Full-Duplex Massive MIMO with Physical Layer Network Coding for the Two-Way Relay Channel

João S. Lemos*, Francisco A. Monteiro†
*Instituto de Telecomunicações, and Instituto Superior Técnico, Universidade de Lisboa, Portugal
†Instituto de Telecomunicações, and ISCTE - Instituto Universitário de Lisboa, Portugal
joao.sande.lemos@tecnico.ulisboa.pt , francisco.monteiro@lx.it.pt

***Abstract*—The role of interference in wireless networks has recently been profoundly re-thought with the emergence of new techniques for combating it and exploit it to maximize the use efficiency of the physical resources. This paper presents a two-way relay channel using a lattice-based physical layer network coding scheme, a massive MIMO array, and in-band full-duplex, taking into account the residual self-interference that results after applying recently developed cancellation techniques for the loopback interference. The proposed scheme is able to ultimately exchange information across the TWRC in only one time slot, whereas four time slots would be needed in a conventional TWRC. The system's performance is shown to be mostly dependent on the number of antennas at the relay, and also dependent on the channel state information of all the channel matrices, including the one describing the loopback interference at the relay. For base-stations and relays with a few hundred antennas, the proposed scheme is feasible for wireless systems.**

***Index Terms*—In-band full-duplex, massive multiple-input multiple-output (MIMO), Physical Layer Network Coding (PLNC), Two-way Relay Channel**

I. Introduction

Higher data rates and lower latencies are a major drive for the 5G mobile systems [1]. In this generation, the long-held assumption that radios can only simultaneously transmit and receive in different frequency bands (i.e., imposing orthogonality for multiplexing) will end. This idea of splurging spectrum was until recently deemed necessary to avoid interference. The recent concept of in-band full-duplex communications makes use of the same frequency band to both transmit and receive data in wireless nodes. This concept is expected be incorporated in the upcoming wireless generation [2], providing a leap forward in terms of spectral efficiency.

Full-duplex may ideally double a link's capacity or, equivalently, reduce by half the allocated frequency band, when it is compared with the current half-duplex or out-of-band full-duplex modes. However, since both frequency and time resources are used simultaneously, the limitations of in-band full-duplex operation arise from the existing self-interference, which is reflects the leakage of the transceiver's outgoing signal to its reception side, a problem that is enhanced by the high power unbalance between both these signals, hence potentially causing inadmissible levels of interference that deteriorate the system's performance [3]. Thus, self-interference must be mitigated, and this is typically done at three different independent stages [4]. The first cancellation stage is performed within the wireless propagation domain, essentially by using passive techniques that can electromagnetically isolate signals. Then, analog radio circuits are employed at a broadband level to further reduce the self-interference signal power. These circuits create a delayed and phase rotated version of the outgoing signal that is subtracted to the incoming one, aiming at tracking and simulating the effect of the channel [5]. Finally, the third (digital) stage is required in the signal processing domain in order to provide a fine mitigation of the residual interference still present after the first two steps. The use of multiple-input multiple-output (MIMO) filters have been deeply explored at this stage, where optimal power allocation, adaptive filtering and adaptive beamforming are all efficient ways of mitigating interference [6], [7].

Another cornerstone technology in 5G is the use of massive arrays (possibly employing hundreds of antennas) at the base stations and relays, which allows serving more users, i.e., increasing the overall system's capacity. Massive MIMO upscales the attractiveness of MIMO by reducing noise, fading, and interference [8], and in this paper massive MIMO is exploited to mitigate self-interference.

Physical layer network coding (PLNC) has emerged as a new way of thinking interference in multi-hop networks. The idea is to treat multi-user interference as a necessary effect, rather than avoid it by allocating different channel resources to different users [9]. PLNC applies the principle of network coding taking in consideration the additive property of wireless channels, and was simultaneous proposed in three independent works [10]–[12]. Afterward, a more practical approach to the problem emerged, which explores the capacity of a relay to decode a combination of symbol constellations [13], [14]. Also, an information theoretic approach emerged, taking advantage of codebooks and lattice network coding [15], [16]. The integration of PLNC with in-band full-duplex is still at an embryonic stage, mainly explored in [17], [18].

This paper accesses the combination of the three aforementioned technologies in a two-way relay channel (TWRC). The orthogonal properties of massive MIMO relaying are conjugated with PLNC, allowing to increase the amount of information exchanged per channel use, and also to further cancel the inherent loopback interference at the relays. Furthermore, a simple lattice-based PLNC scheme is implemented, and the dependency of the system's performance of number of antennas at the relay antennas is assessed. The proposed scheme is able to exchange ultimately information across the TWRC in only one time slot, whereas four time slots would be needed in a conventional TWRC.

II. System Model

A full-duplex two-way relay channel (FD-TWRC) is considered, i.e., a system where the relay and both terminals transmit and receive simultaneously in the same frequency band. Additionally, the relay is considered to have a massive array, and one also considers that the two terminals operate a in-band full-duplex mode. Given that the system is symmetric, only one of the two communication directions is assessed.

Consider that a terminal $\mathcal{A}$ and a terminal $\mathcal{B}$, both with N_T receive and N_T transmit antennas, exchange information via a relay station $\mathcal{R}$, which is assumed to have $M_R >> N_T$ antennas to receive and $M_T = N_T$ antennas to transmit, as Fig. 1 depicts.

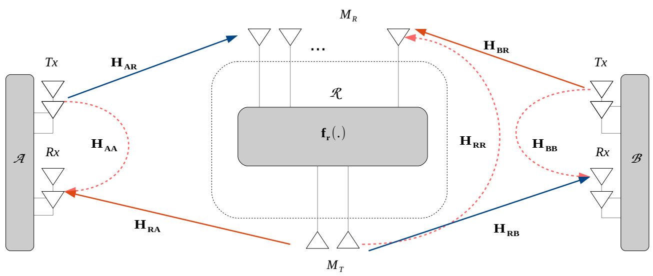

Figure 1. Massive MIMO relay system for the FD-TWRC, for a large M_R and for $M_T = N_T = 2$.

The received signals at each element of the system are expressed by

$$\mathbf{y_R}(n) = \sqrt{p_A}\mathbf{H_{AR}}\mathbf{x_A}(n) + \sqrt{p_B}\mathbf{H_{BR}}\mathbf{x_B}(n) + \sqrt{p_R}k_R\mathbf{H_{RR}}\mathbf{x_R}(n) + \mathbf{n_R}(n), \tag{1}$$

$$\mathbf{y_A}(n) = \sqrt{p_R}\mathbf{H_{RA}}\mathbf{x_R}(n) + \sqrt{p_A}k_A\mathbf{H_{AA}}\mathbf{x_A}(n) + \mathbf{n_A}(n), \tag{2}$$

$$\mathbf{y_B}(n) = \sqrt{p_R}\mathbf{H_{RB}}\mathbf{x_R}(n) + \sqrt{p_B}k_B\mathbf{H_{BB}}\mathbf{x_B}(n) + \mathbf{n_B}(n), \tag{3}$$

where $\mathbf{x_A}(n)$, $\mathbf{x_B}(n)$ and $\mathbf{x_R}(n)$ are the terminal $\mathcal{A}$, terminal $\mathcal{B}$ and relay transmit signals, respectively. Matrices $\mathbf{H_{AR}} \in \mathbb{C}^{M_R \times N_T}$, $\mathbf{H_{BR}} \in \mathbb{C}^{M_R \times N_T}$, $\mathbf{H_{RA}} \in \mathbb{C}^{N_T \times M_T}$ and $\mathbf{H_{RB}} \in \mathbb{C}^{N_T \times M_T}$ represent the channels from $\mathcal{A}$ and $\mathcal{B}$ to $\mathcal{R}$, and vice-versa. Matrices $\mathbf{H_{AA}} \in \mathbb{C}^{N_T \times N_T}$, $\mathbf{H_{BB}} \in \mathbb{C}^{N_T \times N_T}$ and $\mathbf{H_{RR}} \in \mathbb{C}^{M_R \times M_T}$, represent the self-interference channels, while $\mathbf{n_A}(n)$, $\mathbf{n_B}(n)$ and $\mathbf{n_R}(n)$ account for the N_T-dimensional complex circularly symmetric Gaussian noise vectors. The transmit average power of the elements involved in the system are given by p_A, p_B and p_R, respectively. Furthermore, the self-interference is mitigated through parameters k_A, k_B and k_R, that translate the suppression levels, with respect to the NI case. In this case, we assume that

$$\begin{aligned}
\mathbf{H_{AA}}\mathbf{x_A}(n) - \widehat{\mathbf{H}_{AA}\mathbf{x}}_A(n) &:= k_A\mathbf{H_{AA}}\mathbf{x_A}(n), \\
\mathbf{H_{BB}}\mathbf{x_B}(n) - \widehat{\mathbf{H}_{BB}\mathbf{x}}_B(n) &:= k_B\mathbf{H_{BB}}\mathbf{x_B}(n), \\
\mathbf{H_{RR}}\mathbf{x_R}(n) - \widehat{\mathbf{H}_{\mathbf{RR}}\mathbf{x}}_R(n) &:= k_R\mathbf{H_{RR}}\mathbf{x_A}(n),
\end{aligned} \tag{4}$$

where $\widehat{\mathbf{H}_{AA}\mathbf{x}_A}(n)$, $\widehat{\mathbf{H}_{BB}\mathbf{x}_B}(n)$ and $\widehat{\mathbf{H}_{RR}\mathbf{x}_R}(n)$ are the estimations of the self-interference components at terminal $\mathcal{A}$, $\mathcal{B}$ and relay $\mathcal{R}$, respectively. The typical values for k_A, k_B and k_R have been recently derived for different types of signal processing canceling techniques in [7], [19], [20].

III. Compute-and-Forward with Massive MIMO

In order to allow the exchange of information a particular form of PLNC, dubbed compute-and-forward (CF) [12], is implemented. The main concept is that the relay forwards a function of the superimposed received symbols and that an isomorphism exists between the transmitted codewords and the symbols mapped onto a lattice, relying on the closeness of group codes under addition and on the the additive superposition of electromagnetic waves. By receiving a combination of the sent codewords, and by knowing its own codeword, a terminal may be able to decode the codeword from the other pair based on these properties. In practice, this isomorphism is captured when using nested lattice codes [15], defined as

$$\mathcal{L} = \Lambda_\mathrm{F} \cap \mathcal{V}_{\Lambda_\mathrm{C}} = \{\lambda = [\lambda_\mathrm{F}]\mathrm{mod}_{\Lambda_\mathrm{C}}, \lambda_\mathrm{F} \in \Lambda_\mathrm{F}\},$$

where Λ_F is a fine lattice that falls within the fundamental Voronoi region, $\mathcal{V}_{\Lambda_\mathrm{C}}$, of a coarse lattice, Λ_C, and where mod_Λ returns the quantization error with respect to Λ [21].

A. Nested Lattice Code

Each terminal generates data streams from a two-dimensional integer set with field size $Q = 3$, i.e., $S_{A,i}, S_{B,i} \in \mathbb{Z}_3^2$, for $i = 1, \cdots, N_T$, which are then mapped onto $\mathbf{x_A}, \mathbf{x_B}, \in \mathbb{C}^{N_T}$, with function

$$\begin{aligned}
&\phi : \mathbb{Z}_3^2 \to \mathcal{L} = \Lambda_\mathrm{F} \cap \mathcal{V}_{\Lambda_\mathrm{C}}(\in \mathbb{C}), \\
&S_{A,i}, S_{B,i} \to \mathbf{x_A}, \mathbf{x_B}.
\end{aligned} \tag{5}$$

A fine nested Gaussian lattice is considered, defined as $\Lambda_\mathrm{F} = \{x \in \mathbb{C} : x = 2 \cdot z_1 + 3 \cdot z_2\ j;\ z = (z_1, z_2) \in \mathbb{Z}^2\}$, and function $\phi(\cdot)$ is defined such that the transmitted codewords from each antenna of each are mapped onto $\{(0,0),(0,1),(1,0),(1,1),(1,2),(2,1),(2,2)\} \in \mathbb{Z}_3^2$ onto the points $\{0+0j, 0+3j, 2+0j, 2+3j, 2-3j, -2+3j, -2-3j\}$, respectively, and as Fig 2 depicts. Finally, the sequence of lattice points are normalized to be transmitted with unit power.

B. Proposed Protocol

The task of the relay in the CF protocol is to obtain an integer combination of the transmitted symbols, in the form:

$$\Big[\mathbf{D_A}\mathbf{x_A}(n) + \mathbf{D_B}\mathbf{x_B}(n)\Big]\mathrm{mod}_{\Lambda_\mathrm{C}}, \tag{6}$$

where $\mathbf{D_A}, \mathbf{D_B} \in \mathbb{Z}^{N_T \times N_T}$ are diagonal matrices with integer entries forming the network code that interprets the effect of a complex channel as an integer one. The relay starts by applying a zero-forcing (ZF) filter to remove the interference, which, given the orthogonality created by the massive array, is a quasi-optimal approach [8]. Therefore the pseudo-inverses

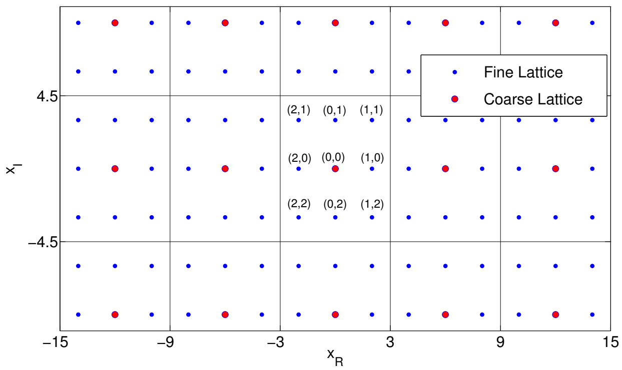

Figure 2. The used Gaussian nested lattice code with mapping function $\mathbb{Z}_{Q=3}^2 \rightarrow \mathbb{C}: \ x = \phi(S)$.

of $\mathbf{H_{AR}}$ and $\mathbf{H_{BR}}$ are calculated and the received vector at the relay is given by

$$\begin{aligned} \mathbf{y_P}(n) =& \mathbf{H_{AR}}^\dagger \mathbf{y_R}(n) + \mathbf{H_{BR}}^\dagger \mathbf{y_R}(n) \\ =& \left(\mathbf{H_{AR}}^\dagger \mathbf{H_{AR}} \mathbf{x_A}(n) + \mathbf{H_{BR}}^\dagger \mathbf{H_{BR}} \mathbf{x_B}(n)\right) + \\ & \left(\mathbf{H_{BR}}^\dagger \mathbf{H_{AR}} \mathbf{x_A}(n) + \mathbf{H_{AR}}^\dagger \mathbf{H_{BR}} \mathbf{x_B}(n)\right) + \\ & \left(\mathbf{H_{AR}}^\dagger + \mathbf{H_{BR}}^\dagger\right)\left(k_R \mathbf{H_{RR}} \mathbf{x_R}(n) + \mathbf{n_R}(n)\right) \\ =& \underbrace{\mathbf{D_A} \mathbf{x_A}(n) + \mathbf{D_B} \mathbf{x_B}(n)}_{\text{desired component}} + \underbrace{\tilde{\mathbf{n}}^\star_{\mathbf{R}}(n)}_{\text{equivalent total noise}} , \end{aligned} \tag{7}$$

where $\mathbf{y_P}(n) \in \mathbb{C}^{N_T \times 1}$ is the desired linear combination of the terminals' signals that arrive at the relay, and where $(\cdot)^\dagger$ represents the pseudo-inverse. Additionally, without loss of generality, consider the case of $p_A = p_B = p_R = 1$, where the network code takes the unitary value.

It is interesting to look at the equivalent noise in (7) given the ZF noise enhancement. Once again, the properties of massive MIMO ensure all these asymptotic properties for the matrices: $\mathbf{H_{BR}}^\dagger \mathbf{H_{AR}} \rightarrow \mathbf{0}, \mathbf{H_{AR}}^\dagger \mathbf{H_{BR}} \rightarrow \mathbf{0}$, as $M_R \rightarrow \infty$. In addition, the self-interference is mitigated by the orthogonality between $\left(\mathbf{H_{BR}}^\dagger + \mathbf{H_{AR}}^\dagger\right)$ and $\mathbf{H_{RR}}$, while the ZF detector under massive MIMO transmissions reduces the AWGN power. Finally, the proposed CF protocol for in-band full-duplex relaying with massive MIMO is detailed in algorithm 1.

IV. Numerical Results

The performance of the proposed protocol is numerically evaluated in terms of the symbol error rate (SER), and the effect of M_R on SER is studied, as well as the robustness to estimation errors of all the channel matrices involved in the system. All results are obtained via Monte Carlo simulation, using uncoded MIMO.

A. Impact of Different M_R Antennas at the Receiver

The orthogonalization of the MIMO channel assumes that the number of antennas at the relay growing to infinity. We start by evaluating the effect of having a finite number of receive antennas at the relay. To that end, consider that each channel has entries generated from a normalized complex Gaussian distribution: $\mathcal{CN}(0,1)$.

Algorithm 1 PLNC Scheme for Massive MIMO Relaying

Processing stage at the relay for each $\mathbf{y_R}(n)$:
1) Zero forcing processing of the received signal:
$\mathbf{y_P}(n) = \mathbf{H_{AR}}^\dagger \mathbf{y_R}(n) + \mathbf{H_{BR}}^\dagger \mathbf{y_R}(n)$;
for $i = 1, \cdots, N_T$ **do**
  2) Quantize $y_{P,i}$ to the closest Λ_F point: $\mathrm{Q}_{\Lambda_\mathrm{F}}(y_{P,i})$;
  3) Obtain back a point of the nested lattice code:
  $x_{R,i} = [\mathrm{Q}_{\Lambda_\mathrm{F}}(y_{P,i})]\mathrm{mod}_{\Lambda_\mathrm{C}}$.
end for
5) Transmit the signal $\mathbf{x_R}(n) = [x_{R,1}, \cdots, x_{R,N_T}]$;
Processing stage at terminal $\mathcal{A}$ (similar for $\mathcal{B}$):
1) Decode the relay transmitted signal:
$\hat{\mathbf{x}}_\mathbf{R} = \arg\min_{\lambda \in (\Lambda_\mathrm{F} \cap \mathcal{V}_{\Lambda_\mathrm{C}})^{N_T}} \parallel \mathbf{y_A} - \mathbf{H_{RA}} \lambda \parallel$;
for $i = 1, \cdots, N_T$ **do**
  2) Map the information back to the finite field for each dimension: $u_{1,i} = \phi^{-1}(\mathcal{R}(\hat{x}_{R,i})) = [S_{A,1,i} + S_{B,1,i}]\mathrm{mod}Q$ and $u_{2,i} = \phi^{-1}(\mathcal{I}(\hat{x}_{R,i})) = [S_{A,2,i} + S_{B,2,i}]\mathrm{mod}Q$;
  3) Subtract own information to obtain:
  $\hat{S}_{B,1,i} = [u_{1,i} - S_{A,1,i}]\mathrm{mod}Q$ and
  $\hat{S}_{B,2,i} = [u_{2,i} - S_{A,2,i}]\mathrm{mod}Q$;
end for
4) Obtain $\hat{S}_{B,i} = (\hat{S}_{B,1,i}; \hat{S}_{B,2,i})$ for $i = 1, \cdots, N_T$.

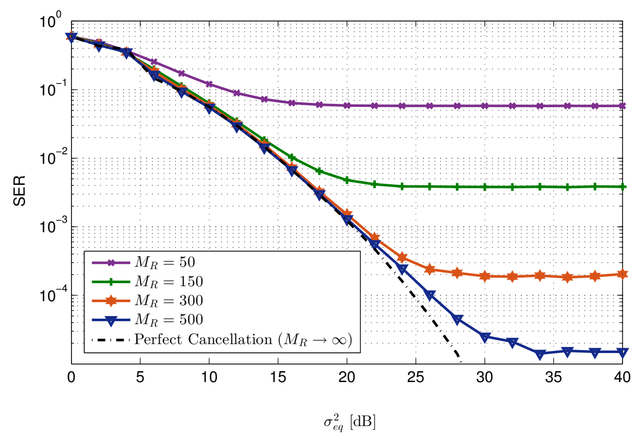

Figure 3. SER performance of the bidirectional CF massive MIMO protocol for different numbers of relay receiving antennas M_R and different equivalent noise power levels σ^2_{eq}.

Fig. 3 , for different number of antennas M_R. Also, the SER is depicted against the equivalent noise power, which considers a fixed self-interference mitigation gain, and equal for the three system elements $k = k_A = k_B = k_R$, and a varying thermal noise $\sigma^2 = \sigma^2_{\mathbf{n_A}} = \sigma^2_{\mathbf{n_B}} = \sigma^2_{\mathbf{n_R}}$, i.e., such that $\sigma^2_{eq} = k + \sigma^2$.

The asymptotic effect is clear in Fig. 3, where, for a low number of antennas, the orthogonal properties of large dimension arrays do not hold. For the different M_R antennas considered at the relay, the SER curves stall at an error floor (caused by the loopback interference) that decreases with M_R and is caused by the interference components that are not properly canceled due to the reminiscent orthogonality-defect.

When considering a very larger number of antennas, for example $M_R = 500$, the effect of imperfect cancellation of the leak between the MIMO spacial channels (i.e., when a perfect orthogonalization of the channel is not achieved) tends to be negligible, as the orthogonal property is valid for a large range of σ_{eq}^2, up to close to 25dB. Moreover, the noise floors appear at acceptable values of SER, and when $M_R = \infty$, the SER tends to the asymptotic case of perfect interference cancellation.

B. Impact of Imperfect Channel Estimation

Another interesting aspect is to evaluate how imperfect channel state information (CSI) may deteriorate the performance. To that end, consider that the relay only has access to erroneous estimations of the channel matrices, i.e., each entry of the channel matrices is known at the relay apart from some error component. Thus, we assume for all channel matrices that

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathcal{E}_{\mathbf{H}},$$

where the error component is generated from a complex Gaussian distribution as $\mathcal{CN}(0, \sigma_{\mathbf{H}}^2)$, and where $\sigma_{\mathbf{H}}^2$ accounts for the estimation error power. Fig. 4 depicts the average SER performance for different values of equivalent noise, different numbers of antennas, and different estimation errors power.

Imperfect estimation of the channel matrices is still a major drawback in the proposed CF protocol with massive MIMO. For $M_R = 150$ antennas (blue curves in Fig. 4), when the relay does not exactly know the channel matrices, the SER curves for $\sigma_{\mathbf{H}}^2 = 10^{-5}$ and for $\sigma_{\mathbf{H}}^2 = 10^{-3}$ tend to an error floor. This is mainly caused by the noise enhancement at the ZF filtering stage, that limits the performance in the presence of imperfect channel estimations, since $\mathbf{D}_{\mathbf{A}} = \mathbf{D}_{\mathbf{B}} = \mathbf{I}$ will no longer be true (i.e., a unitary network code is never achieved). In this case, the codewords become a complex number, introducing a phase rotation in the geometry of the lattice constellation that cannot be handled by the CF scheme.

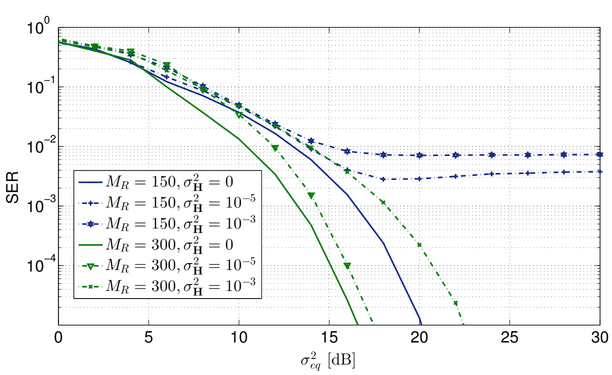

Figure 4. SER curves of the CF massive MIMO protocol for different numbers of relay receiving antennas M_R, different interference power levels σ_{eq}^2 and different channel estimation errors power $\sigma_{\mathbf{H}}^2$.

The interference terms in (7) are not affected by these errors, given the orthogonality between large random matrices is assured. When a larger number of antennas is considered, as the green curves in Fig. 4 show for $M_R = 300$, and for the same power of the estimation errors, the error floor disappears (for the depicted SER values). This effect is also explained by the orthogonal property that comes with large dimensional Gaussian matrices, which minimizes the propagation of errors due to ZF filtering. Nevertheless, these SER curves will eventually stall at an error floor for lower values of SER.

One should note that when increasing the number of antennas M_R to a few hundreds antennas, the SER floor decreases to the desired typical values in wireless links ($\approx 10^{-3}$).

V. Conclusions

The paper proposes a two-way relay channel using in-band full-duplex and employing a massive array for reception at the relay which allows to asymptotically exchange information using just one time slot in a two-hop communication link. Furthermore, lattice-based physical-layer network coding allows to reduced the number of time-slots required to establish the bidirectional information flow.

Massive MIMO plays a central role in reducing the protocol inherent interference between the two data flows, but also helps overcoming the self-interference at the relay as the number of receiving antennas at the relay increases to a few hundreds. The latter effect was observed via SER curves using typical powers for the remaining self-interference when using state-of-the-art cancellation techniques. Finally, the impact of imperfect channel state information (primary links and loopback interference link) was also analyzed.

Acknowledgments

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