On the integrated behaviour of non-stationary volatility in stock markets

Andreia Dionisio*, Rui Menezes** and Diana A. Mendes**

*University of Evora, Center of Business Studies, CEFAGE-UE, Largo Colegiais, 2, 7000 Evora, Portugal, E-mail: andreia@uevora.pt; **ISCTE, Av. Forcas Armadas, 1649-025 Lisboa, Portugal

Abstract

This paper analyses the behaviour of volatility for several international stock market indexes, namely the SP 500 (USA), the Nikkei (Japan), the PSI 20 (Portugal), the CAC 40 (France), the DAX 30 (Germany), the FTSE 100 (UK), the IBEX 35 (Spain) and the MIB 30 (Italy), in the context of non-stationarity. Our empirical results point to the evidence of the existence of integrated behaviour among several of those stock market indexes of different dimensions. It seems, therefore, that the behaviour of these markets tends to some uniformity, which can be interpreted as the existence of a similar behaviour facing to shocks that may affect the worldwide economy. Whether this is a cause or a consequence of market globalization is an issue that may be stressed in future work.

Key words: Cointegration, nonstationarity, exogeneity, fractional integration, FIGARCH models.

Introduction

The persistence of stock price volatility is a well-known stylized fact in the financial literature. Much of the empirical tests of volatility presented in the literature rely on the standard GARCH approach proposed by Bollerslev and Wooldrigde (1992), and often produce evidence that the conditional volatility is highly persistent. The stock prices volatility also presents some attributes that are typically non-stationary, an issue that requires the consideration of a special class of conditional heteroskedasticity models based on the IGARCH specification proposed by Engle and Bollerslev (1986). Under this specification, there is no need to differentiate the series when they prove to be non-stationary in order to apply the conditional heteroskedasticity models, thus retaining the richness of information contained in the original series.

The main purpose of this paper is to compare the volatility between several international stock market indexes, namely the S&P 500 (USA), the Nikkei (Japan), the Hang-Seng (Hong-Kong), the PSI 20 (Portugal), the CAC 40 (France), the DAX 30 (Germany), the FTSE 100 (UK), the IBEX 35 (Spain), the ASE (Greece) and the MIB 30 (Italy), in the context of non-stationarity. We use the daily closing prices of these indexes to perform the tests and to present the empirical results.

In this paper we applied Johansen tests (Johansen, 1988) in order to test cointegration between non-stationary variables, along with tests for weak exogeneity (Johansen and Juselius, 1990). The results were then compared to those obtained by the Granger causality tests in order to get evidence on strong exogeneity of the variables. Besides, stochastic integrated conditional heteroskedasticity specifications based on IGARCH and FIGARCH (Meddahi and Renault, 2004) models were also attempted in order to capture the likely non-stationary attribute of the series under the context of conditional volatility.

Our empirical results show evidence of existence of integrated behaviour among several stock market indexes of different dimensions. It seems, therefore, that the behaviour of these markets tends to some uniformity, which can be interpreted as the existence of a similar behaviour facing to shocks that may affect the worldwide economy.

The rest of the paper is organized as follows. In Section 1 we present a brief discussion of the background theory. Section 2 discusses the results of testing for the long-run relationship in stock indexes using cointegration techniques. Next, we present in Section 3 the results of fractional volatility in stock returns using GARCH, IGARCH and FIGARCH specifications. Finally, Section 4 presents the conclusions.

1 Background Theory

The standard GARCH framework (Bollerslev *et al.*, 1992) often produces evidence that the conditional volatility process is highly persistent and possibly not covariance-stationary, suggesting that a model in which shocks have a permanent effect on volatility might be more appropriate. This is a property of the integrated GARCH (IGARCH) model (Engle and Bollerslev, 1986) which has infinite memory.

Following Engle (1982), we consider the time series y_t with the associated error

$$e_t = y_t - E_{t-1} y_t \tag{1}$$

where E_{t-1} is the expectation operator conditioned on time $t-1$. A generalized autoregressive conditional heteroskedasticity (GARCH) model where

$$e_t = z_t \sigma_t, \quad z_t \sim N(0,1) \tag{2}$$

is defined as

$$\sigma_t^2 = w + \alpha(L) e_t^2 + \beta(L) \sigma_t^2, \tag{3}$$

where $w > 0$, and $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator L $(L^i x_i = x_{t-i})$ of order q and p respectively. Expression (3) can be rewritten as the infinite-order ARCH process,

$$\Phi(L) e_t^2 = w + [1 - \beta(L)] v_t, \tag{4}$$

where $v_t \equiv e_t^2 - \sigma_t^2$ and $\Phi(L) = [1 - \alpha(L) - \beta(L)]$. One limitation of this process applied to financial data, is that GARCH model has short-memory model because volatility shocks decay at a fast geometric rate. So, a way to represent the observed persistence of volatility on the rate of returns is to approximate a unit root, resulting from that the integrated GARCH (IGARCH) model. The specification of the IGARCH model is

$$\Phi(L)(1 - L) e_t^2 = w + [1 - \beta(L)] v_t. \tag{5}$$

Perron (1989) demonstrated that standard tests tend to under-reject the null of unit-root in the presence of structural breaks in the mean. In a similar way, breaks in the conditional variance could lead to spuriously high estimates of its degree of persistence. However, and according to Vilasuso (2002) the IGARCH model is not an entirely satisfactory description of the rate of returns volatility because one property of the model is infinite memory.

Motivated by the presence of apparent long-memory in the autocorrelations of squared or absolute returns of various financial assets, Baillie *et al.* (1996) have introduced the fractionally integrated GARCH, the FIGARCH model. Analogously for the ARFIMA (k, d, l) process for the mean described by

$$a(L)(1 - L)^d y_t = b(L) e_t, \tag{6}$$

where the $a(L)$ and $b(L)$ are polynomials in the lag operator of order k and l and $e_t \sim N(0,1)$. The FIGARCH (p, d, q) process for $\{e_t\}$ is defined by

$$\Phi(L)(1 - L)^d e_t^2 = w + [1 - \beta(L)] v_t \tag{7}$$

where $0 \leq d \leq 1$ is the fractional difference parameter.

The primary purpose of this approach is to develop a more flexible class of processes for the conditional variance that are more capable of explaining and representing the observed temporal dependencies in financial market volatility.

For example, the special cases: $d = 0$ corresponds to modelling a GARCH process and $d = 1$ corresponds to an IGARCH process. For d>0 the process is long-memory. The ARFIMA model essentially disentangles the short-run and the long-run dynamics by modelling the short-run behaviour through the conventional ARMA lag polynomials $[a(L)$ and $b(L)]$ while the long-run characteristic is captured by the fractional differencing parameter (d).

The FIGARCH process combines many of the features of the fractionally integrated process for the mean [ARFIMA(k, d, l) process] together with the regular GARCH process for the conditional variance. It implies a slow hyperbolic rate of decay for the lagged squared innovations in the conditional variance function, although the cumulative impulse response weights associated with the influence of volatility shocks on the optimal forecasts of the conditional variance tend to zero (Baillie *et al.*, 1996).

The common approach for estimation of ARCH models, assumes a conditional normality of the process. Under this assumption, Maximum Likelihood Estimates (MLE) for the parameters of FIGARCH(p, d, q) can be considered the most efficient estimation process. For the FIGARCH(p, d, q) model with $d > 0$ the population variance is not finite. However, subject to the regularity conditions specified by Baillie *et al.* (1996), conditioned on the pre-sample values will not affect the asymptotic distributions of the resulting estimators and test statistics. In most practical applications using financial data, the standardized innovations $z_t = e_t\sigma_t^{-1}$ are leptocurtik and not *i.i.d.* normally distributed through time. In this situations, Baillie *et al.* (1996) point to the use of the robust *Quasi-MLE* (QMLE).

2 Testing for long-run relationships in stock indexes

We start our empirical analysis by testing for long-run equilibrium relationships between the stock indexes variables used in our study. The analysis is based on cointegration techniques using autoregressive (VAR) systems in order to ascertain the extent of integration in these stock market indexes. Our goal is to identify common behaviour and dependencies in these markets, regardless of the occurrence of peaks, slumps, or periods of price stability.

The integration of international capital markets has received a large amount of attention from financial researchers over the past few years, mainly because of factors such as the relaxation of exchange controls and increased international information flows, technological developments in communications and trading systems, and the introduction of innovative financial products.

Cointegration tests provide useful information for strategic asset allocations.

One of the arguments in favour of the international diversification is that it lowers portfolio risks without sacrificing expected returns on the presumption that world stock prices are independent. Examples of some recent studies on this topic include Darrat, Elkhal and Halkim (2000) and Phylaktis and Ravazzolo (2002).

In fact, the literature on stock markets has been mainly focused on international portfolio diversification.

An increased correlation between stock market indexes is usually interpreted as a rise in the extent of market integration. This leads to a higher tendency for a shock in one country being transmitted to another one. But a straightforward use of this approach may sometimes give misleading conclusions, because of the non-stationary nature of most stock market price variables.

The recognition of the importance of the nonstationarity property of stock prices led some researchers to explore possible long-run relations among national and international stock markets using the notion of cointegration as defined by Engle and Granger (1987).

The data set used in this paper contains 4221 daily closing prices spanning the period from January, 5, 1990 to April, 7, 2006 for 10 international stock indexes, namely he S&P 500 (USA), the Nikkei (Japan), the Hang-Seng (Hong-Kong), the PSI 20 (Portugal), the CAC 40 (France), the DAX 30 (Germany), the FTSE 100 (UK), the IBEX 35 (Spain), the ASE (Greece) and the MIB 30 (Italy).[1] Figure 1 presents the general behaviour and trend of the stock indexes covering the period under study.

As may be seen, there are similarities in the behaviour and general trend in many of the series presented in Figure 1, namely the PSI 20, IBEX 35, CAC 40, FTSE 100, DAX 30, MIB 30 and S&P 500, although they may differ in the short-run. The statistical results reveal that all stock indexes are strongly leptocurtik, non-normally distributed and exhibit evidence of hereroskedasticity.

The ADF and the Elliott-Rothenberg-Stock tests for unit-roots were applied. The unit-root hypothesis was not rejected at standard significance levels in any case, for the series in levels, independently of the inclusion of a constant term and a deterministic trend in the ADF and ERS regressions. On the other hand, for first differences, the null hypothesis of a unit root was strongly rejected, indicating that each of the first-differenced series is stationary.

In order to evaluate the possible cointegration in these stock indexes, we applied Johansen tests (Johansen, 1988) for cointegration between these non-

[1] All the variables are transformed into natural logarithms.

Fig. 1. Behaviour and trend of the natural logarithm of the stock indexes. January, 5, 1990 - April, 7, 2006.

stationary variables. Firstly, we tested for bivariate cointegration in each pair of stock indexes. We performed this test for 45 different pairs of stock indexes. We also computed weak exogeneity tests and Granger causality tests.

The finding of a cointegrating vector between pairs of series indicates that over the sample the series move together in an equilibrium relationship. The term equilibrium in the cointegration literature is sometimes synonymous that the series maintain a constant relationship throughout the sample. It does not mean that over specific sub-periods the series did not move apart.

The main results of these tests point to the existence of 18 pairs of stock indexes that show signs of cointegration. Table 1 presents the results of the cointegration tests and the exogeneity tests for the pairs of stocks that show evidence of statistical significant bivariate cointegration. [2]

[Please insert DionisioFigure2 here]

[2] Under the null hypothesis of cointegration, $r = 0$ corresponds to the case where there are no cointegrating vectors, and $r <= 1$ corresponds to the case where there is at most one cointegrating vector [column (1)]. The next three columns provide information about the eigenvalues and the trace and maximum eigenvalue test statistics for each hypothesis. Columns (5) and (6) give the results of the exogeneity tests, where the parameter denoting the speed of adjustment to long-run equilibrium is tested for being zero in the null hypothesis.

The results of the weak exogeneity tests are in conformity with the results obtained with the Granger causality tests,[3] leading us to conclude that there exists strong exogeneity in the reported cases.

Our results also show that there is a close integration within the European stock markets and also between some European stocks and the US and the Asian stock markets. Several factors may explain this situation. One of these factors is that the companies around the world strongly exposed to the global business cycle, leading the national stock markets to move together more tightly [Barton *et al.* (2002)].[4]

3 Volatility in stock returns

We now turn to consider the fractional property of the stock returns volatility. We fit the conditional hereroskedasticity models using the estimation method proposed by Chung (2001) based on *Quasi–Maximum Likelihood Estimation* (QMLE) methods: GARCH $(1,1)$, IGARCH $(1,1)$ and FIGARCH $(1,d,1)$.The rate of returns of the stock indexes was computed as follows:

$$R_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) = \mu + R_{i,t-1} + e_{i,t}, \tag{8}$$

where $P_{i,t}$ is the value of the underlying stock index i at time t, μ is a constant and $e_{i,t}\sigma_{i,t}^{-1}$ is $i.i.d. \sim N(0,1)$, using the model

$$\sigma_{i,t}^2 = w + \beta_1\sigma_{i,t-1}^2 + \left[1 - \beta_1(L) - (1-\Phi)(1-L)^d\right] e_{i,t}^2. \tag{9}$$

The results displayed in Table 2 show that for all stock index returns the parameter d of the FIGARCH model is always statistically significant, with values between 0.3 and 0.5. On the other hand, the GARCH model, which

[3] The Granger causality results are available upon request to the authors.

[4] We also performed multivariate cointegration tests for all the stock indexes contained in our database. We found for the entire set of our variables 3 cointegrating vectors. The speed of adjustment to the long-run equilibrium relationship is statistically significant for the PSI 20 (0.002), MIB 30 (0.001), DAX 30 (0.002), ASE (0.003) and CAC 40 (0.001) indexes. It seems therefore that there is a stronger interaction among the European continental indexes (except the IBEX 35), which goes in same direction of the macroeconomic behaviour of the underlying economies. The weak exogeneity tests for the whole set of indexes showed no evidence of exogeneity in the PSI 20, DAX 30 and ASE indexes. This may indicate that the corresponding stock markets can receive more influences from the other stock markets, being more exposed to shocks in the global economy than other stock markets.

assumes that $d = 0$, produces a lower log-likelihood statistic most of the times. We can also see that the estimated GARCH (1,1) parameters do not differ much from the estimated IGARCH (1,1). The results show that the dynamics of the conditional variance of stock index returns are best represented by the FIGARCH model.

Our results indicate that the residuals $(e_{i,t})$ are not stationary, since for all cases we reject the hypothesis that $d = 0$ while at the same time we can see that it is precisely the FIGARCH model, where $d < 1$ that presents the highest values for the log-likelihood parameter. This seems to reveal that the IGARCH model is not the best alternative to estimate the volatility of the index rate of returns. This result seems to indicate that the stock index series are not $I(1)$ and the first differences are not $I(0)$, and so the main conclusions and results obtained with the traditional stationary tests and cointegration analysis could be misleading.

[Please insert DionisioFigure3 here]

Note: The conditional mean of each rate of return is modelled as a constant μ, and w is constant in the conditional variance. The values in brackets refer to the standard-deviation. For the IGARCH (1,1), the estimation of Φ is unbounded.

It is important to note that the European stock indexes, whose underlying economies are more developed (CAC 40, DAX 30 and FTSE 100) present the highest levels for the parameter d, with values of, respectively, 0.5158, 0.5494 and 0.5135. Since for these index returns $d > 0.5$, there are signals of mean-reverting behaviour, which seems to contradict the efficient market hypothesis.

On the other hand, the stock markets characterized by lower dimension and levels of liquidity, such as PSI 20, ASE, MIB 30 and Hang-Seng as well as the S&P 500 and the Nikkei (which are well developed markets), present values of d smaller than 0.5, which means that the corresponding return series are stationary. Given these results, we cannot conclude that the dimension of the market and its level of liquidity are factors that can promote stationarity on the volatility, being closer to the efficient market hypothesis.

4 Conclusions

In this paper we study the dynamics of the stock price indexes and of the rate of returns volatility.

For the cointegration long-run relationships the results obtained are mixed, with some market indexes being bivariately cointegrated and pothers not. Out of the 45 bivariate models tested, 18 show signs of cointegration. This is the case of most European markets, and also the case of the S&P 500 with some of the other markets. In the Europe, this is especially so among some continental stock indexes, namely the PSI 20, DAX 30, ASE, MIB 30 and FTSE 100, in which the ASE and MIB 30 are essentially endogenous variables and the DAX 30, FTSE 100 and PSI 20 present weak exogeneity.

Relating to the volatility analysis, the results show that the FIGARCH model is better suited to capture the behaviour of stock indexes returns than the commonly used GARCH model and also the IGARCH model. All stock index returns exhibit evidence of fractional integration.

Indeed, we found signs of long-memory effects, which seems to indicate that there is evidence of dynamics, not only in the dimension of prices, but also on the volatility. The finding of a significant integration parameter $0 < d < 1$ seems to indicate that the original series of stock index prices are not integrated of order 1 [$I(1)$], being around $I(1.5)$. This result indicates that the conventional "linear" cointegration analysis may pose some problems since it is based on the assumption that the time series are $I(1)$ and the residuals resulting from the estimation of a long-run equilibrium model are stationary [$I(0)$].

A possible alternative would be a model of fractional cointegration, which is however out of the scope of this research work.

Our empirical results point to the evidence of the existence of integrated behaviour among several stock market indexes of different dimensions. It seems, therefore, that the behaviour of these markets tends to some uniformity, which can be interpreted as the existence of a similar behaviour facing to shocks that may affect the worldwide economy.

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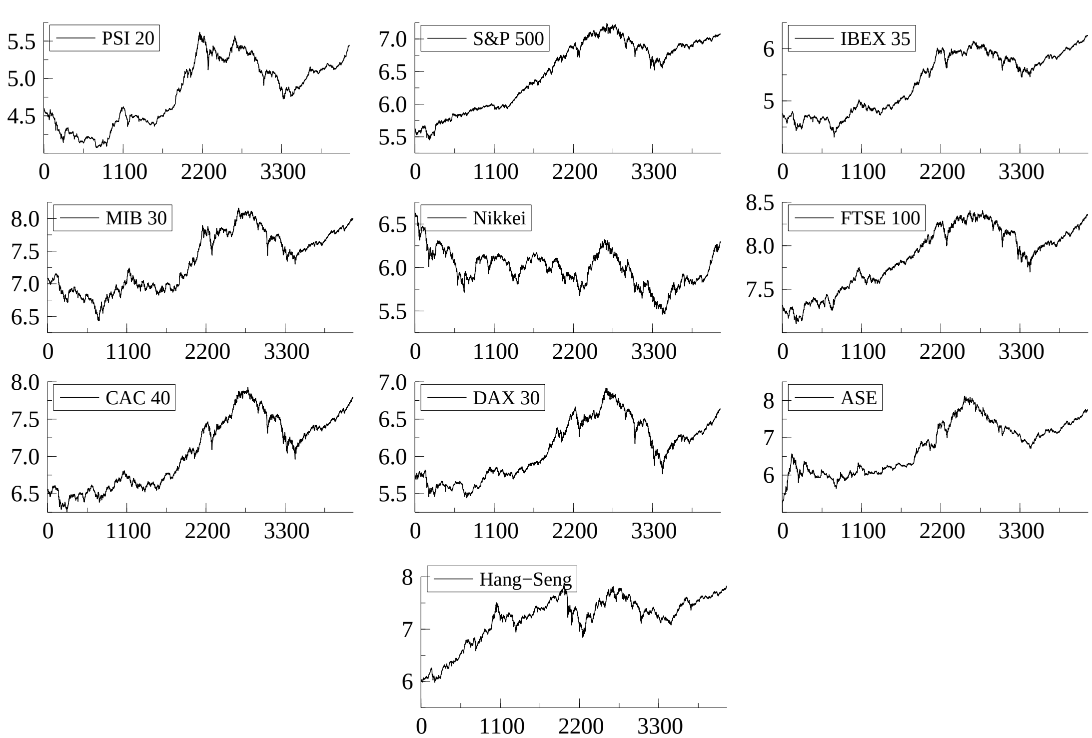
PSI 20
5.5
5.0
4.5
S&P 500
7.0
6.5
6.0
5.5
IBEX 35
6
5
MIB 30
8.0
7.5
7.0
6.5
Nikkei
6.5
6.0
5.5
FTSE 100
8.5
8.0
7.5
CAC 40
8.0
7.5
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DAX 30
7.0
6.5
6.0
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ASE
8
7
6
Hang−Seng
8
7
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0
1100
2200
3300

						Exogeneity	
Index. 1	Index. 2	Rank (1)	Eigenvalue (2)	Trace (3)	Max eigenv. (4)	Index. 1 Stat (5)	Index. 2 Stat (6)
S&P 500	CAC 40	r=0	0.0044	25.574**	18.610*	8.767**	7.963**
		r<=1	0.0016	6.964	6.964		
S&P 500	ASE	r=0	0.0036	18.218**	16.700**	4.916*	11.799**
		r<=1	0.0003	1.517	1.517		
PSI 20	FTSE 100	r=0	0.0038	17.021*	16.187*	10.824**	0.266
		r<=1	0.0002	0.833	0.834		
PSI 20	DAX 30	r=0	0.0036	15.454*	15.383*	0.0035	11.833**
		r<=1	0.00002	0.072	0.073		
PSI 20	MIB 30	r=0	0.0053	23.205**	22.670**	3.840*	22.120**
		r<=1	0.0001	0.535	0.536		
PSI 20	ASE	r=0	0.0069	29.455**	29.373**	0.0934	28.411**
		r<=1	0.00002	0.082	0.083		
PSI 20	Hang-Seng	r=0	0.0062	32.625**	26.441**	10.915**	4.493*
		r<=1	0.0014	6.184	6.184		
IBEX 35	MIB 30	r=0	0.0046	20.476**	19.636**	8.639**	18.771**
		r<=1	0.0002	0.840	0.841		
IBEX 35	ASE	r=0	0.0060	25.623**	25.569**	1.033	25.482**
		r<=1	0.00001	0.053	0.054		
CAC 40	ASE	r=0	0.0045	19.932*	19.225*	4.307*	10.002**
		r<=1	0.0001	0.706	0.706		
FTSE 100	DAX 30	r=0	0.0035	18.359*	15.143*	3.010	11.352**
		r<=1	0.0007	3.217	3.217		
FTSE 100	MIB 30	r=0	0.0036	16.267*	12.958	1.055	8.784**
		r<=1	0.0007	3.309	3.309		
DAX 30	ASE	r=0	0.0047	20.711**	20.039**	1.715	14.048**
		r<=1	0.0002	0.671	0.672		
DAX 30	Hang-Seng	r=0	0.0029	16.003*	12.622	2.325	3.693*
		r<=1	0.0008	3.381	3.381		
MIB 30	ASE	r=0	0.0073	31.741**	31.315**	5.946*	19.506**
		r<=1	0.0001	0.426	0.427		
MIB 30	Hang-Seng	r=0	0.0030	16.423*	13.151	3.584	3.860*
		r<=1	0.0007	3.272	3.272		
ASE	Nikkei	r=0	0.0030	15.542*	14.123*	1.784	9.015**
		r<=1	0.0004	1.419	1.419		
Nikkei	Hang-Seng	r=0	0.0041	22.723*	17.204*	5.452*	2.796
		r<=1	0.0011	5.018	5.018		

Table 1. Cointegration and weak exogeneity tests. * significant at 5%; ** significant at 1%.

Index returns	PSI 20			IBEX 35			CAC 40			DAX 30			FTSE 100	
Model	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH
μ	0.0003	0.0002	0.0002	0.0008	0.0008	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
	(0.00003)	(0.00003)	(0.00002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
w	0.5920	0.0139	0.0133	1,3670	0.0122	0.0168	1,6268	0.0077	0.0135	1,6243	0.0076	0.0093	1,2981	0.0048
	(0.1991)	(0.0089)	(0.0061)	(0.4940)	(0.0034)	(0.0044)	(0.5078)	(0.0026)	(0.004)	(0.6928)	(0.0024)	(0.003)	(0.6928)	(0.0015)
ß	0.2588	0.1532	0.1608	0.1835	0.1006	0.0929	0.2002	0.074	0.0683	0.1293	0.0909	0.0871	0.1955	0.0778
	(0.1151)	(0.060)	(0.0329)	(0.0455)	(0.013)	(0.0119)	(0.0373)	(0.0113)	(0.0109)	(0.0376)	(0.0124)	(0.0121)	(0.040)	(0.0113)
Φ	0.4123	0.8467	0.8461	0.6023	0.8984	0.8967	0.6691	0.9259	0.9214	0.6299	0.9091	0.9081	0.6434	0.9221
	(0.1184)		(0.029)	(0.0614)		(0.013)	(0.0554)		(0.011)	(0.0558)		(0.012)	(0.0544)	
d	0.3709			0.4965			0.5158			0.5494			0.5135	
	(0.0365)			(0.0453)			(0.0519)			(0.0486)			(0.0454)	
log-lik	15021.2	14989.85	14990.1	13487.8	13480.9	13483.0	13446.5	13442.6	13445.9	13765.4	13756.2	13756.9	14452.1	14446.7

Index returns	MIB 30			ASE			S&P 500			Nikkei			Hang-Seng	
Model	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH	GARCH	FIGARCH	IGARCH
μ	0.0005	0.0005	0.0005	0.0003	0.0002	0.0002	0.0007	0.0005	0.0005	0.0007	0.0003	0.0003	0.0007	0.0007
	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
w	1,3219	0.0103	0.0120	3,2702	0.0565	0.0588	0,8272	0.0025	0.0033	1,3511	0.0214	0.0306	1,4059	0.0151
	(0.5417)	(0.0036)	(0.004)	1,9024	(0.0156)	(0.016)	(0.2763)	(0.0012)	(0.0016)	(0.5563)	(0.0054)	(0.0077)	(0.4352)	(0.0056)
ß	0.2090	0.0869	0.0847	0.0723	0.1621	0.1551	0.1882	0.0496	0.0483	0.2094	0.1018	0.0914	0.2680	0.0663
	(0.052)	(0.0136)	(0.0133)	(0.067)	(0.0266)	(0.026)	(0.049)	(0.0101)	(0.0097)	(0.065)	(0.0123)	(0.0115)	(0.077)	(0.0129)
Φ	0.6067	0.9131	0.9121	0.3521	0.8379	0.8382	0.5937	0.9503	0.9494	0.5453	0.8982	0.8931	0.5636	0.9336
	(0.0637)		(0.014)	(0.0793)		(0.026)	(0.0683)		(0.010)	(0.0849)		(0.013)	(0.0893)	
d	0.4863			0.4280			0.4315			0.4156			0.3709	
	(0.0403)			(0.0428)			(0.0417)			(0.0514)			(0.0426)	
log-lik	13072.8	13065.5	13065.7	12249.5	12229.9	12230.2	14181.3	14173.6	14174.0	13060.7	13059.6	13062.6	12575.7	12563.3

Table 2. Quasi-Maximum likelihood parameter estimates of heteroskedasticity models.