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Entropy: A new measure of stock market volatility?

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Abstract. When uncertainty dominates understanding stock market volatility is vital. There are a number of reasons for that. On one hand, substantial changes in volatility of financial market returns are capable of having significant negative effects on risk averse investors. In addition, such changes can also impact on consumption patterns, corporate capital investment decisions and macroeconomic variables. Arguably, volatility is one of the most important concepts in the whole finance theory. In the traditional approach this phenomenon has been addressed based on the concept of standard-deviation (or variance) from which all the famous ARCH type models – Autoregressive Conditional Heteroskedasticity Models– depart. In this context, volatility is often used to describe dispersion from an expected value, price or model. The variability of traded prices from their sample mean is only an example. Although as a measure of uncertainty and risk standard-deviation is very popular since it is simple and easy to calculate it has long been recognized that it is not fully satisfactory. The main reason for that lies in the fact that it is severely affected by extreme values. This may suggest that this is not a closed issue. Bearing on the above we might conclude that many other questions might arise while addressing this subject. One of outstanding importance, from which more sophisticated analysis can be carried out, is how to evaluate volatility, after all? If the standard-deviation has some drawbacks shall we still rely on it? Shall we look for an alternative measure? In searching for this shall we consider the insight of other domains of knowledge? In this paper we specifically address if the concept of entropy, originally developed in physics by Clausius in the XIX century, which can constitute an effective alternative. Basically, what we try to understand is, which are the potentialities of entropy compared to the standard deviation. But why entropy? The answer lies on the fact that there is already some research on the domain of Econophysics, which points out that as a measure of disorder, distance from equilibrium or even ignorance, entropy might present some advantages. However another question arises: since there is several measures of entropy which one since there are several measures of entropy, which one shall be used? As a starting point we discuss the potentialities of Shannon entropy and Tsallis entropy. The main difference between them is that both Renyi and Tsallis are adequate for anomalous systems while Shannon has revealed optimal for equilibrium systems.

1. Introduction

Financial market volatility is central to theory and practice in finance domain. Although its actuality this is not however an entirely new issue and has emerged in a systematic way when [1] first argued that the observed stock market volatility were inconsistent with the predictions of the present value models, quite popular in the past. Furthermore, [2] found out that the intemporal variation appeared to be inexplicably high and could not be rationalized even in models with a stochastic discount factor.

Even thought some authors questioned about the conclusion of excessive volatility, like [3], latter tests accounting for dividend nonstationarity and small sample bias continued to lend support to Shiller's initial claim (see [4-5]).

In this brief overview we have tried to shed some light on the theme and to unfold some of its major implications. Nevertheless, given the impracticability of analyzing the volatility as a whole we focus on its particular aspect of measurement. Here, however we face an obstacle: since volatility is not observed, there has been no agreement on how to measure it, thus emerging a plethora of techniques. Another conclusion that appeared to have raised is that volatility is volatile.

The main contribution of this paper is to compare two different approaches: one based on the statistical measure of the standard deviation or variance and the other one centered on the concept of entropy. In this regard, we particularly focus on the concept of Tsallis entropy which constitutes a possible generalization of the Boltzmann-Gibbs or Shannon entropy. These measures were both generated in the domain of physics, although the latter is also attributed to the Information Theory, and their application to financial phenomena falls in the domain of the so-called econophysics. In an analogy with terms like biophysics, geophysics and astrophysics this word was originally introduced by [6] in an attempt to legitimize the study of economics by physicists. One argument is that there were found some regularities between these two areas. Other, refers to the benefits of the experimental method commonly used in physics which departs from the observed data without imposing any prior model. Also, it is worthy to note the evidence of common research interests between these two areas. As [7] pointed out an active domain of research in physics is the characterization of the process of prices changes, *i.e.*, volatility. In our particular investigation we apply the concept of entropy to capture the presence of nonlinear dynamics in seven stock market indexes since the standard deviation evidence some limitations. The empirical analysis is conducted with data from different countries for comparative purposes.

The remainder of the paper is organized as follows: Section 2 describes the most commonly used measure of volatility - the standard deviation. Section 3 presents two different measures of entropy: the Shannon entropy and a possible generalization of it - the Tsallis entropy. Section 4 exhibits the empirical findings and Section 5 draws the conclusions.

2. Volatility and Entropy Measures: Some Concepts

In this Section we define various measures of volatility. We begin with the standard deviation and then analyze the Tsallis entropy and a special case of it - the Shannon entropy. Before proceeding further on we shall first clarify the term volatility. According to a wide range of research, volatility is popular as a synonymous of risk and uncertainty.

Based on the possibility that volatility could be not constant over time, *i.e.*, "volatility is volatile", some authors have divided the various techniques in two different categories: time invariant (or independent) and time variant (or dependent) measures. In the first group we include the techniques studied in this paper since they do not depend on time. The other one clearly exceeds the scope of our investigation and it is related with, for example, the ARCH (Autoregressive Conditional Heteroscedastic) models, and their subsequent derivations.

2.1. A traditional measure of volatility

A conventional way of measuring volatility is to compute the returns r_t of an asset:

$$r_t = \ln P_t - \ln P_{t-1}, \tag{1}$$

where P_t and P_{t-1} denote the prices at time t and $t-1$, respectively, and then estimate the corresponding standard deviation over some historical period T, given by

$$\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{T}\left(r_t - \langle r \rangle\right)^2}{T-1}}, \tag{2}$$

with $\langle r \rangle$ representing the sample average return, $\langle r \rangle = \sum r_t / T$.

Although this measure has some advantages since it is simple to estimate and has the ability to capture the probability of occurring extreme events it also shows some drawbacks. One is that it could lead to an abrupt change in volatility once shocks fall out of the measurement sample. And, if shocks are still included in a relatively long measurement sample period, then an abnormally large observation will imply that the forecast will remain in an artificial high level even thought the market is subsequently tranquil. Finally, it only captures linear relationships, ignoring all kind of nonlinear dynamics among data. In the light of this, some more sophisticated measures has emerged in a way to improve the understanding of volatility. Regarding to this, a measure that appears to be particularly relevant is the concept of entropy, which constitutes our major aim in this study.

However, it is worthy to note that in spite of all the flaws that have been recognized by a wide body of research the standard deviation is still the most popular measure of volatility being used as a benchmark for comparing the forecast ability of more complex models.

3. Theoretical framework: the concept of entropy

An alternative way to study stock market volatility is by applying concepts of physics which significant literature has already proven to be helpful in describing financial or economic problems, such as, the concept of entropy. Although there are many different understandings of this notion the most commonly used in literature is as a measure of ignorance, disorder, uncertainty or even lack of information (see [8]).Later, in a subsequent investigation Shannon [9] provided a new insight into this matter showing that entropy wasn't only restricted to thermodynamics but could instead be applied in any context where probabilities can be defined.

3.1. Shannon entropy

For a given a probability distribution $p_i \equiv p(X = i)$, $(i = 1,...,n)$ of a given random variable X, Shannon entropy $S(X)$, for the discrete case can be defined as

$$S(X) = -\sum_{i=1}^{n} p_i \log p_i, \tag{3}$$

where $0 \log 0$ is defined as 0 and the normalized associated probabilities $\sum_{i=1}^{N} p_i = 1$. Shannon entropy has been most successful in the treatment of equilibrium systems in which short/space/temporal interactions with ergodicity and independence dominate. However, there are many anomalous systems in nature that do not verify the simplifying assumption of ergodicity and independence. To overcome this kind of drawback Tsallis [11] derived a new measure of entropy, known as Tsallis entropy.

3.2. Tsallis entropy

For any non-negative real number q and considering the probability distribution $p_i \equiv p(X = i)$, ($i = 1,...,n$) of a given random variable X, Tsallis entropy (Tsallis, [10]) denoted by $S_q(X)$ for the discrete case, is defined as

$$S_q\left(X\right)=k\frac{1-\sum_{i=1}^{W}p_i^q}{q-1}, \quad (4)$$

where the q-exponential function is defined by

$$y=\left[1+\left(1-q\right)x\right]^{1/(1-q)}\equiv \mathrm{e}_q^x\left(x\right) \quad ,\left(x\geq 0,q\in \circ\right), \quad (5)$$

whose inverse is the q-logarithm function

$$y=\frac{x^{1-q}-1}{1-q}\equiv \ln_q\left(x\right)\ \left(x\geq 0,q\in \circ\right). \quad (6)$$

The entropic index q characterizes the statistics we are dealing with; as $q\rightarrow 1$, $S_q\left(X\right)$ recovers $S\left(X\right)$ since the q-logarithm uniformly converges to a natural logarithm as $q\rightarrow 1$. This index may be regarded as a biasing parameter since $q<1$ privileges rare events and $q>1$ privileges common events ([11]). A concrete consequence of this is that while the Shannon entropy yields exponential equilibrium distributions, Tsallis entropy yields power-law distributions.

4. Empirical Analysis

This Section explores the empirical relevance of the theoretical results obtained by both perspectives. To do so we gathered data from several different countries in order to detect whether some similarities among all of them can be found. This is especially relevant in the context of globalization, where we living in, which also constitutes an active area of research.

4.1. Data

In our empirical research the data set compounds the daily returns of the FTSE 100 (U.K), SP 500 (U.S.A.), CAC 40 (France), MIB 30 (Italy), NIKKEI 225 (Japan), IBEX 35 (Spain) and PSI 20 (Portugal) extending from 8 January 1990 to 7 April 2006. Each index contains 4240 observations, which is large enough to make our analysis meaningful. These data were collected on a daily basis without considering the re-investment of dividends and were computed in accordance to Eq. (1) where the closing prices were the inputs.

4.2. Standard deviations results

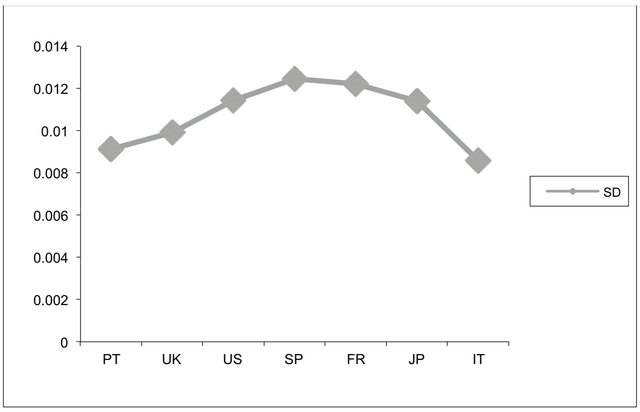

Fig. 1. Standard deviation of stock index returns

We observe that the higher volatility indexes are: first the IBEX 35, second CAC 40 and third, NIKKEI 225. Accordingly, PSI 20 and FTSE 100 are the lower ones. The ranking also shows that all values are close to zero which may suggest that all of them exhibit low volatility in spite of their particular values. However, these outcomes should not be regarded in a strict sense since they exhibit some drawbacks, already mentioned in Section 2 of this research. Indeed, they are influenced by abnormally high observations and are not able to capture nonlinear dynamics.

4.3. Entropy results

In the domain of the econophysics approach we have computed the Tsallis and Shannon entropies, which are depicted in Fig. 2.

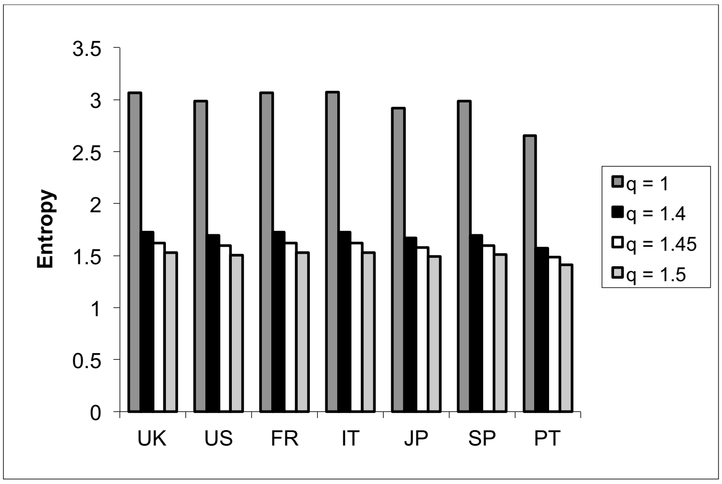

Fig. 2 Entropy Results

All entropies were estimated with histograms based on equidistant cells. For the calculation of Tsallis entropy we have set values at 1.4, 1.45 and 1.5 for the index q, which is consistent with the finding that when considering financial data their values lie within the range $q \simeq 1.4$-1.5 (see [11]). It is worthy to note that MIB 30 index (Italy) attained the highest levels along with CAC 40 and NIKKEI 225. As for the others the results are somewhat mixed since they vary according to technique adopted.

In the overall, it appears that the use of entropy as a measure of uncertainty allows better insights over the identification of volatile markets, by distinguishing them more sharply, than simply using the standard deviation. This leads us to the conclusion that entropy is more general and better suited for describing stock market volatility with its generality arising directly from the fact that it can be computed from metric and non-metric data. Apart from that the major advantages of entropy when compared to the standard deviation can be summarized as follows [12]: (i) it incorporates much more information than the latter; (ii) it is not dependent upon any particular distribution; in other words, it is distribution free, thus avoiding the introduction of errors through the fitting of the distribution of returns to a normal-like distribution. This is especially true when dealing with non-symmetric distributions with generally non-normal additional moments, which seems to be the case in some phenomena in Finance; (iii) Since entropy is independent of the mean for all types of distributions, it satisfies the first order conditions and (iv) finally, due to its common understanding of mean uncertainty, it also serves as a measure of dispersion.

Nevertheless, some disadvantages have also to be weighted when considering the use of any kind of entropy. First one has to do with its inherent complexity when compared to the simple standard deviation. Second, is related to the amount of statistical bias in theses measures due to the degrees of freedom allowed in an experiment.

5. Conclusions

In this paper we have investigated the volatility of seven indexes: CAC 40, MIB 30, NIKKEI 225, PSI 20, IBEX 35, FTSE 100 and SP 500. Our major goal was to compare two different perspectives: one based on the standard deviation and other supported by the concept of entropy. For our purpose two variants of this notion were regarded: the Tsallis and the Shannon entropies.

In particular, the results from both entropies have shown nonlinear dynamics in the volatility of all indexes and must be understood in complementarity since they point out to the same conclusions. However, most of the outcomes are not in accordance with the statistics produced by the standard deviation, which emphasizes that this method is not able to capture the overall behaviour of dispersion. This is especially relevant for the decision making process in which all the information is regarded as necessary and useful. Nonetheless, in spite of all the divergences encountered there is still something in common that relates to the fact that regardless of the method applied the CAC 40 and NIKKEI 225 returns are always the most volatile ones.

In this study we specially address the concept of entropy as an alternative to the standard deviation since it can captures the uncertainty and disorder in a time series without imposing any constraints on the theoretical probability distribution, which constitutes its major advantage. Additionally, in order to capture global serial dependence one should use a specific measure such as, for example, mutual information.

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