51e-08	***
MBSHoldings.l.d.114	-0.0208632	0.0065458	-3.187	0.00192	**
const	0.0033547	0.0003408	9.843	2.41e-16	***
---					
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					
Residual standard error: 0.001483 on 99 degrees of freedom					
Multiple R-squared: 0.6737, Adjusted R-squared: 0.6276					
F-statistic: 14.6 on 14 and 99 DF, p-value: < 2.2e-16					

Figure 23 – Estimation Results for significant regressors in the CPI Equation

	Estimate	Std. Error	t value	Pr(> t )	
INDPROD.l.d.11	-0.2581858	0.0865982	-2.981	0.003653	**
CPI.l.d.12	0.5904010	0.2037092	2.898	0.004670	**
INDPROD.l.d.12	-0.2310744	0.0841456	-2.746	0.007227	**
CPI.l.d.13	0.9463219	0.2167299	4.366	3.25e-05	***
Treasury5to10.l.d.13	0.0496799	0.0140601	3.533	0.000638	***
CPI.l.d.14	0.6055912	0.2367441	2.558	0.012126	*
MBSHoldings.l.d.14	0.2791181	0.0534652	5.221	1.06e-06	***
Treasury10orMore.l.d.14	-0.1115649	0.0375858	-2.968	0.003799	**
MBSHoldings.l.d.15	-0.2151793	0.0602679	-3.570	0.000564	***
Treasury1to5.l.d.15	-0.0357643	0.0122138	-2.928	0.004277	**
Treasury10orMore.l.d.15	0.0861054	0.0380037	2.266	0.025764	*
CPI.l.d.17	0.4745130	0.2016774	2.353	0.020716	*
MBSHoldings.l.d.18	0.1105404	0.0334265	3.307	0.001337	**
CPI.l.d.110	0.8556733	0.2017551	4.241	5.20e-05	***
CPI.l.d.111	0.5434864	0.2079139	2.614	0.010422	*
Treasury1to5.l.d.112	0.0349661	0.0116733	2.995	0.003504	**
CPI.l.d.113	0.5248093	0.1917897	2.736	0.007428	**
Treasury5to10.l.d.115	-0.0381255	0.0127222	-2.997	0.003490	**
const	-0.0049639	0.0008992	-5.521	2.98e-07	***
---					
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					
Residual standard error: 0.003528 on 94 degrees of freedom					
Multiple R-squared: 0.5625, Adjusted R-squared: 0.4741					
F-statistic: 6.362 on 19 and 94 DF, p-value: 3.552e-10					

Figure 24 – Estimation Results for significant regressors in the IP Equation

```

> VAR15.arch #dont reject H0
      ARCH (multivariate)
data: Residuals of VAR object VAR15.ser
Chi-squared = 2218.1, df = 2205, p-value = 0.4178

> VAR15.norm$jb.mu1 #DONT REJECT H0
$JB
      JB-Test (multivariate)
data: Residuals of VAR object VAR15.ser
Chi-squared = 10.961, df = 12, p-value = 0.5322

$Skewness
      Skewness only (multivariate)
data: Residuals of VAR object VAR15.ser
Chi-squared = 3.6059, df = 6, p-value = 0.7298

$Kurtosis
      Kurtosis only (multivariate)
data: Residuals of VAR object VAR15.ser
Chi-squared = 7.3553, df = 6, p-value = 0.2892

> VAR15.BG #DONT Reject H0
      Breusch-Godfrey LM test
data: Residuals of VAR object VAR15.ser
Chi-squared = 205.81, df = 180, p-value = 0.09092

```

Figure 27 – Goodness of Fit Tests (ARCH-LM, JB, BG-LM)

```

> VAR.fevd
$CPI.1d
      CPI.1d  INDPROD.1d  MBSHoldings.1d  Treasury1to5.1d  Treasury5to10.1d  Treasury10orMore.1d
[1,] 4.500000 0.0000000000 0.0000000000 0.000000e+00 0.00000000 0.00000000
[2,] 4.500000 0.0000000000 0.0000000000 0.000000e+00 0.00000000 0.00000000
[3,] 4.419246 0.0002125782 0.0003519933 3.872494e-06 0.0556418 0.00000000
[4,] 4.216329 0.1720492537 0.0004378253 4.816785e-06 0.0692098 0.00000000
[5,] 3.887443 0.1880620640 0.0011489108 6.766044e-03 0.2736229 0.005858285
[6,] 3.753661 0.2086643886 0.0011876245 3.835857e-02 0.3240633 0.011832318
[7,] 3.600472 0.3096159847 0.0318533168 5.578452e-02 0.3169530 0.011281664
[8,] 3.570814 0.3316260275 0.0403605153 5.894833e-02 0.3118354 0.011142195
[9,] 3.516641 0.3450867474 0.0456510098 6.260295e-02 0.3264427 0.018070112
[10,] 3.374303 0.3975880467 0.0586561516 6.132123e-02 0.3357906 0.055054936
[11,] 3.278626 0.3859114415 0.0611427810 7.506381e-02 0.3550328 0.099442040
[12,] 3.241452 0.3854486672 0.0607317495 9.313660e-02 0.3659188 0.102530840
[13,] 3.214232 0.3797683860 0.0605335944 9.179156e-02 0.3882588 0.103581735
[14,] 3.158187 0.3920989597 0.0742464326 9.100499e-02 0.3888992 0.121190009
[15,] 3.109542 0.3869464431 0.0741375516 9.093215e-02 0.3855962 0.160915790

$INDPROD.1d
      CPI.1d  INDPROD.1d  MBSHoldings.1d  Treasury1to5.1d  Treasury5to10.1d  Treasury10orMore.1d
[1,] 0.03489981 4.237827 0.0000000000 0.000000e+00 0.00000000 0.00000000
[2,] 0.03489981 4.237827 0.0000000000 0.000000e+00 0.00000000 0.00000000
[3,] 0.24657725 4.037636 0.0000000000 0.000000e+00 0.00000000 0.00000000
[4,] 0.60334763 3.608532 0.0004298322 4.728848e-06 0.06794629 0.00000000
[5,] 0.60362559 3.336593 0.1658208898 9.895272e-05 0.07993170 0.04324609
[6,] 0.60187709 3.312870 0.1782834163 1.621634e-03 0.08044167 0.04918063
[7,] 0.63908201 3.089664 0.2031303458 3.960796e-02 0.11095277 0.10242678
[8,] 0.76746402 2.886812 0.1923532062 6.325315e-02 0.16433501 0.10130286
[9,] 0.75554007 2.869110 0.1900745836 7.705181e-02 0.16845924 0.11113999
[10,] 0.74404568 2.841195 0.2037692846 7.773342e-02 0.17106480 0.12398447
[11,] 1.01507378 2.587184 0.1849619609 8.446899e-02 0.18205128 0.12518161
[12,] 1.01437211 2.547815 0.1947745112 8.597897e-02 0.20195974 0.12504087
[13,] 1.02651701 2.468752 0.1894331617 1.056834e-01 0.22366778 0.14077152
[14,] 1.03671480 2.429497 0.1868418705 1.366513e-01 0.21960364 0.14664923
[15,] 1.13693681 2.278530 0.1778970824 1.840083e-01 0.24021863 0.14084626

```

Figure 31 – Forecast error variance decomposition