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# The choice of tax shields' discount rate on firm valuation – Cruz Vermelha Portuguesa - Sociedade Gestora de Hospitais, S.A. case study

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## ABSTRACT

This dissertation suggests that the tax savings, in firm valuation, are discounted at a rate computed through a model presented in the literature review[1], which is different from the rates usually used for this purpose either by the top text books from, for example, Neves (2002), Ross, Westerfield and Jaffe (2005), Berk and DeMarzo (2011), and Brealey, Myers and Allen (2007) or the investment banks.

In this model the necessity to fix *a priori* important parameters such as the interest rate, the debt level or the leverage ratio, and, consequently, the tax shields' discount rate is challenged, assumptions that are required by Modigliani & Miller (1963), Harris & Pringle (1985), Miles & Ezzell (1980), among others. In this model all these parameters are *endogenized*.

The only assumptions necessary are the risk free rate and the unlevered cost of capital, whereas the capital structure of the company is computed iteratively by the model.

A graphic representation of the case study was built from which were drawn theoretical and practical inferences that can be generally used in any case. Moreover, an assessment on the pertinence of the model that determines the tax shields' discount rate was made, as well as on how the model reconciles with the state of the art.

Cruz Vermelha Portuguesa – Sociedade Gestora de Hospitais, S.A. (CVP-SGH,S.A.), a publicly unlisted company, became an interesting valuation case study when, in 2012, the cooperation agreement between CVP-SGH, S.A. and Administração Regional de Saúde de Lisboa e Vale do Tejo (ARSLVT) was suspended after the recommendation of the Portuguese Audit Court of non-renewal of such agreement. In this scenario, CVP-SGH,S.A.'s EBIT drops abruptly to levels that no longer cover totally the interest expenses, which, as we will see, jeopardizes the adoption of the standard WACC valuation method.


**Key words:** Discounted Cash Flow, Tax Shields, Discount Rates, Cost of Equity, Cost of Capital, Tax Shield Risk, Adjusted Present Value, Equity Cash Flow, Capital Cash Flow

**JEL Classification System:** G24, G30, G31, G32, I10.

[1] Ansay (2010): $K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V}$

**RESUMO**

Esta dissertação propõe que as poupanças fiscais, na avaliação de empresas, sejam atualizadas a uma taxa calculada de acordo com um modelo apresentado para o efeito na revisão da literatura[2], que é diferente das taxas usadas para este fim quer nos livros de texto de, por exemplo, Neves (2002), Ross, Westerfield e Jaffe (2005), Berk e DeMarzo (2011) e Brealey, Myers e Allen (2007), quer pela banca de investimento.

Neste modelo a necessidade de fixar *a priori* parâmetros tão importantes como a taxa de juro, o nível da dívida ou o rácio de endividamento e, consequentemente, a taxa de atualização das poupanças fiscais é desafiada, pressupostos requeridos por Modigliani & Miller (1963), Harris & Pringle (1985), Miles & Ezzell (1980), entre outros. Neste modelo todos estes parâmetros são endogeneizados

É apenas necessário, como pressuposto, a taxa livre de risco e o custo do capital não alavancado, não sendo necessário a estrutura de capital da empresa, uma vez que esta é determinada iterativamente pelo modelo.

É feita uma representação gráfica do estudo de caso, a partir da qual são retiradas ilações teórico-práticas aplicáveis genericamente a qualquer outro caso e feito um juízo sobre a pertinência da utilização do modelo de determinação da taxa de atualização especifica para as poupanças fiscais e a forma como ele se concilia com o estado da arte.

A Cruz Vermelha Portuguesa - Sociedade Gestora de Hospitais, SA (CVP-SGH, SA), uma empresa não cotada em bolsa, tornou-se um caso de estudo interessante quando, em 2012, o acordo de cooperação entre a CVP-SGH, S.A. e Administração Regional de Saúde de Lisboa e Vale do Tejo (ARSLVT) foi suspenso devido a uma recomendação do Tribunal de Contas Português de não renovação de tal contrato. Neste cenário, o EBIT da CVP-SGH, S.A. cai abruptamente para níveis que não cobrem totalmente os encargos financeiros, o que põe em risco a adoção do standard WACC como método de avaliação.

**Palavras chave:** Discounted Cash Flow, Tax Shields, Discount Rates, Cost of Equity, Cost of Capital, Tax Shield Risk, Adjusted Present Value, Equity Cash Flow, Capital Cash Flow

**Classificações do JEL:** G24, G30, G31, G32, I10.

[2] Ansay (2010): $K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V}$

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**Formulas:**

| | |
|---|---|
| $K_E$ | Market Value Discount Rate for Equity ; Cost of Levered Equity |
| $K_U$ | Market Value Discount Rate for the Unlevered Firm ; Cost of Unlevered Equity |
| $K_D$ | Corporate Interest Rate ; Cost of Debt |
| $R_F$ | Risk-free Interest Rate |
| $K_{E-V_{TS}}$ | Discount Rate relevant to the Market Value difference $E - V_{TS}$ |
| $K_{TS}$ | Market Value Discount Rate for Tax Shields |
| $E$ | Market Value of the Equity |
| $D$ | Market Value of the Debt (assumed equal to its Book Value) |
| $V_U$ | Unlevered Market Value of the Firm |
| $V_{TS}$ | Present (or Market) Value of the Tax Shields |
| $V$ | Market Value of the Firm |
| $DCF$ | Discounted Cash Flow |
| $WACC$ | Weighted Average Cost of Capital |
| $APV$ | Adjusted Present Value Valuation Method |
| $ECF$ | Equity Cash Flows |
| CCF | Capital Cash Flow |
| CFd | Debt Cash Flow |
| $ROE$ | Return on Equity |
| $ROIC$ | Return on Invested Capital |
| $FCF$ | Free Cash Flow |
| $TS$ | Debt Tax Shield Flow |
| $PVITS$ | Present Value of Interest Tax Shields |
| $EBIT$ | Earnings Before Interest and Taxes |
| $NOPLAT$ | Net Operating Profit Less Adjusted Taxes |
| $NI$ | Net Income |
| $MVA$ | Market Value Added |
| $OMVA$ | Operating Market Value Added |

| | |
|---|---|
| $FMVA$ | Financing Market Value Added |
| $E_{Book}$ | Book Value of the Equity |
| $V_{Book}$ | Book Value of the Firm |
| $n$ | Marginal Debt Risk Factor or Number of Years of the Explicit Period |
| $\tau$ | Corporate Tax Rate |
| $t$ | Time Index |
| $g$ | Growth Rate Beyond The Explicit Period |

**Others:**

| | |
|---|---|
| ADSE | Assistência na Doença aos Servidores Civis do Estado |
| ARSLVT | Administração Regional de Saúde de Lisboa e Vale do Tejo |
| Caixa BI | Caixa Banco de Investimento |
| CVP | Cruz Vermelha Portuguesa |
| CVP-SGH | Cruz Vermelha Portuguesa - Sociedade Gestora de Hospitais |
| EC | European Commission |
| ECB | European Central Bank |
| EU | European Union |
| EFSM | European Financial Strabilization Mechanism |
| ERS | Entidade Reguladora da Saúde |
| ESS | Espírito Santo Saúde |
| GDP | Gross Domestic Product |
| HCVP | Hospital Cruz Vermelha Portuguesa |
| HPP | Hospitais Privados de Portugal |
| IMF | International Monetary Fund |
| INE | Instituto Nacional de Estatistica |
| MoU | Memorandum of Understanding |
| OECD | Organization for Economic Co-operation and Development |
| PPP | Private Public Partnership |
| SGPS, S.A. | Sociedade Gestora de Participações Sociais, Sociedade Anónima |

## 1. INTRODUCTION

This thesis was guided by the identification of controversial areas in the managerial science. Thus, we observed that there is still no consensus regarding the tax shields' discount rates and found a research and dissertation topic.

After reading in Luehrman (1997: 7) that "Academics agree that tax shields, (…), should be discounted at an "appropriate" risk-adjusted rate – that is, a rate that reflects riskiness. Unfortunately, they don't agree how risky tax shields are." Luehrman after stating that "A common expedient is to use the cost of debt as a discount rate", says that even if the company affords the interest payments, it might not realize the tax shields. Then claims that "This suggests that tax shields are a bit more uncertain and so deserve a somewhat higher discount rate." and in an *ad hoc* manner adds to the cost of debt 0.5% in order to obtain the rate that reflects the riskiness of tax shields.

It was that controversy and this *ad hoc* solution of establishing the riskiness of tax shields that motivated us to search for a non-*ad hoc* solution for the computation of the tax shields' discount rate and the main reason for this dissertation.

To study this controversial topic it was necessary to study a problematic case. The valuation of Cruz Vermelha Portuguesa – Sociedade Gestora de Hospitais, S.A. (CVP-SGH, S.A.) was not a problematic case in 2012 with the renewal of the cooperation agreement with Administração Regional de Saúde de Lisboa e Vale do Tejo (ARSLVT), but became a challenging case with the recommendation of the Audit Portuguese Court of non-renewal of this contract. Indeed, in this second scenario the CVP-SGH, S.A.'s EBITs do not allow the total or partial realization of tax shields in the year in which the interest expenses are paid. This fact makes all the difference in our case study because it allows to uncover some weaknesses (in our opinion, relevant) of the standard WACC method used by Caixa Banco de Investimentos (Caixa BI).

In a first phase, in point 2.1, we gave a general overview of the discounted cash flows' firm valuation methods and presented all important formulas we know from the literature. We detailed the five main valuation methods (Weighted Average Cost of Capital (WACC), Adjusted Present Value (APV), Equity Cash Flow (ECF), Capital Cash Flow (CCF) and Market Value Added (MVA)) used to get the market value of firms. In point 2.4, we described the contributions of the authors who most contributed and still

contribute, in our opinion, on discounted cash flow valuation methods. Then we gave a special emphasis to tax shields' value, the main topic of the dissertation. In point 2.6 we made a short literature review on the multiple valuation methods, which were not used in the case study because they did not seem to be necessary for what we wanted to demonstrate, i.e. the tax shields' valuation. In point 2.8 we revisited some parameters considered in discounted cash flow valuation. Finally, in 2.9 we gave a short overview about country risk due to the Portuguese economic situation.

In a second step, we started the case study by describing the Portuguese macroeconomic and demographic environment, as well as the description of the Portuguese healthcare industry. Afterwards, we gave an overview of the Caixa BI's CVP-SGH, S.A. firm valuation methodology. We revaluated the company using the five main methods described in the literature review. In this revaluation we *endogenized* the leverage ratio, the interest rate, and tested the model that computes the tax shields' discount rate by Ansay (2010)[3] presented in the literature review, that we will show empirically in the conclusions that incorporates the main inherent risks of tax shields (the risk of default in debt and the operational/realization risk).

## 2. LITERATURE REVIEW

### 2.1. CASH FLOWS STATEMENTS

From an accounting perspective, the year after year actual results of a firm have to be taken into consideration. To further assess them, one needs to refer to the income statement of the firm when valuing it, using a discounted cash flow model. The firm's accounting results are commonly *modelized* as one can see below:

| Income Statement | | | |
|---|---|---|---|
| Items | *Modelization* | | |
| $EBIT_t$ | | | |
| - Operating $Taxes_t$ | = | $EBIT_t.T_c = (EBT_t + (K_D.D)_t).T_c = EBT_t.T_c + K_D.D)_t.T_c = I_t + TS_t$ | (1) |
| = $NOPAT_t$ | = | $EBIT_t - OperatingTaxes_t = EBIT_t.(1 - T_c)$ | (2) |
| - Debt $Interests_t$ | = | $(K_D.D)_t$ | (3) |

[3] $K_{TS} = K_D + (K_{E-V_{TS}} - K_D)\frac{D}{V}$

| | | | |
|---|---|---|---|
| = $EBT_t$ | = | $EBIT_t - (K_D.D)_t$ | (4) |
| - Accum.LCF | = | $Accum.LCF_{t-1} + LCF_t + Max((EBT_{t-1} - TI_{t-1}), 0)$ | (5) |
| = Taxable $Income_t$($TI_t$) | = | $Max((EBT_t - ALCF_t), 0)$ | (6) |
| - $Taxes_t$ ($I_t$) | = | $TI_t.T_c$ | (7) |
| = Net $Income_t$ | = | $EBT_t - I_t$ | (8) |

Five kinds of cash flows are used for firm valuation purposes:

1. **The Free Cash Flow (FCF):**

$$FCF_t = NOPAT_t + Depreciation_t - Investment_t - \Delta WorkingCapital_t \quad (9)$$

The FCF is a measure of the after-tax operating funds produced by a firm, assuming that it is an all-equity firm. It is important to note that the FCF has to be available for distribution to the debt holders and to the equity holders. The ideal FCF is rarely equal to the actual FCF. As Tham and Velez-Pareja (2004: 13) state, "The ideal FCF, without retention of excess cash, is equivalent to the investment of excess cash at the cost of capital rather than the rate of interest on short term securities. Strictly speaking, the change in short-term marketable securities is <u>not</u> part of the operating cash flow" – (underlined added for emphasis).

2. **Tax Shields (TS):**

$$TS_t = (K_D.D)_t.T_c \;\; ; \;\; if\ EBIT > K_D.D \quad (10)$$

The TSs are the tax savings attributed to the tax deductibility of the debt interests (Kemsley and Nissim, 2002). Thus, TSs are an important factor influencing the company's capital structure choice.

3. **The Equity Cash Flow (ECF):**

$$ECF_t = FCF_t - (K_D.D)_t + (K_D.D)_t.T_c + \Delta D_t \quad (11)$$

The ECF is a fraction of the FCF that reverts to equity holders after the debt holders have been paid. "If there is debt financing and TSs are realized, the equity holder also receives the stream of future TSs." – Tham and Velez-Pareja (2004: 14). The equity holders only receive some fraction of the FCF after the claim of the debt holders have been paid.

4. **The Debt Cash Flow (CFd):**

$$CFd_t = (K_D.D)_t - \Delta D_t \quad (12)$$

The CFd is the fraction of the FCF that reverts to the current and future debt holders. The debt holders are the senior claimants of the FCF. The CFd reflects the leverage policy of the firm (Tham and Velez-Pareja, 2004: 14).

5. **The Capital Cash Flow (CCF):**

$$CCF_t = ECF_t + CFd_t = FCF_t + TS_t \quad (13)$$

The CCF is a cash flow used for the Ruback's Capital Cash Flow approach (Ruback, 2002).

The inclusion of the LCF – Loss Carried Forward – is due to the fact that the tax shields are not always realized in the year in which the interest expenses occur and, in this case, the tax shields are added to the free cash flow only in the years in which the tax shields are in effect realized. In the income statement *modelization*, the LCF is calculated in the following way:

$$LCF_t = Max(((K_D.D)_t - EBIT_t), 0) \quad (14)$$

### 2.2. THE ACCOUNTING RETURNS

Return on invested capital (ROIC) is a profitability ratio that measures the return that an investment generates for those who have provided capital – debt holders and equity holders (Kachru, 2005). From the comparison of the company's return on capital (ROIC) with its cost of capital (WACC), one knows whether the invested capital was used effectively. The difference between ROIC and WACC (ROIC – WACC) is called economic spread (Laopodis, 2013).

The ROIC is calculated in the following manner:

$$ROIC_t = \frac{NOPAT_t}{Invested\ Capital_{t-1}} = \frac{NOPAT_t}{(E_{Book} + D_{Book})_{t-1}} = \frac{NOPAT_t}{V_{Book_{t-1}}} \quad (15)$$

Return on equity (ROE) is a profitability ratio that measures the return that an investment on equity generates for the equity holders. From the comparison of the company's return on equity (ROE) with its cost of equity ($K_E$), one knows whether the investment on equity was used effectively.

The ROE is calculated in the following manner:

$$ROE_t = \frac{NI_t}{Invested\ Capital_{t-1}} = \frac{ROIC_t \times V_{Book_{t-1}} - (K_D.D)_t.(1 - T_c)}{E_{Book_{t-1}}} \quad (16)$$

The ROE depends on the difference between the $EBIT_t$ and the $(K_D D)_t$ and can be rewritten in the following ways:

(i) If $EBIT_t \geq (K_D D)_t$:

$$ROE_t = ROIC_t + (ROIC_t - K_{D_t}(1 - T_c))\frac{D_{Book_{t-1}}}{E_{Book_{t-1}}} \quad (16.1)$$

(ii) If $(K_D D)_t > EBIT_t \geq 0$:

$$ROE_t = ROIC_t + \left(ROIC_t - K_{D_t}\left(1 - T_c.\frac{EBIT_t}{(K_D.D)_t}\right)\right)\frac{D_{Book_{t-1}}}{E_{Book_{t-1}}} \quad (16.2)$$

(iii) If $EBIT_t < 0$:

$$ROE_t = ROIC_t + (ROIC_t - K_{D_t})\frac{D_{Book_{t-1}}}{E_{Book_{t-1}}} \quad (16.3)$$

### 2.3. MAIN VALUATION METHODS

In firm valuation, there are five methods that are predominately used. The first four are discounted cash flow methods *stricto sensu*, while the remaining one is a value creation method based on discounting the excess return on capital over the cost of capital.

All these methods can be derived directly from the MM formula (3) (1963: 436):

$$"V_L = \cdots = V_U + \tau D_L"[4] = E + D[5] = V_{Book} + (V_U - V_{Book}) + V_{TS} \quad (17)$$

Where each element of the equation can be calculated as follows:

$$V_L = \sum_{t=1}^{\infty}\frac{FCF_{unlevered_t}}{(1+WACC)^t} = \frac{FCF_{unlevered}}{WACC} \quad (17.1)$$

$$V_U = \sum_{t=1}^{\infty}\frac{FCF_{unlevered_t}}{(1+K_U)^t} = \frac{FCF_{unlevered}}{K_U} \quad (18)$$

$$T_c D = \sum_{t=1}^{\infty}\frac{T_c.(K_D.D)}{(1+K_D)^t} = \frac{T_c.(K_D.D)}{K_D}[6] \quad (19)$$

$$E = \sum_{t=1}^{\infty}\frac{FCF_{levered\ or\ to\ equity_t}}{(1+K_E)^t} = \frac{FCF_{levered\ or\ to\ equity}}{K_E} \quad (20)$$

[4] $\tau$ will be denoted from now on by $T_c$.
[5] The valuation models assume that the market value of D is its book value.
[6] Rigorously $K_D$ should be written as $K_{TS}$ since the discount rate applied to tax shields is still object of great academic controversy.

$$(V_U - V_{Book})^7 = \sum_{t=1}^{n} \left( \frac{(ROIC_t - K_U) \times Invested\ Capital_{t-1}}{(1 + K_U)_t} \right) \quad (21)$$

### 2.3.1. The Weighted Average Cost of Capital method (WACC)

The equation that represents the standard general formula of this method is:

$$V_L = \sum_{t=1}^{\infty} \frac{FCF_{(unlevered)t}}{(1 + WACC)^t} = \frac{FCF_{unlevered}}{WACC} \quad (17.1)$$

The tax shield flow (TS), which is a flow to shareholders and is assumed to effectively exist, is implicitly taken into consideration in the discounted rate (WACC), and not in the free cash flow. The WACC implies, therefore, premises about the tax shields' discount rate and the leverage levels.

The WACC method formula consists of two different components. One corresponds to the n-year period when the cash flows are explicitly forecasted year by year – the so-called explicit period, and the second component corresponds to a terminal value or continuing value. This is the value of the company's expected cash flows created beyond the explicit forecast period and which is based on assumptions established for two value drivers: the growth rate (g) and the return on investment capital of the firm (ROIC).

(i) Thus, if it is assumed that the FCF will grow at a constant rate g after the explicit forecast period and that the ROIC will remain superior to the WACC (g > 0 and ROIC > WACC), then the terminal value (TV) is:

$$TV = \frac{FCF_{n+1}}{WACC_n - g} \quad (22)$$

This expression can be rewritten as follows:

$$TV = \frac{NOPAT_{n+1}(1 - \frac{g}{ROIC_n})}{WACC_n - g} \quad (22.1)$$

(ii) If g = 0 or ROIC = WACC beyond the explicit forecast period – whatever the value of g – the terminal value of the enterprise can be written as shown below:

$$TV = \frac{NOPAT_{n+1}}{WACC_n} \quad (22.2)$$

[7] ($V_u$-$V_{Book}$) will be referred as Operating Market Value Added (OMVA), while $V_{TS}$ = ($T_c$D) will be mentioned as Financing Market Value Added (FMVA). Thus, MVA = OMVA + FMVA.

These equations can be found for example in Copeland, Koller, and Murrin (2000).

Thus,

$$V = \sum_{t-1}^{n} \frac{FCF_t}{(1 + WACC_t)^t} + \frac{1}{(1 + WACC_{n+1})^n} \times TV \tag{23}$$

If the terminal value is using non-growth perpetuity, it is usually referred as the convergence approach.

There are several approaches to calculate the WACC that will be mentioned ahead in this paper.

### 2.3.2. The Adjusted Present Value method (APV)

The general formula of the APV approach is composed by the equations (18) and (19):

$$V = \sum_{t=1}^{\infty} \left( \frac{FCF_t}{(1 + K_U)^t} + \frac{TS_t}{\left(1 + K_{TS_t}\right)} \right) = V_U + V_{TS} \tag{24}$$

In this formulization, we wanted, on purpose, to follow the MM approach. Miles and Ezzell (1980) refer to this method as the "MM-APV valuation model". Myers (1974), to whom this method is credited, formulated it in a broader way:

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1 + K_U)^t} + PV(Financing\ effects) \tag{25}$$

Where the financing effects arise from: flotation costs, tax shields on debt issued, and effects of financing subsidies.

Just like the WACC method, this formula should be broken down into a forecast explicit period and a terminal or continuing value.

(i) If g > 0 and ROIC > $K_U$:

$$TV\ of\ V_U = \frac{FCF_{n+1}}{K_U - g} \tag{26}$$

And:

$$TV\ of\ V_{TS} = \frac{TS_{n+1}}{K_{TS_n} - g} \tag{27}$$

Highlighting the value drivers, the terminal value of $V_u$ can be rewritten the following way:

$$TV\ of\ V_U = \frac{NOPAT_{n+1}\left(1 - \frac{g}{ROIC_n}\right)}{K_U - g} \tag{26.1}$$

(ii) If g = 0 and ROIC = $K_U$:

$$TV\ of\ V_U = \frac{NOPAT_{n+1}}{K_U} \tag{26.2}$$

And:

$$TV\ of\ V_{TS} = \frac{TS_{n+1}}{K_{TS_n}} \tag{27.1}$$

Thus,

$$V = \sum_{t=1}^{n}\left(\frac{FCF_t}{(1+K_U)^t} + \frac{TS_t}{(1+K_{TS})^t}\right) + \left(\frac{1}{(1+K_U)^t} \times TV\ of\ V_U\right) + \left(\frac{1}{(1+K_{TS_n})^t} \times TV\ of\ V_{TS}\right) \tag{24.1}$$

Most of the approaches of APV suggest that the tax shields are discounted at the cost of debt, for example Luehrman (1997).

### 2.3.3. The Cash Flow to Equity method (ECF)

The Cash Flow to Equity method was preceded by the Gordon model valuation:

$$Value\ per\ share\ of\ stock = \sum_{t=1}^{\infty}\frac{Expected\ dividends\ per\ share_t}{(1+K_E)^t} \tag{28}$$

This model has a narrow view of equity cash flows and only considers the dividends to be cash flows to equity. The Equity Cash Flow method does not represent a radical deviation from the Gordon model valuation, since this method represents a model where potential dividends are discounted rather than actual dividends. Indeed, the Cash Flow to Equity method takes into consideration all the flows that go to equity, whether or not these are distributed as dividends. Therefore, in the Cash Flow to Equity method, the expected dividends per share are substituted by the equity cash flow.

The Cash Flow to Equity method can be formulated, by analogy with the MM approach, as follows:

$$V = \sum_{t=1}^{\infty} \left( \frac{ECF_t}{(1 + K_{E_t})^t} + \frac{CFd_t}{(1 + K_{D_t})^t} \right) = E + D \tag{29}$$

However, this valuation model usually assumes that the value of $\sum_{t=1}^{\infty} \frac{CFd_t}{(1+K_{D_t})^t} = debt\ book\ value$. Therefore, the major issue consists in determining the market value of equity E. Whereby, the valuation of the market value of the debt will be neglected from now on.

This formula should be break down into a forecast explicit period and a terminal value.

(i) If g > 0 and ROIC > $K_E$:

$$TV = \frac{ECF_{n+1}}{K_{E_n} - g} \tag{30}$$

Or, explicitly on value drivers:

$$TV = \frac{NI_{n+1}\left(1 - \frac{g}{ROE_n}\right)}{K_{E_n} - g} \tag{30.1}$$

(ii) If g = 0 and ROE = $K_E$:

$$TV = \frac{NI_{n+1}}{K_{E_n}} \tag{30.2}$$

These formulas can be found, for example, in Koller, Goedhart and Wessels (2005, p.127).

Thus,

$$V = \sum_{t=1}^{n} \frac{ECF_t}{(1 + K_{E_t})^t} + \frac{1}{(1 + K_{E_n})^n} \times TV \tag{29.1}$$

### 2.3.4. The Capital Cash Flow method (CCF)

As referred in Ruback (2002: 86), "Stewart Myers suggested the term "Compressed APV" to describe the CCF method because the APV method is equivalent to CCF when the interest tax shields are discounted at the cost of assets" ($K_U$). Indeed, this method only differentiates from the APV method in the discount rate applied to the tax shields. In the

[8] $ECF_t = FCF_t + TS_t - CFd_t$

APV method, the tax shields are discounted at the cost of debt. Therefore, the formulation is very similar:

$$V = \sum_{t=1}^{\infty} \left( \frac{CCF_t}{(1+K_U)^t} + \frac{TS_t}{(1+K_{TS})^t} \right) = V_U + V_{TS} \tag{31}$$

This formula should be break down into a forecast explicit period and a terminal or continuing value.

(i) If g > 0 and ROIC > $K_U$:

$$TV\ of\ V_U = \frac{CCF_{n+1}}{K_U - g} \tag{32}$$

And:

$$TV\ of\ V_{TS} = \frac{TS_{n+1}}{K_{TS_n} - g} \tag{33}$$

Explicating the value drivers, the terminal value of $V_u$ can be rewritten the following way:

$$TV\ of\ V_U = \frac{NOPBT_{n+1}\left(1 - \frac{g}{ROIC_n}\right)}{K_U - g} \tag{32.1}$$

(ii) If g = 0 and ROIC = $K_U$:

$$TV\ of\ V_U = \frac{NOPBT_{n+1}}{K_U} \tag{32.2}$$

And:

$$TV\ of\ V_{TS} = \frac{TS_{n+1}}{K_{TS_n}} \tag{33.1}$$

Thus,

$$V = \sum_{t=1}^{n} \left( \frac{CCF_t}{(1+K_U)^t} + \frac{TS_t}{(1+K_{TS})^t} \right) + \left( \frac{1}{(1+K_U)^t} \times TV\ of\ V_U \right) + \left( \frac{1}{(1+K_{TS_n})^t} \times TV\ of\ V_{TS} \right) \tag{31.1}$$

### 2.3.5. The Market Value Added method (MVA) [9]

Management's objective is to maximise the difference between the market value of a company (V) and the invested capital ($V_{Book}$). This difference is called market value added or MVA. The concept of the MVA for a fairly valued company is: MVA = sum of

[9] The MVA method is included in the abnormal valuation methods.

all future annual economic value added (EVAs). EVA's concept is based on the following idea: if an investment achieves a return higher than the one required by the investor, then value has been added to the investment. Thus, EVA is defined as the difference between the return on invested capital and the cost of capital (the return spread) multiplied by the invested capital, i.e.:

$$EVA = (return\ on\ invested\ capital - cost\ of\ capital) \times invested\ capital \quad (34)$$

The development of this concept is normally attributed to Stern Stewart & Company in the early 1990s (Stewart, 1991). However, earlier, other authors, such as Rappaport (1986), talked about a similar concept.

Consequently, the MVA is calculated as follows:

$$MVA = \sum_{t=1}^{\infty} \left( \frac{(ROIC_t - WACC_t) \times Invested\ Capital_{t-1}}{(1 + WACC_t)^t} \right) \quad (35)$$

Thus:

$$V = Invested\ Capital_0 + \sum_{t=1}^{\infty} \left( \frac{(ROIC_t - WACC_t) \times Invested\ Capital_{t-1}}{(1 + WACC_t)^t} \right) \quad (36)$$

As this approach is similar to the WACC method, the WACC can be used as the discount rate.

The numerator $(ROIC_t - WACC_t)$[10] $\times\ Invested\ Capital_{t-1}$ represents the excess return on capital over the cost of capital – this difference is called economic spread –, and conducts, year after year, to add economic value to the book value of the company, in order to get the market value of the company.

As done in the previous approaches, decomposing the general formula into a forecast period and a terminal value, one gets:

(i) If g > 0 and ROIC > WACC:

$$V = Invested\ Capital_0 + \sum_{t=1}^{\infty} \left( \frac{(ROIC_t - WACC_t) \times Invested\ Capital_{t-1}}{(1 + WACC_t)^t} \right) + \frac{1}{(1 + WACC_{n+1})^n} \times \frac{(ROIC_{n+1} - WACC_{n+1}) \times Invested\ Capital_n}{WACC_{n+1} - g} \quad (36.1)$$

or, based explicitly on value drives:

[10] Assuming this difference is positive.

$$V = Invested\ Capital_0 + \sum_{t=1}^{\infty}\left(\frac{(ROIC_t - WACC_t)\times Invested\ Capital_{t-1}}{(1+WACC_t)^t}\right) + \frac{1}{(1+WACC_{n+1})^n} \times \frac{NOPAT_{n+1}\frac{g}{ROIC_{n+1}}(ROIC_{n+1} - WACC_{n+1})}{WACC_{n+1} - g} \quad (36.2)$$

(ii) If g = 0 or ROIC = WACC:

$$V = Invested\ Capital_0 + \sum_{t=1}^{\infty}\left(\frac{(ROIC_t - WACC_t)\times Invested\ Capital_{t-1}}{(1+WACC_t)^t}\right) + \frac{1}{(1+WACC_{n+1})^n} \times \frac{(ROIC_{n+1} - WACC_{n+1})\times Invested\ Capital_n}{WACC_{n+1}} \quad (36.3)$$

This formula can be rewritten in order to identify the sources of value creation:

$$V = Invested\ Capital_0 + \sum_{t=1}^{\infty}\left(\frac{(ROIC_t - K_U)\times Invested\ Capital_{t-1}}{(1+K_U)^t}\right) + \sum_{t=1}^{n}\left(\frac{TS_t}{(1+K_{TS_t})^t}\right) \quad (36.4)$$

The first sum represents the operating excess return over the "operation costs", which contributes, year after year, for the operating value creation, and is referred as the operating MVA. The difference between $ROIC_t$ and $K_u$ is called the operating economic spread. The second sum is the present value of tax shields and is referred as financing MVA.

If n is ∞ and the invested capital is the book value of the firm, the following expression is equivalent to the previous one:

$$V = V_{Book} + (V_U - V_{Book}) + V_{TS} \quad (36.5)$$

And:

$$MVA = V - V_{Book} = (V_U - V_{Book}) + V_{TS} \quad (35.1)$$

Where $(V_u - V_{Book})$ is the operating MVA and $V_{TS}$ is the financing MVA.

## 2.4. MAIN APPROACHES TO DISCOUNTED CASH FLOW VALUATION METHODS

### 2.4.1. Modigliani and Miller

Modigliani and Miller (1958: 268-269) prove, in their Proposition I, that in a world without taxes the value of the firm is not affected by its leverage policy. In their words: "the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate $\rho_k$ appropriate to its class". Where $\rho_k$, the

average cost of capital, "is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class".

$$V_L = V_U \tag{37}$$

From Proposition I, MM derived their Proposition II, in which the rate of return on common stock in levered companies "is a linear function of leverage". In other words, is positively related with debt-to-equity ratio.

$$K_E = K_U + \frac{D}{E}(K_U - K_D) \tag{38}$$

"The expected yield of a share of stock is equal to the appropriate capitalization rate $\rho_k$ for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between $\rho_k$ and r." Where r is "the rate of interest (…) for sure streams" (MM, 1958: 271).

In 1958, MM already acknowledged the existence of tax shields. However, they disregarded them, saying: "with a corporate income tax under which interest is a deductible expense, gains can accrue to stockholders from having debt in the capital structure, even when capital markets are perfect. The gains however are small, (…)"(p 294).

In 1963, Modigliani and Miller introduced the effect of the tax shields on firm valuation. The advantage of debt taxes was exclusively, due to the fact that companies could deduct the interest payments and consequently have a higher level of after-tax income. However, according to MM, "there is an additional gain due to the fact that the extra after-tax earnings, $\tau R$". Meaning that in addition to the effect of tax shields, since they are considered a sure income, these tax shields would be discounted at r instead of $\rho^{\tau}$ applicable to uncertain streams. Thus, since $r < \rho^{\tau}$, $\frac{\tau R}{r} = \tau D$ is higher than $\frac{r\tau D_L}{\rho^{\tau}}$ (from equation (4), p 436).

$$K_E = K_U + \frac{D}{E}(1 - T_C)(K_U - K_D) \tag{39}$$

In 1977, Miller retreats from this line of thought by stating "that even in a world in which interest payments are fully deductible in computing corporate income taxes, the value of the firm, in equilibrium will still be independent of its capital structure." (p 262). Therefore, if for Miller the firm value is independent from the debt-to-equity ratio, the present value of the tax shields is equal to zero.

From Inselbag and Kaufold (1997), we can derive an "adjusted proposition II" for $K_E$ when the level of debt fluctuates:

$$K_E = K_U + (K_U - K_D)\frac{D - V_{TS}}{E} \quad (40)$$

### 2.4.2. Myers

Myers (1974) introduced the Adjusted Present Value (APV) method. According to this author, the value of the levered firm is equal to the value of the unlevered firm plus the present value of the tax shields, due to the deductibility of interest payments. Myers suggests that tax savings are discounted at the cost of debt. The reasoning behind this statement is that the risk associated with the tax savings derives from the use of debt and, therefore, tax shields have the same risk of the debt.

$$V_L = V_U + Present\ Value\ of\ Tax\ Shields \quad (41)$$

However, later, Myers – co-author with Brealey and Allen (2011: 487) – says that "the risk of interest tax shields is the same as the risk of the project. (…), we will discount the tax shields at the opportunity cost of capital (r).", referring to the unlevered WACC. Thus, if the firm is continuously rebalancing its debt, the tax shields are discounted at the unlevered WACC. Though, if the project keeps a fixed debt, "we assume the risk of tax shields is the same as the risk of debt and we discount at (…) rate of debt", because "With fixed debt, the interest tax shields are safe and therefore worth more." We can conclude that the discount rate depends on the leverage policy:

1. Maintain a constant debt-to-equity ratio over the years;
2. Keep a fixed debt over the years.

Regarding the first leverage policy, the discount rate is the unlevered WACC, and concerning the second one it is the cost of debt.

Luerhman (1997: 7) endorses Myers' methodology – the Adjusted Present Value. Concerning the discount rate applied to tax shields, Luerhman says that the tax savings should be discounted at a rate that reflects their riskiness. The author states "a common expedient is to use the cost of debt as a discount rate". However, it can happen that a company can afford its interest payment, but cannot use the tax shields. Since the tax shields are slightly more uncertain than the debt, they should be discounted also at a

slightly higher discount rate. Meaning that the cost of debt should be used as the discount rate, but adjusted to this slightly higher uncertainty.

### 2.4.3. Miles and Ezzell

As stated by Miles and Ezzell (1980), a firm that wants to keep a constant debt-to-equity ratio needs to, at the end of each period, undertake financial rebalancing to restore its level of debt and has to be valuated in a different way from a firm that has a debt-repayment schedule. In this case, the correct tax shields discount rate is the cost of debt during the first year and the unlevered cost of capital for the following years (assumption 4 of the ME article, p.723). This process is dynamic, since it does the debt rebalancing every single year (assumption 3 of the ME article, p.722). The present value of the tax shields interests (PVITS) with a constant leverage ratio is valuated as follows:

$$V_{TS} = PV(1^{st} year\ tax\ shield) + PV(Remaining\ years\ tax\ shields) \tag{42}$$

$$V_{TS_0} = \frac{K_D D_{L,t=1} T_c}{1+K_D} + \frac{K_D D_{L,t=2} T_c}{(1+K_D)(1+K_U)^{2-1}} + \frac{K_D D_{L,t=3} T_c}{(1+K_D)(1+K_U)^{3-1}} + \cdots + \frac{K_D D_{L,t=n} T_c}{(1+K_D)(1+K_U)^{n-1}} \tag{42.1}$$

$$V_{TS_0} = \frac{1}{(1+K_D)} \sum_{t=1}^{n} K_D D_{L,t=1} T_c + \frac{K_D D_{L,t=2} T_c}{(1+K_U)^{2-1}} + \frac{K_D D_{L,t=3} T_c}{(1+K_U)^{3-1}} + \cdots + \frac{K_D D_{L,t=n} T_c}{(1+K_U)^{n-1}} \tag{42.2}$$

From these equations, one can easily show that:

$$V_{TS_0} = PV\ of\ TS@K_U . \left(\frac{1+K_U}{1+K_D}\right) \tag{42.3}$$

When the firm has no constant debt-to-equity ratio policy, the MM-APV model is more adequate.

Lewellen and Emery (1986), Arzac and Glosten (2005) – see formula 13 – and Cooper and Nyborg (2006) corroborated Miles and Ezzell findings. Inselbag and Kaufold (1997), say that if the company targets a specific amount of money as debt outstanding, the $V_{TS}$

is given by Myers (1974) equation, but if the company targets a constant debt/value ratio, the $V_{TS}$ is given by the Miles and Ezzell (1980) equation.

### 2.4.4. Harris and Pringle

Harris and Pringle (1985) calculate the present value of tax shields by discounting the interest tax shields ($K_D$.T.D) at the unlevered required return $K_u$ (in the original text denoted as $k_0$[11]). The reasoning behind this is that these tax savings have the same systematic risk as the firm's underlying cash flows and, consequently, should be discounted at the unlevered required return ($K_u$):

$$V_{TS} = \sum_{t=1}^{\infty} \frac{K_D D_t T_c}{(1 + K_U)^t} \tag{43}$$

Harris and Pringle (1985: 242) say that "The MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves, because it ignores the costs of financial distress, and because it implies that the firm should use very large amounts of debt. The Miller position is too extreme because it implies that debt cannot benefit the firm at all." For this reason, Harris and Pringle adopt a solomonic position saying that "the truth about the value of tax shields lies somewhere between the MM and Miller positions" and conclude that both HP or ME reach results for required return to assets between those of MM and Miller. The advantage of the HP approach is that it is simpler and more intuitive than the ME approach, in HR opinion.

### 2.4.5. Damodaran

Damodaran (2006) says that the expected tax benefits from issuing debt is a function of the tax rate applied to it and has to be discounted at a rate that reflects the riskiness of the debt cash flow. Therefore, "the appropriate discount rate for this cash flow is the interest rate on the debt because it reflects the riskiness of the debt" ($K_D$) (Damodaran, 1994: 360).

### 2.4.6. Fernandez

Fernandez (2007: 5) developed a valuation formula for companies that maintain a fixed book-value leverage ratio, because he considers that this assumption is more realistic than

[11] In the original article, the HP approach to calculate the unlevered required return is: $(k_0 = w_e k_e + w_d k_d)$. Where "$w_e$,$w_d$ = proportions of financing done by equity and debt", and "$k_e$,$k_d$ = market costs (required rates of return) of equity and debt".

the assumptions of MM and ME. Indeed, he argues that "when managers have a target capital structure, it is usually in book-value terms (as opposed to market-value terms)", which is partially explained by the fact that "this is what credit agencies pay attention to". According to Fernandez (2004), the $V_{TS}$ is the present value of $D.T.K_U$ discounted at the unlevered cost of equity ($K_U$). This results from considering the value of tax shields as the difference between two present values: the value of the levered firm, with the tax savings, and the value of the same firm without leverage.

Damodaran (2006) seems to agree with Cooper and Nyborg (2006) when they argue that Fernandez is wrong and that the value of tax savings is the present value of the interest tax savings, discounted at the cost of debt.

### 2.4.7. Valuation equations according to the main theories

Market value of the debt = Nominal value.

#### *2.4.7.1. The WACC equations according to the main theories*

The formula of the WACC generally presented in Corporate Finance textbooks, like the one from Berk and DeMarzo (2011), is:

$$WACC = \frac{E}{E+D}.K_E + \frac{D}{E+D}.K_D.(1-T_C) \tag{44}$$

One can also rewrite this formula in order to highlight the interest tax shields, to which we will give special emphasis in this paper, in this way:

$$WACC = \underbrace{\frac{E}{E+D}.K_E + \frac{D}{E+D}.K_D}_{\text{Pretax WACC}} \underbrace{- \frac{D}{E+D}.K_D.T_C}_{\text{Reduction due to interest tax shield}} \tag{44.1}$$

There are several different formulas to calculate the WACC, fundamentally due to the different discount rates applied to tax shields and to the debt policy of the company. The value of tax shields determines the increase in the firm's value as a result of the tax savings realized by the payment of interests. However, up to date, there is still no consensus in the literature regarding the correct way of computing the value of tax shields. As previously mentioned, MM (1963), Myers (1974), Allen, Brealey and Myers (2007), and Damodaran (2005) suggest to discount the interest tax shields at cost of debt ($K_D$), whereas Harris and Pringle (1985) and Ruback (2002) propose discounting these interest

tax shields at the unlevered cost of capital ($K_U$). Miles and Ezzell (1985) suggest discounting the tax shields the first year at $K_D$ and the following years at $K_U$.

From these different approaches result different related formulas for the WACC, $K_E$, $V_{TS}$, and $\beta_L$.

| | **Modigliani-Miller (1963)** | **MM+Inselbad and Kaufold (1997)** |
|---|---|---|
| $\boldsymbol{K_E}$ | $K_E = K_U + (K_U - K_D).(1 - T_C).\frac{D}{E}$ (45) | $K_E = K_U + (K_U - K_D).\frac{D - V_{TS}}{E}$ (49) |
| $\boldsymbol{\beta_L}$ | $\beta_L = \beta_U + (\beta_U - \beta_D).(1 - T_C).\frac{D}{E}$ (46) | $\beta_L = B_U + (\beta_U - \beta_D).\frac{D - V_{TS}}{E}$ (50) |
| $\boldsymbol{WACC}$ | $WACC = K_U.\left(1 - T_C.\frac{D}{E + D}\right)$ (47) | $WACC = K_U.\left(1 - \frac{V_{TS}}{E + D}\right)$ (51) |
| $\boldsymbol{V_{TS}}$ | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_D.D.T_C}{(1 + K_D)} = \frac{K_D.D.T_C}{K_D} = T_C.D$ (48) | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_D.D_t.T_C}{(1 + K_D)^t}$ (52) |

| | **Myers (1974)** | **Miles-Ezzell (1980)** |
|---|---|---|
| $\boldsymbol{K_E}$ | $K_E = K_U + \frac{V_U.E}{E}.(K_U - K_D)$ (53) | $K_E = K_U + \left(K_U - K_D.\left(1 - T_C\frac{K_U - K_D}{1 + K_D}\right)\right).\frac{D}{E}$ (57) |
| $\boldsymbol{\beta_L}$ | $\beta_L = \beta_U + \frac{V_U - E}{E}.(\beta_U - \beta_D)$ (54) | $\beta_L = \beta_U + \left(\beta_U - \beta_D.\left(1 - T_C\frac{\beta_U - K_D}{1 + K_D}\right)\right).\frac{D}{E}$ (58) |
| $\boldsymbol{WACC}$ | $K_U - \frac{V_{TS}.(K_U - K_D) + D.T_c.K_D}{E + D}$ (55) | $WACC = K_U - \frac{D.T_c.K_D}{E + D}.\frac{1 + K_U}{1 + K_D}$ (59) |
| $\boldsymbol{V_{TS}}$ | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_D.D_t.T_C}{(1 + K_D)^t}$ (56) | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_D.D.T_C}{(1 + K_{TS_t}{}^{12})^t}$ (60) |

| | **Harris-Pringle (1985)** | **Damodaran (1994)** |
|---|---|---|
| $\boldsymbol{K_E}$ | $K_E = K_U + (K_U - K_D).\frac{D}{E}$ (61) | $K_E = K_U + \frac{D.(1 - T_C)}{E}.(K_U - K_F)$ (65) |
| $\boldsymbol{\beta_L}$ | $\beta_L = \beta_U + (\beta_U - \beta_D).\frac{D}{E}$ (62) | $\beta_L = \beta_U + \frac{D.(1 - T_C)}{E}.\beta_U$ (66) |
| $\boldsymbol{WACC}$ | $WACC = K_U - \frac{D.T_c.K_D}{E + D}$ (63) | $WACC = K_U.\left(1 - \frac{D.T_C}{E + D}\right) + D.\frac{(K_D - K_F).(1 - T_C)}{E + D}$ (67) |

[12] $K_{TS} = K_U + (K_D - K_U).\frac{T_C.K_D.\frac{D}{E+D}}{1+K_D}.\frac{(E+D)_t}{V_{TS_t}}$

| | | | | |
|---|---|---|---|---|
| $V_{TS}$ | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_D . D_t . T_C}{(1+K_U)^t}$ | (64) | $V_{TS}$[13] $= \sum_{t=1}^{\infty} \frac{K_U . D_t . T_C - D_t . (K_D - K_F) . (1-T_C)}{(1+K_U)^t}$ | (68) |

| | **Fernandez (2004)** | |
|---|---|---|
| $K_E$ | $K_E = K_U + \frac{D.(1-T_C)}{E} . (K_U - K_D)$ | (69) |
| $\beta_L$ | $\beta_L = \beta_U + \frac{D.(1-T_C)}{E} . (\beta_U - \beta_D)$ | (70) |
| $WACC$ | $WACC = K_U . \left(1 - \frac{T_C . D}{E+D}\right)$ | (71) |
| $V_{TS}$ | $V_{TS} = \sum_{t=1}^{\infty} \frac{K_U . D_t . T_C}{(1+K_U)^t}$ | (72) |

### 2.5. TAX SHIELDS: A SOURCE OF CONTROVERSY

A company only has the right to earn tax savings when interests are deducted in the income statement and not when the interests are paid. It will only receive tax shields when it pays its taxes. The interest tax shields reduce the amount of taxes paid. Let's see what happens when losses carried forward are allowed, which is the case in Portugal. When EBIT is zero or negative, the tax shields are apparently lost because taxes are zero. Though, when losses carried forward are allowed, the tax shields corresponding to the year when EBIT was negative can be recovered when losses from previous years are carried forward to a future year where the company has enough EBT (earnings before taxes) to offset previous losses.

Vélez-Pareja (2010: 219-220) says that the interest tax shields have three sources of risk: risk of default in debt, market cost of debt risk, and operational or realization risk of tax shields:

(i) Risk of default in debt: this risk exists when there is a possibility that the company had not enough cash to pay interests and/or principal. If the default is such that the company cannot pay the taxes, it will not earn the tax shields.

(ii) Market cost of debt risk: the market rate ($K_D$) can change and is a source of risk for tax shields. Though, what generates the tax saving is not the market rate, but

[13] This formula results from Fernandez interpretation of Damodaran's leverage beta: "Although Damodaran does not mention what the value of tax shields should be, … relating the leverage beta to the asset beta implies that the value of tax shields is: … VTS = PV[$K_U$; D T $K_U$ – D ($K_d$ – $R_F$)(1 – T)]"

the contractual rate. Therefore, the main issue is the variability of the contractual rate used to pay interests.

(iii) Operational or realization risk of tax shields: tax shields are completely dependent on the EBIT. EBIT has to be positive for the company to realize tax shields, and to realize the total amount of tax shields, EBIT has to be higher than interest charges.

As said in the literature review, MM (1963), HP (1985), and ME (1980), the state of the art suggests that there is some unanimity in the following hypothesis:

(a) If the level of debt (D) is expected to be constant ($D_t = D$), then the tax shields should be discounted at cost of debt ($K_D$[14]).

(b) If the leverage ratio (D/E) is expected to be constant, then the tax shields should be discounted at the unleveraged cost of equity ($K_U$).

"Unfortunately, finance theory doesn't tell us unequivocally that this is the correct rate to use. Some would argue that the tax benefit of interest expense should be discounted at the cost of debt or some rate between the cost of debt and the unlevered cost of equity." (Copeland, Koller and Murrin, 2000, p 477), i.e.:

$$K_D < K_{TS} < K_U$$

As said previously, when we defined the cash flows, the tax shields flow entirely to equity. Therefore, we can break down the market value of equity into two different components: the market value of equity without the market value of interest tax shields and the market value of tax shields:

$$E = (E - V_{TS}) + V_{TS} \tag{73}$$

And

$$V = V_U + V_{TS} = E + D = (E - V_{TS}) + V_{TS} + D \tag{74}$$

$$K_U V_U + K_{TS} V_{TS} = K_E E + K_D D \leftrightarrow K_U V_U + K_{TS} V_{TS} = K_{E-V_{TS}}(E - V_{TS}) + K_{TS} V_{TS} + K_D D \tag{75}$$

It can be illustrated graphically in the following way:

[14] In this case, however, the free cash flow discounted at the textbook WACC and adjusted present value leads to different valuations.

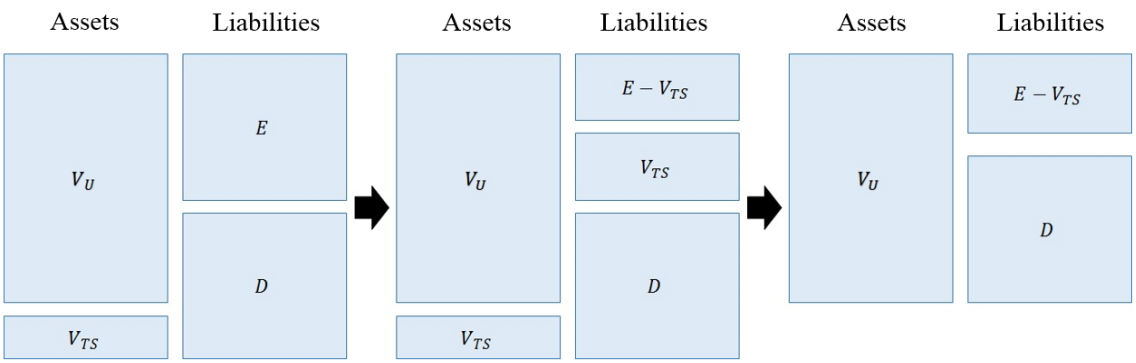


*Figure 1: Graphical representation of equation 75*

The challenge is to find the formula to calculate $K_{E-V_{TS}}$ and $K_{TS}$. The discount rate $K_{E-V_{TS}}$ can be considered as the rate of return that shareholders require if they would not benefit from the interest tax shields.

$$K_U V_U = K_{E-V_{TS}}(E - V_{TS}) + K_D D \tag{76}$$

Since $V_U = E + D - V_{TS}$:

$$K_{E-V_{TS}} = K_U + (K_U - K_D)\frac{D}{E - V_{TS}} \tag{77}$$

Taking into consideration the illustration below:

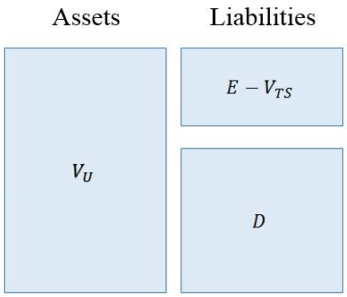


*Figure 2: Part of Figure 1: Graphical representation of equation 75*

and the portfolio theory that claims that the return of any asset is the weighted average of its constituting elements' returns, the cost of leverage equity $K_E$ can now also be calculated in the following way:

$$K_E = K_{E-V_{TS}}\frac{E - V_{TS}}{E} + K_{TS}\frac{V_{TS}}{E} \tag{78}$$

To reach to the general formula of $K_{TS}$, we have to *endogenize* $K_D$into the model. The interest rate required ($K_D$) by debt holders increases proportionally to the size of the company and the financial risk of default. Hence, increases with the leverage ratio. Thus, $K_D$will be equal to the risk-free rate increased by a debt risk premium. For $K_D$, we will adopt the following formula:

$$K_D = K_F + Debt\ risk\ premium \tag{79}$$

$$K_D = K_F + (K_U - K_F)\left(\frac{D}{V_U}\right) \tag{79.1}$$

Following this path, Ansay (2010: 57-58) proposes that "the market value discount rate for tax shields $K_{TS}$ has to be *modelized* as:

$$K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V}," \tag{80}$$

Where "$K_{TS}$is a function of the leverage ratio D/V, whose initial value is the cost of debt $K_D$ and which tends towards the theoretical market value discount rate $K_{E-V_{TS}}$ – which is the return shareholders would require assuming they do not benefit from the debt tax shields – as the leverage ratio D/V tends to be one."

Graphically, the equation can be illustrated as follows:

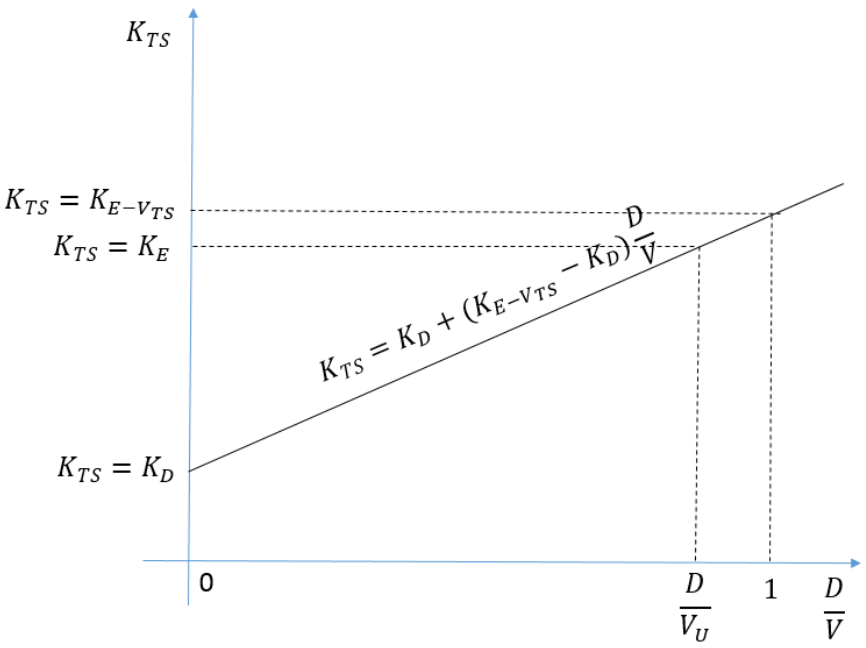


*Figure 3: Graphical representation of equation 80*

A further explanation of the previous expression should be made. The tax shields have at least the risk of the debt. Since the tax shields and the interest expenses increase with the

level of debt, we can state that the minimum initial risk for the tax shields is the same of the debt. Thus, when the debt is equal to zero or when D/V equals zero, then $K_{TS} = K_D$. This way we have the intercept on the axis of $K_{TS}$. We will try to obtain the remaining necessary information to complete the equation previously referred, through the relation between $K_E$ and $K_{E-V_{TS}}$. As we know, $K_{E-V_{TS}}$ is always higher than $K_E$. Consequently, whichever is period t and for whichever positive level of debt for period t, we have:

$$K_{E-V_{TS}} = K_U + (K_U - K_D)\frac{D}{E - V_{TS}} > K_E \qquad (77.1)$$

$$= K_U + (K_U - K_D)\frac{D}{E} - (K_U - K_{TS})\frac{V_{TS}}{E}$$

Assuming that $0 < D < V$ or $0 < D/V < 1$, we can isolate the $K_{TS}$:

$$\Leftrightarrow K_{TS} < \frac{E}{V_{TS}}(K_U + (K_U - K_D)\frac{D}{E - V_{TS}} - K_U - (K_U - K_D)\frac{D}{E} + K_U\frac{V_{TS}}{E} \qquad (77.2)$$

Developing the right side of this inequality, we conclude that the right side is equal to $K_{E-V_{TS}}$ and that, if D/V is higher than zero and lower than 1, $K_{TS} < K_{E-V_{TS}}$. Consequently, if D/V higher than 1, $K_{TS} > K_{E-V_{TS}}$ (note that, in this case, D would be higher than V, a hypothesis which should only be made theoretically[15]).

If D = V, i.e. E = 0 (D/V = 1), since $K_{E-V_{TS}}$ is defined in E = 0 and that $K_{TS}$ is a continuous function, then, when E = 0, $K_{TS} = K_{E-V_{TS}}$.

Finally, from the equation:

$$K_E = K_{E-V_{TS}}\frac{E - V_{TS}}{E} + K_{TS}\frac{V_{TS}}{E} \qquad (78)$$

When $D/V_U = 1$, which implies $V_{TS}/E = 1$ or alternatively $(E - V_{TS}) = 0$,

$$K_E = K_{TS} \qquad (78.1)$$

We, therefore, defined all geometric locus, with which we made the above graphic, and defined the general equation for $K_{TS}$.

### 2.6. MULTIPLES METHODS

A multiple is applied to a specific financial metric of a company to calculate the business' valuation or assess its reasonability. In a summarized way, one can say that if the multiple

[15] In the numerical application: $D = MIN(D_{Book}, V)$

is applied to a pre-debt number the resulting valuation is the estimated enterprise value, and if the multiple is applied to an after debt number the resulting valuation is the estimated equity value. Thus, there are two basic types of multiples: enterprise multiples and equity multiples (Suozzo, Cooper, Sutherland and Deng, 2001).

### 2.6.1. Relative valuation

The usual approach is to compare the current multiple to a historical multiple or, as an alternative approach, one can compare current multiples to multiples of other companies, a sector or a market, and compare the current spread between them to a historical spread (Suozzo, Cooper, Sutherland and Deng, 2001).

### 2.6.2. The most used enterprise value multiples

In these multiples the denominator characterises the flow to all claimants on enterprise cash flow (Suozzo, Cooper, Sutherland and Deng, 2001).

#### *2.6.2.1.EV/Sales*

Definition: Core Enterprise Value[16]/Sales.

Formula:

$$\frac{EV}{Sales} = \frac{ROIC - g}{ROIC \times (WACC - g)} \times (1 - T_C) \times EBIT\ Margin\ as\ a\ \%\ of\ sales \tag{81}$$

#### *2.6.2.2.EV/EBITDA*

Definition: Core EV/earnings before associates, interest, tax, depreciation, amortization, non-cash changes in provisions, and before reported exceptional items.

Formula:

$$\frac{EV}{EBITDA} = \frac{(ROIC - g)}{ROIC\ \times (WACC - g)} \times (1 - T_C) \times (1 - Depreciation\ as\ a\ \%\ of\ EBITDA) \tag{82}$$

#### *2.6.2.3.The most used equity multiples*

An equity multiple is the expression of the market value of equity holders' stake in a firm, relative to a key statistic relating to that value. An equity multiple is the one that represents residual profit, cash flow, assets or another residual measure.

[16] Total enterprise value less the value of non-core assets.

#### *2.6.2.4.Price/Earnings*

Definition: current market capitalization/net income attributable to common shareholders or alternatively, price per share/attributable earnings per share.

Formula:

$$\frac{Market\ cap}{Net\ income} = \frac{Stock\ price}{Earning\ per\ share} = \frac{ROE - g}{ROE \times (K_E - g)} \tag{83}$$

#### *2.6.2.5.Price/Book value*

Definition: market capitalization/book value (alternatively price per share/book value per share).

Formula:

$$\frac{Market\ cap}{Book\ value} = \frac{Price\ per\ share}{Book\ value\ per\ share} = \frac{ROE - g}{K_E - g} \tag{84}$$

#### *2.6.2.6.Price/Earnings growth*

Definition: the PEG ratio is the potential P/E divided by the average forecast earnings growth. This ratio is based on the assumption that the P/E ratio is positively linearly correlated to the expected growth rate in earnings, in other words, PEG is constant.

Formula:

$$\frac{P/_E}{Growth} = \frac{ROE - g}{100 \times g \times ROE \times (K_E - g)} \tag{85}$$

#### *2.6.2.7.Dividend Yield*

Definition: forecast dividend/current market capitalization. Dividend yield can be compared to the market's required yield to determine how a stock should be priced. The "dividend multiple" is 1/market's required dividend yield.

Formula:

$$Dividend\ yield = \frac{Dividends\ per\ share}{Market\ value\ per\ share} \tag{86}$$

#### *2.6.2.8.Some remark on how to use the multiples methods*

According to Goedhart, Koller and Wessels (2005: 8), there are four principles that help companies use multiples appropriately: "the use of peers with similar ROIC and growth

projections, of forward-looking multiples, and of enterprise-value multiples, as well as the adjustment of enterprise-value multiples for non-operating items."

Regarding the enterprise-value multiples, the P/E multiples have two major flaws: one is that they are systematically affected by capital structure, the other is that the P/E ratio is based on earnings, which comprise many non-operating items.

An alternative to the P/E ratio is the enterprise value to EBITDA. This ratio is less vulnerable to manipulation by changes in capital structure. Only when the change in capital structure lowers the cost of capital, it will lead to a higher multiple.

Concerning the enterprise-value-to-EBITDA multiples, these must be adjusted for non-operating items hidden within the enterprise value and EBITDA, which have to be adjusted for these non-operating items, such as excess cash, operating leases, employee stock options, and pensions.

The PEG ratios allow the expected level of growth to vary across companies. However, these ratios have disadvantages that conduct to errors in valuation, since there is no standard time frame for measuring expected growth, and because these ratios assume a linear relation between multiples and growth, in a way that no growth implies zero value.

The multiples methods only give a relative valuation of the company, meaning that they just measure a company's valuation compared to another's. For Goedhart, Koller and Wessels (2005), the discounted cash flow analysis delivers more accurate results, though the multiples analysis has also merit if used thoughtfully. Moreover, the multiples analysis can be used to stress-test a company's cash flow forecasts, to understand discrepancies between its performance and that of its competitors, and to help determine if the company is strategically positioned to create more value than its competitors. For Mauboussin (2006: 2), "Multiples are not valuation; they represent shortland for the valuation process. Like most forms of shortland, multiples come with blind spots and biases that few investors take the time and care to understand."

## 2.7. OTHER VALUATION PARAMETERS

### 2.7.1. Estimating the leverage beta and the cost of equity

We can break down the total risk of financial security into market risk (or systematic risk) and specific risk (or diversifiable risk). These two risks are independent.

The market risk is dependent on its beta coefficient, which measures the correlation between the return on security and market return. Mathematically, the security $\beta_i$ can be calculated as follows:

$$\beta_i = \frac{Cov(K_i, K_M)}{\sigma^2(K_M)} \quad (87)$$

Where:

- $K_i$ is the company's stock returns; and
- $K_M$ is the market's returns.

According with Vernimmen, Quiry and others (2009: 414), "the β coefficient depends on:

- the sensitivity of the company's business sector;
- the economic situation;
- the company's operating costs structure (the higher the fixed costs, the higher the β);
- the financial structure (the greater the group's debts, the higher the β);
- the quality and quantity of information provided to the market (the greater visibility there is over future results, the lower the β); and
- earnings growth rates (the higher the growth rate, the higher the β)."

The firm cost of equity capital represents the return demanded by investors to apply their money on a company's equity, reflecting the company's risk. The most widely used asset pricing model is the CAPM (Goedhart et al. 2005). In the CAPM framework, two of the required parameters are common to all companies considered as part of the market: the risk-free rate ($K_F$) and the market risk premium ($K_M - K_F$). Consequently, $K_E$ can be estimated under the CAPM as follows:

$$K_E = K_F + \beta_i \times (K_M - K_F) \quad (88)$$

Where:

- $K_F$ is the risk-free rate;
- $K_M$ - $K_F$ is the market-risk premium; and
- $\beta_i$ is the company's beta.

### 2.7.2. Estimating the risk free rate and the market risk premium

As sovereign governments can raise taxes to pay the debt it incurs, its debt is virtually free of risk of default. Thus, the bonds issued by sovereign governments (Treasury bills or T-bills) have what we call risk-free return over a short time (one year or less).

Therefore, in order to estimate the risk-free rate, government's default-free bonds have to be looked at. However, sometimes Treasury bonds with longer maturity life are used as risk-free return rate.

Ideally, cash flows should be discounted at a rate that used a risk-free rate based on a government bond of identical maturity. Though, due to practicality reasons, since in company valuation the time frame is infinite, long term government bonds rates are used, as risk-free return (Goedhart et al. 2005; Carabias and Fernández, 2006).

A market risk premium measures the extra return that investors demand for shifting their money from a risk-free investment to an average risk investment. According with Damodaran (1999), there are two ways to estimate the market risk premium:

(i) Look at past and estimate the difference between the investors' returns on stocks and the investors' return on government bonds. To the obtained spread, we call "Historical Premium" and should only be used in mature markets with enough historical data; or

(ii) By using the premium extracted by looking at how markets price risky assets today. This is called the "implied premium". The implied return can be obtained through the following formula, solving it in order to the required return on equity and estimating the remaining variables from publicly available information, i.e.:

$$Value = \frac{Expected\ dividends\ next\ period}{(Required\ return\ on\ equity - Expected\ growth\ rate)} \quad (89)$$

### 2.7.3. Estimating the cost of debt and debt's beta

According with Damodaran (2002), the cost of debt is dependent on three factors: the risk-free rate, the default risk of the firm, and the tax benefit from debt. As the risk-free rate and the tax benefits from debt were already addressed previously, we will focus on estimating the default risk.

There are three alternatives to estimate the default risk of a company (Damoradan 2002):

(i) If a firm is financed through the bond market, it is possible to calculate the yield to maturity, which can be used as cost of debt.

(ii) If a firm is financed by long term loans and there are not significant changes in the market and the company itself, the historical spreads can be incorporated in the cost of debt, i.e.:

$$K_D = K_D \; associated \; with \; recent \; long \; term \; financing \tag{90}$$

(iii) Cost of debt can be estimated by taking the ratios used by the rating agencies and add to the risk-free rate the spread that is coupled with the assigned rating, i.e.:

$$K_D = K_F + Rating \; spread \tag{91}$$

The risk of debt capital is measured by the beta of debt which is calculated by regressing market returns on debt returns. Debt betas are positively correlated with credit ratings and in the long-term have been in the range between 0.30 and 0.40 (Skardziukas, 2010).

According with Benninga (2008, p.737), through the CAPM's security market line (SML), if we know the cost of debt ($K_D$), the risk-free rate ($K_F$) and the expected rate of return on the market ($K_M$), we can compute the beta of debt as follows:

$$K_D = K_F + \beta_D(K_M - K_F) \tag{92}$$

If we use the tax-adjusted version of security market line, then the bond SML becomes:

$$K_D = K_F + \beta_D(K_M - K_F(1 - T_C)) \tag{93}$$

### 2.7.4. Non-equity claims and other non-operating assets

A DCF valuation gives the value of the entire business, therefore, to know the value of the equity, the value of non-equity claims – such as debt, unfunded pension liabilities or minority interests – need to be subtracted from the value of the whole business.

The debt's market value (or book value, if equivalent) is directly subtracted from the enterprise value (V).

Pensions and other postretirement liabilities, if they are designed on a contribution basis, have no valuation effects. If they are designed on a benefit basis, they should be either added to the enterprise value if it results in a surplus or subtracted from the enterprise value if it results in a deficit (Damodaran 2002, Goedhart et al. 2005).

Regarding the minority interests, the best practice determines that their value can be estimated either by using their share price – if listed, or by performing a DCF valuation – if not listed.

The Free Cash Flows should only include the cash flow generated by the operating assets, which excludes therefore excess cash and marketable securities. These must be valuated separately and added to the enterprise value to determine the equity value.

### 2.7.5. Terminal value or continuing value estimation

The formulas of the terminal value were presented in the chapter "Main valuation methods". Thus, we will focus on the growth in terminal value. There are two main approaches: the sustainable advantage approach and the convergence approach.

The sustainable advantage approach considers that the firm keeps creating incremental value beyond the explicit forecast period (Ansay, 2010). The terminal value of a firm assumes that from the explicit forecast period onwards the firm will grow perpetually at a given stable rate (g). Unfortunately, long term growth rates are often hard to forecast. If a company is already in a steady state (i.e. constant return on equity and capital), the growth rate of the company can be inferred from the growth rate of the market to which the company belongs to, since in perpetuity no company can be expected to grow at a faster rate than that of the economy.

In the convergence approach, beyond the explicit forecast period, it is assumed that ROIC = WACC or equivalently ROE = $K_E$ (Ansay, 2010). In this case, no incremental value is created and g = zero. This assumes that, at the end of the explicit forecast period, the firm earns an economic return on future investments identical to the cost of capital. In other words, no value is added from new investments.

If opting for the first approach, the growth rate to be determined depends on the valuation model. If d is the percentage of profits kept in the firm after distribution of dividends, when focusing on the valuation of the firm (WACC, MVA or APV), $g \approx d \times ROIC$; when focusing on the financing side of the firm (ECF), $g \approx d \times ROE$.

For all these methods, the terminal value – since it is a perpetuity – assumes a fixed market value leverage ratio and, consequently, a fixed market value discount rate. Note that the terminal value is a significant part of the total market value of the firm.

## 2.8. COUNTRY RISK

The country risk appears to be systematic and non-diversifiable even in a global portfolio, thus making evident the cross-market correlation. The country premium reflects the extra

risk that equity holders take in a specific market, taking as reference the less risky country (usually US Treasury bonds or German Treasury bonds).

According to Damodaran (1999) to estimate the country risk premium one must:

(i) Measure the country risk;
(ii) Covert the country risk measure into a country risk premium; and
(iii) Evaluate how individual companies in that country are exposed to country risk.

Regarding how to measure country risk, one can simply look at the rating assigned to a country's debt by a rating agency, which measure the risk of default that take into consideration many factors that drive equity risk (such as country's currency, budget and trade balance, and political stability). Moreover, ratings' advantage lies on the fact that they come with default spreads over the US Treasury bond. One critic that can be made to the use of ratings to measure country risk is that rating agencies often cannot keep up when it comes to responding to changes in the underlying default risk. As an alternative, there are numerical country risk scores. Indeed, for example, The Economist has a score that ranges from 0 to 100, corresponding 0 with no risk and 100 with most risky.

Concerning the estimation of the country risk premium, one must look at the volatility of the equity market in a country relative to the volatility of the country bond that is used to estimate the spread. Thus, the country equity risk premium can be written:

$$Country\ Equity\ Risk\ Premium = Country\ Default\ Spread\ \left(\frac{\sigma Equity}{\sigma Country\ Bond}\right) \quad (94)$$

About the estimation of asset exposure to country risk premiums, there are three alternatives to address this issue:

a) Assuming that all companies in a country are equally exposed to country risk, i.e.:

$$Expected\ Cost\ of\ Equity = K_F + \beta \times MP + CRP \quad (95)$$

b) Assuming that a company's exposure to country risk is proportional to its exposure to all other market risk, which is measured by the beta, i.e.:

$$Expected\ Cost\ of\ Equity = K_F + \beta \times (MP + CRP) \quad (96)$$

c) Allow each company to have an exposure to country risk that is different from its exposure to all other market risk, i.e.:

$$Expected\ Cost\ of\ Equity = K_F + \beta \times Market\ Equity\ Risk\ Premium + \lambda \times CRP \quad (97)$$

# 3. CASE STUDY ANALYSIS

## 3.1. PORTUGUESE MACROECONOMIC ENVIRONMENT AND DEMOGRAPHIC PROFILE

*Instituto Nacional de Estatística (2013) de Portugal and European Union Commission (2013)*

### 3.1.1. Macroeconomic environment

Portugal exited the Economic Adjustment Program on May 17th 2014. In 2011, Portugal had been feeling increasing pressures from the financial markets, which raised concerns regarding the sustainability of its public finances due to a sharp increase of its sovereign spreads. Moreover, the consecutive downgrading by credit rating agencies of the Portuguese sovereign bonds caused difficulties for Portugal to refinance itself at rates compatible with its long-term fiscal sustainability. In addition, the Portuguese banking sector, which was greatly dependent on external financing, also experienced difficulties to borrow money from the international market fund and, consequently, became dependent on the Eurosystem for funding (European Union Commission, 2013).

On the 7th of April 2011, the Portuguese Government requested financial assistance from the International Monetary Fund (IMF), the European Commission (EC), and the European Central Bank (ECB) – collectively referred as "Troika". On May 2011, an Economic Adjustment Program was negotiated and, on the 17th of May 2011, the Portuguese Government and Troika's official authorities formally signed the Memorandum of Understanding (MoU) and the Loan Agreement. The Economic Adjustment Program for Portugal included a joint financing package of €78 billion from EU/EFSM, Euro Area and IMF, and the commitment of the Portuguese Government to implement a series of structural reforms to promote economic growth, the reduction of public debt and deficit, as well as to ensure the stability of the financial sector during the 2011-2014 period (European Union Commission, 2013).

The economic recession affected all sectors of activity, mainly the ones that were dependent on domestic demand. In 2012, Portuguese GDP decreased 3.2% and there was

a 6.9% decrease of the domestic demand. The private consumption diminished 5.4% due to a permanent decrease of available income, to a deterioration of the labour market, and to a feeling of uncertainty regarding the future fiscal measures (Instituto Nacional de Estatística, 2013).

In 2012, Portuguese GDP was €165.4 billion and he GDP *per capita* was €15.702. The Portuguese GDP *per capita* on a purchasing power parity basis represented only 72% (€19.169) of the EU average (€25.569) (Instituto Nacional de Estatística, 2013).

For the first time in decades, in 2012, there was a rebalance of the country's external accounts registering a positive current and capital balances of 0.8% of GDP. The predictions for 2013 were that GDP would contract 1.5%, which is explained by a reduction of economic activity in Portugal since 2011. These figures can be understood by the low inflation rate, both internally and externally, by a high unemployment rate, and a reduction of salaries (Instituto Nacional de Estatística, 2013).

In 2014, it was expected for Portugal to grow. According to the IMF, there were already positive signs in this direction in 2013. In the second semester of 2013, the real GDP grew for the first time since 2010 and the unemployment rate decreased. It was expected that most of the Portuguese macroeconomic indicators would improve: i.e. decrease of the unemployment rate, production was expected to remain stable or slightly increase, and so on (Instituto Nacional de Estatística, 2013).

The IMF predicted an average global inflation of 0.6% in 2014, which reflected a persistent weak domestic demand (Instituto Nacional de Estatística, 2013).

### 3.1.2. Demographic profile

By the end of the year 2012, the population residing in Portugal was estimated to be of 10.49 million of inhabitants, which represents a negative growth rate of 0.52% compared with 2011, due to a negative natural growth rate (the difference between total number of births and deaths) and a negative migration growth rate (Instituto Nacional de Estatística, 2013).

Following the general trend in EU, the Portuguese demographic aging process persists over the years, which has changed the profile of the Portuguese age pyramid in the recent years. There is a narrowing of the base of the pyramid caused by an ongoing reduction of the birth rate, an increase of the life expectancy, and an increase of the emigration flow.

It is estimated that this trend will continue during the following years (Instituto Nacional de Estatística, 2013).

Between the years of 2007 and 2012, the young population (with ages between 0 and 14 years old) decreased relatively to the total of population residing in Portugal. In addition, in the same period, the working-age population (with ages between 15 and 65 years old) also decreased, in contrast there was an increase of the elderly population (more than 65 years old). In 2012, there were 131 elderly people for 100 young people (Instituto Nacional de Estatística, 2013).

Most of the Portuguese population resides along the coastline and 44% of the total of inhabitants are distributed in the metropolitan areas of Lisbon and Porto (Instituto Nacional de Estatística, 2013).

The distribution of the purchasing power follows the same trends as the geographical distribution of the Portuguese population, since the two mentioned metropolitan areas gather half the purchasing power in Portugal, representing 52% of the total national purchasing power, and Lisbon alone represents 35% (Instituto Nacional de Estatística, 2013).

## 3.2. HEALTHCARE INDUSTRY IN PORTUGAL OVERVIEW

*Based on the Espírito Santo Saúde – SGPS, S.A. Prospectus for the public offer of distribution and admission to trading of the Shares on the Euronext Lisbon regulated market managed by Euronext Lisbon – Sociedade Gestora de Mercados Regulamentados, S.A. made by Espírito Santo Investment Bank (2014)*

### 3.2.1. The Portuguese healthcare sector overview

In Portugal, there is the co-existence of three sub-systems (Espírito Santo Saúde – SGPS, S.A., 2014):

(i) National Healthcare Service;
(ii) The special public and private health plans specific to certain sectors of activity;
(iii) Voluntary private healthcare insurance plans.

Every citizen residing in Portugal has access to healthcare services provided by the National Healthcare Service. Around 40% of the population benefit additionally from the

special public and private healthcare plans financed by their employers or from private healthcare insurance plans (Espírito Santo Saúde – SGPS, S.A., 2014).

The Portuguese State, in this case represented by the Ministry of Health, is the sole responsible for the development of the healthcare policies and for the control and valuation of the implementation of such policies. Its main responsibility is to regulate, plan and manage the National Healthcare Service, as well as the regulation, audit and inspection of the private healthcare providers, even if they are not part of the National Healthcare Service (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.1.1. The National Healthcare Service*

The National Healthcare Service is a public service of universal medical care that is financed mainly through taxes. The Ministry of Health (or Department of Health) receives a budget from the Ministry of Finance that then is allocated to the different institutions composing the National Healthcare Service. The funding of hospitals is done based on global budgets, but there has been an increasing emphasis on the Homogeneous Diagnostic Groups – these are pre-established pricing structures for standardized health procedures for which fixed prices are charged. Measures such as the implementation of payments of fixed fees by the patient for the service provided have been made in order to share the costs associated with the National Healthcare Service and to raise public awareness on the costs of the healthcare service. The co-payment system has, in its basis, the objective of contention of the demand for public healthcare services. These user charges are applied in medical appointments, medical urgencies, and medical appointment to the domicile, diagnostic tests, and therapeutic procedures (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.1.2. Public and Private healthcare plans financed by the employer*

The public and private healthcare plans are mainly financed by contributions made by the employees and the employers (including the state, as a public sector employer) (Espírito Santo Saúde – SGPS, S.A., 2014).

The biggest healthcare sub-system, ADSE, is controlled by the Ministry of Finance. This sub-system has around 1.3 million of beneficiaries, including public workers, their families, and retired public workers. ADSE is financed by both public workers – 2.5% of their salary, and by the State – 1.25% of the salaries paid. Nowadays, two thirds of the ADSE expenses is financed by its beneficiaries and no transferences are directly made

from the State's Budget to this sub-system, but only by public entities. This plan allows the beneficiaries to choose among a variety of healthcare providers with which ADSE has agreements and has no defined maximum limits to the incurred expenses by each beneficiary. Currently, new public employees can choose to join or not the ADSE sub-system. However, the number of beneficiaries has remained stable (Espírito Santo Saúde – SGPS, S.A., 2014).

Note that, in 2014, the contribution plan for ADSE changed. The Ministry of Finance announced a possible increase in the contribution to 3.5%. This ADSE restructuring might lead the beneficiaries with higher salaries, which will see their contribution reach levels that might justify to seek for alternatives in the private healthcare insurance plans. If a great number of beneficiaries quit the ADSE healthcare plan, this could lead to the termination of the plan (Espírito Santo Saúde – SGPS, S.A., 2014).

The majority of the private sector healthcare sub-systems are financed by the employers and are usually associated with big enterprises that were privatized during the 1980s and 1990s. The contributions made by the employers to the private healthcare sub-systems can be included in the tax income calculation, as equivalent to the contributions to the social security system. The contributions vary accordingly with the salary. Nowadays, there has been a convergence between the private healthcare sub-systems to private healthcare insurance plans (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.1.3. Private healthcare insurance*

There has been an increasing number of private healthcare insurance plans and it has had a positive impact on the healthcare market growth. In the end of 2012, around 20% of Portuguese people had a private health insurance plan, half of which were financed by employers (Espírito Santo Saúde – SGPS, S.A., 2014).

There are only 5 main players in the insurance sector in Portugal: Caixa Seguros, Espírito Santo Financial Group, BCP, Santander, and Allianz. These companies represented alone 67% of the national market in 2012 (Espírito Santo Saúde – SGPS, S.A., 2014).

Insurance companies define the insurance premiums based on measured risk, namely: age, and health situation. The healthcare charges usually paid through reimbursement of expenses to the beneficiary. The health plans have limits regarding the expenses coverage by establishing ceilings or by demanding co-payment of expenses incurred (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.1.4. Spending's evolution on healthcare services in Portugal*

The healthcare spending in Portugal had been increasing consistently and in 2011 represented 10.2% of the GDP, which was above the OECD countries average. Although, the healthcare spending per capita was below the average of the OECD countries. Despite the continuous growth in healthcare expenses, the current expenditures decreased from 2011 to 2012, due to the austerity measures adopted by the government. Such measures included healthcare budget and salaries cuts (Espírito Santo Saúde – SGPS, S.A., 2014).

Regardless the decrease in current expenditure in 2012, the private current expenditure proved to be resistant to the economic crisis by registering an increase of around 2.4% in 2012 (Espírito Santo Saúde – SGPS, S.A., 2014).

### 3.2.2. Overview of the Portuguese healthcare market

The National Healthcare Service includes mainly in primary healthcare and hospital care, the former working as a service that should try to prevent and control the access to the latter. The private healthcare sector mainly provides pharmaceutical products, dental practice, diagnostic technology and private medical practice either outpatient or inpatient care (Espírito Santo Saúde – SGPS, S.A., 2014).

In the current Portuguese economic environment and budget restrictions, the State has been resorting to the private sector to reduce its necessity to allocate public resources in the healthcare sector, while maintaining the public healthcare assistance (Espírito Santo Saúde – SGPS, S.A., 2014).

The main private healthcare players are private hospitals and clinics, private practice doctors and the Misericórdias (mercies – non-profit organizations). Most medical specialty consultations are performed in the private sector, whereas most general practice medical consultations are performed in the public sector. In the primary healthcare centers, there are general practitioners doctors that guide the patients – if necessary – to further healthcare services provided by speciality doctors. Almost all public specialty appointments are performed in an outpatient basis in hospitals. The patients with minor health problems and higher income or private healthcare plans also have the alternative to resort to the private specialist practitioners for outpatient treatments (Espírito Santo Saúde – SGPS, S.A., 2014).

Additionally, the private sector has been increasing its presence due to conventions made with the National Healthcare Service to serve its beneficiaries. These agreements usually

refer to laboratorial tests and exams, such as diagnostic exams, radiology, kidney dialysis and physiotherapy (Espírito Santo Saúde – SGPS, S.A., 2014).

The secondary and tertiary public healthcare services are mainly provided by hospitals that can be classified as: central hospitals, which provide highly specialized healthcare with very advanced technology; specialized hospitals, which provide a variety of specialized healthcare services; district hospitals, which are located in the main administrative districts and provide a variety of specialized services (Espírito Santo Saúde – SGPS, S.A., 2014).

In 2011, the private healthcare sector provided 46% of the totality of the healthcare services provided in Portugal, which represented €5.5 billion. This also represented an increase when compared for example with 2007, when it only represented 40% of the total healthcare services provided in the country. Conversely with the public sector, the private sector has been increasing over the years (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.2.1. Competitive environment: the private players*

The private healthcare sector is very fragmented. The three biggest players in the market are Espirito Santo Saúde (ESS), José de Mello Saúde, and HPP – Hospitais Privados de Portugal (now called Lusíadas Saúde), which had together only 11,7% of the market in 2011. The market consists in a large number of private medical offices and in a large number of small diagnostic exams and treatment clinics. Most of these market players depend largely on the State and on the expenses directly supported by the patient (Espírito Santo Saúde – SGPS, S.A., 2014).

The three biggest players developed a model based on hospitals providing general healthcare services, then complementing it with small outpatient clinics. Unlike the large majority of the players in the market, these ones only depend a little on the State, due to the fact that they celebrated a large amount of agreements with insurance companies and healthcare sub-systems. They have been experiencing a significant increase in the last decade explained by their professional management, high quality medical personnel, and use of advanced technology (Espírito Santo Saúde – SGPS, S.A., 2014).

The current private market leader is ESS in terms of income when excluding public private partnerships (PPP), with a market share of 5% in 2011, and is the second biggest operator when including PPP (Espírito Santo Saúde – SGPS, S.A., 2014).

Several private operators manage public hospitals through public private partnerships. This business model has increasingly been adopted and has been contributing for the increasing of private enterprises' income (Espírito Santo Saúde – SGPS, S.A., 2014).

The current economic situation in Portugal results in a higher competitive environment between private players, forcing them to increase the quality of the services provided while decreasing prices charged for those same services, due to a decrease of family disposable income. Despite this situation, the private sector prospects of growth remains favourable, due to phenomena, such as increase of average life expectancy – aging population and increasing of chronical diseases associated with it, greater acceptance of the healthcare insurance and due to increasing difficulties from the public sector to respond with proper quality to the current demand (Espírito Santo Saúde – SGPS, S.A., 2014).

The existing economic crisis pushed private players out of the market, which is leading to an increasing market concentration. Those that are extremely dependent on the State have been experiencing difficulties to compete with bigger players or players that are not as dependent on the State. The tendency is for bigger companies to acquire smaller ones and therefore consolidate the private healthcare market (Espírito Santo Saúde – SGPS, S.A., 2014).

#### *3.2.2.2. Growth drivers of the Portuguese healthcare market*

##### 3.2.2.2.1. Aging population

The Portuguese population is rapidly aging. The percentage of people under 15 years old decreased from 16.3% in 2000 to 14.8% in 2012, while the percentage of people over 65 years old increased from 16.3% to 19.4% in the same period (Instituto Nacional de Estatística, 2013).

However, the Portuguese national institute of statistics (called INE) projected that in 2060, compared with numbers from 2008, the fertility rate will increase from 1.3% to 1.6%, the average age for the first born child go from 29.5 years old to 30.4 years old, and the average life expectancy will also grow from 75.4 to 82.3 years old for men and 82 to 87.9 for women. It is expected that the percentage of people under 15 years old, as well as the percentage of working age population, will keep decreasing and the percentage of people above 65 years old will keep increasing (Instituto Nacional de Estatística, 2013).

#### 3.2.2.2.2. Chronic diseases and other diseases related with lifestyle

The aging level and the increasing average life expectancy is leading to an increase on the impact of chronic diseases, which will lead to an increase of healthcare spending. Indeed, the economic growth, rapid urban development, lack of physical activity, lack of a healthy diet, among other factors related with a more sedentary life style, all cause the emergence of chronic diseases (Espírito Santo Saúde – SGPS, S.A., 2014).

Portugal has a high incidence of cerebrovascular diseases, being the highest among the OECD countries, 10% of the adults in Portugal have diabetes, which leads to greater risk of cardiovascular diseases, such as heart attacks and strokes (the two main causes of death in Portugal) (Espírito Santo Saúde – SGPS, S.A., 2014).

#### 3.2.2.2.3. Potential for improvements of healthcare infrastructures

In order to assess if a country has the appropriate medical infrastructures one has to look at the hospital bed availability, as well as medical and nurse staff availability (Espírito Santo Saúde – SGPS, S.A., 2014).

In Portugal there is a lack of bed availability in comparison with other European countries, which indicates a necessity to rebalance the demand to match the number of beds available by either creating new facilities or extending the existing ones (Espírito Santo Saúde – SGPS, S.A., 2014).

The number of doctors *per capita* has been increasing gradually and is one of the highest among the European countries and the OECD countries. In 2011, there were 4 doctors per 1000 inhabitants, whereas the ratio on the OECD countries was just a little above 3 doctors per 1000 inhabitants (Espírito Santo Saúde – SGPS, S.A, 2014).

Portugal is among the highest scoring countries from OECD in medical training, with a ratio of 12.2 newly graduated doctors per 1000 inhabitants, whereas the ratio on the OECD countries is 10.6 per 1000 inhabitants (Espírito Santo Saúde – SGPS, S.A., 2014).

There has been an increase in nursing personal. In 2011, Portugal had 6.1 nurses per 1000 inhabitants. However, this ratio is below the OECD countries ratios of 8.8 per 1000 inhabitants (Espírito Santo Saúde – SGPS, S.A., 2014).

#### 3.2.2.2.4. A broader coverage of healthcare insurance and similar plans is a determinant factor to increase demand for private healthcare services

The current restriction on services provided by the National Healthcare Service and the increasing of users' fees to access the public healthcare services can contribute to an increasing demand for private healthcare services by patients looking for alternative ways of healthcare financing and healthcare providers (Espírito Santo Saúde – SGPS, S.A., 2014).

The private healthcare plans have been including more affordable options, and together with the increasing difference in quality between private and public healthcare services, it is expected that the number of people applying for private healthcare plans is likely to increase. This will stimulate the competition, the demand, and the quality of the services provided by the private players (Espírito Santo Saúde – SGPS, S.A., 2014).

#### 3.2.2.2.5. Favourable legal environment

The legal environment of the healthcare sector has been evolving favourably for the private healthcare players. Some of the recommendations of the Portuguese health regulator (called ERS – Entidade Reguladora da Saúde) can result in an increase of services provided by private players, by broadening the access to provide certain medical procedures and services to private entities (Espírito Santo Saúde – SGPS, S.A., 2014).

#### 3.2.2.2.6. Potential access from the private sector to the management of the public sector through public private partnerships

The current State budget restrictions that lead to cuts to the annual National Healthcare Service budget, followed by the difficulty to access medical appointments, the increasing waiting lists for surgeries, the increasing demand for health services due to the aging population phenomenon, the maintenance costs of the public hospital network, among others, is leading to use private entities to manage the public health assets in order to optimize them and avoid additional public financing to the public healthcare sector (Espírito Santo Saúde – SGPS, S.A., 2014).

Therefore, contracts such as public-private partnerships between the State and private entities can be seen as an opportunity for the private healthcare market to grow (Espírito Santo Saúde – SGPS, S.A., 2014).

## 3.3. CASE STUDY – VALUATION OF HOSPITAL CRUZ VERMELHA PORTUGUESA

### 3.3.1. Cruz Vermelha Portuguesa – Sociedade de Gestão Hospitalar S.A. and Hospital Cruz Vermelha Portuguesa

*Based on Auditoria operacional ou de resultados à execução do Acordo de Cooperação entre a ARSLVT, I.P., e a CVP-SGH, S.A. made by Tribunal de Contas de Portugal (2011) and its appendixes, in which is included Caixa BI's CVP-SGH, S.A. valuation.*

#### *3.3.1.1. Overview*

Cruz Vermelha Portuguesa – Sociedade de Gestão Hospitalar S.A. (CVP-SGH) is owned 54.97% by CVP and 45% by Parpública Participações Públicas, SGPS, S.A. (Parpública). Parpública is a state owned company, created in 2000, that manages the state's stakes in companies in process of privatization (Tribunal de Contas de Portugal, 2011).

CVP-SGH was created in May 1998 by Cruz Vermelha Portuguesa (CVP) and its activity is the management and exploitation of healthcare units, as well as the provision of hospital services. Additionally, CVP-SGH can also participate in corporate businesses which activities are similar or complementary with its business activity (Tribunal de Contas de Portugal, 2011).

The CVP-SGH's activity is mainly focused on the management of the Hospital Cruz Vermelha Portuguesa (HCVP) through the exploitation contract signed between CVP-SGH e Cruz Vermelha Portuguesa (CVP) in August 1998 (Tribunal de Contas de Portugal, 2011).

CVP-SGH also owns 100% of the subsidiary Servihospital – Sociedade de Serviços de Apoio Hospitalar, Unipessoal, Lda., which provides similar and complementary services to the management and operation of hospitals (Tribunal de Contas de Portugal, 2011).

Additionally, CVP-SGH is shareholder of the following companies: 26% of Sociedade Portuguesa de Ressonância Magnética, 14.9% of Sociedade Portuguesa de Diálise, 10% of Compromisso Certo, S.A. (Tribunal de Contas de Portugal, 2011).

#### *3.3.1.2. Exploitation contract between CVP and CVP-SGH*

The exploitation contract was signed between CVP and CVP-SGH in August 1998 valid for 25 years (ends in August 2023). CVP-SGH was created in the same year by CVP in order to separate the management of HCVP (Tribunal de Contas de Portugal, 2011).

Under this contract, CVP gave to CVP-SGH the exploitation of HCVP, comprising the exploitation of the building, the transference of the assets and liabilities of HCVP, as well as the staff working there at that time (Tribunal de Contas de Portugal, 2011).

The price of transference of the exploitation of the hospital was € 5.5 million, paid by CVP-SGH in exchange of accepting HCVP's liabilities (Tribunal de Contas de Portugal, 2011).

CVP-SGH is obliged to manage, maintain and develop HCVP's activity in the healthcare sector, as well as to keep the building and equipment in good state of preservation, provided by CVP and perform all necessary renovation work. All improvements made in the building or equipment by CVP-SGH revert to CVP in the end of the exploitation contract. Though, CVP-SGH is entitled to the book value, after depreciation, of the improvements made. In the end of the contract, it also reverts to CVP all the assets and liabilities needed to ensure the maintenance of the HCVP's activity. CVP has a purchase option of the whole or part of the HCVP financial stakes (Tribunal de Contas de Portugal, 2011).

The 7$^{th}$ clause of the exploitation contract determines that the terminal value of HCVP is equal to the sum of the values of HCVP's tangible fixed assets and net working capital in August 2023 (Tribunal de Contas de Portugal, 2011).

CVP-SGH pays a compensation to CVP for the exploitation of the HCVP, 1.5% of the annual turnover of the previous year, up to its EBT (Tribunal de Contas de Portugal, 2011).

#### *3.3.1.3. CVP-SGH and ARSLVT*

The cooperation agreement between Administração Regional de Saúde de Lisboa e Vale do Tejo and CVP was signed in December 2012 and was valid for one year (except in the event of termination by either one of the parties) (Tribunal de Contas de Portugal, 2011).

Under the cooperation agreement, HCVP ensured, complementarily to the National Healthcare Service institutions, healthcare services in orthopaedics, vascular surgery, cardiothoracic surgery and ophthalmology, as well as screening for diabetic retinopathy and breast cancer, to patients living in the geographical area of Lisbon and Tagus Valley (Tribunal de Contas de Portugal, 2011).

The access, from patients living in Lisbon and Tagus Valley region and belonging to the National Healthcare Service, to the CVP healthcare services was limited to the production signed a contract for the period between 1st December 2012 and 30th November 2013 (Tribunal de Contas de Portugal, 2011).

CVP agreed to implement the agreement in exchange of a retribution of € 505 million for the period mentioned above, and an annual retribution of € 7.1 million for the period from 1st January 2013 to November 30th, 2013 – to be paid in twelve equal monthly payments. ARSLVT did not assume any extra charges than the ones agreed in the contract (Tribunal de Contas de Portugal, 2011).

Following the recommendations made by the Portuguese Audit Court (2011), the State did not renovate the cooperation agreement between ARSLVT and CVP. The Portuguese Audit Court concluded that the National Healthcare Service has the capacity to provide the healthcare services provided by HCVP and that the State should only resort to CVP if its costs were lower than the marginal costs of those of the National Healthcare Service (Tribunal de Contas de Portugal, 2011).

#### *3.3.1.4. Hospital Cruz Vermelha Portuguesa*

The HCVP opened in 1965 and is located in Lisbon. In 1998 it went through an extensive restructuring and modernization and is seen today as a healthcare center of excellence in numerous areas of expertise. It is equipped with the most modern last generation technology and has a permanent prestigious practitioners' staff in all clinical areas (Cruzvermelha.pt, 2015).

Besides the cooperation agreement with ARSLVT, CVP-SGH did a 3-year partnership in the end of 2010 with Clínical Girassol in Angola. This partnership includes the provision of healthcare services and on-the-job training by HCVP's practitioners and it is expected to be renewed and expanded in the future. This partnership opened up the doors to other future potential partnerships either in Angola or in other Portuguese speaking countries (Tribunal de Contas de Portugal, 2011).

HCVP provides the following healthcare services (Hospitalcruzvermelha.pt, 2015):

(i) Intensive healthcare unit for adults, that is composed by a permanent medical team highly specialized and gives 24/7 daily health care service during 365 days a year in a space that can receive 12 patients.

(ii) Medical appointment center that offers different areas of expertise.

(iii) Operating room that is composed by 6 surgery rooms equipped with high technology.

(iv) Day hospital center that works for 12 hours a day in a schedule appointment system for patients in need of surgery in all areas of expertise or of special medical exams in an outpatient system.

(v) Inpatient units composed by 4 inpatient units designed to provide the best healthcare services to the patient and his/hers relatives by giving them the maximum comfortable environment possible.

(vi) Renal transplant unit that is considered a pioneer unit in Portugal in this area of expertise.

### 3.3.2. Firm valuation methodology adopted by Caixa BI

#### *3.3.2.1. Overview*

The firm valuation made by Caixa BI to HCVP aimed the economic-financial assessment of the 45% stake of CVP-SGH belonging to Parpública. All projections and assumptions made were based on CVP-SGH's reports and audited accounts, as well as the non-audited financial statements reported to December 2012 and the year budget for 2013 and all other information given by management (Tribunal de Contas de Portugal, 2011).

The projections indicated a relevant operating revenue growth in 2013 justified by the possible renegotiation of the cooperation agreement with ARSLVT (that we know today ended in the end of 2013) (Tribunal de Contas de Portugal, 2011).

The projections estimate that CVP-SHG has a turnover of around € 50.4 million in 2023, which represents an annual average rate of growth of 1.3% between 2013 and 2023, and an EBITDA of € 6.1 million, which represents an annual average rate of growth of 4.7%. These projections are justified by a growth in turnover, but also by gains in operational efficiency (Tribunal de Contas de Portugal, 2011).

The valuation predicted an increase in net working capital over the explicit forecast period lower than in previous periods. It was not predicted that relevant investments in tangible fixed assets over the explicit forecast period were made, just investments regarding the maintenance of the tangible fixed assets (Tribunal de Contas de Portugal, 2011).

The valuation refers to December 31st, 2012, including all the information available at the time, namely the situation of the financial market, among others (Tribunal de Contas de Portugal, 2011).

The methodology used is income statement based, i.e. in the valuation of the weighted average cost of capital (WACC) of the future cash flows (Tribunal de Contas de Portugal, 2011).

The explicit forecast period taken into consideration was until 2023, when the exploitation contract between CVP and CVP-SGH ends. The terminal value is determined that the clause number 7 of the exploitation contract, corresponding to the reversion value of HCVP's assets and liabilities (i.e. net working capital) (Tribunal de Contas de Portugal, 2011).

CVP-SGH is entitled to be compensated in the book value of the improvements made in the existing or new buildings and equipments, as well as to the value of the assets and current liabilities (i.e. net working capital) necessary to the maintenance of the hospital's activity (Tribunal de Contas de Portugal, 2011).

According to the projections made, considering the operating profitability and rather small level of debt in 2023, the initial objective of the exploitation contract – overcome the accumulated debt and revitalize the hospital's activity – will be met (Tribunal de Contas de Portugal, 2011).

It is assumed that the purchasing option of CVP is likely to happen, since the reversion value is usually lower than the HCVP's perpetuity value, thus making sense to CVP to repurchase the hospital (Tribunal de Contas de Portugal, 2011).

Finally, the successive renegotiations between CVP-SGH and ARSLVT show that the State is resorting less and less to HCVP, CVP-SGH is trying to create the conditions to channel its installed capacity to serve the private sector (Tribunal de Contas de Portugal, 2011).

Caixa BI assumed the following scenarios (Tribunal de Contas de Portugal, 2011):

(i) That the cooperation agreement with ARSLVT would be renewed over the forecast period in conditions similar to those negotiated for 2013;
(ii) The renewal of the contract with Clínica Girassol in Angola in conditions similar to those negotiated for the period 2010-2013.

(iii) An investment in fixed tangible assets of 1.9% in 2013 and 3.6% of the revenues from 2018 on.

(iv) The recovery of the net working capital estimated for 2023, according with the exploitation contract reversion value, having as reference its book value in 2023 (it is estimated that the net working capital in August 2023 will be € 18 927 million, where € 28 202 million refer to customers' debts).

#### *3.3.2.2. Free cash flow*

The methodology used by Caixa BI for firm valuation is a discounted cash flow method. The enterprise value is estimated by discounting its expected future cash flows – free cash flow – resulting from the company's business activity, at a discount rate appropriate to the risk associated to the company (Tribunal de Contas de Portugal, 2011).

The free cash flow are the flows available for distribution to equity holders and debt holders, before dividend distribution, interest payments and debt principal payments (Tribunal de Contas de Portugal, 2011).

$(+)$ Earnings before interest and taxes (EBIT)

$(-)$ Taxes on EBIT

$(+)$ Depreciation, amortization and other non-cash expenses

$(-)$ Increase in net working capital

$(-)$ Capital expenditure (CAPEX)

$=$ Free Cash Flow

#### *3.3.2.3. Discount rate*

The free cash flow discount rate is the weighted average cost of capital (WACC), in other words, the weighted average of the cost of equity and the cost of debt (Tribunal de Contas de Portugal, 2011):

$$WACC = \frac{D}{(E+D)} \times K_D \times (1 - T_C) + \frac{E}{(E+D)} \times K_E \quad (98)$$

| | |
|---|---|
| $E$ | Market value of Equity |
| $D$ | Market value of Debt |
| $CAPM$ | Capital Asset Pricing Model |

| | |
|---|---|
| $WACC$ | Weighted average cost of capital |
| $D/(E+D); E/(E+D)$ | Market value of capital structure |
| $K_D$ | Cost of debt |
| $T_C$ | Tax rate |
| $K_E$ | Cost of equity |
| $CAPM$ | $K_E = K_F + \beta_L \times (K_M - K_F)$ |
| $K_F$ | Risk-free rate |
| $(K_M - K_F)$ | Market risk premium |
| $\beta_L$ | $\beta_{asset} \times (1 + (1 - T_C) \times D/E)$ |

The free cash flows were discounted to year 2012.

#### *3.3.2.4. Terminal value*

The terminal value is determined by the exploitation contract between CVP-SGH and CVH, which ends in august 2023. Therefore, the firm valuation made by Caixa BI does not assume a perpetuity, but rather assumes a terminal value equivalent to the reversion value of HCVP's assets and liabilities (i.e. net working capital) determined by the 7th clause of the exploitation contract (Tribunal de Contas de Portugal, 2011):

$$\begin{aligned} Terminal\ Value &= (Tangible\ fixed\ assets_{August\ 2023} \\ &+ Net\ working\ Capital_{August\ 2023}) \end{aligned} \tag{99}$$

#### *3.3.2.5. Equity value*

The value obtained by discounting the free cash flows corresponds to the firm value, which will be distributed to the equity holders and debt holders. To obtain the equity value, one has to subtract the net debt market value to the firm value, add the non-operating assets and subtract the value of off-balance sheet contingent liabilities (Tribunal de Contas de Portugal, 2011):

$$\begin{aligned} \sum_{n=1}^{t} \frac{Cash\ Flow_n}{\prod_{i=1}^{n}(1+WACC_i)} + \frac{Terminal\ Value}{\prod_{n=1}^{t}(1+WACC_t)} - Net\ Debt + Investments \\ - off\text{-}balance\ sheet\ contigent\ liabilities \end{aligned} \tag{100}$$

Note that, even though this is the formula presented by Caixa BI in its HCV firm valuation report, the Π feature was not used, since the WACC was computed in order to be equal every year.

#### *3.3.2.6. Discount rate calculation: inputs and outputs (Tribunal de Contas de Portugal, 2011)*

| Inputs | |
|---|---|
| Assumptions | |
| Risk-free rate | German 10-year treasury bond yield at February 1st, 2013. |
| Country risk premium | Spread Portuguese 10-year treasury bond yield VS. German 10-year treasury bond yield at February 1st, 2013. |
| Market risk premium | Caixa BI analysis of: implicit premium in mature markets, regulatory precedents in Europe, historical data in mature markets and academic professional studies. |
| D/E | Fixed target capital structure of the peer market players. |
| Tax rate | Tax rate applied at the time according with the State Budget for 2013. |
| $\beta_{asset}$ | Market peers. |
| Debt spread | Average spread of debt. |

| Outputs | |
|---|---|
| WACC | |
| Risk-free rate | 1.67% |
| Country risk premium | 4.51% |
| Market risk premium | 5.5% |
| $\beta_{asset}$ | 0.54 |
| $\beta_{equity}$ | 0.69 |
| $K_E$ | 9.98% |
| Spread risk-free interest rate VS. Portuguese interest rate | 6.05% |
| $K_D$ (after taxes) | 5.49% |
| D/(D+E) | 29.1% |
| D/E | 41% |
| Tax rate | 28.9% |

| | |
|---|---|
| WACC | 8.68% |

### 3.3.3. Caixa BI tax shield valuation approach versus our approach to tax shield valuation

#### *3.3.3.1. Hypothesis and scope*

The standard WACC only allows an accurate valuation when the taxable income covers completely the interest charges, because only then tax shields are completely realized, which is a standard WACC's assumption.

Our critical analysis will focus on the WACC calculation in the perspective of the value of tax shields captured by WACC, more precisely on the assumptions concerning:

- the leverage ratio,
- the tax rate, and
- the discount rate applied to tax shields adopted by Caixa BI in their firm valuation.

$$WACC = \frac{E}{E+D}.K_E + \frac{D}{E+D}.K_D \underbrace{- \frac{D}{E+D}.K_D.T_C}_{\text{Reduction due to interest tax shield – scope of the thesis}} \quad (98.1)$$

#### *3.3.3.2. Methodology*

Comparison of the valuation made by Caixa BI with our revaluations of HCVP-SGH, SA. in two scenarios:

- Scenario 1 – renewal of the cooperation agreement with ARSLVT (only scenario considered by Caixa BI),
- Scenario 2 – no renewal of the cooperation agreement with ARSLVT (scenario recommended by the Portuguese Audit Court and the one that ended up becoming real):
    - Scenario 2.1. – no renewal of the cooperation agreement with ARSLVT using Caixa BI's model considering only the tax savings effectively realized,
    - Scenario 2.2. – no renewal of the cooperation agreement with ARSLVT using our model.

Comparison of our revaluations with the valuations made by Caixa BI and the simulation without the cooperation agreement with ARSLVT using Caixa BI´s model.

To revaluate the firm we built our own model, which fundamentally differs on the following parameters:

- Leverage ratio,
- Tax rate, and
- Discount rate applied to tax shields.

#### 3.3.3.2.1. Leverage ratio

Concerning the leverage ratio (D/E+D), even though Caixa BI adopts the target capital structure of the peer market players, one knows that this ratio does not remain constant over the years. This is an understandable technical simplification that, nevertheless, would benefit from a more sophisticated approach, since it influences the value of tax shields, which is the object of this paper.

However, to compute the market value of the firm, we need to know the market value discount rates and to obtain these discount rates, we need to know the market value of the firm in order to use the appropriate leverage ratios. This is a circularity issue of discounted cash flow models. The circularity is inherent to these models, however, for practical reasons, this difficulty is circumvented, on firm valuation, by assuming target levels for both equity and debt.

The problem is that, even though this "solution" is convenient, it is not realistic and leads to less accurate approximations for discount rates. Therefore, since current spreadsheets solve this circularity issue, we will use this feature to calculate the WACC year by year.

#### 3.3.3.2.2. Tax rate

Another WACC method weakness, as it is computed by Caixa BI, lies on the fact that this investment bank assumes that the effective tax rate remains unchangeable year by year. Thus, Caixa BI's WACC assumes that the tax shields are always one hundred per cent realized every year. As already said previously, the tax shields only have effect on the WACC when they lead to real tax savings. Therefore, it is sufficient that in one of the years in the forecast period the EBIT is lower than the interest expenses for the WACC – computed using Caixa BI's model – to no longer be the most appropriate method.

From our proposal to the Hospital Cruz Vermelha tax shield valuation, it becomes clear that for the WACC to take into consideration the real tax savings, it is necessary to adjust the tax rate used in the WACC year by year in the following way:

(i) $If\ EBIT_t < 0, then\ T_{c_t} = 0;$

(ii) $If\ 0 < EBIT_t < K_D D_{t-1}, then\ T_{c_t} = \frac{EBIT_t \times T_C}{K_D D_{t-1}};$

(iii) $If\ EBIT > K_D D_{t-1}, then\ T_{c_t} = T_c.$

#### 3.3.3.2.3. Discount rate applied to tax shields

On what concerns the discount rate applied to tax shields, the WACC method adopted by Caixa BI (with a fixed leverage ratio) assumes that the tax shields are discounted at the unlevered cost of capital $K_U$. This is easy to verify when comparing the results for the firm value obtained by the methods WACC, APV and CCF. Note that the tax shields in the APV and CCF methods will be discounted at $K_U$. As a final step, being the scope of this thesis the discount rate applied to tax shields, we will propose a different tax shield discount rate, already presented in the literature review.

### 3.3.4. Scenario 1 – renewal of the cooperation agreement with ARSLVT

#### *3.3.4.1. The WACC method*

##### 3.3.4.1.1. Caixa BI's valuation

| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|---|
| **Discount Rate** | | | | | | | | |
| **COST OF EQUITY** | | | | | | | | |
| Risk-free interest rate | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Country risk premium | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% |
| Adjusted risk-free interest rate | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% |
| Asset beta | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| Financial leverage (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| Financial leverage (D/(D+E)) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| Tax rate | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% |
| Equity beta | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 |
| Market risk premium | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% |
| Equity beta | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 |
| Risk premium | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% |
| **Cost of Equiy (ke)** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** |
| **COST OF DEBT** | | | | | | | | |
| Risk-free interest rate | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Spread | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% |
| Tax rate | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% |
| **Cost of debt after tax (kd)** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** |
| **CAPITAL STRUCTURE** | | | | | | | | |
| Financial leverage (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| D / (D+E) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| E / (D+E) | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% |
| **WACC** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** |

*Table 1: Scenario 1 – Caixa BI Firm valuation's assumptions (retrieved and adapted from Caixa BI's Firm valuation included as appendix in the Portuguese Audit Court report, 2011)*[17]

[17] Original in Appendix 1

| Free Cash Flow | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
| **Fixed Assets** | **10,703,499** | **10,566,099** | **10,534,043** | **10,640,627** | **10,890,205** | **11,247,686** | **14,168,338** | **14,461,338** |
| **Working Capital** | **16,430,751** | **16,149,135** | **16,303,128** | **16,418,585** | **16,849,926** | **17,305,564** | **18,775,162** | **18,927,325** |
| **Sales and Services Provided** | **32,603,820** | **44,097,054** | **44,538,683** | **45,102,382** | **45,693,490** | **46,329,825** | **49,662,244** | **29,376,652** |
| **EBITDA** | **-385,318** | **3,852,991** | **3,793,043** | **4,298,509** | **4,815,849** | **5,348,640** | **6,076,946** | **3,572,431** |
| Mg EBITDA | | 8.7% | 8.5% | 9.5% | 10.5% | 11.5% | 12.2% | 12.2% |
| Amortizations | | 989,400 | 1,042,277 | 1,068,006 | 1,093,976 | 1,160,491 | 1,223,433 | 768,242 |
| EBIT | | 2,863,591 | 2,750,766 | 3,230,503 | 3,721,873 | 4,188,149 | 4,853,513 | 2,804,189 |
| Tax on EBIT | | 799,759 | 766,476 | 907,998 | 1,052,952 | 1,190,504 | 1,386,786 | 801,236 |
| Tax rate | | 27.9% | 27.9% | 28.1% | 28.3% | 28.4% | 28.6% | 28.6% |
| **NOPLAT** | | **2,063,832** | **1,984,290** | **2,322,505** | **2,668,920** | **2,997,645** | **3,466,727** | **2,002,953** |
| Amortizations | | 989,400 | 1,042,277 | 1,068,006 | 1,093,976 | 1,160,491 | 1,223,433 | 768,242 |
| CAPEX | | 852,000 | 1,010,221 | 1,174,590 | 1,343,554 | 1,517,973 | 1,794,066 | 1,061,242 |
| %Sales and Services Provided | | 1.9% | 2.3% | 2.6% | 2.9% | 3.3% | 3.6% | 3.6% |
| Working Capital Investment | | -281,616 | 153,993 | 115,457 | 431,341 | 455,638 | 256,876 | 151,950 |
| %Sales and Services Provided | | -0.6% | 0.3% | 0.3% | 0.9% | 1.0% | 0.5% | 0.5% |
| **Free Cash Flow** | | **2,482,848** | **1,862,353** | **2,100,463** | **1,988,002** | **2,184,525** | **2,639,218** | **1,558,003** |
| Discount Factor | | 0.92 | 0.85 | 0.78 | 0.72 | 0.66 | 0.44 | 0.41 |
| **Discounted Free Cash Flow** | | **2,284,562** | **1,576,769** | **1,636,341** | **1,425,045** | **1,440,860** | **1,148,166** | **645,672** |
| | | | | | | | | **Terminal Value** |
| **NPV FCF 2012-22E** | | **15,632,889** | | | | | | **33,388,663** |
| **NPV Liquidation Value** | | **13,837,025** | | | | | | |
| **Enterprise Value** | | **29,469,914** | | | | | | |
| Non-Operating Assets | | 1,686,660 | | | | | | |
| Net Debt | | 18,040,184 | | | | | | |
| Shareholders Loans | | 731 | | | | | | |
| **Equity Value** | | **13,115,659** | | | | | | |
| **Equity Vaue x 45% (Stake Parpublica)** | | **5,902,047** | | | | | | |

*Table 2: Scenario 1 – Caixa BI Firm's valuation at beginning of the year (retrieved and adapted from Caixa BI's Firm valuation included as appendix in the Portuguese Audit Court report, 2011)*[18]

### 3.3.4.1.2. Our revaluation

| Assumptions | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| **Risk Free Rate** | | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% |
| **Market Risk Premium** | | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% |
| **Tax Rate** | | 28.90% | 28.90% | 28.90% | 28.90% | 28.90% | 28.90% | 28.90% |
| **Asset Beta** | | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| **Debt Beta** | | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 |
| **EBIT** | | 2863591 | 2750766 | 3230503 | 3721873 | 4188149 | 4853513 | 2804189 |
| **Depreciation** | | 989400 | 1042277 | 1068006 | 1093976 | 1160491 | 1223433 | 768242 |
| **Capex** | | 852000 | 1010221 | 1174590 | 1343554 | 1517973 | 1794066 | 1061242 |
| **Increase in NWC** | | -281616 | 153993 | 115457 | 431341 | 455638 | 256876 | 151950 |
| **Debt @ beginning of the year** | | 18,040,183 | 17,708,069 | 17,029,347 | 16,132,604 | 15,375,268 | 8,531,171 | 6,628,290 |

*Table 3: Scenario 1 – Our revaluation's assumptions*

[18] Original in Appendix 1

| WACC Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| EBIT | 2,863,591 | 2,750,766 | 3,230,503 | 3,721,873 | 4,188,149 | 4,853,513 | 2,804,189 |
| - Tax on EBIT | 799,759 | 766,476 | 907,998 | 1,052,952 | 1,190,504 | 1,386,786 | 801,236 |
| = EBIAT (NOPAT) | 2,063,832 | 1,984,290 | 2,322,505 | 2,668,920 | 2,997,645 | 3,466,727 | 2,002,953 |
| + Depreciation | 989,400 | 1,042,277 | 1,068,006 | 1,093,976 | 1,160,491 | 1,223,433 | 768,242 |
| - Capex | -852,000 | -1,010,221 | -1,174,590 | -1,343,554 | -1,517,973 | -1,794,066 | -1,061,242 |
| - Increase in NWC | 281,616 | -153,993 | -115,457 | -431,341 | -455,638 | -256,876 | -151,950 |
| = FCF | 2,482,848 | 1,862,353 | 2,100,463 | 1,988,002 | 2,184,525 | 2,639,218 | 34,946,666 |
| | | | | | | | |
| Debt ratio | 59.650% | 58.796% | 55.635% | 52.171% | 48.971% | 26.608% | 20.611% |
| Cost of Debt (Kd) | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% |
| After Tax Cost of Debt | 5.492% | 5.492% | 5.492% | 5.492% | 5.492% | 5.492% | 5.492% |
| Percent Equity | 40.350% | 41.204% | 44.365% | 47.829% | 51.029% | 73.392% | 79.389% |
| Return on Assets (Ku) | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% |
| Cost of Equity (Ke) | 11.201% | 11.129% | 10.886% | 10.657% | 10.474% | 9.636% | 9.491% |
| WACC | 7.796% | 7.815% | 7.885% | 7.963% | 8.034% | 8.533% | 8.667% |
| Factor | 1.078 | 1.162 | 1.254 | 1.354 | 1.462 | 2.180 | 2.369 |
| PV | 2,303,292.3 € | 1,602,445.2 € | 1,675,228.9 € | 1,468,597.8 € | 1,493,766.2 € | 1,210,638.4 € | 14,751,866.0 € |
| =Firm Value (V) @ beginning of the year | **30,243,140.0 €** | **30,117,924.2 €** | **30,609,188.0 €** | **30,922,331.8 €** | **31,396,535.6 €** | **32,062,631.8 €** | **32,159,385.6 €** |
| +Non-operating Assets @ beg. of the year | **1,686,660.0 €** | | | | | | |
| -Debt @ beg. of the year | **18,040,183.0 €** | | | | | | |
| -Shareholders Loans @ beg. of the year | **731.0 €** | | | | | | |
| =Equity value @ beginning of the year | **13,888,885.99 €** | | | | | | |

*Table 4: Scenario 1 – Our revaluation (WACC method)*

For the scenario that includes the contract with ARSLVT, the only one presented by Caixa BI, the firm value is slightly different than ours. Caixa BI's firm value was € 29.5 million and ours is € 30.243 million. These values could never be the same, since the WACC was computed differently. From an academic point of view, the difference of the methodology used is not negligible, since we processed to the *endogenization* of, not only, the debt ratio, but also the cost of equity, which is calculated using the MM proposition II, which is also a function of the leverage ratio. In our model, the debt ratio varies every year, conversely to Caixa BI´s ratio, which is fixed. Additionally, we had to *modelize* the computation of the debt service in order for the model to become more flexible. All these issues explain the firm values difference.

#### *3.3.4.2. Other Discounted Cash Flow valuation methods*

##### 3.3.4.2.1. APV method

This is the method that better illustrates the importance and effective realization of the tax savings, by decomposing the firm value in value of the unlevered firm and value of interest tax shields.

| APV Valuation | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Y1** | **Y2** | **Y3** | **Y4** | **Y5** | **Y9** | **Y10** | **Y11** |
| FCF | 2,482,847.6 € | 1,862,353.2 € | 2,100,463.0 € | 1,988,002.4 € | 2,184,525.1 € | 2,607,165.1 € | 2,639,217.8 € | 34,946,666.4 € |
| Return on Assets (Ku) | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| Factor | 1.0913 | 1.191 | 1.300 | 1.418 | 1.548 | 2.195 | 2.395 | 2.614 |
| PV | 2,275,187.2 € | 1,563,853.9 € | 1,616,278.6 € | 1,401,797.1 € | 1,411,537.3 € | 1,187,878.8 € | 1,101,909.3 € | 13,370,371.0 € |
| **Value of Unlevered Firm @ beginning of the year** | **28,051,817.5 €** | | | | | | | |
| | | | | | | | | |
| Interest Tax Shield | 402,714.2 € | 395,300.4 € | 380,149.1 € | 360,131.0 € | 343,224.8 € | 228,544.2 € | 190,442.8 € | 147,964.5 € |
| Return on Assets (Ku) | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| Factor | 1.091 | 1.191 | 1.300 | 1.418 | 1.548 | 2.195 | 2.395 | 2.614 |
| PV | 369,032.0 € | 331,941.4 € | 292,519.8 € | 253,938.6 € | 221,775.7 € | 104,129.5 € | 79,512.5 € | 56,610.3 € |
| **Value of Interest Tax Shield @ Ku @ beginning of the year** | **2,191,322.5 €** | | | | | | | |
| | | | | | | | | |
| **Value of Firm with TS @ Ku @ beginning of the year** | **30,243,140.0 €** | | | | | | | |

*Table 5: Scenario 1 – Our revaluation (APV method)*

#### 3.3.4.2.2. CCF method

This method – also called compressed adjusted present value – assumes that the tax savings are discounted at the cost of unlevered equity and the firm value confirms the results obtained by using the WACC and APV methods when using the same assumptions.

| CCF Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Y1** | **Y2** | **Y3** | **Y4** | **Y5** | **Y10** | **Y11** |
| **EBIT** | 2,863,591.00 | 2,750,766.11 | 3,230,503.33 | 3,721,872.75 | 4,188,149.14 | 4,853,513.01 | 2,804,189.24 |
| **- Tax on EBIT** | 799,759.35 | 766,476.00 | 907,998.48 | 1,052,952.46 | 1,190,504.00 | 1,386,786.34 | 801,236.41 |
| **= EBIAT** | 2,063,831.66 | 1,984,290.11 | 2,322,504.84 | 2,668,920.29 | 2,997,645.14 | 3,466,726.67 | 2,002,952.84 |
| **+ Depreciation** | 989,400.00 | 1,042,277.08 | 1,068,005.61 | 1,093,976.34 | 1,160,490.91 | 1,223,432.94 | 768,241.77 |
| **- Capex** | -852,000.00 | -1,010,221.21 | -1,174,589.98 | -1,343,553.70 | -1,517,972.53 | -1,794,065.56 | -1,061,241.62 |
| **- Increase in NWC** | 281,615.94 | -153,992.74 | -115,457.44 | -431,340.55 | -455,638.41 | -256,876.30 | -151,949.76 |
| **= FCF** | 2,482,847.60 | 1,862,353.24 | 2,100,463.03 | 1,988,002.38 | 2,184,525.11 | 2,639,217.75 | 34,946,666.36 |
| **+ Net Cash Flow to Debt (TS)** | 402,714.22 | 395,300.39 | 380,149.15 | 360,130.98 | 343,224.84 | 190,442.85 | 147,964.49 |
| **=CCF** | **2,885,561.82** | **2,257,653.63** | **2,480,612.18** | **2,348,133.37** | **2,527,749.95** | **2,829,660.60** | **35,094,630.85** |
| **Return on Assets (Ku)** | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| **Factor** | 1.091 | 1.191 | 1.300 | 1.418 | 1.548 | 2.395 | 2.614 |
| **PV** | 2644219.2 | 1895795.3 | 1908798.4 | 1655735.7 | 1633313.0 | 1181421.8 | 13426981.3 |
| | | | | | | | |
| **Firm Value @ beginning of the year** | **30,243,140.0** | | | | | | |

*Table 6: Scenario 1 – Our revaluation (CCF method)*

#### 3.3.4.2.3. ECF method

This method allows us to obtain immediately the equity value and also confirms both the equity value and firm value obtained by the three previous methods when using the unlevered cost of equity as the tax shield discount rate.

| ECF Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| EBIT | 2,863,591.00 € | 2,750,766.11 € | 3,230,503.33 € | 3,721,872.75 € | 4,188,149.14 € | 4,853,513.01 € | 2,804,189.24 € |
| - Tax on EBIT | 799,759.35 € | 766,476.00 € | 907,998.48 € | 1,052,952.46 € | 1,190,504.00 € | 1,386,786.34 € | 801,236.41 € |
| = EBIAT ( NOPAT) | 2,063,831.66 € | 1,984,290.11 € | 2,322,504.84 € | 2,668,920.29 € | 2,997,645.14 € | 3,466,726.67 € | 2,002,952.84 € |
| + Depreciation | 989,400.00 € | 1,042,277.08 € | 1,068,005.61 € | 1,093,976.34 € | 1,160,490.91 € | 1,223,432.94 € | 768,241.77 € |
| - Capex | - 852,000.00 € | - 1,010,221.21 € | - 1,174,589.98 € | - 1,343,553.70 € | - 1,517,972.53 € | - 1,794,065.56 € | - 1,061,241.62 € |
| - Increase in NWC | 281,615.94 € | - 153,992.74 € | - 115,457.44 € | - 431,340.55 € | - 455,638.41 € | - 256,876.30 € | - 151,949.76 € |
| = FCF | 2,482,847.60 € | 1,862,353.24 € | 2,100,463.03 € | 1,988,002.38 € | 2,184,525.11 € | 2,639,217.75 € | 34,946,666.36 € |
| - Interest Charges | 1,393,474.81 € | 1,367,821.42 € | 1,315,394.98 € | 1,246,127.97 € | 1,187,629.20 € | 658,971.79 € | 511,987.85 € |
| + Tax Shields | 402,714.22 € | 395,300.39 € | 380,149.15 € | 360,130.98 € | 343,224.84 € | 190,442.85 € | 147,964.49 € |
| + Δ Debt | - 332,113.58 € | - 678,722.31 € | - 896,743.56 € | - 757,335.93 € | - 939,724.66 € | - 1,902,881.29 € | - 6,628,289.55 € |
| =ECF | 1,159,973.43 € | 211,109.90 € | 268,473.64 € | 344,669.46 € | 400,396.09 € | 267,807.52 € | 27,954,353.45 € |
| Cost of Equity (Ke) | 11.20% | 11.13% | 10.89% | 10.66% | 10.47% | 9.64% | 9.49% |
| Factor | 1.112 | 1.236 | 1.370 | 1.516 | 1.675 | 2.693 | 2.948 |
| PV | **1,043,130.80 €** | **170,833.02 €** | **195,923.46 €** | **227,303.91 €** | **239,020.84 €** | **99,456.18 €** | **9,481,530.90 €** |
| PV ECF @ Ke @ beg. of the year | **12,202,956.99 €** | | | | | | |
| +Non-operating assets @ beg. of the year | **1,686,660.00 €** | | | | | | |
| -Shareholders loans @ beg. of the year | **731.00 €** | | | | | | |
| Equity Value @ beginning of the year | **13,888,885.99 €** | | | | | | |
| Debt @ beg. | **18,040,183.00 €** | | | | | | |
| Firm Value @ beginning of the year | **30,243,140.0 €** | | | | | | |

*Table 7: Scenario 1 – Our revaluation (ECF method)*

### 3.3.5. Scenario 2 – no renewal of the cooperation agreement with ARSLVT

If we recalculate the free cash flows without the cooperation agreement with ARSLVT, we obtain the table below:

| € | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|
| EBITDA | 3,852,991 | -282,443 | 733,895 | 1,096,779 | 1,467,669 | 1,769,498 | 1,748,920 |
| Amortizations | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 1,116,217 |
| EBIT | 2,863,591 | -1,311,382 | -305,447 | 48,842 | 372,716 | 723,422 | 632,704 |
| Tax on EBIT | 799,759 | 0 | 0 | 12,943 | 98,770 | 191,707 | 167,666 |
| **NOPLAT** | **2,063,832** | **-1,311,382** | **-305,447** | **35,899** | **273,946** | **531,715** | **465,037** |
| Amortizations | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 1,116,217 |
| CAPEX | 852,000 | 787,366 | 915,106 | 1,046,311 | 1,181,631 | 1,393,638 | 1,412,649 |
| Investment on Working Capital | -281,616 | -8,172,469 | 263,181 | 271,813 | 283,740 | 117,993 | 119,818 |
| **Unlevered Free Cash Flow** | **2,482,848** | **7,102,660** | **-444,392** | **-234,288** | **-96,472** | **66,161** | **48,787** |
| Financial Results | -1,607,385 | -1,427,664 | -1,253,344 | -1,361,963 | -1,471,032 | -1,766,709 | -1,799,459 |
| Tax Shield | 466,865 | 0 | 0 | 12,943 | 98,770 | 191,707 | 167,666 |
| **Free Cash Flow** | **1,342,327** | **5,674,996** | **-1,697,736** | **-1,583,308** | **-1,468,734** | **-1,508,841** | **-1,583,006** |

*Table 8: Scenario 2 – Portuguese Audit Court FCF simulation of non-renewal of the cooperation contract with ARSLVT using Caixa BI's valuation model (retrieved and adapted from Portuguese Audit Court's Firm valuation simulation included as appendix in the Portuguese Audit Court report, 2011)*[19]

Looking at the spreadsheet above, we can verify that the annual tax shields are much lower than the tax shields that would result from $K_D.D.T_C$. This is not coherent with the tax shields implied in the WACC computation, since Caixa BI's WACC assumes that the tax shields are one hundred per cent realized. Therefore, we found that in 2013 the tax shields were completely realized, in 2014 and 2015 there were no realization of tax shields, from 2016 on the tax shields are just partially realized. For example, in 2016, tax

[19] Original in Appendix 1

shields of € 14 115[20] were realized, which is equivalent to a tax rate of 1.04% on interest expenses (table 9).

| € | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|
| EBITDA | 3,852,991 | -282,443 | 733,895 | 1,096,779 | 1,467,669 | 1,769,498 | 1,748,920 |
| Amortizations | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 1,116,217 |
| EBIT | 2,863,591 | -1,311,382 | -305,447 | 48,842 | 372,716 | 723,422 | 632,704 |
| Tax on EBIT | 799,759 | 0 | 0 | 12,943 | 98,770 | 191,707 | 167,666 |
| TS with 28,9% | 464534 | | | 14,115 | 107,715 | 209,069 | 182,851 |
| **NOPLAT** | **2,063,832** | **-1,311,382** | **-305,447** | **35,899** | **273,946** | **531,715** | **465,037** |
| Amortizations | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 1,116,217 |
| CAPEX | 852,000 | 787,366 | 915,106 | 1,046,311 | 1,181,631 | 1,393,638 | 1,412,649 |
| Investment on Working Capital | -281,616 | -8,172,469 | 263,181 | 271,813 | 283,740 | 117,993 | 119,818 |
| **Unlevered Free Cash Flow** | **2,482,848** | **7,102,660** | **-444,392** | **-234,288** | **-96,472** | **66,161** | **48,787** |
| Financial Results | -1,607,385 | -1,427,664 | -1,253,344 | -1,361,963 | -1,471,032 | -1,766,709 | -1,799,459 |
| **TS in % of interests** | **28.90%** | **0.00%** | **0.00%** | **1.04%** | **7.32%** | **11.83%** | **10.16%** |
| Tax Shield | 466,865 | 0 | 0 | 12,943 | 98,770 | 191,707 | 167,666 |
| **Interest . Tc - TS** | **2,330** | **-412,595** | **-362,217** | **-380,664** | **-326,359** | **-318,872** | **-352,377** |
| **Non-realized Interest TS in % of Interest Expenses** | **-0.14%** | **28.90%** | **28.90%** | **27.95%** | **22.19%** | **18.05%** | **19.58%** |
| **Free Cash Flow** | **1,342,327** | **5,674,996** | **-1,697,736** | **-1,583,308** | **-1,468,734** | **-1,508,841** | **-1,583,006** |

*Table 9: Scenario 2 – Portuguese Audit Court FCF simulation of non-renewal of the cooperation contract with ARSLVT with effective tax rate adjustment using Caixa BI's valuation model (retrieved and adapted from Portuguese Audit Court's Firm valuation simulation included as appendix in the Portuguese Audit Court report, 2011)*[21]

As we can see, the valuation from Caixa BI does not resist to a stress test to the cash inflow when the scenario of a non-renovation of the contract with ARSLVT, a plausible hypothesis due to the recommendation of the Portuguese Audit Court, occurs. As you can check, in this hypothesis, EBIT is either lower than the interest expenses or is between 0 and the interests expenses. This is incompatible with an unchangeable tax rate of 28.9%.

#### *3.3.5.1. Scenario 2.1. – no renewal of the cooperation agreement with ARSLVT using Caixa BI's model considering only the tax savings effectively realized*

[20] We recomputed the annual tax shields applying the tax rate assumed in the Caixa BI's WACC formula.
[21] Data in red added by us.

| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|---|
| **Discount Rate** | | | | | | | | |
| **COST OF EQUITY** | | | | | | | | |
| Risk-free interest rate | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Country risk premium | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% |
| Adjusted risk-free interest rate | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% |
| Asset beta | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| Financial leverage (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| Financial leverage (D/(D+E)) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| Tax rate | 28.90% | 28.9% | 0.0% | 0.0% | 1.0% | 7.3% | 11.8% | 10.2% |
| Equity beta | 0.69 | 0.69 | 0.75 | 0.75 | 0.75 | 0.74 | 0.73 | 0.73 |
| Market risk premium | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% |
| Equity beta | 0.69 | 0.69 | 0.75 | 0.75 | 0.75 | 0.74 | 0.73 | 0.73 |
| Risk premium | 3.80% | 3.80% | 4.15% | 4.15% | 4.14% | 4.06% | 4.01% | 4.03% |
| **Cost of Equiy (ke)** | **9.98%** | **9.98%** | **10.33%** | **10.33%** | **10.32%** | **10.25%** | **10.19%** | **10.21%** |
| **COST OF DEBT** | | | | | | | | |
| Risk-free interest rate | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Spread | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% |
| Tax rate | 28.90% | 28.90% | 0.00% | 0.00% | 1.04% | 7.32% | 11.83% | 10.16% |
| **Cost of debt after tax (kd)** | **5.49%** | **5.49%** | **7.72%** | **7.72%** | **7.64%** | **7.16%** | **6.81%** | **6.94%** |
| **CAPITAL STRUCTURE** | | | | | | | | |
| Financial leverage (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| D / (D+E) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| E / (D+E) | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% |
| **WACC** | **8.68%** | **8.68%** | **9.58%** | **9.58%** | **9.54%** | **9.35%** | **9.21%** | **9.26%** |

*Table 10: Scenario 2 – Caixa BI valuation model's assumptions with tax saving effectively realized*

As you can verify, the only assumptions modified were the effective tax rates, which now varies every year, and the WACC, which also varies every year as a consequence of the amendment of the effective tax rate.

| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|---|
| **Fixed Assets** | **10,703,499** | **10,566,099** | **10,324,526** | **10,200,291** | **10,198,665** | **10,285,343** | **11,923,647** | **12,096,566** |
| **Working Capital** | **16,430,751** | **16,149,135** | **7,976,666** | **8,239,847** | **8,511,659** | **8,795,399** | **9,546,783** | **9,616,677** |
| **Sales and Services Provided** | **32,603,820** | **44,097,054** | **34,713,453** | **35,138,617** | **35,584,453** | **36,064,401** | **38,577,839** | **22,810,722** |
| **EBITDA** | **-385,318** | **3,852,991** | **-282,443** | **733,895** | **1,096,779** | **1,467,669** | **1,769,498** | **1,020,204** |
| Mg EBITDA | | 8.7% | -0.8% | 2.1% | 3.1% | 4.1% | 4.6% | 4.5% |
| Amortizations | | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 651,126 |
| EBIT | | 2,863,591 | -1,311,382 | -305,447 | 48,842 | 372,716 | 723,422 | 369,077 |
| Tax on EBIT | | 799,759 | 0 | 0 | 12,943 | 98,770 | 191,707 | 97,805 |
| Tax rate | | 27.9% | 0.0% | 0.0% | 26.5% | 26.5% | 26.5% | 26.5% |
| **NOPLAT** | | **2,063,832** | **-1,311,382** | **-305,447** | **35,899** | **273,946** | **531,715** | **271,272** |
| Amortizations | | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 651,126 |
| CAPEX | | 852,000 | 787,366 | 915,106 | 1,046,311 | 1,181,631 | 1,393,638 | 824,045 |
| %Sales and Services Provided | | 1.9% | 2.3% | 2.6% | 2.9% | 3.3% | 3.6% | 3.6% |
| Working Capital Investment | | -281,616 | -8,172,469 | 263,181 | 271,813 | 283,740 | 117,993 | 69,768 |
| %Sales and Services Provided | | -0.6% | -23.5% | 0.7% | 0.8% | 0.8% | 0.3% | 0.3% |
| **Free Cash Flow** | | **2,482,848** | **7,102,660** | **-444,392** | **-234,288** | **-96,472** | **66,161** | **28,585** |
| Discount Factor | | 0.92 | 0.84 | 0.77 | 0.70 | 0.64 | 0.41 | 0.39 |
| **Discounted Free Cash Flow** | | **2,284,562** | **5,964,324** | **-340,560** | **-163,905** | **-61,721** | **27,311** | **11,205** |
| | | | | | | | | **Terminal Value** |
| **NPV FCF 2012-22E** | | **7,933,745** | | 15632889 | | | | **21,713,243** |
| **NPV Liquidation Value** | | **8,511,702** | | 13837025 | | | | |
| **Enterprise Value** | | **16,445,447** | | 29469914 | | | | |
| Non-Operating Assets | | 1,686,660 | | | | | | |
| Net Debt | | 18,040,184 | | | | | | |
| Shareholders Loans | | 731 | | | | | | |
| **Equity Value** | | **91,192** | | | | | | |
| **Equity Vaue x 45% (Stake Parpublica)** | | **41,036** | | | | | | |

*Table 11: Scenario 2 – Caixa BI's firm valuation model with tax saving effectively realized at beginning of the year*

The result is an abrupt decrease of the firm value to €16.445 million and of the equity value close to 0 (€91k).

The Portuguese Audit Court estimated an equity value of €627k, because the court made the same assumption as the investment bank, that the tax shields were one hundred per cent realized every year.

#### *3.3.5.2. Scenario 2.2. – no renewal of the cooperation agreement with ARSLVT using our model*

| Assumptions | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| Risk Free Rate | | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% |
| Market Risk Premium | | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% |
| Tax Rate | | 28.90% | 0.00% | 0.00% | 1.04% | 7.32% | 11.83% | 10.16% |
| Asset Beta | | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| Debt Beta | | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 | 0.280 |
| EBIT | | 2863591 | -1311382 | -305447 | 48842 | 372716 | 723422 | 369077 |
| Depreciation | | 989400 | 1028939 | 1039342 | 1047937 | 1094954 | 1046076 | 651126 |
| Capex | | 852000 | 787366 | 915106 | 1046311 | 1181631 | 1393638 | 824045 |
| Increase in NWC | | -281616 | -8172469 | 263181 | 271813 | 283740 | 117993 | 69768 |
| Debt @ beg. of the year | | 18,040,183 | 17,708,069 | 12,075,632 | 12,188,572 | 12,304,663 | 12,608,487 | 12,629,426 |
| Equity @ beg. of the year | | 9,879,995 | 10,697,477 | 7,958,432 | 6,399,641 | 5,086,520 | 532,431 | -510,856 |
| Assets @ beg. of the year | | 27,920,178 | 28,405,547 | 20,034,064 | 18,588,213 | 17,391,183 | 13,140,918 | 12,118,571 |

*Table 12: Scenario 2 – Our revaluation assumptions*

| WACC Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| EBIT | 2,863,591 | -1,311,382 | -305,447 | 48,842 | 372,716 | 723,422 | 369,077 |
| - Tax on EBIT | 799,759 | 0 | 0 | 12,943 | 98,770 | 191,707 | 97,805 |
| = EBIAT ( NOPAT) | 2,063,832 | -1,311,382 | -305,447 | 35,899 | 273,946 | 531,715 | 271,272 |
| + Depreciation | 989,400 | 1,028,939 | 1,039,342 | 1,047,937 | 1,094,954 | 1,046,076 | 651,126 |
| - Capex | -852,000 | -787,366 | -915,106 | -1,046,311 | -1,181,631 | -1,393,638 | -824,045 |
| - Increase in NWC | 281,616 | 8,172,469 | -263,181 | -271,813 | -283,740 | -117,993 | -69,768 |
| = FCF | 2,482,848 | 7,102,660 | -444,392 | -234,288 | -96,472 | 66,161 | 21,741,828 |
| | | | | | | | |
| Precent Debt | 106.022% | 112.912% | 120.614% | 107.200% | 97.407% | 68.130% | 63.102% |
| Cost of Debt (Kd) | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% | 7.724% |
| After Tax Cost of Debt | 5.492% | 7.724% | 7.724% | 7.644% | 7.159% | 6.810% | 6.939% |
| Percent Equity | -6.022% | -12.912% | -20.614% | -7.200% | 2.593% | 31.870% | 36.898% |
| Return on Assets (Ku) | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% | 9.127% |
| Cost of Equity (Ke) | -15.573% | -3.141% | 0.919% | -11.761% | 61.820% | 12.126% | 11.526% |
| WACC | 6.760% | 9.127% | 9.127% | 9.041% | 8.576% | 8.504% | 8.632% |
| Factor | 1.068 | 1.165 | 1.271 | 1.386 | 1.505 | 2.243 | 2.437 |
| PV | € 2,325,625.14 | € 6,096,460.15 | -€ 349,534.27 | -€ 168,998.36 | -€ 64,091.19 | € 29,491.43 | € 8,921,394.66 |
| **Firm Value (V) @ beginning of the year** | **€ 17,015,551.01** | **€ 15,683,029.31** | **€ 10,011,787.77** | **€ 11,369,973.50** | **€ 12,632,262.01** | **€ 18,506,511.40** | **€ 20,014,221.88** |
| +Non-operating assets @ beg. of the year | € 1,686,660.00 | | | | | | |
| -Net Debt @ beg. of the year | € 18,040,183.00 | € 17,708,069.42 | € 12,075,632.10 | € 12,188,571.58 | € 12,304,663.12 | € 12,608,487.01 | € 12,629,426.36 |
| -Shareholders Loans @ beg. of the year | € 731.00 | | | | | | |
| =Equity Value @ beginning of the year | € 661,297.01 | -€ 2,025,040.11 | -€ 2,063,844.32 | -€ 818,598.08 | € 327,598.89 | € 5,898,024.38 | € 7,384,795.52 |

*Table 13: Scenario 2 – Our revaluation (WACC method)*

In this stage, as we already mentioned, the difference between our model and the one used for our simulation using the Caixa BI's model is fundamentally one:

- The leverage ratio is *endogenized*, i.e. it is computed by our model year by year.

The different firm and equity values are mainly due to this fact.

As we can see, the equity value now obtained (€661k) is very close to the one obtained by the Portuguese Audit Court (€627k). This is a mere coincidence, since the computation of the firm value made by the Portuguese Audit Court was different regarding the computation of the leverage ratio and the tax shields.

### *3.3.5.3. Other Discounted Cash Flow methods*

#### 3.3.5.3.1. APV method

| APV Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Y1** | **Y2** | **Y3** | **Y4** | **Y5** | **Y10** | **Y11** |
| FCF | 2,482,847.6 € | 7,102,660.0 € | - 444,391.8 € | - 234,288.0 € | - 96,471.9 € | 66,161.0 € | 21,741,827.7 € |
| Return on Assets (Ku) | 9.127% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| Factor | 1.091 | 1.191 | 1.300 | 1.418 | 1.548 | 2.395 | 2.614 |
| PV | 2,275,187.2 € | 5,964,240.6 € | - 341,953.6 € | - 165,203.1 € | - 62,335.6 € | 27,623.1 € | 8,318,284.2 € |
| **Value of Unlevered Firm @ beg. of the year** | **16,230,633.2 €** | | | | | | |
| | | | | | | | |
| **Debt @ beg. of the year** | **18,040,183 €** | **17,708,069 €** | **12,075,632 €** | **12,188,572 €** | **12,304,663 €** | **12,608,487 €** | **12,629,426 €** |
| Interest | 1,393,475 € | 1,367,821 € | 932,756 € | 941,480 € | 950,447 € | 973,915 € | 975,533 € |
| Tax | 28.90% | 0.00% | 0.00% | 1.04% | 7.32% | 11.83% | 10.16% |
| Interest Tax Shield | 402,714.2 € | 0.0 € | 0.0 € | 9,757.5 € | 69,595.5 € | 115,251.3 € | 99,128.4 € |
| Return on Assets (Ku) | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| Factor | 1.091 | 1.191 | 1.300 | 1.418 | 1.548 | 2.395 | 2.614 |
| PV | 369,032.0 € | - € | - € | 6,880.3 € | 44,969.3 € | 48,119.0 € | 37,925.9 € |
| **Value of Interest Tax Shield @ Ku @ beg. of the year** | 784,917.8 € | | | | | | |
| | | | | | | | |
| **Value of Firm with TS @ Ku @ beg. of the year** | **17,015,551.0 €** | | | | | | |

*Table 14: Scenario 2 – Our revaluation (APV method)*

#### 3.3.5.3.2. CCF method

| CCF Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Y1** | **Y2** | **Y3** | **Y4** | **Y5** | **Y10** | **Y11** |
| **EBIT** | 2,863,591.00 | -1,311,381.70 | -305,446.53 | 48,842.12 | 372,715.58 | 723,422.20 | 369,077.14 |
| **- Tax on EBIT** | 799,759.35 | 0.00 | 0.00 | 12,943.16 | 98,769.63 | 191,706.88 | 97,805.44 |
| **= EBIAT** | 2,063,831.66 | -1,311,381.70 | -305,446.53 | 35,898.96 | 273,945.95 | 531,715.32 | 271,271.70 |
| **+ Depreciation** | 989,400.00 | 1,028,938.97 | 1,039,341.67 | 1,047,937.22 | 1,094,953.67 | 1,046,076.18 | 651,126.41 |
| **- Capex** | -852,000.00 | -787,366.49 | -915,106.14 | -1,046,311.47 | -1,181,631.26 | -1,393,637.63 | -824,045.14 |
| **- Increase in NWC** | 281,615.94 | 8,172,469.19 | -263,180.75 | -271,812.67 | -283,740.21 | -117,992.91 | -69,768.12 |
| **= FCF** | 2,482,847.60 | 7,102,659.98 | -444,391.75 | -234,287.96 | -96,471.86 | 66,160.96 | 21,741,827.72 |
| **+ Net Cash Flow to Debt (TS)** | 402,714.22 | 0.00 | 0.00 | 9,757.49 | 69,595.51 | 115,251.32 | 99,128.39 |
| **=CCF** | **2,885,561.82** | **7,102,659.98** | **-444,391.75** | **-224,530.48** | **-26,876.35** | **181,412.28** | **21,840,956.10** |
| **Return on Assets (Ku)** | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% | 9.13% |
| **Factor** | 1.091 | 1.191 | 1.300 | 1.418 | 1.548 | 2.395 | 2.614 |
| **PV** | 2644219.2 | 5964240.6 | -341953.6 | -158322.8 | -17366.2 | 75742.1 | 8356210.1 |
| | | | | | | | |
| **Firm Value @ beginning of the year** | **17,015,551.0** | | | | | | |

*Table 15: Scenario 2 – Our revaluation (CCF method)*

#### 3.3.5.3.3. ECF method

| ECF Valuation | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Y1 | Y2 | Y3 | Y4 | Y5 | Y10 | Y11 |
| EBIT | 2,863,591.00 € | - 1,311,381.70 € | - 305,446.53 € | 48,842.12 € | 372,715.58 € | 723,422.20 € | 369,077.14 € |
| - Tax on EBIT | 799,759.35 € | - € | - € | 12,943.16 € | 98,769.63 € | 191,706.88 € | 97,805.44 € |
| = EBIAT ( NOPAT) | 2,063,831.66 € | - 1,311,381.70 € | - 305,446.53 € | 35,898.96 € | 273,945.95 € | 531,715.32 € | 271,271.70 € |
| + Depreciation | 989,400.00 € | 1,028,938.97 € | 1,039,341.67 € | 1,047,937.22 € | 1,094,953.67 € | 1,046,076.18 € | 651,126.41 € |
| - Capex | - 852,000.00 € | - 787,366.49 € | - 915,106.14 € | - 1,046,311.47 € | - 1,181,631.26 € | - 1,393,637.63 € | - 824,045.14 € |
| - Increase in NWC | 281,615.94 € | 8,172,469.19 € | - 263,180.75 € | - 271,812.67 € | - 283,740.21 € | - 117,992.91 € | - 69,768.12 € |
| = FCF | 2,482,847.60 € | 7,102,659.98 € | - 444,391.75 € | - 234,287.96 € | - 96,471.86 € | 66,160.96 € | 21,741,827.72 € |
| - Interest Charges | 1,393,474.81 € | 1,367,821.42 € | 932,756.01 € | 941,479.78 € | 950,447.02 € | 973,915.23 € | 975,532.65 € |
| + Tax Shields | 402,714.22 € | - € | - € | 9,757.49 € | 69,595.51 € | 115,251.32 € | 99,128.39 € |
| + Δ Debt | - 332,113.58 € | - 5,632,437.32 € | 112,939.48 € | 116,091.54 € | 119,925.75 € | 20,939.34 € | - 12,629,426.36 € |
| =ECF | 1,159,973.43 € | 102,401.24 € | - 1,264,208.28 € | - 1,049,918.71 € | - 857,397.62 € | - 771,563.61 € | 8,235,997.10 € |
| Cost of Equity (Ke) | -15.57% | -3.14% | 0.92% | -11.76% | 61.82% | 12.13% | 11.53% |
| Factor | 0.84 | 0.82 | 0.83 | 0.73 | 1.18 | 2.42 | 2.70 |
| PV | **1,373,935.56 €** | **125,222.24 €** | **- 1,531,873.53 €** | **- 1,441,787.15 €** | **- 727,603.77 €** | **- 318,457.49 €** | **3,048,022.80 €** |
| PV ECF @ Ke @ beg. of the year | - **1,024,631.99 €** | | | | | | |
| +Non-operating assets @ beg. of the year | **1,686,660.00 €** | | | | | | |
| -Shareholders loans @ beg. of the year | **731.00 €** | | | | | | |
| Equity Value @ beginning of the year | **661,297.01 €** | | | | | | |
| Debt @ beg. | **18,040,183.00 €** | | | | | | |
| Firm Value @ beginning of the year | **17,015,551.0 €** | | | | | | |

*Table 16: Scenario 2 – Our revaluation (ECF method)*

#### 3.3.5.3.4. Remarks

From these three discounted cash flow methods, the first two – Adjusted Present Value and Capital Cash Flow – are specially important because they allow us to test if our "theory" about the WACC computation concerning the annual adjustment of the effective tax rate is correct, since these two methods consider explicitly the interest tax shields effectively realized in the moment of the tax payments. There are only tax savings, if they are realized at the same moment as the tax payments. On the other hand, by showing the value of interest tax shields autonomously, it brings attention to the importance of the discount rate applied to this cash flow.

The different discounted cash flow methods confirm our WACC computation method.

### 3.3.6. A most accurate way to compute the present value of tax shields

Finally, we will introduce the model already presented in the literature review (Ansay, 2010) to compute the discount rate of tax shields [22] and we will *endogenize* the computation of the interest rate ($K_D$). Note that in the assumption table (table 17), conversely with what happened with the assumptions of the previously presented valuations, $K_D$ varies every year and decreases as the leverage ratio increases, being here a false assumption, since it is determined by the model itself. Therefore, $K_D$ is presented

[22] Ansay (2010): $K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V}$. This model will be revisited in the conclusions of this paper.

in the assumptions only to contrast with the interest rates assumed for the previously presented valuations, in which the interest rate was a constant and an exogenous variable. For example, in scenario 1, the *endogenized* interest rate varies between 8.04% and 6.79% (table 17), whereas the interest rate previously used was of 7.724% (table 4).

As we will see, in the scenario with no renewal of the cooperation agreement with ARSLVT, since the leverage ratio in this scenario aggravates considerably, the maintenance of the same interest rates in both scenarios is unrealistic.

The tax rate was maintained at 28.9%. The tax savings effectively realized were taken into consideration in this model through the Loss Carry Forward feature, as one can see in the income statement (Table 19). Recall that previously, we opted to adjust this tax rate year by year to obtain the real tax saving realized (table 9).

We maintained the country risk premium adopted by Caixa BI because we consider that the country risk used by Caixa BI is correct and was correctly used. Caixa BI's valuation, as all valuations, refers to a specific point in time. Therefore, one needs to take into consideration the macroeconomic environment at the moment when the valuation was made. The WACC is the cost of opportunity of capital determined by the market in a certain point of time, being the expected rate of return on a portfolio of all firm's securities (bonds and stocks), adjusted for tax shields as a result of interest payments. The country risk has a direct influence on the adjusted risk-free rate and, therefore, on the cost of equity and cost of debt. The cost of opportunity of a Portuguese Treasury bond, with a similar maturity of valuation timeframe, was at the moment of the valuation 6.18%. This means that the investors have this investment alternative at their disposal at that moment in time. Thus, one could not consider a different adjusted risk-free rate than this one to compute the WACC. Indeed, the country risk premium was very high at the moment of the valuation and it would be, probably, unrealistic to stay at that level over the following years. However, this only reflects the existing macroeconomic environment at the time, which led to a contraction on investment and, as a result, to its decreasing price value. Though, as this is a current "hot topic" since the beginning of the last sovereign crisis, a simulation was made of the equity value in function of the country risk for scenario 2 using WACC method from Caixa BI, as in table 11, which you can see in appendix 2.

#### *3.3.6.1. Scenario 1 – renewal of the cooperation agreement with ARSLVT*

| Assumptions | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| Risk Free Rate | 6.18% | 6.18% | 6.18% | 6.18% | - |
| Market Risk Premium | 5.50% | 5.50% | 5.50% | 5.50% | - |
| Tax Rate | 28.90% | 28.90% | 28.90% | 28.90% | 28.90% |
| Asset Beta | 0.54 | 0.54 | 0.54 | 0.54 | - |
| Ku | 9.13% | 9.13% | 9.13% | 9.13% | - |
| Debt Beta | 0.34 | 0.32 | 0.29 | 0.11 | - |
| Interest rate (Kd) | 8.04% | 7.92% | 7.80% | 0.00% | - |
| EBIT | | 2,863,591 | 2,750,766 | 4,853,513 | 2,804,189 |
| Depreciation | | 989400 | 1042277 | 1223433 | 768242 |
| Investments | | 852000 | 1010221 | 1794066 | 1061242 |
| Increase in NWC | | -281616 | 153993 | 256876 | 151950 |

*Table 17: Scenario 1 – Revaluation Assumption (using our model)*

| Balance Sheet | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| VBook = | 27,920,178 | 28,405,547 | 28,525,481 | 34,608,573 | - |
| EBook = | 9,879,995 | 10,697,477 | 11,496,133 | 27,980,283 | - |
| DBook = | 18,040,183 | 17,708,069 | 17,029,347 | 6,628,290 | - |

*Table 18: Scenario 1 – Our revaluation Balance Sheet (using our model)*

| Income Statement | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| EBIT | | | 2,863,591 | 2,750,766 | 4,853,513 | 2,804,189 |
| -Debt Interests | =KD.D | | 1,456,935 | 1,423,031 | 594,880 | 450,167 |
| = EBT | | | 1,406,656 | 1,327,735 | 4,258,633 | 2,354,023 |
| Losses Carried Forward | $= MAX(K_{D(t)}D_{(t)} - EBIT_{(t)}, 0)$ | | 0 | 0 | 0 | 0 |
| -Accum. Losses Carried Forward | $= ALCF_{(t-1)} + LCF_{(t)} - MAX((EBT_{(t-1)} - \text{Taxable Income}_{(t-1)});0)$ | | 0 | 0 | 0 | 0 |
| =Taxable Income | =MAX((EBT-ALCF);0) | | 1,406,656 | 1,327,735 | 4,258,633 | 2,354,023 |
| - Taxes | | | 406,524 | 383,715 | 1,230,745 | 680,313 |
| = Net Income | | | **1,000,133** | **944,019** | **3,027,888** | **1,673,710** |
| | | | | | | |
| Tax Shield | =EBIT.Tc -Taxes | | **421,054** | **411,256** | **171,920** | **130,098** |

*Table 19: Scenario 1 – Our revaluation Income Statement (using our model)*

| Free Cash Flow | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| NOPAT[=EBIT.(1-Tc)] | | 2,063,832 | 1,984,290 | 3,466,727 | 2,002,953 |
| +Depreciation | | 989,400 | 1,042,277 | 1,223,433 | 768,242 |
| -Δ Working Capital | | -281,616 | 153,993 | 256,876 | 151,950 |
| - Investments | | 852,000 | 1,010,221 | 1,794,066 | 1,061,242 |
| = Free Cash Flow | | **2,482,848** | **1,862,353** | **2,639,218** | **34,946,666** |

*Table 20: Scenario 1 – Our revaluation Free Cash Flow (using our model)*

| Equity Cash Flow | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| Free Cash Flow | | | 2,482,848 | 1,862,353 | 2,639,218 | 34,946,666 |
| -Debt Interest | | | 1,456,935 | 1,423,031 | 594,880 | 450,167 |
| +Tax Shield | | | 421,054 | 411,256 | 171,920 | 130,098 |
| + Δ Debt | | | - 332,114 | - 678,722 | - 1,902,881 | - 6,628,290 |
| Equity Cash Flow | | | **1,114,854** | **171,856** | **313,377** | **27,998,308** |
| PV ECF @ KE | | **12,180,248** | | | | |
| +Non-Operating Assets | | **1,686,660** | | | | |
| -Shareholders Loans | | **731** | | | | |
| =Equity Value | | **13,866,177** | | | | |
| =Firm Value | Equity Value+Net Debt - Non-Operating Assets + Shareholders Loans | **30,220,431** | | | | |

*Table 21: Scenario 1 – Our revaluation ECF Method (using our model)*

| Capital Cash Flow | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| **Free Cash Flow** | | | 2,482,848 | 1,862,353 | 2,639,218 | 34,946,666 |
| **+ Tax Shield** | | | 421,054 | 411,256 | 171,920 | 130,098 |
| **= Capital Cash Flow** | | | **2,903,902** | **2,273,609** | **2,811,138** | **35,076,764** |
| **PV FCF @ KU** | **28,051,817** | | | | | |
| **+TS @ KTS** | **2,168,613** | | | | | |
| **=Firm Value** | **30,220,431** | | | | | |

*Table 22: Scenario 1 – Our revaluation CCF Method (using our model)*

| Cash Flow to debt | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| **Debt Interest** | | | 1,456,935 | 1,423,031 | 594,880 | 450,167 |
| **-Δ Debt** | | - | 332,114 | - 678,722 | - 1,902,881 | - 6,628,290 |
| **=Cash Flow to debt** | | | 1,789,048 | 2,101,754 | 2,497,761 | 7,078,456 |
| **Debt Value = PV CFd @ KD** | **18,040,183** | | | | | |
| **-Non-Operating Assets** | **1,686,660** | | | | | |
| **+Shareholders Loans** | **731** | | | | | |
| **+Equity Value** | **13,866,177** | | | | | |
| **= Firm Value** | **30,220,431** | | | | | |

*Table 23: Scenario 1 – Our revaluation CCd Method (using our model)*

| Market Value Discount Rates | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| KU= | $Ku=KF+KM.\beta u$ | 9.13% | 9.13% | 9.13% | 9.13% | - |
| KD = | $K_{D(t)} = K_F + (K_U - R_F)*(D_{(t)}/V_{U(t)})$ | 8.08% | 8.04% | 7.92% | 6.79% | - |
| KE-VTS = | KE-VTS(t) = KU + (KU - KD(t))*(D(t)/(E - VTS)(t)) | 11.02% | 10.98% | 10.87% | 9.74% | - |
| KTS = | KTS(t) = KD(t) + (KE-VTS(t) - KD(t))*(D(t)/V(t)) | 9.83% | 9.77% | 9.56% | 7.40% | - |
| KE(1) = | $K_{E(t)} = K_U + (K_U - K_{D(t)})*(D_{(t)}/E_{(t)}) - (K_U - K_{TS(t)})*(V_{TS(t)}/E_{(t)})$ (1) | 10.81% | 10.79% | 10.70% | 9.73% | - |
| KE(2)= | $K_{E(t)} = K_{E\text{-}VTS(t)}*(E - VTS)_{(t)}/E_{(t)}+K_{TS(t)}*(V_{TS(t)}/E_{(t)})$ (2) | 10.81% | 10.79% | 10.70% | 9.73% | - |

*Table 24: Scenario 1 – Our revaluation Market Value Discount Rates (using our model)*

| Market Value Balance Sheet | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| V = VU + VTS = | $V_{(t)} = E_{(t)} + D_{(t)} = V_{U(t)} + V_{TS(t)} = V_{Book} + MVA_{(t)}$ | 30,220,431 | 30,090,134 | 30,575,497 | 32,144,932 | - |
| VU = | $V_{U(t)} = Sum_{n=1,\dots,\infty}(FCF_{(t+n)}/(1+K_U)^n)$ | 28,051,817 | 28,129,310 | 28,834,370 | 32,023,797 | - |
| VTS = | $V_{TS(t)} = Sum_{n=1,\dots,\infty}(TS_{(t+n)}/(1+K_{TS(t+n)})^n)$ | 2,168,613 | 1,960,824 | 1,741,127 | 121,135 | - |
| V = E + D = | | 30,220,431 | 30,090,134 | 30,575,497 | 32,144,932 | - |
| E = | $E_{(t)} = E_{(t)} - V_{TS(t)} + V_{TS(t)}$ | 12,180,248 | 12,382,065 | 13,546,150 | 25,516,642 | - |
| E - VTS = | | 10,011,634 | 10,421,241 | 11,805,023 | 25,395,507 | - |
| VTS = | | 2,168,613 | 1,960,824 | 1,741,127 | 121,135 | - |
| D = | $D_{(t)}= MIN(D_{Book(t)}, V_{(t)})$ | 18,040,183 | 17,708,069 | 17,029,347 | 6,628,290 | - |
| V = VBOOK + MVA = | | 30,220,431 | 30,090,134 | 30,575,497 | 32,144,932 | - |
| VBook = | | 27,920,178 | 28,405,547 | 28,525,481 | 34,608,573 | - |
| MVA = | $MVA_{(t)}$ = Operating $MVA_{(t)}$ + Financing $MVA_{(t)}$ | 2,300,253 | 1,684,588 | 2,050,016 | - 2,463,641 | - |
| erating MVA = | Operating $MVA_{(t)} = Sum_{n=1,\dots,\infty}((ROIC_{(t+n)} - K_U)V_{Book(t+n-1)}/(1+K_U)^n)$ | 131,640 | - 276,237 | 308,889 | - 2,584,776 | - |
| nancing MVA = | Financing $MVA_{(t)} = V_{TS(t)}$ | 2,168,613 | 1,960,824 | 1,741,127 | 121,135 | - |

*Table 25: Scenario 1 – Our revaluation Market Value Balance Sheet (using our model)*

If we compare the firm value (€29,469,914) and the equity value (€13,115,659), obtained by Caixa BI, with those we now obtained of €30,220,431 and €13,866,177[23] respectively,

[23] Equity book value + MVA + Non-operating assets - Shareholders loans

even though different, they are not as different as we would expect. However, this is not completely unexpected due to the fact that the tax savings are one hundred per cent realized and that they were discounted to $K_{TS}$ (discount rate that includes the operational/realization risk and the risk of default in debt), which varies between 9.83% and 7.40% (table 24). Whereas Caixa BI, by using the WACC method, discounted the tax shields to $K_U$, which was of 9.13%[24] (as we can easily infer from the assumptions presented by Caixa BI). Since the discount rate used by Caixa BI is not substantially different from the one used now to discount the tax shields and since, in this scenario, they were completely realized, the firm and equity values could not be substantially different. However, as we will see further ahead, since the discount rate used by Caixa BI ($K_U$) is slightly higher than the $K_{TS}$ computed by our model, the Caixa BI´s standard WACC undervalues to some extent CVP – SGH, S.A..

In the table 22, we computed the capital cash flow, but in reality we did not compute the firm value by the CCF method, since the CCF method always assumes that the tax shields are discounted at $K_U$. As we now have a more adequate discount rate that better captures the risk characteristics of tax shields – $K_{TS}$, we had to discount the tax shields to this discount rate. Thus, the computation made is more similar to the APV method than to the CCF method, though both methods are already similar.

The market value added method shows the value creation sources year by year. Thus, the firm value is equal to the book value plus the market value added. This method consists in computing the operating market value added (OMVA), to which we add the present value of tax shields (FMVA). The Market Value Added can be obtained by the difference between the Equity Market Value and the Equity Book Value. This difference between the Equity Market Value and the Equity Book Value can be explained by the company's ability to create value from the business activity itself and by the value added by the leverage. Note that to obtain an equity market value – €13,866,177– that can be compared with the ones obtained by the other discounted cash flow methods, it is necessary to add to the equity book value the MVA and the non-operating assets, and subtract the shareholders loans. The equity value obtained in the table 25 is the same as the present value of the equity cash flows discounted at $K_E$ (€12,180,248). The MVA method allows us to identify that almost all of the MVA has its source on the financing MVA, whereas

[24] $K_U = K_F + K_M \cdot \beta_U$

only a slightly part has its source on the operating MVA[25] (table 25). This means that the enterprise value is not created by the company's business activity, but by its leverage. In other words, the business itself almost does not have value.

#### *3.3.6.2. Scenario 2 – no renewal of the cooperation agreement with ARSLVT*

| Assumptions | | | | | |
|---|---|---|---|---|---|
| | **Y0** | **Y1** | **Y2** | **Y10** | **Y11** |
| **Risk Free Rate** | 6.2% | 6.2% | 6.2% | 6.2% | - |
| **Market Risk Premium** | 5.5% | 5.5% | 5.5% | 5.5% | - |
| **Tax Rate** | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% |
| **Asset Beta** | 0.54 | 0.54 | 0.54 | 0.54 | - |
| **Ku** | 9.13% | 9.13% | 9.13% | 9.13% | - |
| **Debt Beta** | 0.60 | 0.62 | 0.68 | 0.34 | - |
| **Interest rate (Kd)** | 9.46% | 9.61% | 9.92% | 8.05% | - |
| **EBIT** | | 2,863,591 | -1,311,382 | 723,422 | 369,077 |
| **Depreciation** | | 989400 | 1028939 | 1046076 | 651126 |
| **Investments** | | 852000 | 787366 | 1393638 | 824045 |
| **Increase in NWC** | | -281616 | -8172469 | 117993 | 69768 |
| **D/VU** | | 1.163 | 1.269 | 0.634 | - |
| **D/V** | | 1.117 | 1.181 | 0.631 | - |

*Table 26: Scenario 2 – Revaluation Assumption (using our model)*

| Balance Sheet | | | | | |
|---|---|---|---|---|---|
| | **Y0** | **Y1** | **Y2** | **Y10** | **Y11** |
| **VBook =** | 27,920,178 | 28,405,547 | 20,034,064 | 12,118,571 | **-** |
| **EBook =** | 9,879,995 | 10,697,477 | 7,958,432 | -510,856 | **-** |
| **DBook =** | 18,040,183 | 17,708,069 | 12,075,632 | 12,629,426 | **-** |

*Table 27: Scenario 2 – Our revaluation Balance Sheet (using our model)*

| Income Statement | | | | | |
|---|---|---|---|---|---|
| | **Y0** | **Y1** | **Y2** | **Y10** | **Y11** |
| **EBIT** | | 2,863,591 | -1,311,382 | 723,422 | 369,077 |
| **-Debt Interests** | | 1,705,796 | 1,701,139 | 1,035,061 | 1,016,536 |
| **= EBT** | | 1,157,795 | -3,012,520 | -311,639 | -647,459 |
| **Losses Carried Forward** | | 0 | 3,012,520 | 311,639 | 647,459 |
| **-Accum. Losses Carried Forward** | | 0 | 3,012,520 | 7,856,082 | 8,503,541 |
| **=Taxable Income** | | 1,157,795 | 0 | 0 | 0 |
| **- Taxes** | | 334,603 | 0 | 0 | 0 |
| **= Net Income** | | **823,192** | **-3,012,520** | **-311,639** | **-647,459** |
| | | | | | |
| **Tax Shield** | | 492,975 | 0 | 209,069 | 106,663 |

*Table 28: Scenario 2 – Our revaluation Income Statement (using our model)*

[25] OMVA = $V_U$ – Book Value

| Free Cash Flow | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| EBIT | | 2863591 | -1311382 | 723422 | 369077 |
| +Depreciation | | 989400 | 1028939 | 1046076 | 651126 |
| -Δ Working Capital | | -281616 | -8172469 | 117993 | 69768 |
| - Investments | | 852000 | 787366 | 1393638 | 824045 |
| = Free Cash Flow | | 2482848 | 7102660 | 66161 | 21741828 |
| Unlevered Firm Value | 16,230,633 | | | | |

*Table 29: Scenario 2 – Our revaluation Free Cash Flow (using our model)*

| Equity Cash Flow | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| Free Cash Flow | | 2,482,848 | 7,102,660 | 66,161 | 21,741,828 |
| -Debt Interest | | 1,705,796 | 1,701,139 | 1,035,061 | 1,016,536 |
| +Tax Shield | | 492,975 | - | 209,069 | 106,663 |
| + Δ Debt | - | 332,114 | - 5,632,437 | 20,939 | - 12,629,426 |
| Equity Cash Flow | | **937,913** | **- 230,916** | **- 738,892** | **8,202,529** |
| PV ECF @ KE | - 816,096 | | | - | - |
| +Non-Operating Assets | 1,686,660 | | | | |
| -Shareholders Loans | 731 | | | | |
| = Equity Value | 869,833 | | | | |
| =Firm Value | 17,224,087 | | | | |

*Table 30: Scenario 2 – Our revaluation ECF Method (using our model)*

| Capital Cash Flow | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| Free Cash Flow | | 2482848 | 7102660 | 66161 | 21741828 |
| + Tax Shield | | 492975 | 0 | 209069 | 106663 |
| = Capital Cash Flow | | **2975823** | **7102660** | **275230** | **21848491** |
| PV FCF @ KU | 16,230,633 | | | | |
| + PV TS @ KTS | 993,454 | | | | |
| = Firm Value | 17,224,087 | | | | |

*Table 31: Scenario 2 – Our revaluation CCF Method (using our model)*

| Cash Flow to Debt | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| Debt Interest | | 1,705,796 | 1,701,139 | 1,035,061 | 1,016,536 |
| -Δ Debt | - | 332,114 | - 5,632,437 | 20,939 | - 12,629,426 |
| =Cash Flow to debt | | **2,037,910** | **7,333,576** | **1,014,121** | **13,645,962** |
| Debt Value = PV CFd @ KD | 18,040,183 | | | | |
| - Non-Operating Assets | 1,686,660 | | | | |
| + Shareholders Loans | 731 | | | | |
| + Equity Value | 869,833 | | | | |
| = Firm Value | 17,224,087 | | | | |

*Table 32: Scenario 2 – Our revaluation CFd Method (using our model)*

| Market Value Discount Rates | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| KU= | 9.13% | 9.13% | 9.13% | 9.13% | - |
| KD = | 9.46% | 9.61% | 9.92% | 8.05% | - |
| KE-VTS = | 12.40% | 12.55% | 12.86% | 10.99% | - |
| KTS = | 12.54% | 12.90% | 13.40% | 9.91% | - |
| KE(1) = | #DIV/0! | #DIV/0! | #DIV/0! | 10.98% | - |
| KE(2)= | #DIV/0! | #DIV/0! | #DIV/0! | 10.98% | - |

*Table 33: Scenario 2 – Our revaluation Market Value Discount Rates (using our model)*

| Market Value Balance Sheet | | | | | |
|---|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 | Y11 |
| V = VU + VTS = | 17224086.79 | 15854244.99 | 10222188.83 | 20020433.21 | - |
| VU = | 16230633.17 | 15229184.81 | 9516520.07 | 19923384.39 | - |
| VTS = | 993453.63 | 625060.18 | 705668.76 | 97048.81 | - |
| | 0.00 | 0.00 | 0.00 | 0.00 | - |
| V = E + D = | 17224086.79 | 15854244.99 | 10222188.83 | 20020433.21 | - |
| E = | 0.00 | 0.00 | 0.00 | 7391006.85 | - |
| E - VTS = | -993453.63 | -625060.18 | -705668.76 | 7293958.04 | - |
| VTS = | 993453.63 | 625060.18 | 705668.76 | 97048.81 | - |
| D = | 17224086.79 | 15854244.99 | 10222188.83 | 12629426.36 | - |
| | 0.00 | 0.00 | 0.00 | 0.00 | - |
| V = VBOOK + MVA = | 17224086.79 | 15854244.99 | 10222188.83 | 20020433.21 | - |
| VBook = | 27920177.54 | 28405546.67 | 20034063.85 | 12118570.63 | - |
| MVA = | -10696090.75 | -12551301.68 | -9811875.02 | 7901862.58 | - |
| Operating MVA = | -11689544.37 | -13176361.86 | -10517543.78 | 7804813.76 | - |
| Financing MVA = | 993453.63 | 625060.18 | 705668.76 | 97048.81 | - |

*Table 34: Scenario 2 – Our revaluation Market Value Balance Sheet (using our model)*

As we did previously, we introduced in the assumptions table some parameters which are not real assumptions, but endogenous variables: the debt beta, the interest rate ($K_D$) and the leverage ratio. As can be seen, the interest rate now varies between 9.46% and 8.05%, which contrasts with the interest rates assumed by Caixa BI of 7.72% and with the interest rates from the first scenario that varied between 8.04% and 6.79%. This is due to the fact that the interest rates are now being *endogenized* in function of the financial risk. Visible when looking at the debt beta that varies between 0.60 and 0.34 (table 26) or at the leverage ratio (D/V) that varies between 1.05 and 0.63 (table 26). In the first scenario, the debt beta varied between 0.34 and 0.11 and the leverage ratio varied between 0.60 and 0.21 (table 17).

We know that the interest rate varies accordingly with financial leverage. Therefore, strictly speaking, one should not use a fixed interest rate. However, if that is acceptable in the first scenario, it is completely unacceptable in the second scenario (without renewal

of the cooperation agreement with ARSLVT) given the fact that leverage increases abruptly, as one can verify by the leverage ratios.

We believe, therefore, that the debt ratio and interest rate's *endogenizations,* introduced by us, are indispensable to the firm valuation in this second scenario.

Another relevant point, in which this scenario differs from the first one, lies on the fact that the tax savings are not realized or just partially realized. The loss carried forward feature allows us to take that into consideration. This point is visible in the income statement (table 27).

Since the model was built with a restriction, so that the debt will never exceed the assets, and since the equity value, excluding the non-operating assets, is negative, the model computes the equity value as zero for the first years. Thus, the cost of equity ($K_E$) for year 0, 1, 2 and 3 tends to infinity because the equity value (E) is close to zero. This fact makes the direct computation of the present value of the equity cash flows (table 30) impossible. This value can be, however, obtained in an indirect way through the other discounted cash flow methods or through the MVA method. Therefore, if we subtract the net debt to the firm value or add to the equity book value the MVA, we obtain - € 816,096, which corresponds to the present value of the equity cash flows (table 30). The present value of the equity cash flows allows us to compute the equity value of € 869 833[26].

By the MVA method, the book value of € 27,920,178 is reduced by 42% by CVP-SGH S.A.'s operating activity. The adverse effect of the operating MVA is only slightly compensated by the financing MVA in such way that the firm value of € 17,224,087 is only 61% of the book value, which shows the importance of the cooperation agreement with ARSLVT with CVP-SGH S.A..

Therefore, and since the recommendation of the Portuguese Audit Court was implemented, the CVP-SGH S.A. will have to diversify its markets in order to overcome the void left by the non-renewal of the agreement. We know today that this was CVP-SHG S.A.'s strategy.

[26] The Equity value computed through the firm value and the equity book value:

| | |
|---|---|
| **Firm Value** | 17,224,087 |
| **-Net Debt** | 18,040,183 |
| **+Non-Operating Assets** | 1,686,660 |
| **-Shareholders Loans** | 731 |
| **=Equity Value** | **869,833** |

| | | |
|---|---|---|
| **Equity Book Value** | | 9,879,995 |
| **+MVA** | - | 10,696,091 |
| **+Non-Operating Assets** | | 1,686,660 |
| **- Shareholders Loans** | | 731 |
| **= Equity Value** | | **869,833** |

The fact that the firm value was computed year by year for all years of the forecast period allows us to see that, despite the fact that MVA does not have significant changes over the forecast period, the composition of MVA evolves positively, relatively to operating MVA, even though only reaching positive values in the last two years of the forecast period. In other words, the operating MVA is substituting slowly the role of the financing MVA in the MVA's structure.

#### *3.3.6.3. Discount rate*

The biggest differences between this valuation model and Caixa BI's valuation approach were the *endogenization* of the leverage ratio and interest rate, as we already referred previously, as well as the *endogenization* of the discount rate applied to tax shields, which will be the object of this topic.

As already mentioned in the literature review, the academic world still did not reach an agreement regarding this discount rate. Some authors defend that the tax shields should be discounted at $K_D$, which includes the risk of default in debt. Other authors defend discounting at $K_U$, which incorporates the business risk. There are still other authors who have been proposing models that in some extent intend to incorporate both risks.

The model presented in the literature review is (Ansay, 2010):

$$K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V} \tag{80}$$

Which, of course, being this a controversial topic, we tested in this case study

In scenario 1, $K_{TS}$ varies between 9.83% and 7.40%, while $K_D$ varies between 8.08% and 6.79% and $K_U$ is 9.13%, which are the other rates also used to discount the tax shields. Since $K_D$ already incorporates the risk of default. The $K_{TS}$ in scenario 2 varies between 12.54% and 9.91%, while $K_D$ varies between 9.46% and 8.05% and $K_U$ is 9.13%. The difference between the $K_{TS}$ of scenario 1 and scenario 2 is due to the huge increase of debt. The difference between $K_D$ and $K_{TS}$ is mainly associated with the business risk in proportion of the leverage, i.e. the higher the financial leverage the higher the credit spread, but also the higher the level of debt the higher the financial expenses and the higher the risk of total or partial non-realization of tax savings.

## 4. CONCLUSION: THE CHOICE OF THE DISCOUNT RATE APPLIED TO TAX SHIELDS

In most financial literature, the volatility of interest tax shields is frequently associated with debt risk. This dissertation proposes that the volatility of tax shields is also associated with the operational risk. It is the existence of EBIT that allows the company to earn tax shields. Although interest expenses are at the origin of debt tax shields, its realization depends on the amount of EBIT.

Without going into details already addressed in the literature review, it can be stated that there are two main general discount tax shields approaches. The first states that tax shields should be discounted at cost of debt (MM (1963), Myers (1974), Inselbag and Kaufold (1997), Luehrman (1997), among others). The second states that tax shields should be discounted at the unlevered cost of equity (HP (1985), Ruback (2002), Tham and Velez-Pajera (2001, 2004), and others). Miles and Ezzell (1985) suggest to discount tax shields at cost of debt for year t and at the cost of unlevered equity for all subsequent years from t+1. Kaplan and Ruback (1995) agree that tax shields are relevant and might be an important part for of the firm's value. One knows that some companies cannot use their tax shields in the current period, however receive them in the future when losses carried forward are allowed.

The state of the art can be summarized as follows:

(a) If D is a fixed amount, then the tax shields should be discounted at $K_D$,
(b) If D/V is fixed, then the tax shields should be discounted at $K_U$.

In this paper we did not want to confine ourselves to solving just one case study, but rather take inferences susceptible to be used in most cases. Thus, we believe we can conclude that:

(a) We showed the advantages of *endogenizing* the financial leverage (D/V) and the interest rate ($K_D$), and
(b) Knowing that this topic is highly controversial and will remain like that, we have proposed the use of a reliable model to determine the discount rate of tax shields.

We will now present the closing arguments regarding the choice of the discount rate applied to tax shields.

To achieve that desideratum, we built a graph that relates the different discount rates: $K_D$, $K_{TS}$, $K_{E\text{-}VTS}$, $K_E$ and $K_U$, on the ordinate, with the leverage ratio: D/V and $D/V_U$, on the abscissas. For this purpose, we used the values obtained in the scenario 2 case study.

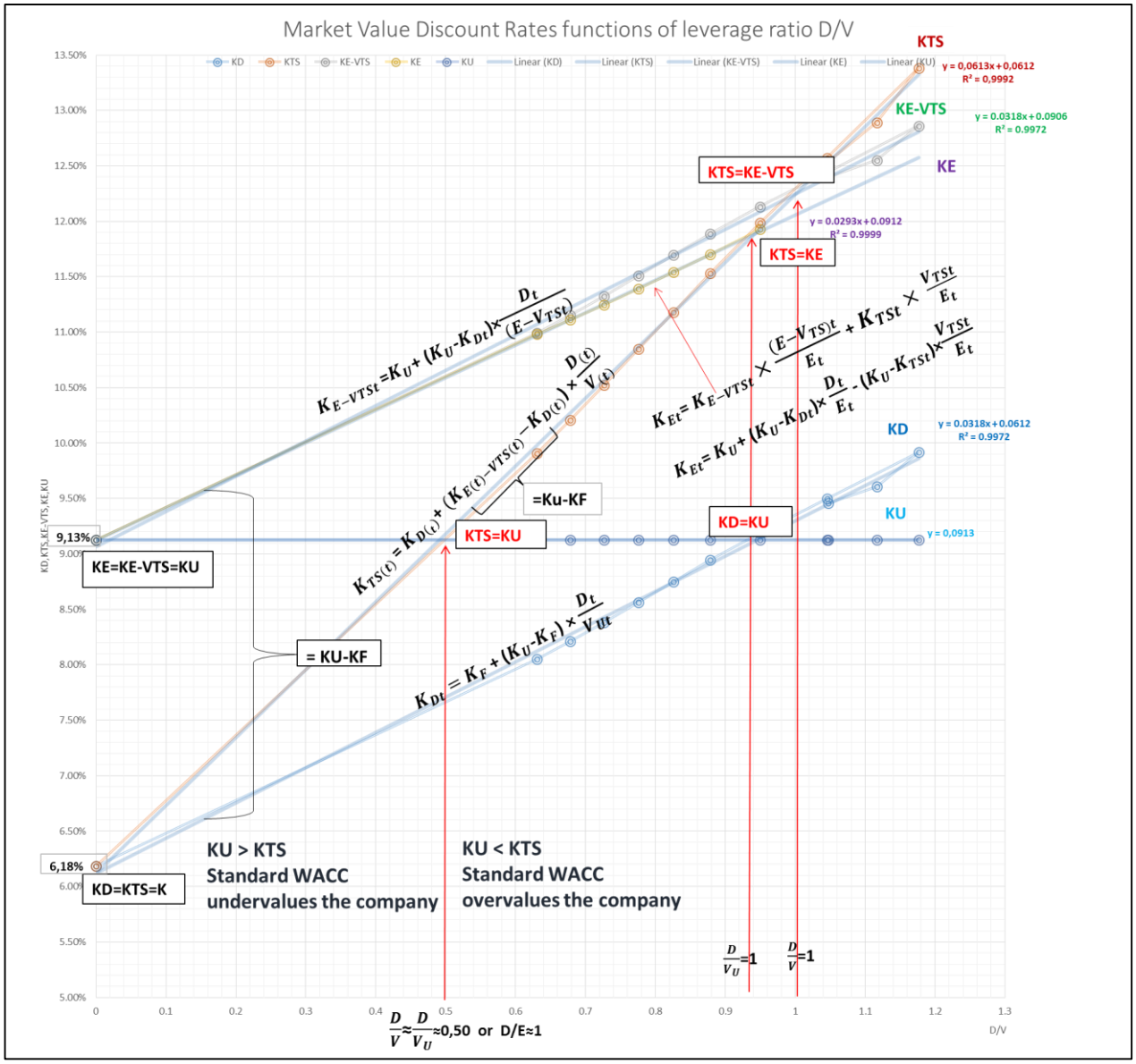


*Figure 4: Graphical representation of the case study – Market Value Discount Rates function of leverage ratio D/V*

| Market Value Discount Rates | | | | | | |
|---|---|---|---|---|---|---|
| | | Y0 | Y1 | Y2 | Y10 | Y11 |
| KU= | | 9.13% | 9.13% | 9.13% | 9.13% | - |
| KD = | $K_{D(t)} = R_F + (K_U - R_F)*(D_{(t)}/V_{U(t)})$ | 9.46% | 9.61% | 9.92% | 8.05% | - |
| KE-VTS = | KE-VTS(t) = KU + (KU - KD(t))*(D(t)/(E - VTS)(t)) | 12.40% | 12.55% | 12.86% | 10.99% | - |
| KTS = | KTS(t) = KD(t) + (KE-VTS(t) - KD(t))*(D(t)/V(t)) | 12.54% | 12.90% | 13.40% | 9.91% | - |
| KE(1) = | $K_{E(t)} = K_U + (K_U - K_{D(t)})*(D_{(t)}/E_{(t)}) - (K_U - K_{TS(t)})*(V_{TS(t)}/E_{(t)})$ (1) | #DIV/0! | #DIV/0! | #DIV/0! | 10.98% | - |
| KE(2)= | $K_{E(t)} = K_{E\text{-}VTS(t)}*(E - VTS)_{(t)}/E_{(t)} + K_{TS(t)}*(V_{TS(t)}/E_{(t)})$ (2) | #DIV/0! | #DIV/0! | #DIV/0! | 10.98% | - |

*Table 35: Scenario 2 – Our revaluation Market Value Discount Rates (using our model)*

| MARKET VALUE RATIOS | | | | |
|---|---|---|---|---|
| | Y0 | Y1 | Y2 | Y10 |
| $D/V_U$ = | 1.111 | 1.163 | 1.269 | 0.634 |
| D/V = | 1.047 | 1.117 | 1.181 | 0.631 |

*Table 36: Scenario 2 – Our revaluation Market Value Ratios (using our model)*

As can been seen in the graph:

- $K_{TS} = K_D$, when D = 0
- $K_{TS} < K_{E\text{-}VTS}$, when D/V < 1
- $K_{TS} = K_E$, when $D/V_U = 1$
- $K_{TS} = K_{E\text{-}VTS}$, when D/V = 1
- $K_{TS} > K_{E\text{-}VTS}$, when D/V > 1

Existing a broad consensus that $K_{TS}$ is between $K_D$ and $K_E$ or more precisely, according to the literature review, that $K_{TS}$ is between $K_D$ and $K_{E\text{-}VTS}$, wherein $K_E < K_{E\text{-}VTS}$, the solution will be to find a model that links $K_D$ and $K_{E\text{-}VTS}$.

The $K_{TS}$ determined according with the following formula (Ansay, 2010):

$$K_{TS} = K_D + \left(K_{E-V_{TS}} - K_D\right)\frac{D}{V} \tag{80}$$

Links linearly (observation *a posteriori*) $K_D$ to $K_{TS}$, when:

- D = 0 and the discount rate is equal to the tax free rate ($K_F$), which is 6.18% in this case, and

Links linearly $K_{E\text{-}VTS}$ to $K_{TS}$, when:

- D = V and the discount rate is equal to 12.25%.

Observing anatomically the model, $K_{TS}$ is obtained by adding to $K_D$ (interest rate), which includes the risk of default, the part $(K_{E-V_{TS}} - K_{D_t})$. It can be observed that the difference between the shareholders required rate of return and the interest rate ($K_D$) is equal to $K_U$ – $K_F$, meaning the business risk premium. This is how business risk is incorporated by $K_{TS}$ and this risk premium varies proportionally with the degree of indebtedness. This is taken into consideration in the model by multiplying $\left(K_{E-V_{TS}} - K_D\right)$ by D/V. The logic behind, is that the tax shield realization risk is proportional to the weight of debt.

From the observation of the graph, we conclude that from the two discount rates most applied to tax shields: $K_U$ and $K_D$, $K_D$ is the least credible. Only intersects $K_{TS}$ when D = 0 and when $K_D$=$K_F$.

An important inference that can be drawn from the graphical representation of the case study is that $K_{TS}$ interests $K_U$ when $D/V \approx 0.5$ or $D/E \approx 1$. Additionally, since the WACC discounts the tax shields to $K_U$, through the observation of the graph we can conclude that if the $D/V < 0.5$ and $K_U > K_{TS}$, then the standard WACC undervalues the company. If $D/V > 0.5$ and $K_U < K_{TS}$, then the standard WACC overvalues the company.

Regarding the suggestions of the state of the art, our empirical study only allows us to endorse the second assumption when $D/V \approx 0.5$ and the recommendation of point (a) only applies to APV and CCF methods but not to the WACC method. Moreover, since fixed debt and fixed leverage ratios are rarely found in the real world and since we did not find in the literature consolidated recommendations for the case when the leverage ratio varies, we therefore believe that the suggestion of the model used to discount the tax shields is an adequate compromise.

## 5. LIMITATIONS AND FURTHER DEVELOPMENTS

The discounted cash flow methods have been validated through the share prices on the stock market. Therefore, the next step will be to verify if the values obtained by the DCF valuation methods with the tax shields discounted at the rate proposed in this dissertation ($K_{TS}$) introduces robustness to the DCF valuation methods when checked with the share prices with statistical significance.

However, this goes beyond the scope of this thesis. Moreover, note that CVP-SGH S.A. is not listed on the stock market.

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# APPENDIX

## APPENDIX 1 – ORIGINAL TABLES FROM CAIXA BI'S FIRM VALUATION AND THE PORTUGUESE AUDIT COURT FIRM VALUATION SIMULATION

| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|---|
| **Taxas de Atualização** | | | | | | | | |
| **CUSTO DOS CAPITAIS PRÓPRIOS** | | | | | | | | |
| Taxa de juro sem risco | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Prémio de risco do país | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% | 4.51% |
| Taxa de juro sem risco ajustada | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% | 6.18% |
| Beta dos Ativos | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 |
| Alavancagem Financeira (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| Alavancagem Financeira (D/(D+E)) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| Taxa de imposto | 28.90% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% |
| Beta dos Capitais Próprios | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 |
| Prémio de risco do mercado | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% | 5.50% |
| Beta dos capitais próprios | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 |
| Prémio de risco | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% | 3.80% |
| **Custo dos capitais próprios (ke)** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** | **9.98%** |
| **CUSTO DA DÍVIDA** | | | | | | | | |
| Taxa de juro sem risco | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% | 1.67% |
| Spread | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% |
| Taxa de imposto | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% | 28.9% |
| **Custo líquido da dívida (kd)** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** | **5.49%** |
| **ESTRUTURA DE CAPITAL** | | | | | | | | |
| Alavancagem Financeira (D/E) | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% | 41.0% |
| D / (D+E) | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% | 29.1% |
| E / (D+E) | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% | 70.9% |
| **WACC** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** | **8.68%** |

*Table 37: Scenario 1 – Caixa BI Firm valuation's assumptions original table (retrieved from Portuguese Audit Court's Firm valuation simulation included in appendix in the Portuguese Audit Court report, 2011)*

| Free Cash Flow | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2012 | 2013E | 2014E | 2015E | 2016E | 2017E | 2018E | 2019E | 2020E | 2021E | 2022E | 2023E |
| **Imobilizado** | **10,703,499** | **10,566,099** | **10,534,043** | **10,640,627** | **10,890,205** | **11,247,686** | **11,686,509** | **12,238,802** | **12,956,967** | **13,597,706** | **14,168,338** | **14,461,338** |
| **Fundo de Maneio** | **16,430,751** | **16,149,135** | **16,303,128** | **16,418,585** | **16,849,926** | **17,305,564** | **17,770,900** | **18,016,214** | **18,265,323** | **18,518,286** | **18,775,162** | **18,927,325** |
| **Vendas e Serviços Prestados** | **32,603,820** | **44,097,054** | **44,538,683** | **45,102,382** | **45,693,490** | **46,329,825** | **46,976,005** | **47,632,180** | **48,298,507** | **48,975,141** | **49,662,244** | **29,376,652** |
| **EBITDA** | **-385,318** | **3,852,991** | **3,793,043** | **4,298,509** | **4,815,849** | **5,348,640** | **5,895,145** | **5,939,554** | **5,984,650** | **6,030,444** | **6,076,946** | **3,572,431** |
| Mg EBITDA | | 8.7% | 8.5% | 9.5% | 10.5% | 11.5% | 12.5% | 12.5% | 12.4% | 12.3% | 12.2% | 12.2% |
| Amortizações | | 989,400 | 1,042,277 | 1,068,006 | 1,093,976 | 1,160,491 | 1,258,202 | 1,168,435 | 1,026,636 | 1,128,505 | 1,223,433 | 768,242 |
| EBIT | | 2,863,591 | 2,750,766 | 3,230,503 | 3,721,873 | 4,188,149 | 4,636,943 | 4,771,119 | 4,958,014 | 4,901,939 | 4,853,513 | 2,804,189 |
| Imposto sobre EBIT | | 799,759 | 766,476 | 907,998 | 1,052,952 | 1,190,504 | 1,322,898 | 1,362,480 | 1,417,614 | 1,401,072 | 1,386,786 | 801,236 |
| Taxa de imposto | | 27.9% | 27.9% | 28.1% | 28.3% | 28.4% | 28.5% | 28.6% | 28.6% | 28.6% | 28.6% | 28.6% |
| **NOPLAT** | | **2,063,832** | **1,984,290** | **2,322,505** | **2,668,920** | **2,997,645** | **3,314,045** | **3,408,639** | **3,540,400** | **3,500,867** | **3,466,727** | **2,002,953** |
| Amortizações | | 989,400 | 1,042,277 | 1,068,006 | 1,093,976 | 1,160,491 | 1,258,202 | 1,168,435 | 1,026,636 | 1,128,505 | 1,223,433 | 768,242 |
| CAPEX | | 852,000 | 1,010,221 | 1,174,590 | 1,343,554 | 1,517,973 | 1,697,024 | 1,720,729 | 1,744,800 | 1,769,244 | 1,794,066 | 1,061,242 |
| %Vendas e Serviços Prestados | | 1.9% | 2.3% | 2.6% | 2.9% | 3.3% | 3.6% | 3.6% | 3.6% | 3.6% | 3.6% | 3.6% |
| Investimento em Fundo de Maneio | | -281,616 | 153,993 | 115,457 | 431,341 | 455,638 | 465,336 | 245,314 | 249,109 | 252,963 | 256,876 | 151,950 |
| %Vendas e Serviços Prestados | | -0.6% | 0.3% | 0.3% | 0.9% | 1.0% | 1.0% | 0.5% | 0.5% | 0.5% | 0.5% | 0.5% |
| **Free Cash Flow** | | **2,482,848** | **1,862,353** | **2,100,463** | **1,988,002** | **2,184,525** | **2,409,887** | **2,611,031** | **2,573,127** | **2,607,165** | **2,639,218** | **1,558,003** |
| Fator Desconto | | 0.92 | 0.85 | 0.78 | 0.72 | 0.66 | 0.61 | 0.56 | 0.51 | 0.47 | 0.44 | 0.41 |
| **Discounted Free Cash Flow** | | **2,284,562** | **1,576,769** | **1,636,341** | **1,425,045** | **1,440,860** | **1,462,562** | **1,458,084** | **1,322,162** | **1,232,665** | **1,148,166** | **645,672** |
| **NPV FCF 2012-22E** | | **15,632,889** | | | | | | | | | | |
| **NPV Valor de Liquidação** | | **13,837,025** | | | | | | | | | | **33,388,663** |
| **Enterprise Value** | | **29,469,914** | | | | | | | | | | |
| Ativos Não Operacionais | | 1,686,660 | | | | | | | | | | |
| Dívida Financeira Líquida | | 18,040,184 | | | | | | | | | | |
| Empréstimos Accionistas | | 731 | | | | | | | | | | |
| **Equity Value** | | **13,115,659** | | | | | | | | | | |
| **Equity Vaue x 45% (Stake Parpublica)** | | **5,902,047** | | | | | | | | | | |

*Table 38: Scenario 1 – Caixa BI's Firm valuation original table (retrieved from Portuguese Audit Court's Firm valuation simulation included in appendix in the Portuguese Audit Court report, 2011)*

| € | 2013E | 2014E | 2015E | 2016E | 2017E | 2022E | 2023E |
|---|---|---|---|---|---|---|---|
| EBITDA | 3 852 991 | -282 443 | 733 895 | 1 096 779 | 1 467 669 | 1 769 498 | 1 748 920 |
| Amortizações | 989 400 | 1 028 939 | 1 039 342 | 1 047 937 | 1 094 954 | 1 046 076 | 1 116 217 |
| EBIT | 2 863 591 | -1 311 382 | -305 447 | 48 842 | 372 716 | 723 422 | 632 704 |
| Imposto sobre EBIT | 799 759 | 0 | 0 | 12 943 | 98 770 | 191 707 | 167 666 |
| **NOPLAT** | **2 063 832** | **-1 311 382** | **-305 447** | **35 899** | **273 946** | **531 715** | **465 037** |
| Amortizações | 989 400 | 1 028 939 | 1 039 342 | 1 047 937 | 1 094 954 | 1 046 076 | 1 116 217 |
| CAPEX | 852 000 | 787 366 | 915 106 | 1 046 311 | 1 181 631 | 1 393 638 | 1 412 649 |
| Investimento Fundo Maneio | -281 616 | -8 172 469 | 263 181 | 271 813 | 283 740 | 117 993 | 119 818 |
| **Unlevered Free Cash Flow** | **2 482 848** | **7 102 660** | **-444 392** | **-234 288** | **-96 472** | **66 161** | **48 787** |
| Resultados Financeiros | -1 607 385 | -1 427 664 | -1 253 344 | -1 361 963 | -1 471 032 | -1 766 709 | -1 799 459 |
| Tax Shield | 466 865 | 0 | 0 | 12 943 | 98 770 | 191 707 | 167 666 |
| **Free Cash Flow** | **1 342 327** | **5 674 996** | **-1 697 736** | **-1 583 308** | **-1 468 734** | **-1 508 841** | **-1 583 006** |

*Table 39: Scenario 2 – Portuguese Audit Court's FCF simulation of non-renewal of the cooperation contract with ARSLVT using Caixa BI's valuation model original table (retrieved from Portuguese Audit Court's Firm valuation simulation included in appendix in the Portuguese Audit Court report, 2011)*

## APPENDIX 2 – SENSIBILITY ANALYSIS OF EQUITY VALUE TO COUNTRY RISK

To illustrate the consequences of the country risk premium in the equity value, we used the Caixa BI's model in the second scenario (with no renewal of the cooperation agreement with ARSLVT). Thus, we varied the country risk premium by decreasing the

rate considered by Caixa BI of 4.51% by 0.25%, 0.5%, 1%, 1.25% and so on, until a variation of 3%.

The results obtained were the following:

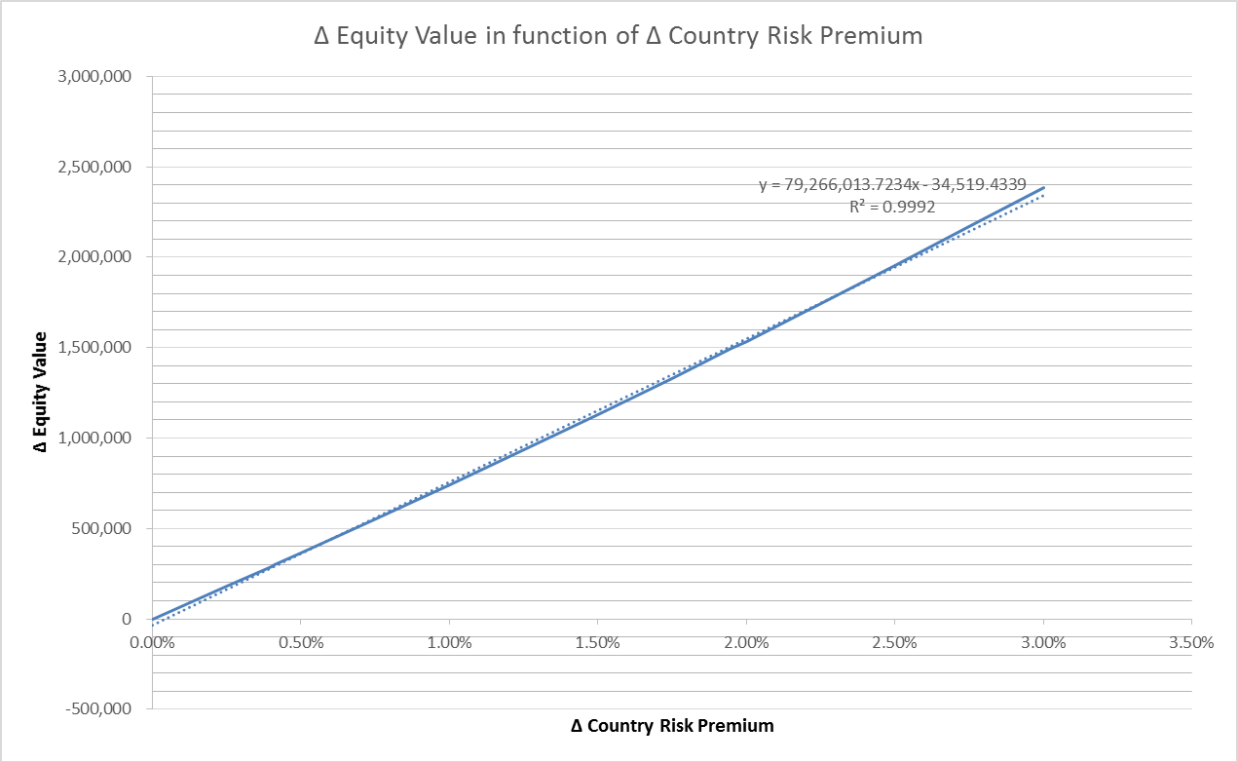


*Figure 5: Δ Equity Value in function of Δ% Country Risk Premium*

| Δ Country Risk | 0.00% | 0.25% | 0.50% | 0.75% | 1.00% | 1.25% | 1.50% | 1.75% | 2.00% | 2.25% | 2.50% | 2.75% | 3.00% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ Equity Value (Caixa BI) | 0 | 180,360 | 363,853 | 550,542 | 740,494 | 933,774 | 1,130,450 | 1,330,593 | 1,534,272 | 1,741,562 | 1,952,536 | 2,167,270 | 2,385,843 |

*Table 40: Δ% Country Risk Premium and Δ Equity Value data*

$$\frac{\Delta\ Equity\ Value}{\Delta\%\ Country\ Risk} \approx € 79\ 266\ 013.72$$

This means that a variation in 1% in the country risk premium reflects a variation on equity value of € 792 661 or a country risk base point variation reflects a variation on equity value of € 7 926.61.

One can observe that the Portuguese country risk premium in December 31st 2013, December 31st 2014 and May 11th 2015 was respectively of: 4.1%, 2.15% and 1.75%, to which corresponds respectively to a: Δ% = 0.41%, 2.36%, 2.76% (Countryeconomy.com (2015) – this website allows us to consult the daily country risk premiums).