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# Designing and Building for Extreme Environments

## A multi-criteria decision model to evaluate architecture for extreme temperatures

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*ABSTRACT: The purpose of this research is to present an assessment methodology that validates architecture designs for environments with extreme temperatures, considering structural and energy demands, as well as sustainability-related concerns. This is achieved using multi-criteria decision analysis modelling. This study presents the results of a MCDA model built for this purpose, and evaluated through four variations representing different project scenarios. A total of 11 criteria (regarding energy efficiency, material performance, architectural performance and circularity) are used to analyse four building assemblies, in order to understand which is more appropriate for an environment with extremely cold and extremely hot temperatures. This allows the validation of the proposed multi-criteria analysis framework, which will lead to further research on extreme climate design.*

*KEYWORDS: Extreme Environments; Sustainable Architecture; Multi-criteria Evaluation.*

## 1. INTRODUCTION

This paper is part of an on-going research that aims at optimizing the mediation between extreme environmental conditions and the architectural habitat. This requires an evaluation methodology for the validation of the architectural design proposals for extreme temperatures. This paper briefly presents the groundwork already undertaken related to designing for extreme environments and describes a new methodology to evaluate architectural designs for extreme temperatures. The evaluation methodology is based on a multi-criteria decision analysis (MCDA) framework, including a set of criteria grouped according to energy efficiency, material performance, architectural performance, and circularity. This paper is focused on the MCDA modelling structure and on an application for preliminary evaluation purposes.

## 2. DESIGNING FOR EXTREME ENVIRONMENTS

An extreme environment can be considered as any environmental setting in which human life is hard or even impossible. Such extreme characteristics can be temperature, humidity, pressure, salinity, lack of oxygen or no air quality, extreme pH values (mainly acidity) and dangerous levels of radiation [1]. This study is focused only on extreme temperatures and because of this, two locations among those with the lowest and highest temperatures on permanently inhabited places on earth have been selected. The two locations with the lowest and highest temperatures ever recorded on earth are Oymyakon, in Siberia, Russia [2], with -62°C recorded in 2018, and Furnace Creek in Death Valley, California, United States [3], with a temperature of +57°C recorded in 1913. Due to the lack of accurate annual climate data for these locations, similar places were chosen for this study instead. For the coldest climate Yakutsk, in Russia, was selected, being one of the coldest large cities in the world, where the lowest temperature ever recorded was -64°C [4], and Needles in California, close to Death Valley, USA, with the highest recorded temperature of +52°C. It also holds the record for the highest minimum temperature in the world, +38°C [5].

Due to previous research [6], three types of characteristics of architectural design were considered as crucial for extreme design: the building morphology for the optimization of energy requirements; the building materials and construction assemblies that are able to effectively withstand these extreme conditions; and the internal spatial configuration for an effective use of space, integrating minimal space requirements and functionality.

### 2.1 Building Morphology

Building morphology is addressed through the heat transfer form factor of the architectural form. The form factor is used to assess the compactness of a building and may be obtained by dividing the external envelope surface area by either the internal volume of the building or the internal treated floor area. In this research, the second option was chosen because, for a given functional floor area, larger volumes mean more energy to keep the internal volume comfortable. Therefore, it was considered that the floor area should be the reference to

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calculate the form factor [7]. The lower the form factor, the lesser the heat transfer. Previous work carried out in this project led to the conclusion that the best outside shape for a building in an extreme environment would be either a prism or semi-ellipsoid; this latter shape would be better for aerodynamic purposes, taking into account environments with high wind speeds [6]. In this paper, the prismatic shape is used to validate the proposed multi-criteria model.

### 2.2 Materials

The materials and construction assemblies used in this preliminary application of the MCDA model were selected among the default set included in the EnergyPlus software material library [8] (the engine used for the energy performance simulations). These material assemblies cover brick, concrete, steel and wood-based construction and are recommended for the climates included in the study, according to ASHRAE Standard 189.1, "Standard for the Design of High-Performance Green Buildings" [9] (climate zone 2 for Needles and climate zone 8 to represent Yakutsk climate [10]).

The full set of material assemblies considers 21 solutions, grouped in four types, designated as follows: 'Generic' (brick-based); 'Mass' (concrete-based); 'Metal' (steel-based); and 'Wood' (wood-based). Each type includes floors, walls and roofs, and the recommended thickness of each material layer depends on climate.

### 2.3 Interior Spatial Configuration

In extreme environments, the interior habitat has an increased importance. In these circumstances, inhabitants are more psychologically dependent on the indoor environment. Certain elements such as lightning conditions, colour, perception of safety, separation between public and private spaces, noise and flexibility become essential for people living in these realities [11].

Also, minimum area requirements need to be ensured. Outer space being the ultimate extreme environment, a sound reference for the threshold values for 'minimum area' is NASA's reference for space crews. According to these requirements, a habitat of six people must have a minimum area of 55m$^2$ and a habitable volume of 150m$^3$ [12]. In extreme conditions, architectural design must integrate improved effectiveness with functionality and aesthetics to assure a holistic well-being of the inhabitants. This leads to selecting three essential morphology qualities: floor area (it must ensure the minimal requirements while trying to remain as small as possible); height (must also ensure a minimal comfortable height but the smallest the better), and space organization (which must ensure all the requirements presented by NASA to guarantee the most comfortable living experience possible).

## 3. METHODOLGY

Designing for extreme environments is a peculiar task and it must respond to distinct requirements other than the usual contemporary standards. As mentioned above, there are specific technical, material, and spatial attributes that inform the design process. It seems therefore that an evaluation methodology may be of great interest to validate design proposals for these environments. This study aims at developing a proposal for such an evaluation methodology, based on a multi-criteria decision analysis (MCDA) framework [13].

### 3.1 MCDA Model Criteria

Based on the insights from other research and from previous phases of this project (vd. section 2), a set of four categories (grouping a total of 11 criteria) was established for the evaluation methodology: energy efficiency (including criteria 'energy consumption' and 'free-floating mean internal operative temperature'); material performance (including criteria 'service temperature', 'fracture toughness', 'weight', 'carbon footprint' and 'end-of-life'); architectural performance (including criteria 'minimum areas', 'internal spatial height' and 'space organization'); and circularity. It should be noted that heat transfer properties are not considered individually because these will be the most conditioning factors for the energy performance simulations. Circularity is introduced as a way of including sustainability-related concerns in the design options; it is an index-type criterion built upon energy efficiency, carbon footprint and end-of-life.

### 3.2 Energy Efficiency

The data for this criterion (energy consumption and average free-floating internal temperature) was obtained from EnergyPlus using Ladybug and Honeybee running on a Grasshopper parametric model [14].

For the purpose of this study, data relative to the indoor temperature of the building (for each material assembly presented previously) is retrieved, both in free-floating and within comfort targets, to know how much energy would be necessary to achieve comfort conditions. The indoor comfort target temperatures used in the study are common references for low-energy indoor environments [15]: 16-18 °C for Yakutsk and 26-28 °C for Needles.

In order to run the simulation, it is also required to provide Honeybee other types of information, specially related to space usage, occupancy schedules, lightning conditions, and equipment load. Space usage is in this case six people; the occupancy schedule is defined for a 24-hours period, considering that, in an extreme environment, inhabitants would live and work within the building; the lighting load density is defined at 3 W/m$^2$, which corresponds to efficient LED bulbs; and lastly, equipment load density is defined at 7 W/m$^2$, corresponding to a mid-scale load density (between 2 W/m$^2$ for one laptop and 15 W/m$^2$ for a heavy-equipped officed). As the building would have both a work area and sleeping areas where little to no electronic equipment would be required, maintaining the load at half the scale was considered an adequate depiction of the equipment load. Simulations were run for the most extreme period in each climate: June to August for the hot climate, and December to February for the cold climate. Simulation results were then analysed through Ladybug visualization tools and exported to the MCDA model.

### 3.3 Material Performance

Regarding the data for Material Performance, the first criterion of Service Temperature was only used for the extreme cold temperature scenario, because all the materials could successfully handle the extreme hot temperatures. High resistance to cold temperatures is indeed a mandatory feature for very cold environments. Material fracture toughness was selected as a baseline indicator of mechanical strength and durability. Based on data retrieved from a previous study (a library of 52 materials studied for the range of climate conditions selected in this study [16]) each construction assembly was given a qualitative assessment within a 5-level scale ranging from 'very low' to 'very high'. The weight of each construction assembly was assessed by its total weight computing the sum of each layer weight calculated using density from the EnergyPlus library and volume of material used. Carbon footprint is assessed through the total embodied carbon of the construction assembly, considering the system boundary of cradle-to-gate. Unit values for embodied carbon were retrieved from the Inventory of Carbon & Energy (ICE) [17]. The criterion end-of-life characterizes the reuse and recycle potential of each construction assembly.

### 3.4 Architectural Performance

For this specific group of criteria, in order to facilitate the testing of the model, just one example of architectural morphology and internal design was used: a habitat planned for up to 6 people. The 3D digital model was replicated in Rhinoceros 3D, a prism with interior dimensions of 4.80m per 11.80m, with 56.64m$^2$ of internal area. The height was defined at 2.40m, as it is the minimum height for residential spaces [18]. The interior spatial configuration was defined considering the functional distribution advised by NASA. Therefore, in the MCDA model used for this paper, there is no variation in this criterion, all the options rating very good. The sleeping quarters and the hygiene quarters are on opposite sides of the prism; a social/eating zone and the workspace occupy the central area of the plan; this offers both privacy and noise reduction. Sleeping and workspace zones have the same area, while the social area is the largest space, and the hygiene quarter was divided into two smaller areas, one for bathing and another one for various uses (Fig.1). Category 'architectural performance' is divided in three criteria: Area, Height and Space Organization. The first two criteria are assessed quantitatively against the references recommended by Nasa. The last criterion is qualitative, and it refers to the internal organization of the different functionalities; as a proposal based of the directives of NASA, it rates High on a scale of Low, Medium and High.

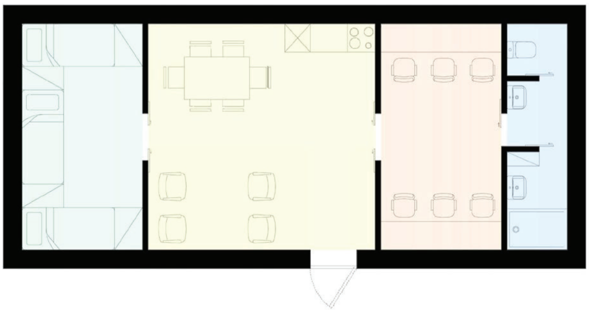

*Figure 1: Floor plan of the base prism with sleeping quarters (green), social area (yellow), workspace (orange) and hygiene quarters (blue).*

### 3.5 Circularity

The circularity criterion was envisaged as a mean to have a single indicator expressing the environmental impact of the options being evaluated by the MCDA model. Following an index-type procedure, it puts together energy consumption, average internal free-floating temperature, carbon footprint and end-of-life data. The assessment of each construction assembly for this criterion was performed in a dedicated MCDA sub-model, and then the results exported to the main MCDA assessment model.

### 3.6 MCDA Model Creation

The main advantage of a multi-criteria decision analysis methodology to evaluate a set of architectural proposals is the easiness with which design scenarios may be represented through the relative weighing of each criterion. For the purpose of this study, the MACBETH approach is used [18], through the computational tool M-MACBETH, offering the ability to include qualitative judgments based on differences of attractiveness.

The first step when planning the M-MACBETH model is to define the value tree composed of the set of criteria that will be used to evaluate the group of options (in this case, options are the material assemblies). The basis for comparison in each criterion may be the options themselves (using or not reference values), qualitative performance levels or quantitative performance levels. Then, the options to assess are uploaded to the model and characterised against each criterion. The difference of attractiveness between criteria is then used to include judgements related to the objective of the assessment process. Further fine-tuning may be accomplished through changing the difference of attractiveness between the performance levels of each criterion, what may prove useful to represent different project scenarios.

The base value tree includes 11 criteria: energy consumption and free-floating mean internal operative temperature (energy efficiency category); service temperature, fracture toughness, weight, carbon footprint and end-of-life (material performance category); minimum areas, height and space organization (architecture performance category); and circularity (as above described).

At the actual stage of the research, it was decided to use a single base architecture morphology and configuration. Therefore, the options being assessed to test the model are the four types of construction assembly above described: ‘Generic’, ‘Mass’, ‘Metal’ and ‘Wood’.

Options characterization for energy consumption and free-floating mean internal operative temperature comes from the simulations using Ladybug and Honeybee. The performance levels, in each case, are set within the range of actual values for the set of options. Energy consumption was measured for the whole 3-month period per unit surface area [kWh/m$^2$].

The performance related to the resistance to extreme low temperatures was characterized through a qualitative basis for comparison using a 3-level scale (low, medium, high). As previously mentioned, mechanical strength and durability, as assessed by the fracture toughness, was also characterized using a 5-level qualitative scale ranging from ‘very low’ to ‘very high’; in this case, the need for a wider qualitative scale arose from a wide range between the best and the least performing options. Weight is a quantitative criterion considering the total weight of each construction assembly [kg]. Carbon footprint is also a quantitative criterion assessing the comparative climate change potential of each option, computed as the total embodied greenhouse gas emissions [kgCO$_2$e]. The reuse and recycle potential (criterion end-of-life) of each option is compared through a qualitative 3-level scale (low, medium and high).

In what concerns the architecture performance category, minimum areas and height are quantitative criteria ([m$^2$] and [m], respectively); for this specific study the dimensions of the model are the same for all four options, 56.64m$^2$ in area and 2.4m in height. Space organization is again a 3-level scale qualitative criterion (low, medium, high).

As above described, the circularity criterion is an index-type assessment using a MCDA sub-model, with scores attributed in the range 0-100. The performances of all options in all criteria are shown in table 1 (Needles) and table 2 (Yakutsk). The two values for ‘Circularity’ are for model variations A and B, respectively (please, refer to next paragraph).

*Table 1: Options performance in each criterion for the hot climate (Needles).*

| Criteria / Assemblies | Generic | Mass | Metal | Wood |
|---|---|---|---|---|
| Energy Consumption (kWh/m$^2$) | 70.7 | 61.4 | 46.6 | 45.9 |
| Free-floating temperature (°C) | 42.6 | 46.4 | 44.7 | 46.3 |
| Fracture Toughness (-) | VL | VL | VH | VL |
| Weight (kg) | 86865 | 45898 | 7239 | 6904 |
| Carbon footprint (kgCO$_2$e) | 21800 | 17379 | 9184 | 7805 |
| End-of-life (-) | L | L | M | M |
| Minimum Areas (m$^2$) | 56.64 | 56.64 | 56.64 | 56.64 |
| Height (m) | 2.40 | 2.40 | 2.40 | 2.40 |
| Space Organization (-) | H | H | H | H |
| Circularity (-) | 25%/5% | 17%/12% | 71%/48% | 63%/48% |

*Table 2: Options performance in each criterion for the cold climate (Yakutsk).*

| Criteria / Assemblies | Generic | Mass | Metal | Wood |
|---|---|---|---|---|
| Energy Consumption (kWh/m$^2$) | 202.5 | 65.4 | 61.5 | 55.6 |
| Free-floating temperature (°C) | -38 | -32 | -33 | -32 |
| Service Temperature (-) | L | L | H | H |
| Fracture Toughness (-) | VL | VL | VH | VL |
| Weight (kg) | 86865 | 45898 | 7239 | 6904 |
| Carbon footprint (kgCO$_2$e) | 21800 | 17379 | 9184 | 7805 |
| End-of-life (-) | L | L | M | M |
| Minimum Areas (m$^2$) | 56.64 | 56.64 | 56.64 | 56.64 |
| Height (m) | 2.40 | 2.40 | 2.40 | 2.40 |
| Space Organization (-) | H | H | H | H |
| Circularity (-) | 0%/0% | 55%/27% | 80%/56% | 88%/59% |

Four variations of the base MCDA model were considered. Model A corresponds to the base model, in which the difference of attractiveness between each criterion was given by an equitable distribution among the four categories (25%). In each category, the weighting factors are evenly distributed: the 25% are divided among the number of criterions within the category, meaning that if one category has more criteria, each will value less than what is the case in another category with less criteria.

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In model B, the sub-model for calculating the scores related to the 'circularity' criterion was changed in how it compares options for the energy consumption and the free-float operative temperature. While in the first case (MCDA model A), this circularity sub-model used the actual values in the range as low and high reference scores, in the MCDA model B the references used correspond to energy efficiency good practices in Europe. In the case of energy consumption, a reference of 11 kWh/m$^2$ final energy was used for the highest score. In what concerns operative temperature, 18°C and 28°C were used for the cold and the hot climates, respectively.

MCDA models C and D represent a change in the assessment approach, from a general evaluation methodology to a more realistic approach related to project scenarios. The criteria 'free-floating operative temperature' and 'circularity' are now disregarded. In the first case, because the values are so extreme and out of the comfort zone that it would not make sense to consider it as an evaluation criterion in real conditions; on the other hand, the differences between options are too small to be significant (from -38°C to -32°C for the cold climate, and from 43° to 46°C for the hot climate). The option of removing 'circularity' was related to the fact that in these two new models (C and D) the difference of attractiveness between criteria now corresponds to real conditions project scenarios and thus the redundancy effect associated with using circularity as an independent criteria is no longer useful. Another important change implemented in these two models (C and D) is that the value scale for evaluating the resistance to extreme low temperatures was modified so that options characterized by 'low' or 'medium' are now evaluated with a score of zero (this was done through changing judgments associated with the performance levels of the criterion). Apart from this specific way of judging criterion 'service temperature' in models C and D, in all other cases for the four models (A, B, C and D) the difference of attractiveness between performance levels of each criterion remained the same, a linear variation scale.

The project scenario associated with MCDA model C is one in which priority is given to the environmental impact, thus giving higher weighting factors to the carbon footprint, the end-of-life and the energy consumption. For MCDA model D, priority was given to the weight and the mechanical strength, thus representing a scenario where ease of transport and strength are critical factors (for instance, considering high wind speeds). In both cases (models C and D), the resistance to extreme low temperatures was the top priority for the cold climate because it is an excluding criterion for construction assemblies.

## 4. RESULTS

The ratings of each construction assembly, in the four MCDA models, are presented below in tables 3 and 4 in a 0-100 scale.

MCDA model A, with all the categories having the same relative importance, delivered different results between the extreme hot and the extreme cold climates. In the first case, the 'Metal' construction assembly rates higher, followed by 'Wood', then 'Generic' and 'Mass'. For the model related to the cold climate however, the 'Wood' option rates higher than the best option for the hot climate, followed closely by the 'Metal' assembly, then 'Mass' and then 'Generic' at the bottom of the scale.

*Table 3: Ratings of the four construction assemblies in the four MCDA models, for the hot climate (Needles).*

| Needles | Model A | Model B | Model C | Model D |
|---|---|---|---|---|
| Generic | 25% | 15% | 11% | 10% |
| Mass | 19% | 16% | 31% | 31% |
| Metal | 76% | 64% | 85% | 91% |
| Wood | 61% | 53% | 77% | 67% |

*Table 4: Ratings of the four construction assemblies in the four MCDA models, for the cold climate (Yakutsk).*

| Yakutsk | Model A | Model B | Model C | Model D |
|---|---|---|---|---|
| Generic | 1% | 1% | 9% | 8% |
| Mass | 56% | 42% | 30% | 28% |
| Metal | 85% | 73% | 88% | 92% |
| Wood | 87% | 73% | 83% | 76% |

In model B, all the assemblies rated lower than in the previous model A but generally kept the same relative position in the overall scale.

In MCDA model C and D, the assemblies rated similarly in the two environments, both models following the same organization of ratings: 'Metal', 'Wood', 'Mass' and 'Generic'.

## 5. DISCUSSION

Considering all the simulations run through the four MCDA models it may be concluded that construction assemblies 'Mass' and 'Generic' are the worst for extreme climates, being that they aren't prepared to handle this environments within controlled parameters, even if the Mass assemblies rates higher in the cold climate than in the hot one. For hot climates, the 'Metal' and 'Wood' are preferential, although 'Metal' rates higher, the biggest difference between the ratings being in model D. This is due to the fact that mechanical properties are given priority in this case, and 'Wood' fracture toughness is much lower than 'Metal'. On model C 'Wood' rated higher due to environmental concerns (with the end-of-life and embodied carbon criteria).

Regarding the cold environment, ‘Wood’ rates higher in model A, when all criteria have the same weight, and in Model B, where high-performance benchmarks were used for the criterion ‘Circularity’, ‘Wood’ and ‘Metal’ rate very similarly. Relative to model C and D, ‘Wood’ rates higher in model C, and lower in model D, exactly for the same reasons as in the hot climate scenario.

It is important to remark that these construction assemblies are preliminary since they were chosen from a default database, and thus are not yet customized. Future work will include running the models with customized construction assemblies to reach a better understanding about the influence of each material.

The ultimate purpose of this study is to validate a multi-criteria approach to evaluate architecture building proposals for extreme environments. The models proposed in this paper, including their capacity to be adapted to different project scenarios, seem to be a valid framework. Further research will include custom materials and assemblies, other types of architectural morphologies (with different areas and heights) and other climate scenarios. This will add to the existent knowledge on architecture for extreme environments, hopefully providing a new methodology for evaluating future building proposals and opening the way for new research.

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