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EURO AREA
INFLATION-LINKED BONDS MARKET:
ANALYSIS AND IMMUNIZATION ABILITIES

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Abstract

This thesis focuses on inflation-linked bonds. These bonds, although not as popular as other types of bonds, are not a new instrument. The first inflation-linked bond issue took place in the United States in the eighteenth century. In Europe, it would be only in the end of the twentieth century that inflation-linked bonds would be issued in a regular basis by European Monetary Union participant countries.

This thesis objective is twofold: to characterize this type of bonds and evaluate their immunization abilities. The first part of this study focuses in defining and describing thoroughly these bonds: their issuers, security design, indexation choice and applications.

The second part is dedicated to investigating which portfolio strategies using inflation-linked bonds achieve better immunization results for single inflation-growing liabilities. Simulations were made for investment horizons between one and five years, with and without transaction costs. The results presented point to the superiority of bullet portfolios, over random and barbell portfolios, in terms of liability coverage. Bullet portfolios also prove to be less volatile. These aforementioned results hold in the presence of transaction costs. It is also visible that the portfolios' return is very sensitive to the evolution of the real term structure of interest rates. As for the comparison between fixed rate bonds and inflation-linked bonds, the latter prove to be better suited to immunize the liabilities under analysis.

Keywords: Inflation-linked Bonds, Immunization, Duration, Real Interest Rates.

JEL Classification: E31, G11, G12, G23

Resumo

A presente tese incide sobre obrigações indexadas à inflação. Apesar de não serem tão populares como outras obrigações, não são um novo activo. A primeira emissão de obrigações indexadas à inflação ocorreu no século XVIII nos Estados Unidos. Na Europa, apenas no final do século vinte estas obrigações passariam a ser emitidas com regularidade por países inseridos na União Económica e Monetária.

Este estudo tem dois objectivos: caracterizar as obrigações indexadas à inflação e avaliar a sua capacidade de imunização. A primeira parte do estudo consiste em definir e caracterizar exaustivamente estas obrigações: emitentes, estrutura, indexação e aplicações.

A segunda parte consiste em aferir que estratégias para carteiras de obrigações indexadas à inflação imunizam melhor responsabilidades únicas que crescem com a inflação. Foram efectuadas simulações para horizontes temporais entre um e cinco anos, com e sem custos de transacção. Quanto ao grau de cobertura de responsabilidades, os resultados alcançados mostram que as carteiras *bullet* obtêm uma performance superior às carteiras aleatórias e *barbell*. As carteiras *bullet* são também menos voláteis. Estes resultados mantêm-se quando se consideram custos de transacção. É também observável que a rentabilidade das carteiras é bastante sensível a variações na estrutura temporal de taxas de juro reais. Relativamente à comparação entre obrigações nominais de taxa fixa e obrigações indexadas à inflação, estas últimas são mais indicadas para imunizar este tipo de responsabilidade.

Palavras-chave: Obrigações indexadas à inflação, Imunização, Duração, Taxas de juro reais.

JEL Classification: E31, G11, G12, G23

Sumário Executivo

A presente tese incide sobre obrigações indexadas à inflação. Este tipo de obrigações permite ao investidor proteger-se contra flutuações no nível de preços, garantindo assim o poder de compra dos seus *cash flows* futuros. Os *cash flows* deste tipo de obrigação têm duas componentes: uma componente fixa, associada ao retorno real, e uma componente variável, que varia com a taxa de inflação associada ao indexante escolhido. O investidor natural deste tipo de instrumentos é um investidor de longo prazo, que pretenda anular o risco associado ao nível de preços na sua carteira de activos ou que pretenda imunizar o seu balanço. Alguns exemplos de entidades tipicamente investidoras em obrigações indexadas à inflação são: fundos de pensões, seguradoras (que emitem Planos de Poupança Reforma) ou mesmo empresas cujas receitas (e custos) estejam fortemente associadas à evolução da inflação.

As obrigações indexadas à inflação não são tão populares entre os investidores como as obrigações de taxa fixa ou as obrigações hipotecárias. Porém, não se trata de um novo tipo de activo. As primeiras emissões de obrigações indexadas à inflação remontam ao século dezoito, nos Estados Unidos. Este tipo de obrigações voltaria a ser emitido em alguns países após a Segunda Guerra Mundial, dado que esta era a única forma de os governos conseguirem emitir e colocar divida junto dos investidores, devido à conjuntura macroeconómica débil do pós-guerra e ao elevado nível de inflação (que contribuía para tornar este tipo de activo interessante face às alternativas). Na Europa, apenas no final do século vinte se viria a verificar a emissão sistemática de obrigações indexadas à inflação. A primeira obrigação foi emitida pela França em 1998 e, até à data, apenas quatro países da Zona Euro emitiram este tipo de activos. São eles a Alemanha, França, Grécia e Itália. Existem também diferentes tipos de indexantes para estas obrigações. Apesar da maioria das obrigações emitidas serem indexadas ao Índice Harmonizado de Preços no Consumidor da Zona Euro (excluído tabaco), a França também emitiu obrigações indexadas ao Índice de Preços no Consumidor local (excluindo tabaco).

Tendo em conta o carácter de investimento das obrigações indexadas à inflação, seria interessante explorar se (e como) é possível aplicar este instrumento na imunização de responsabilidades futuras que cresçam com a inflação. Assim, foram efectuadas simulações de imunização de responsabilidades únicas futuras cuja taxa de crescimento se encontra associada à variação do nível de preços, utilizando uma estratégia de imunização passiva, através de igualdade das durações da responsabilidade e da carteira de activos. Foram considerados horizontes temporais de investimento entre um e cinco anos e estratégias de construção de carteiras de activos aleatórias, *bullet* e *barbell*. As simulações foram efectuadas com e sem custos de transacção e a valores reais. As carteiras consideradas seriam sempre auto-financiadas, ou seja, não existe nenhum tipo de injecção de capital nas carteiras de activos; os únicos *cash flows* investidos na carteira são provenientes de cupões recebidos ou capital associado a títulos que se venceram durante o horizonte temporal de investimento considerado.

Na ausência de custos de transacção, as carteiras *bullet* demonstram uma maior capacidade de cobertura de responsabilidades futuras face às carteiras aleatórias e *barbell* para todos os horizontes temporais considerados. Alem disso, são também as carteiras de activos que apresentam uma menor volatilidade e uma estrutura mais próxima à da responsabilidade que se pretende imunizar. Estes resultados continuam a verificar-se após a introdução de custos de transacção.

Relativamente à rentabilidade das carteiras simuladas, as carteiras *barbell* apresentam rentabilidades bastante atractivas para os horizontes temporais de um e cinco anos. Mas, quando se pretende montar uma estratégia de imunização, o mais importante é o grau de cobertura de responsabilidades que a carteira de activos atinge durante o horizonte temporal analisado, e não a rentabilidade absoluta dessa carteira. Assim, as carteiras *barbell*, tendo uma rentabilidade absoluta superior, são também mais arriscadas e, por isso, não garantem um grau de cobertura de responsabilidades tão elevado como as carteiras *bullet*. São também as carteiras cuja rentabilidade se encontra mais dependente da evolução da estrutura temporal de taxas de juro reais. No que concerne às carteiras aleatórias, a sua estrutura evolui para uma estratégia *bullet* ao longo dos horizontes temporais considerados, o que atesta bem a superioridade das carteiras *bullet*.

Foi também efectuada uma comparação entre este tipo de obrigações e as obrigações de taxa fixa, que são cotadas e avaliadas em termos nominais. Assim, através da evolução das taxas de juro nominais, e tendo presente a Equação de *Fisher*, também a rentabilidade e *cash flows* destas obrigações têm embutida uma expectativa de evolução da inflação futura. Deste modo, foi simulado se estas obrigações também seriam um bom activo para imunizar responsabilidades expressas em termos reais. A comparação foi efectuada somente para carteiras *bullet*, visto que esta é a melhor estratégia de imunização. Foi também expurgada a componente associada à inflação dos *cash flows* das obrigações de taxa fixa a fim de se tornar comparáveis as carteiras de activos de ambos os tipos de obrigações.

Após as simulações verificou-se que as carteiras de obrigações de taxa fixa são mais voláteis e não apresentam um grau de cobertura de responsabilidades futuras tão elevado como as carteiras de obrigações indexadas à inflação. Assim, para imunizar uma responsabilidade futura cuja taxa de crescimento se encontra relacionada com a inflação a melhor abordagem será constituir carteiras de activos com estrutura *bullet*, recorrendo somente a obrigações indexadas à inflação.

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Chapter 1 – Introduction

Although inflation-linked bonds are not a new instrument, its issuance in the Euro Area is fairly new. The first government issue took place in 1998, and, until now, only four Euro Area countries decided to issue these bonds. They are France, Germany, Greece and Italy. With so few issuers, the number of bonds issued is also limited. Currently only twenty four bonds are alive, five of which are linked to the French Consumer Price Index ex-tobacco. The remaining nineteen are linked to the European Harmonized Consumer Price Index ex-tobacco.

Inflation-linked bonds are not as known and as popular between investors as fixed rate nominal bonds because they are not seen as a short term speculative investment. Usually these bonds' natural investors are long term investors that wish to protect their future purchasing power, like pension funds, insurance companies and companies whose revenues grow with inflation.

This way, it would be interesting to access if (and how) they could be used to immunize future liabilities and if the results that would be achieved prove that, for liabilities that grow with inflation, these instruments are a better investment than fixed rate nominal bonds. That is the main driver for this study.

Chapter 2 – Literature Review

The literature available for European inflation-linked bonds and immunization is scarce. Hence, this section main focus will be to review the literature deemed relevant by dividing it in two main subjects: the inflation-linked bond market itself and the findings in immunization techniques.

As for the first theme, the available literature in inflation-linked bonds comprises not only the Euro Area market but also the British and American markets. Notwithstanding, many of the findings in these two markets can be transposed to the European market, since many of the features of this type of instrument are similar. Their main focus lies in pricing structures, trading strategies, inflation forecasting and macroeconomic agents' considerations.

Bardong and Lehnert (2004) focus is driven to the price process of inflation-linked bonds, both in the United States and in Europe. To do this, they focus in two main subjects: the extent to which inflation information revealed today is reflected in the securities pricing (as a measure of efficiency) and how this information could be used to generate trading signals for investors. In order to investigate how the indexation lag can affect the securities pricing, they develop an econometric framework to estimate the break-even inflation[1] inherent to the market prices and to derive inflation forecasts (as an alternative to break-even inflation), either for the short term and medium term, in order to have different inflation forecasts for the remaining maturity of the inflation-linked bonds. These estimates are then used to construct a model which aims to provide signals for trading strategies, based in the differential between break-even inflation and the inflation forecast as an indicator of the expected future change in the bond's observed yield. The information value-added would then be tested by using long-short strategies to trade; this way, by evaluating the results from the trading strategies implemented by using these signals, the authors could assess the degree of efficiency in this market. They did this assessment for markets without restrictions and with restrictions (some of which were short-selling restrictions and trading costs). They found that investors that used a combination of trading strategies with inflation forecasting had better results than uninformed investors; although these results were very pronounced in markets without restrictions, they also hold in markets with restrictions. This way, the inefficiencies existent in the inflation-linked market are not related to the mispricing of the inflation term itself but

[1] Break-even inflation is defined as a time-average consensus estimate of market participants as to the future inflation rate and is derived from the market prices of inflation-linked bonds and nominal bonds with similar maturities.

to the time lag that exists between the reporting and incorporation of the inflation expectations in the securities price. This can happen due to market illiquidity problems and not due to information incorporation problems; if this were the case, the trading model should not work, since the inflation differential would provide erroneous trading signals. However, it seems break-even inflation can be estimated through the inflation forecasts derived from the econometric models; hence, this supports the quality of the trading signals derived and explains the strategies success. This illiquidity problem seems to be also the main driver to explain the reason why long-term strategies produce more favourable results than short-term strategies: these seem to be penalized by high transaction costs. Bardong and Lehnert (2004) used this framework to assess efficiency in the U.S., U.K. and French inflation-linked bond markets, and achieved similar results for the three regions. As for the forecasts' horizon, the most useful horizon is between one and two years, which leads the authors to assume that break-even inflation seems to be a consensus estimate of the average inflation over this period. Nonetheless, they recognize that, to forecast inflation, seasonality patterns are an issue; when not accounted for, they could produce biased estimations and, consequently, make the models and findings invalid.

Ejsing, Garcia and Werner (2007) focus this issue by discussing the impact of seasonality in the term structure of Euro Area break-even inflation rates. They begin their analysis by estimating the term structure of real interest rates and break-even inflation rates, without taking into account the inflation seasonality and the indexation lag. The conventional estimation process for real interest rate term structure is straightforward; it can be derived from the double clean[2] market quotes for the inflation-linked bonds; as for the nominal interest rate structure, it is derived from the transaction (or dirty) price of the inflation-linked bonds, i.e. the double clean price plus the interest accrual and the inflation accrual. However, the authors propose a term structure derived from constant maturity zero-coupon nominal and real interest rates as the proper way to estimate break-even inflation rates. The problem with this framework is how to estimate comparable real and nominal zero-coupon yield curves, namely for longer maturities. This was achieved by estimating real and nominal term structures using the parametrical yield curve approach by Nelson and Siegel[3] instead of using the real and nominal yield to maturity of each security. The benefit of this framework is that it allows for the correction of the seasonality factor inherent to inflation-linked bonds. This is

[2] Inflation-linked bond prices are referred to as double clean because they do not include the interest accrual nor the inflation accrual, that is calculated for both principal and coupon payments.
[3] ECB Working Paper Series No 830, page 17

done by adjusting the dirty prices of inflation-linked bonds for the seasonal factor and the indexation lag, and then used the adjusted priced to estimate the nominal term structure of interest rates. Hence, the break-even inflation term structure computed using these two interest rate structures is unbiased and provides more accurate information that the term structure estimated without taking into account the seasonal bias. This finding is cross-checked with the break-even inflation term structure derived from zero-coupon inflation swaps; by comparing these two term structures the authors conclude that their correlation is very strong when the break-even inflation term structure estimated with the Nelson and Siegel framework is adjusted for seasonality. Since inflation swap rates are annual and, therefore, unaffected by the intra-year seasonality effects, the strong correlation between both term structures allows for the use of the proposed framework to derive intra-year break-even inflation rates, that could be useful not only for market participants but also for other economic agents.

For central banks, inflation-linked bonds could also be useful as a monetary policy instrument. Garcia and Rixtel (2007) discuss how these securities could prove useful for the European Central Bank, as an aid to adequate monetary policy to the (then) 12 Euro Area country participants. Their aim is twofold: they begin discussing the pros and cons of inflation-linked bonds and then move onwards to discuss how these securities' embedded inflation expectations can be used in the implementation of monetary policy. As for the issuance of inflation-linked bonds, some conceptual considerations are presented. On the issuers' side, the argument is that the issuance of inflation-linked bonds allows a government to reduce its cost of financing, by paying a lower real interest rate along with the inflation compensation embodied in these instruments. The *rationale* is that if the total cost associated with these bonds is less than the cost associated with the nominal debt, there is a positive indirect effect on the cost of financing through a reduction of the inflation risk premium embodied on the nominal debt. For investors, the utility of inflation-linked bonds is straightforward: these instruments represent a hedge for inflation risk and allow for investment while preserving their purchasing power. On a social welfare perspective, these instruments can prove useful since they allow to hedge future liabilities whose rate of growth is closely correlated with inflation, like pension funds liabilities. These bonds can also prove useful for balance sheet immunization for companies whose revenues grow with inflation. As for the use of inflation-linked bonds as an instrument for monetary policy, Garcia and Rixtel's (2007) findings state that by deriving break-even inflation rates from these instruments it is possible for central banks to gauge any change in the market participants expectations for inflation and economic growth, aiding the implementation of monetary policy. The existence of these instruments is also beneficial for monetary policy credibility, since it is perceived that if countries are willing to issue inflation-linked bonds this reflects a strong commitment by their governments to keep inflation rates low. In this case, the fact that Euro Area countries are issuing inflation-linked bonds accounts for the credibility of the European Central Bank's monetary policy.

For governments, the important issue is if inflation-linked bonds are a true alternative for their budget and if there is room for its issuance. The Dutch Working Group on the Budget in Real Terms (2005) discusses these aspects thoroughly in their reports. They summarize the reasons why a government should issue inflation-linked bonds in the following way: (1) credibility (as stated before, the issuance of inflation-linked bonds is seen as a reinforcement of an anti-inflation policy); (2) asset-liability management considerations (debt issuance as a way to stabilize the economy as a whole or to hedge the government's balance sheet or budget); (3) expectations to achieve cost-effective debt financing; (4) market demand for these instruments and (5) the possibility to measure the inflationary expectations in these markets by analysing the break-even inflation rates of these securities. They also evaluate if the expected (ex-ante) benefit of inflation-linked bonds is consistent with the realised savings with the issuance of these instruments, using as a benchmark bonds issued by the U.K., Sweden and France. They arrive to a result stated as the 'valuation puzzle', that states that, from a government budget point of view, these securities are cheaper in theory but not in practice.

Siegel and Waring (2004) discuss the importance of the dual duration problem associated with the U.S. Treasury Inflation Protection Securities. Either inflation-linked bonds or nominal bonds have two durations, the inflation duration and the real-interest rate duration, and these two can be seen as the decomposition of a bond's nominal duration. In the case of nominal bonds, the difference is not relevant because both durations are similar to each other and to the nominal duration. This way, by investing in nominal bonds, an investor is unable to bet separately on changes in real interest rates and in inflation. For inflation-linked bonds, the inflation duration is zero, so the nominal duration is only explained by the real-interest rate duration. Therefore, if the liabilities one wishes to hedge are affected by inflation, immunization through real-interest rate duration is the best way to achieve this. The authors suggest this type of immunization to pension funds and individual tax-deferred use. However, one must account for the indexation lag that exists in all inflation-linked bonds in order to make the *formulae* consistent with reality. Notwithstanding, these findings will hold since considering the indexation lag will make the inflation duration different from zero but very close to zero. The major contribution of this working paper lies in addressing something that is quite important for the aim of this thesis: the duration measures of these two types of bonds are not directly comparable. This is an important issue when trying to compare the immunization results between nominal bonds and inflation-linked bonds. Although these findings are for the U.S. market, they can be transposed to the European market, because the inflation-linked bonds' framework is similar in both.

Regarding immunization and duration, Bierwag (1977) proposes investment strategies based on the concept of duration to immunize a portfolio, protecting it from future unpredictable shifts in interest rates. The author shows that the optimal selection of an immunized portfolio depends on the term structure of interest rates observed when the immunization process begins. The immunization efficiency is also dependent on the type of securities used to hedge the portfolio. If one uses zero coupon bonds the result will be different from the one derived from nonzero coupon bonds due to uncertainty about the reinvestment yield for future payments. However, using multiple nonzero coupon bonds allows for a better fit to the term structure of interest rates relevant for the immunization period, allowing the investor to construct a more flexible immunization portfolio. Bierwag (1977) discusses all of these aspects applied to a set of different shifts of the term structure of interest rates that include additive and multiplicative shocks, either discrete or instantaneous. He concludes that the immunization strategy has to be fit not only to the actual interest rate term structure of interest rates but also to the investors' expectations of future evolution of the term structure for the immunization horizon considered.

Ingersoll, Skelton and Weil (1978) analyse the duration concept theoretically, by exposing its uses and abilities. They refute the idea that Macaulay duration measure is a risk proxy, since it introduces bias regarding the impact of interest rate shifts on high and low coupon bonds. Previous literature states the existence of a straightforward positive relationship between interest rate volatility and maturity i.e., other things equal, the impact of a shift in interest rates will be greater in bonds with higher time to maturity. The authors prove that this does not apply for all types of bonds; for instance, a positive parallel interest rate shift will make the price of a low coupon bond vary more than the price of a high coupon bond. The opposite applies for a negative parallel interest rate shift. This way, duration will only be a good risk proxy for uniform infinitesimal interest rate shifts. For other interest rate shifts, duration will only be an approximate measure of the change in the bond's value and this must be taken into account when setting up an immunization strategy. However this does not imply the uselessness of duration, it only draws the attention to a setback in this risk measure that can impact the immunization process and thus force the investor to screen the immunization's quality quite often. The authors present two ways to address this issue: by computing duration with a non-flat term structure of interest rates and with an autoregressive model that (1) accounts for the bias introduced by the diversity of bond coupons and (2) weighs the importance of the yield curve's convexity as a complement measure of interest rate basis risk.

Nonetheless, duration measures, along with convexity and dispersion measures were still used in empirical studies. Bierwag, Kaufman and Toevs (1983) address immunization strategies for multiple liabilities and multiple interest rate shocks. The authors state that, for an immunization strategy to be effective the asset portfolio and the liability portfolio must fulfil three conditions: (1) the asset portfolio and the liability portfolio must have the same present value; (2) both portfolio durations must be similar and (3) the difference between assets and liabilities must be minimal in the stable interest rate scenario. Under the multiple liability funding hypothesis, the authors extend the results further by proving that all the above conditions will be met if the asset portfolio can be divided in two sub-portfolios such that (1) the lowest duration portfolio has a duration lower than the first liability, (2) the highest duration portfolio has a duration higher than the last liability and (3) the duration of the overall asset portfolio matches the durations of the overall liability portfolio. This way, by applying these rules, one can use multiple bonds with different maturities to immunize a stream of predictable future liabilities.

Agca (2005) also addresses the immunization strategy issue by comparing the traditional duration and convexity measures and the single factor Heath-Jarrow-Morton (HJM) framework for multiple portfolio strategies and multiple immunization horizons. His findings can be divided in several propositions. Regarding the risk measures that serve as a base for the immunization strategy, Agca (2005) found that the traditional measures, such as Fisher-Weil duration and convexity, perform better than the HJM framework because the parameters of the latter have to be estimated. When considering the choice between a duration matching immunization strategy and a duration and convexity matching strategy, the author considers the second strategy superior when there are no transaction costs, for both short term and medium to long term immunization horizons. Yet, in the presence of this restriction, a duration matching strategy tends to produce better results since it implies less rebalancing than a duration and convexity strategy for medium to long term immunization horizons; for short term portfolios the duration and convexity matching strategy still achieves better immunization results. Concerning portfolio structures, the findings point for better immunization ability for bullet portfolios if the immunization period is short; however, for medium to long term immunization periods, barbell portfolios attain superior immunization results. These results hold in the presence of transaction costs and are consistent with the immunization strategies chosen. Hence, an investor should adjust its portfolio structure accordingly to the results above while adjusting his immunization strategies for either immunization horizon or existence of transaction costs. These are, in Agca's (2005) point of view, the most important aspects of an immunization framework, since the author results suggest that choosing the appropriate immunization strategy is more important for immunization performance than risk measures or interest rate term structure models.

Oliveira's (2007) study addresses Agca's (2005) findings within the HJM framework but with a deeper analysis by testing the HJM framework with multiple factors and applying it not only to the traditional duration measures but also by using stochastic duration. In this context, Oliveira (2007) shows that for duration matching strategies, and considering three-factor stochastic HJM duration models, immunization results are superior than those achieved using traditional risk measures such as Macaulay and Fisher-Weil. The main reason for this to happen lies in the fact that traditional risk measures are flawed as for their ability to capture adequately interest rate volatility, which makes the three-factor stochastic HJM duration model superior. These findings hold for three and five-year immunization periods (for one-year immunization period the stochastic duration measures do not present themselves as superior) and they remain consistent with and without transaction costs. Oliveira (2007) also concludes that not only the immunization strategy and risk measures are important to achieve a better immunization performance, but it should also be considered and studied how the evolution and shape of the term structure of interest rates during the immunization period can influence the immunization's performance.

Chapter 3 –The European inflation-linked bond market

The aim of this section is to describe thoroughly this market, from its beginning until today. Firstly it is presented a definition of inflation-linked bonds; then follows a description of the market, the issuers and the calculus methodology for these instruments. This chapter concludes with the formulas for duration and breakeven inflation.

3.1. – What are inflation-linked Bonds?

There are several ways to define an instrument; hence two definitions from the literature were chosen in order to do so. Brynjolfsson (2002: 203) defines inflation-linked bonds as “bonds that are contractually guaranteed to protect and grow purchasing power”. A more broad definition to inflation-linked bonds is “securities (…) designed to help protect borrowers and investors alike from changes in the general level of prices in the real economy” (Deacon, Derry and Mirfendereski, 2004: 1). Apart from literature contributions, one can define inflation-linked bonds as bonds which allow for inflation risk protection, since they provide a fixed real interest rate return plus a floating return indexed to a broad inflation measure, in both coupon and principal payments. This way an investor that buys inflation-linked bonds will earn a return that is not only interest rate driven but also protects him from inflation fluctuations, thus not eroding his purchasing power through his investment horizon. For an issuer, inflation-linked bonds allow for bond issuance with a cost structure indexed to inflation, which can be useful since most issuers profits vary with inflation. This way both profit and cost structures are indexed and account for inflation variations and protect the issuer from inflation risk while ensuring more easily debt repayments over time, since both cost and revenue structures grow at the same inflation rate.

3.2. – The inflation-linked bond Market: description and issuer countries

Although not so popular as other types of financial instruments, inflation-linked bonds are not a new instrument. The first bond whose principal and coupon payments were linked to the price of a basked of goods was issued by the State of Massachusetts in 1780. However, it would pass about 200 years before these securities were issued again. In the post World War II inflation-linked bonds became popular again for several reasons. In some countries, this was the only affordable way to issue long term debt, due to the high and volatile inflation environment these economies experienced. Some examples are Chile (1956), Brazil (1964) and Argentina (1973). In the period of 1980-1990, some industrialised countries also started to issue indexed-debt, not because they had to but because the chose to do so. By then, the issuance of inflation-linked bonds was seen as a way to reduce borrowing costs by taking advantage of the high inflation expectations, which made these instruments very attractive when compared to other debt, while also accounting for the issuers' government credibility to commit to control inflation in the future. Some issuers that belong to this group are United Kingdom (1981), Sweden (1994) and New Zealand (1995). More recently, the issue of inflation-linked bonds is associated not with inflation control policies but with social concerns; these securities were presented as a way to provide long-term hedge against inflation, useful namely for pension management. Some of the countries that shared this commitment were Canada (1991), the United States (1997), France (1998), Greece and Italy (2003) and Germany (2006)[4].

However, the issuance of inflation-linked bonds in the Euro Area is fairly new. The first European government to issue bonds linked to a Euro Area consumer price index was France, in October 2001. At this moment, there are four European countries that issue these bonds; a brief description of the issuers is given below. Those descriptions are supported by Deacon, Derry and Mirfendereski, (2004), Bettiss, James and Sooben (2008) and also some information taken from the issuers websites.

3.2.1 –France

The use of indexed bonds in France is not recent. Since 1925 that these instruments have been issued, with several types of linkage; in the pre-Second World War the issues were linked to the exchange rate. However, after the war, normal debt issuance was very difficult due to the lack of confidence in the franc, associated with the poor condition of the French public finances. In the 1950s, government indexation issuance became more frequent, with quite a lot of different linkages, such as the Paris price of the 20-franc Napoleon gold coin (known as the *Rentes Pinay* bond), the main equity index or even the issuance of a state bond that paid interest of 5% plus 0,05% for each point by which industrial production were above the 1955 level. Along with the state issuance, other public corporations and nationalized industries issued indexed bonds whose indexes were mainly associated with the goods they sold. Some examples are in the table below.

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[4] Garcia and Rixtel (2007: 7).

Table 1 : Some examples of French indexed bond issues

Issue date	Issuer	Indexation
1952	Electricité de France	Interest and redemption value linked to the average price to the consumer of 100 kilowatt hours of electricity
1953	Société Nationale de Chemins de Fer	Interest and redemption value linked to the cost of second class rail travel ("free" tickets could be taken in lieu of interest)
1953	Gaz de France	Interest and redemption value linked to the average price of 25 cubic metres of gas
1955	Regié Renault	Interest and redemption value linked to the increase in company sales
1957	Electricité de France	Redemption value included a premium related to the growth in electricity production
1958	Charbonnages de France	Interest and redemption value linked to the price of coal

Data source: Bank of England, (Taken from Deacon, Derry and Mirfendereski (2004: 107).

In 1958, due to the strong devaluation of the franc, indexed issuance was banned because the government feared that the proliferation of these bonds, associated with rising import prices resulting from devaluation, would lead to spiralling inflation. The general ban on indexation was lifted ten years later. Yet, issuance of public indexed debt would only resume in 1973 with a new issuance of the *Rentes Pinay* bond and an issue of a bond whose capital and interest were guaranteed to maintain a fixed relation between the gold content of the franc at the time of issue and that of the EEC Unit of Account used for the Common Agricultural Policy. However, the demise of the EEC Unit of Account in 1979 triggered a clause in the bond's prospectus that stipulated that the bond's cash flows would be linked to the price of gold henceforth; as a result the price of gold rose rapidly, making this bond trade at seven times its face value in 1980 (Deacon, Derry and Mirfendereski, 2004).

There would be no further public issuances of indexed bonds until 1997, when *Agence France Trésor* (*Trésor* henceforth) announced that it was considering issuing index-linked debt, in the form of OAT[5] or BTAN[6], to reduce borrowing costs. Inflation was chosen to be the linkage due to the perceived interest for both issuer and investors in this type of product: the issuer would achieve a more cost efficient debt framework and the investor would be able to purchase a bond that protected its' investment purchasing power throughout the investment's time horizon. The main problem with the setup for inflation-linked bonds was to choose which inflation index to use. There were two possible alternatives: the INSEE's[7] French

[5] OAT stands for *Obligations assimilables du Trésor*, that are defined as fungible securities issued with maturities of seven to 50 years and a coupon paid annually (www.aft.gouv.fr).
6 BTAN stands for *Bons à Taux Annuel Normalisés*, that are fungible Treasury notes with maturities of two and five years and a coupon paid annually (www.aft.gouv.fr).
[7] INSEE is the French National Institute of Statistics and Economic Studies.

Consumer Price Index (CPI) and the Eurostat's European Harmonized Index of Consumer Prices (HICP). The inflation index to use had to be a widely known measure and investors had to be aware of its purpose and how it was assembled and calculated. Although it was feared at the time that the French CPI would not be seen as a good proxy for European inflation and, consequently, limit the bond's attractiveness to investors outside France, the European HICP was very recent, had no track record, and it was still to be determined how it would be calculated, namely regarding each country contribution to the index and the fear of revision risk. Hence, the *Trésor*'s choice was the French CPI because it was, at that time, a broader and better understood inflation measure than the European HICP; but, because under the French law all government contractual arrangements with a link to inflation must not include the price of tobacco, the French CPI ex-tobacco index was used in the end.

This way the *Trésor* issued its first inflation indexed bond (OATi hence forth) on the 15th September 1998, a 10-year bond maturing in 2009, through a syndication process. As mentioned above, the bond was linked to the French CPI ex-tobacco and was designed as a capital indexed bond[8] with a three month indexation lag and paid annual interest. It also had a guarantee that, at redemption, the principal payment would not be less than the original par value, thus protecting the nominal value of the principal should deflation occur during the life of the bond (this guarantee applies only for principal; it does not apply for coupon payments). On the 21st September 1999, the *Trésor* issued the second OATi, a 30-year bond maturing in 2029, also through a syndication process and with a similar design.

On 2001, the *Trésor* reassessed the structure for the new inflation-linked bonds. By then, the main problems for the usage of the European HICP had ceased to exist: the coverage of the index was far more complete than previously expected and the measure itself became very well known to investors since the European Central Bank used it as the basis for its inflation target since the European currency was created in 1999. Notwithstanding, one problem remained with this measure: the revision risk. This problem was also an issue in the United States system: the inflation measure used in the U.S. was also subject to revisions; however, the U.S. Treasury decided during the course of 2000 to manage this issue by using the first value published for the index in each month and not considering possible future revisions in the Treasury Inflation-Protection Securities' valuation. The *Trésor* also decided to do this and, in the 23rd October 2001, issued its first inflation-linked bond indexed to the European HICP ex-tobacco (OAT€i henceforth). The structure of OAT€is is equal to the former OATis; the

[8] The definition and technicalities of this security design will be addressed in the next section.

only difference lies in the inflation index. This even caused some substitution effect, since when the first two OAT€is were issued, in 2001 (OAT€i 2012) and 2002 (OAT€i 2032), many of the purchases in the primary market was done through the exchange from OATi 2009 and OATi 2029 to the newly issued bonds. From then on, the *Trésor* has been issuing bonds with both linkages, in an attempt to construct a benchmark real and inflation yield curves with both French and European inflation indexes. All inflation-linked bonds issued by the French *Trésor* until now can be found in the table below.

Table 2 : French Inflation-linked bonds (OATis and OAT€is)

Bond	Issue date	Maturity date	Nominal Amount Outstanding (€ million)
3,00% OATi 2009	15-Sep-1998	25-Jul-2009	13.761
1,25% BTAN€i 2010	04-Oct-2006	25-Jul-2010	9.325
1,60% OATi 2011	06-Jul-2004	25-Jul-2011	15.313
3,00% OAT€i 2012	23-Oct-2001	25-Jul-2012	14.494
2,50% OATi 2013	24-Jan-2003	25-Jul-2013	15.718
1,60% OAT€i 2015	29-Oct-2004	25-Jul-2015	13.417
!,00% OATi 2017	02-Sep-2005	25-Jul-2017	18.485
2,25% OAT€i 2020	15-Jan-2004	25-Jul-2020	17.551
2,10% OATi 2023	13-Fev-2008	25-Jul-2023	6.077
3,40% OATi 2029	21-Sep-1999	25-Jul-2029	7.451
3,15% OAT€i 2032	23-Oct-2002	25-Jul-2032	9.369
1,80% OAT€i 2040	01-Mar-2007	25-Jul-2040	6.096

Data taken from Bloomberg and *Agence France Trésor* (24-Oct-2009).

For the purpose of this thesis only inflation-linked bonds indexed to the European HICP ex-tobacco (HICPx hereafter)[9] will be used in the analysis, in order to avoid discrepancies arising from differences in the inflation index.

3.2.3 – Germany

Since the hyperinflation crisis experienced in the 1920s, Germany has been very averse to all types of debt indexation. The Federal Republic's Constitution of 1949 enforced this prohibition on indexation, which was in force until Germany entered the European Monetary Union (EMU) in 1999. The first issues of inflation-linked bonds in Germany were done by regional banks. In 2002 the Landesbank Baden-Württemberg issued a 6-year euro denominated bond indexed to the European HICP and the Federal State of Saxony-Anhalt issued a 7-year euro denominated bond linked to the European HICPx; this state would, in 2003 issue another bond with the same design, but with only 5 years to maturity. The

[9] A brief description of the HICPx can be found in Annex 1.

intention of issuing euro HICP ex-tobacco linked bonds was announced to the market in 2004 by the German Finance Agency (*Bundesrepublik Deutschland – Finanzagentur GmbH*). The structure of these securities was equal to the one adopted by the French *Trésor*, with the same cash flow structure and inflation index linkage. The inaugural issue took place only in 2006, with the syndication of a 10-year bond maturing in 2016. The second issue was a 5-year bond maturing in 2013 and took place in October 2007, but through an auction process. In June 2009 took place another 10-year issue of these bonds. Notwithstanding, it is not clear yet if the German Finance Agency had any intention of constructing an inflation-linked bond benchmark curve with further issues of different maturities. Similarly to the French government, these bonds were named accordingly to their maturity and with similar names given to regular fixed rate bonds, but with the "€i" suffix in order to account for the indexation. Hence the 10-year bonds are knows as Federal Bonds or "Bunds" (Bund€i for inflation-linked bonds) and the 5-year notes are known as Federal Notes or "Bobls" (Obl€i for inflation-linked bonds).[10] The table below summarizes all the issues done in Germany so far.

Table 3 : Germany Inflation-linked Bonds

Bond	Issue date	Maturity date	Nominal Amount Outstanding (€ million)
2,25% Obl€i 2013	16-Oct-2007	15-Apr-2013	13.000
1,25% Bund€i 2016	07-Mar-2006	15-Apr-2016	9.000
1,75% Bund€i 2020	04-Jun-2009	15-Apr-2020	3.000

Data taken from Bloomberg and *Bundesrepublik Deutschland – Finanzagentur GmbH* (24-Oct-2009).

3.2.3 – Greece

The first inflation-linked bonds issued in Greece were referred to as State Bonds Index Linked (SBIL) and its issuance took place on the 19th May 1997. Two bonds were launched, with 5 and 10-year maturities. Both of them were designed to be real return bonds[11] with annual interest payment with the addition of the deflation floor for the principal redemption. These bonds were linked to the Greek CPI, since, at that time, it was the most well known and widely accepted measure for inflation. On the 11th August of the same year the Greek Ministry of Finance issued two more SBILs with the same maturities. Since inflation started to fall in early 1999, these instruments stopped being appealing to investors and the State stopped its issuance and even started to accept these bonds as a means of payment for fixed

[10] http://www.deutsche-finanzagentur.de

[11] Real return bonds are indexed bonds with a design similar to the capital indexed bonds but pay coupons semi-annually.

rate bond auctions. However, in March 2003, the Greek Public Debt Management Agency issued, via syndication, an inflation-linked bond maturing in 2025, with the design of a capital indexed bond linked to the European HICPx, just like the French bonds, that were used as a benchmark for this bond's pricing in the syndication process. In 2007, Greece launched its second inflation-linked bond maturing in 2030 and also made a private placement for a 50-year bond linked to the European HICPx. Although having built both major issues to benchmark[12] size, the Greek Public Debt Management Agency already made clear that it has no intention of building a benchmark curve for these bonds. In the table below are the details of the Greek inflation-linked bonds issued so far.

Table 4 : Greek inflation-linked bonds

Bond	Issue date	Maturity date	Nominal Amount Outstanding (€ million)
2,90% GGB€i 2025	18-Mar-2003	25-Jul-2025	8.200
2,30% GGB€i 2030	03-Apr-2007	25-Jul-2030	7.500

Data taken from Bloomberg and the Greek Public Debt Management Agency (24-Oct-2009).

3.2.4 – Italy

The first time Italy considered issuing inflation-linked bonds was in 1981, with the threesome objectives of extending the maturity of the government debt, enhance monetary policy credibility and lower the cost of borrowing. The first issue was a 10-year bond with principal and interest linked to the GDP deflator at factor cost, made in 1983. Although the objectives proposed were achieved with this issue, it was not a successful issue because it was done very quickly and most investors were not well informed about its details and technicalities; it also came to the market at a real interest rate of only 2,5%, which was perceived to be very low when compared to market rates at that time. The fact that it was indexed to changes to a measure that was published only once a year, thus having an indexation lag that was very long and that was not broadly known, also explain its lack of popularity among investors. In the years thereafter, many issues were made by Italian financial institutions. Between 1982 and 1992, the *Instituto Italiano di Credito Fondiario* issued several long term inflation-linked bonds with a 15-year maturity and a partial indexation to changes to the Italian CPI of 75%. Other institutions also issued inflation-linked bonds during the 1990s and early 2000, like the commercial banks *Monti dei Paschi di Siena* and *Banco Ambrosiano*.

[12] A benchmark bond issue is a liquid issue with nominal amount outstanding over 1 billion euros.

Nonetheless, the idea of issuing inflation-linked debt was not forgotten by the Italian Government, and in 2002, the Italian *Dipartimento del Tesoro* announced that was evaluating whether to launch an inflation-indexed bond program in addition to its fixed rate bond program (referred to as BTP).[13] In September 2003, the *Tesoro* syndicated its first inflation-linked bond, a 5-year maturity BTP€i linked to the European HICPx. The design of the Italian inflation-linked bonds is very similar to the French bonds with the difference that Italian bonds pay semi-annual coupons. From 2004 onwards the Italian *Tesoro* started to build a benchmark curve with more issues of different maturities and, by 2005, Italy had surpassed France as the European country with the largest stock of debt issuance linked to the European HICPx. In the table below are listed all the BTP€i issued so far.

Table 5 : Italian inflation-linked-bond issues

Bond	Issue date	Maturity date	Nominal Amount Outstanding (€ million)
1,65% BTP€i 2008	10-Sep-2003	15-Sep-2008	13.400
0,95% BTP€i 2010	26-Jan-2005	15-Sep-2010	14.301
1,85% BTP€i 2012	23-Mar-2007	15-Sep-2012	10.438
2,15% BTP€i 2014	10-Out-2004	15-Sep-2014	14.500
2,10% BTP€i 2017	21-Jun-2006	15-Sep-2017	12.488
2,35% BTP€i 2019	21-Mai-2008	15-Sep-2019	12.377
2,60% BTP€i 2023	20-Jun-2007	15-Sep-2023	13.652
2,35% BTP€i 2035	19-Oct-2004	15-Sep-2035	13.685
2,55% BTP€i 2041	21-Oct-2009	15-Sep-2041	3.500

Data taken from Bloomberg and *Dipartimento del Tesoro* (24-Oct-2009).

3.3 – Inflation-linked bonds security design

The security design of these bonds is very technical; this way this whole section is dedicated to a thorough description of all the *minutiae* related with inflation-linked bonds. The section begins with the description of the cash flows structures that can be adopted. Then it is presented the details associated with the indexation choice and how it is applied to the bond's principal and coupon cash flows, followed by the market price and yield construction. The section concludes with the explanation on how to calculate these bonds' duration and breakeven inflation considerations.

[13] BTP stands for *Buoni del Tesoro Poliennali* and are medium and long term securities with fixed coupons particularly useful for those investors requiring constant payments every 6 months (www.dt.tesoro.it).

3.3.1. – Cash Flow design

The main objective of inflation-linked bonds is to protect investors against an erosion of his investments' purchasing power caused by inflation. However, the design of these securities can take various forms. In this section five different design forms will be presented.[14] The most common are the capital indexed bond (CIB henceforth) and the interest indexed bond (IIB henceforth), but there are three other types of design that are used and worth mentioning: current pay bonds, indexed annuity bonds and indexed zero-coupon bonds.

- ***Capital Indexed Bonds***

This structure has a fixed real coupon rate and a nominal principal value and both rise with inflation. The nominal interest payments are computed as the real coupon rate multiplied by the compounded inflation rate since the bond was issued, as shown below

$$C_t = \frac{r}{c} \times \frac{P_t}{P_0} \tag{3.1}$$

where C_t is the nominal coupon payment at time t (%), r is the annual real coupon rate (%), c is the number of coupons the bond pays *per* year, P_t is the value of the price index at time t and P_0 is the value of the price index when the bond is issued. The ratio $\frac{P_t}{P_o}$ is known as the Index Ratio and allows for the real coupon $\frac{r}{c}$ to be adjusted for inflation.

As for the principal repayment it is also adjusted for inflation. However, to prevent loss of value if deflation occurs, some securities (like the issued by France), have a deflation floor. This way the principal payment can be computed as follows

$$\textit{Without Deflation Floor}: \quad F = 100 \times \frac{P_T}{P_0} \tag{3.2}$$

$$\textit{With Deflation Floor}: \quad F = \max\left(100, 100 \times \frac{P_T}{P_0}\right) \tag{3.3}$$

where F is the principal payment, P_T is the value of the price index at maturity and P_0 is the value of the price index when the bond is issued. This way the indexation allows for small adjustments in the coupon payments but only pays the inflation uplift in the principal

[14] Taken from Deacon, Derry and Mirfendereski, (2004).

repayment. Another important aspect of this design relates to the possibility of introducing a deflation floor: as only the principal payment is protected against deflation, if this occurs during the life of the bond, the coupon payments will reflect it, since they will be smaller than the real coupon rate at issuance.

This is the most widely used structure to design inflation-linked bonds. In some literature it is also referred to as the *Canadian Model*,[15] because Canada was the first country to adopt this design for inflation-linked bonds.

This structure is also known as real return bond structure when the coupon payments are semi-annual, like the Italian case. The design is the same; the only thing that changes is the models' denomination. All the issuer countries referred in the previous section use this security design for their bonds, as well as the USA, the UK, Australia, Sweden and Canada, among others.

- ***Interest Indexed bonds***

In this design, all the inflation adjustments come through with the coupon payments. So every time a coupon is paid, it is adjusted in order to reflect the coupon rate of the bond[16] plus the periodic inflation rate, as follows

$$C_t = r + 100 \times \left(\frac{P_t}{P_{t-1}} - 1 \right) \tag{3.4}$$

where C_t is the coupon payment at time t, r is the annual real coupon rate, P_t is the value of the price index at time t and P_{t-1} is the value of the price index at time t-*1*. The principal of the bond is paid at par value (i.e. 100) without any further adjustments, like conventional fixed rate bonds. These bonds are seen as a form of inflation-protected floating rate bonds and do not provide purchasing power protection of all future cash flows because the principal repayment is not adjusted. This way, this structure provides a lower degree of inflation protection than CIBs, and this explains its lack of popularity among investors.

The only country that used this model to issue inflation-linked bonds was Australia in the 1980s, in parallel with the issuance of inflation-linked bonds under the CIB model. As the former issues proved to be less popular than the issues based in the CIB model, the issuance of inflation-linked bonds with this security design ceased in 1988.

[15] James and Levy-Yeyati (2008: 8).
[16] Only CIBs security design accounts for coupon payments more than once *per* year. Hence, all other types of indexed bonds *formulae* will consider annual coupon payments only.

- ***Current Pay Bonds***

Like the former model, in this security design the inflation adjustments are done in the coupon payments. However, in these bonds' coupon, the adjustment is done on two ways: the coupon itself is inflation-adjusted but it is also summed up an indexation of the fixed principal, as shown below

$$C_t = r \times \frac{P_t}{P_{t-1}} + 100 \times \left(\frac{P_t}{P_{t-1}} - 1 \right) \tag{3.5}$$

where C_t is the coupon payment at time t, r is the annual real coupon rate (%), P_t is the value of the price index at time t a and P_{t-1} is the value of the price index at time t-1. The principal of the bond is paid at par value (i.e. 100) without any further adjustments, like conventional fixed rate bonds.

This structure was considered by the U.S. Treasury when the decision to issue inflation-linked bonds was taken; however they chose to design the securities according the CIB model. The only country that effectively issued securities based in this design was Turkey, between 1997 and 1999.

- ***Indexed Annuity Bonds***

These bonds' design is based on a fixed annuity payment plus a variable payment that compensates for inflation. The base annuity payment (*B*) is calculated using an annual pre-established real interest rate (*r*) and the face value of the bond (*F*) as follows

$$B = \frac{F}{a_n} \quad : a_n = \frac{1}{r}\left(1 - \frac{1}{(1+r)^n}\right) \tag{3.6}$$

Where a_n is the constant annuity formula for a bond where n stands for the number of years the bond will be alive. Hence the coupon payment for this bond is simply calculated by multiplying the base amount by the Index Ratio of the bond.

$$C_t = B \times \frac{P_t}{P_0} \tag{3.7}$$

where C_t is the coupon payment at time t, P_t is the value of the price index at time t and P_0 is the value of the price index when the bond is issued.

Since there is an annuity payment in each coupon, there is no principal repayment at maturity; the investor only receives the last annuity multiplied by the Index Ratio derived from the variation of inflation during the life of the bond. The issue of this type of securities was done in Australia by public corporations.

- ***Indexed Zero-Coupon Bond***

As the name suggests, these bonds pay an inflation adjusted principal at maturity and pay no coupons. The principal repayment is simply the par value of the bond times the Index Ratio as the expression below shows

$$N_t = 100 \times \frac{P_T}{P_0} \tag{3.8}$$

where N is the final payment at maturity, P_T is the value of the price index at maturity and P_0 is the value of the price index when the bond is issued. This type of bonds has been issued in Poland, Iceland and Sweden.

3.3.2. – Indexation choice and cash flow application

As it has already been mentioned several times, the choice of the price index is of the upmost importance to make these bonds an appealing investment. Recalling Deacon, Derry and Mirfendereski, (2004), there are four characteristics the index chosen must have. The first one is matching both investors and issuers' inflation hedging requirements. When using inflation-linked bonds, investors will be exposed to basis risk, i.e. the difference between the inflation measure associated with the liabilities to be hedged and the inflation measure to be used in the bond's cash flows. The higher the mismatch between these two measures, the higher the basis risk and the lower the investors' interest in using these bonds for liability hedging. Secondly, the index must be reliable. In order to be so, the calculation and revision rules for the index must be very explicit and clear and the bond's prospectus must have a description of how an eventual revision, rebasing or other changes in the index will affect the investors. For instance, in the case of the country issuers presented above, revisions do not affect the value of the bond, since only the first number published is considered for the calculus of the Index Ratio; in the case of index rebasing, the Index Ratio will be converted to the new base when the rebasing occurs in order to prevent loss of value and make all past cash flows comparable with the new index base.[17] If the is a delay in the index publication an average of the last 12 numbers published will be calculated and used to compute the Index Ratio. Thirdly, an integrity issue arises; the index provider must be independent from the bond issuer, in order to

[17] The rebasing process is explained in Annex 1.

safeguard the possibility of index manipulation. In the European case this is not a problem since the bonds are issued by country government agencies and the index is published by *Eurostat*, which is under the jurisdiction of the European Commission and not any of the issuer countries; hence, the possibility of index manipulation is residual. The index publication must also be disseminated as quickly and widely possible, in order to allow for the market prices to incorporate it as promptly as possible. Last but not least, index seasonality is also an aspect to take into account to value the bond's cash flows, since it will affect not only the expected nominal size of the future cash flows by the seasonality pattern (accordingly to the time of the year when the interest payments are done) as well as market nominal yield calculations, because these entail a future inflation forecast that is not constant over time. This way, quoted nominal yields will fluctuate throughout the year with seasonality changes. The method to eliminate these problems would be correcting the seasonality patters in the index, but that would imply back month revision of the index, something that is not acceptable for the cash flows calculation. Although not being corrected in the cash flows valuation, the issuers and investors are aware of the seasonality patterns, since they are relatively stable over time and are a pure reflection of economic reality.

One thing that can also be considered is whether to fully index the security's cash flows or to allow for partial indexation. Taking into account that if partial indexation is to be applied, then the bond will not reflect price changes in full, the truth is that in some cases partial indexation can make inflation-linked bonds more attractive to investors. One example of partial indexation is the introduction of a deflation floor at redemption for principal payment. This way, if deflation occurs during the life of the bond, it will not erode the principal's real value, allowing for some extra return for investors. Although this represents a mismatch in the bond's inflation hedging abilities, in this case it is considered benign, since it produces a higher return than the one reflected in the inflation-linked liability portfolios that the investor wanted to hedge.

Other important feature to be considered for the security design of these bonds is the indexation lag. One of the reasons why inflation-linked bonds were created was because they can allow investors to have a high degree of real value certainty in their future cash flows. Total real value certainty would be achieved if all cash flows were adjusted for inflation right up to the moment they are paid. However, this is not possible because it takes time to compile and publish price indices. One would also have to take into account the instantaneous inflation variation between the trade date and the settlement date for accrued interest calculation, and that is not possible as well. Notwithstanding, the longer the indexation lag, the lesser the inflation hedging the bond provides; this point becomes even more important for short term inflation-linked bonds, because the proportion of the bond's remaining life as a fixed rate bond security increases as the security approaches its time to maturity. The indexation lag will correspond to the time to maturity in which the security is exposed to inflation risk, i.e. if the indexation lag is six months, in the last six months the security is alive, all the inflation changes that occur will not be transferred to the security cash flows, since when that was supposed to happen the security will already have matured.

As mentioned above, one way to minimize the indexation lag is to make it as short as possible. As James and Levy-Yeyati (2008) point out, in the case of the CIB structure used for the euro denominated inflation indexed bonds, the lag applied is of three months. Between index publications the reference index is computed by linear interpolation,[18] like the formula below shows.

$$I_t = P_{m-3} + \frac{(t-1)}{M_m} \times (P_{m-2} - P_{m-3}) \tag{3.9}$$

where I_t is the reference index for day t, P_{m-3} is the value of the price index at time m-3 months, P_{m-2} is the value of the price index at time m-2 months, M is the number of days in month m, t is the day of the month m when settlement occurs and m is the month on which settlement takes place. Applying formula (3.9) to the day in which the inflation accrual for the bond begins, by substituting the m day for the first day when interest starts to accrue in the bond (the *base* day), allows for the calculation of the base index (I_{base}). This way, it is possible to compute a daily Index Ratio to adjust for daily inflation changes in the bond and whenever it is traded, making the inflation accrual steadily over each month instead of adjusting only once a month, when the new number of the price index is published. The daily adjusted Index Ratio is given by the expression

$$IR_t = \frac{I_t}{I_{base}} \tag{3.10}$$

where IR_t stands for Index Ratio in day t. Both indices used to compute IR are truncated to six decimal places and then rounded to five decimal places. In order to compute cash settlement amounts, real accrued interest is computed as done for nominal fixed rate bonds. Then, clean price and real accrued interest are each multiplied by the Index Ratio. As for coupon and principal amounts, the process is the same: each is multiplied by the Index Ratio computed with reference to the day when they are calculated. However, if at maturity the Index Ratio is

[18] Since all euro denominated bonds follow the CIB structure, from now onwards the only *formulae* presented is the one that applies to this security design.

less than one (this will happen if deflation occurs), if the bonds have this option, the deflation floor will be triggered and the principal amount will be redeemed at par value.

One thing that stands out from this design is that for, both annual and semi-annual coupons, the inflation accrual varies between coupon payments, since it reflects the monthly changes in the index. Nonetheless, the indexation lag means that the nominal yield does not include all known inflation information from a time lag that could vary between two weeks and a month and a half of inflation in the current CPI measure, which has not yet been incorporated in the calculus of the Index Ratio.

The complete *formulae* for the CIB security design can be found in Annex 2.

3.3.3. – Duration calculation

As Brynjolfsson (2002: 209), summarizes, “Duration is the measure of a bond’s market value sensitivity to changes in specific yields – real or nominal”. Hence, duration measures, as defined by Macaulay and Fisher-Weil, also apply for inflation-linked bonds, with some variations. These bonds have two types of duration[19]: real-interest rate duration and inflation duration, which can be seen as a decomposition of a bond’s nominal duration. Although this decomposition is very hard (as it does not make sense) for nominal bonds, since all the cash flows associated to the bond are expressed in nominal terms, the same does not apply for inflation-linked bonds. Since inflation-linked bonds’ cashflows are stated in real terms, the usual duration formulae can be used to compute the bonds real duration. This way, for a non-flat interest rate term structure the inflation-linked bond’s real duration (DFW) will be computed as follows

$$DFW = \frac{\sum_{j=1}^{n} \tau_j \times \frac{r/c}{\left[1 + y_R\left(0,\tau_j\right)\right]^{\tau_j}} + \tau_n \times \frac{100}{\left[1 + y_R\left(0,\tau_n\right)\right]^{\tau_n}}}{B(0)} \tag{3.11}$$

where τ_j is the remaining time to the real coupon’s payment (in years), r is the annual real coupon rate, y_R is the annual real spot rate, n is the time to maturity of the bond (in years), c is the number of coupons the bonds pays *per* year and *B(0)* is the bond’s real fair value, computed as expressed below:

$$B(0) = \sum_{j=1}^{n} \frac{r/c}{\left[1 + y_R\left(0,\tau_j\right)\right]^{\tau_j}} + \frac{100}{\left[1 + y_R\left(0,\tau_n\right)\right]^{\tau_n}} \tag{3.12}$$

[19] Siegel and Waring (2004: 52)

In real terms, the formula is straightforward, but in nominal terms the calculation can be tricky. There are two ways to compute these bonds' nominal duration, most commonly referred to as *effective duration* (ED). One of them is by adjusting the real duration by a factor denominated *yield beta* that arises from the β coefficient of a linear regression that attempts to measure the yield sensitivity of the real interest rate to a change in the equivalent nominal interest rate.[20] The formula is beneath:

$$ED = \beta \times DFW \tag{3.13}$$

This is a very simple way to estimate inflation-linked bonds effective duration; however it has important flaws that need to be considered. Pond (2008) addresses some of these flaws. Although nominal interest rates variations can be explained by real interest rates variations, this is not the only factor that makes nominal interest rates vary. Inflation changes also contribute for variations in nominal interest rates and, in a broader level, anything that can lead to inflation changes can have an effect on nominal interest rates. Another flaw is related to the unsteadiness of the *yield beta* estimate: the inflation-linked bonds' yield sensitivity to changes in the equivalent nominal yield is not stable. This arises because normally real interest rates are less volatile than nominal interest rates, but it is also an information issue: market movements and the arrival of new information can be incorporated in both yields in different ways. For instance a new inflation rate release should not affect real interest rates, only nominal interest rates and, for this specific event the *yield beta* is zero. This way, to correctly use the *yield beta* to compute effective duration, new estimates would be needed on a daily basis and the information effects needed to be purged from the estimates, which can prove to be very difficult. Taking into account all these aspects, to simply adjust the *effective duration* formula using the *yield beta* seems to be an oversimplification and can lead to a high level of basis risk since all that is not incorporated in the *yield beta* can be attributed to inflation changes.

The other way is to compute the *effective duration* adjusting the real cash flows of the bond with an inflation expectation until the bond's maturity and then discount the cash flows using nominal rates. This approach is more complicated and it also has the setback of being dependent from a forecast of future yearly inflation rate (and, this way, also entailing some room for error) but it seems more realistic than the *yield beta* approach. For this purpose, assuming that the best estimate for future inflation is the *year-on-year* actual inflation rate

[20] Pond (2008: 164).

(depicted as π) and bearing in mind the relation between nominal interest rates and real interest rates as portrayed in Fisher's Equation:

$$\left(1+y_N\right)=\left(1+y_R\right)\times\left(1+\pi\right) \tag{3.14}$$

where y_R stands for real annual spot rate and y_N stands for nominal annual spot rate, formulas 3.11 and 3.12 can be converted in order to allow the calculus of duration in nominal terms.[21]

$$DFW=\frac{\sum_{j=1}^{n}\tau_j\times\frac{C_j}{\left[1+y_N\left(0,\tau_j\right)\right]^{\tau_j}}+\tau_n\times\frac{FV_n}{\left[1+y_N\left(0,\tau_n\right)\right]^{\tau_n}}}{B(0)}, \tag{3.15}$$

$$where \quad B(0)=\sum_{j=1}^{n}\frac{C_j}{\left[1+y_N\left(0,\tau_j\right)\right]^{\tau_j}}+\frac{FV_n}{\left[1+y_N\left(0,\tau_n\right)\right]^{\tau_n}} \tag{3.16}$$

where C_j is the *j*-th coupon of the bond and *FV* is the face value of the bond.

Although it can be argued that the Index Ratio affects expressions (3.15) and (3.16) in the same way and, hence, the inflation adjustment is redundant, that is not entirely true due to the indexation lag. This way the future inflation expectations are reflected instantaneously in the nominal interest rates but reflected with a three month lag in the coupon and principal valuation, since the Index Ratio computation allows for that lag, as stated in equations (3.9) and (3.10). Anyway, although not being redundant, it is possible that the difference between both duration measures might be small.

The calculus of the *effective duration* is important to allow comparisons between inflation-linked bonds and nominal bonds in a portfolio context, since all risk measures computed in a portfolio must derive from similar individual bond measures, i.e. it is not possible to calculate accurately the portfolio's duration using nominal duration for fixed rate bonds and real duration for inflation-linked bonds.

3.3.4. – Breakeven inflation calculation

By comparing fixed rate bonds and inflation-linked bonds, it is clear-cut to see that they are related. Fixed rate bonds only allow yield calculations in nominal terms; as for inflation-linked bonds, they allow for price and yield calculations in real terms and, with a conversion factor (the Index Ratio), transform them in nominal terms. So, by using fixed rate bonds (*FRB*) and inflation-linked bonds (*ILB*) with similar maturities it possible to derive an

[21] The calculus detail is depicted in Annex 3.

inflation expectation that comes from the difference between nominal and real interest rates. This estimate is named breakeven inflation (*BEI*) and can be obtained by the expression below.

$$BEI\ rate = \frac{\left(1+y_N^{FRB}\right)}{\left(1+y_R^{ILB}\right)} - 1 \tag{3.17}$$

This expression is derived from Fisher's equation (3.14) and it simply allows computing the annual inflation assumption that will make hold-to-maturity investors indifferent to having fixed rate bonds or inflation-linked bonds with similar maturities. To illustrate this we can take a simple theoretical example. Suppose we have two bonds maturing in June 2011, a fixed rate bond with a nominal *yield-to-maturity* of 3,5% and an inflation-linked bond with a real *yield-to-maturity* of 1,2%. By computing the breakeven inflation accordingly to expression (3.17),

$$BEI\ rate = \frac{(1+3{,}5\%)}{(1+1{,}2\%)} - 1 = 2{,}273\%$$

we find that the annual breakeven inflation rate is 2,273%, i.e. if this was the inflation rate until June 2011, for an investor that wants to be protected against inflation fluctuations from now until that date it will be completely indifferent to buy the fixed rate bond or the inflation-linked bond at the given *yield-to-maturity* rates.

Although this reasoning might seem naive, since the probability that future inflation will equal current breakeven inflation is very low, this is what the breakeven inflation concept stands for. This measure is used mainly to access market expectations for future inflation; it also works as a barometer for central banks to evaluate future expectations of market participants, helping to implement future monetary policy.

Chapter 4 – Immunization Procedures

Assuming that one has liabilities in euros that need to be liquidated in the future and that those liabilities' growth rate is equal to the European inflation rate, what is the best way to immunize them? The aim of this section is to search immunization strategies using inflation-linked bonds. The first section discusses the term structure of interest rates. The chapter proceeds with liability definition, immunization strategies and an in-depth presentation of the hypothesis assumed for the asset portfolio. Then the results of the immunizing strategies using inflation-linked bonds are revealed. The last section compares immunization results between inflation-linked bonds and fixed rate bonds.

4.1. – Term Structure of Interest Rates

One of the main inputs for this analysis is interest rates and since the analysis can be done in real terms or nominal terms, this implies that both types of interest rates must be collected. For nominal interest rates the procedure is easy: it is only necessary to gather zero coupon rates for the Euro Area from any market information source.[22] This information is widely known and used by market agents. Conversely, to collect information for real interest rates is not so simple since the information available in the market comes from interest rate swaps, where the market source is not sovereign bonds and the rates published are subject to the swap counterparty risk level. This way, the only way to get real interest rates that are based on sovereign bonds is to estimate them from the bonds that are quoted in real terms, i.e. inflation-linked bonds. From all the issuers presented, only two have enough bonds to accurately estimate the term structure for interest rates: France and Italy. By analysing both issuers, one can infer that, although Italy has more bonds issued now, France was the first issuer for this type of bonds in the Euro Area. Hence, by using French bonds it is possible to estimate the real term structure of interest rates for a wider investment horizon; and this is the main reason for the choice of French bonds as input for real interest rate estimation.

The term structure extraction chosen was the parametric approach first introduced by Nelson and Siegel (1987) that consists in minimising the sum of squared price errors, defined beneath

$$\min_{\beta_0,\beta_1,\beta_2,\beta_3} \sum_{t=1}^{n} \left[SQ_t(0) - B_t(0)\right]^2 \qquad (4.1)$$

[22] All market data used in this thesis has been collected from Bloomberg.

where $SQ_t(0)$ is the settlement quote for bond *t*, computed as the sum of the clean price and the real accrued interest for the bond, $B_t(0)$ is the fair value of the bond, computed as shown in equation (3.12) and β_0, β_1, β_2, β_3 represent parameters that need to be estimated in order to compute the real interest rates through the equation below

$$y_R(0,\tau) = \beta_0 + \beta_1\left(\frac{1-\exp\left(\frac{-\tau}{\beta_3}\right)}{\frac{-\tau}{\beta_3}}\right) + \beta_2\left(\frac{1-\exp\left(\frac{-\tau}{\beta_3}\right)}{\frac{-\tau}{\beta_3}} - \exp\left(\frac{-\tau}{\beta_3}\right)\right) \tag{4.2}$$

where y_R is the real spot interest rate and τ stands for the rate's investment horizon. The real term structure of interest rates was estimated on a weekly basis and for investment horizons that vary from *overnight* to forty years.

4.2. – Liability definition and immunization strategies

The first step is to define the liability we wish to immunize. As Fabozzi (2000: 449) defines, "a liability is a cash outlay that must be made at a specific time". This author classifies liabilities in four types: (1) one for which both the cash outlay's amount and timing are certain in the beginning of the immunization process, (2) one for which the cash outlay's amount is certain but the timing is not known, (3) one for which the cash outlay's timing is known but the amount is uncertain and (4) one for which both the cash outlay's amount and time are uncertain. In this case we wish to immunize a liability whose nominal final amount is unknown in the future, since it grows at the European annual inflation rate (measured by the HICPx): this way, according to Fabozzi (2000) we are in the presence of a type 3 liability. It could also be considered a choice between single liability or multiple liability immunization procedures. For the purpose of this thesis, a single liability is considered.

Now that the liability to immunize has been defined it matters to identify the correct immunization strategy. According to Christensen, Fabozzi and LoFaso (1997: 929), setting up a single-period immunization implies that at least two conditions are met in the beginning of the immunization period: (1) equalling the duration of the asset portfolio to the duration of the liability one wants to immunize, (2) ensure the present value of the asset portfolio equals the value of the liability. As for the rebalancing of the asset portfolio, this should be done in a way that the duration of the asset portfolio is always equal to the duration of the remaining life of the liability. This implies that every cash flow generated by the asset portfolio (i.e. coupons, bond redemptions) will have to be reinvested into the portfolio. However the duration-matching strategy described above is not the only immunization strategy that could be used. One could also opt for a duration and convexity immunization approach that has the advantage of accounting for variations in the term structure of interest rates that are not parallel changes. As for the immunization procedure itself, one could choose between three approaches: (1) passive immunization, where the goal is to keep the liability immunized at all times, regardless of the asset portfolio's return, (2) active immunization, where a strategy is devised to attain a minimum realized return rate for the asset portfolio, while keeping the liability immunized and (3) a contingent immunization, which is a mixture of active and passive approaches, i.e. one establishes a "trigger" return rate for the asset portfolio and proceeds with an active immunization approach; if the realized rate of return of the asset portfolio falls bellow the defined "trigger" rate, a passive immunization approach is adopted thereafter.

For the purpose of this thesis, it is considered a simple plan: single liability that will be immunized by applying a duration matching strategy and a passive approach. This will be done by creating an asset portfolio, according to the structure described in the next section.

4.3. – Asset portfolio structure assumptions

The asset portfolio structure that will be implemented to immunize our liability is presented below. It will also be discussed the main assumptions that will take place in the immunization process.

4.3.1. – Cash outlay

The first step is calculating the amount we need to invest to immunize the liability in the beginning of the process. In this case, the investment amount will equal the present amount of the liability to immunize. The asset portfolio will also be self-refinanced, i.e. there will not be any type of cash injections during the immunization process, unless they are generated within the portfolio *via* coupon payments and/or principal redemptions than occur during the immunization process.

4.3.2. – Eligible securities

Since the liability is defined as growing at a rate equal to the inflation rate and is in euros, the security universe to be considered are inflation-linked bonds. Since there are two different inflation indexes for which exist inflation-linked bonds issued (the HICPx and the French CPI ex-tobacco), in order to avoid discrepancies between the growth rate for the liability and the asset portfolio, the inflation-linked securities that have a linkage to the French CPI ex-tobacco are excluded from the set of eligible securities. However, the market securities available do not include short term securities that could prove useful for this analysis. To account for this, synthetic inflation-linked zero coupon bonds were created, with six-month and one-year maturities and accordingly to the Indexed Zero-Coupon Bond design presented in Chapter 3. The duration and fair value calculations are done in real terms accordingly to expressions (3.11) and (3.12), but accounting for the fact that no coupons are paid.

4.3.3. – Investment Horizon

The investment horizon for the asset portfolio will be equal to the time remaining to liquidate the liability one wishes to immunize. In order to decide which investment horizons should be considered for the simulations it is necessary to observe the horizon for which it is possible to collect data for inflation-linked bonds. This way, and considering that the first eligible inflation-linked bond was issued only in 2001, the investment horizons chosen were one, three and five years. However, after analysing the results from the simulations, it was visible that for the five-year horizon some results seem to be odd. So, taking into account that real bonds are used and the last two years we went through one of the most serious financial market crisis ever, a new investment horizon of four years was introduced in order to access if the five-year investment horizon results were dependent (or not) of the market environment.

4.3.4. – Rebalancing frequency

The asset portfolio will be rebalanced on a weekly basis, ensuring this way sufficient data for all the investment horizons chosen. This rebalancing frequency also allows for faster incorporation of any kind of cash flows generated by the invested securities into the asset portfolio. It also makes possible to collect enough data for the smaller investment horizons, something that would not be attained with a wider rebalancing frequency.

4.3.5. – Additional Constraints

It is also possible to set up any additional constraints for the immunization process that might be relevant, namely immunization risk, transaction costs, and as some assumptions related to the security universe chosen (i.e. nominal or real immunization, country spreads). The above mentioned features are those that allow for more freedom of choice and testing of the immunization procedures. The choices made are explained below.

- ***Immunization risk assumptions***

Although it is being applied a duration-matching passive strategy, one has to bear in mind that only by chance the interest rates changes will be parallel changes for the investment horizons considered. This way it is possible to choose several positioning approaches for the asset portfolio in order to test which will produce better results in each investment horizon. Since only single liability immunization is being considered, the creation of bullet portfolios is a necessary condition for the simulations, since it is foreseeable that these are the portfolios that achieve higher results in this scenario, because they allow matching both asset portfolio and liability cash flows.[23] However, to complement the analysis, barbell and random portfolios will also be simulated in order to evaluate if there is value added in choosing any of these types of portfolio positioning (which are riskier in terms of immunization strategies), and, if so, why that occurs. Each portfolio positioning will be simulated for all investment horizons considered.

- ***Transaction costs assumptions***

The base scenario for all simulations will be to assume that there are no transaction costs, and all securities will be bought and sold at mid-market prices. However, inflation-linked bonds are fairly illiquid, so it would be interesting to evaluate if the results that will be obtained for all the portfolios estimated under the “no transaction costs” assumption will hold with transaction costs. This way, a second scenario will be implemented by recomputing all simulations, for all investment horizons and portfolio designs with transaction costs, such that bond purchases will be done at offer prices and bond sales at bid prices. Under this scenario, and considering that, in the presence of transaction costs excessive rebalancing might be harmful for the immunization performance,[24] for all investment horizons and portfolio designs, a monthly rebalancing horizon will also be considered in order to gauge if there is superior performance with less portfolio rebalancing. In order to be possible to simulate the portfolios with both real and synthetic bonds, the latter ones’ prices will be adjusted to reflect different bid and offer prices. This will be done by calculating the bid-offer spread,[25] in cents, for each rebalancing date, using the shorter real inflation-linked bond and then applying that

[23] Fabozzi (2000: 462-463).
[24] Agca (2005: 648) and Fabozzi (2000: 461-462).
[25] The bid offer spread in computed by the difference between offer and bid prices.

value to the estimated mid prices. The offer price will be created by adding the half bid-offer spread to the mid price and the bid price will be created by subtracting it.

- ***Inflation-linked bonds assumptions***

Regarding the set of eligible securities, two issues must be addressed before proceeding to the simulations.

The first one is to choose between an immunization process in nominal terms or real terms. Since both asset portfolio and liability are set to grow at the same inflation rate, it could be straightforward to choose an immunization process in real terms. In the liability side it is possible to go either way since the only adjustment to be made is to add the growth rate to the real value of the liability for each investment horizon in analysis. Yet a difficulty arises when one considers the possibility of comparing these results with results derived from immunization with fixed rate bonds. If the simulation using inflation-linked bonds is done in real terms and the simulation using fixed rate bonds is done in nominal terms the results will not be comparable. Hence, there are two alternatives: (1) convert the transaction prices and duration values for inflation-linked bonds in nominal terms or (2) convert the transaction price and duration values for fixed rate bonds in real terms. As already addressed in Chapter 3, the differences between the nominal values and the real values might be small. However, this issue must be considered before proceeding to the simulations, because it does not make sense to compare results expressed in real terms with results expressed in nominal terms. In order to evaluate this, values in real terms and nominal terms were estimated for the inflation-linked bonds' duration and fair values. The results are presented beneath.

Table 6 : Inflation-linked Bonds comparison: nominal vs. real estimation

Country	Average	Maximum	Minimum
Panel A: Differences between nominal and real duration (absolute percentage)			
France	0,272%	0,816%	0,007%
Germany	0,050%	0,087%	0,013%
Greece	0,210%	0,693%	0,008%
Italy	0,430%	0,497%	0,363%
Panel B: Differences between nominal and real bond fair values (absolute percentage)			
France	7,814%	12,056%	5,107%
Germany	5,888%	6,848%	4,929%
Greece	6,193%	8,601%	4,408%
Italy	8,827%	9,738%	7,916%

It is possible to see that, for the duration measure, the differences are small. The same does not occur for the bonds' fair values, since they vary a lot, as expected, due to the inflation accrual. Hence, it is shown that ignoring this issue would introduce an unnecessary bias to the study that could reduce its accuracy and credibility. This way, the choice lied in an immunization process in real terms and, consequently, the disinflation of the fixed rate bonds' cash flows.

The other issue is related with the issuers country spreads. As visible in the table below, the sovereign issuer ratings suggest different risk levels between the issuers.

Table 7 : Sovereign Issuer Ratings for inflation-linked bond issuers

Country	S&P	Moody's	Fitch
France	AAA	Aaa	AAA
Germany	AAA	Aaa	AAA
Greece	A-	A1	A-
Italy	A+	Aa2	AA-

Data taken from Bloomberg in 24-Oct-2009. The ratings present were attributed by the most reliable rating agencies worldwide. Rating definitions can be found in Annex 4.

This issue is relevant because different risk levels require different interest rate term structures (in both real and nominal terms) that reflect the inherent issuer risk of the bond. This is inconsistent with the simulation of values and durations for the asset portfolio, because the portfolios' duration and present value can only be computed accurately if all future cash flows from all the bonds included in the portfolio are discounted using the same term structure of interest rates. There are several ways to address this matter. One of then is to estimate term structures for interest rates accordingly to the sovereign risk level. However, this cannot be adopted because it does not solve the portfolios issue, so a solution where only one interest rate term structure is used must be adopted. Hence two options can be considered. One of them is using for the simulations only bonds with the sovereign rating similar to France, since this is the country from which the real term structure of interest rates was derived. The set of eligible securities would be narrowed to the ones issued by France and Germany. Since the set of eligible bonds is by definition quite small, another option can be considered. In order to make the values comparable, one can discount the country spread in the bonds coupons to lower the future cash flows to levels consistent with a higher credit quality.

The *rationale* behind this approach is simple: if a country has a perceived higher risk level, in order to be able to sell its debt it must pay a premium that compensates that higher risk. This premium is reflected in the bond's coupon, since the bond's principal is always 100%, no matter the issuer. This way, similarly to what was done for the latter issue, values for the higher risk bonds were estimated considering the coupon correction and compared with the values estimated without the coupon correction. The country spreads were calculated from the difference between nominal sovereign yield curves that reflect the risk premium associated with both rating structures.[26] Both duration and fair value differences were evaluated for the set of eligible bonds. The results are presented in Table 8.

Table 8 : Inflation-linked Bonds comparison: Country Spreads

Country	Average	Maximum	Minimum
Panel A: Country Spread duration differences (absolute percentage)			
Greece	0,0702%	0,1088%	0,0316%
Italy	0,0066%	0,0220%	0,0000%
Panel B: Country Spread fair values differences (absolute percentage)			
Greece	1,0922%	1,3009%	0,8835%
Italy	0,0740%	0,1906%	0,0093%

As it can be observed, the differences are quite small. This can allow us to conclude that, contrarily to the nominal *versus* real terms issue, this, although relevant, is not a serious question that could bias the results. This way the set of eligible securities will remain the same, and, although this issue is acknowledged, the calculus of data for the inflation-linked bonds will be done without taking into account the country spread.

All scenarios and hypothesis for the simulations are summarized in Table 9.

Table 9 : Inflation-linked Bonds' simulations structure

	No Transaction Costs	With Transaction Costs
Investment Horizon (years)	1,3,4,5	1,3,4,5
Rebalancing Frequency	Weekly	Weekly and Monthly
Immunization Strategies	Duration-matching bullet, barbell and random portfolios	Duration-matching bullet, barbell and random portfolios
Inflation-linked Bond Options	Real Approach without country spread correction	Real Approach without country spread correction

[26] Although the calculation was done in real terms, the country spreads can be derived from the difference between nominal interest rates and then applied to the real coupons because, as already discussed in Chapter 3, the only difference between nominal and real interest rates is the inflation rate. The choice to collect country spreads from market rates instead of estimating them from the inflation-linked bonds' term structures was done to avoid calculation bias as well as to obtain market values that evolve through time.

4.3. – Immunization using inflation-linked bonds

Having discussed all the strategies and hypothesis implemented, the simulations were done and the results were interpreted accordingly to three sets of indicators:

- Liability coverage – to access if, when, and in what conditions the liability is fully covered.
- Asset returns – to evaluate asset returns for all portfolio strategies and compare them with the spot real interest rate in the beginning of each investment horizon.
- Portfolio composition statistics – this set of indicators will only be presented for the random portfolios and its objective is to gauge how the portfolios are composed under this strategy.

Before discussing the results I present some statistics related with the liability to immunize in the table below:

Table 10 : Liability returns

	1 year	3 years	4 years	5 years
Annualized Return	0,70%	1,03%	1,18%	1,32%
Standard Deviation	0,34%	0,96%	1,37%	1,91%

- ***Results without Transaction Costs***

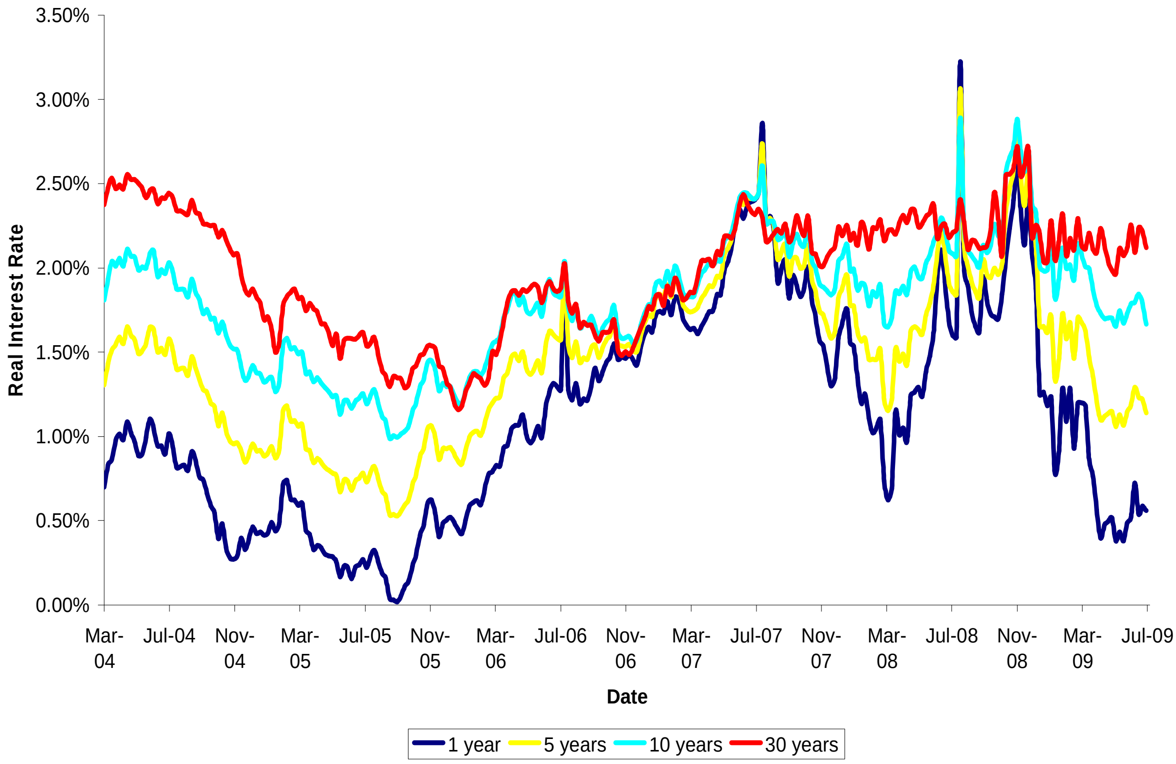
Annex 5 contains the tables with the results for this scenario. The table within this annex is divided in four panels, one for each investment horizon considered, and inside each panel are presented the three sets of indicators listed above. Table 11 shows only the comparison between the asset portfolio's realized rate of return and the real spot rate for the investment horizon analyzed.

Table 11 : Asset Portfolios Realized Rate of Return vs. Spot Rate (without transaction costs)

	Random	Bullet	Barbell
Panel A: One-year Immunization Period			
Real spot rate	0,6990%	0,6990%	0,6990%
Realized Return Rate	0,6405%	0,6721%	2,3170%
Panel B: Three-year Immunization Period			
Real spot rate	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0543%	1,0601%	0,8450%
Panel C: Four-year Immunization Period			
Real spot rate	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,1611%	1,1123%	1,3255%
Panel D: Five-year Immunization Period			
Real spot rate	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,3106%	1,3185%	2,2467%

The first thing that is visible is that the difference between both measures is not very high for random and bullet portfolios. However, for barbell portfolios it is quite significant, namely in the one-year and five-year investment horizons, where this strategy outperformance is outstanding. Is this difference so significant that proves this strategy far superior to all others? Unfortunately the answer is no. A barbell strategy consists of investing in short term and long term bonds in order to immunize a liability by constructing an asset portfolio whose duration matches the liabilities duration, and, as already discussed in section 4.3.5, stands for a riskier approach to immunization. This is the case because the performance of these portfolios is highly dependent from variations in real interest rates. As depicted in Figure 1, real short-term interest rates are more volatile than long-term interest rates, and they do not evolve in the same way.

Figure 1 : Real Interest Rate Evolution



This ultimately explains why for some investment horizons these portfolios outperform. Barbell portfolios achieve higher returns by investing in long-term bonds because their interest rates seem to be more stable and not because the strategy is superior; if long term interest rates had been more volatile during the investment horizon depicted in Figure 1, the barbell portfolios would easily underperform the other strategies. This also explains the outperformance of five-year investment horizons mentioned in section 4.3.3.

In terms of liability coverage, bullet portfolios achieve superior results. This is particularly visible for one-year and three-year immunization periods where bullet portfolios have higher performance and lower risk when compared with the other two strategies. In order to access this I emphasise the 25th percentile results, since it allows for a worst-case scenario analysis for all portfolios' liability coverage abilities. For one-year and three-year horizons (respectively) bullet portfolios' 25th percentile exceed in 4 basis points (b.p.) and 15 b.p. the random portfolios and in 44 b.p. and 18.b.p. the barbell portfolios. For the four-year and five-year portfolios this dominance does not occur for random portfolios but it is still visible when comparing the performance of bullet portfolios versus barbell portfolios, where bullet portfolios' 25th percentile exceeds the barbell portfolios in 13 b.p. for the four-year horizon; for the five-year horizon the barbell portfolios outperform the bullet portfolios by 1 b.p.. This accounts for the stability of bullet portfolios since they are indeed less sensible to outlier returns.

This is corroborated by the analysis of the liability coverage standard deviations. Here the picture is straightforward; bullet portfolios tend to have lower standard deviations in the one-year (0,01%) and three-year (0,15%) investment horizons and random portfolios have the lowest standard deviations in the four-year (0,16%) and five-year (0,15%) investment horizons. Notwithstanding, standard deviation differences between bullet and random portfolios vary between 1 b.p. and 2 b.p. which is indeed a small value for this measure. Contrarily, barbell portfolios' standard deviations are between two and ten times higher than the random and bullet portfolio' standard deviations.

As for random portfolios, the portfolio composition statistics point to an evolution to a bullet structure as time goes by, within each investment horizon, which also supports the superiority of the bullet strategies. This is particularly visible for the longer investment horizons; by analysing the portfolios variations by bucket for the four-year and five-year investment horizon one can conclude that there is a migration from higher duration buckets to lower duration buckets which is consistent with a migration to a bullet strategy towards the end of the investment horizon simulations. This behaviour of the random portfolios explains partially their outperformance to bullet portfolios within the mentioned investment horizons.

- ***Results with Transaction Costs***

Annex 6 contains the tables with the results for this scenario. As for the presentation, it is equal to the previous scenario: the table within this annex is divided in four panels, one for each investment horizon considered, and inside each panel are presented the three sets of indicators listed above. Table 12 shows only the comparison between the asset portfolio's realized rate of return and the real spot rate for the investment horizon analyzed. In this case, and for each strategy, results for both weekly and monthly rebalancing approaches are presented.

Table 12 : Asset Portfolios Realized Rate of Return vs. Spot Rate (with transaction costs)

	Random		Bullet		Barbell	
	Weekly	**Monthly**	**Weekly**	**Monthly**	**Weekly**	**Monthly**
Panel A: One-year Immunization Period						
Real spot rate	0,6990%	0,6990%	0,6990%	0,6990%	0,6990%	0,6990%
Realized Return Rate	0,8157%	0,7677%	0,6671%	0,9337%	2,3168%	2,3168%
Panel B: Three-year Immunization Period						
Real spot rate	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0738%	1,0601%	1,0601%	1,0668%	1,3982%	0,6236%
Panel C: Four-year Immunization Period						
Real spot rate	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,1901%	1,1120%	1,2124%	1,1461%	1,2574%	1,4795%
Panel D: Five-year Immunization Period						
Real spot rate	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,2708%	1,2495%	1,2837%	1,3425%	2,1055%	1,7876%

With transaction costs, the conclusions one can draw are not very different from the ones that were derived from the scenario where transaction costs were not considered. Once again, bullet and random portfolios seem to have similar returns while barbell portfolios stand out as the ones with higher return and also higher risk. The reason for the barbell portfolios outperformance is the same as in the previous scenario.

As for the rebalancing frequency testing, there is no clear evidence that a monthly rebalancing strategy is superior to a weekly one. Besides, the monthly rebalancing has the setback of not allowing the re-investment of cash received from coupons and/or redemptions into the portfolio as quickly as in the weekly rebalancing strategy, which can introduce duration mismatch risk into the asset portfolio due to the creation of a cash position between rebalancing periods. This is not desirable for the immunization analysis, because re-investment rates of return are clearly influenced by the point in time when the re-investment occurs, influencing the portfolios' realized rate of return. In terms of realized return rates, the monthly rebalancing portfolios only achieve higher realized rates of return for five-year bullet portfolios and four-year barbell portfolios, and this can happen to different cash flow re-investment rates of return and not due to lower transaction costs that derive from a lower rebalancing frequency. This way, although the results are presented, the analysis will proceed taking into account only the weekly rebalancing portfolios.

As for liability coverage, bullet portfolios and random portfolios stand out as those whose results are superior. Analysing once again the 25th percentile for these portfolios, it is visible that for one-year and four-year investment horizons (respectively), the random portfolios outperform bullet portfolios in 5 b.p. and 1 b.p. and barbell portfolios in 49 b.p. and 61 b.p.. As for the three-year and five-year investment horizons, bullet portfolios outperform random portfolios in 6 b.p. (three-year) and 14 b.p. (five-year) and barbell portfolios in 15 b.p. (three-year) and 10 b.p. (five-year).

Regarding liability coverage standard deviations, once again bullet portfolios prove to be less risky and barbell portfolios riskier than all other strategies. With transaction costs, bullet portfolios tend to have lower standard deviations in the one-year (0,01%), four-year (0,12%) and five-year (0,18%) investment horizons and random portfolios have the lowest standard deviations in the three-year (0,13%) investment horizons. Once again, standard deviation differences between bullet and random portfolios are between 1 b.p. and 7 b.p., which is small, even though a little bit higher than in the previous scenario. Barbell portfolios' standard deviations stand out again as being three to ten times higher than the values for random and bullet portfolios.

Random portfolios' composition statistics also point to an evolution to a bullet structure as time goes by within the investment horizon, particularly for long time investment horizons, which, like in the previous scenario, also supports the superiority of the bullet strategies.

However, there is one thing that stands out when comparing both scenarios. The truth is that the return differences between portfolios simulated without and with transaction costs are not significant. Sometimes portfolios' simulated with transaction costs have better results than those achieved without transaction costs and this is totally unexpected. When comparing both scenarios, what would be expected is an underperformance with transaction costs. How is this possible? One would assume that there is something wrong with the model, but, if that were the case, the model would be corrected and the right results would be presented. The explanation is far more expressive and can be visible in the table below, which presents a comparison between inflation-linked bonds (ILB) and fixed rate bonds (FRB) bid-offer spreads.

Table 13 : Bid-Offer Spreads: Inflation-linked bonds vs. Fixed rate bonds

Bonds approximate maturity at issue (years)	Average ILB Bid-Offer Spreds (%)	Average FRB Bid-Offer Spreads (%)
5 years	0,129%	0,064%
10 years	0,216%	0,092%
15 years	0,357%	0,080%
20 years	0,551%	0,087%
30 years	0,338%	0,090%

The bid-offer spreads presented are computed as the difference between offer and bid prices, divided by the bid price, for both ILB and FRB. The ILB prices used are equal to the bond prices used as input for the simulations. The FRB prices used were collected from bonds that paired each ILB, i.e. have the same issuer and maturity. Only real bonds are used for this analysis.

By comparing the bid-offer spreads for inflation-linked bonds with the ones for fixed rate bonds, it is clear-cut that they are far higher (two to six times higher). How can this help to explain the results achieved? The first thing that is noticeable is that the awkward higher results in the presence of transaction costs appear in the shorter investment horizons for only random and bullet portfolios. Analysing the portfolios' composition and variation statistics for random portfolios, it is visible that the portfolios structures tend to be "more bullet" with transaction costs and the investment transfers between buckets are also lower. This is somewhat a "liquidity paradox": in this case the transaction costs are so high that less trading is done which leads to less portfolio turnover and ultimately to portfolio returns that are not as different than those achieved without transaction costs. Since lower investment horizons tend to be more sensible to transaction costs, in this particular case, the simulations show portfolios with hold-to-maturity positions and that is the reason why in these cases the portfolios realized rate of return is higher.

4.4. – Immunization using fixed rate bonds

After presenting the simulations for inflation-linked bonds one faces an important question: what if this kind of bonds is not available to immunize liabilities that grow with inflation? Would it be possible to do such immunization with fixed rate bonds? Is there any advantage in using inflation-linked bonds?

As already discussed, a fixed rate bond is expressed in nominal terms, and, although this is not usual, its cash flows can be decomposed in two components: real cash flows and inflation compensation. So in order to evaluate if there is any value added in using inflation-linked bonds, this section compares both bond types. To do this, nominal fixed rate bonds with the same issuer and maturity of the set of eligible inflation-linked bonds were selected. For the shorter maturities, synthetic bonds were also created with the same maturities used for inflation-linked synthetic bonds. In order to make both bond portfolios comparable, and taking into account the issue discussed in point 4.3.5, all cash flows (coupon and/or principal) were converted from nominal cash flows to real cash flows by purging the inflation component as explained in Annex 7.

As for the simulations carried, all the assumptions made before were kept. The only difference concerned portfolio strategies; only bullet portfolios were considered, since they are the best strategy for single liability immunization. To keep in mind the assumptions, they are once again expressed in the table below.

Table 14 : Fixed Rate Bonds' simulations structure

	No Transaction Costs	**With Transaction Costs**
Investment Horizon (years)	1,3,4,5	1,3,4,5
Rebalancing Frequency	Weekly	Weekly and Monthly
Immunization Strategies	Duration-matching bullet portfolios	Duration-matching bullet portfolios
Fixed Rate Bond Options	Real Approach without country spread correction	Real Approach without country spread correction

As for the simulation results, these are grouped in two sets of indicators (already defined in the previous section): liability coverage and asset returns. For this purpose, the results will be presented not on a stand alone basis but in comparison with the inflation-linked results for bullet portfolios. The full table is in Annex 8.

- ***Results without Transaction Costs***

Table 15 presents the realized return rate and the spot rate for each type of bond and investment horizon.

Table 15 : Inflation-linked bonds vs. fixed rate bonds (without transaction costs)

	Inflation-linked	Fixed Rate
Panel A: One-year Immunization Period		
Real spot rate	0,6990%	0,6990%
Realized Return Rate	0,6721%	0,9982%
Panel B: Three-year Immunization Period		
Real spot rate	1,0280%	1,0280%
Realized Return Rate	1,0601%	1,0666%
Panel C: Four-year Immunization Period		
Real spot rate	1,1728%	1,1728%
Realized Return Rate	1,1123%	1,1774%
Panel D: Five-year Immunization Period		
Real spot rate	1,3047%	1,3047%
Realized Return Rate	1,3185%	1,2166%

Looking at the differences between both asset portfolios, it seems that fixed rate bonds are indeed a better asset to immunize real liabilities, since those portfolios returns are higher than the returns achieved with inflation-linked bonds. Even though, the difference is only significant in the one-year period, where the fixed rate bond portfolio outperforms the inflation-linked bond portfolio by 22 b.p..

However, in terms of liability coverage, inflation-linked bond portfolios achieve superior results. Analysing once again the 25th percentile results, for all investment horizons this measure is higher for inflation-linked bonds. For the one-year investment horizon the difference is very high: inflation-linked bonds exceed fixed rate bonds in an astonishing 90 b.p. and this partially proves that the outperformance in terms of realized rate of return is solely due to a riskier approach. For three, four and five-year horizons (respectively) inflation-linked bonds' portfolios' 25th percentile exceed in 25 b.p., 52 b.p. and 12 b.p. the fixed rate bond portfolios' 25th percentile.

As for the liability coverage standard deviations, the picture is the same; inflation-linked bonds have lower standard deviations for all investment horizons, proving to be a less risky approach to single liability immunization than fixed rate bonds. The standard deviations observed for inflation-linked bond portfolios are 1 b.p. for one-year, 15 b.p. for three-year, 19 b.p. for four-year and 17 b.p. for five-year investment horizons. The standard deviations observed for fixed rate bond portfolios are about three to fifty four times higher (and the higher difference between standard deviations is observed in the one-year investment horizon). Hence it is proved that, in that case, the high return is derived from a riskier strategy.

Overall, the implied risk for fixed rate bonds portfolios is very high and, although the returns might also be higher, the returns achieved do not compensate for the higher risk involved in these simulations. This way, the best immunization strategy is achieved by constructing bullet portfolios using inflation-linked bonds.

- ***Results with Transaction Costs***

Table 16 presents the realized return rate and the spot rate for weekly and monthly rebalancing frequencies for each type of bond and investment horizon.

Table 16 : Inflation-linked bonds vs. fixed rate bonds (with transaction costs)

	Inflation-linked		Fixed Rate	
	Weekly	**Monthly**	**Weekly**	**Monthly**
Panel A: One-year Immunization Period				
Real spot rate	0,6990%	0,6990%	0,6990%	0,6990%
Realized Return Rate	0,6671%	0,9337%	0,8975%	0,9016%
Panel B: Three-year Immunization Period				
Real spot rate	1,0280%	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0601%	1,0668%	1,0499%	1,0676%
Panel C: Four-year Immunization Period				
Real spot rate	1,1728%	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,2124%	1,1461%	1,1189%	1,1187%
Panel D: Five-year Immunization Period				
Real spot rate	1,3047%	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,2837%	1,3425%	1,1959%	1,2573%

In the presence of transaction costs, inflation-linked bond portfolios have higher returns than fixed rate portfolios for the longer investment horizons. Only in the one-year investment horizon the fixed rate bonds achieve higher returns. Regarding the rebalancing frequencies, once again there is no evidence that higher results are achieved with monthly rebalancing. Hence from this point forward only the weekly rebalancing portfolios are considered for the analysis.

As for liability coverage percentiles and standard deviations, previous results hold in the presence of transaction costs. In terms of liability coverage, the 25th percentile results point for an outperformance of inflation-linked bonds by 89 b.p. for one-year, 30 b.p for three-year, 100 b.p. for four-year and 29 b.p for five-year investment horizons; the only news here is that the differences are higher, also something consistent with a transaction costs scenario. As for the liability coverage standard deviations, values observed for inflation-linked bond portfolios are 1 b.p. for one-year, 14 b.p. for three-year, 12 b.p. for four-year and 18 for five-year investment horizons. The standard deviations observed for fixed rate bond portfolios are also higher, in about three to thirty nine times.

One thing that occurs as expected with fixed rate bonds is that the realized returns are lower with transaction costs in all cases, something that is expected and has already been acknowledged by Agca (2005: 665). The differences between both approaches are still not very high but recalling the bid-offer spreads presented for fixed rate bonds (Table 14), that vary between 6 and 9 basis points, one can conclude that these values are consistent with the differences between fixed rate bond portfolios estimated with and without transaction costs.

Chapter 5 – Conclusion

In order to access what is the best approach to immunize a single liability in real terms, simulations were made using inflation-linked bonds and assuming a passive duration-matching strategy. Several investment horizons were considered; for each, bullet, barbell and random strategies were simulated with and without transaction costs. The results presented point to an outperformance of bullet portfolios over all other portfolio strategies in terms of liability coverage for all investment horizons. They also prove to be more consistent and less volatile than the alternatives. The conclusions are the same, with and without transaction costs. Notwithstanding, with transaction costs, asset portfolios' turnover decreases due to high bid-offer spreads.

In terms of portfolio returns, barbell portfolios are superior for one-year and five-year investment horizons. However, the objective of a passive immunization strategy is not to achieve higher returns, but to ensure liability coverage. Barbell strategies, although superior in return, are riskier in terms of passive immunization implementation and do not ensure liability coverage as well as bullet portfolios because they are more volatile. As for random portfolios, their structure evolves during the investment horizons to a bullet structure, giving a wider support to this strategy's immunization superiority.

When compared to nominal fixed rate bonds immunization procedures, inflation-linked bonds prove to be a better asset to immunize liabilities that grow with inflation. Hence, even taking into account that nominal fixed rate bonds have an implicit inflation growth rate, given by Fisher's Equation (3.14), they prove to produce volatile portfolios that do not immunize this type of liability as well as inflation-linked bond portfolios do.

There are some setbacks to this study. One of them is related with the investment horizons chosen. Normally immunization strategies are used to ensure that it is possible to cover future liabilities in medium to long term investment horizons. So, considering simulation for one-year to five year-investment horizons seems very limitative. However, for now, these are the only investment horizons that can be considered, since these bonds' issuance is fairly new and, consequently, the time horizon for which is possible to collect data is small. Anyway, it would be interesting to do this analysis for longer investment horizons, like seven-year or ten-year horizons, when enough time as passed to do so.

The other setback is related with the liability structure and immunization strategy chosen. It is also possible to do this study for multiple liability immunization and for active or contingent immunization approaches. It could also be considered a duration and convexity immunization approach, in order to adjust the asset portfolios' to different types of shifts in the term structure of interest rates. One could also test stochastic duration models with this type of bonds.

Regarding geographic coverage, this study could also be done for other issuers and other currencies, like the United States or the United Kingdom. The present study is only a gauge to potential future research that could be done with inflation-linked bonds.

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Financial Information

All market data (bond prices and descriptions, nominal interest rates, country spreads) was collected from Bloomberg in the 3rd and 4th of July 2009.

Information about the inflation-linked bonds and fixed rate bonds were collected from Bloomberg and the following Country agencies websites:

Dipartimento del Tesoro: www.dt.tesoro.it
Greek Ministry of Finance Public Debt Management Agency: www.pdma.gr
Agence France Trésor: www.aft.gouv.fr
Bundesrepublik Deutschland – Finanzagentur GmbH: www.deutsche-finanzagentur.de

Information and data about the European HICPx Índex was collected from Eurostat's website http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home

Information about issuers' ratings was collected from Bloomberg and the following rating agency websites:

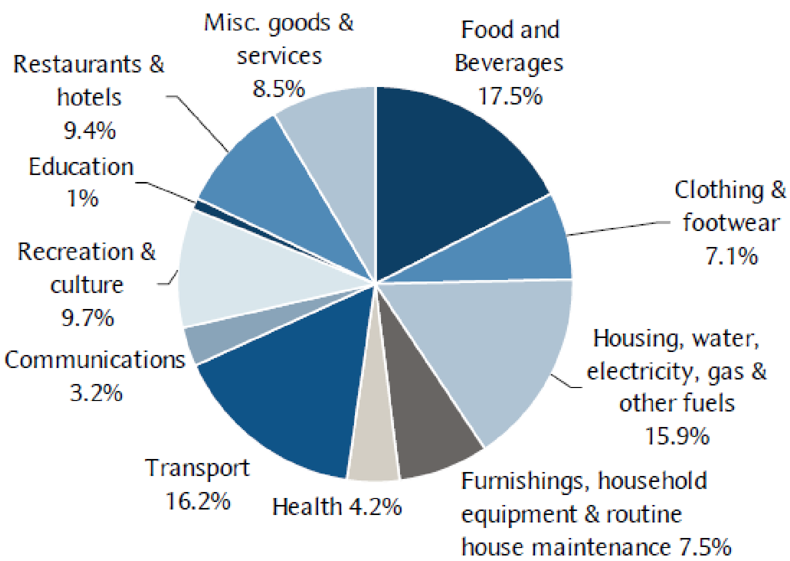
Standard and Poor's: www.standardandpoors.com
Moody's Investor Services: www.moodys.com
Fitch Ratings: www.fitchratings.com

Annexes

Annex 1 – European HICPx Index

The European HICPx Index is computed as a weighted average of the individual euro area countries' HICP indices (that are computed as geometric chain-weighted Laspeyres indices). The weights are determined according to each country's share of consumption expenditure within the euro area as measured by the "household final monetary consumption expenditure" in national accounts data. The country weights are reviewed yearly and applied in the January inflation figures published in February. Countries joining the European Monetary Union are also added to the index. The graphic below shows the Index breakdown by major category.

Breakdown of Euro HICPx components



Source: Eurostat (taken from Bettiss, James and Sooben, 2008: 45)

The index rebasing process affects both reference index and Index Ratio calculus that have been done with the old base index, since they both need to be converted with a reference key. The conversion is done as follows:

$$RK = \frac{P_{bm}^{new\ base}}{P_{bm}^{old\ base}} \quad : \tag{A1.1}$$

$$I_j^{new\ base} = I_j^{old\ base} \times RK \quad \wedge \quad IR_j^{new\ base} = IR_j^{old\ base} \times RK \wedge \quad j = t, base$$

where: RK = reference key

IRj = Index Ratio in day j

Ij = reference index for day j

P_{bm} = value of the price index in the rebasing month (for $P^{new\ base}$ and $P^{old\ base}$)

t = any valuation date (in this case prior to the rebasing month)

$base$ = bond's first interest accrual day

The last time this index was rebased was in January 2006 (base year 2005).

Annex 2 – Capital Interest Bonds *formulae*

1. Index Ratio calculation

For both valuation and settlement purposes the Index Ratio is computed as follows:

$$IR_t = \frac{I_t}{I_{base}} \quad : \quad I_j = P_{m-3} + \frac{(t-1)}{M_m} \times (P_{m-2} - P_{m-3}),\ j = t, base \qquad \text{(A2.1)}$$

where:
IR_t = Index Ratio in day t
I_t = reference index for day t
I_{base} = reference index for the first interest accrual day
P_{m-3} = value of the price index at time m-3 months
P_{m-2} = value of the price index at time m-2 months
M = number of days in month m
t = day of the month m when settlement occurs
m = month on which settlement takes place
base = bond's first interest accrual day

The formula shown here is for an Index Ratio with an indexation lag of 3 months. However, other indexation lags can be considered by substituting variables P_{m-3} and P_{m-2} with the values for the indexation lag required, i.e. if the indexation lag wanted is 8 months then the price indices to consider are P_{m-8} and P_{m-7}.

2. Interest payment calculation

The nominal interest payment in euros is computed as shown below.

$$C_t = \frac{1}{100} \times \frac{r}{c} \times IR_t \times FV \qquad \text{(A2.2)}$$

where:
C_t = interest payment in day t (€)
r = annual real coupon rate (%)
c = number of coupons the bond pays *per* year
IR_t = Index Ratio in day t
FV = face value of the bond (€)
t = day of the interest payment

The real interest payment in euros is computed in a similar way, but does not include the Index Ratio.

$$C_t = \frac{1}{100} \times \frac{r}{c} \times FV \qquad \text{(A2.3)}$$

3. Principal payment calculation

Calculation of the principal's value to be redeemed at maturity is done as shown below

$$\textit{Without Deflation Floor}: \quad Principal = FV \times IR_T \qquad \text{(A2.4)}$$

$$\textit{With Deflation Floor}: \quad Principal = FV \times \max\{IR_T, 1\} \qquad \text{(A2.5)}$$

where: FV = face value of the bond (€)

IR_T = Index Ratio in maturity day T

4. Settlement price calculation

Settlement price calculation is more complex than for nominal fixed rate bonds, since it implies adjusting for inflation both clean price and real accrued interest. The formulas are presented beneath.

$$NQ_t = IQ_t + IAI_t \quad : \qquad \text{(A2.6)}$$

$$IQ_t = RQ_t \times IR_t \qquad \text{(A2.7)}$$

$$RQ_t = \left(\frac{1}{1+y_R}\right)^{f/d} \times \left[r + r.\sum_{j=1}^{n} \left(\frac{1}{1+y_R}\right)^{j} + 100.\left(\frac{1}{1+y_R}\right)^{n} \right] - RAI_t \qquad \text{(A2.8)}$$

$$RAI_t = r \times \frac{(d-f)}{d} \qquad \text{(A2.9)}$$

$$IAI_t = RAI_t \times IR_t \qquad \text{(A2.10)}$$

where: NQ_t = nominal price per €100 face value for day t

IQ_t = inflation adjusted price for day t

IAI_t = inflation adjusted accrued interest for day t

RQ_t = real price for day t

IR_t = Index Ratio in day t

y_R = annual real spot rate

f = number of days from the settlement date to the next interest payment date

d = number of days in the regular annual coupon period ending on the next interest payment date

r = annual real coupon rate (%)

n = number of full annual coupon periods between the next interest payment date and the maturity date

RAI_t = real accrued interest for day t

Annex 3 – Nominal duration calculation for inflation-linked bonds

Recalling the Fisher-Weil duration concept, the real duration of an inflation-linked bond can be computed as follows:

$$DFW = \frac{\sum_{j=1}^{n} \tau_j \times \frac{r/c}{\left[1+y_R\left(0,\tau_j\right)\right]^{\tau_j}} + \tau_n \times \frac{100}{\left[1+y_R\left(0,\tau_n\right)\right]^{\tau_n}}}{B(0)}, \qquad \text{(A3.1)}$$

$$where \quad B(0) = \sum_{j=1}^{n} \frac{r/c}{\left[1+y_R\left(0,\tau_j\right)\right]^{\tau_j}} + \frac{100}{\left[1+y_R\left(0,\tau_n\right)\right]^{\tau_n}} \qquad \text{(A3.2)}$$

The relationship between nominal interest rates and real interest rates is given by Fisher's equation:

$$\left(1+y_N\right) = \left(1+y_R\right) \times \left(1+\pi\right) \qquad \text{(A3.3)}$$

where y_R stands for real annual spot rate, y_N stands for nominal annual spot rate and π is the *year-on-year* actual inflation rate. In order to transform the duration formula (A3.1) it is necessary to adjust both numerator and denominator in it. The denominator adjustment is straightforward: to obtain the nominal spot interest rate y_N it is only necessary to multiply the real spot interest rate y_R by the *year-on-year* inflation π by applying Fisher's Equation. The numerator adjustment is, however, more complicated.

As illustrated in section 2 of Annex 2, the nominal interest rate calculation for inflation-linked bonds requires the annual real coupon rate at time t to be adjusted for inflation. This is done by multiplying it by the Index Ratio (IR) at time t. The main issue here is that, in order to be able to compute the *effective duration* it is necessary to estimate the future Index Ratios. One way to do so is by assuming that future *year-on-year* inflation will be equal to the most recent data on present *year-on-year* inflation, under the hypothesis that future inflation will not be much different than present inflation due to control mechanisms placed by the central banks. This way, the Index Ratio for $t + f$ can be calculated as shown below:

$$IR_{t+f} = IR_t \times \left(1+\pi\right)^{(t+f-t)/d} \qquad \text{(A3.4)}$$

where f is the number of days from the settlement date to the next interest payment date and d is the number of days in the regular coupon period ending on the next interest payment date accordingly to market convention ACT/ACT. By using the formula above it is possible to compute the nominal annual coupon rate for all future coupons as well as the nominal future principal redemption value. To do so it is only necessary to apply formula 3.1 and 3.3 using the future Index Ratios. Hence, the nominal future payments can be computed as below:

$$Coupon\ Payments: \quad C_j = \frac{r}{c} \times IR_j \ : \ j = t + f.,...,n \tag{A3.5}$$

$$Principal\ Payments: FV_n = \max\left\{1, 1 \times IR_n\right\} \tag{A3.6}$$

This way, the formula that should be applied to compute the *effective duration* of an inflation-linked bond is as follows:

$$DFW = \frac{\sum_{j=1}^{n} \tau_j \times \frac{C_j}{\left[1 + y_N\left(0, \tau_j\right)\right]^{\tau_j}} + \tau_n \times \frac{FV_n}{\left[1 + y_N\left(0, \tau_n\right)\right]^{\tau_n}}}{B(0)}, \tag{A3.7}$$

$$where \quad B(0) = \sum_{j=1}^{n} \frac{C_j}{\left[1 + y_N\left(0, \tau_j\right)\right]^{\tau_j}} + \frac{FV_n}{\left[1 + y_N\left(0, \tau_n\right)\right]^{\tau_n}} \tag{A3.8}$$

Annex 4 – Long Term Issuer Rating Definitions

Below is a summary of the long term issuer rating definitions according to each rating agency mentioned.

1. Standard & Poor's
   1.1. **AAA –** An obligor rated 'AAA' has extremely strong capacity to meet its financial commitments. 'AAA' is the highest issuer credit rating assigned by Standard & Poor's.
   1.2. **AA** – An obligor rated 'AA' has very strong capacity to meet its financial commitments. It differs from the highest-rated obligors only to a small degree.
   1.3. **A** – An obligor rated 'A' has strong capacity to meet its financial commitments but is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligors in higher-rated categories.

Standard and Poor's may append symbols "+" or "-" to each rating classification.

2. Moody's
   2.1. **Aaa –** Obligations rated Aaa are judged to be of the highest quality, with minimal credit risk.
   2.2. **Aa –** Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.
   2.3. **A –** Obligations rated A are considered upper-medium grade and are subject to low credit risk.

Moody's appends numerical modifiers 1, 2, and 3 to each generic rating classification.

3. Fitch Ratings
   3.1. **AAA: Highest credit quality –** 'AAA' ratings denote the lowest expectation of default risk. They are assigned only in cases of exceptionally strong capacity for payment of financial commitments. This capacity is highly unlikely to be adversely affected by foreseeable events
   3.2. **AA: Very high credit quality** – 'AA' ratings denote expectations of very low default risk. They indicate very strong capacity for payment of financial commitments. This capacity is not significantly vulnerable to foreseeable events.

3.3. **A: High credit quality –** ‘A’ ratings denote expectations of low default risk. The capacity for payment of financial commitments is considered strong. This capacity may, nevertheless, be more vulnerable to adverse business or economic conditions than is the case for higher ratings.

The modifiers “+” or “-” may be appended to a rating to denote relative status within major rating categories.

Annex 5 – Inflation-linked bonds' immunization results without transaction costs

Table A.1 : Inflation-linked bonds' immunization results without transaction costs

This table is divided in four panels, one for each investment horizon. The indicators presented are divided in three categories. Liability coverage category displays the level of coverage at maturity, mean coverage level, the 75th and 25th percentile, the standard deviation and the liability coverage deficit, i.e. the percentage of simulations where the present value of the liability exceeded the present value of the asset portfolio. Asset Portfolio Returns displays the spot rate and the portfolio's realized return rate, mean return rate and standard deviation. For random portfolios, asset portfolio composition and variation is also presented. The composition is divided by buckets and the variation shows if a certain bucket's investment was increasing (positive values) or decreasing (negative values) during the investment horizon considered.

Panel A: One-year Immunization Period			
	random	**bullet**	**barbell**
Nº Simulations	53	53	53
Liability Coverage			
At maturity	99,9432%	99,9746%	101,6080%
Mean Coverage	99,9718%	99,9963%	100,6076%
75th percentile	100,0080%	100,0055%	101,7552%
25th percentile	99,9456%	99,9858%	99,5409%
Standard Deviation	0,0337%	0,0106%	1,1055%
Liability Coverage Deficit	64,1509%	58,4906%	37,7358%
Asset Portfolio Returns (annualized)			
1-year spot rate	0,6990%	0,6990%	0,6990%
Realized Return Rate	0,6405%	0,6721%	2,3170%
Mean Return Rate	0,6409%	0,6726%	2,3368%
Standard Deviation	0,3079%	0,3228%	1,9864%
Asset Portfolio Composition (mean duration bucketing)			
>1 year	96,74%		
1-3 years	0%		
3-7 years	3,25%		
7-10 years	0%		
10-15 years	0%		
15-20 years	0%		
<25 years	0%		
Asset Portfolio Variation (by bucket)			
>1 year	99,82%		
1-3 years	0%		
3-7 years	-28,69%		
7-10 years	-15,86%		
10-15 years	-7,61%		
15-20 years	-10,12%		
<25 years	0,00%		

Table A.1 : continued

Panel B: Three-year Immunization Period			
	random	**bullet**	**barbell**
Nº Simulations	157	157	157
Liability Coverage			
At maturity	100,0908%	100,1080%	99,4702%
Mean Coverage	100,0554%	100,0967%	100,3516%
75^{th} percentile	100,2004%	100,2004%	100,9488%
25^{th} percentile	99,9062%	100,0527%	99,8742%
Standard Deviation	0,1568%	0,1479%	0,7859%
Liability Coverage Deficit	33,1210%	19,1083%	40,1274%
Asset Portfolio Returns (annualized)			
3-year spot rate	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0543%	1,0601%	0,8450%
Mean Return Rate	1,0574%	1,0669%	0,8714%
Standard Deviation	0,7873%	1,1683%	2,2953%
Asset Portfolio Composition (mean duration bucketing)			
>1 year	94,58%		
1-3 years	3,53%		
3-7 years	0,96%		
7-10 years	0%		
10-15 years	0%		
15-20 years	0%		
<25 years	0,77%		
Asset Portfolio Variation (by bucket)			
>1 year	478,28%		
1-3 years	-58,03%		
3-7 years	-17,32%		
7-10 years	-10,20%		
10-15 years	-9,08%		
15-20 years	-21,47%		
<25 years	-21,04%		

Table A.1 : continued

Panel C: Four-year Immunization Period			
	random	**bullet**	**barbell**
Nº Simulations	209	209	209
Liability Coverage			
At maturity	99,9680%	99,7752%	100,6192%
Mean Coverage	100,0788%	99,9824%	99,7982%
75th percentile	100,2004%	100,1400%	100,1842%
25th percentile	99,9715%	99,8004%	99,6703%
Standard Deviation	0,1639%	0,1853%	0,5786%
Liability Coverage Deficit	32,0574%	59,3301%	60,7656%
Asset Portfolio Returns (annualized)			
4-year spot rate	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,1611%	1,1123%	1,3255%
Mean Return Rate	1,1677%	1,1235%	1,3586%
Standard Deviation	1,1418%	1,4893%	2,3016%
Asset Portfolio Composition (mean duration bucketing)			
>1 year	89,84%		
1-3 years	7,73%		
3-7 years	0,34%		
7-10 years	0,53%		
10-15 years	0%		
15-20 years	0%		
<25 years	1,31%		
Asset Portfolio Variation (by bucket)			
>1 year	408,70%		
1-3 years	-129,67%		
3-7 years	-6,68%		
7-10 years	-10,90%		
10-15 years	-2,71%		
15-20 years	-24,25%		
<25 years	-0,54%		

Table A.1 : continued

Panel D: Five-year Immunization Period			
	random	**bullet**	**barbell**
Nº Simulations	261	261	261
Liability Coverage			
At maturity	100,0387%	100,0776%	104,7468%
Mean Coverage	100,0471%	100,0463%	100,8334%
75th percentile	100,1963%	100,1925%	101,6518%
25th percentile	99,9411%	99,9207%	99,9297%
Standard Deviation	0,1463%	0,1653%	1,3197%
Liability Coverage Deficit	36,7816%	32,9502%	29,8851%
Asset Portfolio Returns (annualized)			
5-year spot rate	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,3106%	1,3185%	2,2467%
Mean Return Rate	1,3226%	1,3342%	2,3274%
Standard Deviation	1,5445%	1,7677%	3,9758%
Asset Portfolio Composition (mean duration bucketing)			
>1 year	85,22%		
1-3 years	8,50%		
3-7 years	4,16%		
7-10 years	0,95%		
10-15 years	0,60%		
15-20 years	0%		
<25 years	0,48%		
Asset Portfolio Variation (by bucket)			
>1 year	389,57%		
1-3 years	1,79%		
3-7 years	-7,11%		
7-10 years	-7,15%		
10-15 years	-15,93%		
15-20 years	-1,86%		
<25 years	-0,72%		

Annex 6 – Inflation-linked bonds' immunization results with transaction costs

Table A.2 : Inflation-linked bonds' immunization results with transaction costs

This table is divided in four panels, one for each investment horizon. The indicators presented are divided in three categories. Liability coverage category displays the level of coverage at maturity, mean coverage level, the 75th and 25th percentile, the standard deviation and the liability coverage deficit, i.e. the percentage of simulations where the present value of the liability exceeded the present value of the asset portfolio. Asset Portfolio Returns displays the spot rate and the portfolio's realized return rate, mean return rate and standard deviation. For random portfolios, asset portfolio composition and variation is also presented. The composition is divided by buckets and the variation shows if a certain bucket's investment was increasing (positive values) or decreasing (negative values) during the investment horizon considered. Results are presented for both weekly and monthly rebalancing periods.

Panel A: One-year immunization Period						
	random		**bullet**		**barbell**	
	weekly	**monthly**	**weekly**	**monthly**	**weekly**	**monthly**
Nº Simulations	53	14	53	14	53	14
Liability Coverage						
At maturity	100.1171%	100.0695%	99.9696%	100.2344%	101.6078%	101.6078%
Mean Coverage	100.0649%	100.0466%	99.9935%	100.1267%	100.6075%	100.5884%
75th percentile	100.1309%	100.0834%	100.0055%	100.2506%	101.7550%	101.5301%
25th percentile	100.0277%	100.0036%	99.9801%	100.0000%	99.5409%	99.2547%
Standard Deviation	0.0807%	0.0397%	0.0131%	0.1261%	1.1054%	1.0993%
Liability Coverage Deficit	11.3208%	7.1429%	60.3774%	7.1429%	37.7358%	35.7143%
Asset Portfolio Returns (annualized)						
1-year spot rate	0.6990%	0.6990%	0.6990%	0.6990%	0.6990%	0.6990%
Realized Return Rate	0.8157%	0.7677%	0.6671%	0.9337%	2.3168%	2.3168%
Mean Return Rate	0.8166%	0.7093%	0.6676%	0.8629%	2.3366%	2.1572%
Standard Deviation	0.4470%	0.4108%	0.3216%	0.5290%	1.9863%	2.0877%
Asset Portfolio Composition (mean duration bucketing)						
>1 year	96.87%	96.89%				
1-3 years	0%	0%				
3-7 years	3.12%	3.10%				
7-10 years	0%	0%				
10-15 years	0%	0%				
15-20 years	0%	0%				
<25 years	0.00%	0.00%				
Asset Portfolio Variation (by bucket)						
>1 year	99.86%	99.79%				
1-3 years	0%	0%				
3-7 years	-28.69%	-28.68%				
7-10 years	-15.86%	-15.86%				
10-15 years	-7.61%	-7.61%				
15-20 years	-10.12%	-10.12%				
<25 years	0%	0%				

Table A.2 : continued

Panel B: Three-year immunization Period						
	random		bullet		barbell	
	weekly	monthly	weekly	monthly	weekly	monthly
Nº Simulations	157	40	157	40	157	40
Liability Coverage						
At maturity	100,1489%	100,1492%	100,1342%	100,1279%	101,1160%	98,8166%
Mean Coverage	100,1047%	100,1729%	100,1051%	100,0948%	100,7182%	100,0958%
75th percentile	100,2004%	100,2005%	100,2004%	100,1881%	101,3866%	100,6943%
25th percentile	100,0405%	100,0328%	100,1002%	100,0694%	99,9471%	99,4106%
Standard Deviation	0,1335%	0,4970%	0,1401%	0,1416%	0,7827%	0,8767%
Liability Coverage Deficit	19,1083%	17,5000%	12,7389%	10,0000%	29,2994%	55,0000%
Asset Portfolio Returns (annualized)						
3-year spot rate	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0738%	1,0601%	1,0601%	1,0668%	1,3982%	0,6236%
Mean Return Rate	1,0768%	1,0216%	1,0755%	0,9910%	1,4173%	0,5911%
Standard Deviation	0,7661%	2,5133%	1,1428%	1,1703%	1,9526%	1,7799%
Asset Portfolio Composition (mean duration bucketing)						
>1 year	90,08%	93,44%				
1-3 years	7,42%	4,43%				
3-7 years	1,61%	1,10%				
7-10 years	0%	0%				
10-15 years	0%	0%				
15-20 years	0%	0%				
<25 years	0,70%	0,93%				
Asset Portfolio Variation (by bucket)						
>1 year	377,25%	406,79%				
1-3 years	-26,23%	-123,96%				
3-7 years	-8,15%	-8,16%				
7-10 years	-7,60%	-7,60%				
10-15 years	-7,61%	-7,62%				
15-20 years	-9,24%	-10,27%				
<25 years	-0,47%	-0,30%				

Table A.2 : continued

Panel C: Four-year immunization Period						
	random		bullet		barbell	
	weekly	monthly	weekly	monthly	weekly	monthly
Nº Simulations	209	53	209	53	209	53
Liability Coverage						
At maturity	100,0824%	99,7741%	100,1709%	99,9085%	100,3491%	101,2325%
Mean Coverage	100,1106%	100,0511%	100,1202%	100,0898%	99,6576%	100,1010%
75^{th} percentile	100,2004%	100,2004%	100,2252%	100,2046%	100,0800%	100,3972%
25^{th} percentile	100,0312%	99,8520%	100,0219%	99,9598%	99,4243%	99,7511%
Standard Deviation	0,1410%	0,1722%	0,1218%	0,1338%	0,6341%	0,5217%
Liability Coverage Deficit	22,4880%	35,8491%	18,1818%	32,0755%	69,3780%	47,1698%
Asset Portfolio Returns (annualized)						
4-year spot rate	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,1901%	1,1120%	1,2124%	1,1461%	1,2574%	1,4795%
Mean Return Rate	1,1964%	1,0317%	1,2247%	1,0665%	1,5196%	1,3779%
Standard Deviation	1,1239%	1,0675%	1,5615%	1,3544%	2,3084%	1,6171%
Asset Portfolio Composition (mean duration bucketing)						
>1 year	88,04%	89,94%				
1-3 years	9,72%	7,68%				
3-7 years	0,88%	0,91%				
7-10 years	0%	0%				
10-15 years	0%	0%				
15-20 years	0%	0%				
<25 years	0,54%	0,51%				
Asset Portfolio Variation (by bucket)						
>1 year	393,21%	381,78%				
1-3 years	-95,74%	-87,49%				
3-7 years	-8,13%	-21,78%				
7-10 years	-7,38%	-7,34%				
10-15 years	-1,48%	-1,52%				
15-20 years	-2,88%	-3,30%				
<25 years	2,31%	-0,78%				

Table A.2 : continued

Panel D: Five-year immunization Period						
	random		**bullet**		**barbell**	
	weekly	**monthly**	**weekly**	**monthly**	**weekly**	**monthly**
Nº Simulations	261	66	261	66	261	66
Liability Coverage						
At maturity	99,8425%	99,7373%	99,9060%	100,1961%	104,0255%	102,4162%
Mean Coverage	100,0453%	99,9487%	100,1110%	100,0725%	100,5967%	100,0286%
75th percentile	100,2506%	100,1925%	100,2497%	100,2492%	100,9634%	100,2510%
25th percentile	99,8617%	99,7507%	99,9920%	99,9236%	99,8925%	99,7507%
Standard Deviation	0,2101%	0,2352%	0,1844%	0,1885%	1,0167%	0,6967%
Liability Coverage Deficit	44,8276%	64,0625%	24,9042%	36,3636%	29,1188%	43,9394%
Asset Portfolio Returns (annualized)						
5-year spot rate	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,2708%	1,2495%	1,2837%	1,3425%	2,1055%	1,7876%
Mean Return Rate	1,2802%	1,1999%	1,3027%	1,2489%	2,1613%	1,6880%
Standard Deviation	1,3675%	1,4412%	1,9405%	1,4402%	3,3133%	2,7979%
Asset Portfolio Composition (mean duration bucketing)						
>1 year	81,04%	76,45%				
1-3 years	13,96%	17,23%				
3-7 years	1,65%	3,77%				
7-10 years	1,46%	1,05%				
10-15 years	0,67%	0,75%				
15-20 years	0%	0%				
<25 years	1,09%	0,73%				
Asset Portfolio Variation (by bucket)						
>1 year	383,43%	267,13%				
1-3 years	53,12%	45,58%				
3-7 years	-7,79%	-7,10%				
7-10 years	-7,37%	-7,46%				
10-15 years	-16,08%	-16,34%				
15-20 years	-1,66%	-1,77%				
<25 years	-0,20%	-0,54%				

Annex 7 – Fixed Rate Bonds disinflation process

First of all, we recall the computation of the Index Ratio for inflation-linked bonds. For both valuation and settlement purposes the Index Ratio is computed as follows:

$$IR_t = \frac{I_t}{I_{base}} \quad : \quad I_j = P_{m-3} + \frac{(t-1)}{M_m} \times (P_{m-2} - P_{m-3}), \; j = t, base \qquad \text{(A7.1)}$$

where: IR_t = Index Ratio in day t

I_t = reference index for day t

I_{base} = reference index for the first interest accrual day

P_{m-3} = value of the price index at time m-3 months

P_{m-2} = value of the price index at time m-2 months

M_m = number of days in month m

t = day of the month m when settlement occurs

m = month on which settlement takes place

base = bond's first interest accrual day

For fixed rate bonds the interest payment is computed in nominal terms, as shown below.

$$C_t^N = \frac{1}{100} \times \frac{i}{c} \times FV \qquad \text{(A7.2)}$$

where: C^N_t = nominal interest payment in day t (€)

i = annual nominal coupon rate (%)

c = number of coupons the bond pays *per* year

FV = face value of the bond (€)

t = day of the interest payment

To convert the coupon to real terms what needs to be done is to divide the nominal coupon by the Index Ratio estimated for day t, as shown below.

$$C_t = \frac{\frac{1}{100} \times \frac{i}{c} \times FV}{IR_t} \qquad \text{(A7.3)}$$

The same can be applied to principal payment calculation. Bear in mind that, for inflation-linked bonds there is a deflation floor in order to prevent the real principal erosion if deflation occurs during the life of the bond. This is also taken into account in this process. Hence, the real principal payment will always be less or equal to 100% of the bonds' face value. It will never be allowed to be more than 100% since that only occurs if the Index Ratio drops below one. Disinflation of the principal's value to be redeemed at maturity is done as shown below

$$\text{Without Deflation Floor}: \quad Principal = \frac{FV}{IR_T} \tag{A7.4}$$

$$\text{With Deflation Floor}: \quad Principal = FV \times \min\left\{\frac{1}{IR_T}, 1\right\} \tag{A7.5}$$

where: FV = face value of the bond (€)

IR_T = Index Ratio in maturity day T

After doing these adjustments the bond's fair value (*B(0)*) and duration (*DFW*) are computed taking into account the real cash flows and assuming that future inflation will equal the present *year-on-year* inflation. This way, future Index Ratios were projected and future real cash flows were estimated. The formulas used are given below:

$$DFW = \frac{\sum_{j=1}^{n} \tau_j \times \frac{C_j/100}{\left[1+y_R(0,\tau_j)\right]^{\tau_j}} + \tau_n \times \frac{FV/100 \times \min\left\{1/IR_T, 1\right\}}{\left[1+y_R(0,\tau_n)\right]^{\tau_n}}}{B(0)}, \tag{A7.6}$$

$$\text{where} \quad B(0) = \sum_{j=1}^{n} \frac{C_j/100}{\left[1+y_R(0,\tau_j)\right]^{\tau_j}} + \frac{FV/100 \times \min\left\{1/IR_T, 1\right\}}{\left[1+y_R(0,\tau_n)\right]^{\tau_n}} \tag{A7.7}$$

For synthetic fixed rate bonds the formulae used is similar. The prices and duration were estimated as for the inflation-linked bonds, always assuming that future inflation will be equal to present *year-on-year* inflation. The expression for the principal, duration and fair value calculation for the synthetic bonds is detailed underneath.

$$Principal = 100 \times \min\left\{\frac{1}{IR_T}, 1\right\} \tag{A7.8}$$

$$DFW = \tau_n \tag{A7.9}$$

$$B(0) = \frac{100 \times \min\left\{1/IR_T, 1\right\}}{\left[1+y_R(0,\tau_n)\right]^{\tau_n}} \tag{A7.10}$$

In order to estimate the transaction costs scenario, the bid-offer spreads (*boS*) were computed, in cents, for each bond, and then the bond's fair values were adjusted in order to reflect market conditions. The formulas for the spread, bid prices (P_{bid}) and offer prices (P_{offer}) are beneath:

$$boS = P_{offer} - P_{bid} \quad : \tag{A7.11}$$

$$P_{bid} = B(0) - \frac{boS}{2} \quad \wedge \quad P_{offer} = B(0) + \frac{boS}{2} \tag{A7.12}$$

This adjustment was made for both real and synthetic fixed rate bonds.

Annex 8 – Bullet strategy comparison: Inflation-linked bonds vs. fixed rate bonds

Table A.3 : Bullet Strategy Comparison: Inflation-linked bonds vs. fixed rate bonds

This table is divided in four panels, one for each investment horizon. The indicators presented are divided in two categories. Liability coverage category displays the level of coverage at maturity, mean coverage level, the 75th and 25th percentile, the standard deviation and the liability coverage deficit, i.e. the percentage of simulations where the present value of the liability exceeded the present value of the asset portfolio. Asset Portfolio Returns displays the spot rate and the portfolio's realized return rate, mean return rate and standard deviation. When accounting for transaction costs, both weekly and monthly rebalancing periods are presented.

Panel A: One-year immunization Period

	Without Transaction Costs		With Transaction Costs			
	Inflation-linked	Fixed Rate	Inflation-linked		Fixed Rate	
	weekly	weekly	weekly	monthly	weekly	monthly
Nº Simulations	53	53	53	14	53	14
Liability Coverage						
At maturity	99,9746%	100,2984%	99,9696%	100,2344%	100,1984%	100,2025%
Mean Coverage	99,9963%	99,6495%	99,9935%	100,1267%	99,5970%	99,5695%
75th percentile	100,0055%	100,0045%	100,0055%	100,2506%	99,9257%	99,9644%
25th percentile	99,9858%	99,0900%	99,9801%	100,0000%	99,0900%	99,0900%
Standard Deviation	0,0106%	0,5465%	0,0131%	0,1261%	0,5143%	0,5173%
Liability Coverage Deficit	58,4906%	71,6981%	60,3774%	7,1429%	83,0189%	85,7143%
Asset Portfolio Returns (annualized)						
1-year spot rate	0,6990%	0,6990%	0,6990%	0,6990%	0,6990%	0,6990%
Realized Return Rate	0,6721%	0,9982%	0,6671%	0,9337%	0,8975%	0,9016%
Mean Return Rate	0,6726%	1,0049%	0,6676%	0,8629%	0,9033%	0,8403%
Standard Deviation	0,3228%	1,1717%	0,3216%	0,5290%	1,0888%	1,3398%

Table A.3 : continued

Panel B: Three-year immunization Period						
	Without Transaction Costs		With Transaction Costs			
	Inflation-linked	Fixed Rate	Inflation-linked		Fixed Rate	
	weekly	weekly	weekly	monthly	weekly	monthly
Nº Simulations	157	157	157	40	157	40
Liability Coverage						
At maturity	100,1080%	100,1273%	100,1342%	100,1279%	100,0778%	100,1305%
Mean Coverage	100,0967%	99,9437%	100,1051%	100,0948%	99,8578%	99,7976%
75th percentile	100,2004%	100,2004%	100,2004%	100,1881%	100,2004%	100,1305%
25th percentile	100,0527%	99,8007%	100,1002%	100,0694%	99,8010%	99,7013%
Standard Deviation	0,1479%	0,4296%	0,1401%	0,1416%	0,4676%	0,5973%
Liability Coverage Deficit	19,1083%	47,1338%	12,7389%	10,0000%	55,4140%	65,0000%
Asset Portfolio Returns (annualized)						
3-year spot rate	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%	1,0280%
Realized Return Rate	1,0601%	1,0666%	1,0601%	1,0668%	1,0499%	1,0676%
Mean Return Rate	1,0669%	1,0822%	1,0755%	0,9910%	1,0743%	1,0102%
Standard Deviation	1,1683%	1,7648%	1,1428%	1,1703%	2,2125%	2,2690%

Table A.3 : continued

Panel C: Four-year immunization Period						
	Without Transaction Costs		With Transaction Costs			
	Inflation-linked	Fixed Rate	Inflation-linked		Fixed Rate	
	weekly	weekly	weekly	monthly	weekly	monthly
Nº Simulations	209	209	209	53	209	53
Liability Coverage						
At maturity	99,7752%	100,0324%	100,1709%	99,9085%	99,8011%	99,8004%
Mean Coverage	99,9824%	99,6265%	100,1202%	100,0898%	99,5213%	99,4475%
75^{th} percentile	100,1400%	100,1267%	100,2252%	100,2046%	100,0706%	100,0859%
25^{th} percentile	99,8004%	99,2800%	100,0219%	99,9598%	99,0248%	98,8347%
Standard Deviation	0,1853%	0,7493%	0,1218%	0,1338%	0,7952%	0,9066%
Liability Coverage Deficit	59,3301%	63,1579%	18,1818%	32,0755%	67,9426%	67,9245%
Asset Portfolio Returns (annualized)						
4-year spot rate	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%	1,1728%
Realized Return Rate	1,1123%	1,1774%	1,2124%	1,1461%	1,1189%	1,1187%
Mean Return Rate	1,1235%	1,2075%	1,2247%	1,0665%	1,1520%	1,0665%
Standard Deviation	1,4893%	2,4447%	1,5615%	1,3544%	2,5667%	2,5667%

Table A.3 : continued

Panel D: Five-year immunization Period						
	Without Transaction Costs		With Transaction Costs			
	Inflation-linked	Fixed Rate	Inflation-linked		Fixed Rate	
	weekly	weekly	weekly	monthly	weekly	monthly
Nº Simulations	261	261	261	66	261	66
Liability Coverage						
At maturity	100,0776%	99,5753%	99,9060%	100,1961%	99,4738%	99,7759%
Mean Coverage	100,0463%	99,6036%	100,1110%	100,0725%	99,5349%	99,4896%
75^{th} percentile	100,1925%	100,1027%	100,2497%	100,2492%	100,0087%	100,1319%
25^{th} percentile	99,9207%	99,8004%	99,9920%	99,9236%	99,7007%	99,0482%
Standard Deviation	0,1653%	0,8928%	0,1844%	0,1885%	0,8708%	0,9194%
Liability Coverage Deficit	32,9502%	61,3027%	24,9042%	36,3636%	73,5632%	68,1818%
Asset Portfolio Returns (annualized)						
5-year spot rate	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%	1,3047%
Realized Return Rate	1,3185%	1,2166%	1,2837%	1,3425%	1,1959%	1,2573%
Mean Return Rate	1,3342%	1,2558%	1,3027%	1,2489%	1,2372%	1,1876%
Standard Deviation	1,7677%	2,6190%	1,9405%	1,4402%	2,6953%	2,3562%