ISCTE IUL
Instituto Universitário de Lisboa

IUL School of Business

Department of Finance

EMPIRICAL ESSAYS ON PORTFOLIO IMMUNIZATION

Cláudia Patrícia Gonçalves Simões

Thesis submitted as partial requirement for the fulfilment of the degree of

Ph.D. in Finance

Supervisor:

Doutor Luís Alberto Ferreira de Oliveira, Professor Auxiliar, ISCTE-IUL

Business School, Departamento de Finanças

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Abstract

This thesis is dedicated to interest rate risk immunization. Several widely known immunization strategies, like the naïve and duration-matching bullet and barbell, will be implemented and tested empirically. Furthermore, the M-Absolute, M-Squared and M-Vector strategies will also be implemented and tested empirically in order to evaluate if their additional complexity adds any value to the immunization process, while bearing in mind that these strategies immunize portfolios against both non-parallel and parallel shocks in the term structure of interest rates. A common methodology will be applied to different bond datasets in order to infer what is the best and most consensual immunization strategy.

Firstly, the aforementioned strategies will be empirically tested with German *bunds*, in order to assess if they cover the future payment of a single known liability. The results show a good performance of the naïve and barbell strategies that are mainly explained by the decreasing interest rate environment. The M-Absolute strategy also produces good results, while showing low transaction costs. This is due to the cash-flow clustering around the date when the liability is due to be paid and explains the lesser need to rebalance the portfolios.

Afterwards, the same strategies will be empirically tested with two bond datasets from the U.S.: Treasuries and Treasury Protected Inflation Bonds (TIPS). The strategies are applied independently to both bond datasets, once again with the aim of covering the future payment of a single known liability. Hence, for U.S. TIPS, only the real component is considered at this stage. The results obtained mimic the results from the previous Chapter: a good performance of the naïve and barbell strategies, mainly explained by the decreasing interest rate environment, and of the M-Absolute strategy, with low transaction costs. It is also noted that transaction costs for U.S. TIPS are very high due to the lack of market liquidity of these securities that, consequently, affects negatively the absolute and risk-based excess return of these portfolios.

Taking into account the previous results for the three datasets, a multiple liability immunization portfolio, where the future liability grows with inflation, is constructed. In this sense, the applied strategy was the M-Absolute because it was the most coherent strategy in the previous empirical tests presented. Once again, the datasets used were the U.S. Treasuries and U.S. TIPS, that were further adjusted to take into account that the liability profile has changed. The inflation accrual has been included in the immunization procedures of the U.S. TIPS, calculated using the U.S. Consumer Price Index for all Urban Consumers. Future liabilities have also been recalculated to account for inflation growth. The process was developed while considering the relationship among nominal and real interest rates and the inflation rate, as portrayed by Fisher (1930). It is possible to infer that the best bond dataset to immunize this type of liability is U.S. TIPS, not only because their cash flow profile resembles the cash flow profile of the future liabilities, but also because the inflation accrual leverages significantly the portfolio returns, namely in the 3-year immunization horizon. This is due to the high inflation rates that are compounding into the payable cash-flows (namely between 2004 and 2006), associated with the decreasing real interest rate environment.

JEL Classification: E31, E43, G11

Keywords: Interest rate risk, Immunization, Duration, M-Absolute, M-Squared, M-Vector, Term Structure of Interest Rates, Inflation

Resumo

Esta tese de doutoramento é dedicada à imunização de risco de taxa de juro e explora a implementação empírica de múltiplas estratégias de imunização de carteiras. Serão implementadas estratégias comuns, como a estratégia de maturidade *naïve* e as estratégias *bullet* e *barbell* que assentam na premissa de equivalência entre a duração das carteiras de ativos e da responsabilidade futura a imunizar. As estratégias *M-Absolute*, *M-Squared* e *M-Vector* serão também implementadas, de modo a aferir se a sua complexidade adicional se justifica, dada a necessidade de acomodar a possibilidade de movimentos não paralelos da estrutura por prazos de taxas de juro durante o processo de imunização de carteiras. Para aferir qual a estratégia de imunização de carteiras mais consensual foi desenvolvida uma metodologia comum a aplicar aos três conjuntos de obrigações considerados nesta dissertação.

Numa primeira fase, as estratégias mencionadas acima são testadas empiricamente com um conjunto de obrigações da Alemanha (*bunds*), para cobrir o pagamento de uma responsabilidade única e conhecida no início do período de imunização. Os resultados obtidos mostram uma boa performance das estratégias *naïve* e *barbell*, esta última muito assente na conjuntura de descida significativa de taxas de juro. A estratégia *M-Absolute* também atinge bons resultados com custos de transação baixos, o que pode dever-se ao efeito de diversificação associado a um investimento num conjunto de obrigações cujos *cash-flows* se encontram próximos da data de pagamento da responsabilidade, o que implica uma menor necessidade de ajustamentos nas carteiras nas datas de rebalanceamento.

O mesmo racional é aplicado a testes empiricos efetuados com recurso a dois conjuntos de obrigações dos E.U.A, obrigações de taxa fixa e obrigações indexadas à inflação. As estratégias de imunização mencionadas acima são aplicadas de forma independente a fim de cobrir o pagamento de uma responsabilidade única e conhecida no início do período de imunização. Assim, neste capítulo e no caso das obrigações indexadas à inflação, só é considerada a componente real e deterministica desta classe de ativos. Os resultados obtidos são muito similares aos do primeiro capítulo, com uma boa performance das estratégias *naïve* e *barbell*, que deriva novamente da conjuntura de descida significativa de taxas de juro. A estratégia *M-Absolute* volta a atingir bons resultados com custos de transação baixos. Os custos de transação associados às obrigações indexadas à inflação são elevados, pelo que a falta de liquidez destes títulos leva a que a sua rentabilidade, quer em termos absolutos, quer em termos relativos, seja afetada negativamente.

Por fim, os resultados dos testes anteriores são aplicados num teste de imunização de carteiras multiperíodo a fim de imunizar um conjunto de responsabilidades anuais futuras, cujo valor varia de acordo com a taxa de inflação. Para este efeito foi testada empiricamente a estratégia *M-Absolute*, por ter sido a estratégia mais consensual nos capítulos anteriores. Foram novamente usados os dois conjuntos de obrigações dos E.U.A, tendo, no entanto, sido aplicados ajustamentos adicionais necessários devido à mudança do perfil das responsabilidades a imunizar. Foi integrada a componente de correção monetária nas obrigações indexadas à inflação (utilizando o Índice de Preços no Comsumidor dos E.U.A.) e nas responsabilidades, de acordo com a relação demonstrada pela equação de Fisher (1930) entre taxas de juro nominais, reais e taxa de inflação. Verifica-se que as obrigações indexadas à inflação são o melhor instrumento a utilizar nestes casos, não só por o seu perfil se aproximar mais do perfil das responsabilidades a imunizar, mas também porque o juro decorrido associado à componente de inflação traz retornos muito significativos às carteiras, nomeadamente no período de imunização mais curto (3 anos), devido ao nível elevado de inflação durante o período em análise (nomeadamente entre 2004 e 2006), associado a um decréscimo significativo do valor das taxas de juro.

Classificação JEL: E31, E43, G11

Palavras-chave: Risco de taxa de Juro, Imunização, Duração, *M-absolute*, *M-squared*, *M-vector*, Estrutura por Prazos de Taxas de Juro, Inflação.

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As one of my dearest friends highlights, the creative process around a Ph.D. thesis requires a huge amount of effort, dedication, discipline, patience and organization. Mind you, if you know me well, you'll know that caffeine has to be in this list! However, doing a Ph.D. thesis with a full time job will also require a decent amount of (sane) lunacy. This was not an easy task for me. Even so, while finishing this thesis and looking back on these five years, I realize now that this has also been an internal growth process for me.

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Sumário Executivo

O tema desta tese de doutoramento consiste em testar empiricamente múltiplas estratégias de imunização de risco de taxa de juro, tendo por base a mesma metodologia mas usando diferentes amostras de títulos de dívida pública alemã e americana. O objetivo é apresentar uma metodologia que possa ser facilmente replicada em contexto real numa sala de mercados ou em fundos de gestão de ativos cujo objetivo seja imunizar responsabilidades futuras em contexto de uma gestão de *Asset and Liability Management* (num fundo de pensões, por exemplo) ou de pagamento de rendas futuras, como é o caso de seguros de capitalização e planos de poupança reforma.

Serão implementadas estratégias comuns, como as estratégias *naïve*, *bullet* e *barbell.* A estratégia *naïve* visa simplesmente replicar a responsabilidade a imunizar, ao comprar a obrigação cuja maturidade se encontra mais próxima da data de liquidação desta. As estratégias *bullet* e *barbell* assentam na equivalência entre a duração destas carteiras e da responsabilidade a imunizar, usando duas obrigações cuja maturidade poderá ser próxima (*bullet*) ou distante (*barbell*) da data de liquidação da responsabilidade. Outras estratégias, como a *M-Absolute*, *M-Squared* e *M-Vector,* são também implementadas, com recurso a carteiras compostas por 8 ou 10 obrigações, atendendo aos dois horizontes temporais a aplicar (3 e 5 anos). A complexidade destas estratégias é acrescida face às mencionadas anteriormente, mas com a vantagem de ter em consideração a possibilidade de movimentos não paralelos da estrutura por prazos de taxas de juro, de segunda ordem (ou ordem superior), enquanto as estratégias de imunização assentes na equivalência entre a duração da carteira e da responsabilidade somente levam em consideração o efeito de primeira ordem de deslocamento paralelo da estrutura por prazos de taxas de juro. O teste empirico destas estratégias visa aferir a sua aplicabilidade e praticalidade em contexto real.

Para todos os testes apresentados é considerada a existência de custos de transação (traduzidos na diferença entre preços de compra e de venda dos títulos) e vedada a possibilidade de efetuar vendas a descoberto. As carteiras são rebalanceadas trimestralmente e os cupões pagos durante o período de imunização serão reinvestidos na carteira em todas as estratégias. Deste modo, pretende-se replicar um processo de imunização de acordo com as condições normais de mercado e atendendo às restrições que um investidor institucional costuma enfrentar, aproximando assim o teste da realidade diária de um gestor de carteiras de investimento. Para avaliar a *performance* das estratégias é calculado o nível de cobertura da responsabilidade bem como a *performance* relativa face à estratégia *naïve*, a mais básica de todas as estratégias implementadas. São ainda medidos os custos de transacção e o *turnover* das carteiras bem como a rentabilidade das mesmas, não só em termos absolutos mas também por unidade de risco incorrido, através do *Reward-to-Risk Ratio*. Serão usados três conjuntos diferenciados de obrigações soberanas, da Alemanha e dos E.U.A., compostos por obrigações de taxa fixa e obrigações indexadas à inflação, de modo a testar empiricamente a imunização de uma única responsabilidade ou de um conjunto de responsabilidades, cujo valor poderá (ou não) ser conhecido no início do período de imunização.

Nos Capítulos 2 e 3 é apresentada a revisão de literatura e a metodologia de seleção de obrigações e implementação das estratégias de imunização, respetivamente. Assim, é feita *a priori* uma caracterização dos elementos comuns a todos os testes empíricos que serão apresentados nesta dissertação. O Capítulo 4 apresenta os resultados da aplicação empírica das estratégias de imunização a um conjunto de obrigações da Alemanha (*bunds*), entre 2001 e 2014. Este período foi caracterizado por uma descida significativa das taxas de juro na Europa, com uma maior proeminência na zona curta da estrutura por prazos de taxas de juro (maturidades residuais até 5 anos), que no final do período em análise chega a registar taxas de juro negativas. O objetivo é aplicar as estratégias de imunização mencionadas acima de modo a cobrir o pagamento de uma responsabilidade única e cujo valor é conhecido no início do período de imunização. Os resultados obtidos mostram uma boa *performance* das estratégias *naïve* e *barbell*, esta última muito assente na conjuntura de descida significativa de taxas de juro. Assim, verifica-se uma valorização substancial, explicada pela descida das taxas de juro e também pelo efeito de reinvestimento de cupões de valor elevado, recebidos de obrigações com maturidade residual mais elevada, que suplanta o efeito da valorização do preço das obrigações associado à descida das taxas de juro. Relativamente ao conjunto de estratégias *M*, a estratégia *M-Absolute* também atinge bons resultados devido aos custos de transação baixos. Como esta estratégia assenta na criação de um *cluster* de obrigações cuja maturidade esteja próxima da data de liquidação da responsabilidade a imunizar, o efeito de diversificação associado a um investimento num conjunto de obrigações leva a uma menor necessidade de ajustamentos nas carteiras nas datas de rebalanceamento. Além disso, esta estratégia leva em consideração choques não paralelos na estrutura por prazos de taxas de juro, ainda que não descurando os choques paralelos, o que também explica a sua boa performance. As estratégias *M-Squared* e *M-Vector* não atingem resultados tão bons quanto a estratégia *M-Absolute*, sendo que nos horizontes de imunização mais longos apresentam performances negativas, ainda que cubram sempre o valor da responsabilidade a imunizar.

A análise empírica é novamente replicada no Capítulo 5 tendo por base dois conjuntos de obrigações dos E.U.A, obrigações de taxa fixa e obrigações indexadas à inflação, entre 2000 e 2014. Tal como na Europa, este período foi caracterizado por uma descida significativa das taxas de juro. No entanto os comportamentos das estruturas de taxa de juro nominal (usada para as obrigações de taxa fixa) e real (usada para as obrigações indexadas à inflação) são diferenciados, pois entre 2004 e 2006 verificou-se uma descida das taxas de juro reais que não se verifica nas taxas de juro nominais, devido ao aumento da inflação. A estrutura por prazos de taxas de juro reais também apresenta uma inversão no curto prazo no final de 2014, que não se verifica na estrutura por prazos de taxas de juro nominais. As estratégias de imunização mencionadas acima são aplicadas aos dois conjuntos de obrigações de forma independente a fim de cobrir o pagamento de uma responsabilidade única e cujo valor é conhecido no início do período de imunização. Neste contexto, as obrigações indexadas à inflação são modelizadas atendendo somente à sua componente real e deterministica, não levando em conta o juro decorrido associado à variação do Índice de Preços no Consumidor. Esta modelização é possível atendendo a que o preço de mercado e respetivos *cash-flows* futuros são deterministicos e expressos em termos reais. O objetivo da utilização deste conjunto prende-se com a possibilidade de testar empiricamente estratégias de imunização num conjunto de obrigações com o mesmo emitente (logo padrão de risco de crédito semelhante) mas com menor liquidez. Os resultados obtidos são muito similares aos do capítulo anterior, com uma boa *performance* das estratégias *naïve* e *barbell*, esta última muito assente na conjuntura de descida significativa de taxas de juro. Relativamente ao conjunto de estratégias M, a estratégia *M-Absolute* volta a atingir bons resultados com custos de transação baixos. As estratégias *M-Squared* e *M-Vector* não atingem resultados tão bons quanto a estratégia *M-Absolute*. É ainda importante notar que, devido à sua menor liquidez, os custos de transação associados às obrigações indexadas à inflação são bastante maiores, pelo que a falta de liquidez destes títulos leva a que a sua rentabilidade, quer em termos absolutos, quer em termos relativos, seja afetada negativamente. Deste modo, tratando-se de uma responsabilidade fixa, o melhor ativo a utilizar para a imunizar serão as obrigações de taxa fixa, cuja liquidez é bastante superior e cujo perfil mais se assemelha à responsabilidade a imunizar, pois não existem incerteza nos *cash-flows* a receber no futuro.

No Capítulo 6 é feita uma extensão das análises empíricas anteriores à imunização multiperíodo. Assim, com base nos resultados obtidos, são construídas carteiras de modo a imunizar um conjunto de responsabilidades anuais futuras cujo valor varia de acordo com a taxa de inflação, o que implica que o valor da responsabilidade a imunizar só seja conhecido no final do horizonte temporal de imunização. Para este efeito, foi testada empiricamente a estratégia *M-Absolute* pois, de todas as estratégias testadas nos capítulos anteriores, foi a que, de uma forma consistente, obteve melhores resultados. Foram novamente usados os dois conjuntos de obrigações dos E.U.A, obrigações de taxa fixa e obrigações indexadas à inflação, entre 2000 e 2014. No entanto, como o perfil das responsabilidades a imunizar muda, neste capítulo foi integrada a componente de inflação nas obrigações indexadas à inflação e nas responsabilidades, atendendo à relação demonstrada entre taxas de juro nominais, reais e inflação através da equação de Fisher (1930) e utilizando o Índice de Preços no Consumidor dos E.U.A.. Assim, quer as responsabilidades, quer o preço e *cash-flows* futuros das obrigações indexadas à inflação irão ser ajustados com a estimativa de inflação futura em cada momento de imunização. Esta estimativa é calculada atendendo à evolução do Índice de Preços no Consumidor dos E.U.A., conforme explicitado no *Index Ratio* destes títulos. Verifica-se que o melhor conjunto de obrigações a utilizar para este fim são as obrigações indexadas à inflação, não só por o seu perfil se aproximar mais do perfil das responsabilidades a imunizar, mas também porque o juro decorrido associado à componente de inflação traz retornos muito significativos às carteiras, nomeadamente no período de imunização mais curto (3 anos). Isto sucede devido ao nível elevado de inflação durante o período em análise (nomeadamente entre 2004 e 2006), associado a um decréscimo significativo do valor das taxas de juro. Este efeito será anómalo e não será expectável que se verifique numa conjuntura de subida de taxas de juro e baixa inflação. No entanto, não invalida os resultados principais dos testes empiricos apresentados: que quando estamos perante uma responsabilidade cujo valor é conhecido *a priori* o ativo mais indicado para a imunizar serão obrigações de taxa fixa; no entanto, se a responsabilidade cresce a uma taxa variável e semelhante à taxa de inflação, a melhor classe de ativos a utilizar serão obrigações indexadas à inflação.

As conclusões e sugestões para análises futuras são apresentadas no Capítulo 7.

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CHAPTER 1

Introduction

This dissertation's main theme is interest rate immunization and its transposition to market applications in asset management. In this sense, several portfolio strategies will be applied empirically to this end, relying not only in the most common portfolios techniques, based on duration matching strategies, but also in more complex techniques such as the M-derived immunization measures. The tests carried out were quite extensive, as they were applied to sovereign bond datasets from different countries (Germany and the U.S.) and also to different bonds designs (nominal treasury bonds and real inflation linked-bonds). The main goal was to test empirically the immunization abilities of these instruments and see if any kind of immunization strategy stood out as clearly preferable, regardless of the country or bond type chosen. To do so, a common methodology was developed and implemented for all the datasets considered, in order to eliminate any kind of methodological bias that could induce different results for each bond dataset. This was done for term structure estimation, immunization procedures and measures applied to infer the results, with the aim of minimizing methodological risk that could be held accountable for different results in each dataset. If not, the results obtained would be less robust, since they could be due to other factors than the immunization strategy applied. Consequently, the portfolio immunization strategies have been implemented in the same way while also replicating the normal constraints an investor faces while buying bonds, such as the disallowance of short selling and transaction costs. In this sense, we are able to obtain our ultimate goal that is to set up a straightforward procedure that can be applied by asset and investment managers that need to immunize their future liabilities (for insurance or pension funds, for instance).

The first dataset refers to the German *bund* market, the most liquid market in Europe. The objective was to test empirically immunization strategies for a single known liability in a decreasing interest rate environment, where we even have negative values in the short sector of the term structure of interest rates in the end of the sample period. This way, the main purpose was to see if the more complex M-derived immunization measures proved to be more efficient, in both excess return (absolute and risk adjusted) and implementation cost, than the more straightforward duration techniques such as naïve maturity bond, bullet and barbell strategies. When one normally addresses immunization, is to gain protection from interest rate volatility, that can erode the future value of an asset. The decreasing interest rate environment was challenging because it was not clear beforehand what immunization techniques would be preferable, given that most of the time (if not all the time), shifts in the term structure of interest rates are not parallel.

The same framework and methodology is also applied to the U.S. bond market. Here we take a step forward regarding the universe of bonds chosen by applying the single liability immunization strategies to two independent datasets of bonds: U.S. nominal Treasury bonds, whose issuance and liquidity is very high (higher than the German *bunds*) and U.S. real Treasury Inflation Protected Securities (TIPS), whose liquidity and issuance is quite low, as these securities are used by a very specific subset of institutional investors, like insurance companies and pension funds. Bear in mind that for this empirical test the bond datasets were treated independently. Hence, for U.S. TIPS only the real deterministic part is modelled at this stage. This is possible due to the security design of these bonds. Their market quotes and cash-flows are depicted in real terms, while the inflation component added to market information (i.e. clean market price and real accrued interest) and future cash-flows through the computation of the Index Ratio, is determined by the non-seasonally adjusted U.S. Consumer Price Index for all urban consumers. The typical investment profile of these companies is characterized by buy-and-hold strategies (they are not active traders), hence the lower liquidity of these instruments. Both nominal and real term structures of interest rates underlying tendency is decreasing throughout the sample period, even considering that during some years the interest rates were diverging due to increasing inflation. Hence, the aim of the tests carried out for these datasets was not only an attempt to replicate the results obtained in the previous Chapter with a different dataset of bonds but also to see to what extent illiquidity plays a role in the implementation of these strategies, by inducing higher transaction costs to the portfolios.

The results from the aforementioned empirical tests are also extended taking into account the U.S. subset of bonds, by applying the most consensual immunization strategy while introducing two innovations. The first one is to the liability profile. We build portfolios that immunize multiple annual liabilities throughout the considered investment horizon, whose final value is unknown in the beginning of the implementation of the immunization strategies. The growth rate applied to the payable liability is the annual inflation rate. The second innovation is taking into account the inflation accrual of U.S. TIPS in order to allow a direct comparison between the immunization results obtained for the Treasury and the TIPS portfolios, while recalling Fisher (1930) equation that explains the relation between nominal and real interest rates and the inflation rate that has to hold in order to eliminate the emergence of arbitrage opportunities between these two instruments. This way, the U.S. TIPS cash-flows and market quotes (clean price and accrued interest) were adjusted taking into account the embedded inflation expectation in the non-seasonally adjusted U.S. Consumer Price Index for all urban consumers in order to allow for the comparison of both datasets. The main contribution of this empirical test is to outline that the immunization strategy implemented should be adjusted to the type of liability we wish to immunize. This way, for known future liabilities the best securities to immunize against interest rate shocks are fixed rate bonds while for unknown future liabilities one should build a portfolio composed of variable rate bonds whose coupon and principal index rate growth is close to the growth rate of the liability we wish to immunize.

CHAPTER 2

Literature Review

As the common theme for the Chapters presented in this thesis is portfolio immunization, we present an extensive literature review on this subject, that will be complemented in the forthcoming Chapters when necessary.

Immunizing a portfolio implies the generation of sufficient funds to satisfy a single liability (or a stream of liabilities) regardless of the course of future interest rates (Fabozzi (2000)). As for duration, Nawalkha, Soto and Beliaeva (2005) present a straightforward definition, by stating that duration gives the planning horizon at which the future value of a bond portfolio remains immunized from an instantaneous parallel shift in the term structure of interest rates.

The concept of immunization was first introduced by Redington (1952) when addressing how to choose securities to immunize a company's net worth. The immunization theorem this author introduced broadly states that if one considers that the present value of the assets and liabilities of a company can be modelled as functions of interest rates, then, for any interest rates shock, when the asset and liability cash-flows are chosen appropriately, the mean term of the assets and the mean term of the liabilities will be equal. Redington (1952) work has been developed by several authors. For instance, Grove (1974) extended this approach to balance sheet hedging techniques, by acknowledging that if a company's asset duration equals its liability duration adjusted by the company's capital structure, the company's value will not be affected by interest rate shifts. This approach is also presented by Bierwag, Fooladi and Roberts (2000), that extend the duration analysis as a risk management tool for effective management of balance sheet duration risk for governments and state owned companies, extending its use to real assets.

Fisher and Weil (1971) transpose Redington (1952) reasoning to portfolio management by applying immunization strategies based in the maturity and duration of bonds to build portfolios that would allow to receive in a given point in future time a fixed monetary amount regardless of the evolution of interest rates. They propose the so-called naïve investment strategy, based in the investment on bonds whose maturity is close or equal to the maturity of the liability one wishes to immunize. The naïve strategy was also applied to portfolios whose duration (instead of maturity) was close or equal to the maturity of the liability. The reasoning is straightforward: by investing in assets that mature when (or close to) the date the liability has to be paid, it is possible to eliminate most of the interest rate risk during the investment horizon. According to Fisher and Weil (1971), the return of these bond portfolios will be equal to the return of the immunized liability if the bonds used are zero-coupon bonds and above the return of the immunized liability if the bonds used are coupon bearing bonds.

Bierwag and Kaufman (1978) support Fisher and Weil (1971) findings by addressing the price and reinvestment effects that affect the market value of a bond portfolio where the bond proceeds are reinvested. If the portfolio is fully immunized against interest rate shifts, the price effect will be cancelled by the coupon reinvestment effect. These effects are broadly similar to the substitution and income effects presented by the demand theory when one considers financial assets as the objects of choice. This was stated beforehand in Bierwag and Grove (1968), where this extension is thoroughly explained. The Hicksian substitution effect for financial assets can be seen as a price effect because it represents the variation of the market value of the bond due to an interest rate shift. In the same vein, the income effect for financial assets can be seen as the return from the reinvestment of coupon payments at a different interest rate than the one from bond purchase. Bierwag and Grove (1968) further acknowledge that the relation between the price effect and the coupon reinvestment effect is an extension of the "Marshallian Proposition" that states that these effects will tend to cancel each other if the bonds available for investment are both complementary and substitutes for the immunization portfolio setup. However, this will only be the case if the portfolio is fully immunized. Bierwag and Kaufman (1977) develop this reasoning further by analyzing these price and coupon reinvestment effects as opposite forces that drive the overall portfolio immunization performance. If the interest rate decreases the price effect will be positive as the market value of the bond will be higher. However, the coupon reinvestment effect will be negative due to the lower return of coupons paid and reinvested into the portfolio. The authors further extend their analysis by stating that coupons that are reinvested at a yield different from the yield of purchase will affect the overall return of the bonds.

Bierwag (1977) also proposes investment strategies based on the concept of duration to immunize a portfolio, protecting it from future unpredictable shifts in interest rates. The author shows that the optimal selection of an immunized portfolio depends on the term structure of interest rates observed when the immunization process begins. The immunization efficiency is also dependant on the type of securities used to hedge the portfolio. If one uses zero coupon bonds the result will be different from the one derived from nonzero coupon bonds due to uncertainty about the reinvestment interest rate for future payments. However, using multiple coupon bearing bonds allows for a better fit to the term structure of interest rates relevant for the immunization period, allowing the investor to construct a more flexible immunization portfolio. Bierwag (1977) discusses all of these aspects applied to a set of different shifts of the term structure of interest rates that include additive and multiplicative shocks, either discrete or continuous. He concludes that the immunization strategy has to be fit not only to the actual term structure of interest rates but also to the investors' expectations of future evolution of the term structure for the considered immunization horizon.

Ingersoll, Skelton and Weil (1978) analyze the duration concept theoretically, by exposing its uses and abilities. They refute the idea that Macaulay (1938) duration measure is a risk proxy, since it introduces bias regarding the impact of interest rate shifts on high and low coupon bonds. Previous literature states the existence of a straightforward positive relationship between interest rate volatility and maturity i.e., other things equal, the impact of a shift in interest rates will be greater in bonds with higher time to maturity. The authors prove that this does not apply evenly for all types of bonds; for instance, a positive parallel interest rate shift will make the price of a low coupon bond fluctuate more than the price of a high coupon bond. The opposite applies for a negative parallel interest rate shift. As a consequence, for bonds that mature in the same date, the duration measure of the high coupon bond will be lower than the duration measure of the low coupon bond. Furthermore, Ingersoll et al. (1978) state that duration will only be a good risk proxy for uniform infinitesimal interest rate shifts. For other interest rate shifts, duration will only be an approximate measure of the change in the bond's value and this must be taken into account when setting up an immunization strategy. However this does not imply the uselessness of duration, it only draws the attention to a setback in this risk measure that can impact the immunization process and thus force the investor to screen the immunization's quality quite often. The authors present two ways to address this issue: by computing duration with a non-flat term structure of interest rates and with an autoregressive model that (1) accounts for the bias introduced by the diversity of bond coupons and (2) weights the importance of the term structure of interest rates' convexity as a complementary measure of interest rate basis risk. This way, Ingersoll et al. (1978) addresses immunization with a stochastic approach to duration and convexity.

Cox, Ingersoll and Ross (1979) also address the inability of traditional duration measures, namely Macaulay (1938) duration, to account for basis risk. The authors present their own measure of basis risk, that is expressed in time units and fits adequately to non-flat term structures of interest rates. Khang (1983) proves that an immunization strategy where the portfolios are regularly adjusted in order to reduce the duration gap between the portfolio and the liability associated with the immunization process, is a global minimax strategy. This strategy, that consists of building a portfolio whose return is the highest possible while minimizing the difference between its duration and the residual maturity of the liability being immunized, is valid irrespective of the timing or dimension of the shift in the term structure of interest rates. This paper addresses one of the main limitations identified so far in Fisher and Weil (1971) and Bierwag and Kaufman (1978) where this strategy is applied only taking into account that the shift in interest rates occurs only once and immediately after the immunization strategy is implemented. This way, by considering jointly the possibility of several interest rates changes occurring and at any time during the immunization process, Khang (1983) expands the findings of Fisher and Weil (1971) and Bierwag and Kaufman (1978) to a more realistic setting to the immunization process. Gaya and Arribas (1991) also build up in the work developed by Khang (1983) and develop a minimax strategy for portfolio selection using linear programming, while introducing a real common investor restriction: the existence of transaction costs. The authors find that transaction costs can take a toll on the immunization process by not allowing the investor to achieve full immunization. This can be induced into the immunization process if these costs are too high and affect the portfolio value due to the number of times the portfolio is rebalanced and the amounts of the bonds traded.

To address the aforementioned shortcomings of duration as an immunization measure, more models have been put forward by the literature. Soto (2001) presents a comparison between the models previously explained from Fisher and Weil (1971) and Bierwag (1977) with other empirical unifactorial models based in logarithmic and time dependent movements of the term structure of interest rates developed by Khang (1979) and Babbel (1983). These models try to address several features of the term structure of interest rates but fail to further explain the dynamics of the term structure of interest rates, like the decreasing interest rate volatility as the time to maturity increases and the correlation between short term, medium term and long term interest rates. Even though the added complexity, the aforementioned models failed to become a standard when compared with the univariate models put forward by Fisher and Weil (1971) and Bierwag (1977). Soto (2001) also presents several models of stochastic duration that aim at hedging interest rate risk while addressing the shortcomings of the univariate models.

Besides the aforementioned model put forward by Ingersoll et al. (1978), Soto (2001) presents several models that aim at building portfolios whose shifts are fully correlated to the risk factors from synthetic zero coupon bonds. These synthetic bonds are build such that their market value is totally correlated to the shifts of the term structure of interest rates. This way, these models attempt to implement immunization strategies that mimic the assumed stochastic movements of the term structure of interest rates, achieving full immunization against interest rate risk. The one-factor model from Cox et al. (1979), the two-factor models developed by Brennan and Schwartz (1983), Nelson and Schaefer (1983) and Moreno (1999) and the three-factor model of Chen (1996) are highlighted as meaningful theoretical contributions to the stochastic approach to model the term structure of interest rates. However, Soto (2001) stresses that the empirical results from these models fail to address their purpose and often lead to results that are not economically meaningful or adherent to the term structure of interest rates, thus failing the possibility to be transposed into a new market standard.

In the vein of immunization measures that are build taking into account multi-factor arbitrage models, Agca (2005) addresses the immunization strategy issue by comparing the traditional duration and convexity measures and the single factor Heath, Jarrow and Morton (1992) framework for multiple portfolio strategies and multiple immunization horizons. The author's findings can be divided in several propositions. Regarding the risk measures that serve as a base for the immunization strategy, Agca (2005) found that the traditional measures, such as the Fisher and Weil (1971) duration and convexity, perform better than the Heath et al. (1992) models. When considering the choice between a duration matching immunization strategy and a duration and convexity matching strategy, the author considers the second strategy superior when there are no transaction costs, for both short term and medium to long term immunization horizons. Yet, in the presence of this restriction, a duration matching strategy tends to produce better results since it implies less rebalancing than a duration and convexity strategy for medium to long term immunization horizons; for short term portfolios the duration and convexity matching strategy still achieves better immunization results. Furthermore, in Agca (2005) research, the most important aspects of an immunization framework lie in the correct choice of the immunization strategy to be implemented, the existence of transaction costs and the holding period H for the immunization process rather than risk measures or interest rate term structure models. Oliveira, Nunes and Malcato (2014) study addresses Agca (2005) findings within the Heath et al. (1992) framework while testing the Heath et al. (1992) with multiple factors and applying it not only to the traditional duration measures but also by using stochastic duration. In this context, Oliveira et al. (2014) shows that for duration matching strategies, and considering three-factor stochastic Heath et al. (1992) duration models, immunization results are superior than those achieved using traditional risk measures such as Macaulay (1938) and Fisher and Weil (1971). The main reason for this to happen lies in the inability if traditional risk measures to capture adequately interest rate volatility, which makes the three-factor stochastic Heath et al. (1992) duration model superior. These findings hold for three and five-year immunization periods (for one-year immunization period the stochastic duration measures do not present themselves as superior) and they remain consistent with and without transaction costs. Oliveira et al. (2014) also point out that the main issue with Agca (2005) research is the application of a single factor Heath et al. (1992) framework. In this sense, it has to be considered that the application of stochastic models should occur in an appropriate setting and that reducing the factors used in the Heath et al. (1992) framework can account for the fact that the risk measures or interest rate term structure models are deemed less important in the immunization process.

Bravo (2007) also points out one of the main issues with stochastic models, like the Heath et al. (1992), that is related to the ability to correctly infer and model the stochastic process behind each model. Hence, these models also have stochastic process risk, which means that an incorrectly specified stochastic process can reduce substantially the effectiveness of the immunization strategy. This way, Bravo (2007) takes a different avenue and develops a theoretical immunization measure that combines first- and second-order effects in accounting for shifts in the term structure of interest rates parametric approach specified by Nelson and Siegel (1987) and Svensson (1995). The author's view is that successful immunization models account for both duration and convexity risk measures while capturing the sensitiveness of bond returns to changes in one or more interest rate risk factors.

Nawalkha and Soto (2009) also discuss several classes of multivariate models that can be used to deal with the hedging of interest risk that arises from large nonparallel term structure of interest rates shifts, such as (i) M-Absolute/M-Squared models, (ii) duration vector/M-Vector models, (iii) key rate duration models and (iv) Principal Component Analysis (PCA) models as well as their extension to fixed income derivatives. The key question these authors try to address is "how do managers of financial institutions hedge their portfolios composed of fixed income securities and their derivatives, against the effects of non-parallel term structure of interest rates shifts?". Key rate duration models introduced by Ho (1992) are based on a discrete vector composed on the key spot rates of various maturities that serve as anchor points for the term structure estimation (done via linear interpolation) and are considered as extensions to traditional duration models, while not relying on stationary covariance structures of interest rate changes. PCA assumes that the term structure of interest rate movements can be summarized by a few composite variables, at least three (level, slope and curvature), representing the parallel change in the term structure of interest rates, the change in the steepness of the term structure of interest rates and the change of the humpness of the term structure of interest rates. However, in order to obtain robust results while applying PCA the covariance structure of interest rates has to be stationary.

The models (i) and (ii) mentioned in Nawalkha and Soto (2009) article originated from a new avenue in immunization research, that aims at addressing eventual arbitrage opportunities that might be implicit in the Fisher and Weil (1971) traditional duration measures, by studying the lower bounds in portfolio value changes and consequently, deriving new risk control strategies. The M-Squared model, introduced by Fong and Vasicek (1983a), Fong and Vasicek (1984) and Fong and Fabozzi (1985), consists of a linear transformation of convexity in order to allow obtaining a portfolio that will be immunized against parallel movements of the term structure of interest rates while protecting the investor from interest rate risk resulting from non-parallel term structure of interest rate shifts. In this sense, the M-Squared can be interpreted as a measure of immunization risk. Hence, the M-Squared of a bond is defined as the weighted average of the squared differences of the bond's cash-flows from a horizon point. This is achieved by clustering the portfolios' bonds cash-flows around the planning horizon date for the immunization. This approach has been found preferable when compared with the normal second-term immunization strategy (composed by duration and convexity) by Lacey and Nawalkha (1993), whose empirical study using U.S. Treasuries between 1976-1987 finds evidence that high convexity portfolios lead to higher immunization risk without maximizing bond portfolio returns, whereas high M-Squared adds return to bond portfolios, while minimizing the immunization risk more efficiently than in the duration and convexity approach.

The M-Absolute of a bond, developed by Nawalkha and Chambers (1996) is defined as the weighted average of the absolute distances of the bond's cash-flows from a given horizon point. It serves as a measure of dispersion of the bond portfolios cash-flows around the portfolio immunization horizon and is designed to provide a powerful and practical univariate risk measure immunization in particular circumstances. M-Absolute is similar to M-Squared but is derived as a first order interest rate risk hedging model, whose main concern is to address non-parallel interest rate shifts, in opposition to the traditional duration model, that only addresses parallel shifts of the term structure of interest rates. The M-Absolute will only immunize partially against level shifts, while reducing the interest rate risk caused by shifts in the slope, curvature and all other term structure parameters. This is achieved by building portfolios whose bond cash-flows are clustered around the planned horizon date, in a similar way to the M-Squared model.

The relative desirability of this model will depend on the nature of the term structure shifts that are expected. If level shifts are the most influent factor in the term structure of interest rates, the traditional duration model will outperform the M-Absolute model. However, if slope, curvature and other higher order shifts prevail then the M-Absolute will outperform the traditional duration model. Empirical tests done by these authors show that M-Absolute reduces the interest rate risk inherent in the traditional duration model by more than half, namely in the case of general shifts in the level, slope, curvature and other higher order term structure shape parameters (i.e. non-parallel shifts).

The M-Squared and M-Absolute models are also addressed in Bierwag and Fooladi (2006) as models that minimize stochastic process risk when applied as immunizing stochastic movements of the term structure of interest rates. Therefore, these models application is widespread, since they can be applied to both parametric and stochastic approaches of interest rate movements.

The M-Vector model - Nawalkha and Chambers (1997) and Nawalkha, Soto and Zhang (2003) - is a multivariate model derived as an extension of the M-Squared model and it demonstrates near-perfect hedging performance, eliminating more than 95% of interest rate risk inherent to the traditional duration model. Vector models arise from the fact that most interest rate risk models, duration or M-derived, only achieve perfect immunization for zero coupon bonds maturing at (or close to) the horizon date. These models attempt to immunize interest rate risk by applying a vector of higher-order risk measures than duration and convexity to achieve perfect immunization for any kind of changes of the term structure of interest rates. The main difference between duration vector models - Granito (1984), Nawalkha (1995) and Chambers, Carleton and McEnally (1988) - and M-Vector models is that the former are more restrictive by imposing a polynomial functional form to the term structure shifts while the M-Vector approach is based on a Taylor series expansion of the bond price function. Like the M-Squared model, the M-Vector does not impose strong assumptions on the particular stochastic processes for the term structure movements. The M-Vector model requires short positions in order to obtain immunizing solutions. Nevertheless, explicit short spot positions in bonds are not necessary for applying the M-Vector model since forward contracts can be used as short positions. However, if short positions are allowed, the cash-flows of the portfolio will not be clustered around the planned horizon.

Crack and Nawalkha (2000) have build up from several other previous contributions - Macaulay (1938), Fisher and Weil (1971), Fong and Vasicek (1984) and Chambers et al. (1988) - and present generalized expressions of the sensitivity of several risk measures (duration, convexity and higher order bond risk measures) to non-parallel interest rate changes. As Crack and Nawalkha (2000) acknowledge, shifts in the term structure of interest rates' level, slope and curvature are not independent. This way, this results can prove useful in a volatile interest rate environment due to its ability to capture the combined effects of level, slope and curvature shifts. They support their analysis with some simple and convincing numerical examples. However, it could prove useful to test empirically this strategy, in order to infer its immunization results with real market data.

Several empirical studies have been done regarding the application of these models, whose main findings are listed below. Bierwag, Fooladi and Roberts (1993) compare the performance of traditional duration models and the M-Squared model using U.S. Treasury Bond and Canadian Government Bond data. The authors find that, when using the maturity bond[1], a two-bond duration bullet strategies exhibits results that are equal or even better than the results of the application of the M-Squared model.

Soto and Prats (2002) also evaluate several traditional and M-derived models using data from the Spanish Government debt market. The authors main finding is that the portfolio composition is not trivial for the success of the implemented immunization strategy. This way, portfolios that include the maturity bond have lower exposition to non parallel shifts of the term structure of interest rates and mimic the perfect immunization asset behavior, thus facilitating the implementation of any immunization strategy. The empirical results presented show that the performance of the M-Absolute strategy

[1]The maturity bond is the bond whose maturity is closest or equal to the planning horizon date.

and of bullet and barbell portfolios outweigh the performance of the M-Squared strategy. Soto and Prats (2002) also discuss the importance of the inclusion of the maturity bond, implying that duration models fail *per se* to grasp the interest rate risk embedded in bond portfolios. The maturity bond is needed as an anchor point to stabilize the immunization strategies applied. Soto and Prats (2002) mention that the implementation of multifactorial models as a possible way forward to address this issue.

Soto (2004) uses Spanish government bond data to compare single and multiple factor duration-matching strategies in order to evaluate to what extent the success of these duration strategies are primarily attributable to (1) the particular model chosen or (2) to the number of risk factors considered. In this sense, Soto (2004) concludes that three-factor immunization strategies offer the highest immunization benchmark and that the number of risk factors considered in the immunization strategy is more important than the model applied in the immunization strategy.

Bravo and Silva (2006) evaluate the relative immunization performance of the M-Vector model using market data for the Portuguese government debt. Their main findings are that the application of single and multifactor immunization models has the benefit of removing most of the interest rate risk embedded in a naïve maturity strategy and that duration-matching strategies that include the maturity bond and the use of a single-factor immunization models achieve the best overall performance in highly volatile term structure environments and shorter holding periods, as the maturity bond acts as an anchor for the applied immunization strategy.

Díaz, González and Navarro (2008) argue that the implementation of the M-derived strategies leads to form portfolios that tend to concentrate their investment in a few bonds whose maturity is close to the maturity of the immunized liability. This entails an elevated level of idiosyncratic risk, related with taxes, principal and interest rate strips, status changes from on-the run to off-the-run and changes in cheapest-to-deliver in future contracts, among others. These factors can be harmful to the immunization strategy and only be addressed by inducing diversification in the portfolio setup. The authors use Spanish Public debt market data, including all bond types in their analysis. When building the immunization portfolios, Díaz et al. (2008) apply several diversification procedures *via* linear programming to the M-Absolute strategy to obtain fully diversified portfolios and benchmark the results of these portfolios to the M-Absolute strategy as developed by Nawalkha and Chambers (1996). The empirical results show that the performance of the diversified portfolios based in the M-Absolute strategy immunization is superior to the application of the M-Absolute strategy *per se.* In the opinion of the authors this is due to the more realistic approach taken to build the bond dataset, where no bond is discarded regardless of its liquidity level or embedded features.

Kittithawornkul (2008) applies the M-Vector model to the Thai Government debt market. The results presented are twofold. When the immunization strategy is not conditioned to include the maturity bond, the M-Vector strategy's immunization performance is better than the traditional duration strategy. As for the length of the M-Vector, Kittithawornkul (2008) finds that for the Thai Government debt market the optimal length is the M6-vector. However, when the immunization strategy includes the maturity bond the M-Vector strategy is outperformed by the bullet strategy. The author attributes this underperformance to the non parallel shifts in the term structure of interest rates, since the maturity bond is less vulnerable to these effects throughout the immunization horizon.

Bravo and Fonseca (2012) evaluate the immunizations performance of a multifactor parametric interest rate risk model based on the Nelson and Siegel (1987) and Svensson (1995) parametric extraction methods for the term structure of interest rates using European Central Bank estimated data for spot, forward, and par yield curves in order to evaluate if this approach improved immunization performance in high volatile interest rate environment. Their findings show that the empirical duration vector model built around the Nelson and Siegel (1987) and Svensson (1995) parameters outperforms several duration matching strategies as the portfolios, while capturing the level, slope and curvature shifts of the term structure of interest rates, achieve a very high degree of immunization, eliminating substantially interest rate risk. The traditional duration-based strategies and the naïve strategy do not outperform the multifactor parametric interest rate risk model. Even so, the authors acknowledge that the good performance of duration-based strategies that include the maturity bond can be taken into account if transaction costs and frequent reallocations are an issue for the investor.

CHAPTER 3

Data Selection and Methodology

This section contains a review of some term structure of interest rate parametric approaches and models and the theoretical framework applied throughout this thesis.

The term structure of interest rates was derived using the exponential functional form method developed by Nelson and Siegel (1987) parametric approach estimated using daily interest rates from each dataset used in Chapters 4, 5 and 6. From all the approaches available, this was the most feasible taking into account the main objective of applying the same methodology for the term structure of interest rates to the three datasets considered in this thesis. In this sense, the lower liquidity and bond availability for U.S. TIPS[1] was an active constraint since it would not allow the application of an interest rate term structure estimation using a parametric approach that would imply estimating more than four parameters. The low availability of U.S. TIPS bonds induced the inexistence of estimates for maturities below the 5-year by the U.S. Department of the Treasury, due to the lack of on-the-run bonds below that maturity. This additional constraint will be revisited in Chapter 5.

3.0.1. Term structure approaches and framework applied

Choosing an approach to estimate the term structure of interest rates can be quite difficult, since many approaches have been developed so far, even though all have failed to become an undisputable market standard. Even though it is not the aim of this thesis, some meaningful work, that has been developed regarding the estimation of the term structure of interest rates, is presented.

[1]U.S. TIPS dataset only includes 19 bonds whose maturity spans between 2 and 30 years.

In this sense, we recall the analysis made by Martellini, Priaulet and Priaulet (2003)[2], that state the pros and cons of the cubic spline and the Nelson and Siegel (1987) parametric approach. The cubic spline parametric approach is deemed as one of the best approaches to extract a smooth term structure of interest rates, even though it is highly dependant on the choice of the smoothing points (where the splines meet) and the number of parameters to estimate. The functional form of the spline (i.e. polynomial or exponential) also plays a role in the effectiveness of the smoothing. The most common approach relies in three splines in order to ensure that the second derivative of the model (used to compute the convexity) is continuous. This means estimating five to eight parameters for the term structure of interest rates. Furthermore, applying a smoothing process to the term structure of interest rates can induce higher estimation errors when a small shock is introduced in the evolution of interest rates, thus reducing the quality of the fit of these models to the real world. Even so, Martellini et al. (2003) states that, for portfolio pricing purposes, this parametric approach applied to the discount factor functions seems to be preferable.

Regarding the Nelson and Siegel (1987) parametric approach, Martellini et al. (2003) see its merits due to the economic intuition associated with the parameters that are estimated, allowing the investor to better understand the shifts on the term structure of interest rates by interpreting the parameters estimated. Its mathematical tractability is also a plus, which is also the reason this approach is widely used by investors and central bankers[3]. This parametric approach is also deemed better for risk management and hedging purposes because it allows the direct extraction of a discount function curve, while the unsmoothness of the approach is also a plus, since risk management aims at mitigating risk in markets under distress, which is the case for all the bond datasets analyzed. A smoothed dataset could fail to promptly identify a market distress

[2] Please refer to Chapter 4.

[3] The European Central Bank and several Eurosystem national central banks use Nelson and Siegel (1987) and Svensson (1995) extraction methods to estimate daily term structures of interest rates. Also several data vendors (i.e. Bloomberg) and portfolio management tools have the Nelson and Siegel (1987) model embedded in their softwares.

event and, consequently, induce an adjustment in the portfolios that could occur in a later phase, reducing the effectiveness of the immunization procedures. This parametric approach is also deemed superior when the source of risk is associated with nonparallel shifts of the term structure of interest rates. In this sense, the economic intuition behind the estimated parameters plays a very important role, because it allows reducing and clearly identifing the source of the risk factors that are inducing the shift.

The Nelson and Siegel (1987) parametric approach has also been analyzed and further developed by several authors. Diebold and Li (2006) enhance this parametric approach by estimating vector autoregressive models for the Nelson and Siegel (1987) parameters. This dynamic model achieves a better fit of the term structure of interest rates above the 10-year maturities, while keeping the properties the authors deem necessary for a good model of the term structure of interest rates, namely, persistency in long term interest rates and yield dynamics, increasing and concave term structures of interest rates, that are able to assume upward sloping, downward sloping, humped and inverted humped shapes and higher volatility in the short end of the curve. This dynamic model is not, however, an affine model, since Diebold and Li (2006) believe that it is not obvious that these models are necessary to produce good forecasts, which also explains why they have failed to become market standards so far.

Christensen, Diebold and Rudebusch (2009) also infer that the Nelson and Siegel (1987) and Diebold and Li (2006) provide a remarkably goodness of fit to the cross section of yields for many countries but show a lower ability to fit the long maturity yields to the term structure of interest rate due to convexity effects. In addition, these models do not impose a non arbitrage condition. This way, Christensen et al. (2009) develop a model that includes a second slope factor and imposes the absence of arbitrage. The main drawback of the five factor generalization of the Nelson and Siegel (1987) presented rests in its assumption that the state factors are independent, which implies that any given interest rate shock will only affect one factor. This might not be the case in reality where, for instance, an interest rate shock that implies the steepening of the yield curve can also affect both the slope and curvature of the term structure of interest rates at the same time. Furthermore, the imposition of the non-arbitrage condition enhances the computational burden of the model. Christensen, Diebold and Rudebusch (2011) continue their studies of the Nelson and Siegel (1987) and Diebold and Li (2006) by yet again addressing the main drawbacks of these models against the affine class of interest rates models. The aforementioned parametric approaches have a higher forecasting power and goodness of fit but they admit arbitrage opportunities, while non-arbitrage models normally exhibit a poor goodness-of-fit and forecasting ability. In order to settle this matter, Christensen et al. (2011) develop an affine Nelson and Siegel (1987) that, once again, achieve a higher goodness-of-fit for maturities above the 10-year threshold and is arbitrage free. However, as in the previous article from these authors, the affine model has the same drawbacks: the assumption that the state factors are independent and the computational burden of the model, that implies that it lacks the simplicity to be applied in a quick and effective manner. Alfaro (2011) also presents an affine discrete-time version of the Diebold and Li (2006) specification of the Nelson and Siegel (1987), using the Euler equation as the main tool for pricing. This is however, a theoretical article that only presents the model and does not apply any empirical goodness-of-fit and forecasting ability test to market data.

The non-arbitrage condition of the Nelson and Siegel (1987) has also been studied by Coroneo, Nyholm and Vidova-Koleva (2011), whose work consists of addressing the comparability of the normal Nelson and Siegel (1987) and its Gaussian affine non-arbitrage specification. The authors conclude that, although not being explicitly a non-arbitrage model, the Nelson and Siegel (1987) parametric approach is compatible with the hypothesis of no arbitrage, since the parameters derived with the latter hypothesis are not statistically different from the parameters derived with the original Nelson and Siegel (1987) approach. Furthermore, if it is possible to explore arbitrage opportunities with the Nelson and Siegel (1987) parametric approach, this is due to data bias, i.e. the markets themselves are not in equilibrium and exploiting this arbitrage opportunity will be necessary to drive the markets to a new equilibrium.

A final note on the Nelson and Siegel (1987) parametric approach literature review goes to the empirical tests performed by Nyholm and Vidova-Koleva (2012), that perform a horse race among affine term structure models, quadratic term structure models and the dynamic Nelson and Siegel (1987) by Diebold and Li (2006). The authors fail to prove that a given model class is superior to the others, even though they demonstrate that quadratic term structure models achieve the best in-sample goodness-of-fit, while the affine term structure models and dynamic Nelson and Siegel (1987) parametric approach by Diebold and Li (2006) achieve the highest out-of-sample goodness-of-fit.

Taking into account that, for all datasets, several market distress events have occurred due to the U.S. subprime crisis and the European sovereign debt crisis, cubic splines seem not to be the way to go when estimating the term structure of interest rates. Hence, we stick to the application of the Nelson and Siegel (1987) parametric approach since it seems to be the one that better suits our needs:

(i) it is highly tractable and economically intuitive;

(ii) serves our main purpose of testing for immunization procedures while still capturing the distressed events that stir interest rates, and that could jeopardize the effectiveness of the applied immunization strategies;

(iii) it allows to clearly identify the cause of a given shift in the term structure of interest rates;

(iv) it is possible to estimate for all datasets analyzed, thus not inducing any bias to the empirical tests we wish to perform.

We apply the original approach because, even though we see merits in the several developments that have been discussed above, these have yet again failed to become a market standard. Furthermore, the main advantages seem to rest above the 10-year maturity, which is not used for our immunization procedures. In this sense, and keeping the aim to develop a strategy that can be easily replicated by investors and market agents, we choose to keep it simple and apply the concrete specification of the Nelson and Siegel (1987) parametric approach. We also acknowledge the fact, mentioned before, that several institutions, such as the European Central Bank and several Eurosystem national central banks (namely the Deutsche Bundesbank, whose nominal interest rates are used in Chapter 4) estimate daily term structures of interest rates using the Nelson and Siegel (1987) and Svensson (1995) extraction methods.

3.0.2. Preliminary Notation

In this section we put forward a list of notation that will be applied throughout this thesis.

We define $(0, n)$ as the time interval, expressed in years, for the occurrence of bond cash-flows, where the coupon payments occur at a given time t and the principal payments occur at time n, such that $0 < t \leq n$.

The investor's immunization planning horizon - H - will be defined as the time difference, expressed in years, between the beginning of the rebalancing period and the date the liability is due.

We will apply the immunization strategies to a portfolio of m bonds, where a given bond will be indexed as the j-th of the portfolio.

In the rebalancing periods, the spot interest rates will be defined as $y(0, t)$ and the discount factors as $\delta(0, t)$, and since we will be estimating nominal and real continuous compounded interest rates, the index N will be used to refer to nominal values and the index R will be applied to real values, according to the examples below:

$y_N(0, t)$ - nominal continuous compounded spot interest rate between $t = 0$ and $t = t$

$y_R(0, t)$ - real continuous compounded spot interest rate between $t = 0$ and $t = t$

$\delta_N(0, t) -$ nominal discount factor between $t = 0$ and $t = t$

$\delta_R(0, t) -$ real discount factor between $t = 0$ and $t = t$

3.0.3. Term Structure of Interest Rates Specification

Let $y(0,t)$ be defined as the continuous compounded spot rate maturing at time t, and γ_0 to γ_3 represent parameters that need to be estimated in order to compute the term structure of interest rates through the equation below,

$$y(0,t) = \gamma_0 + \gamma_1 \left(\frac{1 - \exp\left(\frac{-t}{\gamma_3}\right)}{\frac{-t}{\gamma_3}} \right) + \gamma_2 \left(\frac{1 - \exp\left(\frac{-t}{\gamma_3}\right)}{\frac{-t}{\gamma_3}} - \exp\left(\frac{-t}{\gamma_3}\right) \right). \qquad (3.1)$$

As mentioned before, and also acknowledged by Martellini et al. (2003)[4] and Nawalkha et al. (2005)[5], the parameters estimated for the Nelson and Siegel (1987) parametric approach have economic intuition. γ_0 can be interpreted as the consol interest rate, since it represents the level of interest rates when $y(0,t)$ goes to infinity. The other three parameters explain the shape of the term structure of interest rates: γ_1 represents the slope of the term structure of interest rates, by measuring the spread between the short term interest rate and the long term interest rate. In this sense, the short term interest rate can be easily estimated by adding these two parameters (i.e. $y(0) = \gamma_0 + \gamma_1$). The curvature of the term structure of interest rates is represented by γ_2. If $\gamma_2 < 0$ the term structure will have a convex shape and, consequently, if $\gamma_2 > 0$ the shape of the term structure will be concave. Finally, γ_3 represents the velocity at which the short and medium term components of the term structure of interest rates converge to the consol rate.

The correspondent discount function can be computed as

$$\delta(0,t) = \exp - (y(0,t) \times t). \qquad (3.2)$$

[4]Please refer to Chapter 4.
[5]Please refer to Chapter 3.

This way, it is possible to extract a continuous function for the term structure of interest rates from the discrete data points gathered, which gives us a great amount of flexibility to our immunization process. When no rates were available through market sources, we had to estimate them from the dataset collected to implement the strategies. The process is similar to the one described above, as it consists of minimizing the mean squared price error as defined below, where $P_{mid}^{j}(0)$ is the dirty mid price[6] of bond j and $B^{j}(0)$ is the estimated fair value of the bond, computed as the discounted future coupons c, paid x times during the year and principal amount 100 for bond j discounted by $\delta(0,t)$.

$$\min_{\gamma_0,\gamma_1,\gamma_2,\gamma_3} \frac{\sum_{j=1}^{m}\left[P_{mid}^{j}(0) - B^{j}(0)\right]^2}{m} \tag{3.3}$$

where,

$$B^{j}(0) = \sum_{t=1}^{n}\frac{c_t}{x} \times \delta(0,t) + 100 \times \delta(0,n) \wedge P_{mid}^{j} = \frac{\left(P_{bid}^{j} + P_{ask}^{j}\right)}{2}. \tag{3.4}$$

These methodologies were used for the estimation of the nominal and real term structure of interest rates without any loss of generality.

3.1. Theoretical Framework

The first step is to define the liability we wish to immunize. As Fabozzi (2000, p. 449) defines, “a liability is a cash outlay that must be made at a specific time”. This author classifies liabilities in four types: (1) one for which both the cash outlay’s amount and timing are certain in the beginning of the immunization process, (2) one for which the cash outlay’s amount is certain but the timing is not known, (3) one for which the cash outlay’s timing is known but the amount is uncertain and (4) one for which both the cash outlay’s amount and time are uncertain. In this case we wish to

[6]The dirty price is the sum of the clean quoted price plus the bond’s accrued interest. When referring to prices throughout the article, P will always be a dirty price.

immunize a liability whose nominal final amount and timing is known in the beginning of the planning horizon H. This way, and according to Fabozzi (2000) classification, we will be immunizing a type (1) liability in Chapters 4 and 5 and a type (3) liability in Chapter 6. Fabozzi (2000) and Siegel and Waring (2004) mention that, to achieve better immunization results, it is beneficial to adjust the immunizing portfolio composition to the liability one is covering. This way, we can state *ex-ante* that for known future liability values, defined as type (1), the usage of fixed-rate bonds will be more suitable whereas for unknown future liability values, defined as type (3), the usage of floating-rate bonds indexed to the rate of growth applied to the liability will be preferable. In the specific case of liabilities whose growth rate is indexed to inflation, Fogler (1984) suggests the usage of stocks, real estate and any type of inflation-indexed bonds. The usage of TIPS would only be possible several years later, as the U.S. Government only started issuing these bonds in January 1997, as highlighted by Wrase (1997).

3.1.1. Risk Measures

Even though the traditional duration measures have some setbacks that have been addressed in the literature by the development of the non-arbitrage multifactorial models and new risk control strategies based in the lower bound of immunized portfolios, the proposed models from Fisher and Weil (1971) and Bierwag (1977) continue to be the main market standard and will be the starting point of our analysis. The methodology applied to immunize this kind of liability is presented in Nawalkha et al. (2005).[7]

The most widely known strategies (naïve, bullet and barbell) were applied by optimizing the portfolios' duration in order to match (or to be close to) the duration of the liabilities. Duration is a measure that aims to immunize against infinitesimal and parallel (level) shifts of the term structure of interest rates, i.e. the percentage change in interest rates is the same, regardless of their maturity. Hence, the bonds in the portfolio

[7] Please refer to Chapters 2, 4 and 5.

will suffer the same adjustment in their valuation even though their cash flow structure and maturities differ.

The duration of a given j bond is computed as the weighted average time to maturity t of the future cash-flows c_t^j, paid x times per year, where the weights w_k are defined as the present value of the cash-flows of a given bond divided by their fair price $B^j(0)$, already defined in equation (3.4). A straightforward *formulae* for the duration of the j-th bond is presented below:

$$D_j = \sum_{t=1}^{n} t \times w_t : w_t = \frac{\left[\frac{c_t^j}{x} \times \delta(0,t)\right]}{B^j(0)}. \tag{3.5}$$

In order to account for second order effects in the parallel shifts of the term structure of the interest rates, we can compute the convexity of a bond, which is the continuous-time weighted average of the squared time to maturity of the future j-th bond cash flows.

$$C_j = \sum_{t=1}^{n} t^2 \times w_t : w_t = \frac{\left[\frac{c_t^j}{x} \times \delta(0,t)\right]}{B^j(0)} \tag{3.6}$$

However, how often are the shifts in the term structure of interest rates parallel? The most common term structure adjustments imply different absolute adjustments to interest rates accounting to their time to maturity. Convexity *per se* does not address nonparallel shifts in the term structure of interest rates, it only immunizes a portfolio against large and parallel shifts in the term structure of interest rates when used with the duration measure. In this sense, the M-Squared measure was developed as a linear transformation of convexity, by relating the convexity measure to the slope shifts in the term structure of interest rates.

The M-Squared measure is computed as a weighted average of the squared time distance between the maturities of the bonds' cash-flows and the liability immunization planning horizon H, where the weights are defined in the same way as in equation (3.5).

$$M_j^2 = \sum_{t=1}^{n} (t-H)^2 \times w_t : w_t = \frac{\left[\frac{c_t^j}{x} \times \delta(0,t)\right]}{B^j(0)} \tag{3.7}$$

The immunization strategy based on the M-Squared measure allows building portfolios where the selected bonds' maturities are clustered around the liability horizon H. This way, the portfolio becomes immunized against both level and slope shifts in the term structure of interest rates. However, if the level shifts in the term structure prevail, the M-Squared measure might not be enough to fully immunize a portfolio, and, like convexity, will require the use of duration to get a better immunization result. Therefore, although M-Squared is a more elaborate measure than convexity, it can still be dependent on duration to achieve a better immunization performance.

M-Absolute was developed to address this shortcoming, by condensing in a single measure the ability to immunize against nonparallel term structure of interest rates shifts, while partially immunizing against level shifts of the term structure of interest rates. The M-Absolute measure is computed as a weighted average of the absolute time distance between the bonds' cash-flows and the immunization planning horizon, where, yet again, the weights are defined in equation (3.5).

$$M_j^A = \sum_{t=1}^{n} abs(t-H) \times w_t : w_t = \frac{\left[\frac{c_t^j}{x} \times \delta(0,t)\right]}{B^j(0)} \tag{3.8}$$

The principle is similar to the M-Squared model: immunization strategies that minimize the M-Absolute measure are those whose cash-flows are nested around the planned liability horizon H. Even so, the M-Absolute model will only achieve better results than the traditional duration model if the term structure of interest rates evolves in a such a way that non parallel shifts (slope, curvature and other higher order shifts) dominate the parallel height shifts in the planned liability horizon.

However, as already stated, it is more likely that non-parallel shifts occur in the term structure of interest rates and these shifts could have a higher order than second order shifts or even be a combination of several higher order shifts. The M-Vector model was developed to address each order shift with a different M-related measure. This model is built with i rows, each row related to a M-derived measure. For the purpose of this thesis a vector of up to five order M-Vectors was derived, meaning that $1 \leq i \leq 5$. This way, to immunize a portfolio using this model is to compute its composition while setting each row of this vector to zero. Hence, M1 is created to address parallel level shifts, while M2 addresses slope shifts, M3 curvature shifts, and so on.

$$\overrightarrow{M_j^i} = \begin{bmatrix} \sum_{t=0}^{n} (t-H)^1 \times w_t \\ \vdots \\ \sum_{t=0}^{n} (t-H)^i \times w_t \end{bmatrix} : w_t = \frac{\left[\frac{c_t^j}{x} \times \delta(0,t)\right]}{B^j(0)} \wedge i = 1, ..., 5 \tag{3.9}$$

Nawalkha and Chambers (1997, p. 9) state that the fifth order M-Vector model allows for the elimination of over 95% of the interest rate risk when compared to the traditional duration model (that can be seen as the M1 vector when $H = 0$). Moreover, these authors argue that by extending the M-Squared model and including higher order effects, the immunization performance of the portfolio almost doubles by increasing the elimination of risk from over 50% to the 95% stated above. We test the M-Vector model until the fifth order shift to evaluate what are the most significant shifts in the term structure of interest rates and how these shifts can affect the immunization performance.

Please note that all the aforementioned measures can be computed for a portfolio composed by m bonds. The broad formula for that computation can be found below:

$$Risk\ Measure_{Ptf} = \sum_{j=1}^{m} k_j \times Risk\ Measure_j : \tag{3.10}$$

$$Risk\ Measure_j \in \left\{D_j, C_j, M_j^2, M_j^A, \overrightarrow{M_j^i}\right\} \tag{3.11}$$

where $Risk\ Measure_{Ptf}$ is the risk measure for the portfolio, k_j is the percentage of the money invested in the j-th bond of the portfolio and $Risk\ Measure_j$ is the risk measure for the j-th bond of the portfolio, that can be any of the measures included in the set defined above.

3.1.2. Portfolio and Immunization Setup

In order to empirically test what is the best strategy to immunize a single fixed liability, immunization portfolios were built throughout the sample period with overlapping periods and quarterly rebalancing. Two immunization planning horizons were considered, 3- and 5-years. For the German dataset, 44 and 36 portfolios were estimated for the 3-year and 5-year immunization periods respectively (with 12 and 20 quarterly rebalancing dates after the strategy was initially set up). For the U.S. bonds datasets, 48 portfolios were estimated for the 3-year immunization period and 40 portfolios were estimated for the 5-year immunization period (with equal quarterly rebalancing dates after each strategy was implemented). Short-selling was not allowed and transaction costs were considered, as the bonds bought to the portfolio were valued at the ask price, while bonds sold were valued at the bid price, in order to replicate the constraints that the usual investor normally faces. The coupons that are received during the investment horizon are reinvested in the portfolio. For Chapters 4 and 5 we applied nine strategies, summarized in Table 3.1. In Chapter 6 we will focus on the M-Absolute strategy.

(insert Table 3.1 here)

All the immunization strategies applied in this thesis have been defined as minimax strategies applying linear or quadratic minimization programming[8], since their aim is to achieve the highest possible value for each portfolio while minimizing the difference between the risk measure applied in the immunization procedure and the residual maturity of the liability we wish to immunize. The naïve strategy is set up as an investment in the maturity bond while the bullet and barbell strategies are set up to minimize the

[8]For other types of optimization criterions that can be applied in an immunization process please refer to Kondratiuk-Janyska and Kaluszka (2005).

duration of the portfolio, whose objective is to be closest to the residual maturity of the liability. These strategies were designed to replicate the most common immunization behaviors.

The naïve strategy is the most cautious; by placing all the available resources in a single bond whose maturity is closest to the liability's maturity, the investor is seeking only to fulfill its obligation in the end of the investment period, no matter what happens to the term structure of interest rates during the immunization period. The bullet portfolio also shares this cautionary level, the main difference is that by using a combination of two bonds the difference between the duration of the portfolio and the maturity of the liability is trimmed further. The objective is to test if (and to what extent) the transaction costs embedded in the quarterly adjustments needed in this portfolio outweigh the immunization performance. The barbell portfolio is assumed to be the more risky and volatile but whose results might also be the best considering the underlying tendency for interest rate decrease across the sampled period.

The naïve strategy is straightforward and consists of using the bond whose maturity is closest to the date when the liability will need to be reimbursed. The immunization condition can thus be formulated as below:

$$D_j = H \tag{3.12}$$

Since we are using market bonds and not theoretical bonds, when the maturity of the bond is different from the maturity of the liability, the investment was allocated to the closest maturity bond above the liability date. This means that for some cases the condition above is modelled as $D_j \geq H$.

The other strategies are built taking into account the condition that forces the risk measure used in the immunization process to equal the residual maturity of the liability to be paid and the common restrictions an institutional investor normally faces, like the impossibility of short selling and the obligation of investing the overall monetary amounts, including coupons received, into the portfolio. The bullet and barbell strategies are applied using a portfolio composed of $m = 2$ bonds with the usual restrictions to investment. The formulation of these strategies is shown below.

$$\min_k \sum_{j=1}^{m} k_j D_j \tag{3.13}$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{m} k_j D_j = H \\ & \sum_{j=1}^{m} k_j = 1 \\ & k_j \geq 0, \forall j = 1, 2 \end{aligned}$$

Bear in mind that the immunization objective of the bullet and barbell strategies is the same. The difference lies in the way the two bonds used for the immunization process are chosen. In the bullet strategy we use two bonds whose maturity is closest to the liability and in the barbell strategy the portfolio includes the maturity bond and a bond whose maturity occurs at least 5 years after the liability is due to be reimbursed.

The M-derived strategies are set-up to minimize the difference between the M-Risk figure that is being tested and the residual maturity of the liability. The objective here is to test among the different strategies and evaluate what is the best. It will also be interesting to evaluate if these strategies perform better than the most common immunization strategies applied by investors. The M-derived strategy portfolios are built with eight to ten bonds whose maturity ranges between the setup date for the portfolio and at least five years after the liability is due to be reimbursed. Each strategy has its own linear programming for the immunization procedure. The M-Squared strategy's linear programming is explicit below, and has been adapted from Soto and Prats (2002).

$$\min_k \sum_{j=1}^{m} k_j M_j^2 \tag{3.14}$$

$$
\begin{aligned}
s.t. \quad & \sum_{j=1}^{m} k_j M_j^2 = H \\
& \sum_{j=1}^{m} k_j = 1 \\
& k_j \geq 0, \forall j = 1, ..., m
\end{aligned}
$$

The M-Squared strategy is tested in two ways, by itself, and named as M-Squared, and within the M-Vector approach (i.e. the minimization of the duration M1 measure and the M-Squared measure at the same time), and named as M2.[9] As for the M-Absolute strategy, we also explicit its linear programming taking into account the formulation from Soto and Prats (2002).

$$
\min_{k} \sum_{j=1}^{m} k_j M_j^A \tag{3.15}
$$

$$
\begin{aligned}
s.t. \quad & \sum_{j=1}^{m} k_j M_j^A = H \\
& \sum_{j=1}^{m} k_j = 1 \\
& k_j \geq 0, \forall j = 1, ..., m
\end{aligned}
$$

As for the M-Vector strategies, these will be tested modelling up to five factors. The formulation of the immunization procedure is based in Nawalkha and Chambers (1997) quadratic minimization. This difference is important for the programming of this strategy since this is the only strategy where we are minimizing more that one risk measure

[9]This difference explains the adaptation from the original formulation present in Soto and Prats (2002), since the linear programming depicted in the mentioned article for the M-Squared measure will be tested in M2-Vector strategy.

at the same time and this could induce multiple solutions for the chosen portfolios. This procedure is applied to ensure that the estimated portfolios's are unique and achieve maximal diversification across the dataset selected, thus minimizing unsystematic interest rate risk.

$$\min_k \sum_{j=1}^{m} k_j^2 \tag{3.16}$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{m} k_j = 1 \\ & \sum_{j=1}^{m} k_j \overrightarrow{M_j^i} = 0, \forall i = 1, ..., 5 \wedge \forall j = 1, ..., m \end{aligned}$$

These portfolios' implementation do not have a restriction to include the maturity bond, allowing for free allocation of the investment among the potentially selectable bonds. In order to clearly assess the performance of the strategy in itself, the same set of bonds is used in each rebalancing period. This way, we eliminate another factor that could introduce calculation bias: different portfolio compositions.

3.1.3. Immunization and Performance measures

The results have been evaluated using absolute and relative measures. The absolute measures are straightforward and aim to assess coverage and return. We use an average liability coverage measure ($\overline{LC}$) to assess if, on average, the results obtained with the strategies are sufficient to cover the liability payment in the end of the projected liability horizon. Let λ be any portfolio built to empirically test each immunization strategy and, consequently, let Λ be the total number of portfolios built to empirically test each immunization strategy S. We also define V as the value of the portfolio in the beginning (V_0) and in the end (V_H) of the immunization horizon. If the value, expressed as a percentage, is below 100%, then the portfolio strategy used would not allow to cover the liability payment at maturity.

$$\overline{LC}\left(S\right)=\frac{\sum\limits_{\lambda=1}^{\Lambda}\frac{V_H^{\lambda}}{V_0^{\lambda}}}{\Lambda} \tag{3.17}$$

Another absolute measure computed was the average excess return ($\overline{ER}$) of the portfolio. The excess return evaluates, on average, if the return of the portfolio, taking into account the reinvestment of the coupons that were received during the investment horizon, was above (or below) the spot rate for the liability horizon (3 or 5 years) observed in the beginning of the immunization period, i.e. if the portfolio beats the simple strategy of doing a continuous compounded time deposit of H years in the beginning of the immunization period.

$$\overline{ER}\left(S\right)=\frac{\sum\limits_{\lambda=1}^{\Lambda}\left(\frac{\ln\left(\frac{V_H^{\lambda}}{V_0^{\lambda}}\right)}{H}\right)-y\left(0,H\right)}{\Lambda} \tag{3.18}$$

In order to evaluate the stability and liquidity of the portfolios, average turnover ($\overline{T}$) and average transaction costs ($\overline{TC}$) have also been computed for each strategy. Turnover aims to evaluate, on average, if (and to what extent) there was the need to rebalance the portfolio very often in order to keep the immunization strategy, whether this was done as a bond buy, bond sale or coupon (and bond) reinvestment. Transaction costs are computed as the absolute value of the product between the bid-offer spread $\left(P_{ask}^{j}-P_{bid}^{j}\right)$ and the transaction amount, in units, for each bond (Q^{j}) in the portfolio.

As already mentioned, each strategy is implemented with quarterly rebalancing for both the 3- and 5-year horizon. Let z denote the total number of portfolio rebalancing quarters for each strategy, such that α_z stands for the current quarterly rebalancing period for the portfolio. The transaction amount for each bond is computed as the difference between the amount held in the portfolio in the last rebalancing period and the amount estimated in the next quarterly rebalancing optimization procedures. If the

j-th bond was not held in the portfolio before the current quarterly rebalancing took place, the respective transaction amount is set to zero. The amount to be reinvested includes any coupons received since the last rebalancing period and any amount bought or sold in the rebalancing date. These two measures are positively related, since a higher (average) absolute turnover (expressed in units) will lead to higher (average) absolute transaction costs (expressed as a percentage of each bond's value). For the strategy S the average turnover $\left(\overline{T}\right)$ have been computed as

$$\overline{T}\left(S\right)=\frac{\sum_{j=1}^{m} abs\left(Q_{\alpha_z}^{j}-Q_{\alpha_{z-1}}^{j}\right)}{m\times z} \tag{3.19}$$

and the average transaction costs $\left(\overline{TC}\right)$ have been computed as

$$\overline{TC}\left(S\right)=\frac{\sum_{j=1}^{m} abs\left[\left(Q_{\alpha_z}^{j}-Q_{\alpha_{z-1}}^{j}\right)\right]\times\left(P_{ask}^{j}-P_{bid}^{j}\right)}{m\times z} \tag{3.20}$$

Based in the aforementioned absolute measures, some relative measures have also been computed in order to allow ranking the several strategies. The natural and easiest strategy to implement is the naïve strategy of buying the bond whose maturity is closest to the liability we have to cover. Therefore, the relative coverage (RC) has been computed taking the naïve strategy as the benchmark and aims to measure how many times the average liability coverage $\left(\overline{LC}\right)$ of strategy S exceeds the average liability coverage $\left(\overline{LC}\right)$ of the naive strategy. If the relative coverage is positive, this means that the strategy S we are evaluating was better than the naïve strategy, if the measure is negative, the conclusion is the opposite. For the strategy S, the relative coverage is computed as

$$RC\left(S\right)=\frac{\overline{LC}(S)}{\overline{LC}(Naive)} \tag{3.21}$$

Turnover and transactions costs have also been computed as multipliers when compared to the naïve strategy. This is the strategy that by design is expected to have the lowest transaction costs and turnover, since it will only account for the coupon reinvestments. These measures are not by themselves informative of portfolio performances but can help explaining the causes of the performance of other strategies. For instance, a portfolio whose relative turnover and transaction costs are quite high might achieve a lower excess return than a portfolio whose relative turnover and transaction costs are lower, since high turnovers (while triggered by the need to adjust the portfolio immunization measure), can erode the excess return obtained by a portfolio strategy. The turnover multiplier (T_X) and the transaction costs multiplier (TC_X) are computed taking the naïve strategy has the benchmark. For the strategy S, the turnover multiplier is defined as

$$T_X(S) = \frac{\overline{T}(S)}{\overline{T}(Naive)} \tag{3.22}$$

and the transaction costs multiplier is defined as

$$TC_X(S) = \frac{\overline{TC}(S)}{\overline{TC}(Naive)} \tag{3.23}$$

Another relative measure that is used to assess portfolio strategies is the Reward-to-Risk Ratio (R/R), that is computed by the portfolio's excess return divided by the volatility of the portfolio's returns. The aim of this measure is to rank strategies controlling for the volatilities of their returns. This allows evaluating what is the most efficient immunization measure, i.e. the measure that achieves the highest return by unit of risk incurred, defined as volatility (σ_S) and computed as the standard deviation of the portfolio returns. For the strategy S the Reward-to-Risk Ratio is defined as

$$R/R(S) = \frac{\overline{ER}(S)}{\sigma_S} \tag{3.24}$$

Table 3.1: Immunization Strategies Description

The table presents a description of the immunization strategies applied throughout this thesis.

Strategy Name	**Strategy Design**
Naive	Single bond whose maturity equals (or is closest to) the horizon date.
Bullet	Portfolio with two bonds whose maturity is closest to the horizon date.
Barbell	Portfolio with two bonds where one of the bonds is close to the horizon date and the other matures at least 5 years after the horizon date.
M-Absolute	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date.
M-Squared	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date.
M1	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date. Immunizes the first element of the M-vector.
M2	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date. Immunizes two elements of the M-vector.
M3	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date. Immunizes three elements of the M-vector.
M4	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date. Immunizes four elements of the M-vector.
M5	Portfolio with 8 (3-year estimation) or 10 bonds (5-year estimation) with maturities spread between the setup date and 5 years after the horizon date. Immunizes five elements of the M-vector.

CHAPTER 4

Single Liability Immunization: strategies for the German bond market

The purpose of this Chapter is to test empirically several immunization techniques using the German bond market in order to assess which produces the best results. In this sense we will apply the most common strategies, based in bullet and barbell portfolios as well as some less known techniques, like the M-derived immunization strategies (M-Absolute, M-Squared and M-Vector), testing a wide range of single and multiple duration measures as already described in the previous Chapter.

The Chapter is structured as follows: section 4.1 contains a characterization of the German bond market and a thorough analysis of the German term structure of interest rates within the sample period. Section 4.2 discusses the empirical results obtained. The last section summarizes our conclusions and proposes a way forward for future research.

4.1. German bond data and term structure of interest rates

The dataset applied in this study is composed of German treasury bond data gathered from Bloomberg and interest rates computed by the Deutsche Bundesbank[1] using the Nelson and Siegel (1987) and Svensson (1995) parametric extraction method.

The bond data used comprises daily bid and offer prices from 34 bonds with maturities of less than 30 years, selected taking into account the total amount issued and the bid-offer spread. This way the objective was to use on-the-run bonds with a high degree of liquidity in order to minimize the impact of liquidity risk in the immunization results. The case for on-the-run bonds is obvious as these are far more liquid than off-the-run

[1]Data can be collected from https://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/Money_and_capital_markets/money_and_capital_markets_node.html?anker=GELDZINS

bonds, that are used as buy-and-hold securities for long term investors like pension funds. Price data was gathered between January 2001 and December 2014. The German bond market is one of the most liquid among European issuers, and its issuances comprises Federal and Regional bonds. The latter were not considered for the estimation. All the issued bonds pay an annual coupon and have maturities at issuance ranging from 2 to 30 years. Normally the most liquid issuances are included in the delivery basket of the futures contracts issued on these bonds[2].

For the purpose of this empirical study the maturity at issuance of the bonds chosen is 10 and 30 years (*Bunds*), as these proved to be more liquid that the 5 (*Bobl*) and 2 (*Schaetze*) year maturity bonds. This is consistent with the German debt issuance profile, where *Bunds* are considered the most important security used as a mean of Government funding, accounting for about half of the total German federal debt issuance[3]. Zero coupon bonds and principal or interest rate strips were also not considered in this analysis. In the same vein, bonds selected were plain vanilla bonds. Thus, bonds with embedded options (i.e. callable and puttable bonds) have been discarded from the dataset. Even though we are using real bonds, the idiosyncratic risks discussed by Díaz et al. (2008) are not a concern since the bonds selected allow to build a homogeneous dataset. This way, any differences from the implementation of the portfolio strategies will not be due to idiosyncratic risk.

All the bonds considered pay a coupon whose value ranges from 1,75% to 9%. Since the former tend to be more sensitive to negative interest rate shifts, it is expected that the immunization process proves to be somewhat challenging. Table 4.1 contains the main features of the chosen bonds.

(insert Table 4.1 here)

[2]The futures contracts issued by EUREX have maturities of 2, 5 10 and 30 years and are named after the bonds' maturities they target.

[3]For more information on German debt issuance please refer to http://www.deutsche-finanzagentur.de/en/institutional-investors/federal-securities/

Since the data available comprises a subset of residual maturities datapoints, that range from 0,5 to 30 years, we need to extract a continuous discount function curve to apply to the intermediate residual maturities for the immunization process. We applied the Nelson and Siegel (1987) parametric extraction method defined in equation (3.1) to infer the parameters using the known daily interest rates computed by the Deutsche Bundesbank, whose residual maturities (t) range from 6 months to 30 years, to obtain the continuous discount function curve as depicted in equation (3.2). This means that, for standard maturities, the interest rates from the Deustche Bundesbank were applied, while for non-standard maturities, the rates were estimated with the Nelson and Siegel (1987) discount function curve. For instance, if we use in one of the rebalancing periods a bond whose residual time to maturity is 2,2 years we do not have an interest rate available. Two options would then be feasible. We could interpolate[4] this rate from the rates for the 2 and 3 year published maturities or we could extract that rate parametrically. We chose the latter and opted for using the Nelson and Siegel (1987) method for consistency reasons, as this will be the method applicable to the entire thesis. This method was chosen because, even though there are models available that seem to show a better fit to the data, has already discussed in this thesis, many of these models have failed to prove themselves as a market standard, due to its mathematical intractability or lack of economic intuition. The Nelson and Siegel (1987) is widely used in financial market and central banking activity and is highly tractable, quick and easy to estimate and the parameters have a strong economic intuition associated.

The differences between the observed rates available from the Bundesbank and our estimated rates from the Nelson and Siegel (1987) parametric approach can be found in Figure 4.1 and Panel A of Table 4.2. The absolute average difference between the observed and the estimated rates lies between 3,6 and 35,5 basis points and the standard

[4]The linear interpolation method seems to be too blunt to be applied to this end. Even so, data for the linear interpolation of the spot rates for non-standard (and non-published) residual maturities is available upon request.

deviation of the difference ranges from 9,14 to 36,88 basis points. Even though it would be desirable to have the lowest difference possible, these differences are acceptable.

(insert Figure 4.1 and Table 4.2 here)

In order to grasp to what extent we would be compromising the adherence of the data estimated by the Nelson and Siegel (1987) model to the data collected from the Deutsche Bundesbank website, we computed the daily correlation rates and the t-test of equality of means[5] for the maturities used to fit the model. The values for the correlations are also depicted in Panel A of Table 4.2 and show a very high correlation between observed and estimated daily interest rates, namely in the diagonal of the matrix, which implies that the estimated rates do not differ much from market rates. The correlation coefficients lie between a minimum of 97,34% for the 2-year maturity and a maximum of 99,91% for the 30-year maturity. The results of the t-test of equality of means are included in Panel B of Table 4.2. The t-statistic and respective *p-value* is presented for each maturity. This test is designed to infer of there is statistical evidence that the means of the observed interest rates $y\left(0,t\right)$ and the estimated rates $\hat{y}\left(0,t\right)$ are equal or sufficiently close to each other and relies on the following formulation:

$$\begin{aligned} H_0 &: E\left[\hat{y}\left(0,t\right)\right] = E\left[y\left(0,t\right)\right] \\ H_1 &: E\left[\hat{y}\left(0,t\right)\right] \neq E\left[y\left(0,t\right)\right] \end{aligned} \tag{4.1}$$

The test is performed using a t student statistical distribution and assumes that interest rates follow a Normal distribution. We reject the null hypothesis H_0 if the *p-value* is equal or below the 0,05 threshold. If that is the case then the means will be considered statistically different. This implies that the distributions of the observed and estimated interest rates are different. We can see in Panel B of Table 4.2 that we fail to

[5]This test is included and has been performed in the Eviews 9 software package.

reject the null hypothesis for every maturity, even though we have *p-values* that range from 0,1431 (5-year maturity) to 0,8834 (30-year maturity).

As visible in Figure 4.2, several adjustments to interest rates have occurred throughout the sample period but the underlying tendency is clearly towards low interest rates. In this sense two periods stand out, between January 2001 and June 2005 and between July 2008 and December 2014, where a significant downward movement occurred. In this last period, from 2012 onwards, it is visible that interest rates are closer to zero or in negative territory for the shorter maturities. This can also be seen in Figure 4.3, that depicts the term structure of interest rates of the aforementioned years. The yield curve remained quasi-flat until 2003, while interest rates were slightly decreasing. Between 2003 and 2007 the yield curve steepened while interest rates were increasing. From 2007 onwards the main tendency was for a sharp interest rate decrease allied with a strong yield curve steepening. It can be noted the impressive downward shift in interest rates between 2008 and 2015 (about 4 percentage points throughout the selected maturities) and that after 2013 the short sector of the curve has negative interest rates.

(insert Figure 4.2 and Figure 4.3 here)

4.2. Results

The results obtained for both 3- and 5-year immunization horizons are presented in Tables 4.3 and 4.4.

(insert Table 4.3 and Table 4.4 here)

Several conclusions can be inferred. The first one is the naïve strategy good performance, that allows for an excess liability coverage of about 10% and 20% in each immunization horizon. This is due to the coupon reinvestments and can be explained not only by the high coupon rates of the bonds used but also due to the decreasing interest rates throughout the time horizon of the immunization. It is possible to see in

Table 4.1 that, due to its issuance date, several bonds have coupon rates above 4% while the average spot rate for the 3-year period is 3% and for the 5-year period is 3,6%. When reinvesting these coupons into the portfolio, we obtain a gain associated with the excess coupon value because the interest rates are decreasing throughout the investment horizon. This is known as the coupon reinvestment effect. Notwithstanding, high coupon bond prices are higher and more volatile than low coupon bond prices, which means that high coupon bond prices also decrease more with decreasing interest rates, eroding the portfolio value. This is known as price effect. These two effects tend to cancel each other out. As highlighted by Bierwag and Grove (1968), these effects behave in a similar way and can be viewed as the income and price effect from consumer demand theory. However, when we transpose this effects to financial assets, and particularly to our analysis, what we verify is that the coupon reinvestment effect outweighs the price effect, allowing the portfolio to gain some extra value from the reinvestment of these coupons. This corroborates the findings of Fisher and Weil (1971) and Bierwag and Kaufman (1978). The valuation driven by the coupon reinvestment effect will also show up in all other strategies, even though transaction costs and higher turnover volumes might erode these gains in some cases.

As for the bullet and barbell portfolios, it is visible that the barbell portfolio is tangently superior to the naïve strategy while the bullet portfolio has lower results. The latter even shows a negative Reward-to-Risk Ratio in both immunization horizons, that is explained by the fact that these portfolios achieve a lower excess return than the naïve strategy portfolios, while having higher turnover volumes (that also lead to higher transaction costs) and similar standard deviations. The barbell portfolios, while having a slightly higher excess return, exhibit the highest transaction costs and turnover volumes (about 3 times and 8 times higher than the naïve portfolios for both immunization horizons). The higher return of this portfolios is explained by the decreasing interest rate environment, that is normally very favorable to barbell strategies, since the long term positions tend to achieve higher realized returns when interest rates decrease. This finding corroborates the findings from Bierwag and Kaufman (1978), that state that the performance of a barbell portfolio strategy is highly dependant on both the movements of the term structure of interest rates and the overall portfolio composition (i.e. if the longer bond selected had a lower coupon or if we had opted by building *pseudo* barbell portfolios using bonds with lower maturities the result could have been very different). Our analysis also shows that these are also the most volatile portfolios due to their higher cash-flow dispersion around the planned horizon H, therefore these results could easily have been worse in an increasing interest rate environment. Even so, they achieve fairly good Reward-to-Risk Ratios, thus proving to be a good immunization strategy if the market expectations are of a general interest rate decrease throughout the planned immunization horizon.

As for the M-strategies, the M-Absolute shows the best results in both immunization horizons, which can be inferred from all the indicators computed. The excess return of these strategies is 1,9% for the 3-year investment horizon and 2,4% for the 5-year investment horizon and both investment horizons achieve Reward-to-Risk Ratios above 80%, due to the high excess returns and similar standard deviations when compared with the naïve portfolios. This bond clustering strategy also allows for lower turnover and transaction costs than the naïve portfolio, which is quite surprising since the only turnover that the naïve portfolio has is coupon reinvestments. This pattern can be explained by the lower coupon amounts and reinvestment positions in each rebalancing period. Unlike the naïve, bullet and barbell portfolios, where only two bonds were considered, the set of possible bonds to reinvest in each rebalancing period is higher. Since we have more bonds to choose from, the reinvestments are done in a more efficient way, which leads to lower transaction costs and turnover, while we still achieve total immunization due to the spreaded positions around the maturity of the portfolio. These results are in line with the results from Soto and Prats (2002).

The M-Squared and the M-Vector strategies do not achieve better results than the naïve portfolio strategies in the 3-year and 5-year investment horizon. The M-Squared strategy has lower excess returns than the naïve portfolio, although it achieves a positive Reward-to-Risk Ratio for the 3-year immunization horizon of about 8% (the 5-year horizon Reward-to-Risk Ratio is -1,93%). These results are in line with Bierwag et al. (1993). For the 3-year investment horizon, the M-Vector strategies only achieve positive results for the M4-vector and M5-vector, with Reward-to-Risk Ratios of about 15% and 18% and excess returns of 0,30%. In what concerns the 5-year investment horizon, the M-Vector always shows negative excess returns and Reward-to-Risk Ratios. These results do not support previous empirical results presented by Nawalkha and Chambers (1997) and Kittithawornkul (2008), as the M-Vector strategy ability to eliminate interest rate risk is not corroborated.

One last remark regarding the maturity bond. Even though the M-derived portfolios were built without this restriction, the immunization process applied for every strategy will choose the maturity bond as a part of the portfolio towards the end of the immunization horizon. Since the naïve, bullet and barbell portfolios include the maturity bond by design, our analysis corroborates the empirical results from Soto and Prats (2002) and Kittithawornkul (2008) as the maturity bond does seem to play a role for the anchoring of the M-derived strategies in the portfolio setup when non-parallel shifts in the term structure of interest rates occur.

4.3. Concluding Remarks

This Chapter presents the results of several empirical tests for immunization strategies applied to German bonds in a period characterized by decreasing interest rates.

The superior M-Absolute performance shows that this is the best immunization strategy to be applied in this environment, because it immunized non-parallel shifts while not disregarding the parallel component, in line with the empirical results from Soto and Prats (2002). In this sense, the added complexity of implementing the M-Absolute strategy is clearly outweighed by its superior immunization results.

The findings of Soto and Prats (2002) and Kittithawornkul (2008) regarding the maturity bond are corroborated by this empirical analysis. Furthermore, the barbell strategy's results, explained by the high coupon rate bonds reinvestment in a favorable environment, allied with the dominance of the coupon reinvestment effect over the price effect corroborate the findings stated by Ingersoll et al. (1978) regarding the parallel interest rate shifts on low and high coupon bonds. We also acknowledge that the fact that strategies based in the clustering of cash-flows around the maturity date have the best performance shows that the term structure of interest rates has experienced both parallel and non-parallel shifts.

As with any empirical study, the aforementioned conclusions cannot be extended beyond the dataset and methodology applied. Hence, it would be interesting to see if these results can be confirmed in an increasing interest rate environment. This could be a useful empirical test to the performance of the barbell portfolio, as this seems to clearly be highly dependable of the interest rate environment that characterizes this dataset.

Other hypothesis could be stressed further, like the rebalancing frequency, to infer if a lower rebalancing frequency (i.e. semi annual or annual) could improve the performance of some strategies and, consequently, its immunization results, that seem to be burdened with high transaction costs. It could also be tested if with other term structure model estimation (either parametric or stochastic) the same results would be obtained in order to assess to what extent the estimation results might be influenced by the method used to estimate the term structure of interest rates. As for the M-Vector strategies, the results shown here do not confirm the near-perfect hedging performance stated by Nawalkha and Chambers (1997) and Kittithawornkul (2008). In this sense, different specifications for the M-Vector, such as logarithms, polynomials or other generalizations, could also be applied.

Table 4.1: German Bunds Dataset

This table contains a description of the overall bondset selected to implement the immunization strategies described in Chapter 3. The bond subsets used in each portfolio were selected taking into account the restrictions included in Table 3.1.

ISIN	Description	Coupon	Issue Date	Maturity Date
DE0001134864	DBR 8 07/22/02	8,00%	14/07/1992	22/07/2002
DE0001134906	DBR 6 1/2 07/15/03	6,50%	06/08/1993	15/07/2003
DE0001134930	DBR 6 3/4 07/15/04	6,75%	22/07/1994	15/07/2004
DE0001134989	DBR 6 1/2 10/14/05	6,50%	20/10/1995	14/10/2005
DE0001134997	DBR 6 01/05/06	6,00%	08/01/1996	05/01/2006
DE0001135036	DBR 6 07/04/07	6,00%	25/04/1997	04/07/2007
DE0001135051	DBR 5 1/4 01/04/08	5,25%	09/01/1998	04/01/2008
DE0001135127	DBR 4 1/2 07/04/09	4,50%	04/07/1999	04/07/2009
DE0001135150	DBR 5 1/4 07/04/10	5,25%	05/05/2000	04/07/2010
DE0001135184	DBR 5 07/04/11	5,00%	25/05/2001	04/07/2011
DE0001135200	DBR 5 07/04/12	5,00%	05/07/2002	04/07/2012
DE0001135218	DBR 4 1/2 01/04/13	4,50%	10/01/2003	04/01/2013
DE0001135259	DBR 4 1/4 07/04/14	4,25%	28/05/2004	04/07/2014
DE0001135267	DBR 3 3/4 01/04/15	3,75%	26/11/2004	04/01/2015
DE0001135291	DBR 3 1/2 01/04/16	3,50%	25/11/2005	04/01/2016
DE0001135317	DBR 3 3/4 01/04/17	3,75%	17/11/2006	04/01/2017
DE0001135358	DBR 4 1/4 07/04/18	4,25%	30/05/2008	04/07/2018
DE0001135374	DBR 3 3/4 01/04/19	3,75%	14/11/2008	04/01/2019
DE0001135390	DBR 3 1/4 01/04/20	3,25%	13/11/2009	04/01/2020
DE0001135424	DBR 2 1/2 01/04/21	2,50%	26/11/2010	04/01/2021
DE0001135473	DBR 1 3/4 07/04/22	1,75%	13/04/2012	04/07/2022
DE0001134922	DBR 6 1/4 01/04/24	6,25%	04/01/1994	04/01/2024
DE0001135044	DBR 6 1/2 07/04/27	6,50%	04/07/1997	04/07/2027
DE0001135069	DBR 5 5/8 01/04/28	5,63%	23/01/1998	04/01/2028
DE0001135143	DBR 6 1/4 01/04/30	6,25%	21/01/2000	04/01/2030
DE0001135176	DBR 5 1/2 01/04/31	5,50%	27/10/2000	04/01/2031
DE0001135226	DBR 4 3/4 07/04/34	4,75%	31/01/2003	04/07/2034
DE0001135275	DBR 4 01/04/37	4,00%	28/01/2005	04/01/2037

Table 4.1: continued

ISIN	Description	Coupon	Issue Date	Maturity Date
DE0001135325	DBR 4 1/4 07/04/39	4,25%	26/01/2007	04/07/2039
DE0001135366	DBR 4 3/4 07/04/40	4,75%	25/07/2008	04/07/2040
DE0001135432	DBR 3 1/4 07/04/42	3,25%	23/07/2010	04/07/2042
DE0001135481	DBR 2 1/2 07/04/44	2,50%	27/04/2012	04/07/2044

Table 4.2: German Bunds Observed and Estimated Interest Rates Comparison

This table is composed by two panels. Panel A contains the absolute average and standard deviation of the difference between market interest rates collected from Deustsche Bundesbank and estimated interest rates using the Nelson and Siegel (1987) parametric extraction method (values in basis points) and the correlations between the aforementioned interest rates for each maturity included in this dataset. It is possible to see that the values are very high, which implies a very strong positive correlation between estimated and observed interest rates. Panel B presents the results for the t-test of equality of means. For all the maturities presented we do not reject the null hypothesis of equality of the observed and estimated interest rate means.

Panel A - Absolute Differences and Correlations								
	Maturities (years)							
	0,5	**1**	**2**	**3**	**5**	**10**	**20**	**30**
Average (b.p.)	3,60	10,78	11,94	28,25	35,50	15,01	6,33	1,95
Standard deviation (b.p.)	28,66	32,44	34,88	36,88	35,40	9,14	21,01	13,13
Correlation (%)	98,35	98,34	97,38	97,40	98,17	99,82	99,39	99,91

Panel B - Mean interest rate equality tests								
	Maturities (years)							
	0,5	**1**	**2**	**3**	**5**	**10**	**20**	**30**
t-statistic	-0,2115	-0,6146	0,7318	0,8728	1,4677	1,1854	-0,5084	0,1468
P-value	0,8326	0,5392	0,4648	0,3834	0,1431	0,2367	0,6115	0,8834

Table 4.3: German Bunds 3-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 3-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	109,84%	—	0,29%	14,43%
Barbell	110,22%	100,36%	0,65%	23,08%
Bullet	109,18%	99,41%	-0,31%	-17,73%
M-Absolute	111,62%	101,63%	1,92%	87,33%
M-Squared	109,70%	99,88%	0,16%	8,47%
M1	109,22%	99,45%	-0,27%	-1,59%
M2	109,49%	99,68%	-0,04%	-0,31%
M3	109,40%	99,62%	-0,10%	-5,98%
M4	109,85%	100,02%	0,30%	15,14%
M5	109,92%	100,08%	0,36%	18,54%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,189%	12,10	—	—
Barbell	1,534%	40,19	8,1	3,32
Bullet	0,536%	37,69	2,8	3,11
M-Absolute	0,151%	11,58	0,8	0,96
M-Squared	0,127%	10,00	0,7	0,83
M1	0,334%	24,85	1,8	2,05
M2	0,223%	16,72	1,2	1,38
M3	0,485%	34,16	2,6	2,82
M4	0,503%	33,84	2,7	2,80
M5	0,507%	32,05	2,7	2,65

Table 4.4: German Bunds 5-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 5-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

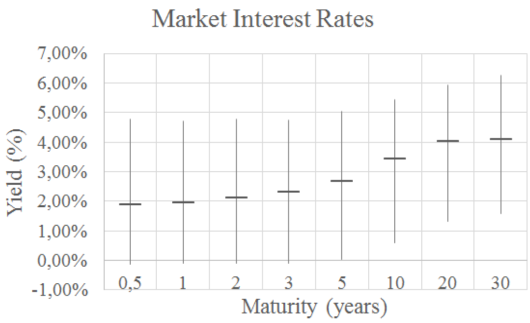
Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	120,27%	—	0,26%	9,50%
Barbell	120,45%	100,19%	0,45%	13,09%
Bullet	119,40%	99,30%	-0,41%	-15,88%
M-Absolute	122,90%	102,21%	2,37%	85,27%
M-Squared	119,89%	99,70%	-0,05%	-1,93%
M1	119,42%	99,31%	-0,44%	-17,45%
M2	119,50%	99,38%	-0,38%	-14,73%
M3	119,27%	99,21%	-0,55%	-10,82%
M4	118,76%	98,78%	-0,98%	-36,44%
M5	118,93%	98,93%	-0,83%	-27,29%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,183%	12,31	—	—
Barbell	1,449%	39,66	7,9	3,22
Bullet	0,655%	48,86	3,6	3,97
M-Absolute	0,091%	7,85	0,5	0,64
M-Squared	0,072%	6,30	0,4	0,51
M1	0,215%	18,46	1,2	1,50
M2	0,154%	13,06	0,8	1,06
M3	0,341%	25,29	1,9	2,05
M4	0,439%	29,70	2,4	2,41
M5	0,488%	31,64	2,7	2,57

Figure 4.1: German Bunds Nelson-Siegel Estimation Errors

The figure contains a comparison between Market Interest Rates from Deutsche Bundesbank and Estimated Interest Rates with the Nelson-Siegel parametric extraction method for the selected monthly interest rates. The chart can be interpreted as follows: the horizontal bar is the average rate for that maturity and the vertical bar contains the minimum and maximun interest rates, during the sample period for each maturity. It is visible that market interest rates and estimated interest rates have a similar pattern in all the maturities presented. This means that their distributions are very close, which is confirmed by the results shown in Table 4.2.



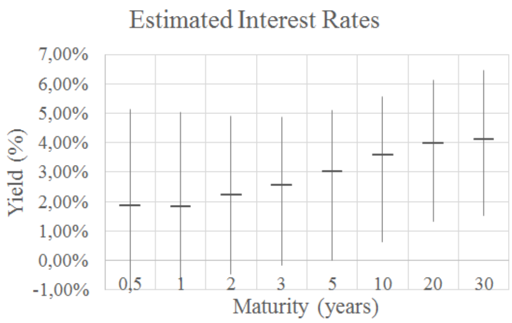


Figure 4.2: German Bunds Interest Rates

This figure contains the nominal yields for the 1, 3, 5 and 10-year maturities between January 2001 and December 2014. It is observable that in the beginning of 2001 interest rates are fairly high and tend to decrease until December 2003 in a highly volatile interest rate environment. Between December 2003 and December 2007 interest rate increase with some flattening. From December 2007 until December 2012 we face a sharpe decrease towards negative interest rates in the short sector (1- and 3-year maturities), while the 5- and 10-year maturities also exhibit a decreasing tendency but with a small inflexion in 2009. For the medium sector we see a slight increase in interest rates in 2013 followed by a sharp decrease in interest rates until year-end 2014.

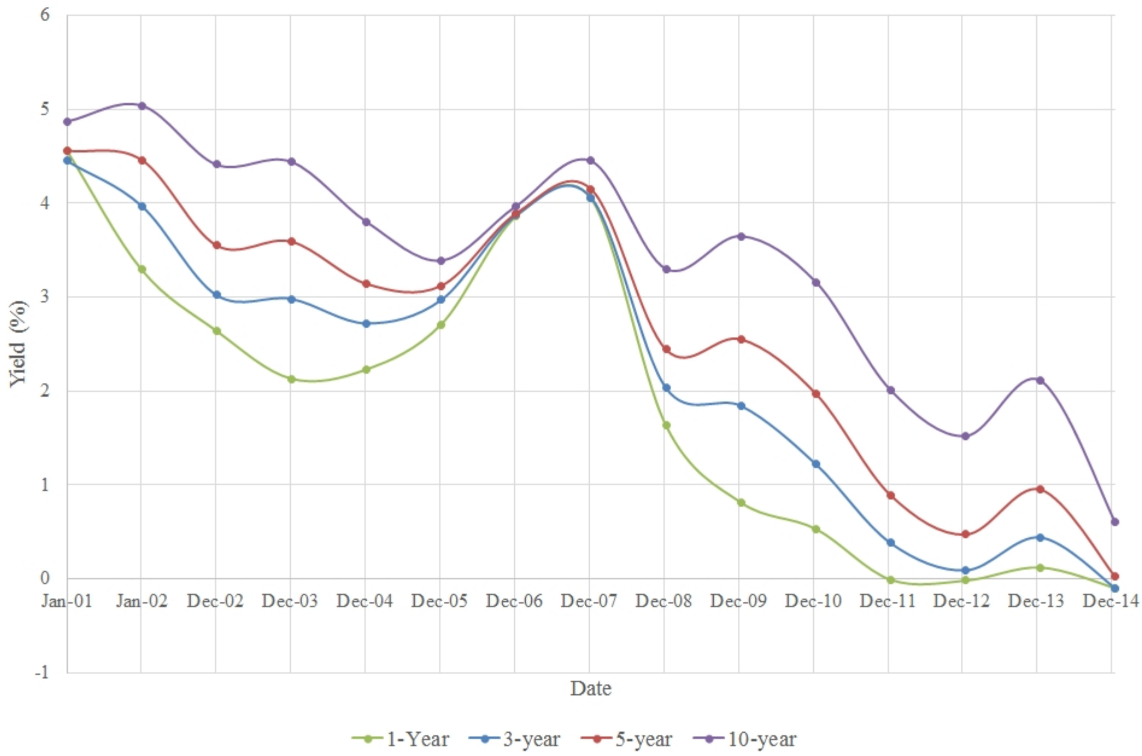
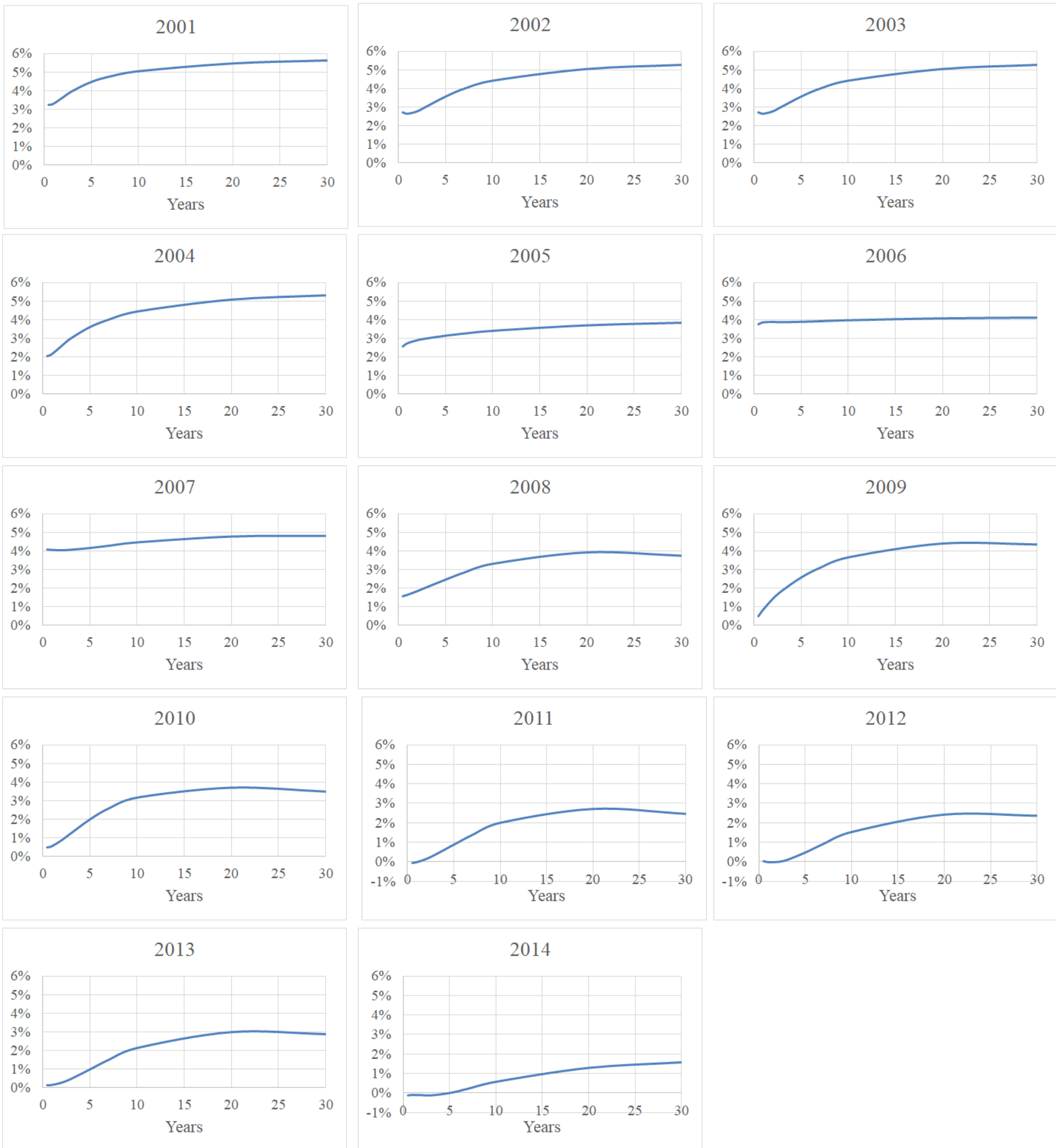


Figure 4.3: German Bunds Yield Curve

This figure contains the year-end term structure of interest rates from 2001 to 2014, as estimated by the Deutsche Bundesbank. It is possible to observe that the term structure of interest rates assumes several shapes during the sample period. Between 2001 and 2004 the curve has a concave shape with some flattening movements in the short sector, evolving to a quasi-flate structure between 2005 and 2007. From 2008 to 2013 we resume the concave curve with a sharp steepening in the maturities below 10 years, explained by the significant decrease in interest rates. The curve starts to flatten again in 2014 while showing negative interest rates in the short sector.



CHAPTER 5

Single Liability Immunization: Strategies for U.S. Treasuries and U.S. Treasury Inflation Protected Securities

This Chapter's purpose is twofold: it aims to apply several immunization techniques to the U.S. bond market in order to (i) assess which produces the best results and (ii) assess to what extent the results vary when we use nominal or real bonds.

The immunization strategies described in Chapter 3 will be applied to nominal U.S. Treasury Bonds (Treasuries) and to real U.S. Treasury Inflation Protected Securities (TIPS). The comparison with nominal and real bonds is aimed at seeing which bond type produces the best immunization results for each strategy. In this sense, the aim is not to compare these bond datasets with each other, but only to see to what extent the results may vary, since U.S. TIPS are less liquid than U.S. Treasuries. Therefore, in this Chapter we will only focus on the real component of TIPS. The inflation component will be included in Chapter 6. Will the liquidity factor allow for different immunization results? Will the asset turnover erode the returns due to higher transaction costs? These are some of the queries we aim to answer in this Chapter.

The Chapter is structured as follows: section 5.1 defines the U.S. Treasuries and U.S. TIPS security design. Section 5.2 contains a characterization of the U.S. Treasury and TIPS bond market and a thorough analysis of the U.S. nominal and real term structure of interest rates within the sample period and Section 5.3 discusses the empirical results obtained for each dataset. The last section summarizes our conclusions and proposes a way forward for future research.

5.1. U.S. Security Design

Treasuries are defined by Fabozzi and Fleming (2002, p. 186) as "coupon securities (...) issued with a stated rate of interest, pay interest every six months, and are redeemed at par value (or principal value) at maturity". Hence, Treasuries are bonds that pay a fixed coupon and principal amount and whose implied rate of return is nominal (i.e. its rate accounts for both real investment return and inflation accrual). The estimated fair value of a U.S. Treasury (B_{UST}) can be found below[1]:

$$B_{UST}(0) = \sum_{t=1}^{n} \frac{c_t}{x} \times \delta_N(0,t) + FV \times \delta_N(0,n), \tag{5.1}$$

where c_N is the annual nominal coupon paid at time t, x is the number of times *per* year the coupon is paid, FV is the principal amount due at time n when the bond is redeemed and $\delta_N(0,t)$ is the nominal spot discount factor for the residual maturity where every cash-flow is due.

A broad definition proposed by Deacon, Derry and Mirfendereski (2004, p. 1) for securities that are linked to inflation is "securities (...) designed to help protect borrowers and investors alike from changes in the general level of prices in the real economy". Brynjolfsson (2002, p. 203) provides a more specific definition for inflation-linked bonds, as "bonds that are contractually guaranteed to protect and grow purchasing power". Wrase (1997) states the main objectives surrounding the issuance of these bonds, that started in January 1997, as useful for investors that want to protect their investments from inflation or that wish to diversify their portfolios and also for investors whose liability structure varies with inflation, such as insurers, pension funds or companies whose revenues are indexed to inflation. Apart from literature contributions, one can define TIPS as bonds which allow for inflation risk protection, since they provide a fixed real interest rate return plus a floating return indexed to a broad inflation measure, in both coupon and principal payments. This way an investor that buys inflation-linked bonds will earn

[1] Please refer to section 3.2.1. Preliminary Notation.

a return that is not only interest rate driven but also protects him from inflation fluctuations, thus not eroding his purchasing power through his investment horizon. Hence, these bonds can be seen as a combination of two instruments: one with a real deterministic component and other that accounts for inflation. This way, the real estimated fair value of a U.S. TIPS (B_{TIPS_R}) can be defined as stated below.

$$B_{TIPS_R}(0) = \sum_{t=1}^{N} \frac{c_{R_t}}{x} \times \delta_R(0,t) + FV \times \delta_R(0,n), \tag{5.2}$$

where c_{R_t} is the annual real coupon paid at time t, x is the number of times *per* year the coupon is paid, FV is the principal amount due at time n when the bond is redeemed and $\delta_R(0,t)$ is the real discount factor for the residual maturity where every cash-flow is due. As stated above, we will only consider the real component of the U.S. TIPS in this Chapter. This is possible because this bonds' market quotes, collected from Bloomberg, do not include the inflation accrual. It is also this fact that allows for a straightforward computation of the real term structure of interest rates using the Nelson and Siegel (1987) parametric approach and for the straightforward application of the methodology defined in Chapter 3, by adjusting coupon c_R and the j-th bond's value by $B_{TIPS_R}(0)$. Regarding the total value paid in nominal terms, in a broad sense, the settlement price of this bonds is computed as

$$P_{TIPS}(0) = [RQ(0) + RAI(0)] \times IR(0), \tag{5.3}$$

where $P_{TIPS}(0)$ is the nominal dirty payable price for a U.S. TIPS, $RQ(0)$ is the real quoted price for a U.S. TIPS, $RAI(0)$ is the real coupon accrued interest and $IR(0)$ is the Index Ratio for the settlement date. As depicted above, the inflation component of these bonds is estimated and included in the price to pay through the Index Ratio and it is not known at time t. Further elaboration on this matter is done in Chapter 6, but, for the sake of clarity, further details on the valuation of these bonds are included in Appendix A.

There have been several contributions in the literature regarding the pricing of TIPS. Our work is focused on the immunization abilities of these bonds and we have available all the market information needed to achieve this purpose. With market prices, inflation rates and interest rates available, we do not need to theoretically value these bonds. The application of the above formulas will be enough. However, it is of the utmost importance to mention the most important pricing methods related to these bonds and we will do so in the remainder of this section.

One of the most meaningful articles regarding TIPS pricing is the Jarrow and Yildirim (2003) model. This model is based in a foreign currency analogy, where the nominal rates are modelled as the domestic currency rates, the real rates are modelled as the foreign currency rates and the inflation rate is assumed to be the spot exchange rate that links both economies. Both nominal and real economies are modelled to be Gaussian Heath et al. (1992) economies, since the volatility and the drift of the instantaneous forward rate are assumed to be deterministic. Moreover, the volatility of the inflation index is also assumed to be deterministic, which implies that the inflation index follows a geometric Brownian motion. Hence, the logarithm of the inflation index process is normally distributed.

Chen, Liu and Cheng (2005) Value TIPS by applying an analytical two-factor Cox, Ingersoll and Ross (1985) model with correlated real rates and inflation, which has the advantage of allowing the explicit modelling of the structure of inflation risk premium by estimating endogenously the correlation between real instantaneous interest rates and inflation. This addresses one of the limitations of the Jarrow and Yildirim (2003) model of not including risk premium modelling, by assuming the real interest rates and inflation are not independent, even though this means that Chen et al. (2005) model will not benefit from the mathematical tractability that the Jarrow and Yildirim (2003) has. Chen et al. (2005) also acknowledge that the difference between the nominal and real interest rates, perceived as expected inflation, will also contain an embedded inflation risk premium. This is one of the reasons put forward by Chen et al. (2005) for the non-compliance of their model with the Fisher (1930) equation. The authors also state that this could be due to other factors they do not consider in their research such as the TIPS liquidity premium or the estimation of the real term structure of interest rates directly from TIPS that compare with the constant maturity treasury rates used for nominal instantaneous interest rates computed by the St. Louis Federal Reserve Board. Hence, the real instantaneous rates estimation might not be as smooth and will most likely be noisier than the nominal instantaneous rates used.

Falbo, Paris and Pelizzari (2010) put forward a mixed model using a Vasiček (1977) model for instantaneous inflation rate processes and a Cox et al. (1985) model for the nominal instantaneous interest rate, extracting the real instantaneous interest rates by taking the difference between nominal and inflation rates. The authors try to address some shortcoming of the previous models by using this approach. For instance, they use the Vasiček (1977) model for inflation because this model will allow inflation rates to become negative, thus being more realistic. They also apply the Cox et al. (1985) model to eliminate two strong hypothesis from the Jarrow and Yildirim (2003) model regarding interest rate modelling: (i) Gaussian independence and (ii) non-negativity of interest rates. The non-negativity of interest rates and inflation rates is also an assumption of the Chen et al. (2005) that Falbo et al. (2010) do not consider. The authors succeed in achieving a closed-form equation for the price of TIPS that accounts for the modelling of economies during deflationary periods but the implied computational burden of this model is also assumed to be one of the major setbacks, which ultimately will impair this approach to become a market standard for valuation of TIPS.

Other authors have also used econometric approaches to value TIPS, from which we highlight Campbell, Shiller and Viceira (2009) consumption-based pricing model using a vector autoregressive approach that relates the modelling of the real interest rate, while accounting for the economic contribution behind Consumer Theory. Their goal is to relate the evolution of the real interest rates with the implications of the expectations hypothesis and to infer how short term shocks in the real interest rate can be propagated along the real term structure of interest rates. In this sense, Campbell et al. (2009) findings show that TIPS, when added to mixed asset portfolios, reduce their variance in the long run by eliminating idiosyncratic risks, thus being risk-efficient assets.

There is also a wide array of research available regarding the pricing of inflation-linked bonds through the modelling of the interest rates and inflation. Ang, Bekaert and Wei (2008) apply an econometric regime-switching affine autoregressive and moving average model to study the correlation between real rates, expected inflation and inflation risk premiums and determine to what extent these effects explain the structure of the U.S. nominal term structure of interest rates. This formulation allows for the stochastic modelling of inflation and real interest rates. Even though Ang et al. (2008) find that the dynamics of these variables are constant over time, their drifts are not, and the inclusion of the regime-switching component allows for a more reality driven modelling which is fit to the very long horizon of the studied dataset (1952 to 2004). Their main findings imply that even taking into account Jensen's inequality and convexity bias associated with inflation compensation (that is assumed to be the difference between nominal and real interest rates), the one-year inflation risk premium is estimated to be 1 basis point. When considering estimating longer term inflation risk premiums, this value will be increasing with the maturity of the bonds. They also find that the inflation compensation is the main driver for changes in the nominal interest rates, accounting for about 80% of the variation of nominal interest rates, irrespective of their maturities, and also for the nominal interest rate spread for long horizons.

D' Amico, Kim and Wei (2014) work also study the liquidity risk premium using no-arbitrage term structure models and find that there is a persistent liquidity premium component in TIPS that could hamper the ability to use the inflation compensation estimates as a proxy for year-on-year future inflation. This finding does not undermine the findings from Ang et al. (2008) since D' Amico et al. (2014) dataset relies on a smaller time period (from 1999 to 2013). In addition, the authors also acknowledge that other research, with different datasets, produces different values for the inflation risk premium, often negative at shorter maturities. Hence, the results presented by D' Amico et al. (2014) seem to be highly dependent on the business cycle embedded in the dataset used. Chen, Liu and Cheng (2010) also study inflation risk and the term structure of inflation risk premium in U.S. TIPS by applying a two-factor correlated Cox et al. (1985) model to a dataset that spans between 1998 and 2007. Their work also addresses the aforementioned limitation of the Jarrow and Yildirim (2003) by assuming that real interest rates and inflation are correlated and thus estimating all parameters within the model. The authors find that the correlation between the instantaneous real interest rate and the instantaneous inflation factor is positive and significant and influences the estimated inflation risk premium, that is estimated to be 1,95 basis points for one-year horizon. Chen et al. (2010) also find that, although the inflation risk premium tends to be stable over time, its term structure is positively sloped, which confirms the findings from Ang et al. (2008).

The research regarding inflation risk premium is not confined to the U.S. TIPS market. Hördahl and Tristani (2012) use a joint macroeconomic and term-structure model to estimate the dynamics of inflation risk premium in the Euro Area between 1999 and 2007, with a dataset of bonds issued by the French Government. The authors estimate the inflation risk premium over nominal Euro Area 10-year yields to be about 20 basis points and also acknowledge that the term structure for this inflation risk premium is upward sloping, thus confirming that the findings of Ang et al. (2008) and D' Amico et al. (2014) regarding the term structure of the inflation risk premium can be transposed to other markets.

Pericoli (2014) estimates the real term structure of interest rates using smoothing B splines to price French inflation linked bonds, indexed to both the French CPI and the Euro Area ex-tobacco Harmonized CPI[2]. The author finds that the smoothing B splines are superior to other parametric approaches to extract the term structure of real interest rates, such as the Nelson and Siegel (1987) as the obtained results are more stable over time. However, Pericoli (2014) also acknowledges that the aim of his work is to obtain smooth forward curves that are flexible enough to capture movements in the term structure of interest rates as a way to supply a market measure that can be used for monetary policy purposes and not to achieve high price precision for all bonds available in the markets. This way, notwithstanding the meaningful contribution for monetary policy and central banking purposes, the works of Pericoli (2014) might be difficult to transpose to asset pricing and financial markets, where the aim is to price accurately all assets and develop models that adhere the reality and movement of markets as much and as quick as possible.

Evans (2003) introduces a Markov-switching Cox et al. (1985) model to estimate the term structure of nominal and real interest rates and inflation compensation using data from the United Kingdom from 1983 to 1995. The author's aim is to assess how accurately the term structure captures changes in future yields and inflation and it also addresses the estimation of the term structure of inflation risk premium. However, Evans (2003) model goes further by introducing the Markov-switching component that allows for the identification of three distinct inflation regimes: (i) slowing rising inflation, (ii) quickly rising inflation and (iii) slowly falling inflation, while allowing for the study of the behavior of the inflation risk premium in the United Kingdom in the three different regimes. Evans (2003) acknowledges that the ability to infer a good proxy for the inflation expectations from nominal and real interest rates depends on the size of the inflation risk premium, thus implying that it is not possible to estimate the level of future inflation by taking the difference between nominal and real interest rates due to the size of the inflation risk premium across states and horizons. Hence, to get a

[2]The French Government issues inflation-linked bonds using two different series of inflation indexes. The first issues were linked to the French CPI and, after the introduction of the Euro currency, other series of issues linked to the Euro Area ex-tobacco HICP was introduced.

feasible estimation of the inflation risk premium and inflation compensation, one needs to acknowledge beforehand what is the state behind the evolution of inflation.

5.2. U.S. bond data and term structure of interest rates

The datasets applied in this study are composed of U.S. Treasuries and Treasury Inflation Protected Securities (TIPS) data gathered from Bloomberg and nominal and real interest rates gathered from the U.S. Department of the Treasury[3]. These rates are computed using the cubic splines parametric extraction approach.

The bond datasets used comprises daily bid and offer prices from 52 U.S. Treasuries and 19 U.S. TIPS with maturities that range from 2 to 30 years, selected taking into account the total amount issued and the bid-offer spread. As in the previous Chapter (and for the same reasons) the objective was to use on-the-run bonds with a high degree of liquidity. Price data was gathered between January 2000 and December 2014. All the issued bonds pay semi-annual coupons, as this is the standard in the U.S. markets.

As in the German market, the most liquid U.S. Treasury issuances are included in the delivery basket of the futures contracts issued on these bonds[4] and, for this dataset, this criteria was crucial to select the most liquid (on-the-run) bonds, taking into account that the eligible universe comprised over 500 bonds, as the U.S. Treasuries is currently the biggest bond market with a total outstanding debt of $19,5 trillion as of September 2016. As in the German dataset, zero coupon bonds, principal or interest rate strips and bonds with embedded options (i.e. callable and puttable bonds) have been discarded from the dataset. All the bonds considered have nominal coupons whose annual value ranges from 0,25% to 11,625%. The huge discrepancy among coupons shows the evolution of interest rates throughout the considered period. As in the previous Chapter, it is expected that high coupon bonds could introduce some challenges to the immunization process, as

[3]Interest rate data is available in https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/default.aspx
[4]The CBOT futures contracts issued have maturities of 2, 5 10 and 30 years, being the most liquid the 5- and 10-year series.

these bonds tend to be more sensitive to negative interest rate shifts. Table 5.1 contains the main features of the chosen U.S. Treasury bonds.

(insert Table 5.1 here)

As for the U.S. TIPS issuances, the selection process was simpler because its issuance is more recent and less widespread than U.S. Treasuries. U.S. TIPS address a specific segment of investors, like insurance companies and pension funds, whose primary investment objective is to protect long-term investments from changes in the inflation rate. This also explains the lower liquidity, as these investors have a buy-and-hold profile (hence the stock available for regular trading activities is lower for these bonds). Consequently, all the 19 on-the-run bonds alive from January 2000 to December 2014 were selected[5]. The annual coupon value ranges from 0,125% to 4,25%, showing less amplitude than the U.S. Treasury peer bonds. Even so, the coupons are also decreasing with time (i.e. U.S. TIPS issued throughout the considered sampled period tend to have lower annual coupons as time goes by). Table 5.2 contains the main features of the chosen U.S. TIPS bonds.

(insert Table 5.2 here)

Once again, the idiosyncratic risks discussed by Díaz et al. (2008) are not a concern since the bonds selected for the U.S. Treasuries and U.S. TIPS dataset are very similar. In this sense the issue could arise only for the U.S. TIPS dataset, where no pre-screening of the bonds is done but these bonds characteristics are homogeneous. No strips, embedded options or futures contracts exist for these bonds.

As in the previous Chapter, for nominal rates, we applied the Nelson and Siegel (1987) parametric extraction method defined in equation (3.1) to infer the parameters

[5]The eligible universe for U.S. TIPS comprises 48 bonds. However, 29 bonds have been excluded due to severe liquidity issues (i.e. lack of prices for several weeks or months). No further selection criteria have been aplied.

using the known daily interest rates computed by the U.S. Department of the Treasury, whose residual maturities (t) range from 6 months to 30 years, to obtain the continuous discount function curve as depicted in equation (3.2). The reason is the same - the need to obtain a continuous discount function for the term structure of interest rates for the immunization process, as depicted in equation (3.2) to infer the discount factors for non-standard and non-published maturities. We recall the reasoning presented in Chapter 3 for the application of the Nelson and Siegel (1987) parametric approach. Once again, we will apply the Nelson and Siegel (1987) parametric approach since it seems to be the one that better suits our needs:

(i) it is highly tractable and economically intuitive;

(ii) serves our main purpose of testing for immunization procedures while still capturing the distressed events that stir interest rates, and that could jeopardize the effectiveness of the applied immunization strategies;

(iii) it allows to clearly identify the cause of a given shift in the term structure of interest rate;

(iv) it is possible to estimate for all datasets analyzed, thus not inducing any bias to the empirical tests we wish to perform.

Furthermore, we have an active constraint to the estimation of real interest rates due to the lower liquidity and number of bonds issued when compared to the nominal interest rates.

For real interest rates the approach depicted in equation (3.2) would not be feasible due to the fact that no real rates prior to the 5-year maturity were available. The inexistence of estimates for maturities below the 5-year is explained by the use of on-the-run bonds for the estimation (as is done for the U.S. Treasuries) by the U.S. Department of the Treasury. As highlighted above, these bonds are used by specific market segments and this explains the inexistence of on-the-run bonds for maturities below 5 years. Hence, this term structure has been estimated using U.S. TIPS instead of the rates gathered from the U.S. Department of the Treasury. We minimize the mean square price errors, as depicted in equation (3.3), for the overall U.S. TIPS dataset in order to obtain estimated interest rates for maturities below the 5-year maturity. Although the extraction process is different, since it is applied to bonds instead of spot interest rates, the objective is similar, i.e. to obtain a continuous discount function for the term structure of interest rates for the immunization process, as depicted in equation (3.2).

The mean square price error obtained for the overall sampling period and dataset are 0,0016 basis points and the standard deviation of this measure is 0,021 basis points. The disaggregated values for the mean square prince error (and respective standard deviation) for each U.S. TIPS are included in Table 5.3. The mean square prince error between the observed and the estimated prices range from 0 to 0,0079 basis points and the standard deviation varies between 0 and 0,1664 basis points. Table 5.3 also includes the correlations between estimated and real prices for each U.S. TIPS. These values are fairly high, ranging from 76,98% to 99,89%.

(insert Table 5.3 here)

We also replicate the analysis presented in the previous Chapter for observed market rates from the U.S. Department of the Treasury and estimated rates from the Nelson and Siegel (1987) parametric extraction method. The results are depicted in Figure 5.1 and in Panel A of Table 5.4 for the nominal interest rates. The absolute average difference between the observed and the estimated rates lies between 10,5 basis points (0,5-year maturity) and 27,7 basis points (3-year maturity), which deems a high goodness-of-fit for the Nelson and Siegel (1987) estimation. The standard deviation of the difference is also low, ranging from 3,32 basis points (10-year maturity) to 27,74 basis points (0,5-year maturity). The correlation coefficients lie between a minimum of 98,40% for the 20-year maturity and a maximum of 99,98% for the 5-year maturity.

(insert Figure 5.1 and Table 5.4 here)

Panel B of Table 5.4 contains the results of the t-test of equality of means for the maturities used to fit the Nelson and Siegel (1987) model. The test is performed as formulated in equation (4.1) and the null hypothesis H_0 is rejected if the *p-value* is equal or below the 0,05 threshold. Again, we fail to reject the null hypothesis for every maturity, even though we have *p-values* that range from 0,2196 (30-year maturity) to 0,9588 (5-year maturity).

As for the real interest rates, although we have already presented the results for U.S. TIPS market and estimated prices, we replicate this analysis for the sake of completeness and include the results in Figure 5.2 and Table 5.5. The analysis is presented for maturities above 5-years, for the reasons explained previously. The absolute average difference between the observed and the estimated rates lies between 0,55 basis points (10-year maturity) and 28,41 basis points (5-year maturity) and the standard deviation of the difference is also low, ranging from 9,16 (20-year maturity) basis points to 25,24 basis points (5-year maturity).

(insert Figure 5.2 and Table 5.5 here)

As for the t-test of equality of means between the observed market interest rates and the estimated interest rates, we fail to reject the null hypothesis for every maturity, even though we have *p-values* that range from 0,3141 (5-year maturity) to 0,9604 (10-year maturity).

Figure 5.3 depicts the evolution of nominal interest rates. It can be observed that the underlying tendency is towards low interest rates during the sample period, with particular emphasis between January 2000 and January 2004 and between January 2007 and December 2008, where a significant downward movement occurred, namely in the 1- and 3-year maturities. The huge adjustment in nominal interest rates is also visible in Figure 5.4, namely in the short end of the term structure of interest rates (i.e. maturities below 5 years), where the downward movements have been more pronounced.

(insert Figure 5.3 and Figure 5.4 here)

A similar analysis is done for real interest rates, as shown in Figures 5.5 and 5.6. The underlying tendency towards low interest rates is also visible for real interest rates. However, the pattern is not similar. The periods where real interest rates decrease sharply are between 2000 and 2005 and again between 2009 and 2012. The difference between the two term structures is the evolution of inflation, which increased between 2004 and 2006. The term structures of interest rates also show this pattern. However, the decrease seems to be similar throughout the whole term structure of interest rates except in 2014, where an inversion is visible in the short end of the term structure of interest rates.

(insert Figure 5.5 and Figure 5.6 here)

5.3. Results

Prior to addressing the results for each dataset independently, we reiterate the remark done in the previous Chapter regarding the maturity bond. The empirical results from Soto and Prats (2002) and Kittithawornkul (2008) are once again replicated in this analysis, as the maturity bond is once again included in all the portfolios built for all the immunization strategies tested for both datasets. This way, the inclusion of the maturity bond is not trivial and seems to be as important as the immunization strategy chosen. We also reiterate that, in this Chapter, TIPS have been addressed taking into account only their real component and that the portfolios whose results are presented below have been estimated independently and comprise only Treasuries or TIPS bonds as we are not creating mixed portfolios nor attempting to compare Treasuries and TIPS. We are only comparing their immunization abilities towards a single fixed liability with the objective of assessing to what extent liquidity plays a role in the immunization procedure applied.

5.3.1. U.S. Treasuries

The results obtained for both 3- and 5-year immunization horizons are presented in Tables 5.6 and 5.7.

(insert Table 5.6 and 5.7 here)

In what regards the immunization results, the first conclusion is the same derived from the German dataset used in the previous Chapter: the good performance of the naïve strategy, that allows for an excess liability coverage of about 11% and 23% in each immunization horizon. This is mainly due to the coupon reinvestments and can be explained not only by the high coupon rates of the bonds used but also due to the decreasing interest rates throughout the time horizon of the immunization sampled period. Hence, this allows to further conclude that carrying high coupon bonds in a low interest environment is beneficial to the immunization process. As in the German case, when reinvesting these coupons into the portfolio, we obtain a gain associated with the excess coupon value because the interest rates are decreasing throughout the investment horizon. We also obtain a loss associated with bond prices increases that erode the portfolio value. These two effects - coupon reinvestment effect and price effect - behaving similarly to the income and price effect of consumer demand theory, as stated by Bierwag and Grove (1968), and tend to cancel each other out. However, what we verify in our analysis is that, as high coupons were reinvested at rates that were decreasing throughout the investment horizon, the portfolio gained extra value that was yet again reinvested in the portfolio. In this sense, the coupon reinvestment effect will not cancel the price effect associated with these bonds' rising prices and the high coupon will allow an extra gain for the portfolio, corroborating the findings of Fisher and Weil (1971) and Bierwag and Kaufman (1978). The valuation driven by the coupon reinvestment effect will also show up in all other strategies, even though transaction costs and higher turnover volumes might erode these gains in some cases.

As for the bullet and barbell portfolios, it stands out the barbell portfolio's superior performance while the bullet portfolio has lower results. The amazing returns associated with the barbell portfolio are due to the expressive downward movement of the interest rates, as shown in Figure 5.3. Besides that, the term structure of interest rates has also steepened quite sharply, and this explains the positive carry associated with both longer maturing bonds and also high coupons paid and reinvested into the portfolio. This allows for an average excess return of 5,44% and 3,33% for the 3-year and 5-year investment horizons and high Reward-to-Risk Ratios. Even so, this is also achieved with high turnover and transaction costs (i.e. for the 3-year immunization horizon this strategy shows the highest transaction costs and turnover). These are also the most volatile portfolios, therefore these results could easily have been worse in an increasing interest rate environment. These results confirm the empirical results presented by Bierwag and Kaufman (1978). The bullet portfolios perform marginally worse than the naïve strategy but exhibit lower average excess return and Reward-to-Risk Ratio statistics. This can be explained by the higher transaction costs and turnover that erode the returns (bear in mind that the naïve strategy transaction costs are only due to coupon reinvestments).

As for the M-strategies, the M-Absolute shows the best results in both immunization horizons, as it had already been seen in the German immunization case. The excess return of the strategy is 0,76% for the 3-year investment horizon and 1,44% for the 5-year investment horizon and the correspondent Reward-to-Risk Ratios are 34% and 48%. This bond clustering strategy also allows for lower turnover than the naïve portfolio, even though, naturally, the transaction costs are higher. This is consistent with lower quantities of each bond reinvested in each rebalancing period to achieve total immunization due to the nested positions around the maturity of the portfolio and even though the transaction costs are higher, this does not seem to have an impact in the average excess return for the 5-year investment horizon, where the results are close to the naïve strategy. This is in line with the results from Soto and Prats (2002).

The M-Squared strategies achieve lower results than the M-Absolute strategy, namely in the 5-year immunization horizon. The average transaction costs and turnover are lower in the 3-year strategy, which could imply that the M-Squared could be more cost efficient than the M-Absolute strategy. This is not the case because the M-Squared strategy returns are slightly more volatile (2,52% and 2,99% in the 3-year and 5-year immunization horizons, respectively) than the M-Absolute strategy returns (2,23% and 2,95% in the 3-year and 5-year immunization horizons, respectively), supporting Bierwag et al. (1993). Since this strategy accounts for the second order effect of the term structure of interest rates movements, this pattern also supports the barbell strategy outperformance being related to the steepening of the term structure of interest rates in the short sector, as this is an example of a slope adjustment, corroborating Bierwag (1977).

As for the M-Vector strategies, for the 3-year horizon, the M1 strategy has better results than the M-Absolute strategy, but the average transaction costs and turnover are three times higher. This outperformance is also not visible in the 5-year M1 strategy. As for the other M-Vector strategies, in the 3-year immunization horizon the M1 and M4 strategy also achieve good results, with higher average excess returns and Reward-to-Risk Ratios than the M-Absolute strategy (the M1 strategy even achieves higher results than the naïve strategy). Yet again, this also comes at average transaction costs and turnover, deeming these strategies less cost efficient. In the 5-year immunization horizon the M-Vector strategies overall underperform the M-Absolute and naïve strategies. Once again, these results are not in line with the empirical results presented by Nawalkha and Chambers (1997) and Kittithawornkul (2008).

As in the German case, it is also shown that the M-Squared strategy can achieve good immunization results without accounting for the first order effect of immunization, as this strategy outweighs the M2 strategy in both immunization horizons. This finding is also consistent with the yield curve slope adjustment observed throughout the sample period.

5.3.2. U.S. TIPS

The results obtained for both 3- and 5-year immunization horizons are presented in tables 5.8 and 5.9.

(insert Table 5.8 and 5.9 here)

In what regards the immunization results, the first conclusion is, yet again, the good performance of the naïve strategy, that allows for an excess liability coverage of about 7% and 12% in each immunization horizon. Bear in mind that this only accounts for the real interest rate immunization. Once again, the reason is straightforward and has already been widely discussed in the previous section and in the previous Chapter. This is mainly due to the coupon reinvestments and can be explained not only by the high coupon rates of the bonds used but also due to the decreasing interest rates throughout the time horizon of the immunization. Hence, this allows to further conclude that carrying high coupon bonds in a low interest environment is beneficial to the immunization process, since the reinvestment effect outweighs the coupon effect. As high coupons were reinvested at rates that were decreasing throughout the investment horizon, the portfolio gained extra value that was yet again reinvested in the portfolio. This valuation also appears in all other strategies, even though transaction costs and higher turnover volumes might erode these gains in some cases.

As for the bullet and barbell portfolios, these once again stand out but for different reasons. The barbell portfolio has, again, an outstanding performance for both immunization horizons. This is explained not only by the reinvestment of higher coupons but also due to the aggressively steepening of the term structure of interest rates in the short end, as we can recall from Figure 5.5, and as it was also acknowledged from the U.S. Treasuries results. This way, the longer bonds' return more than compensated the decreasing interest rates in the short end of the term structure of interest rates, that also lead to higher average excess returns (1,26% and 1,4% for the 3-year and 5-year investment horizons respectively) and Reward-to-Risk Ratios (39,51% and 36,31% for the 3-year and 5-year investment horizons respectively). Even so, this is only true due to the combination of a decreasing interest rate environment and a pronounced steepening of the term structure of interest rates. If it were the opposite, the most likely scenario would be that the barbell portfolio would be the lowest performer. The bullet portfolio has a similar performance to the naïve portfolio, but exhibits fairly high turnover (that also account for the high transaction costs). This can be explained by the lower liquidity of these bonds. Taking into account that the bullet portfolio is only composed by two bonds, keeping the duration of the portfolio equal to the target duration of the liability implies a higher turnover and transaction cost for this strategy and clearly this takes its toll on performance.

As for the M-strategies, the M-Absolute shows again the best results in both immunization horizons, and stands out as the only strategy achieving two-digit Reward-to-Risk Ratios (13,42% for the 3-year horizon and 10,55% for the 5-year horizon) due to the low standard deviation and positive average excess return. It also exhibits low turnover and transaction costs. Even so, it does not achieve the returns obtained by the barbell portfolio but the significantly lower turnover and transaction costs seem to imply that, from a cost efficiency perspective, this strategy might be better for immunization. The difference between the average excess return of these strategies is 0,85 percentage points for the 3-year immunization horizon and 1 percentage point for the 5-year immunization horizon while average transaction costs are about 1,6 and 1,3 percentage points higher for the 3- and 5-year immunization horizon. This way, the loss in transaction costs clearly do not compensate the higher return of the barbell portfolios when compared to the M-Absolute portfolios. The M-Squared strategy also achieves a good coverage and low transaction costs and turnover. However, the excess return and Reward-to-Risk Ratios are negative.

The M-Vector strategies do not achieve better results than the naïve portfolio strategies. For the 3-year investment horizon, the M-Vector strategies never have positive returns or Reward-to-Risk Ratios while for the 5-year investment horizon it achieves positive excess returns of 0,22% and 0,05% in the M4 and M5 strategies, with fairly high turnover and transaction costs when compared to other strategies. When comparing the M-Squared with M2, the results are different. In U.S. TIPS dataset, M2 strategies have similar or higher average excess returns. This finding can be explained, in this case, by the dominant parallel effect in the downward movement of interest rates. Although a slope adjustment also occurs towards the end of the sampled period, this adjustment is embedded with an inversion in the short end of the term structure of interest rates (the hump observable in the 1-year residual maturity in Figure 3.4), that could also explain this result.

As in the U.S. Treasuries dataset, the empirical results from Fisher and Weil (1971) and Bierwag and Kaufman (1978) regarding coupon reinvestments and barbell portfolios are also verified in this dataset. Soto and Prats (2002) results favouring the M-Absolute strategy when compared with the M-Squared strategy and Bierwag et al. (1993) results favouring the traditional bullet and barbell strategies when compared with the M-Squared strategy are also acknowledged. Once again, the M-Vector results are not in line with the empirical results presented by Nawalkha and Chambers (1997) and Kittithawornkul (2008).

5.4. Concluding Remarks

This Chapter presents the results of several empirical tests for immunization strategies applied to U.S. Treasuries and U.S. TIPS from 2000 to 2014, where significant downward interest rate movements have occurred. In this sense the immunization strategies that achieve the best overall results are the barbell strategy and the M-Absolute strategy.

These results hold whether we use nominal or real bonds. As for the strategies based in the clustering of cash-flows around the maturity date, it is not clear cut that the M-Absolute is the best strategy in the U.S. Treasuries dataset for the 3-year immunization horizon. Even so, this is not the case for the 5-year horizon portfolio. In the U.S. TIPS case this strategy achieves superior results as it allows to immunize non-parallel term structure of interest rates shifts while not disregarding the parallel component. For the U.S. TIPS dataset we see that liquidity does play a role in the immunization strategies, as the higher rebalancing costs take its toll on the strategies return. However, there is also an upside to this effect: we also see that the results from this dataset are more stable which could also mean that illiquidity may work as a catalyst that avoids excessive turnover in some strategies.

Other meaningful results are consistent with the empirical results achieved in the German bond dataset presented in the previous Chapter. In both datasets the barbell strategy's good results are explained by the high coupon rate bonds reinvestment in a favorable environment, corroborating again the findings stated by Ingersoll et al. (1978) regarding the parallel interest rate shifts on low and high coupon bonds. The inclusion of the maturity bond in all the immunization portfolios built for each immunization strategy also corroborate the findings of Soto and Prats (2002) and Kittithawornkul (2008).

As in the German case, it would be interesting to see if these empirical results can be confirmed in an increasing interest rate environment or with semi-annual rebalancing frequency. This could help evaluating if the performance of some strategies is dependent on the interest rate environment and if some strategies would have better immunization results if the rebalancing frequency was lower. This could be a good empirical test in favour of the M-Absolute strategy. It could also be empirically tested if with another term structure model estimation (either parametric or stochastic) the same results would be obtained in order to assess to what extent the estimation results might be influenced by the method used to estimate interest rates. As for the M-Vector results, we verify again that the results shown here do not confirm the near-perfect hedging performance stated by Nawalkha and Chambers (1997) and Kittithawornkul (2008), namely in the U.S. TIPS dataset, where the lower liquidity of the bonds seems to erode the results of the strategies. In this sense, different specifications for the M-Vector, such as logarithms, polynomials or other generalizations, could also be empirically assessed, as they could prove more efficient for bonds that show less market liquidity.

Appendix A: U.S. TIPS Valuation

This appendix is based on Deacon et al. (2004), pages 176 to 178. Some adjustments have been applied to the notation, according to Section 3.1.2, for the sake of consistency throughout this thesis.

Index Ratio calculation

For both valuation and settlement purposes the Index Ratio in day t is computed as follows:

$$IR_t = \frac{I_t}{I_{base}} : I_t = CPI_{v-3} + \frac{(d-1)}{D_v} \times (CPI_{v-2} - CPI_{v-3}), \tag{A.1}$$

where

IR_t = Index Ratio in day t

I_t = Reference Index for day t

I_{base} = Reference Index for the first interest accrual day of the bond

CPI_{v-3} = value of the price index at time v-3 months

CPI_{v-2} = value of the price index at time v-2 months

D = number of days in month v

d = day of the month v when settlement occurs

v = month on which settlement takes place

$base$ = bond's first interest accrual day

The formula shown here is for an Index Ratio with an indexation lag of 3 months. However, other indexation lags can be considered by substituting variables CPI_{v-3} and CPI_{v-2} with the values for the indexation lag required, i.e. if the indexation lag wanted is 8 months then the price indices to consider are CPI_{v-8} and CPI_{v-7}.

Interest payment calculation

The nominal interest payment at time t is computed as shown below.

$$cpn\,(t)_N = \frac{c_{R_t}}{x} \times IR_t, \tag{A.2}$$

where

$cpn\,(t)_N$ = (%) interest payment in day t

c_{R_t} = annual real coupon rate in day t

x = number of coupons the bond pays per year

IR_t = Index Ratio in day t

The real interest payment at time t is computed in a similar way, but does not include the Index Ratio.

$$cpn\,(t)_R = \frac{c_{R_t}}{x} \tag{A.3}$$

Principal payment calculation

Calculation of the principal's value to be redeemed at maturity is done as shown below

$$Redemption(n) = FV \times \max\{1, IR_n\}\,, \tag{A.4}$$

where

FV = Face Value of the bond

IR_n = Index Ratio in maturity day n

The redemption payment contains an embedded option that means that this payment shall not be inferior to the face value of the bond, i.e. the inflation accrual will only be taken into account if it is positive. Please note that this feature is only applicable to the redemption payment.

Settlement price calculation

Settlement price calculation is more complex than for nominal fixed rate bonds, since it implies adjusting for inflation both clean price and real accrued interest. The formulas are presented beneath.

$$
\begin{aligned}
P_{TIPS_t} &= IQ_t + IAI_t : \qquad (A.5)\\
IQ_t &= RQ_t \times IR_t \\
RQ_t &= \left(\frac{1}{1 + \frac{f}{d} \frac{y_R(0,t)}{x}} \right) \times \\
& \left[\frac{c_{R_t}}{x} + \frac{c_{R_t}}{x} \sum_{t=1}^{n} \left(\frac{1}{1 + \frac{y_R(0,t)}{x}} \right)^t + \left(\frac{1}{1 + \frac{y_R(0,n)}{x}} \right)^n \right] - RAI_t \\
RAI_t &= \frac{c_{Rt}}{x} \times \frac{(g - f)}{g} \\
IAI_t &= RAI_t \times IR_t
\end{aligned}
$$

where

P_{TIPS_t} = Nominal Dirty Price for day t

IQ_t = Inflation adjusted Price for day t

RQ_t = Real Quoted Price for day t

IAI_t = Inflation adjusted Accrued Interest for day t

RAI_t = Real Accrued Interest for day t

IR_t = Index Ratio in day t

$y_R(0,t)$ = annual real spot rate for residual maturity t

$y_R(0,n)$ = annual real spot rate for residual maturity n

f = number of days from the settlement date to the next interest payment date

g = number of days in the regular annual coupon period ending on the next interest payment date

c_{R_t} = annual real coupon rate payment in day t

x = number of coupons the bond pays per year

Table 5.1: U.S. Treasury Bond Dataset

This table contains a description of the overall bondset selected to implement the immunization strategies described in Chapter 3. The bond subsets used in each portfolio were selected taking into account the restrictions included in Table 3.1.

ISIN	Description	Coupon	Issue Date	Maturity Date
US912810DM72	T 11.625 11/2004	11,625%	30/10/1984	15/11/2004
US912810DR69	T 10.75 8/2005	10,750%	02/07/1985	15/08/2005
US9128272C54	T 5.875 11/2001	5,875%	02/12/1996	30/11/2001
US912827Y554	T 7 7/2006	7,000%	15/07/1996	15/07/2006
US9128273L45	T 5.75 10/2002	5,750%	31/10/1997	31/10/2002
US9128273E02	T 6.125 8/2007	6,125%	15/08/1997	15/08/2007
US9128274V18	T 4.75 11/2008	4,750%	16/11/1998	15/11/2008
US9128274T61	T 4 10/2000	4,000%	02/11/1998	31/10/2000
US9128273V27	T 5.5 1/2003	5,500%	02/02/1998	31/01/2003
US9128275N82	T 6 8/2009	6,000%	16/08/1999	15/08/2009
US9128275E83	T 5 4/2001	5,000%	30/04/1999	30/04/2001
US9128275Z13	T 6.5 2/2010	6,500%	15/02/2000	15/02/2010
US9128275X64	T 6.375 1/2002	6,375%	31/01/2000	31/01/2002
US9128277H96	T 3.25 12/2003	3,250%	31/12/2001	31/12/2003
US9128277B27	T 5 8/2011	5,000%	15/08/2001	15/08/2011
US9128277E65	T 2.75 10/2003	2,750%	31/10/2001	31/10/2003
US912828AG57	T 2.25 7/2004	2,250%	31/07/2002	31/07/2004
US912828AL43	T 1.875 9/2004	1,875%	30/09/2002	30/09/2004
US9128277L09	T 4.875 2/2012	4,875%	15/02/2002	15/02/2012
US912828AX80	T 1.625 4/2005	1,625%	30/04/2003	30/04/2005
US912828AW08	T 1.625 3/2005	1,625%	31/03/2003	31/03/2005
US912828BH22	T 4.25 8/2013	4,250%	15/08/2003	15/08/2013
US912828CF56	T 2.25 4/2006	2,250%	30/04/2004	30/04/2006
US912828CD09	T 1.5 3/2006	1,500%	31/03/2004	31/03/2006
US912828CA69	T 4 2/2014	4,000%	17/02/2004	15/02/2014
US912828DS68	T 3.625 4/2007	3,625%	02/05/2005	30/04/2007
US912828EE63	T 4.25 8/2015	4,250%	15/08/2005	15/08/2015
US912828DQ03	T 3.75 3/2007	3,750%	31/03/2005	31/03/2007
US912828EU06	T 4.375 1/2008	4,375%	31/01/2006	31/01/2008

Table 5.1: continued

ISIN	Description	Coupon	Issue Date	Maturity Date
US912828FF20	T 5.125 5/2016	5,125%	15/05/2006	15/05/2016
US912828HF02	T 3.625 10/2009	3,625%	31/10/2007	31/10/2009
US912828HA15	T 4.75 8/2017	4,750%	15/08/2007	15/08/2017
US912828JP65	T 1.5 10/2010	1,500%	31/10/2008	31/10/2010
US912828JR22	T 3.75 11/2018	3,750%	17/11/2008	15/11/2018
US912828LT59	T 1 10/2011	1,000%	02/11/2009	31/10/2011
US912828LU23	T 3.125 10/2016	3,125%	02/11/2009	31/10/2016
US912828LY45	T 3.375 11/2019	3,375%	16/11/2009	15/11/2019
US912828NB24	T 1 4/2012	1,000%	30/04/2010	30/04/2012
US912828MH03	T 2.25 1/2015	2,250%	01/02/2010	31/01/2015
US912828NA41	T 3.125 4/2017	3,125%	30/04/2010	30/04/2017
US912828MP29	T 3.625 2/2020	3,625%	16/02/2010	15/02/2020
US912828QE36	T 0.625 4/2013	0,625%	02/05/2011	30/04/2013
US912828QG83	T 2.625 4/2018	2,625%	02/05/2011	30/04/2018
US912828RR30	T 2 11/2021	2,000%	15/11/2011	15/11/2021
US912828SR21	T 0.25 4/2014	0,250%	30/04/2012	30/04/2014
US912828ST86	T 1.25 4/2019	1,250%	30/04/2012	30/04/2019
US912828SF82	T 2 2/2022	2,000%	15/02/2012	15/02/2022
US912810EW46	T 6 2/2026	6,000%	15/02/1996	15/02/2026
US912810EV62	T 6.875 8/2025	6,875%	15/08/1995	15/08/2025
US912810ES34	T 7.5 11/2024	7,500%	15/08/1994	15/11/2024
US912828UL23	T 1.375 1/2020	1,375%	31/01/2013	31/01/2020
US912828UN88	T 2 2/2023	2,000%	15/02/2013	15/02/2023

Table 5.2: U.S. Treasury Inflation Protected Securities Dataset

This table contains a description of the overall bondset selected to implement the immunization strategies described in Chapter 3. The bond subsets used in each portfolio were selected taking into account the restrictions included in Table 3.1.

ISIN	Description	Coupon	Issue Date	Maturity Date
US9128273A89	TII 3.625 7/2002	3,625%	15/07/1997	15/07/2002
US9128272M37	TII 3.375 1/2007	3,375%	06/02/1997	15/01/2007
US9128273T70	TII 3.625 1/2008	3,625%	15/01/1998	15/01/2008
US9128274Y56	TII 3.875 1/2009	3,875%	15/01/1999	15/01/2009
US9128275W81	TII 4.25 1/2010	4,250%	18/01/2000	15/01/2010
US912828CZ11	TII 0.875 4/2010	0,875%	29/10/2004	15/04/2010
US9128276R87	TII 3.5 1/2011	3,500%	16/01/2001	15/01/2011
US912828FB16	TII 2.375 4/2011	2,375%	28/04/2006	15/04/2011
US912828GN45	TII 2 4/2012	2,000%	30/04/2007	15/04/2012
US912828AF74	TII 3 7/2012	3,000%	15/07/2002	15/07/2012
US912828BD18	TII 1.875 7/2013	1,875%	15/07/2003	15/07/2013
US912828KM16	TII 1.25 4/2014	1,250%	30/04/2009	15/04/2014
US912828DH04	TII 1.625 1/2015	1,625%	18/01/2005	15/01/2015
US912828QD52	TII 0.125 4/2016	0,125%	29/04/2011	15/04/2016
US912828SQ48	TII 0.125 4/2017	0,125%	30/04/2012	15/04/2017
US912828HN36	TII 1.625 1/2018	1,625%	15/01/2008	15/01/2018
US912828NM88	TII 1.25 7/2020	1,250%	15/07/2010	15/07/2020
US912828UH11	TII 0.125 1/2023	0,125%	31/01/2013	15/01/2023
US912810FH69	TII 3.875 4/2029	3,875%	15/04/1999	15/04/2029

Table 5.3: U.S. TIPS Observed and Estimated Price Comparison

The table contains the mean square price error (MSPE) and respective standard deviation obtained for the real term structure of interest rates estimation process using the Nelson and Siegel (1987) parametric extraction method on the U.S. TIPS dataset. The values, in basis points, are presented for each bond as well as for the global dataset. The correlations between the market prices collected from Bloomberg and estimated prices using the Nelson and Siegel (1987) parametric extraction method for each bond included in this dataset are also presented.

Description	**Mean Square Price Error (b.p.)**	**Standard Deviation of the MSPE (b.p.)**	**Correlation (%)**
TII 3.625 7/2002	0,0079	0,1664	91,91
TII 3.375 1/2007	0,0037	0,1025	98,08
TII 3.625 1/2008	0,0032	0,0961	99,37
TII 3.875 1/2009	0,0028	0,0908	99,50
TII 4.25 1/2010	0,0021	0,0843	99,19
TII 0.875 4/2010	0,0036	0,1103	94,65
TII 3.5 1/2011	0,0009	0,0287	98,26
TII 2.375 4/2011	0,0040	0,1173	98,43
TII 2 4/2012	0,0025	0,0491	96,37
TII 3 7/2012	0,0013	0,0349	93,82
TII 1.875 7/2013	0,0026	0,0847	93,70
TII 1.25 4/2014	0,0004	0,0005	76,98
TII 1.625 1/2015	0,0002	0,0004	97,74
TII 0.125 4/2016	0,0003	0,0004	65,54
TII 0.125 4/2017	0,0002	0,0002	90,91
TII 1.625 1/2018	0,0003	0,0003	98,00
TII 1.25 7/2020	0,0002	0,0003	99,53
TII 0.125 1/2023	0,0000	0,0000	99,89
TII 3.875 4/2029	0,0004	0,0008	99,70
Global Dataset	0,0016	0,0210	—

Table 5.4: U.S. Observed and Estimated Nominal Interest Rates Comparison

This table is divided in two panels. Panel A contains the absolute average and standard deviation of the difference between market interest rates collected from U.S. Department of the Treasury and estimated interest rates using the Nelson and Siegel (1987) parametric extraction method (values in basis points) and the correlations between the aforementioned interest rates for each maturity included in this dataset. It is possible to see that the values are very high, which implies a very strong positive correlation between estimated and observed interest rates. Panel B presents the results for the t-test of equality of means. For all the maturities presented we do not reject the null hypothesis of equality of the observed and estimated interest rate means.

Panel A - Absolute Differences and Correlations

	Maturities (years)							
	0,5	**1**	**2**	**3**	**5**	**10**	**20**	**30**
Average (b.p.)	10,50	3,55	1,23	3,67	0,84	2,29	7,84	27,70
Standard deviation (b.p.)	27,74	17,12	6,97	6,46	3,94	3,32	22,60	27,30
Correlation (%)	99,20	99,67	99,94	99,95	99,98	99,97	98,40	96,36

Panel B - Mean interest rate equality tests

	Maturities (years)							
	0,5	**1**	**2**	**3**	**5**	**10**	**20**	**30**
t-statistic	-0,4798	-0,1686	0,0624	0,1983	-0,0516	0,1820	0,7437	1,2299
P-value	0,6317	0,8662	0,9503	0,8429	0,9588	0,8556	0,4575	0,2196

Table 5.5: U.S. Observed and Estimated Real Interest Rates Comparison

This table is divided in two panels. Panel A contains the absolute average and standard deviation of the difference between market interest rates collected from U.S. Department of the Treasury and estimated interest rates using the Nelson and Siegel (1987) parametric extraction method (values in basis points) and the correlations between the aforementioned interest rates for each maturity included in this dataset. It is possible to see that the values are very high, which implies a very strong positive correlation between estimated and observed interest rates. Panel B presents the results for the t-test of equality of means. For all the maturities presented we do not reject the null hypothesis of equality of the observed and estimated interest rate means.

Panel A - Absolute Differences and Correlations					
	Maturities (years)				
	5	**7**	**10**	**20**	**30**
Average (b.p.)	28,41	9,92	0,55	1,48	6,15
Standard deviation (b.p.)	25,24	18,21	13,28	9,16	18,60
Correlation (%)	97,81	98,72	99,12	99,19	94,17
Panel B - Mean interest rate equality tests					
	Maturities (years)				
	5	**7**	**10**	**20**	**30**
t-statistic	1,0085	0,8151	-0,0497	0,1377	0,7195
P-value	0,3141	0,4157	0,9604	0,8906	0,4724

Table 5.6: U.S. Treasuries 3-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 3-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	111,22%	—	1,42%	64,46%
Barbell	115,59%	103,97%	5,44%	122,25%
Bullet	110,22%	99,15%	0,54%	28,68%
M-Absolute	110,43%	99,38%	0,76%	33,99%
M-Squared	110,37%	99,31%	0,70%	27,74%
M1	111,80%	100,54%	1,95%	68,96%
M2	110,17%	99,11%	0,49%	26,27%
M3	110,19%	99,13%	0,51%	27,60%
M4	110,75%	99,62%	1,01%	49,08%
M5	110,65%	99,52%	0,92%	45,16%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,365%	15,36	—	—
Barbell	2,187%	80,24	6,0	5,22
Bullet	0,926%	37,65	2,5	2,45
M-Absolute	0,415%	12,84	1,1	0,84
M-Squared	0,276%	10,29	0,8	0,67
M1	1,172%	31,47	3,2	2,05
M2	1,697%	16,72	4,6	1,09
M3	1,171%	29,15	3,2	1,90
M4	0,638%	25,62	1,7	1,67
M5	0,706%	26,29	1,9	1,71

Table 5.7: U.S. Treasuries 5-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 5-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	122,84%	—	1,54%	48,32%
Barbell	124,85%	101,75%	3,33%	100,63%
Bullet	121,62%	99,04%	0,56%	19,33%
M-Absolute	122,74%	99,92%	1,44%	48,22%
M-Squared	121,73%	99,14%	0,65%	22,01%
M1	121,54%	98,94%	0,44%	15,62%
M2	121,35%	98,80%	0,30%	10,78%
M3	121,48%	98,92%	0,42%	14,87%
M4	122,18%	99,49%	1,00%	33,53%
M5	122,05%	99,38%	0,90%	29,92%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,374%	14,55	—	—
Barbell	1,140%	49,23	3,1	3,38
Bullet	1,097%	44,05	2,9	3,03
M-Absolute	0,715%	9,10	1,9	0,63
M-Squared	0,208%	7,00	0,6	0,48
M1	1,786%	31,22	4,8	2,15
M2	2,264%	12,69	6,1	0,87
M3	1,114%	22,79	3,0	1,57
M4	0,592%	23,65	1,6	1,62
M5	1,052%	24,30	2,8	1,67

Table 5.8: U.S. TIPS 3-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 3-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	106,86%	—	0,94%	28,93%
Barbell	107,17%	100,38%	1,26%	39,51%
Bullet	105,74%	99,08%	-0,07%	-2,48%
M-Absolute	106,25%	99,56%	0,41%	13,42%
M-Squared	105,18%	98,47%	-0,33%	-11,24%
M1	105,57%	98,92%	-0,24%	-7,97%
M2	105,48%	98,86%	-0,32%	-10,57%
M3	105,57%	98,95%	-0,23%	-7,70%
M4	105,71%	99,08%	-0,10%	-3,13%
M5	105,71%	99,09%	-0,09%	-2,96%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,232%	7,80	—	—
Barbell	1,914%	45,27	8,3	5,80
Bullet	2,820%	81,77	12,2	10,48
M-Absolute	0,354%	12,53	1,5	1,61
M-Squared	0,380%	13,85	1,6	1,78
M1	0,597%	20,16	2,6	2,58
M2	1,195%	39,83	5,2	5,10
M3	1,556%	54,32	6,7	6,96
M4	1,794%	61,81	7,7	7,92
M5	1,619%	56,29	7,0	7,21

Table 5.9: U.S. TIPS 5-Year Immunization Results

This table is divided in two panels that include several metrics for the strategies defined in Table 3.1 for the 5-year horizon. Panel A contains the immunization coverage and performance metrics explained in Chapter 3. $\overline{LC}$ is the average Liability Coverage Ratio, RC is the Relative Coverage Ratio, $\overline{ER}$ is the average Excess Return and R/R is the Reward-to-Risk Ratio. The Relative Coverage compares with the Naive Strategy (i.e. if the value for a given Strategy exceeds 100% then the Strategy's Average Liability Coverage is higher than the Naive Strategy). Panel B contains the Immunization Costs metrics. Average Transaction Costs ($\overline{TC}$) is expressed as a percentage of the bond's value and Average Turnover ($\overline{T}$) is expressed in quantities. The Transaction Costs Multiplier (TC_X) and Turnover Multiplier (T_X) compare with the Naive strategy (i.e. if the value for a given strategy exceeds 1 then that strategy has higher Transaction Costs and Turnover than the Naive strategy).

Panel A - Immunization Coverage and Performance

Strategy S	$\overline{LC}\,(S)$	$RC\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$
Naive	112,72%	—	0,84%	21,31%
Barbell	113,33%	100,58%	1,40%	36,31%
Bullet	111,68%	99,14%	-0,06%	-1,64%
M-Absolute	112,15%	99,62%	0,40%	10,55%
M-Squared	111,41%	98,94%	-0,28%	-7,40%
M1	111,44%	98,93%	-0,28%	-7,35%
M2	111,55%	99,06%	-0,16%	-4,17%
M3	111,49%	99,02%	-0,20%	-5,30%
M4	111,98%	99,42%	0,22%	5,60%
M5	111,80%	99,26%	0,05%	1,40%

Panel B - Immunization Costs

Strategy S	$\overline{TC}\,(S)$	$\overline{T}\,(S)$	$TC_X\,(S)$	$T_X\,(S)$
Naive	0,239%	7,90	—	—
Barbell	1,686%	43,50	7,0	5,51
Bullet	3,680%	106,69	15,4	13,51
M-Absolute	0,336%	11,50	1,4	1,46
M-Squared	0,267%	9,11	1,1	1,15
M1	0,811%	27,13	3,4	3,44
M2	1,173%	39,84	4,9	5,05
M3	1,471%	50,76	6,1	6,43
M4	1,433%	49,21	6,0	6,23
M5	1,538%	51,91	6,4	6,57

Figure 5.1: U.S. Treasuries Nelson-Siegel Estimation Errors

The figure contains a comparison between Market Interest Rates from the U.S Department of the Treasury and Estimated Interest Rates with the Nelson-Siegel parametric extraction method for the selected monthly interest rates. The chart can be interpreted as follows: the horizontal bar is the average rate for that maturity and the vertical bar contains the minimum and maximun interest rates, during the sample period for each maturity. It is visible that market interest rates and estimated interest rates have a similar pattern in all the maturities presented. This means that their distributions are very close, which is confirmed by the results shown in Table 5.4.

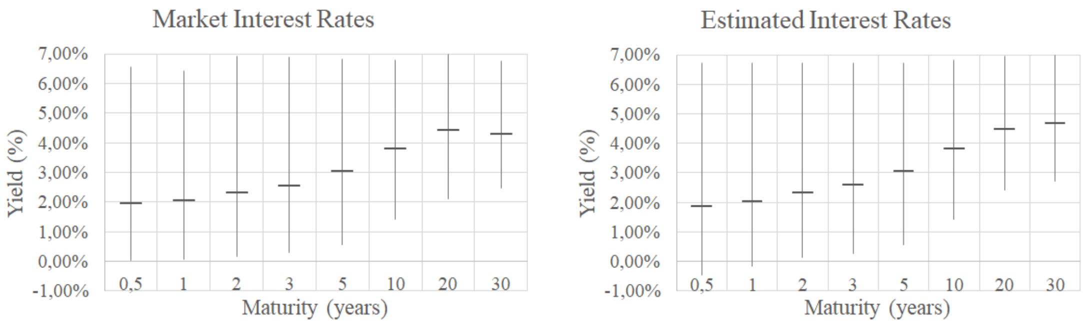
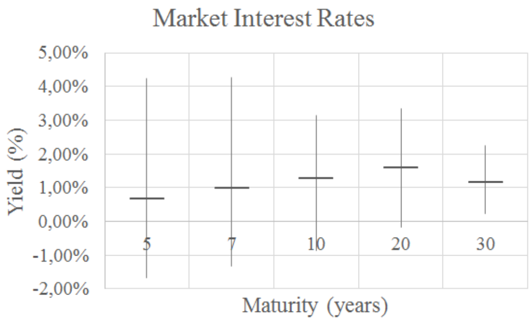


Figure 5.2: U.S. TIPS Nelson-Siegel Estimation Errors

The figure contains a comparison between Market Interest Rates from the U.S Department of the Treasury and Estimated Interest Rates with the Nelson-Siegel parametric extraction method for the selected monthly interest rates. The chart can be interpreted as follows: the horizontal bar is the average rate for that maturity and the vertical bar contains the minimum and maximun interest rates, during the sample period for each maturity. It is visible that market interest rates and estimated interest rates have a similar pattern in all the maturities presented. This means that their distributions are very close, which is confirmed by the results shown in Table 5.5.



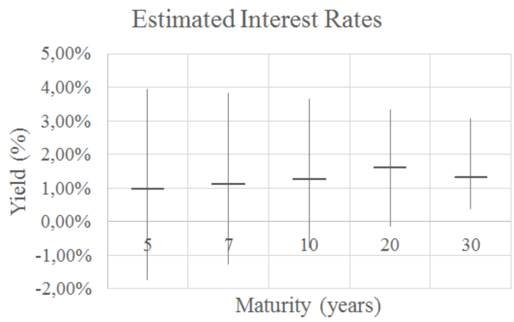


Figure 5.3: U.S. Treasuries Interest Rates

This figure contains the nominal yields for the 1, 3, 5 and 10-year maturities between January 2000 and December 2014. We observe a considerable decrease in interest rates between 2000 and 2002, followed by an increase for all maturities until year-end 2006 in a highly volatile environment, namely in the 1-year maturity. From 2006 to 2008 we see another significan decrease in interest rates due to the subprime crisis. Apart from the 1-year maturity, that stays in low levels until year-end 2014, interest rates rebound for a while in 2009 but subsequently resume their decreasing tendency until 2012. From that year onwards, interest rate above the 3-year maturity tend to increase until the end of the sampling period.

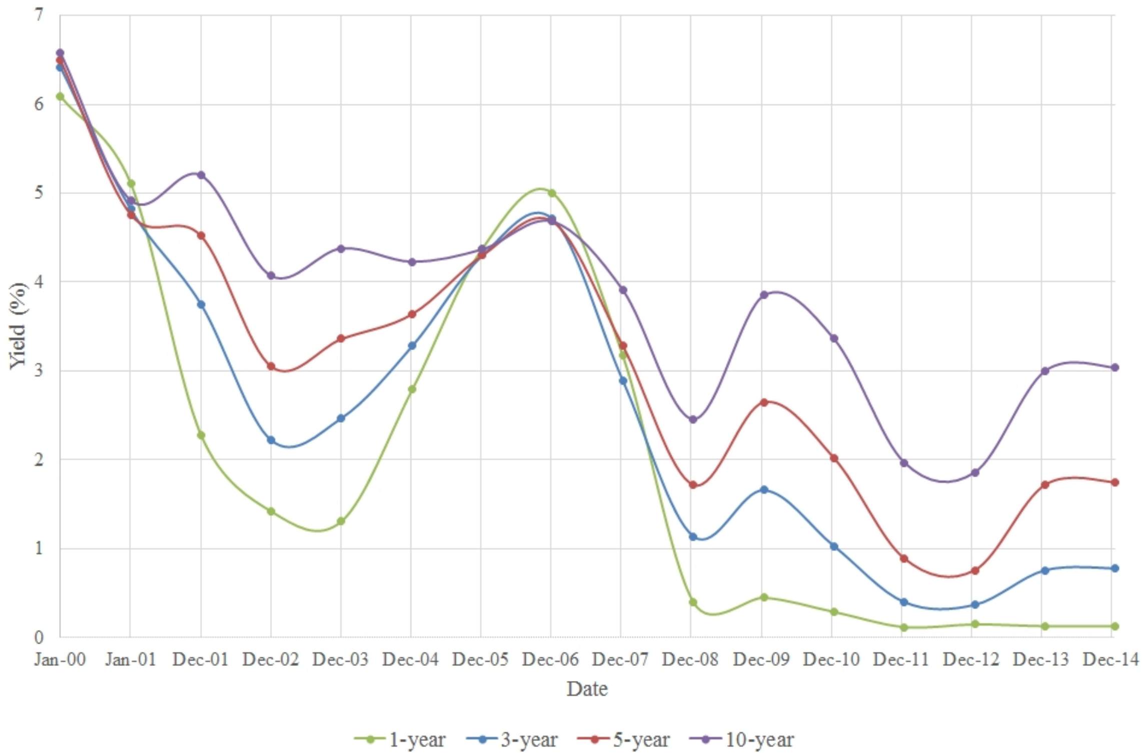


Figure 5.4: U.S. Treasuries Yield Curve

This figure contains the term structure of nominal interest rates in the year-end from 2000 to 2014, obtained from the U.S. Department of the Treasury (please note that between 2003 and 2005 no data is available for the 30-year maturity). As it is visible, the yield curve assumed several shapes, beginning with an inverted shape in 2000, where we can see that the 0,5-year interest rate is above the 5-year interest rate. A steepening movement occurs from 2001 to 2003 with a sharp decrease in interest rates in the short sector of the curve. The curve flattens again between 2005 and 2007, showing a slight inversion in the short sector in the latter year. From 2008 onwards, the curve assumes a concave shape with steepening movement as interest rates in the short sector decrease more than in the long sector of the term structure of interest rates.

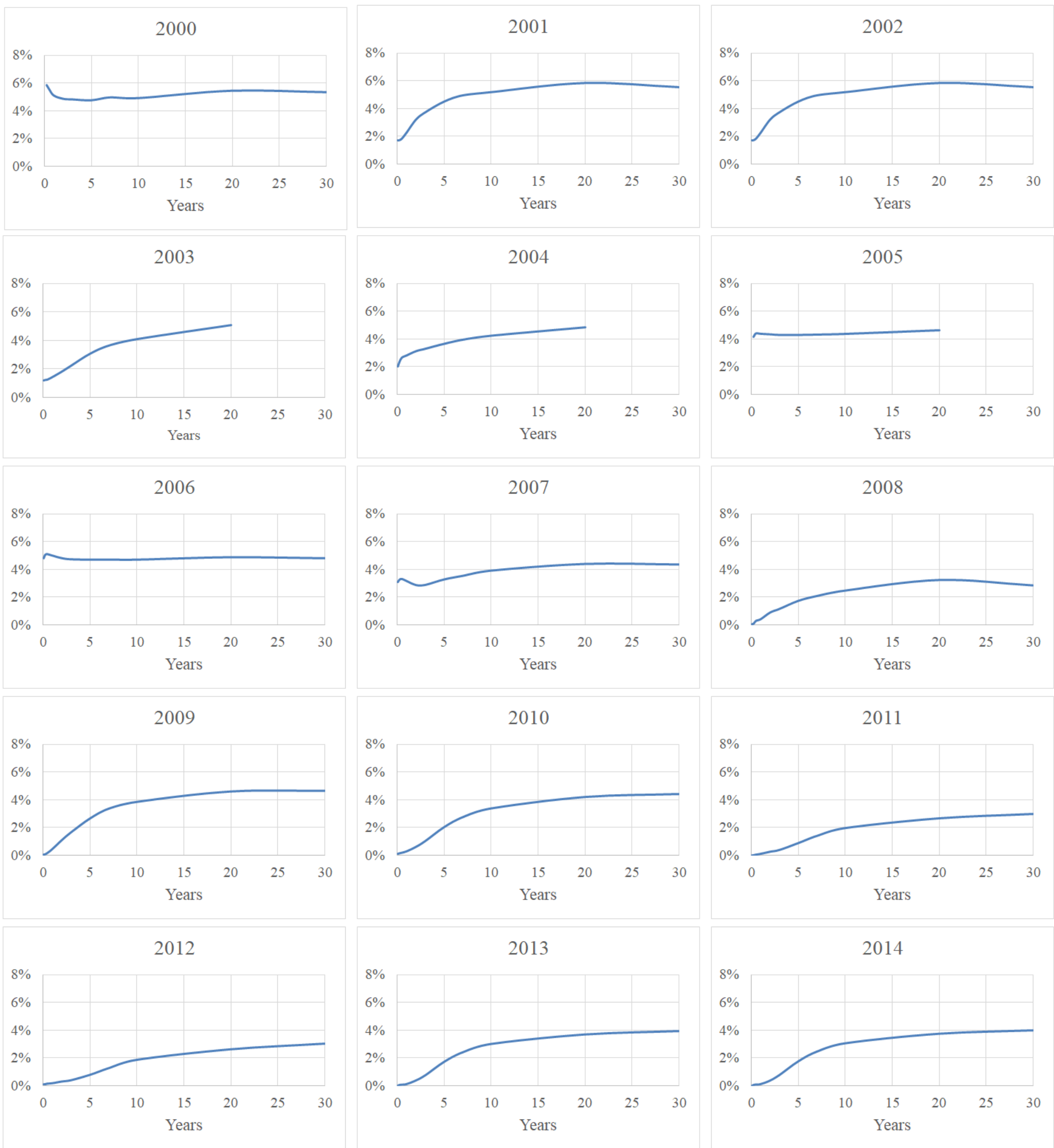


Figure 5.5: U.S. TIPS Interest Rates

This figure contains the real yields for the 1, 3, 5 and 10-year maturities between January 2000 and December 2014. The interest rate evolution hints at a quasi flat structure until year-end 2012. It is visible a downward tendency in interest rates between 2000 and 2005 with some stability from 2002 to 2004. From 2006 onwards a increasing tendency sets is and interest rates peak in 2008. Afterwards, the downward tendency resumes until December 2012 and into negative territory. Rates increase and decouple from 2012 onwards, even though they remain negative in the 1-year maturity until the end of the sample period.

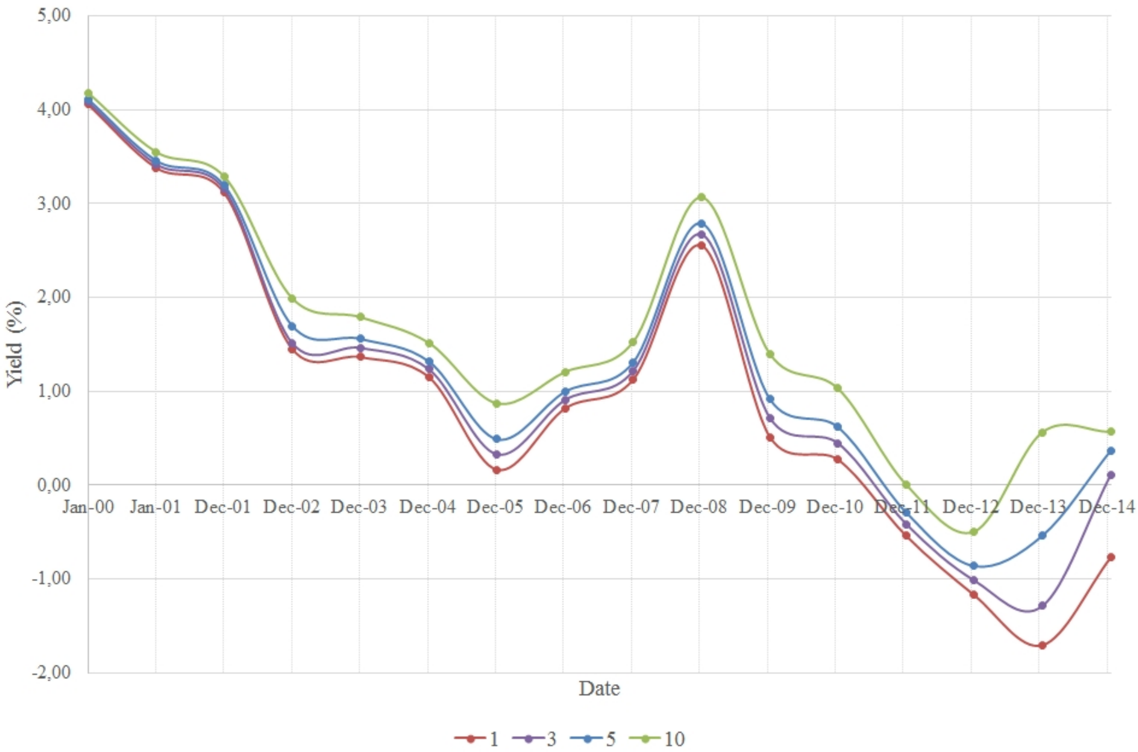
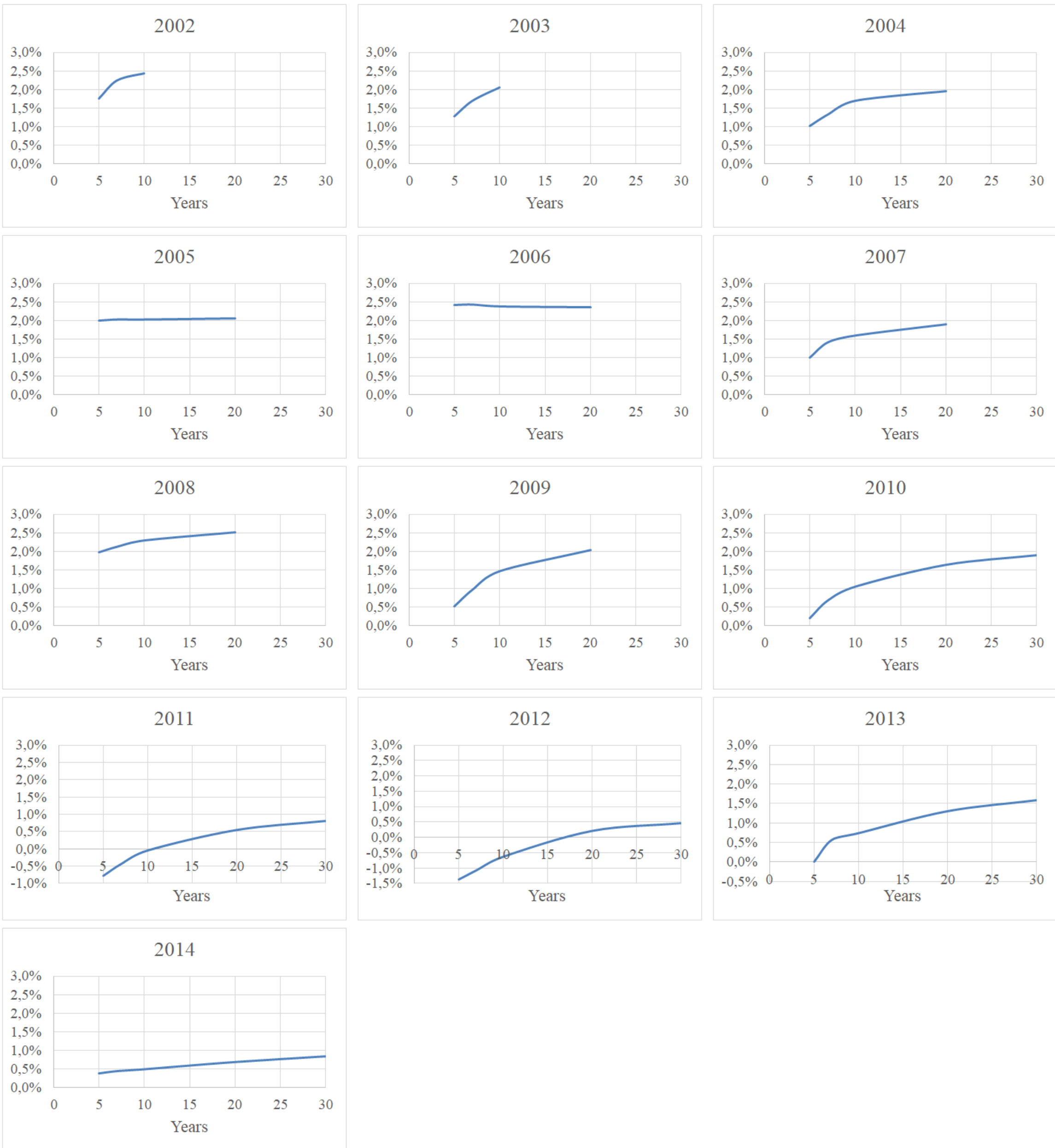


Figure 5.6: U.S. TIPS Yield Curve

This figure contains the term structure of real interest rates in the year-end from 2002 to 2014, as estimated by the U.S. Department of the Treasury. The interest rates were only available from 2002 onwards and for maturities equal or above 5-years and several gaps is the data are visible until 2009. As it is visible, the yield curve assumed several shapes, beginning by what seems to imply a concave shape from 2002 to 2004. The cruve flatten in years 2005 and 2006 with a slight increase in interest rates. From 2007 onwards we see a decrease with flattening at first (year 2008) but evolving onwards to a steep shape with a significant decrease in interest rates, that reach negative values in years 2011 and 2012 for maturities below the 10- to 15-year interest rates. In 2013 interest rates rise with steepening but consequently converge to a quasi-flat term structure in year-end 2014.



CHAPTER 6

Multi Liability Immunization with the M-Absolute model: an approach to the U.S. bond market

This Chapter takes the results presented in the previous Chapters and extends the analysis of the M-Absolute strategies to immunize multi-period liabilities. We apply the strategy to the U.S. datasets used in the previous Chapter while going one step forward and taking into account all the necessary adjustments to make the datasets comparable, in both risk measurement and return.

The M-Absolute strategy will be applied to nominal U.S. Treasury Bonds (Treasuries) and to real U.S. Treasury Inflation Protected Securities (TIPS), whose definitions and main characteristics are included in Chapter 5. However, in the present Chapter, we will compare both datasets, which poses a challenge in what concerns portfolio setup and performance evaluation. As stated in Chapter 3, we will immunize a type (3) liability - one for which the cash outlay's timing is known but the amount is uncertain - according to Fabozzi (2000, p. 449) classification. The comparison with nominal and real bonds is aimed at seeing which bond type produces the best immunization results for each strategy. It will also be possible to compare the results of U.S. Treasuries and U.S. TIPS datasets, since U.S. TIPS have been tested for immunization in their two components (with and without inflation accrual).

Bierwag, Kaufman and Toevs (1983) address immunization strategies for multiple liabilities and multiple parallel interest rate shocks, expanding the findings by Redington (1952). Under the multiple liability funding hypothesis, the authors state that, for an immunization strategy to be effective the asset portfolio and the liability portfolio must fulfill three conditions:

(1) the asset portfolio and the liability portfolio must have the same present value;

(2) both portfolio durations must be similar and

(3) asset dispersion must be higher than liability dispersion in the stable interest rate scenario.[1]

This way, by applying Bierwag et al. (1983) conditions, one can use multiple bonds with different maturities to immunize a stream of predictable future liabilities.

The article by Fong and Vasicek (1983a), that presents the M-Squared as an immunization risk measure, extends Bierwag et al. (1983) reasoning towards multiple liability immunization by establishing the necessary and sufficient conditions for multiperiod immunization with this risk measure. In this sense, conditions (1) and (2) are identified and condition (3) is modified, taking into account the way the M-Squared measure is built. The new Fong and Vasicek (1983a) condition (3) states that the dispersion of the mean absolute deviation of the bond portfolio has to be higher than the dispersion of the mean absolute deviation of the liability portfolio. This is an extension from the single liability immunization problem where we see that for this measure, the bond portfolio's cash-flows are clustered around the date the liability will be paid. For the multiple liability immunization problem, the restated condition (3) implies that the bond portfolio's cash-flows have to be clustered around each liability due to be paid.

Fong and Vasicek (1983b) also extend the immunization multiple liability problem to the M-Squared measure to assess the trade off between risk and return in an immunized portfolio, due to accounting for the composition of the immunization portfolio in its design. The authors state that strictly minimizing risk while applying an immunization strategy may be quite restrictive to the investor, hence they present the risk return minimax problem as the minimization of the difference between the M-Squared measure and the target return of the portfolio, subject to the immunization conditions of equality of

[1] This last condition can be replaced by the Bierwag rule.
The authors extend the results further by proving that the 3-rd condition will be met if the asset portfolio can be divided in two sub-portfolios and the following rules are applied:
(a) one of the asset portfolios has a duration below or equal to the date the first liability is paid;
(b) one of the asset portfolios has a duration equal or above the date the liability is paid;
(c) the combined asset portfolios' duration equals the duration of the liabilities.

the portfolio's duration and residual maturity of the liability and other investment policy requirements (like the absence of short-selling and minimum investment in individual securities, for example).

Barber and Copper (1998a) also acknowledge the work of Fong and Vasicek (1983b) and Fong and Vasicek (1984) and extend it for a multiple liability setting applying a minimax strategy and an infinite factor interest rate model using the M-Squared measure. In this sense, this article differs from Fong and Vasicek (1983b) because the aim here is not to study the trade off between return and risk in a multiple liability immunization problem. Another article from Barber and Copper (1998b) establishes the necessary and sufficient conditions for immunization for addictive term structure models, considering the single and multiple liability immunization problems. The conditions presented broadly state that an immunized portfolio's duration will be equal to the residual maturity of the liability due to be paid and that for each pair of assets maturing around the liability payment date H, and considering that one matures before and the other after date H, the investment combination that achieves full immunization is unique. This last condition is also extended to the case where multiple liabilities have to be paid. The authors state that the added complexity of a multiple liability immunization problem can be handled by separating immunizing each liability, i.e. separately building sub portfolios that contain two assets that fulfill the conditions above. In this sense, the multiple liability immunization problem is considered as an extension of the single liability immunization problem.

Shiu (1988) also confirms the conclusions presented by Bierwag et al. (1983) for multiperiod immunization and acknowledges that the 3-rd condition stated by Bierwag et al. (1983) is a necessary and sufficient condition to immunize multiple liabilities. Shiu (1988) findings are extended by Uberti (1997) to portfolio immunization considering general shifts on the term structure of interest rates and embedding the M-Squared measure in the multiple liability conditions (1) to (3) established by Bierwag et al. (1983), taking into account that the multiple liability immunization problem can be set up minimizing both duration and M-Squared measures, accounting for parallel and nonparallel interest rate shocks.

The multiple liability problem is also addressed by Theobald and Yallup (2005), that extend previous findings to the M-Squared and M-Vector strategies and test these strategies empirically using the United Kingdom guilts market from 1997 to 2003. The authors state that immunization will be successfully implemented if the moments of the asset and liability portfolios are the same or very close, since this will assess to what extent the distribution of cash-flows of the asset portfolio and liability portfolio will agree and present a general framework for the M-Squared and M-Vector risk measures. Their results show that the immunization strategies using these latter measures outweigh the results of the traditional duration strategies.

Alina Kondratiuk-Janyska and Marek Kaluszka have also extensively studied portfolio immunization techniques and procedures. For instance, Kaluszka and Kondratiuk-Janyska (2004) generalize the multiple liability portfolio immunization strategies to other dispersion measures, taking into account that the immunization strategies based in the traditional theory risk measures, such as Macaulay (1938) and Fisher and Weil (1971) duration, do not rule out ex-ante the possibility of arbitrage, thus being inconsistent with modern finance theory. The authors present a stochastic measure that builds from the duration gap and dispersion of portfolio payments that also accounts for the maturity bond. Kaluszka and Kondratiuk-Janyska (2004) state that the immunization of multiple liabilities can be achieved by immunizing separately each liability cash-flow. This is an extension of the rules (a) to (c) stated by Bierwag et al. (1983), since these rules were developed in a setting where one would divide the asset portfolio into two sub-portfolios, no matter the number of liabilities to immunize. Kaluszka and Kondratiuk-Janyska (2004) statement implies, when immunizing with Macaulay (1938) and Fisher and Weil (1971) duration, each sub-portfolio that is built will have a duration equal to the date the liability it is supposed to immunized is due and, consequently, the combined asset portfolios' duration will match the duration of the liabilities.

The authors expand their research in Kondratiuk-Janyska and Kaluszka (2006a) and Kondratiuk-Janyska and Kaluszka (2006b) where an application of immunization strategies to the single factor Heath et al. (1992) framework is presented and the immunization programming for Fong and Vasicek (1984) M-Squared and Nawalkha and Chambers (1996) M-Absolute strategies in a continuous time setting is extended to the multiperiod immunization setting. Regarding the M-Absolute measure, Kondratiuk-Janyska and Kaluszka (2006b) also show that for the multiple liability immunization problem, minimizing the M-Absolute measure implies *per se* the minimization of the absolute duration gap between the asset portfolio and the liability portfolio. This is an important result that has to be taken into account while setting up the linear programming for the multiple liability portfolios. Finally, Kondratiuk-Janyska and Kaluszka (2009), extends the immunization setting for continuous-time single and multiple liability portfolios, using again the M-Squared and M-Absolute strategies, and assuming random shocks to the term structure of interest rates.

The Chapter is structured as follows: section 6.1 contains a brief description of the year-on-year inflation behavior throughout the sampled period[2]. Section 6.2 presents the theoretical framework for the M-Absolute model with the adjustments made to the U.S. TIPS bonds to account for the inflation accrual component and their implications for portfolio design. Section 6.3 presents the methodology and assumptions applied in this empirical study, bearing in mind the necessary adjustments, and section 6.4 discusses the empirical results obtained. The last section summarizes our conclusions and proposes a way forward for future research.

6.1. U.S. inflation data

In order to allow for U.S. TIPS to account for the inflation evolution, the index used as an inflation proxy for these bonds is the non-seasonally adjusted Consumer Price Index for all urban consumers (CPI-U henceforth). The dataset used has been retrieved

[2]The dataset characterization can be found in section 5.2 of the previous chapter.

from the Bureau of Labor Statistics[3] from 2000 to 2014, and comprises the values of this index, as published by the Bureau. These values were used to compute the year-on-year inflation rate and the Index Ratios applied to U.S. TIPS, whose calculation will be explained in the next section. Figure 6.1 shows the evolution of the year-on-year inflation rate throughout the sample period. As it is visible, there is an upward tendency between 2002 and 2006 and 2007-2008, that is followed by a severe disinflation process, that arises as a consequence of the quantitative easing process carried out by the U.S. Federal Reserve in the wake of the subprime crisis. Actually, from 2009 to 2012 inflation picks up again, even though it never reaches the around 4% area from 2008; from 2012 onwards the underlying tendency is for a steady and continuous decline in year-on-year inflation.

(insert Figure 6.1 here)

6.2. Theoretical framework

The formula for computing the fair value of a U.S. Treasury bond can be found in equation (5.1)[4]

$$B_{UST}(0) = \sum_{t=1}^{n} \frac{c_t}{x} \times \delta_N(0,t) + FV \times \delta_N(0,n),$$

The formula for the real fair value of U.S. TIPS has already been defined in equation (5.2).

$$B_{TIPS_R}(0) = \sum_{t=1}^{N} \frac{c_{R_t}}{x} \times \delta_R(0,t) + FV \times \delta_R(0,n),$$

To compute the nominal fair value of a U.S. TIPS, this formula has to be adjusted to include the inflation accrual. Since time is needed to compile and publish the data

[3]Please refer to http://data.bls.gov/pdq/SurveyOutputServlet for more information.

[4]Please refer to Subsection 3.1.2 in Chapter 3 and to Section 5.2 in Chapter 5 for preliminary notation and details on the definitions recalled in this Section.

for the index, the inflation adjustment is done with a lag, i.e., the Index Ratio that is computed today uses the information from the two last published values for the CPI-U as a proxy for the actual inflation rate. This way, it is not possible to have a full hedge against inflation through these bonds; however, they still provide a very high degree of protection against inflation and purchasing power erosion.

The indexation lag is minimized by making it as short as possible. The indexation lag applied to U.S. TIPS is 3 months and the reference index is computed by linear interpolation, between index publications, like equation (6.1) shows.

$$I_t = CPI_{v-3} + \frac{(d-1)}{D_v} \times (CPI_{v-2} - CPI_{v-3}), \tag{6.1}$$

where I_t is the reference index for day t, CPI_{v-3} is the value of the price index at time $v - 3$ months, CPI_{v-2} is the value of the price index at time $v - 2$ months, D is the number of days in month v, d is the day of the month v when settlement occurs and v is the month on which settlement takes place[5]. Applying this formula to the day in which the inflation accrual for the bond begins, by substituting the D_v day for the first day when the bond's inflation component starts to accrue (the base day), allows for the calculation of the base index (I_{base}). This way, it is possible to compute a daily Index Ratio to adjust for daily inflation changes in the bond and whenever it is traded, making the inflation accrual steadily over each month instead of adjusting only once a month, when the new figure of the price index is published. The daily adjusted Index Ratio is given by the expression

$$IR_t = \frac{I_t}{I_{base}} \tag{6.2}$$

where IR_t stands for Index Ratio in day t. Both indices used to compute IR_t are truncated to six decimal places and then rounded to five decimal places. In order to compute cash settlement amounts, real accrued interest is computed as done for nominal

[5]For further details please refer to Appendix A in Chapter 5.

fixed rate bonds. Then, clean price and real accrued interest are each multiplied by the Index Ratio. As for coupon and principal amounts, the process is the same: each is multiplied by the Index Ratio computed with reference to the day when they are calculated. However, if at maturity the Index Ratio is less than one (this will happen if deflation occurs), the inflation floor will be triggered and the principal amount will be redeemed at par value.

One thing that stands out from this design is that, for both annual and semi-annual coupons, the inflation accrual varies between coupon payments, since it reflects the monthly changes in the index. Nonetheless, the indexation lag means that the nominal interest rates do not include all known inflation information from a time lag that could vary between two weeks and a month and a half of inflation in the current CPI measure, which has not yet been incorporated in the calculus of the Index Ratio.

After accounting for the inflation accrual, it is possible to derive the formula for computing the nominal fair value of a U.S. TIPS, which is similar to the calculation of the value of a U.S. Treasury bond. The main difference is the inclusion of the Index Ratio IR_t and of an option against deflation at maturity (i.e. if deflation occurs throughout the life of the bond this is not reflected in the payment of the face value of the bond, thus no capital erosion occurs[6]). Equation (6.3) presents the formula we use to calculate the fair value of a U.S. TIPS, taking into account both real and inflation components.

$$B_{TIPS}\left(0\right) = \sum_{t=1}^{n} \frac{c_{R_t}}{x} \times IR_t \times \delta_N\left(0,t\right) + \max\left\{1, IR_n\right\} \times \delta_N\left(0,n\right) \tag{6.3}$$

Please note that $\delta_N\left(0,t\right)$ and $\delta_N\left(0,n\right)$ refers to a nominal discount factors. As for the component $\max\left\{1, IR_t\right\}$, as already stated above, it is an embedded option that

[6]The study of the deflation option is beyond the scope of this thesis. However, for a meaningful research contribution regarding the embedded deflation option, we would like to refer to Grishchenko, Vanden and Zhang (2016), who assert this option's ability and informational content to account for the evolution of future inflation.

aims at protecting the investor from deflation. This way, if the estimated Index Ratio IR_t is negative, the investor will not lose value when the bond is redeemed.

We now highlight the adjustments needed to compare the U.S. Treasury and U.S. TIPS duration.

As Brynjolfsson (2002, p. 209) summarizes, "Duration is the measure of a bond's market value sensitivity to changes in specific yields - real or nominal". Hence, duration measures, as defined by Macaulay (1938) and Fisher and Weil (1971), also could be applied for inflation-linked bonds. Equation (3.5) can be applied to compute the nominal duration of U.S. Treasuries and the real duration of U.S. TIPS. However, to be able to compare both datasets results, further adjustments need to be done.

Siegel and Waring (2004) discuss the importance of the dual duration problem associated with the U.S. TIPS. Either inflation-linked bonds and nominal bonds have two durations, the inflation duration and the real-interest rate duration, and these two durations can be separately defined and seen as the decomposition of a bond's nominal duration. In the case of nominal bonds, the difference is not relevant because both durations are similar to each other and to the nominal duration, since any change in nominal interest rates influences the nominal bond price in a similar way, whether is arises from changes in inflation or changes in the real interest rate. This way, by investing in nominal bonds, an investor is unable to hedge independently against changes in real interest rates and in inflation. For inflation-linked bonds, the inflation duration is close to zero, so the nominal duration is only explained by the real-interest rate duration. Therefore, if the liabilities one wishes to hedge are indexed to inflation, it seems clear that immunization through real-interest rate duration is the best way to achieve this, because we do not know *ex-ante* the final value of the liability. This way, the investor would be able to hedge directly against real interest rate risk while accounting for (and naturally hedging) inflation risk in both variable components of inflation-linked bonds and inflation-linked liabilities. Siegel and Waring (2004) suggest this type of immunization to pension funds and to individual tax-deferred use. However, one must account for the indexation lag that exists in all inflation-linked bonds in order to make the *formulae* consistent with reality. Notwithstanding, these findings will hold since considering the indexation lag will make the inflation duration different but very close to zero. The major contribution of Siegel and Waring (2004) work lies in addressing something that is quite important for the aim of this Chapter: the duration measures of these two types of bonds are not directly comparable. This is an important issue when trying to compare the immunization results between nominal bonds and inflation-linked bonds, as the risk measures used will have to be adjusted.

Other authors have also focused in the comparison of the real and nominal durations of TIPS. Roll (2004) documents the correlation of TIPS returns with nominal bonds and equity returns and the relationship between TIPS real and effective durations through the estimation of the yield beta (β), by regressing TIPS returns on current changes in nominal yields using data on U.S. TIPS from 1997 to 2003. However, the author points out that the yield β estimation does not take into account the ageing effect of bonds until they mature, and, by assuming that this parameter is stable through time, Roll (2004) infers that the relation between TIPS real duration and effective duration is constant over time, and, consequently, that the response of TIPS to changes to the nominal term structure shape is also the same, irrespective of the bonds residual maturity. This hypothesis is not realistic and this is also demonstrated in the the empirical work carried out by Roll (2004). An interesting empirical work on the factors that influence the estimates of U.S. TIPS yield β is put forward by Cocci (2013), that presents a simple regression-based measure that links real and nominal yields, using data from the U.S. bond market from 2003 to 2013. The author's work also takes into account the research carried out by Roll (2004) and confirms that the estimate of the yield β cannot be considered constant. By studying the evolution of the yield β parameter throughout the aforementioned sample, Cocci (2013) shows that the correlation between nominal and real interest rate yields has dropped during the financial crisis, implying a weaker link between these markets in a distressed environment, which has serious implications to the applicability of the yield β measure to estimate TIPS' effective duration.

Laatsch and Klein (2005) take a different approach and study the applications of the effective duration of TIPS while setting up mixed bonds portfolios. They present a proprietary model[7] to value TIPS that is based in the relation between nominal and real interest rates as portrayed by the Fisher (1930) equation, that is confirmed to hold since TIPS bonds are shown to have little sensitivity to changes in expected inflation. The authors show that the relationship between effective durations and real durations of TIPS is not constant and have to take into account the investors believe regarding the future evolution of nominal and real interest rates and the expected inflation when building portfolios with TIPS and nominal bonds, supporting the findings from Siegel and Waring (2004).

U.S. TIPS bonds have two types of duration: real-interest rate duration and inflation duration, which can be seen as a decomposition of a bond's nominal duration. Although this decomposition is very hard for nominal bonds, since all the cash-flows associated to the bond are expressed in nominal terms, the same does not apply for inflation-linked bonds. Since inflation-linked bonds' cash-flows are stated in real terms, the usual duration *formulae* can be used to compute the bonds real duration (D_R). This way, for a non-flat interest rate term structure the inflation-linked bond's real duration will be computed as depicted in equation (3.5) with the below mentioned adjustments

$$D_{R^j} = \sum_{t=1}^{n} t \times w_t : w_t = \frac{\left[\frac{c_{R_t}}{x} \times \delta_R(0,t)\right]}{B_{TIPS_R}} \tag{6.4}$$

The real duration of a given j-th bond will be computed as the weighted average time to maturity t of the future real cash-flows c_{R_t}, paid x times per year and discounted with the real discount factor $\delta_R(0,t)$ where the weights w_t are defined as the present

[7] The proprietary model is presented in the referenced paper, but, for further details, please refer to Laatsch and Klein (2002).

value of the cash-flows of a given bond divided by their fair price B_{TIPS_R}, already defined in equation (5.2). In real terms, the formula is straightforward, but in nominal terms the calculation can be tricky. There are two ways to compute these bonds' nominal duration, most commonly referred to as effective duration.

First, by adjusting the real duration by a factor denominated yield beta that arises from the yield β coefficient of a linear regression that attempts to measure the yield sensitivity of the real interest rate to a change in the equivalent nominal interest rate - Pond (2008, p. 164). Given a 1% change in nominal yields, the yield β can be interpreted as the expected percent change in TIPS real yields. The same reasoning can be extended to the relationship between TIPS' nominal effective duration (ED) and real duration, as depicted in the formula beneath:

$$ED = \beta \times D_R \tag{6.5}$$

This is a very simple way to estimate inflation-linked bonds effective duration; however we recall that it has important flaws that have been explained by Roll (2004) and Cocci (2013), namely that this value cannot be assumed to be constant over time. Pond (2008) also addresses some of these flaws. Although nominal interest rates changes can be explained by real interest rates changes, this is not the only factor that makes nominal interest rates vary. Inflation changes also contribute for variations in nominal interest rates and, in a broader level, anything that can lead to inflation changes can have an effect on nominal interest rates. Pond (2008) also acknowledges the unsteadiness of the yield beta estimate: the inflation-linked bonds' yield sensitivity to changes in the equivalent nominal yield is not stable. This arises because usually real interest rates are less volatile than nominal interest rates, but it is also an information issue: market movements and the arrival of new information can be incorporated in both yields in different ways. For instance a new inflation rate release should not affect real interest rates, only nominal interest rates and, for this specific event the yield beta is zero. This way, to correctly use the yield beta to compute effective duration, new estimates for this parameter would be needed on a daily basis. Taking into account all these aspects, to simply adjust the effective duration formula using the yield beta seems to be an oversimplification and can lead to a high level of basis risk. See, for instance, for a complete and detailed explanation, the overall findings of Deacon et al. (2004, p. 77-79), whose view on the effective duration computation based in equation (6.5) is quite critical, since estimating the β coefficient with historical data might lead to inducing further bias because future market conditions might (and in most cases do) differ from the past.

Another method, already discussed above, is the model presented by Siegel and Waring (2004) and Laatsch and Klein (2005). This approach seems more feasible than the yield β estimate presented beforehand because, even though we do not intend to build mixed portfolios with both U.S. TIPS and Treasuries, we do mean to compare the results of our multiple liability immunization strategies using both datasets. Hence, this method was applied in this empirical test under the formulation and hypothesis presented by Siegel and Waring (2004) and Laatsch and Klein (2005). In order to compute the effective duration we adjust the real cash-flows of the bond considering the inflation compensation until the bond's maturity and then discount the cash-flows using nominal rates. For this purpose, we assume that the best estimate for future inflation is the year-on-year actual inflation rate (depicted as π) and recall the relation between nominal interest rates and real interest rates as portrayed in Fisher (1930):

$$(1 + y_N(0,t)) = (1 + y_R(0,t)) \times (1 + \pi) \tag{6.6}$$

where $y_R(0,t)$ stands for real annual spot rate and $y_N(0,t)$ stands for nominal annual spot rate. Equation (6.6) is widely known as the Fisher's equation. Deacon et al. (2004, p. 80) propose a more thorough formulation for Fisher's equation, by decomposing the term $(1 + \pi)$ - that accounts for the year-on-year actual inflation rate, but also serves as a proxy for the embedded inflation compensation demanded by investors to hold TIPS, as presented by the CPI-U Index - into two terms that account for expected inflation

π^e and an inflation risk premium ρ that aims to account for the uncertainty of future inflation. Hence, their formulation of equation (6.6) is extended to

$$(1 + y_N(0,t)) = (1 + y_R(0,t)) \times (1 + \pi^e) \times (1 + \rho) \tag{6.7}$$

Even though the decomposition of the inflation compensation shown in equation (6.7) is not the aim of our analysis, we note that this is not a trivial issue. As mentioned in Chapter 5, empirical work conducted by Ang et al. (2008) and Chen et al. (2010) has found that the value for the one-year inflation risk premium for the U.S. TIPS market is estimated to be between one and two basis points. This way, even though the presence of an inflation risk premium might hamper the validity of our hypothesis of considering the actual-year-on-year inflation rate as a proxy of the inflation compensation, the aforementioned research seems to support our assumption. Recalling that our immunization program implies the annual payment or the liability we wish to immunize grows with year-on-year inflation, keeping this assumption seems quite realistic. This would not be the case if the liability we wish to immunize was indexed to a two-year (or higher) inflation growth rate.

In order to transform the duration equation (3.5) it is necessary to adjust both numerator and denominator in it. The denominator adjustment is straightforward: since we have estimated a continuous function for the nominal term structure of interest rates we can use those values as the nominal spot interest rate $y_N(0,t)$. The numerator adjustment will be done by multiplying it by the Index Ratio (IR_t) at time t, assuming that future year-on-year inflation π will be equal to the most recent data on present year-on-year inflation. This hypothesis is consistent with the absence of arbitrage opportunities between both securities at time t. This way the U.S. TIPS effective duration is computed as shown in equation (3.5) with the following adjustments:

$$ED = \sum_{t=1}^{n} t \times w_t : w_t = \frac{\left[\frac{c_{R_t}}{x} \times IR_t + \max\{1, IR_n\}\,|_{t=n}\right] \times \delta_N(0,t)}{B_{TIPS}(0)} \tag{6.8}$$

Although it can be argued that the Index Ratio affects equations (6.3) and (6.8) in the same way and, hence, the inflation adjustment is redundant, that is not entirely true due to the indexation lag. This way the future inflation expectations are reflected instantaneously in the nominal interest rates but reflected with a three month lag in the coupon and principal valuation, since the Index Ratio computation allows for that lag, as stated in equations (6.1) and (6.2). Anyway, although not being redundant, it is expected that the difference between the effective duration and the nominal duration measures might be small.

The calculation of the effective duration is important to allow comparisons between inflation-linked bonds and nominal bonds in a portfolio context, since all risk measures computed in a portfolio must derive from similar individual bond measures, i.e. it is not possible to compare accurately the portfolio's duration using nominal duration for fixed rate bonds and real duration for inflation-linked bonds, as stated by Siegel and Waring (2004).

As stated before by Nawalkha and Chambers (1996), the M-Absolute model has been developed to address both parallel and nonparallel shifts in the term structure of interest rates, while addressing M-Squared shortcoming of being dependant on duration to achieve a better immunization performance. This is done by condensing in a single measure the ability to immunize against nonparallel term structure of interest rates shifts, while partially immunizing against level shifts in the term structure of interest rates. The immunization principle behind the M-Absolute model states that immunization strategies that minimize the M-Absolute measure are those whose cash-flows are nested around the planned liability horizon H. In this Chapter, this will be tested in a multi-liability immunization setup. Once again, for U.S. Treasuries and U.S. TIPS the M-Absolute measure can be computed as shown in equation (3.8). However, for the portfolio where we are using U.S. TIPS accounting for both inflation accrual and real interest rate accrual, the M-Absolute will have to be adjusted in a similar way as done with the effective duration. Hence, the M-Absolute of a bond for the immunization exercise for U.S. TIPS taking into account the inflation accrual will be computed as shown below,

$$M^A = \sum_{t=1}^{n} abs\left(t - H\right) \times w_t : w_t = \frac{\left[\frac{c_{R_t}}{x} \times IR_t + \max\left\{1, IR_n\right\}|_{t=n}\right] \times \delta_N\left(0, t\right)}{B_{TIPS}\left(0\right)}. \quad (6.9)$$

As Kondratiuk-Janyska and Kaluszka (2006a) point out, this measure can be only applied for a single liability immunization problem, where H is when the liability is paid. In order to use the M-Absolute measure for the multiple liability immunization problem, the authors derive the M-Absolute of the multiple asset and liability portfolio in a continuous-time setting as for expression below:

$$M^A_{ptf} = \int_h^H abs\left(V_A\left(h\right) - V_A\left(H\right) + E\left[V_L\left(H\right) - V_L\left(h\right)\right]\right) dh, \quad (6.10)$$

where V_A stands for the value of the asset portfolio, $E(V_L)$ represents the expected value of the liability portfolio (i.e. the set of liabilities we wish to immunize), h is the time at which the intermediate liabilities are due and H is the point in time where the final liability is paid and both portfolios cease to exist, such that $h = h_1, ..., H$.

6.3. Methodology

In the following sections the portfolio setup and the immunization programming and performance measures used in this Chapter are highlighted.

6.3.1. Portfolio setup

The M-Absolute strategy was set-up to minimize the difference between the M-Absolute of the invested amount and the residual maturities of the liability. Since we are in the presence of multiple annual liabilities for the two immunization planning horizons considered, the immunization process has been carried out by taking into account that the stream of liabilities we wish to immunize can be seen as a portfolio of zero coupon bonds. In this sense, the easier way to estimate the amounts to buy and sell of each bond is by decomposing the overall liability into subsets of single liabilities and immunize $\frac{1}{H}$ of the portfolio against each upcoming payable liability, as suggested in Barber and Copper (1998b) and Kaluszka and Kondratiuk-Janyska (2004). In this sense, we refer again to H as the last liability to be paid and h as the intermediate liabilities payable during the immunization planned horizon. This way, each sub-portfolio will comprise a 3-rd or a 5-th of the overall portfolio value, taking into account the final immunization horizon of 3 or 5 years. As before, the portfolios were set up with 8 or 10 bonds (for the 3- and 5-year estimation respectively) with residual maturities spreaded between the setup date and 5 years after the horizon date.

To make the U.S. Treasuries subset comparable with the U.S. TIPS, the final amount of the liability was adjusted in order to account for the year-on-year inflation, as measured by the CPI-U. By applying the same inflation rate to the liabilities and the bonds we wish to use in the immunization process, we eliminate the possibility of inducing any bias in the process that could arise if these metrics were different. The year-on-year and average inflation growth rates, used as a proxy to compute the final value of the liability we wish to immunize, can be found in Table 6.1. Year-on-year inflation growth ranges from 2,15% to 2,62%. As for the cumulative growth values, for the 3-year immunization period, inflation growth is 6,9% on average while for the 5-year immunization inflation growth is 12,35% on average. This means that each year the immunized portfolios have to generate, on average, at least the cumulative liability growth projected for each year in order to allow paying in full the liability due at the end of each year.

(insert Table 6.1 here)

The immunization procedure applied is once again, based on a minimax strategy, whereby we wish to minimize the difference between the M-Absolute risk measure of the invested amount and the residual maturities of the liabilities while maximizing the value of the portfolio in each rebalancing period. The linear programming applied is depicted below and takes into account the conditions stated in Bierwag et al. (1983) and Fong and Vasicek (1983a) in their setup:

$$\min_k \sum_{j=1}^{m} k_j M^A \tag{6.11}$$

$$\begin{aligned}
s.t. \quad & \sum_{j=1}^{m} k_j = 1 \\
& k_j \geq 0, \forall j = 1, .., m \\
(i) \quad & V_A(t) = E\left[V_L(t)\right], \forall t = 0, .., n \\
(ii) \quad & \begin{bmatrix} M_{h_1}^A \\ \vdots \\ M_H^A \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ H \end{bmatrix} \\
(iii) \quad & \sum_{h=1}^{H} M_h^A = \sum_{h=1}^{H} h
\end{aligned}$$

Besides the conditions applied to the weights k_j, that are similar to the ones applied in the single immunization problem, it is also necessary to consider the conditions for the multiple liability immunization problem, while taking into account that the immunization measure used is the M-Absolute and not the duration. As stated in Chapter 2, the M-Absolute is designed as a measure of dispersion of the asset portfolio's cash-flows around the immunization horizon. Since we have multiple liabilities, the M-Absolute will measure the dispersion of the sum of the overall asset portfolio around the liability portfolio, considering that each sub-portfolio's cash-flows are clustered around each payable liability. In this sense, we have adapted the conditions stated by Bierwag et al.

(1983) and Fong and Vasicek (1983a) to the M-Absolute measure. Firstly, we add *ex-ante* necessary conditions that ensure that in the beginning of the immunization process[8]:

(i) the value of the asset portfolio has to be equal to the expected value of the liability we wish to immunize for each liability due to be paid in time $h \geq H$;

(ii) the vector of the M-Absolute measure of each sub-portfolio has to be equal to the residual maturity of each liability due to be paid for all times $h \in [1, ..., H]$;

(iii) The sum of the M-Absolute measures has to be equal to the sum of the residual maturities of each liability due to be paid for all times $h \in [1, ..., H]$.

At this stage, we recall Nawalkha and Chambers (1996) definition of the M-Absolute as a weighted average of the absolute distance of the bond's cash-flows around the liability dates of a given portfolio and its main purpose to serve as an alternative to the traditional duration measures while immunizing a portfolio against non-parallel interest rate shocks. Since this measure takes into account the portfolio's composition, it is already built as a dispersion measure. Taking the simple example of a bullet and barbell portfolio (like the ones built in Chapters 4 and 5), these will, by definition, have the same duration. However, their M-Absolute will be quite different, because the bond cash-flows of the bullet portfolio are nested around the liability date (i.e. low M-Absolute), while the bond cash-flows of the barbell portfolio are spreaded out (i.e. higher M-Absolute). Furthermore, Fong and Vasicek (1983a) have adapted the 3-rd condition to account for the dispersion of the mean absolute deviation between portfolio cash-flows and upcoming liabilities for the M-Squared measure. This condition can also be applied for the M-Absolute measure. This way, it does not appear to make sense to compare the dispersion indices of the asset and liability portfolio based in the duration measure, as mentioned in Bierwag et al. (1983), since these dispersion indices are based in Fisher and Weil (1971) duration and we are applying the M-Absolute, instead of duration, as a risk measure for the immunization process.

[8]The conditions will be ensured only in the beginning of the immunization process because we assume that the received coupons are reinvested into the asset portfolio. If we wished to enforce these conditions in each rebalancing period we would have to drop this hypothesis.

Adding the findings of Kondratiuk-Janyska and Kaluszka (2006b) regarding the relation between the M-Absolute and the duration gap (i.e. minimizing the M-Absolute of a portfolio implies minimizing the absolute duration gap between asset cash-flows and payable liabilities), we are sure that there is no need to compare asset and liability dispersion measures based on duration in order to apply the multiple immunization minimax strategy.

To ensure that immunization is achieved overall we could compare the M-Absolute of the portfolio with the M-Absolute of the multiple liabilities we wish to immunize, applying the formula stated in equation (6.10). Please recall that, applying equation (3.8) to the single liability setting, the M-Absolute of the liability is null, because the date t the liability is due equals the immunization horizon H. This will not be the case in the multiple liability setting because we face several liabilities to be paid in dates $t \geq H$. In this sense, we can treat the multiple liabilities as a portfolio and apply the M-Absolute measure as stated in equation (6.10) to the asset and liability portfolios. Taking a closer look into equation (6.10) we observe that it incorporates the expected value of the liabilities in its calculation. This way, since we enforce *ex-ante* conditions (i) to (iii) stated above, this means that the M-Absolute measure of the assets will be equal to the residual maturity of the liability portfolios and, consequently, the difference between these two measures will be null. This is true because the M-Absolute measure, as already mentioned, is built as a dispersion measure and its anchor (around which the level of dispersion is evaluated) is the liability portfolio.

Consequently, it will be sufficient to ensure that conditions (i) to (iii) apply to guarantee that the dispersion of the asset portfolio is equal or above the dispersion of the liability portfolio. By doing so, we are also ensuring that the 3-rd condition, as defined by Fong and Vasicek (1983a), is fulfilled. However, we would always need to enforce these conditions *ex-ante*, because we are immunize $\frac{1}{H}$ of the portfolio against each upcoming payable liability. The 3-rd condition defined by Fong and Vasicek (1983a) is trivial when immunizing single liabilities, thus it would be always fulfilled for each liability.

The extension that allows to ensure that the overall portfolio complies with all Fong and Vasicek (1983a) conditions implies enforcing conditions (*ii*) and (*iii*) simultaneously. This way, the conditions stated in equation (6.11) are both necessary and sufficient conditions to apply the multiple liability immunization minimax problem with the M-Absolute strategy.

6.3.2. Immunization and Performance measures

The results have been evaluated using some of the measures that have been defined in Chapter 3 and are recalled here. The absolute measures used were the average liability coverage, defined in equation (3.17) and the average excess return of the portfolio, defined in equation (3.18). Please note that $\delta(0,t)$ could refer to the nominal or real discount factor, according to the type of dataset we are evaluating. If we are using the U.S. Treasuries or the U.S. TIPS with inflation adjustment datasets the discount factor will be computed in nominal terms and if we are using the U.S. TIPS without inflation dataset the discount factor will be computed in real terms.

$$\overline{LC}(S) = \frac{\sum_{\lambda=1}^{\Lambda} \frac{V_H^{\lambda}}{V_0^{\lambda}}}{\Lambda}$$

$$\overline{ER}(S) = \frac{\sum_{\lambda=1}^{\Lambda} \left(\frac{\ln\left(\frac{V_H^{\lambda}}{V_0^{\lambda}}\right)}{H} \right) - y(0,H)}{\Lambda}$$

The average turnover - equation (3.19) - and average transaction costs - equation (3.20) - have also been computed for each immunization year.

$$\overline{T}(S) = \frac{\sum_{j=1}^{m} abs\left(Q_{\alpha_z}^{j} - Q_{\alpha_{z-1}}^{j}\right)}{m \times z}$$

$$\overline{TC}\,(S) = \frac{\sum_{j=1}^{m} abs\left[\left(Q^{j}_{\alpha_z} - Q^{j}_{\alpha_{z-1}}\right)\right] \times \left(P^{j}_{ask} - P^{j}_{bid}\right)}{m \times z}$$

Finally, the relative measure that is computed is the Reward-to-Risk Ratio - equation (3.24).

$$R/R\,(S) = \frac{\overline{ER}(S)}{\sigma_S}$$

6.4. Results

We reiterate that the immunization procedure has been conducted taking into account that a portfolio of multiple liabilities can be decomposed in a set of single liabilities and the respective assets can be divided accordingly in order to immunize $\frac{1}{H}$ of the portfolio against each upcoming payable liability, as suggested by Kaluszka and Kondratiuk-Janyska (2004). This is done while fixing for each subset of liabilities the available bonds for the immunization (i.e. in each $\frac{1}{H}$ immunization procedure the portfolios were set up with the same 8 or 10 bonds). This allows us to know in every rebalancing period which is the portion of each bond that has been bought to the portfolio, irrespective of the dates the intermediate h liabilities are due. The conditions (i) and (ii) regarding the value of the assets and liabilities portfolios have also been ensured *ex-ante* since, as discussed above, they are necessary and sufficient conditions for the immunization procedure.

The results for U.S. Treasuries, reported in Table 6.2 and for U.S. TIPS, reported in Table 6.3, show that in all cases the immunization procedures are largely achieved, since, on average, the liability coverage is always above 100%, even taking into account that the average year-on-year inflation growth is 2,2%. This is the case due to the decreasing interest rate environment.

(insert Tables 6.2 and 6.3 here)

Another thing that we observe is the direct relation between the average return and transaction costs, since portfolios with higher average transaction costs tend to have lower average excess returns. We also highlight that for the longer portfolios the average turnover tends to be lower, which is a sign that shorter immunization horizons generate portfolios that are more volatile and need to be adjusted more often to achieve perfect immunization. These two factors are related and can be explained by the fact that most U.S. Treasuries and U.S. TIPS bonds have maturities above 5 years, as can be see in Table 5.1 and Table 5.2.

However, the results for the U.S. TIPS immunization strategy taking into account the inflation accrual, included in Table 6.4, are striking.

(insert Table 6.4 here)

One would expect to see the immunization results from this dataset in line with the immunization results achieved for the U.S. Treasuries but what is observed is that the average liability coverage is much higher. For the 3-year immunization portfolios, the average liability coverage for U.S. TIPS is about 7 to 10 percentage points above the average liability coverage for U.S. Treasuries, while for the 5-year immunization portfolios the average liability difference ranges from 7 to 18 percentage points. If the datasets are similar and the time to maturity of the bonds selected for the clustering process are also in line, what could explain this divergent result? The answer seems to lie in the divergence itself: inflation. Bear in mind that for either coupon or principal amounts the inflation accrual is computed through the Index Ratio, as stated in (6.2), that takes into account the evolution of the CPI-U index since the issuance of the bond. If one looks more closely to the issue dates for U.S. TIPS in Table 5.2, it is possible to see that most bonds were issued prior to 2000. In line with the year-on-year inflation estimates, if we analyze the CPI-U index values, as shown in Figure 6.2, it is possible to see that these are increasing during the sampling period, with an exception for 2008, that even so does not jeopardize the upward tendency of the CPI-U index values. There is even a

2-year period in this sample (2004 to 2006) where inflation is growing and real interest rates are decreasing. In this sense, the coupon reinvestment effect is enhanced with the growing inflation accrual component. Since we had already observed in Chapter 5, that the coupon reinvestment effect outweighs the price effect associated with the decreasing interest rates, the inclusion of the inflation accrual allied with the divergent behavior of inflation and real interest rates leverages the difference that was observed, enhancing portfolio returns.

(insert Figure 6.2 here)

Hence, the Index Ratios for all the bonds are always increasing with time. This effect generates higher inflation accruals as time goes by, that are reinvested into the portfolio and leverage the portfolios' coverage in a similar way as verified with high coupons reinvested from the previous Chapters. The only way this would not be the case was if the Index Ratio applied to U.S. TIPS bonds coupons was computed to account only for the inflation growth from the previous coupon and not since the bond's issuance, as it would make the inflation accrual lower. In theory this would allow for near-perfect hedging of the inflation accrual, since the only bias would come from the indexation lag, as depicted in equation (6.1). However, this is not the way these bonds are built and, even considering it would be a good theoretical exercise, it would not be possible to apply it in the real world. Recalling the cumulative inflation growth rates in Table 6.1, these can be compared with the average cumulative implicit inflation growth rates in U.S. TIPS, derived from their Index Ratios. This information can be found in Table 6.5. As can be observed, the average cumulative inflation growth is above the liabilities' cumulative inflation growth, thus explaining this overvaluation.

(insert Table 6.5 here)

Even so, this allows to conclude that, to immunize liabilities that are uncertain and whose growth is related to inflation, U.S. TIPS seem to be a better instrument than U.S. Treasuries, corroborating Fogler (1984) and Siegel and Waring (2004).

6.5. Concluding Remarks

This Chapter presents the empirical results for the multiperiod immunization applied to U.S. Treasuries and U.S. TIPS from 2000 to 2014, taking into account the results from previous Chapters and extending the M-Absolute strategy empirical tests from single to multiperiod immunization. To the best of our knowledge, this is one of the first empirical tests with the M-Absolute measure considering multiperiod immunization and U.S. TIPS and U.S. Treasuries datasets.

Immunizing growing liabilities with U.S. TIPS seems to be a better strategy than using U.S. Treasuries, even though both achieve above 100% liability coverage, corroborating Fogler (1984) and Siegel and Waring (2004). Furthermore, the security design for U.S. TIPS explains their overperformance in the immunization setting applied, despite their lower liquidity when compared to U.S. Treasuries, which are considered one of the most liquid assets in the world. This is due to the way the inflation component is accounted for in the U.S. TIPS floating component.

The sample period is characterized by significant downward interest rate movements (either nominal or real interest rates) and an upward inflation growth tendency only interrupted in 2008. Other empirical tests could be carried out, such as testing the robustness of these results in a different interest rate environment. It could also be tested empirically if with another term structure model estimation (either parametric or stochastic) the same results would be obtained in order to assess to what extent the estimation results might be influenced by the method used to estimate the term structure of interest rates and if different specifications for the M-Absolute strategy, such as logarithms, polynomials, or other generalizations, could induce different results.

Table 6.1: U.S. Average Inflation Growth Rate

This table contains the average year-on-year inflation growth rates for each payment year and the correspondent cumulative growth rates. These inflation rates are computed taking into account the year-on-year growth rates. For instance, the cumulative growth rate of 4,48% for the 2-year immunization period assumes that inflation grows by 2,15% in the 1^{st} year and 2,28% in the 2^{nd} year.

Inflation	**Payment year**				
Growth (%)	**1**st	**2**nd	**3**rd	**4**th	**5**th
Y-o-Y	2,15%	2,28%	2,35%	2,38%	2,62%
Cumulative	2,15%	4,48%	6,94%	9,48%	12,35%

Table 6.2: U.S. Treasuries Immunization Results

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio (R/R). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

Panel A - 3-year immunization horizon

Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	103,30%	0,76%	49,60%	1,43%	28,57
2-year	107,34%	1,45%	72,76%	0,77%	18,91
3-year	110,43%	0,76%	33,99%	0,63%	19,62

Panel B - 5-year immunization horizon

Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	103,85%	0,82%	52,49%	1,55%	27,53
2-year	108,46%	1,64%	78,56%	0,88%	18,98
3-year	111,70%	0,74%	31,61%	0,72%	19,14
4-year	118,14%	2,05%	73,66%	0,63%	14,82
5-year	122,74%	1,44%	48,22%	1,21%	15,42

Table 6.3: U.S. TIPS Immunization Results

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio (R/R). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

Panel A - 3-year immunization horizon

Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	102,54%	3,16%	128,07%	0,66%	22,65
2-year	104,40%	5,45%	195,68%	0,43%	14,89
3-year	106,25%	7,67%	251,99%	0,35%	12,53

Panel B - 5-year immunization horizon

Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	103,04%	0,92%	59,88%	0,55%	22,94
2-year	105,43%	1,04%	51,98%	0,77%	18,91
3-year	107,70%	0,93%	41,77%	0,63%	19,62
4-year	109,73%	0,52%	18,79%	0,63%	14,82
5-year	112,15%	0,40%	10,55%	0,34%	11,50

Table 6.4: U.S. TIPS with Inflation Accrual Immunization Results

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio (R/R). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

Panel A - 3-year immunization horizon					
Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	110,97%	9,03%	198,81%	0,70%	24,85
2-year	118,19%	13,89%	251,24%	0,46%	16,10
3-year	120,31%	13,67%	238,02%	0,35%	12,31
Panel B - 5-year immunization horizon					
Sub-portfolio horizon	$\overline{LC}\,(S)$	$\overline{ER}\,(S)$	$R/R\,(S)$	$\overline{TC}\,(S)$	$\overline{T}\,(S)$
1-year	110,51%	5,76%	128,67%	0,58%	22,95
2-year	119,37%	9,12%	161,19%	0,44%	16,90
3-year	122,99%	7,36%	125,63%	0,35%	13,34
4-year	127,11%	6,06%	96,20%	0,29%	10,34
5-year	140,54%	11,87%	136,85%	0,46%	14,69

Table 6.5: U.S. TIPS Index Ratios and Implied Inflation Growth

This table contains the average Index Ratios and implicit Inflation Growth for the U.S. TIPS Dataset. Please refer to equation (6.2) to recall how this Index is computed. As visible, this measure is cumulative by design, which means that both average Index Ratio and average implied Inflation Growth are cumulative figures. Residual Maturity represents the number of years for which the Inflation Growth ficure is calculated. It is computed taking into account the sample period. This way, for bonds issued before January 2000, the residual time to maturity is shown. For bonds that mature after December 2014, the residual maturity is cut-off at that date. Please refer to table 5.2) for the Issuance and Maturity Dates of each bond.

ISIN	Description	Average Index Ratio	Average Inflation Growth	Residual Maturity
US9128273A89	TII 3.625 7/2002	1,1174	11,74%	2,53
US9128272M37	TII 3.375 1/2007	1,1687	16,87%	7,04
US9128273T70	TII 3.625 1/2008	1,1653	16,53%	8,04
US9128274Y56	TII 3.875 1/2009	1,1665	16,65%	9,04
US9128275W81	TII 4.25 1/2010	1,1526	15,26%	9,99
US912828CZ11	TII 0.875 4/2010	1,0889	8,89%	5,46
US9128276R87	TII 3.5 1/2011	1,1424	14,24%	10,00
US912828FB16	TII 2.375 4/2011	1,0694	6,94%	4,96
US912828GN45	TII 2 4/2012	1,0685	6,85%	4,96
US912828AF74	TII 3 7/2012	1,1478	14,78%	10,00
US912828BD18	TII 1.875 7/2013	1,1510	15,10%	10,00
US912828KM16	TII 1.25 4/2014	1,0600	6,00%	4,96
US912828DH04	TII 1.625 1/2015	1,1338	13,38%	9,95
US912828QD52	TII 0.125 4/2016	1,0397	3,97%	3,67
US912828SQ48	TII 0.125 4/2017	1,0194	1,94%	2,67
US912828HN36	TII 1.625 1/2018	1,0633	6,33%	6,96
US912828NM88	TII 1.25 7/2020	1,0415	4,15%	4,46
US912828UH11	TII 0.125 1/2023	1,0077	0,77%	1,92
US912810FH69	TII 3.875 4/2029	1,2470	24,70%	15,00

Figure 6.1: U.S. Year-on-year Consumer Price Index Rates

This figure contains the year-on-year inflation rate, computed between January 2000 and December 2014, taking into account the monthly U.S. CPI-U unrevised index statistics, as published by the U.S. Bureau of Labor Statistics. The year-on-year inflation is quite volatile during the sample period, without showing a clear tendency. The only significant movement occurs between 2007 and 2009 where, after an increasing tendency and consequent upper bound above 5%, a sharp decrease occurs into negative inflation of around 2% during 2009, mainly due to the U.S. Federal Reserve Quatitative Easing measures. In late 2009 inflation starts increasing again into positive teritorry.

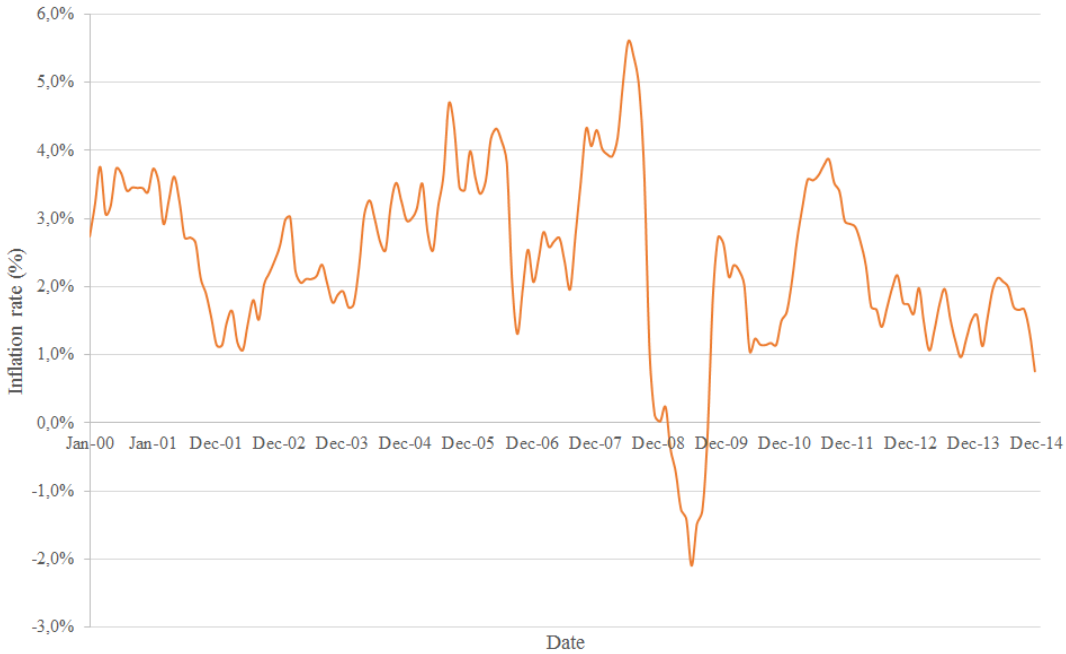
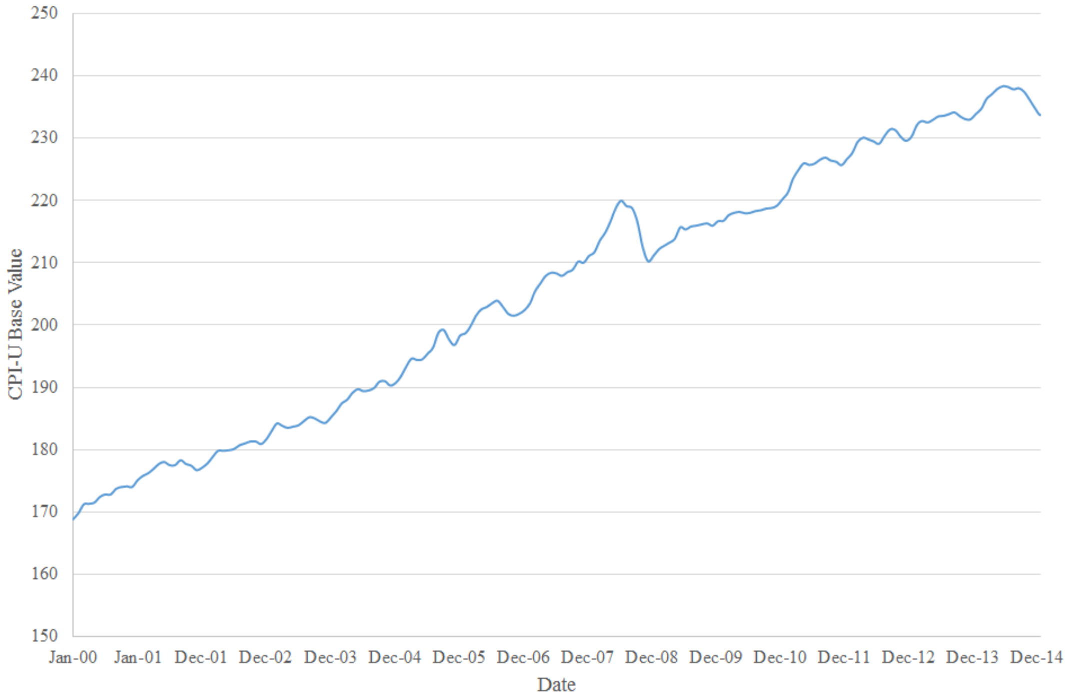


Figure 6.2: U.S. Consumer Price Index Values

This figure contains the U.S. CPI-U unrevised index statistics, published monthly by the U.S. Bureau of Labor Statistics and used for the calculation of the Index Ratio for U.S. TIPS. We can observe the positive tendency of the index that shows that coimpounded inflation is rising throughout the sample, thus explaining the high Index Ratios for the inflation accrual of U.S. TIPS.



CHAPTER 7

Conclusions

In this dissertation several immunization techniques and strategies have been empirically tested with three different datasets of treasury bonds, from different countries and different designs. As with any empirical work, there are some constraints to the results obtained: (1) the underlying dataset, (2) the methodology applied and (3) the overall interest rate environment. The empirical results presented are quite similar, even tough the bond datasets are different, which confers some robustness to the empirical results presented in terms of dataset and methodology applied.

In what concerns the most widely used immunization techniques, it is possible to see that naïve maturity-bond strategies have good performances. Hence, if the objective is to purely immunize a single future liability without taking into account what will happen to the term structure of interest rates, this could be a feasible way to go. In this sense, the same would apply for a ladder strategy for multi-liability portfolios, if one build a portfolio composed of several maturity bonds (one for each due liability). However, this has not been empirically tested in this thesis and could be developed in future articles regarding multiperiod immunization.

Barbell strategies, while being fairly risky, have proven to be good to immunize portfolios in a decreasing interest rate environment for the bond datasets tested. These strategies allowed reinvesting high coupons at decreasing rates and, at the same time, holding long-term bonds that have gained a significant intrinsic value as interest rates decreased. Therefore, it seems to exist a non-negligible bias from the decreasing interest rate environment embedded in the success of these strategies. If we were in the presence of an increasing interest rate environment, this strategy might not have been efficient.

This is also something that could be empirically tested in future research. Bullet strategies, when compared to the naïve maturity bond strategy, prove to be less efficient from a cost perspective, due to the higher rebalancing of the portfolios, as the naïve strategy only has transaction costs derived from coupon reinvestments. This effect is more pronounced in the U.S. TIPS dataset, due to its lower liquidity. This way it seems better to fit the liability to be paid to a single bond.

As for the most complex immunization strategies applied, it is visible that, from all the M-derived immunization strategies, the M-Absolute is the most consensual in what regards immunization abilities. In this sense, previous empirical studies that had acknowledged the good immunization ability of the M-Squared and M-Vector strategies have not been confirmed. The M-Absolute produces good immunization results while accounting for lower transaction costs (i.e. even lower than bullet portfolios). This shows that the bond clustering strategy in which the M-Absolute is based can be a better alternative to the naïve strategy because investing in several bonds diminishes the need to rebalance significantly the portfolios. This can be seem as a diversification effect in the sense that portfolios become more cost efficient if the investments are spreaded throughout several bonds.

This way, the coherence of the results presented for the M-Absolute strategy show that this strategy can be applied by an institutional investor or asset manager whose purpose is to immunize its investment in order to guarantee the payment of future liabilities, as an alternative to the traditional duration matching strategies. In this sense, the added complexity of this strategies is outweighed by the lower turnover and transaction costs, that ultimately also allow for better immunization results. On the opposite side, if the investor does not have the possibility to invest in 8 to 10 bonds, he is better off sticking to the naïve strategy. For example, if an individual investor wishes to invest some money to safeguard future liabilities, instead of buying investment funds or contributing to private pension plans, he could buy the maturity bond.

Regarding the type of liability the investor wishes to immunize, it is also clear that for nominal liabilities, known *ex-ante*, it will be better to use nominal interest rate bonds. The real interest rate bonds achieve good immunization results but this can also be a consequence of the decreasing interest rate environment. Even so, these bonds liability coverage is lower than the one achieved by nominal bonds. If the liability's value is not known in the beginning of the immunization period, the use of variable coupon bonds, whose coupon growth rate is close to the growth rate of the liability we wish to immunize, will be the best asset to build the immunization portfolio. This has been tested for inflation growing liabilities and the portfolios using inflation-linked bonds have shown to be far better at immunizing these liabilities than nominal portfolios. Even so, we would see the possibility of empirically retesting these strategies in an environment of increasing interest rates as a good robustness check for these results since these results can be deeply linked to the decreasing interest rate environment portrayed in all the datasets.

Several other empirical tests can be carried out with these datasets in order to infer if different methodological setups would achieve different results. For instance, a lower rebalancing frequency and longer immunization horizons could be used to test the robustness of these results. Other avenues for future research can be testing for different parametric constellations or even stochastic term structures of interest rates or different designs for the M-Vector strategies, like logarithms and/or polynomials. Regarding the inflation effect, that clearly seems to play an important role in the results obtained in Chapter 6, one could also try to develop a stochastic model for the term structure for inflation rates, that takes into account both tendency and seasonality of this index, in order to assess what is the impact of both effects in the immunization process. For instance, it could be tested if it is indifferent to invest in U.S. TIPS regardless of the month their coupons are paid, or if inflation accruals are affected by inflation seasonality and what role (if any) these effects have in the immunization process.

Even so, our empirical results show that an informed institutional investor will be better off implementing the M-Absolute strategy, since, from all the M-Risk measures tested empirically, this is the one that seems to clearly add value to the immunization process. When compared to the M-Squared and the M-Vector risk measures, the M-Absolute is also the easiest to implement, since it relies only in a single dispersion measure. When compared to the traditional duration-based immunization strategies, the M-Absolute is highly mathematically tractable, thus not adding excessive complexity to the immunization process while achieving better results with lower transaction costs.

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