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Volatility Risk Premium – New insights into the systematic edge in the market for option sellers

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Master in Finance

Supervisor:

PhD Paulo Viegas Ferreira de Carvalho, Invited Assistant Professor,

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BUSINESS
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Resumo

A volatilidade tem sido, desde há muito tempo, uma das variáveis mais estudadas nos mercados financeiros. Para além disso, os produtos financeiros derivados, mais especificamente as opções financeiras, têm desempenhado um papel importante na inovação financeira. Como tal, pretendemos fornecer nova pesquisa académica relativamente às dinâmicas na relação entre volatilidade realizada e volatilidade implícita nos preços das opções, e acerca do denominado "prémio de risco de volatilidade". Confirmamos empiricamente que o prémio de risco de volatilidade está presente nos mercados à data de hoje. Introduzimos alguma inovação ao investigar o prémio de risco de volatilidade singularmente, procurando concluir se é uma série estacionária, bem como se apresenta padrões sazonais numa base trimestral e/ou correlação com outras variáveis financeiras. Neste sentido, concluímos que a série é estacionária, e encontramos correlações acentuadas com outras variáveis, tais como a volatilidade realizada. Procuramos também atualizar a literatura na comparação do prémio de risco de volatilidade entre índices de ações e ativos individuais, e confirmamos as conclusões da literatura anterior sobre o maior prémio de risco presente nos ativos individuais. Por fim, analisamos também os preços de opções, a fim de testar empiricamente se o prémio de risco de volatilidade distorce os preços das opções a um nível monetário material, concluindo que este é precisamente o caso. O estudo abrange o período de 2000 a 2020, com períodos variáveis consoante o ativo em questão. Os dados envolvem três índices de ações, cinco ações individuais e três *ETF's*. Os dados seguem uma frequência diária.

Palavras Chave: Volatilidade Implícita, Prémio de Risco de Volatilidade, Preços de Opções, Volatilidade Realizada

Classificação JEL: D53, G11, G14

Abstract

Financial options have been at the forefront of financial innovation. Their value depends significantly on volatility, one of the most studied variables of financial markets for a long time. Our study provides empirical evidence on the dynamics of the relationship between realized volatility of asset returns and implied volatility extracted from option prices, and on the so-called "volatility risk premium". We confirm that the volatility risk premium is still present in today's market. Most literature looks at the volatility risk premium as a tool to investigate other financial dynamics. We innovate by investigating the patterns of volatility risk premium singularly as a time series, and so assess whether this is a stationary series, as well as if it presents signs of quarterly seasonality and/or marginal effects from two explanatory variables. We find a clear indication of stationarity and valuable marginal effects by financial variables such as realized volatilities. We also update the literature on the comparison between indices and individual equities surrounding the volatility risk premium; our results confirm that previous findings of a larger premium in individual equities are still applicable nowadays. Lastly, by analyzing option prices, we empirically confirm that volatility risk premium provides a monetary option mispricing. The results we obtain are supported by daily observations, from 2000 to 2020, on three equity indices, five individual equities, one commodity exchange-traded fund, one currency exchange-traded fund, and one emerging-market exchange-traded fund.

Keywords: Implied Volatility; Volatility Risk Premium; Option Prices; Realized Volatility

JEL Classification: D53, G11, G14

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Glossary of Acronyms and Symbols

AAPL – Apple

ADF – Augmented Dickey-Fuller

AMZN – Amazon

ATM – At-the-money

BIC – Bayesian Information Criterion

BSM – Black-Scholes-Merton

CBOE – Chicago Board Options Exchange

CRR – Cox, Ross, and Rubinstein

ETF – Exchange-Traded Fund

EWM – Exponentially Weighted Moving Average

EWZ – iShares MSCI Brazil ETF

FXE – Invesco CurrencyShares Euro Currency Trust

GLD – SPDR Gold Shares

GS – Goldman Sachs

GSPC – Standard and Poor’s 500

GOOGL – Alphabet, Inc.

IBM – International Business Machines Corporation

IV – Implied Volatility

KPSS – Kwiatowski, Phillips, Schmidt and Shin

MLR – Multiple Linear Regression

NDX – Nasdaq 100

OLS – Ordinary Least Squares

RUT – Russel 2000

RV – Realized Volatility

S&P 500 – Standard and Poor’s 500

VRP – Volatility Risk Premium

1 INTRODUCTION

"Option Pricing is unique in that it involves only one uncertain variable, namely volatility". (Thaler, 1993:342)

The derivatives market has been at the forefront of financial innovation for the past decades. Within the derivatives spectrum, options gained major popularity amongst many types of investors. Such financial instruments offer limitless functionalities (hedging, arbitrage, or pure gambling) and require low amounts of capital to trade with, as the option almost always takes less capital allocation than the equivalent position in the underlying.

Options allow the buyer of the contract to purchase (call option) or sell (put option) the underlying asset, for a pre-established price (strike) within the contracts' maturity. For pricing options, market participants have long followed the model initially proposed by Black and Scholes (1973), and later expanded by Merton (1973), and the binomial model created by Cox, Ross, and Rubinstein (1979). Black and Scholes (1973) model incorporates the underlying asset price and the related volatility (of the underlying asset returns), the strike price and maturity of the option contract, as well as a dividend yield of the underlying asset and the risk-free interest rate. Merton's intervention provided higher quality to the model, by incorporating dividends, as well as an alternative derivation which holds the model's validity under weaker assumptions.

The volatility of returns, which is taken as the future payoff uncertainty and risk of an investment, has been examined for decades. A widely used measure of future uncertainty in financial markets is implied volatility (Whaley, 2000), derived by solving the Black-Scholes-Merton (BSM) model given certain option prices. Conceptually, this volatility measure is different from the realized or historical volatility that may be applied in the option-pricing model as an input. Shaikh and Padhi (2015) address the rationale of implied volatility and conclude that it serves as an "investor fear gauge", as it signals investors' expectations on future uncertainty.

Within the aforementioned variables of the option-pricing model, this thesis analyses the potential differences between implied and realized volatility. Particularly, it focuses on the so-called "volatility risk premium" (*VRP*), which represents the difference between implied volatility (*IV*) and realized volatility (*RV*). This premium has been analyzed in previous literature which argues that the main driving factors of implied volatility and the *VRP* concern price jump risks, option contract characteristics and/or underlying equity risks, or a specific catastrophe fear related with economic factors and/or the overall market (e.g., Chernov and Gisels, 2000; Pan, 2002; Duarte and Jones, 2007; Bollerslev et al., 2009). However, to this day there is still a lack of knowledge to understand how the *VRP* behaves and relates with other financial variables, and even to justify its existence, since it challenges the efficient markets hypothesis. Therefore, we study the *VRP* to achieve useful conclusions for investors seeking to generate positive returns in the markets, thus contributing to the literature on the volatility risk premium by focusing on the *VRP* itself.

We update the literature on the divergences of its behavior between individual equities and indices, as well as on its actual existence in the markets, as most of the related literature, by our knowledge, is slightly outdated. This is perhaps the most important part, as it provides a foundation for the subsequent studies.

We innovate by looking at the volatility risk premium independently in the form of a time series, whereas most previous literature uses the *VRP* as a tool to explain other factors, mostly ignoring the relevance of the *VRP* itself on option pricing efficiency. We study the predictability of the *VRP* by estimating an auto-regressive model – this is of extreme relevance, as it would allow for adequate timing in deployment of option-selling/buying strategies by market participants. Furthermore, we perform a complementary empirical analysis through econometrical methods to conclude whether the time series of *VRP* is stationary, seasonal, as well as if it correlates with other financial variables.

Finally, we assess the extent to which the volatility risk premium distorts option prices, to conclude whether options have a material mispricing that can potentially be used to generate positive returns. For this, we generate two series of hypothetical option prices, in which the only differentiating factor is the volatility variable used. Contrary to previous research on this subject, which is mostly conceptual and/or focus on testing trading strategies, we apply a volatility-focused methodology. Again, we maintain consistency in the objective of focusing in analyzing the *VRP* with the contribution of other variables, and not the opposite, thus challenging the predominant trend in past literature.

This study does an empirical analysis of the volatility risk premium in multiple assets, namely equities and indices. The empirical analysis of both *IV* and *RV* is done over three equity indices, five individual equities, one commodity Exchange-traded fund (*ETF*), one currency *ETF*, and one emerging-market *ETF*. The periods under analysis vary according to data availability in the different assets.

In sum, this research aims at finding answers to the following set of questions:

1. Does implied volatility overestimate realized volatility?
2. Is the *VRP* higher in individual equities than in indices?
3. Is the *VRP* affected with other financial variables? Is it predictable?

4. Does the *VRP* materialize in option over/underpricing?

The results show confirmation of previous literature in that, not only there is a *VRP* in options markets, but this premium is higher for individual equities than for broad indices. Additionally, we find that for the emerging market used in this study, the premium is negative, suggesting a systematically higher volatility than expected. Alternatively, we conclude that the series is stationary, and in general does not present signs of seasonality. Furthermore, we find realized volatilities have a significant marginal effect on the *VRP*, and that previous *VRP* values are indicative of future ones, thus allowing for some predictability. Finally, we conclude that the volatility overestimation leads to an option over-pricing, and that this mispricing shows different magnitudes across all assets.

The rest of this study is structured in the following manner. Section 2 reviews the literature and provides further contextualization on the subject. Section 3 describes the data utilized and the corresponding sources, as well as the methodology. Section 4 analyzes the empirical results, and Section 5 concludes.

2 LITERATURE REVIEW

The basis of this research is the existence of a variance or volatility risk premium. In simple terms, this premium means that the implied volatility derived from option prices tends to be higher than the actual subsequent realized volatility of the underlying asset returns.

Bakshi and Kapadia (2003) investigate whether the VRP exists and if it varies among individual equities and indices. For this purpose, they empirically analyze 25 individual equities within the S&P 500 index and compute the corresponding realized volatilities, as well as the implied volatilities from at-the-money options. Their results are conclusive, pointing to a volatility risk premium, where the *IV* is greater than *RV*. They also conclude that this dynamic occurs in index options for the S&P 500, but to a lesser degree when compared to individual equities.

Jackwerth and Rubinstein (1996) show that, excluding the 1987 crash period, implied volatility from *ATM* options for the S&P 500 are “almost always biased upward from prior historical realizations” (Jackwerth and Rubinstein, 1996: 1613). Alternatively, Padhi and Shaikh (2014) perform a study about the informational content in implied volatility using options from the S&P CNX Nifty index, from 2001 to 2011. Through empirical analysis, they state that implied volatilities are higher than realized volatilities. Furthermore, they distinguish put-option *IV* from call-option *IV*, and conclude that the volatility risk premium is not found in call options, pointing towards the portfolio insurance theory (Leland and Rubinstein, 1976).

Similarly, Fleming, Ostdiek, and Whaley (1995) confirm the relevance of the volatility premium. Their data ranges from 1986 until 1992 and focuses on the VIX (which, at the time, was the implied volatility index for the S&P 100, and not for the S&P 500 as it is today). According to them, “the average difference between the volatility index and the subsequent, realized stock market volatility is 584 basis points” (Fleming, Ostdiek and Whaley, 1995: 295), and so the *IV* is a biased forecaster of future volatility. Using data from 1990 until the end of 2006, Eraker (2009) explains the volatility premium through the Eraker-Shaliastovich equilibrium model, with the differences between the model results and real data being statistically insignificant. His study confirms once again that the *VRP* is empirically significant, since it “averages 3.3 percent in annualized standard deviation units and 1.5 in variance units” (Eraker, 2009: 8).

One potential explanation for the volatility premium lies in the asymmetry in volatility, due to bad news having a greater impact on volatility than equally good news (Black, 1976). This suggests a negative correlation between stock returns and volatility, meaning that, if prices decrease, volatility tends to increase more than in the alternative scenario (when prices increase volatility tends to decrease). Using model-free measures of realized daily volatility and correlation, Andersen et al. (2001) also confirm an asymmetric relation between returns and the corresponding volatility. They also detect that, in line with expectations, the asymmetric effect is significantly stronger at the individual equities level when compared to indices, as indices are exposed only to systematic risk, whereas individual equities are exposed to both systematic and idiosyncratic risk.

One potential economic explanation for this asymmetric dynamic lies in the leverage effect, as advanced by Christie (1982). He argues that declines in stock prices reduce the market value of equity relatively to the market value of debt, thus increasing the financial leverage, and so diminishing the financial health of the company. This explanation is also confirmed by multiple studies (e.g., Schwert, 1989; Bouchaud and Potters, 2001), although others report opposite evidence, such as Andersen et al. (2001).

Another possible explanation for the asymmetry in volatility exists in the human nature to overreact to threats. Based on daily observations for the S&P 100 index options between 1983 and 1987, Stein (1989) finds a disproportionate reaction (overreaction) of longer-term options' implied volatility when faced with a change in implied volatility from shorter-term options, which strays away from the rational expectation's theory. Additionally, Poteshman (2001) tests for the over/underreaction in the S&P 500 index options market between 1988 and 1997. He uses 4 models in his research – a stochastic variance model, the Heston (1993) model, a nonparametric stochastic variance model (Poteshman, 1998), and a stochastic variance with jumps model (Pan, 2000). The results he reports are for the Heston model, from where he concludes that "investors underreact to information at short horizons and overreact to information at long horizons" (Poteshman, 2001: 874).

By accepting the relevance of VRP, with implied volatility systematically overestimating subsequent realized volatility, one could conclude that implied volatility is not a good forecast of future volatility, although widely used by many as such. Still, there is abundant literature showing that *IV* is an efficient estimator of future volatility. Corrado and Miller (2005) examine the forecast quality of implied volatility indices computed for the Nasdaq 100, S&P 100, and S&P 500, ranging from 1988 and 2003. They compare the implied volatility indices to the sample standard deviation of daily returns (scaled to a 22-day standard deviation), with the results pointing towards *IV* being an efficient forecaster for future volatility. Focusing on the foreign exchange markets, Jorion (1995) also concludes that implied volatility, although upwardly biased and confirming the volatility premium, outperforms both moving average and GARCH models in forecasting future volatility.

Moreover, Blair et al. (2001) tests the informational content of implied volatility by comparing it against ARCH models based on realized volatility computed from both high and low-frequency data. The study is done on the S&P 100 and spans from 1987 to 1999. Their results point towards implied volatility being a more accurate forecaster of future volatility in all cases. Christensen and Prabhala (1998) also study the informational content of implied volatility in the S&P 100 options and reach a similar conclusion.

Focusing on emerging markets, Padhi and Shaikh (2014) examine the informational content of *IV* using options on the S&P CNX Nifty index, from 2001 to 2011. In line with most studies, they conclude that implied volatility subsumes all information about future realized volatility. Likewise, studies surrounding other markets besides the American indices, such as the French stock market (Moraux et al., 1999), the Australian stock market (Frijns et al., 2005), and the Taiwanese stock market (Lee et al., 2012), all conclude that implied volatility is an efficient forecast of future realized volatility. Poon and Granger (2003) analyze 93 different studies about volatility forecasting, and the conclusion is that implied volatility is the winner of the bunch in most cases.

Lastly, we find it relevant to mention that several acknowledged studies have been made on the impact of the *VRP* on option trading strategies and go in agreeance with the idea of a positive *VRP* benefiting option sellers. Coval and Shumway (2001) measured returns for long, zero-delta, *ATM* weekly straddles (long put and long call) on the S&P 500, between 1990 and 1995, and concluded such strategy yielded negative returns (around -3% per week). Similarly, Bakshi and Kapadia (2003) test a delta-hedged options' portfolio in the S&P 500 (long options, short stock) from 1988 to 1995, also testifying the negative returns of such strategy. Studies such as these show that long-premium strategies suffer from overpaying for volatility, i.e., short-premium strategies profit from the *VRP*.

3 DATA AND METHODOLOGY

3.1 DATA

To find answers to the initial questions, this thesis analyzes the empirical evidence on asset prices and implied volatility from the following assets:

1. Standard and Poor's 500 (GSPC)
2. Nasdaq 100 (NDX)
3. Russel 2000 (RUT)
4. Amazon.com, Inc. (AMZN)
5. Apple Inc. (AAPL)
6. International Business Machines Corporation (IBM)
7. The Goldman Sachs Group, Inc. (GS)
8. Alphabet, Inc. (GOOGL)
9. SPDR Gold Shares (GLD)
10. Invesco CurrencyShares Euro Currency Trust (FXE)
11. iShares MSCI Brazil ETF (EWZ)

Prices from indices, *ETF's* and individual equities are obtained from Yahoo Finance, while implied volatility data, again for all assets, is retrieved from the CBOE database.

The data frequency is daily across all variables and assets. The sample period ranges from 2000 to 2020, with varying periods for each asset, depending on data availability. The reasoning for this is that CBOE, as of the time of this research, provides implied volatility data with different periods across the distinct assets.

3.2 REALIZED VOLATILITY

We start by empirically comparing implied and realized volatilities, to assess the volatility risk premium. It is worth mentioning that the implied volatility indices used in this study have a forecast period of 30 days, which is the average maturity of the options used to compute such indices. These values are next annualized, considering 252 trading days in a year, which results in an average near 21 trading days each month. The realized volatility for date *t* is thus computed as the rolling 21-day standard deviation of daily returns and scaled to a yearly basis by multiplying by the square root of $\frac{252}{21}$. This allows a direct comparison with the implied volatility data at moment *t*-21.

The annualized rolling 21-day standard deviation for asset i on day t is computed as follows:

$$\sigma_{it} = \sqrt{\frac{252}{21}\sum_{j=1}^{21}\left(R_{i,t-j} - \bar{R}_{i,t}\right)^2} \tag{1}$$

where R_{it} reflects the daily logarithmic returns, and $\bar{R}_{i,t} = \frac{\sum_{j=1}^{21} R_{i,t-j}}{21}$ is the average daily logarithmic returns in the previous 21 trading days.

3.3 Stationarity Testing and Modelling

We now focus on the series of differences between *IV* and *RV*, with the main question here being whether the series is stationary. For this, we perform a unit-root test, the Augmented Dickey-Fuller test (Dickey and Fuller, 1979).

3.3.1 ADF Test

In the Augmented Dickey-Fuller (ADF) test, the null hypothesis states the presence of a unit root (non-stationary time series), which indicates that the process might well be described by a random walk model. For this test, we estimate one of the following equations (the choice is made based on the graphical analysis of the data in question) by using OLS:

$$\Delta y_t = \gamma y_{t-1} + \sum_{j=2}^{p} \beta_j \Delta y_{t-j+1} + \varepsilon_t \tag{2}$$

$$\Delta y_t = \beta_0 + \gamma y_{t-1} + \sum_{j=2}^{p} \beta_j \Delta y_{t-j+1} + \varepsilon_t \tag{3}$$

$$\Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + \sum_{j=2}^{p} \beta_j \Delta y_{t-j+1} + \varepsilon_t \tag{4}$$

Equation 1, the most basic version of the regression, represents a pure random walk. Equation 2 incorporates a drift (β_0) and Equation 3 incorporates both a drift (β_0) and a linear time trend ($\beta_1 t$). Furthermore, we set the number of lags (Δ) used in the ADF test with the Bayesian Information Criterion (BIC). The BIC formula is as follows:

$$\mathrm{BIC} = \ln(n)k - 2\ln(\hat{L}) \tag{5}$$

where $\hat{L}$ is the maximized value of the log likelihood function for the model, n is the number of occurrences and k is the number of free parameters being estimated. The t-statistic for the ADF test is:

$$t = \frac{\hat{\gamma}}{se(\hat{\gamma})} \tag{6}$$

where $\hat{\gamma}$ is the estimate of the coefficients in a first order autoregression and $se(\hat{\gamma})$ is the Ordinary Least Squares (OLS) coefficients' standard errors. The null hypothesis for non-stationarity is:

$$H_0: \gamma = 0$$

Under this hypothesis, the t-statistic does not follow a student's t distribution. The distribution is nonstandard, asymmetric negative. For this reason, the t-statistic estimate is compared with the critical values proposed by Dickey and Fuller (1981) and Hamilton (1994).

3.3.2 *VRP* Modelling

By modelling the *VRP*, we intend to assess three issues about the volatility risk premium. First, we look to investigate whether the *VRP* has a quarterly seasonal pattern. Second, we intend to conclude about the marginal effects between the *VRP* and two important financial variables. Third, we study the possibility of predicting the volatility risk premium time series with the help of an autoregressive model. To reach evidence, we estimate two linear regressions by using the OLS.

On the one hand, we generate one regression with dummy variables for each quarter to test for their statistical significance and thus conclude about the possibility of quarterly seasonality. Furthermore, based on previous studies on volatility (e.g., Blair et al., 2001; Goyal and Saretto, 2009), we incorporate the underlying asset returns and realized volatility as potential explanatory variables. The introduction of these variables is based on their economic relevance to volatility and the *VRP* - returns are related to volatility based on the assumption of asymmetry and/or the leverage effect, and volatility is a variable incorporated in the computation of the *VRP*. Hence, we model *VRP* as follows:

$$VRP_{it} = \alpha_i + \sum_{j=1}^{3} \beta_{ij} D_{ijt} + \lambda_i R_{it} + \gamma_i \sigma_{it} + \varepsilon_{it} \tag{7}$$

This regression can be decomposed into three explanatory components. The first, which contains the coefficient β, tests for seasonality on a quarterly basis, where D represents the dummy variable for each quarter. The second, characterized by the coefficient λ, tests the influence/effect of the underlying asset daily returns on the *VRP*, where R is the underlying asset return. The third component, containing the coefficient γ, tests the influence/effect of the underlying asset daily realized volatility on the *VRP*, where σ is the underlying asset volatility. As in Section 4.1, *i* represents the underlying asset.

The daily returns (R) are computed as the daily log-returns of the underlying asset, and the realized volatility is computed through an Exponentially Weighted Moving Average (EWMA), which computes the variance as follows:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)R_{t-1}^2 \tag{8}$$

where σ_t^2 is the variance at day t, σ_{t-1}^2 is the variance at day t-1 and R_{t-1}^2 is the squared returns on day t-1. λ is the weighting factor, which we estimate for each asset individually through the Maximum Likelihood method. To transform the variance to volatility, we naturally take the square root of the EWMA variance for each period.

On the other hand, another regression is formulated, but this time focusing on investigating whether it is possible to predict the *VRP*. For this, we generate an autoregressive model of order 1, in which we use both explanatory variables from Equation 7, as well as the *VRP* itself. The regression is as follows:

$$VRP_{it} = \delta_i + \alpha_i VRP_{it-1} + \beta_i R_{it-1} + \gamma_i \sigma_{it-1} + \varepsilon_{it} \tag{9}$$

Furthermore, since one of the main goals of this thesis is to provide valuable insights to generate positive returns, we also estimate both previous regressions on a quarterly time frequency. In this case, the quarterly returns (R) are computed as the logarithmic returns in each quarter, and the realized volatility (σ) follows the same formula as in Equation 1. Both use a 63-day period, which is the nearest integer to the average trading days in one quarter, assuming a non-leap year.

3.4 OPTION PRICING

To test empirically the pricing impact of the volatility risk premium, we consider the options' contracts as close as possible to the ones used by the CBOE when calculating the implied volatility indices. These indices are based on a weighted average of mostly at-the-money options, with an average maturity of 30 days.

Thus, in computing option prices, we use a standard maturity of 30 days, with at-the-money (*ATM*) strikes, which are obtained by rounding the adjusted closing price to the nearest integer. The *IV* and *RV* time series are the same utilized in the rest of the thesis. The dividend yield is considered as zero, and the risk-free rate is the 1-Month LIBOR converted to continuous time. We consider the zero-dividend yield assumption since we use the Adjusted Close prices for the spot prices (which incorporates dividends paid). Also, considering that we are comparing the difference in option prices using IV and RV, the dividend-yield would not have a different effect on this comparison since it would be the same in both cases.

Since index options are European-style options, and individual equity options are American-style options, we use two different pricing models.

3.4.1 Black-Scholes-Merton Model

For the European-style options, we use the Black-Scholes-Merton model (1973) to price the different options. The equations for calls and puts are, respectively:

$$c = S_t N(d_1) - Ke^{-rT} N(d_2) \tag{10}$$

and,

$$p = Ke^{-rT} N(-d_2) - S_t N(-d_1) \tag{11}$$

where d_1 is:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{12}$$

and d_2 is:

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{13}$$

In the previous formulas, S_t is the underlying asset price in date t, K is the strike of the option contract, r is the risk-free rate, σ is the volatility input and T is the options’ residual time to maturity.

3.4.2 *CRR* Model

In the case of American-style options, we use the Cox, Ross, and Rubinstein (1979) pricing model for the call and extrapolate the value of the put through the put-call parity relationship. The equation for the present value of the American-style call is:

$$c = Se^{-N\Delta t}\sum_{j=m}^{N}\binom{N}{j}q^j(1-q)^{N-j}u^jd^{N-j} - Ke^{-N\Delta t}\sum_{j=m}^{N}\binom{N}{j}q^j(1-q)^{N-j} \qquad (14)$$

in which $\Delta t = T/N$ is the length of time in each subinterval during the life of the option contract, obtained by dividing the options' time to maturity (T) by the number of binomial periods until that date (N). m is the smallest integer for which the options' intrinsic value at maturity is greater than zero, implying that $u^m d^{N-m} S \geq K$. The elements $u,\ d,$ and q are as follows:

$$u = e^{\sigma\sqrt{\Delta t}} \qquad (15)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \qquad (16)$$

$$q = \frac{e^{r\Delta t} - d}{u - d} \qquad (17)$$

Finally, the term $\binom{N}{j}q^j(1-q)^{N-j}$ is the computation of the binomial probability of j upward jumps in the underlying assets' price after the first N trading periods, and the term $u^j d^{N-j}$ is the value of the option after j upward jumps of the underlying. The rest of the variables are the same as in the BSM model.

4 EMPIRICAL RESULTS

4.1 REALIZED VOLATILITY AND IMPLIED VOLATILITY COMPARISONS

In this section, we analyze the *IV* and *RV* of all assets, firstly on an individual basis, and then extending the analysis to a cross-asset comparison to conclude about the behavior of the *VRP* depending on the asset class (indices or individual equities). The main goal is to not only to present the data obtained but also to formulate possible reasonable explanations for the dynamics found in the empirical results.

To assess the volatility risk premium, we compute the realized volatility for all assets and compare it with the implied volatility data obtained from CBOE. From this, it is possible to get the *VRP*, which serves as the basis for subsequent sections in this thesis. Table 4.1 presents the averages for both volatility measures of all the assets examined along the periods in question.

Table 4.1 – Implied volatility and realized volatility data.

Ticker	Observations (days)	Period	Average *IV*	Average *RV*	Average Difference[1]
GSPC	5217	1/2000 – 9/2020	19.88	16.53	3.35 *
NDX	4943	2/2000 – 9/2020	24.71	21.51	3.20 *
RUT	4211	1/2004 – 9/2020	24.10	21.03	3.07 *
AMZN	2600	6/2010 – 9/2020	33.21	29.05	4.16 *
AAPL	2600	6/2010 – 9/2020	29.48	25.38	4.10 *
IBM	2600	6/2010 – 9/2020	22.87	19.86	3.01 *
GS	2600	6/2010 – 9/2020	29.25	25.50	3.75 *
GOOGL	2600	6/2010 – 9/2020	25.85	23.35	2.50 *
FXE	3248	11/2007 – 9/2020	10.33	8.96	1.37 *
GLD	3102	6/2008 – 9/2020	19.16	16.14	3.02 *
EWZ	2400	3/2011 – 9/2020	34.58	39.01	- 4.43 *

Note: “*” denotes significance at the 1% significance level.

[1] We test the statistical significance of the difference between averages with a Welch Two Sample t-test (Welch, 1938). As a preliminary check, we test for variance homogeneity with a median-centering Fligner-Killen test (Conover, Johnson, and Johnson, 1981), and conclude for non-homogeneity.

The overestimation of volatility in the options markets is confirmed by the statistically significant differences between the average IV and the average RV in almost all assets. Thus, we find evidence to answer our first question and, in line with previous literature, conclude for the significance of a volatility risk premium. Interestingly, in our sample, the *VRP* is negative only in the case of an emerging market, the Brazilian equity market, which is subject to higher levels of volatility. Indeed, Brazil has been subject to political instability over the years, combined with feeble fiscal and monetary policies, which produce the perfect environment for higher than expected levels of volatility, i.e. implied volatility underestimating actual volatility. This persistence of elevated volatility in emerging markets is well documented in previous literature (e.g., de Santis, 1997; Aggarwal et. al, 1999).

Another interesting fact is that FXE presents the lowest *IV* and *RV*. These low volatilities not only reflect the asset considered (an *ETF* tracking a foreign exchange pair, the Euro/US Dollar), but also the period in question (2007-2020) which includes one of the longest quantitative-easing runs in history, adopted by both the Federal Reserve and the European Central Bank, thus providing strong currency stability.

Similarly, GLD, the *ETF* for gold, also presents one of the lowest average *RV*, confirming one of the main functions of gold – a financial asset that can serve as a store of value. Despite this, the GLD options market does not seem to accommodate the lower *RV* environment as well as in the case of FXE, with the average *IV* being reasonably higher than the average realized volatility.

Moreover, GSPC also presents a low average *RV* relative to the other assets. This index is the largest in our sample, including some of the most dominant and profitable companies in America. Similar to GLD, the average *IV* of GSPC is comfortably above the average *RV*, so the premium is still predominant in this market.

Finally, the case of IBM is also worth mentioning. Although belonging to the same sector of Apple, Alphabet, or Amazon, IBM shows a lower average *RV* for the same period as the others. Figures 4.1, 4.2, and 4.3 show the *IV*, *RV* and the above-described dynamics for GSPC, GLD, and AAPL. Further visualization of the behaviors of *IV* and *RV* for the remaining assets is shown in Annex A.

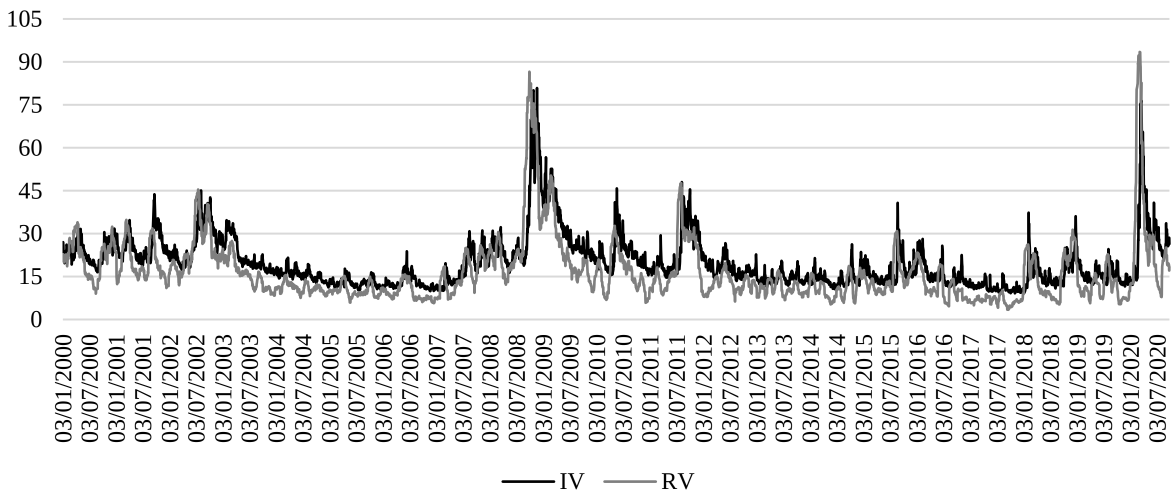

Figure 4.1 - IV and RV for GSPC

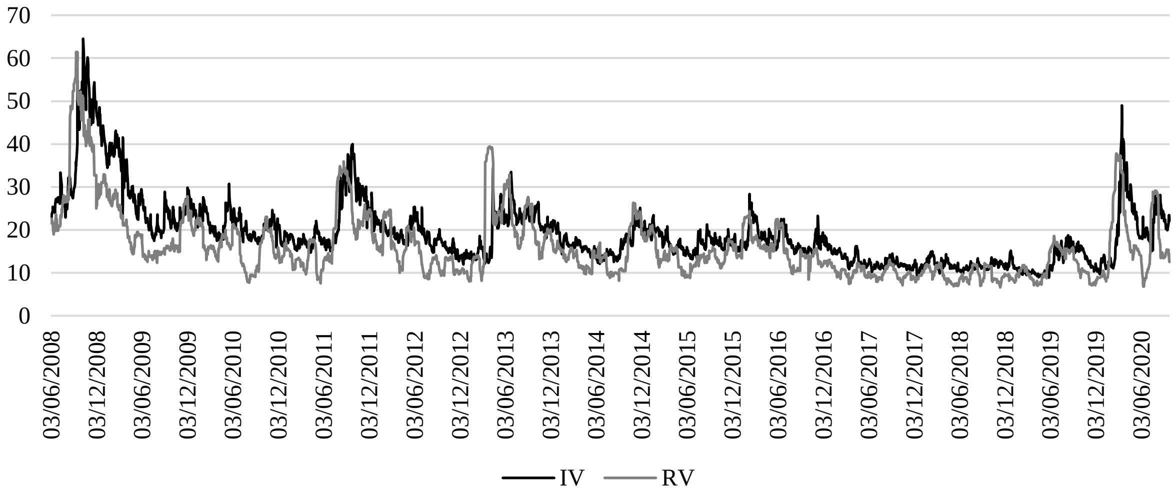

Figure 4.2 - IV and RV for GLD

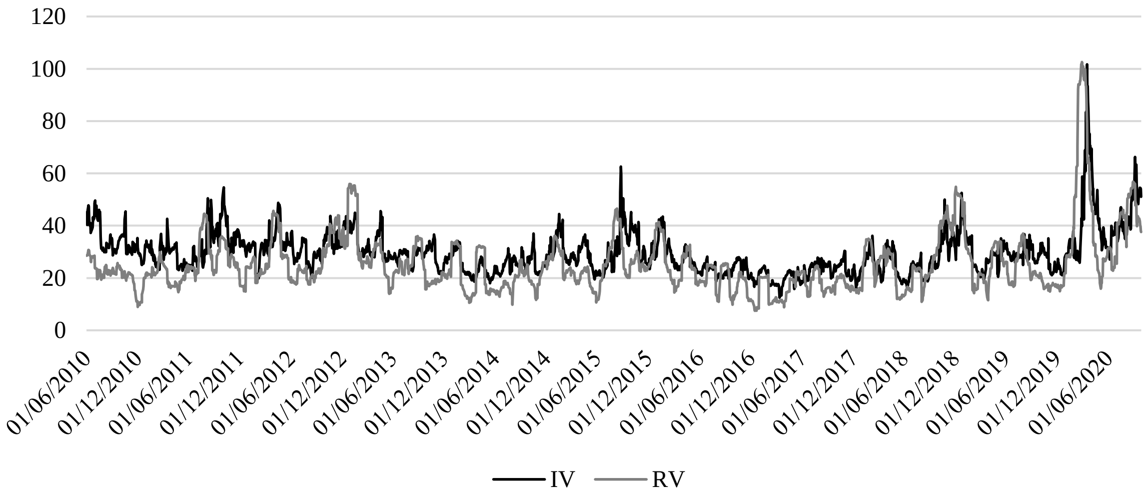

Figure 4.3 - IV and RV for AAPL

In terms of the actual premium, more interesting aspects can be observed. Table 4.2 presents the descriptive statistics for the *VRP* of all assets. From this table, it is possible to make further context on the behavior of the *VRP*.

Table 4.2 – Volatility risk premium descriptive statistics.

Ticker	**First Obs.**	**Min.**	**Max.**	**Median**	**Mean**
GSPC	1/2000	-20.775	12.001	4.005	3.280
NDX	2/2000	-22.358	14.410	3.998	3.447
RUT	1/2004	-25.424	12.150	4.078	3.137
AMZN	6/2010	-11.818	13.677	4.362	3.870
AAPL	6/2010	-17.613	13.812	4.643	3.913
IBM	6/2010	-22.870	8.676	3.920	2.848
GS	6/2010	-23.972	12.313	4.120	3.505
GOOGL	6/2010	-16.383	7.457	4.349	2.328
FXE	11/2007	-2.108	4.693	1.414	1.481
GLD	6/2008	-6.544	11.902	3.315	2.976
EWZ	3/2011	-65.562	7.633	-2.415	-4.446

As seen before, GSPC and GLD, despite having some of the lowest average realized volatilities, also present a similar average *VRP* to the other assets (3.28% and 2.976% respectively). This confirms the perception provided by Table 4.1 that the options market in these assets did not accommodate, proportionally, the low volatility environments.

Some consistencies found in the data were expected. For instance, all three major American indices present similar average premiums, with the Nasdaq-100 edging higher ever-so-slightly, perhaps from the raging bull tech market that has been witnessed for the last two decades. This explanation is supported when we consider that the four technological companies (Amazon, Apple, Alphabet, and IBM) analyzed show very different *VRP* averages (3.87%, 3.912%, 2.328%, and 2.848% respectively).

We highlight that the periods analyzed either contain both the 2008 financial crisis crash and the 2020 Covid-19 crash (cases of GSPC, NDX, RUT, FXE, and GLD), or contain only the latter (AMZN, AAPL, IBM, GS, GOOGL, and EWZ). Therefore, we may attribute the cause of the large negative minimum observed *VRP* values to these extreme events, in which massive realized volatility spikes were observed. Notably, the visual outliers are also expected to behave as such. Namely, EWZ, which contains only the 2020 crash in its data, has a disproportionate negative minimum *VRP* when compared to the other assets, as Brazil was one of the more severely affected countries by the pandemic, thus creating bigger shocks in its equity market. FXE and GLD are in the opposite spectrum, showing a rather small minimum observed *VRP*, which is in line with our previous argument of both the macro-economic policies influencing FXE and the store of value characteristics present in GLD.

At this stage we can answer our second question – "Is the VRP higher in individual equities than in indices?". From Table 4.2, we can see that the three broad indices (GSPC, NDX, RUT) present lower averages for the *VRP* than Amazon, Apple, and Goldman Sachs. Despite this, Alphabet, and IBM present lower average *VRP*'s, which does not allow a conclusive answer to this question. Having that said and considering that previous literature (Bakshi and Kapadia, 2003) points to a bigger premium in individual equities than in broad indices, we may assume that Alphabet and IBM are outliers in this logic.

Figures 4.4, 4.5, and 4.6 show the *VRP* time series plot for GSPC, GLD, and AAPL. From these plots, it is particularly easy to detect the extreme expansions in realized volatility associated with market crashes as we explained before, such as the 2020 Covid-19 crash or the 2008 financial crisis crash. The rest of the assets' VRP plots are shown in Annex B for further context.

Furthermore, to assess the statistical difference between the different *VRP* averages we have just analyzed, we perform a Welch Two Sample t-test (Welch, 1938). Additionally, we run a preliminary median-centering Fligner-Killen test (Conover, Johnson, and Johnson, 1981) to check for variance non-homogeneity. Table 4.3 shows the results for the Welch Two Sample t-test.

Table 4.3 - Welch's Two Sample t-test results

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**
GSPC (1)	-										
NDX (2)	-15.82 *	-									
RUT (3)	-4.20 *	14.45 *	-								
AMZN (4)	57.32 *	46.07 *	57.10 *	-							
AAPL (5)	62.31 *	52.76 *	62.06 *	10.5 *	-						
IBM (6)	29.90 *	13.93 *	29.31 *	35.30 *	43.13 *	-					
GS (7)	1.83	0.07	1.57	10.49 *	9.17 *	-2.08 *	-				
GOOGL (8)	-4.99 *	-6.93 *	-5.28 *	18.55 *	53.0 *	-9.31 *	-4.69 *	-			
FXE (9)	-142.67 *	-105.83 *	-183.12 *	101.91 *	100.2 *	105.56 *	11.36 *	5.69 *	-		
GLD (10)	19.45 *	5.22 *	18.34 *	40.7 *	48.18 *	7.87 *	-0.74	-7.81 *	94.06 *	-	
EWZ (11)	-174.86 *	-167.46 *	-180.91 *	160.58 *	155.47 *	168.23 *	32.64 *	29.53 *	123.46 *	161.19 *	-

Note: “*” denotes significance at the 1% significance level.

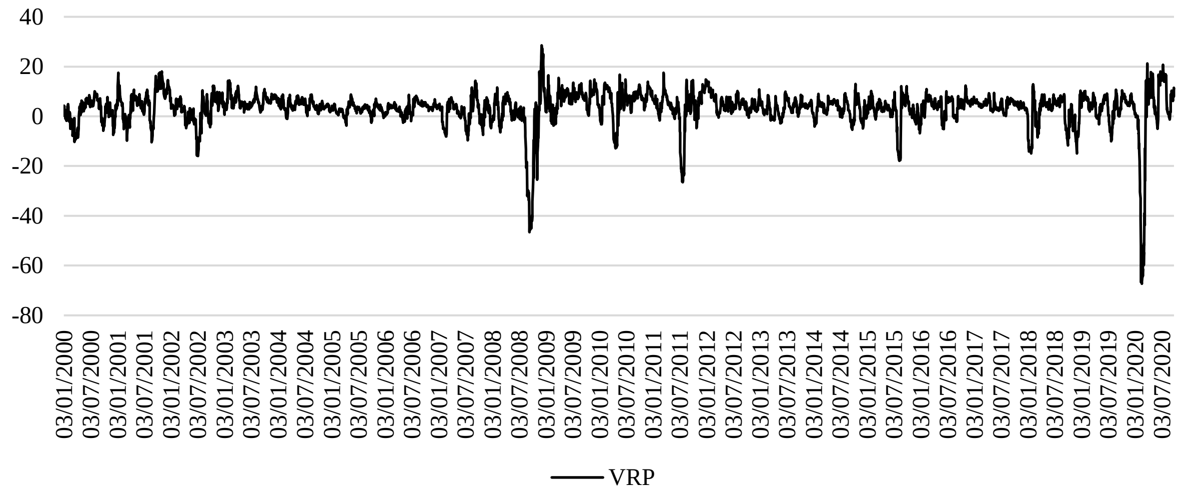

Figure 4.4 – VRP time series plot for GSPC

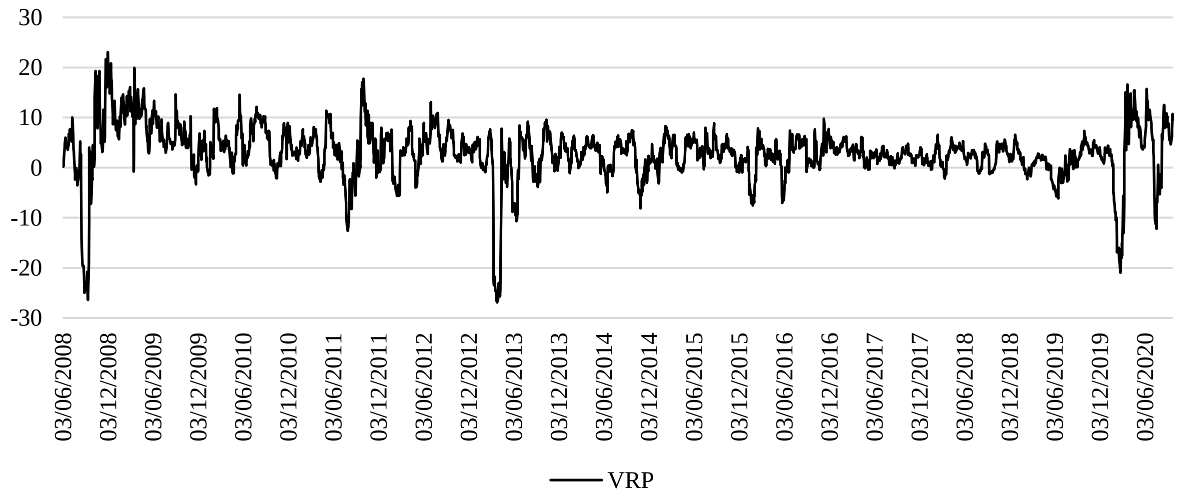

Figure 4.5 – VRP time series plot for GLD

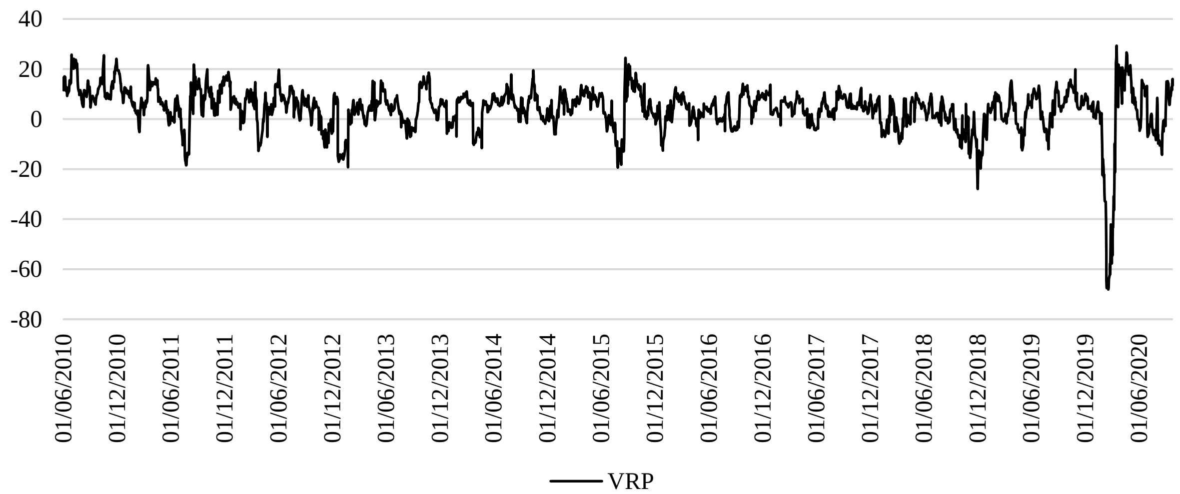

Figure 4.6 – VRP time series plot for AAPL

4.2 STATIONARITY IN THE *VRP*

To answer our third question, we test for stationarity in the volatility risk premium time series. As stated in Section 3.3.1, we use the ADF test, applied by estimating the model in Equation 3, which incorporates a drift. The inclusion of the drift is made after visualizing the time series plot of the *VRP* that reveals a random trend, though presenting a clear non-zero intercept. Furthermore, we set the maximum number of lags for the endogenous variable to 20 and use the BIC to select the number of lags for each regression, as mentioned in Section 3.3.1. Table 4.4 presents the results of the ADF test for all assets.

Table 4.4 – Descriptive Statistics for ADF and KPSS tests.

Ticker	ADF t-stat
GSPC	-12.653 *
NDX	-12.448 *
RUT	-10.642 *
AMZN	-8.529 *
AAPL	-8.031 *
IBM	-7.634 *
GS	-10.695 *
GOOGL	-7.733 *
FXE	-10.281 *
GLD	-8.446 *
EWZ	-6.198 *

Note: "*" denotes a rejection of the null hypothesis at the 1% significance level.

A stationary time series means the series maintains its properties throughout time, i.e., its mean and variance are constant. This is relevant because we are trying to better understand the *VRP*, so it is important that its time series is easily analyzed, which is the case of a stationary time series.

From Table 4.4, we can see that for all assets, the null hypothesis (non-stationarity) is rejected with a 1% significance level. Furthermore, these results are in line with the preliminary visual analysis we obtain when plotting the *VRP* time series.

Based on this, we can present empirical results for modelling the *VRP* in the following Section, under the assumption that the volatility risk premium time series maintains a constant mean and variance through time, i.e., it is stationary.

4.3 SEASONALITY TESTING AND MODELLING

Before looking at the results of this section, we first clarify the precautionary measures taken to ensure the estimators obtained with OLS for each regression are reliable for interpretation.

After estimating the regressions, we verify the necessary assumptions for using OLS, namely the normality of the errors, the absence of multicollinearity, and spherical error variance. Upon checking the regressions, we find that most show, for all assets, problems of heteroskedasticity and/or autocorrelation in the errors. We correct these problems through the Newey-West (Newey and West, 1987) method which provides heteroskedasticity and autocorrelation consistent estimators (HAC) for the covariance matrix of the regression parameters.

4.3.1 Multiple Linear Regression Model

As mentioned in Section 3.3.2, this first regression aims to conclude about possible quarterly seasonal patterns, as well as to check whether the *VRP* has marginal effects from both underlying returns and realized volatilities. To analyze this section, we examine each of the previous focus points of the regressions individually and consecutively. Table 4.5 contains the daily Multiple Linear Regression (MLR) model summaries.

Table 4.5 – MLR Daily Frequency Model Summaries.

Explanatory Variables	Estimates										
	GSPC	NDX	RUT	AMZN	AAPL	IBM	GS	GOOGL	FXE	GLD	EWZ
Intercept	0.039 ***	2.928 ***	4.627 ***	-0.037	0.027 **	0.668	1.13	-0.845	-0.416	-0.087	-0.127
	(0.008)	(0.964)	(0.798)	(0.0231)	(0.013)	(1.67)	(1.478)	(2.051)	(0.339)	(0.287)	(1.754)
D_{3t}	-0.016	-1.769	-0.978	0.027 *	-0.015	1.149	0.794	-1.261	-0.069	-1.046 ***	-0.906
	(0.01)	(1.136)	(1.09)	(0.015)	(0.011)	(0.919)	(1.241)	(1.405)	(0.283)	(0.252)	(1.879)
D_{2t}	-0.005	-0.920	-0.365	0.008	0.000	0.948	0.901	-1.46	-0.039	-0.505 **	-1.043
	(0.007)	(0.953)	(0.732)	(0.014)	(0.01)	(0.88)	(1.285)	(1.167)	(0.275)	(0.22)	(1.927)
D_{1t}	-0.02	-1.986	-2.05	-0.015	-0.039 **	-2.193	-3.765	-1.356	-0.246	-1.27 ***	-7.74 *
	(0.012)	(1.484)	(1.36)	(0.017)	(0.015)	(1.681)	(2.433)	(1.695)	(0.326)	(0.236)	(4.64)
R_t	0.089	75.022 ***	42.385 **	-0.086	-0.032	35.603	-6.864	-32.971 *	-13.982	1.489	40.370
	(0.136)	(12.588)	(17.397)	(0.109)	(0.142)	(22.243)	(20.784)	(18.841)	(8.939)	(11.243)	(37.085)
σ_t	0.394	103.877	-53.893	3.743 ***	1.643 **	171.85	187.191 **	277.787 **	324.972 ***	370.311 ***	-99.893
	(0.885)	(70.897)	(64.565)	(1.143)	(0.824)	(116.352)	(78.471)	(118.168)	(61.763)	(26.926)	(63.54)
Adjusted R^2	0.013	0.039	0.018	0.055	0.036	0.04	0.051	0.02	0.101	0.105	0.032

Note: The values between parentheses represent the standard error of the estimated parameter. ***, **, * denote, respectively, significance at the 1%, 5% and 10% significance levels. Both returns and realized volatilities are computed with a daily frequency.

Regarding seasonal quarterly patterns, the results show mixed evidence depending on the asset in question. For instance, none of the major stock indices display clear signs of quarterly seasonality in the *VRP*, with no dummy variable displaying statistical significance. The individual equities display similar results for seasonality, which makes sense, considering that individual equities belong to the previously mentioned indices. In this case, only Amazon shows statistical significance at a 10% significance level for seasonality in Q3, and Apple displays statistical significance at a 5% significance level for seasonality in Q1. Therefore, it may be prudent to assume that this is a result of in-sample characteristics and that in general there is no quarterly seasonality for these individual equities. As for the ETF's, both FXE and EWZ present similar results to previous assets, with the latter showing statistical significance for Q1 at a 90% significance level. However, contrary to the overall trend, GLD stands out from the sample, presenting statistical significance for all dummy variables (i.e., Q1, Q2, and Q3) ranging from 5% to 1% significance levels, thus presenting as a highly cyclical asset in *VRP* terms.

As for the influence of the underlying asset daily returns on *VRP*, besides Alphabet, which shows a statistical significance for this estimator with a 10% significance level, only the Russel 1000 and the Nasdaq-100 show statistical significance at a 5% and 1% significance levels, respectively. In the case of the two indices, the coefficient estimates are both positive (75.022 for NDX and 42.385 for the RUT), while in the case of Alphabet the results show a negative value (-32.971). The former result (cases of Russel 1000 and Nasdaq-100) is expected. Since volatility is asymmetric, positive returns tend to be related with lower levels of volatility. Also, considering that realized volatility changes immediately as returns evolve while implied volatility requires investors' expectations to change and adapt, positive returns would, in turn, result in a higher *VRP*.

However, the negative coefficient estimate in the case of Alphabet does not confirm this intuition. One potential explanation for this result is that Alphabet's investors react quickly and in excess to changes in returns of this asset, i.e., when returns are positive, investors are quick to adapt their volatility expectations, thus resulting in a rapid, and excessive reduction of *IV* and consequently lower *VRP* (since the reduction in *IV* outpaces the reduction in *RV*). However, there is no reason for investors of Alphabet reacting differently to underlying asset returns when compared to investors in other assets. Another possibility may be linked to investors expecting, *a priori*, predominantly positive returns from Alphabet. If we consider that Alphabet has been a fairly stable and dominant company for almost 2 decades now, it may be fair to say that investors expect, in advance, lower levels of volatility, thus causing a negative relation between asset returns and *VRP*. In other words, if investors expect positive returns and thus low volatility, increasingly positive returns would only intensify this behavior by reassuring them of their bias. Furthermore, if we recall the data presented in Table 4.1, Alphabet presents one of the lowest average *IV* for the last decade, as well as showing the lowest average *VRP* of all individual equities analyzed, which corroborates the idea that investors don't expect high levels of volatility from Alphabet[2].

Finally, as it relates to the marginal effect of realized volatility on the *VRP*, the results are more similar across all assets than the previous variables. AMZN, AAPL, GS, GOOGL, FXE, and GLD present statistical significance for the estimator of the coefficient of *RV*. Furthermore, the positive coefficients provide us with some insight into how responsive investors are in their expectations of future volatility (*IV*) when confronted with an increase in realized volatility. Since the parameter estimates are all positive, this means that when realized volatility increases, investors quickly increase their volatility expectations. And not only do they adjust their expectations, but they adjust with an excess margin, considering that the *VRP* increases, thus maintaining the systematic volatility premium in the market.

As already mentioned, our regressions also consider a quarterly frequency, to allow for possible conclusions that may be useful for investors chasing alpha in the markets. As such, we present our results for our MLR model on a quarterly frequency and cross-check these results with daily frequency MLR model. Like Table 4.5, Table 4.6 contains the model summaries for the quarterly MLR model.

[2] Alphabet's Beta (5Y Monthly) of 1.00 is the lowest of all stocks comprised in our research, thus confirming the idea of a low volatility asset. As a comparison, Goldman Sachs, Apple, and IBM present Betas (5Y Monthly) of 1.5, 1.20, and 1.23, respectively (Source: Yahoo Finance, 01/05/2021)

Table 4.6 – MLR Quarterly Frequency Model Summaries.

Explanatory Variables	Estimates										
	GSPC	NDX	RUT	AMZN	AAPL	IBM	GS	GOOGL	FXE	GLD	EWZ
Intercept	4.722 **	3.468 *	6.213 ***	11.275 ***	11.844 ***	11.081 ***	10.889 ***	12.056 ***	1.052 *	4.33 *	11.257 ***
	(1.653)	(2.034)	(1.56)	(1.609)	(2.047)	(1.41)	(3.838)	(2.053)	(0.553)	(2.32)	(1.63)
D_{3t}	-0.551	-0.763	-0.957	-0.139	-2.218	-1.431	-0.758	-2.175	-0.312	-1.884	0.787
	(1.205)	(1.28)	(1.506)	(1.653)	(1.647)	(1.264)	(2)	(1.613)	(0.385)	(1.708)	(1.498)
D_{2t}	-0.061	0.065	-0.255	-0.653	-0.108	0.688	0.558	-1.136	-0.232	-1.184	1.052
	(0.816)	(0.999)	(1.082)	(1.752)	(1.622)	(1.267)	(1.841)	(1.692)	(0.417)	(1.609)	(1.49)
D_{1t}	-0.691	-0.623	-1.303	-1.319	-2.121	-3.452 *	-2.174	-0.643	-0.481	-2.333	-3.226 **
	(1.343)	(1.867)	(1.477)	(1.651)	(1.7)	(1.368)	(2.137)	(1.693)	(0.474)	(1.71)	(1.541)
R_t	0.275 ***	0.229 **	0.246 ***	0.095 *	0.129 ***	0.233 **	0.058	0.08	0.074 *	0.161	0.31 ***
	(0.043)	(0.078)	(0.069)	(0.054)	(0.041)	(0.053)	(0.065)	(0.055)	(0.04)	(0.096)	(0.032)
σ_t	-0.077	-0.009	-0.136	-0.256 ***	-0.291 ***	-0.347 **	-0.26	-0.375 ***	0.08	-0.01	-0.453 ***
	(0.109)	(0.09)	(0.088)	(0.062)	(0.069)	(0.055)	(0.166)	(0.068)	(0.058)	(0.166)	(0.039)
Adjusted R^2	0.404	0.261	0.543	0.327	0.541	0.702	0.422	0.445	-0.004	0.041	0.922

Note: The values between parentheses represent the standard error of the estimated parameter. ***, **, * denote, respectively, significance at the 1%, 5% and 10% significance. Both returns and realized volatilities are computed with a 63-trading day period.

We cross-check this information with the one provided by the daily regressions, and in the case of consistencies, conclude about potential marginal effects between variables in the quarterly frequency.

Starting with the possibility of a seasonality pattern, we confirm the results in the daily regression, with not many assets showing statistical significance for the dummy variables. This increases our confidence in the fact that quarterly patterns are not observable in the *VRP*.

As for the marginal effect of the underlying asset returns, the coefficient estimates are predominantly statistically significant, except for GS, GOOGL, and GLD. Most coefficients are positive, reinforcing the impact of underlying asset returns on the *VRP*, where positive returns generally imply lower *RV*, and since *IV* does not react immediately, there is a temporary increase in the *VRP*.

Finally, in terms of underlying realized volatility, the statistical significance is also reasonably predominant. However, we note that the coefficients are all negative in a quarterly frequency, as well as being much lower in absolute terms, contrary to the daily frequency. In the previous analysis, we show that the speed of investors' changes in volatility expectations impacts the sign of this coefficient. Given that, the negative sign in the longer timeframe is acceptable. In the short-term, investors most likely react fast and with an aggressive increase of *IV* to increases in realized volatility. In the long-term, these reactions are smoothed, as reflected by the lower absolute values of the coefficients. Additionally, the sign of the coefficients (negative) might reflect that, as time elapses, investors reverse the short-term exaggerated responses to increases in *RV*, thus making the *VRP* negatively correlated with *RV*.

4.3.2 ARX1 Model

One of our goals is to assess whether there is predictability in the *VRP*, namely if past *VRP* values, underlying asset returns, and realized volatility have any predictive power for a future volatility risk premium. Such conclusion would be of the utmost importance for investors, as it would allow for an efficiently timed implementation of option-selling strategies. If the model predicts a decrease (increase) in the future *VRP*, *ceteris paribus*, short (long) option strategies would be a particularly profitable strategy to employ knowing this in advance. Like in the MLR model section, we first present our daily regression results and then compare them with our quarterly model. Table 4.7 contains the model summary for the daily ARX1 model.

Table 4.7 – ARX1 Daily Frequency Model Summaries.

Explanatory Variables	**Estimates**										
	GSPC	NDX	RUT	AMZN	AAPL	IBM	GS	GOOGL	FXE	GLD	EWZ
Intercept	-0.001	-0.175 *	-0.029	-0.003	-0.001	-0.333	-0.064	-0.267	-0.023	-0.041	-0.623 *
	(0)	(0.095)	(0.118)	(0.003)	(0.003)	(1.647)	(0.136)	(0.249)	(0.04)	(0.086)	(0.369)
$\boldsymbol{VRP_{t-1}}$	0.96 ***	0.962 ***	0.963 ***	0.925 ***	0.941 ***	54.591 ***	0.951 ***	0.944 ***	0.924 ***	0.949 ***	0.981 ***
	(0.003)	(0.007)	(0.016)	(0.011)	(0.013)	(11.455)	(0.006)	(0.014)	(0.007)	(0.013)	(0.023)
$\boldsymbol{R_{t-1}}$	0.237 ***	6.292	1.322	0.11	0.278 ***	29.364 *	20.242 ***	21.704 *	-3.444	1.056	2.056
	(0.022)	(4.384)	(4.301)	(0.092)	(0.091)	(16.976)	(3.308)	(12.927)	(2.244)	(7.39)	(3.683)
$\boldsymbol{\sigma_{t-1}}$	0.139 ***	21.594 ***	10.826 *	0.297 **	0.162	82.218	14.861 **	24.811 *	22.022 ***	19.292 *	27.198
	(0.04)	(8.315)	(5.671)	(0.137)	(0.181)	(134.348)	(7.496)	(14.846)	(6.939)	(10.195)	(18.615)
Adjusted R^2	0.925	0.933	0.926	0.859	0.889	0.382	0.909	0.891	0.867	0.909	0.961

Note: The values between parentheses represent the standard error of the estimated parameter. ***, **, * denote, respectively, significance at the 1%, 5% and 10% significance levels. Both returns and realized volatilities are computed with a daily frequency.

The estimates for the influence of the lagged *VRP* are indicative of a clear effect, in that all assets show statistical significance for the coefficient estimate of this explanatory variable, which allows us to conclude that past values of the *VRP* determine future values. Moreover, all estimated coefficients are positive, indicating that the *VRP* is procyclical and tends to follow its previous trend. Also, we note that except for IBM, all estimates are similar in magnitude, pointing to a very similar relationship between present and past *VRP* values across multiple assets and asset classes.

As for past underlying returns, only GSPC, AAPL, IBM, GS, and GOOGL present statistical significance for this estimator. All coefficients are positive, indicating that positive returns tend to increase the *VRP* in the next day.

Finally, in the case of the underlying asset realized volatility, the results are similar to those of previous *VRP* values, in the sense that most assets show statistical significance for the coefficient estimators, with only AAPL, IBM, and EWZ not showing statistical significance for this variable. Furthermore, all statistically significant coefficients are positive, indicating that increases in volatility lead to subsequent increases on the volatility risk premium.

Yet, the previous results allow for conclusions on a daily horizon only. As such, and since we wish to provide valuable information that can be used by market participants, we extend the timeframe of our regression into a more standardized frequency in the context of generating positive returns – i.e., quarterly frequency, as this is the standard timeframe used to measure returns/performance by companies, investment funds, etc.). Thus, and as in the previous section, we now present the same regressions, but in a quarterly timeframe, which is adaptable into an option selling strategy that would look to short options with maturity around one quarter. Table 4.8 contains the quarterly ARX1 model summaries.

By looking at this table, the viable conclusion we can make at this stage is that there is a marginal effect between the realized volatility of one quarter and the subsequent quarter *VRP*. The estimated coefficients are positive, meaning that an increase of realized volatility in one quarter leads to an increase of the volatility risk premium in the subsequent quarter, and vice versa. Based on these results, an investor can focus on option selling strategies in quarters following increases in volatility from previous quarters and can also avoid strategies in the opposite scenarios (i.e., quarters with decreases in volatility).

Table 4.8 – ARX1 Quarterly Frequency Model Summaries.

Explanatory Variables	**Estimates**										
	GSPC	NDX	RUT	AMZN	AAPL	IBM	GS	GOOGL	FXE	GLD	EWZ
Intercept	0.110	0.267	0.158	-4.218	1.361	-2.961	-0.364	-6.101 *	-0.956 **	-2.774 ***	-18.883 **
	(1.283)	(1.328)	(1.789)	(3.624)	(4.006)	(3.625)	(3.262)	(3.3)	(0.432)	(0.91)	(8.411)
$\boldsymbol{VRP_{t-1}}$	0.082	-0.058	0.011	0.368 *	0.119	0.185	0.066	0.299	0.426 ***	0.116	0.927
	(0.144)	(0.129)	(0.19)	(0.198)	(0.233)	(0.277)	(0.232)	(0.216)	(0.109)	(0.098)	(0.601)
$\boldsymbol{R_{t-1}}$	0.081	-0.003	0.099	-0.009	-0.037	0.08	0.072	0.041	-0.026	0.072	-0.288
	(0.08)	(0.059)	(0.083)	(0.057)	(0.066)	(0.098)	(0.059)	(0.068)	(0.032)	(0.048)	(0.209)
$\boldsymbol{\sigma_{t-1}}$	0.172 ***	0.155 ***	0.131 **	0.22 **	0.187	0.255 *	0.134	0.314 **	0.2 ***	0.341 ***	0.553 *
	(0.059)	(0.05)	(0.064)	(0.103)	(0.128)	(0.143)	(0.099)	(0.12)	(0.045)	(0.048)	(0.319)
Adjusted R^2	0.063	0.113	0.032	0.055	0.018	0.04	0.004	0.109	0.455	0.509	0.01

Note: The values between parentheses represent the standard error of the estimated parameter. ***, **, * denote, respectively, significance at the 1%, 5% and 10% significance level. Both returns and realized volatilities are computed with a 63-trading day period.

4.4 OPTION PRICING

In this section, we look at the results from option pricing using both *IV* and *RV*, which allows us to materialize the volatility risk premium into its monetary consequence in option prices. We divide this section into European-style options and American-style options, which vary depending on the assets in question.

4.4.1 European-style options

As mentioned in Section 3.4.1, for European-style options we use the Black-Scholes-Merton model (1973). The assets in our research that use European-style options are the equity indices, i.e., GSPC, RUT, and NDX. Table 4.9 contains the average option prices for puts and calls in the three assets, using both *IV* and *RV* as volatility inputs for the pricing model. It also presents the average volatility risk premiums, both in absolute and relative terms, which is the most important data here.

Table 4.9 – European-style option prices and VRP.

Ticker	**Average Call *VRP* ($)**	**Average Call *VRP* (%)**	**Average Put *VRP* ($)**	**Average Put *VRP* (%)**
GSPC	5.599	17,480	5.599	18.750
NDX	9.177	12.642	9.177	13.202
RUT	3.166	13.969	3.166	14.723

Notes: The average VRP in dollar terms (for calls and puts) is taken by subtracting the average option price computed with IV by the average option price computed with RV. The average VRP in relative terms (for calls and puts) is taken by dividing the average VRP in dollar terms by the average option price computed with IV.

The results are clear in confirming the presence of a systematic *VRP* in option prices (e.g., GSPC shows an average premium of 5.599$, and the NDX an average of 9.177$). Moreover, this premium seems to be consistently bigger, in relative terms, for puts than for calls across the three equity indices. Furthermore, if we compare the assets, it is visible that the S&P 500 presents a bigger relative volatility risk premium than the Russel 2000 or the Nasdaq 100.

Note that the *VRP* in absolute terms is equal for calls and puts in each asset. This is an expectable phenomenon since the options we price are *ATM* options, in which the typical volatility skew (regardless of being on the call side or the put side) is not noticeable.

Figures 4.7, 4.8, and 4.9 present the time series plot for put options priced with both *IV* and *RV* for each asset.

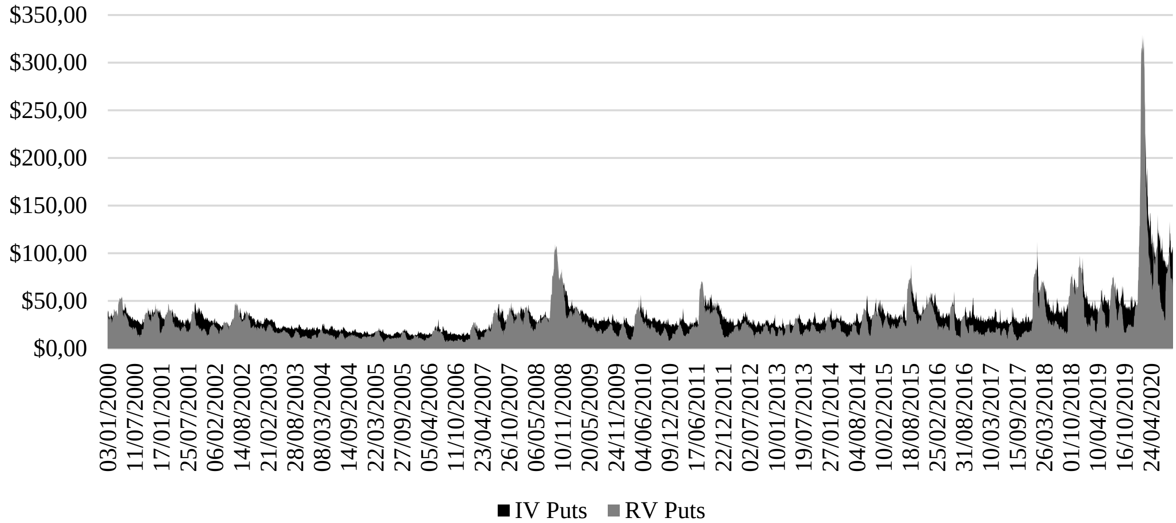

Figure 4.7 – GSPC put options priced with IV and RV time series plot

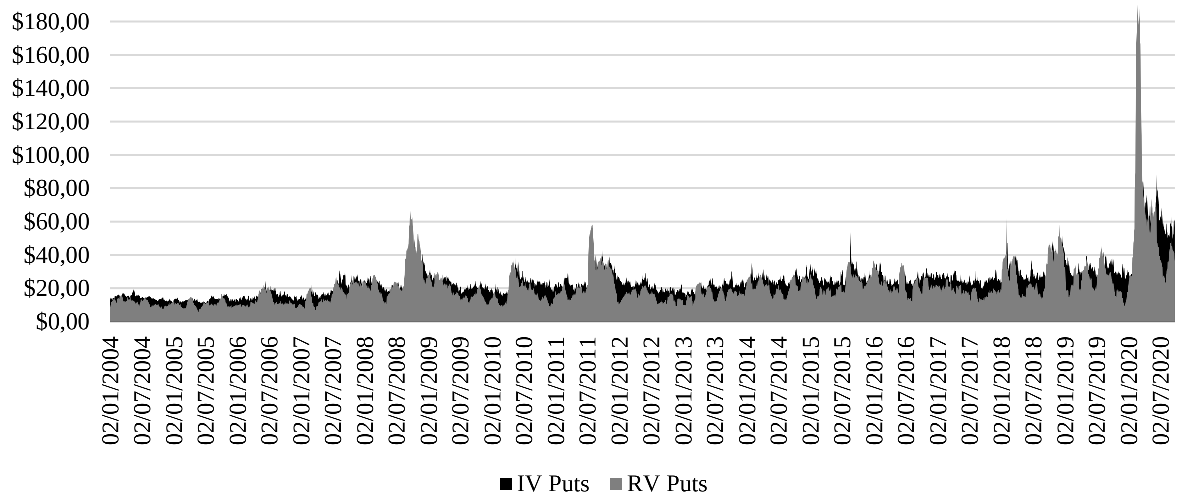

Figure 4.8 – NDX put options priced with IV and RV time series plot

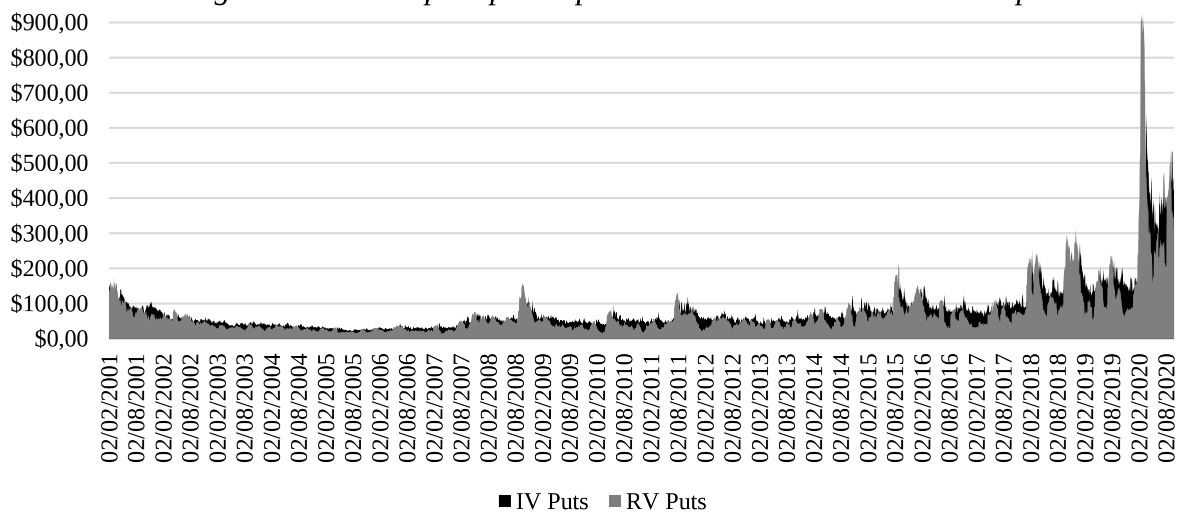

Figure 4.9 – RUT put options priced with IV and RV time series plot

These figures suggest two particularly useful conclusions. First, it reinforces the evidence in Table 4.9, that options are systematically overpriced based on volatility. Second, it is noticeable that after big volatility spikes, such as the 2008 or the 2020 crises, this premium tends not only to increase, but to maintain itself elevated in these particular option markets for a long period (illustrated by the increase in the black area after such moments). The latter aspect offers insights on a very interesting dynamic for investors, since it shows that it is rewarding to sell options under normal circumstances, particularly during and/or after moments of high volatility.

4.4.2 American-style options

For the American-style options, we use the Cox, Ross, and Rubinstein (1979) binomial model, as explained in Section 3.4.2. The assets used in our research that have American-style options are the *ETF's* and individual equities. Like the European-style options section, Table 4.10 contains the average option premiums for puts and calls, as well as the absolute and relative volatility risk premiums.

Table 4.10 – American-style option prices and VRP.

Ticker	**Average Call *VRP* ($)**	**Average Call *VRP* (%)**	**Average Put *VRP* ($)**	**Average Put *VRP* (%)**
AMZN	3.241	12.573	3.24	12.766
AAPL	0.091	14.617	0.091	13.826
IBM	0.44	13.553	0.44	13.756
GS	0.603	12.442	0.603	12.58
GOOGL	1.301	9.141	1.301	9.213
FXE	0.2	12.103	0.2	12.687
GLD	0.414	14.634	0.414	14.858
EWZ	-0.19	-13.308	-0.19	-13.791

Note: The average VRP in dollar terms (for calls and puts) is taken by subtracting the average option price computed with IV by the average option price computed with RV. The average VRP in relative terms (for calls and puts) is taken by dividing the average VRP in dollar terms by the average option price computed with IV.

EWZ confirms what was previously found, of a systematically negative *VRP*. In contrast to the other assets, this shows that on EWZ, it may be profitable to buy options instead of selling options, since the investor will effectively be buying volatility at a discount.

As for the remainder of the assets, the tendency for volatility overpricing is confirmed, with all cases revealing volatility overpricing in the options market. Moreover, the higher relative *VRP* on the puts side demonstrated in the equity indices is also present in these assets, showing consistency with the equity indices. The only outlier in this sense is Apple, which shows a higher relative *VRP* for calls than for puts.

Additionally, when considering the results in Table 4.10, it is noticeable that if we consider the *VRP* in relative terms, assets like Apple, IBM, and GLD would have been particularly interesting for option selling strategies in the samples studied here, since they exhibit the largest average relative *VRP*. Figures 4.10 to 4.17 display put option prices in a time series plot for all individual assets.

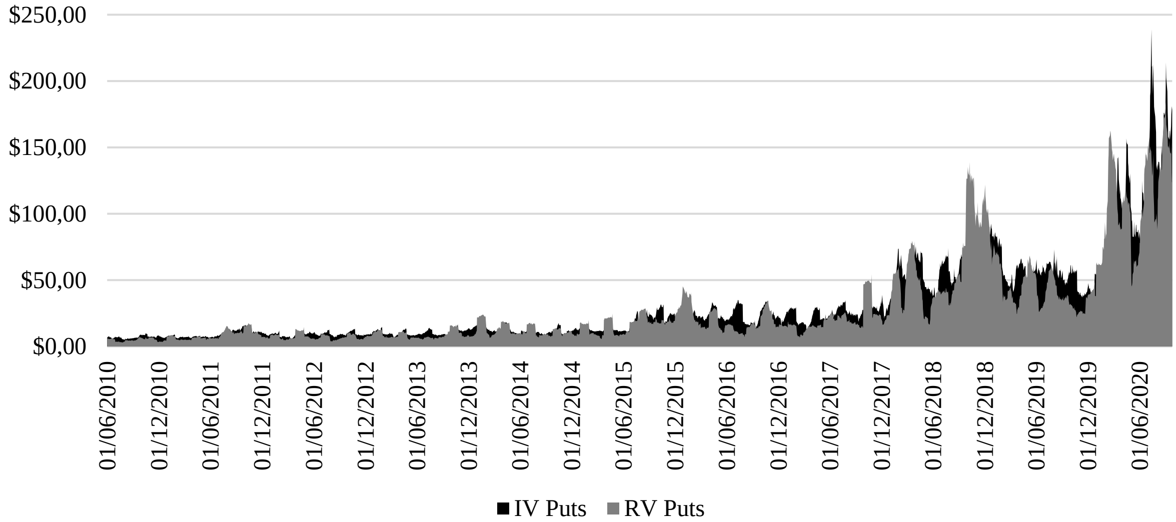

Figure 4.10 – AMZN put options priced with IV and RV time series plot

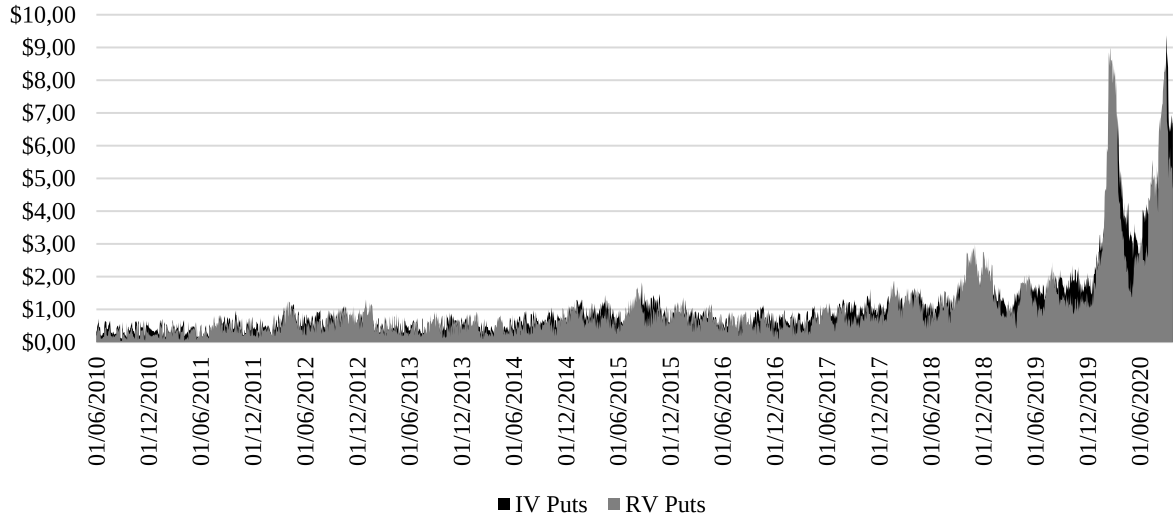

Figure 4.11 – AAPL put options priced with IV and RV time series plot

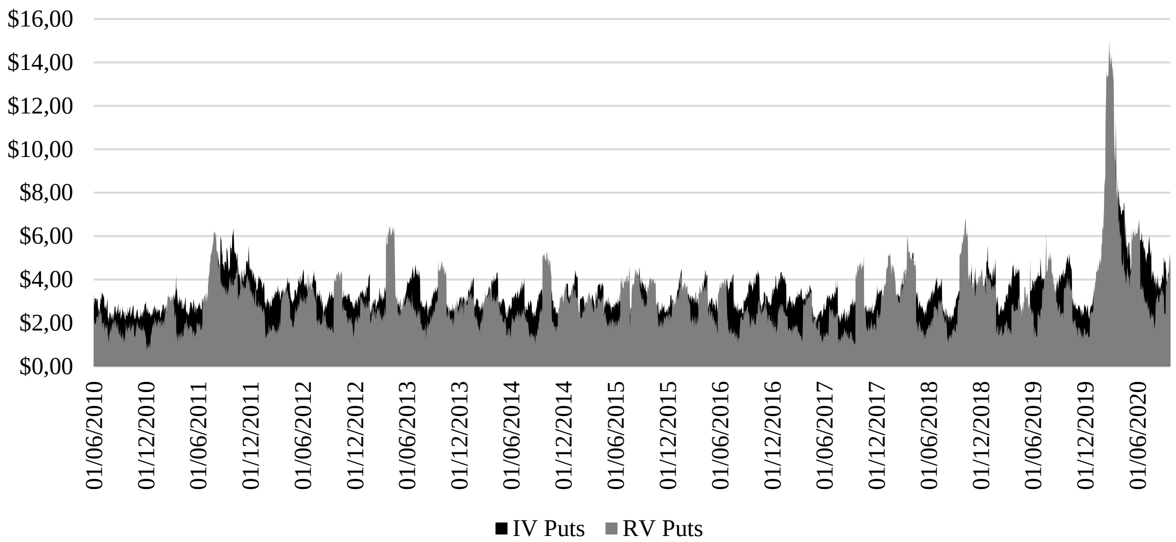

Figure 4.12 – IBM put options priced with IV and RV time series plot

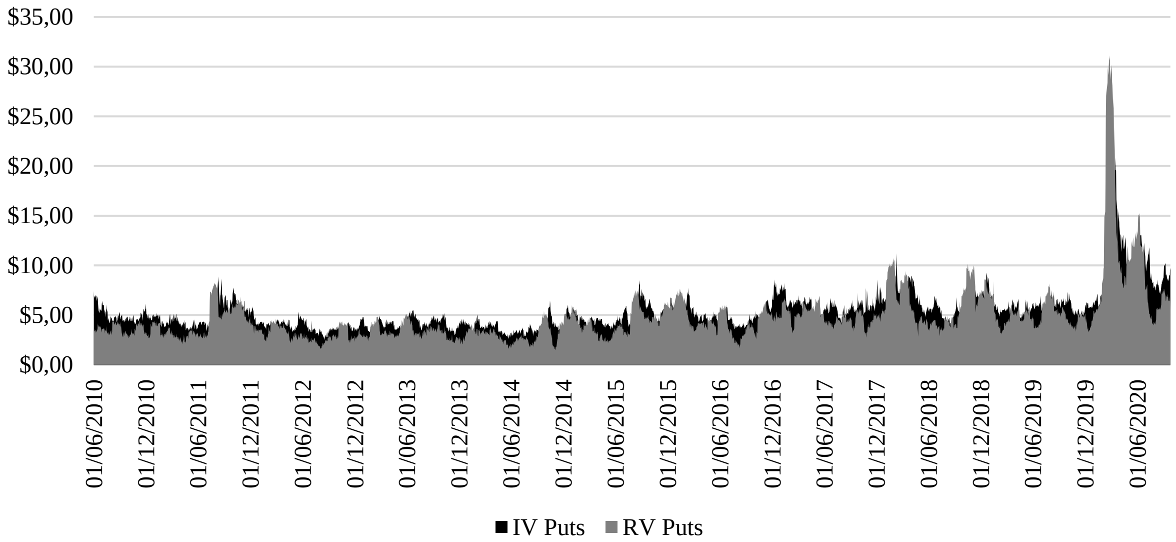

Figure 4.13 – GS put options priced with IV and RV time series plot

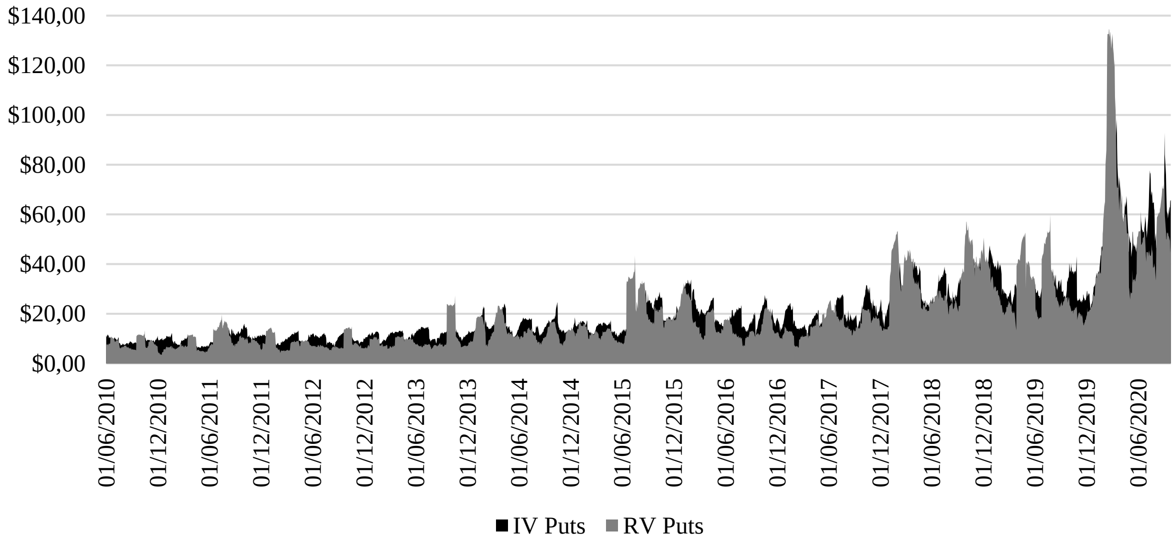

Figure 4.14 – GOOGL put options priced with IV and RV time series plot

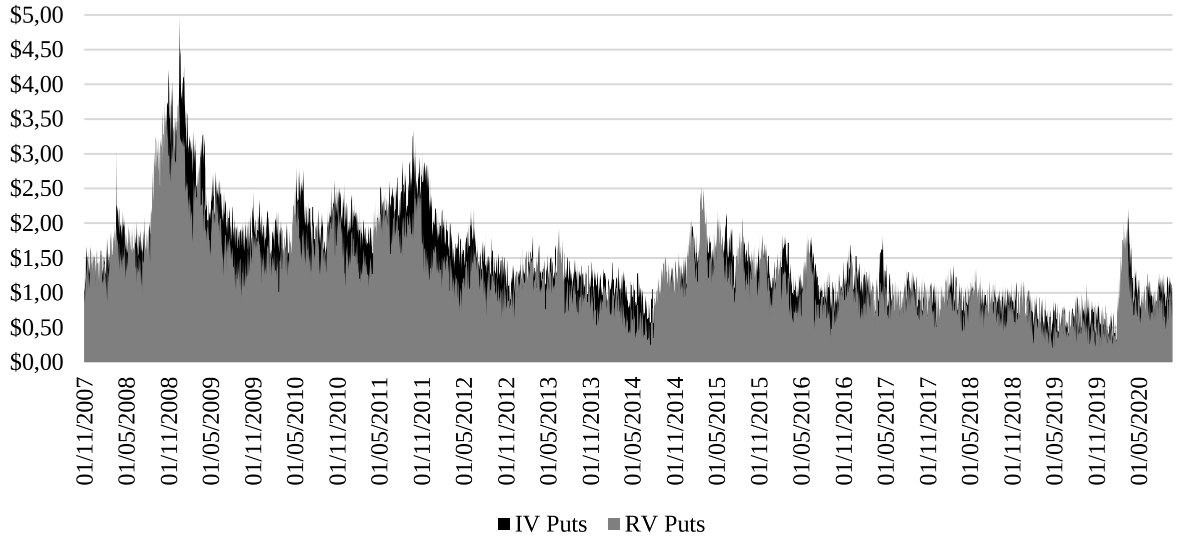

Figure 4.15 – FXE put options priced with IV and RV time series plot

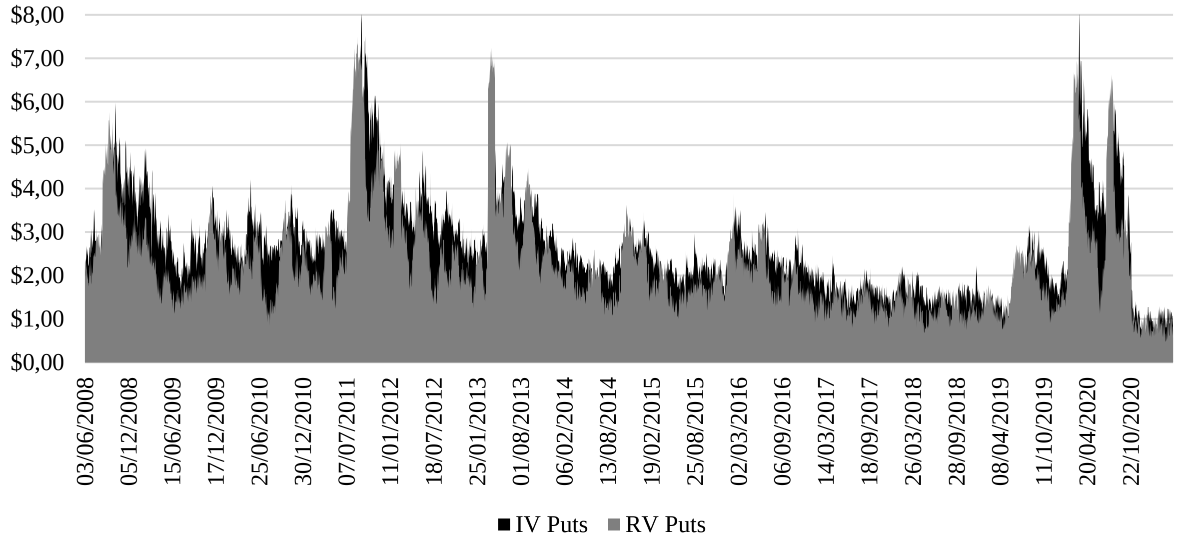

Figure 4.16 – GLD put options priced with IV and RV time series plot

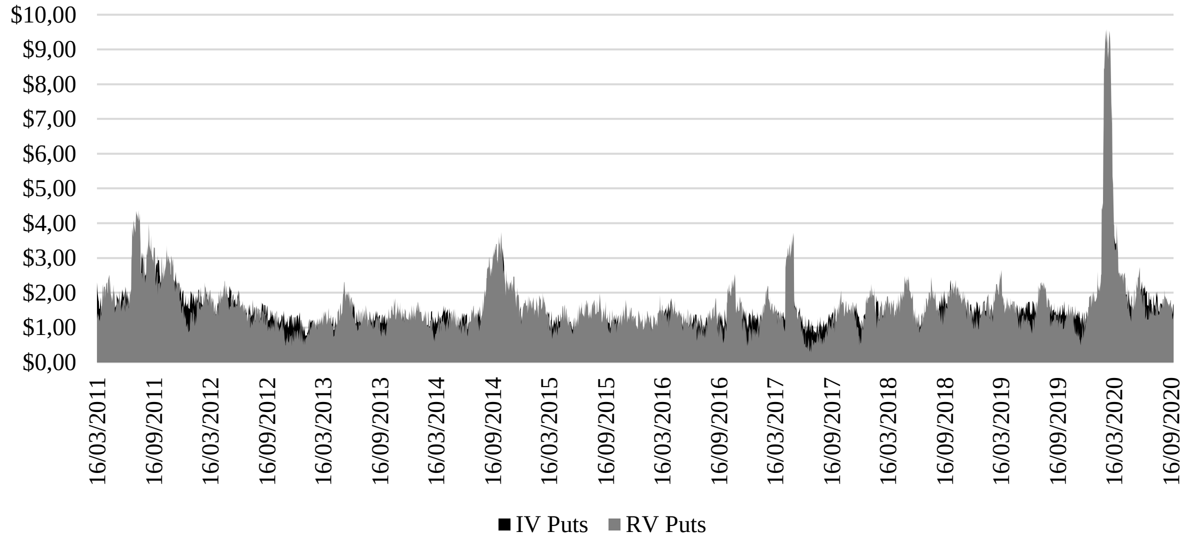

Figure 4.17 – EWZ put options priced with IV and RV time series plot

We can draw further information from the previous figures. For instance, the tech stocks (except IBM) seem to have an increase in the *VRP* from 2015 onwards, which shows a fairly stable volatility risk premium throughout the sample period. GLD and IBM seem to have more exaggerated spikes in implied volatility following realized volatility expansions, indicating that these two assets may be excellent options selling vehicles after volatility increases. Also, some trends are identifiable. GS seems to have been reasonably constant over the sample period, while assets such as AMZN or AAPL show a slight visual increase in the volatility risk premium with time. This may be associated with the fast growth such companies achieved along the observed period. The same about consistency can be found on IBM and FXE, although the latter has seen a decrease in the *VRP* in recent years. EWZ once again confirms the average negative *VRP*.

5 CONCLUDING REMARKS

This thesis does an empirical analysis of the patterns of *VRP* and its determinants. The results suggest some option strategies depending on the *VRP* profile observed relative to the underlying asset.

The empirical findings in this study validate the systematic presence of the *VRP* in the options market. Additionally, these findings confirm the distinguishing magnitudes of the *VRP* when comparing equity indices to individual equities, the latter presenting larger average *VRPs*.

Interestingly, when we price options contracts using either realized or implied volatility, the premium (*VRP*) in relative terms is very similar between both asset classes, with the S&P 500 displaying a greater premium in relative terms to the options prices among all assets studied. Moreover, we find that particularly volatile markets, such as the Brazilian stock market (EWZ), present a negative volatility risk premium, opposite to all other assets.

These findings allow useful conclusions. First, there is a hedge for option sellers in the market, since the volatility implied by option prices is systematically overvalued when compared to realized volatility. Second, when looking at each asset individually, it is possible to conclude that there are assets where this premium is more noticeable. Judging by the option prices obtained, assets such as GSPC, GLD, and RUT present particularly interesting opportunities for option selling strategies, since these show the bigger *VRP* concerning the option prices. In the case of EWZ, buying options seems more attractive, at the very least from a volatility perspective.

Regarding market timing, we find that after moments of high volatility, such as those in market crashes, the *VRPs* tend to increase significantly. This indicates that selling options in/after moments of market downturns and high volatility is more advantageous than selling options in low volatility environments. To human nature, this might seem counterintuitive, since selling options is essentially selling insurance, and insurance loses money when the insured event happens. But the numbers show that this reality is generally compensated in excess and larger proportions after big market downturns.

We reach important statistical findings. First, we find that the *VRP* is heavily affected by realized volatilities of the underlying asset, both on daily (with positive estimated coefficients) and quarterly (with negative estimated coefficients) timeframes. Second, with the results from our ARX1 model, we find that previous *VRP* values (daily frequency) and realized volatility values (daily and quarterly frequency) influence the future *VRP* (both with positive coefficients). This finding is very insightful for investors, as it indicates that timely deployment of option-selling strategies is possible by tracking the behavior of the *VRP*.

Finally, we try to look at our research with a critical sense and identify areas that could be relevant for future research. On the one hand, larger data samples could result in further interesting studies, especially to allow for more robust modelling in the quarterly timeframe. Bigger diversity of assets would also be interesting, as it would allow for more comparisons and insights. On the other hand, additional information about implied volatility would enhance future research. In our case, we use *ATM* implied volatility – although this is still highly useful for our perspective, having implied volatility data on different options contracts, both puts and calls and with different strikes, would open the possibility to study the *VRP* and attest with more detail if there are different behaviors between calls and puts, as well as between different levels of *moneyness* (i.e., different strikes other than *ATM*).

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7 ANNEXES

7.1 ANNEX A – *IV* AND *RV* TIME SERIES PLOTS

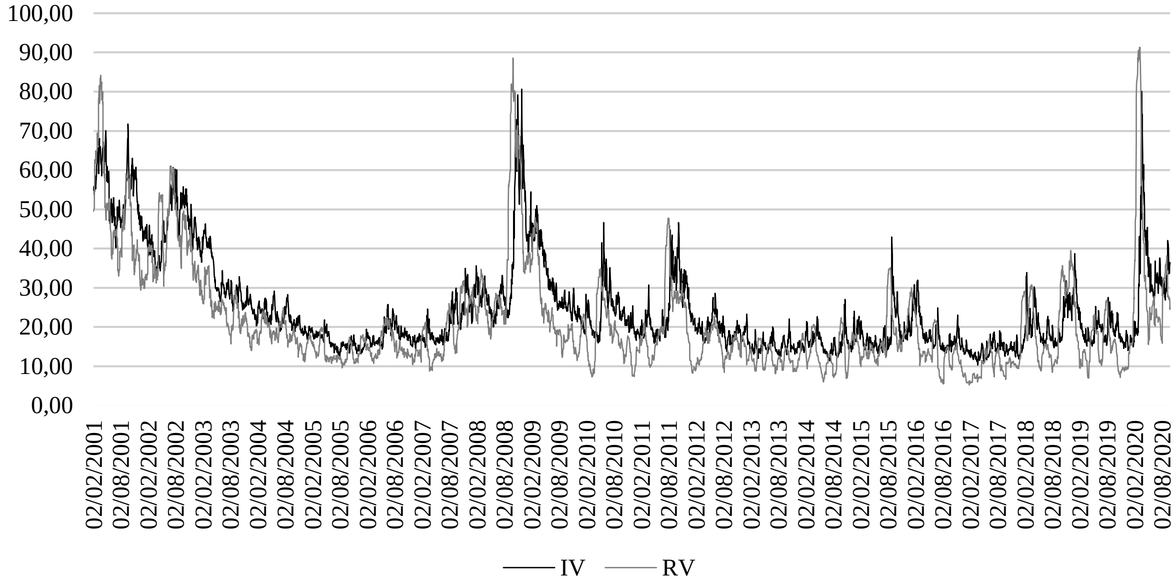

Figure 7.1 – IV and RV for NDX

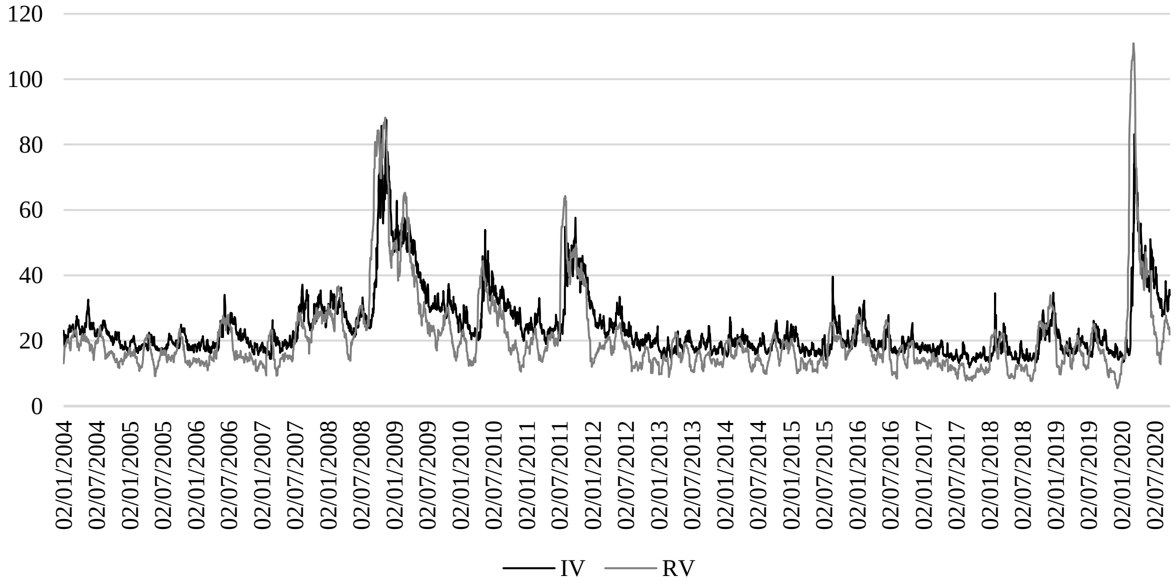

Figure 7.2 – IV and RV for RUT

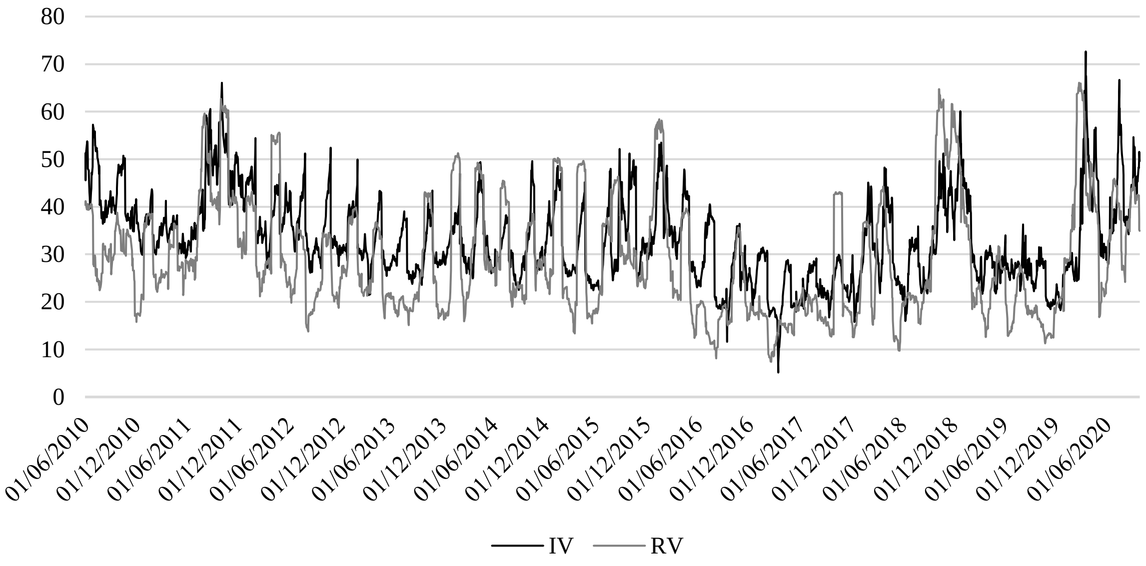

Figure 7.3 – IV and RV for AMZN

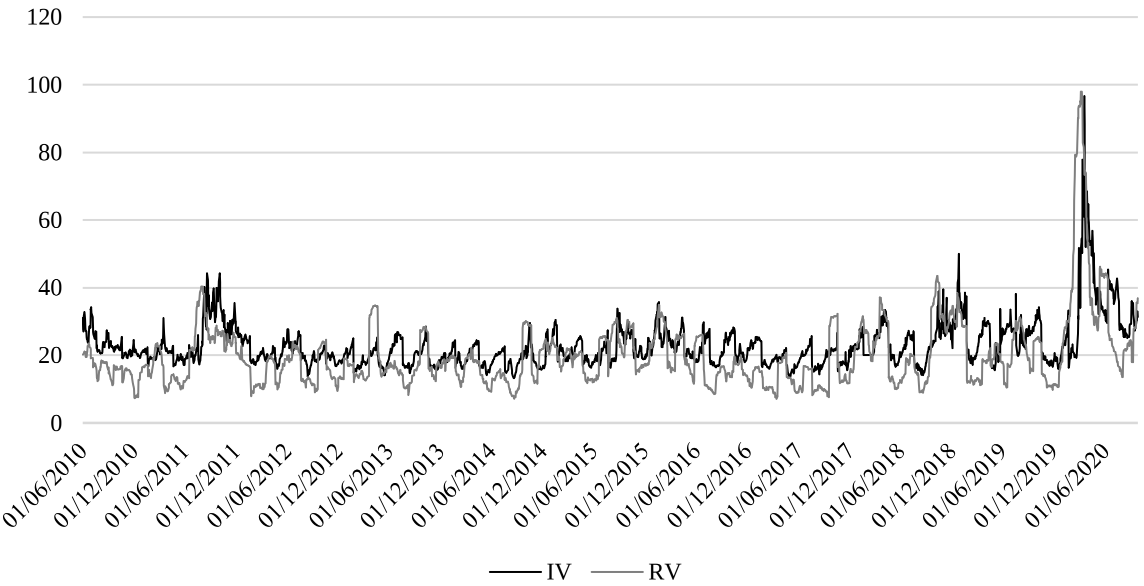

Figure 7.4 – IV and RV for IBM

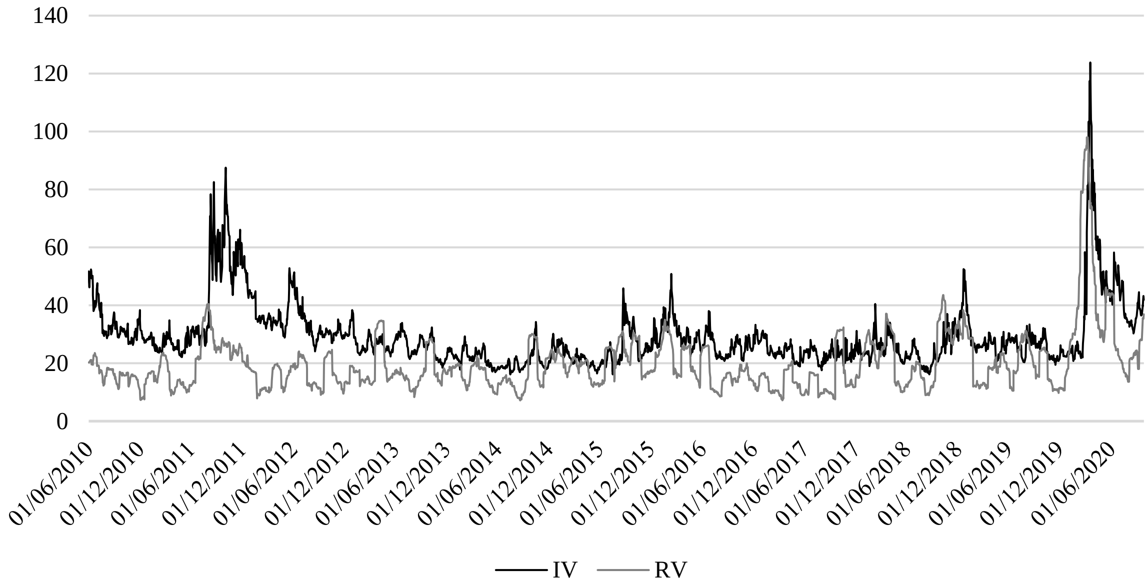

Figure 7.5 – IV and RV for GS

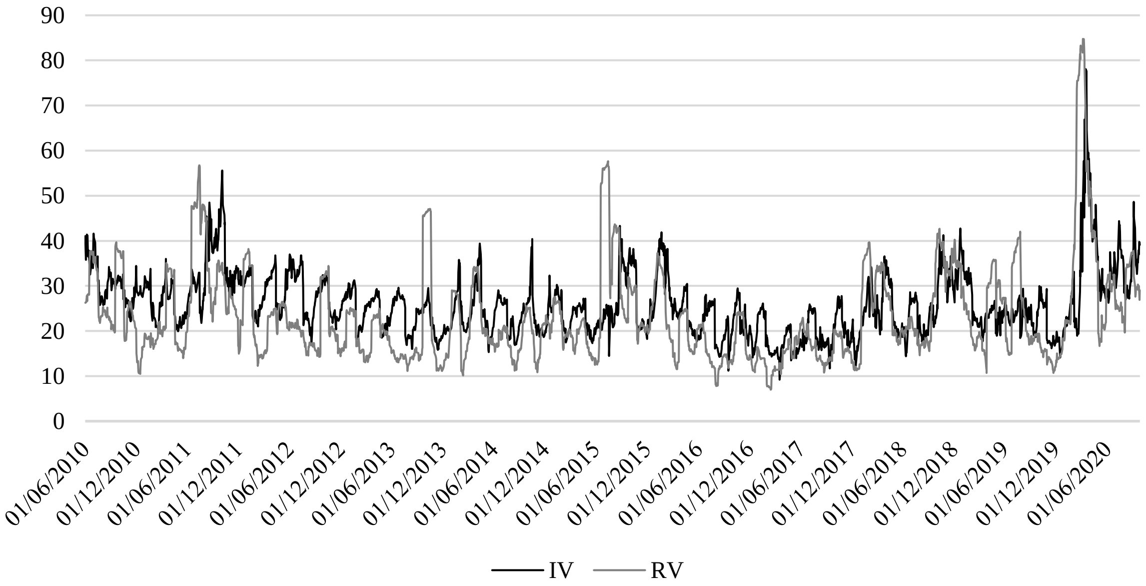

Figure 7.6 – IV and RV for GOOGL

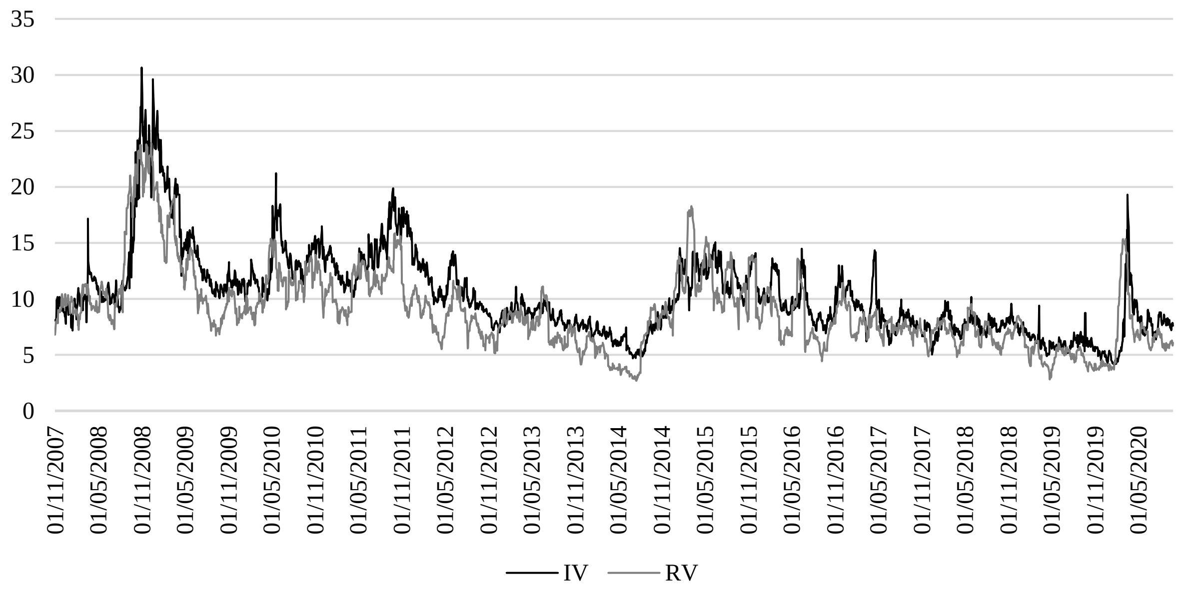

Figure 7.7 – IV and RV for FXE

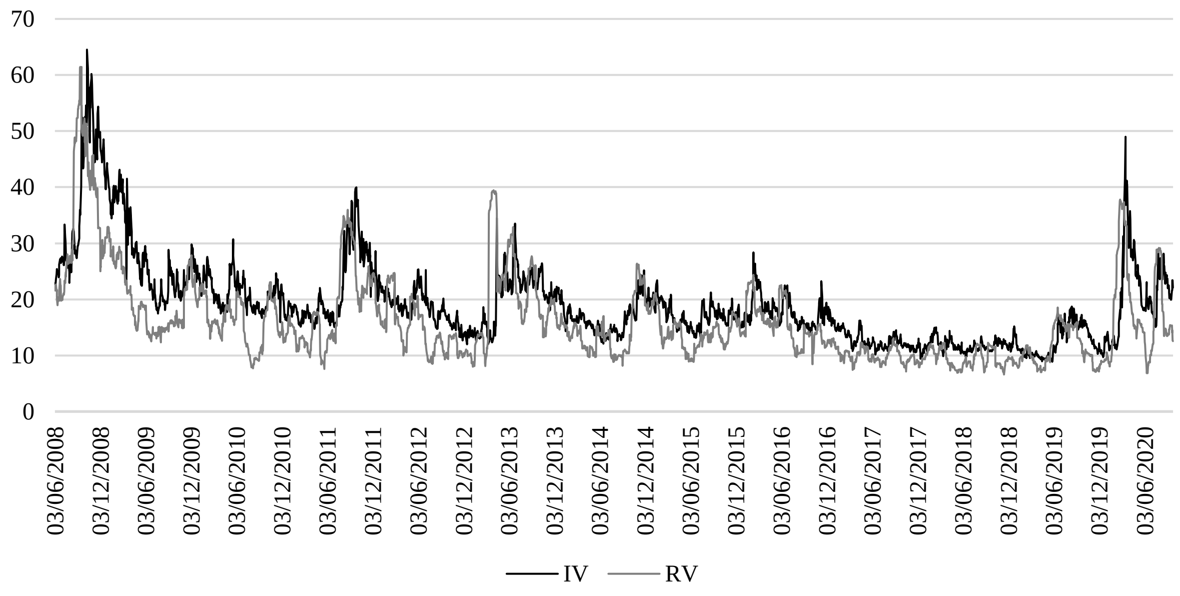

Figure 7.8 – IV and RV for GLD

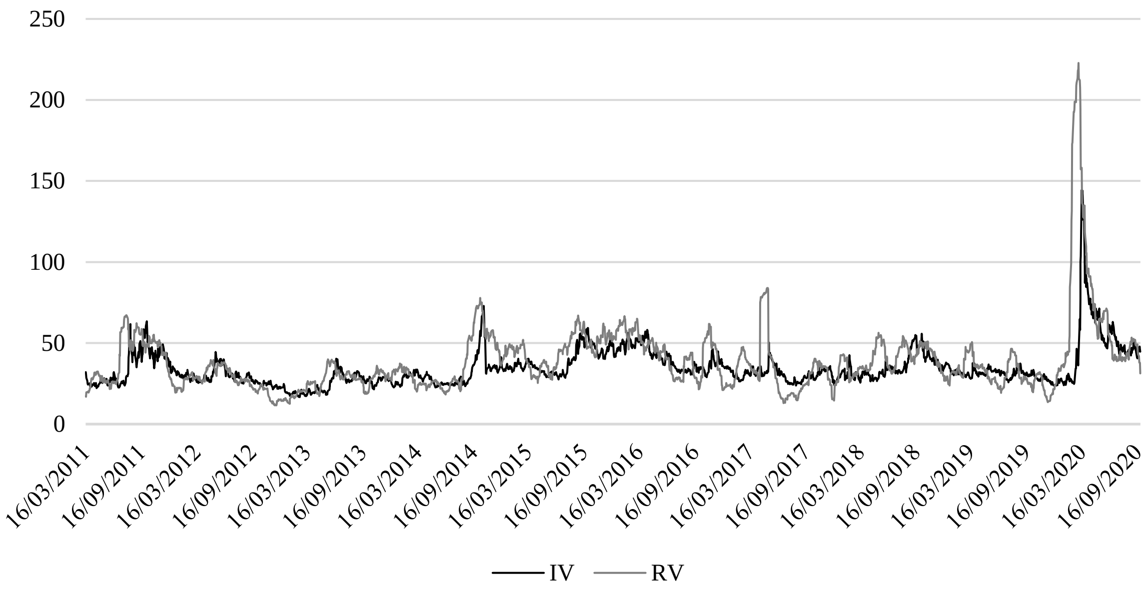

Figure 7.9 – IV and RV for EWZ

7.2 ANNEX B – *VRP* TIME SERIES PLOTS

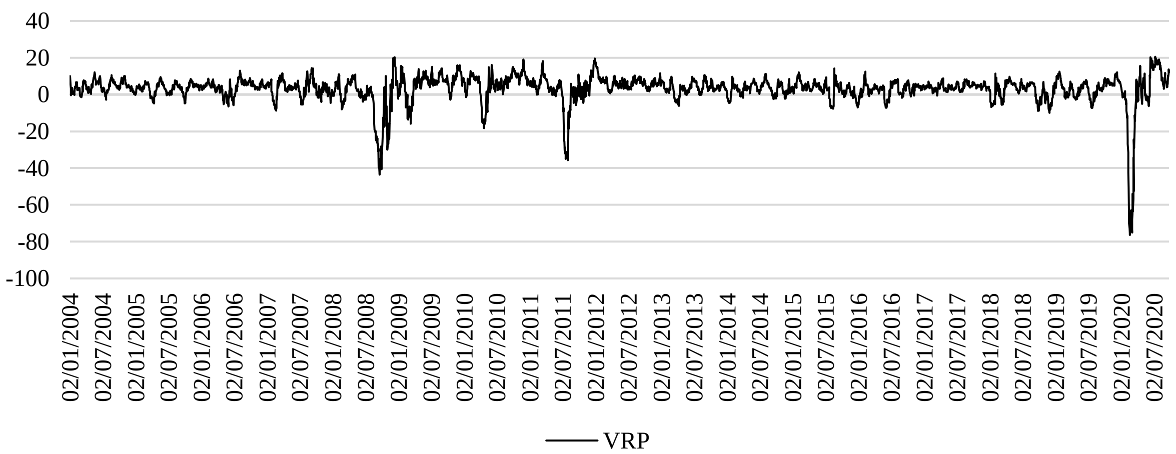

Figure 7.10 – VRP time series plot for RUT

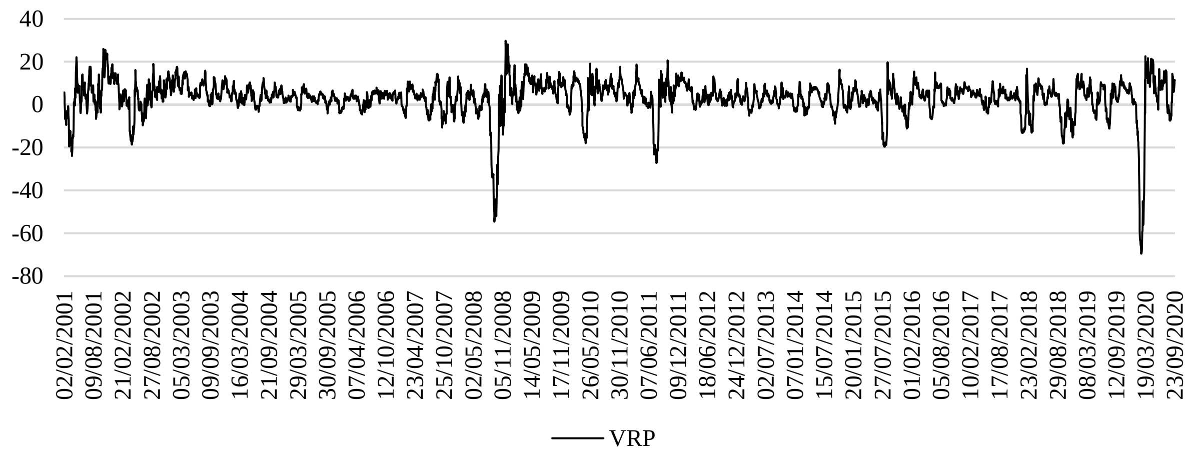

Figure 7.11 – VRP time series plot for NDX

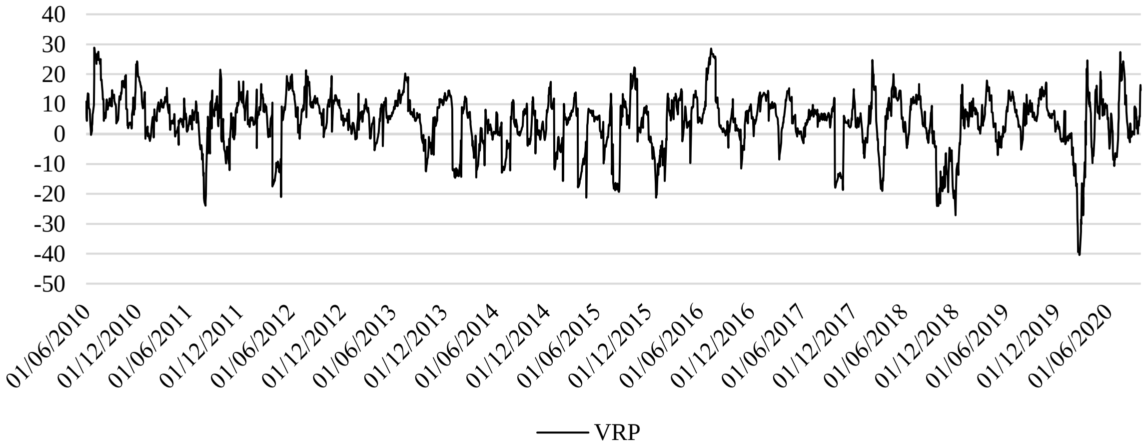

Figure 7.12 – VRP time series plot for AMZN

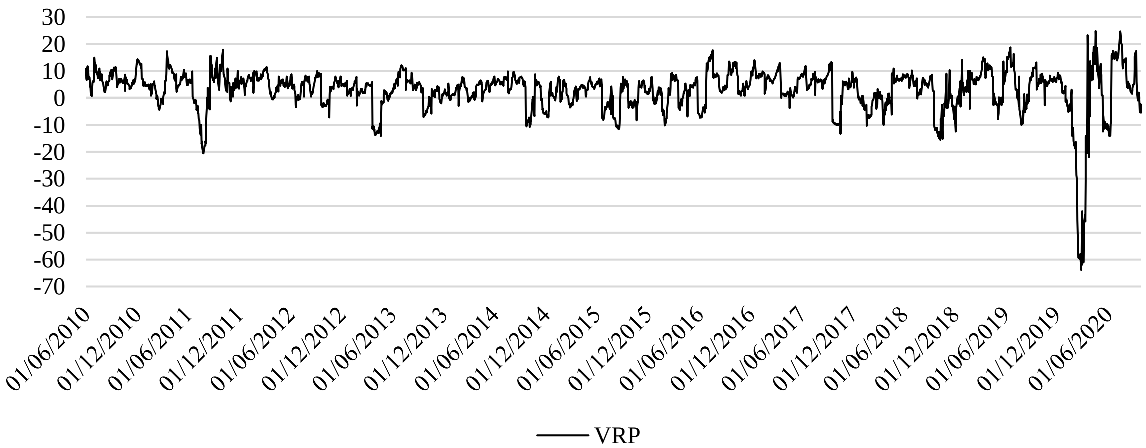

Figure 7.13 – VRP time series plot for IBM

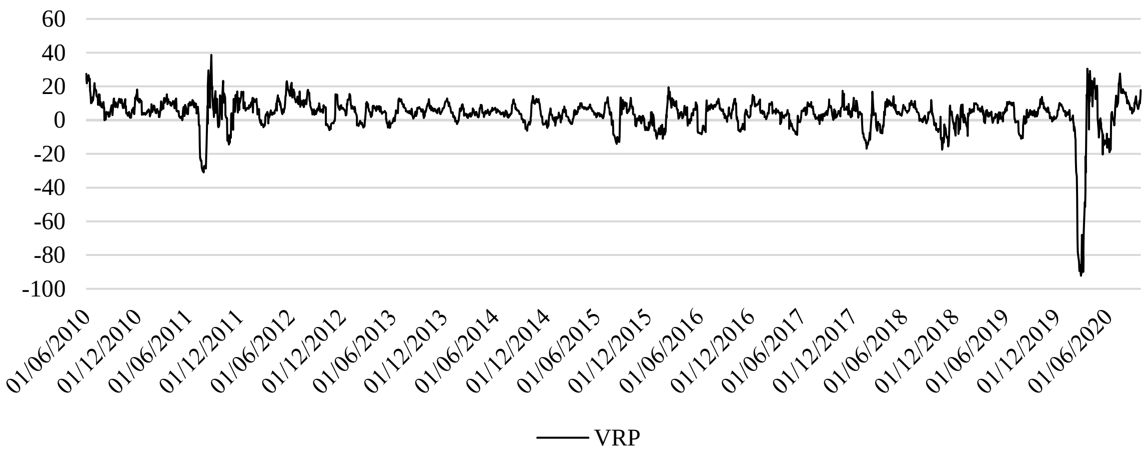

Figure 7.14 – VRP time series plot for GS

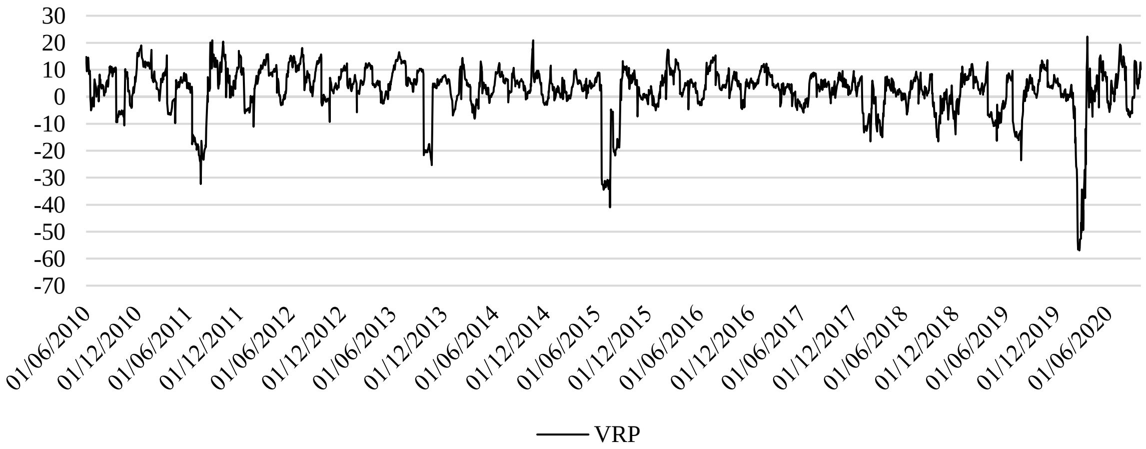

Figure 7.15 – VRP time series plot for GOOGL

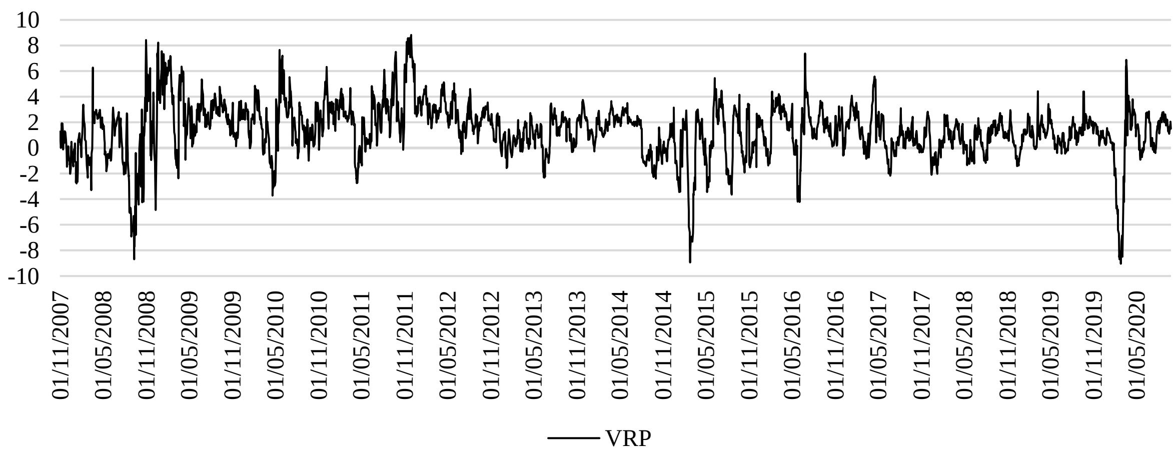

Figure 7.16 – VRP time series plot for FXE

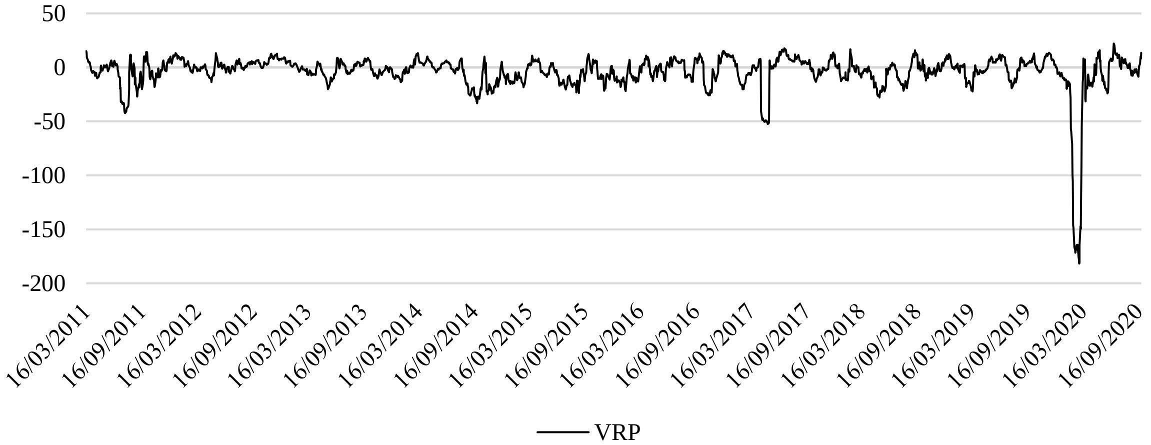

Figure 7.17 – VRP time series plot for EWZ

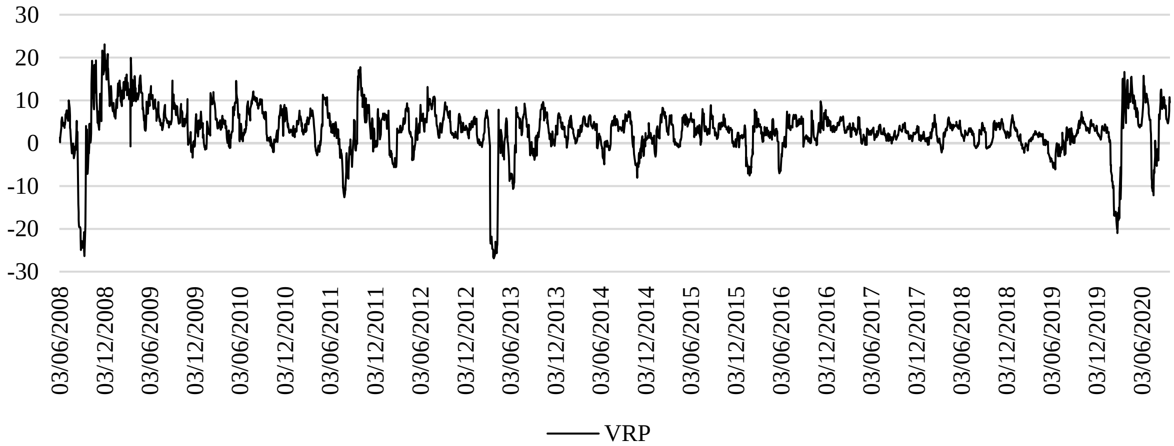

Figure 7.18 – VRP time series plot for GLD