From problem structuring to optimization: A multi-methodological framework to assist the planning of medical training

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Abstract: Medical training is an intricate and long process, which is compulsory to medical practice and often lasts up to twelve years for some specialties. Health stakeholders recognise that an adequate planning is crucial for health systems to deliver necessary care services. However, proper planning needs to account for complexity related with the setting of medical school vacancies and of residency programs, which are highly influenced by multiple stakeholders with diverse perspectives and views, as well as by the specificities of medical training. Aiming at building comprehensive models with a potential to assist health decision-makers, this article develops a multi-methodological framework to assist the planning of medical training under such a complex environment. It combines the structuring of the objectives and specificities of the medical training problem with a Soft Systems Methodology through the CATWOE (Customer, Actor, Transformation, *Weltanschauung*, Owner, Environment) approach, and the formulation of a Mixed Integer Linear Programming model that considers all relevant aspects. Considering the specificities of countries based on a National Health Service structure, a multi-objective planning model emerges, informing on how many vacancies should be opened/closed per year in medical schools and in each specialty. This model aims at i) minimizing imbalances between medical demand and supply; ii) minimizing costs; and iii) maximizing equity across medical specialties. A case study in Portugal is explored so as to illustrate the applicability of the proposed multi-methodology, showing the relevance of proper structuring for planning models having the potential to inform health decision-makers and planners in practice.

Keywords: OR in Health Services, Medical training, Multi-methodology, MILP, CATWOE

Highlights

- Strategic medical training planning using a multi-methodology is addressed.
- The multi-methodology combines optimization model with Soft Systems Methodology.
- A multi-objective model reflects issues identified with Soft Systems Methodology.
- Objectives related to medical force imbalances, costs and equity are considered.
- The model applicability is shown through a case study in Portugal.

1. Introduction

The health care workforce is critical to ensure the delivery of health care services to the population, and to guarantee the overall effectiveness of health systems (Dreesch et al., 2005; Perfilieva & Buchan, 2015). This is especially relevant for countries facing serious imbalances and inefficiencies in the availability of Health Human Resources (HHR). In fact, according to estimates presented by the European Commission, a '*potential shortfall of around 1 million health care workers, rising to 2 million if long-term care and ancillary professions are taken into account (including shortfalls of 230 000 doctors, 150 000 dentists, pharmacists and physiotherapists, and 590 000 nurses)*' is likely to occur across EU members (Perfilieva & Buchan, 2015) (p.10). This is partly explained by an expected rise in the demand for health care due to a significant increase in average life expectancy, as well as by changes in epidemiological and demographics trends (Scheffler et al., 2008). Under such circumstances an adequate planning of HHR needs to address multiple challenges, which include (Fritzen, 2007; Perfilieva & Buchan, 2015): i) the expected increase in the demand for healthcare services; ii) serious skill shortages, across countries/regions and specialties; iii) issues related to the motivation and retention of health professionals; iv) trends and patterns of health care workforce mobility and migration; and v) an ageing workforce with insufficient new recruits. Such concerns led the European Commission to promote several recent European projects and initiatives, such as the *HEALTH PROMeTHEUS – Health Professional Mobility in the European Union Study* (2017), the *Green Paper on the European Workforce of Health* (2008), and the *Joint Action on Health Workforce Planning and Forecasting* (2017).

Being broadly recognised by health policy-makers and literature that an adequate planning of the health care workforce is essential for the proper management of a health system (Dreesch et al., 2005), the need to ensure the right number of people with the right skills in the right place at the right time is at the core of HHR planning. This is valid for all health workers (which include physicians, nurses, hygienists, therapists and other support staff (Amorim-Lopes, Soares, et al., 2016)), and so training turns out to be a central tenet of HHR planning as it conditions entry to the profession (Lavieri & Puterman, 2009), and is strategic, bearing long-term implications (Amorim-Lopes et al., 2015; European Union, 2017). For instance, the admission of too many students to medical schools may result in an oversupply of physicians, leading to inefficiencies in resource allocation, unemployment or increased healthcare costs due to the supplier-induced demand. On the other hand, low admissions may result in a shortage of professionals, with consequences in terms of the quality and quantity of healthcare services, increased waiting lists and professionals' burnout (Amorim-Lopes et al., 2015). Finding the threshold line for how many is too many is therefore critical, being especially relevant to countries with a National Health Service (NHS), where the state plays a key role in financing and organising the training of the health care workforce, as is the case of Portugal (Ministry of Health, 2010; Santana et al., 2014).

Approaches reported in literature to inform the planning of HHR training typically only consider a crude assessment of the vacancies needed (Malgieri et al., 2015). However, the planning of HHR training is far from being a simple topic, as multiple stakeholders have a word on training decisions, and several policy objectives may exist, and may in fact conflict (Malgieri et al., 2015; Monks, 2016). For instance, in Portugal vacancies in medical schools are defined by the Ministry of Science, while residency vacancies are defined by the Ministry of Health, and the Medical Association is a key player that influences training capacity levels. Also, multiple budget constraints apply at different levels of public spending, which limit the room for manoeuvre in adjusting policy levers.

Thus, models with a potential to assist the planning of HHR training need to address the complexity and richness of this context, and should thus be informed by a comprehensive structuring of the planning problem (Mingers & Rosenhead, 2004). Namely, planning models need to answer to a wide range of questions – e.g., which are the key planning decisions? Which stakeholders should be involved in this decision-making process? Which are the main concerns and objectives according to these stakeholders? According to the authors' knowledge, such a comprehensive structuring has not been common practice in the planning literature.

What is commonly found are mathematical programming models that play a key role when the aim is to support the strategic planning of the workforce (Ernst et al., 2004). Still, a small body of literature employing these methods exists in the area of health care workforce planning and training (Hu et al., 2016; Lavieri & Puterman, 2009; Schell et al., 2016; Schell et al., 2015; Senese et al., 2015), with existing studies mainly featuring mono-objective mathematical programming models focused on the minimization of costs. Therefore, these models typically fail to account for the multiplicity of objectives that mirror the concerns of the different stakeholders.

This article thus contributes to the literature by proposing a multi-methodological framework combining an optimization model based on mathematical programming, a traditional hard OR (Operational Research) approach, with Soft Systems Methodology (SSM) so as to build a model to support the planning of HHR training that accounts for a comprehensive analysis of the medical training problem. This multi-methodology will be hereafter called *Multi-PiHLOT (**Multi**-methodology for **P**lann**i**ng **H**ea**l**th care w**O**rkforce **T**raining).* Specifically, this article complements existing research efforts in different ways. Firstly, differently from what has been common practice in workforce planning models based on mathematical programming, the proposed framework makes use of SSM to guide the planning modelling stage, while ensuring that existing multiple opinions and potentially conflicting interests of the stakeholders involved, as well as the specificities of the medical training process, are dealt with in detail (following Checkland (1989, 1999)). In particular, this study uses SSM through the CATWOE approach (with CATWOE standing for Customer, Actor, Transformation, *Weltanschauung*, Owner and Environment) to characterize in detail the HHR training context. The CATWOE approach allows to identify all the stakeholders, processes and external factors involved in the system under study (Mingers & Rosenhead, 2004; Pownall, 2012).

Secondly, it develops existing planning models so as to reflect this initial problem structuring. Particularly, a multi-objective mixed integer linear programming (MILP) model is presented to support the strategic planning of the health care workforce training and inform decision-makers accordingly. The proposed model is developed in the context of NHS-based countries with a public medical education system, and is focused on the training of physicians, informing on how many vacancies should be opened/closed per year in medical schools and in each specialty to avoid imbalances. The planning model provides policy insights, e.g., on how much additional training capacity has to be provided in case of a shortage, while accounting for the minimization of oversupply and shortages in the provision of care, the maximization of equity in the distribution of physicians across specialties, and the minimization of costs incurred in the training process. It also considers other medical training specificities, for instance ensuring a stability in the *numerus clausus* to avoid disruptive changes. A case study in Portugal is explored to illustrate the applicability of the model.

The remainder of this article is organized as follows. A brief literature review on the key topics and methods related to this study is presented in Section 2. The proposed multi-methodology is described in Section 3, with the structuring of the medical training problem being outlined in section 3.1 and the details of the proposed MILP model being presented in section 3.2. Section 4 presents the case study and key results, and conclusions and lines for further research are outlined in Section 5.

2. Literature Review

We herein review the scope and features of models that have been reported in literature to assist workforce planning in general, and of HHR in particular, as well as studies combining soft and hard approaches for planning in health contexts.

Workforce planning literature

Much of the existing literature in health workforce planning has focused on the operational level – as shown in a recent review from Van den Bergh et al. (2013) –, as well as on building forecasts for the evolution of the medical workforce – as reported by Barber and López-Valcárcel (2010), Lagarde and Cairns (2012) and Amorim-Lopes, Almeida, et al. (2016), and in a recent review in Amorim-Lopes et al. (2015). Still, to the best of our knowledge, a lack of research has been devoted to structuring the planning problem and to strategically planning the health care workforce.

With regard to methods, simulation modelling has been extensively used in the healthcare workforce planning literature (with a strong use of System Dynamics (SD) modelling), especially to forecast the supply of health human resources or the demand for health care services. Tomblin Murphy et al. (2016), Lodi et al. (2016) or Barber and López-Valcárcel (2010), to name just a few studies, provide applications of SD for forecasting and planning the workforce. More recently, agent-based simulation modelling (ABM) has gained traction due to its capability of modelling individual features and decision-making, in sharp contrast with SD, which models the system at the macro, aggregate level. A recent example of ABM study for forecasting the medical workforce can be found in Amorim-Lopes, Almeida, et al. (2016).

Concerning strategic workforce planning, despite a large number of studies has been reported in literature in multiple sectors – most of them based upon mathematical programming and specifically adopting MILP models –, such studies are scarce in the health context. Examples of studies in other sectors are workforce planning models to inform hiring, firing and promotions by de la Torre et al. (2016), Horn et al. (2016) and Wishon et al. (2015), and workforce planning models focused on training decisions by Yu et al. (2004) and Srour et al. (2006). A comprehensive review about workforce training models is available in Ernst et al. (2004).

When one compares the adequacy of using simulation or optimization approaches for strategic workforce planning in general, and specifically to inform training decisions, one observes that it is difficult to find the best training levels that either maximize or minimize stakeholders' objectives through the use of simulation. Accordingly, in our modelling context, simulation is in fact particularly useful for generating forecasts, to project how the demand and supply inputs for health workforce planning might evolve under circumstances deemed likely. On the other hand, optimization – based upon mathematical programming – is more adequate to inform on the values of some variables that condition the evolution of the health workforce system (e.g., training variables), so that it evolves in a way that best suits the stakeholders' objectives.

To sum up, taking into account that no study has focused on the strategic planning of health care workforce supported by a comprehensive problem structuring step, this article aims to fill this gap.

Combining problem structuring methods with hard OR approaches for health care planning

As mentioned above, producing useful planning models reflecting the plurality of opinions from different stakeholders with a role in the planning of medical training requires a detailed structuring of the problem in general (Checkland (1989, 1999)) and of the medical training problem in our case. Within this setting, it is worth exploring the combination of 'soft' problem structuring methods with 'hard' OR approaches for health care planning.

When one looks into the health care planning literature one observes that it is now starting to recognize the relevance of combining problem structuring methods with the development of planning models. Although still limited in number, some studies have proposed combining simulation with problem structuring methods – e.g., Tako and Kotiadis (2015) proposed a multi-methodology framework combining Discrete Event Simulation (DES) with SSM so as to incorporate stakeholder involvement in health care planning studies, and Sachdeva et al. (2007) and Pessôa et al. (2015) proposed combining DES with cognitive mapping for similar purposes. Nevertheless, to the authors' knowledge, no health care planning study exists proposing problem structuring methods to inform the development of optimization models in general, and for the health care workforce training context specifically.

Mathematical programming models to support health care workforce planning

Concerning the use of mathematical programming models to support the strategic planning of the health care workforce, we have searched for studies published from 2000 onwards, and analysed their features.

We observe that few studies exist in the area, with these differing mainly in terms of: i) class of clinical staff considered for planning purposes; ii) type of planning decisions accounted for; and iii) planning objectives.

In what concerns the class of clinical staff, there are specificities in the workforce training of physicians and nurses – mostly related with training and career pathways – that require specific modelling. For instance, most Registered Nurses do not enrol on a specialty to become Clinical Nurse Specialists or Advanced Nursing Practitioners (in fact, few countries recognize these specialized roles). As opposed, physicians are encouraged (and in some countries required) to join a residency program to become specialists, with these programs requiring 4 to 6 years of specific training depending on the specialty chosen. As a result, the lead time for a professional to be ready to practice increases considerably: while a nurse can do so in between 3 and 4 years, a physician requires up to 10 years of training in some cases. Also, it is typically the case that residency programs have limited vacancies and that the training system entails specific requirements. Despite these differences, when one looks into health workforce literature, one observes that most studies have been focusing on nurses (Hu et al., 2016; Lavieri & Puterman, 2009; Li et al., 2007; Schell et al., 2016; Schell et al., 2015), with only one study targeting the planning of physicians (Senese et al., 2015).

In terms of planning decisions, Li et al. (2007), Lavieri and Puterman (2009) and Hu et al. (2016) proposed mathematical programming models for determining the optimal staff to be trained, promoted and hired for the coming years. The training component of the model proposed by Lavieri and Puterman (2009) was later adapted by Schell et al. (2016), with this model focusing on the planning of nurses and determining the optimal number of students to be admitted to nursing bachelor's degree and the number of employed nurses to be admitted to master's programs, and with admission to specialties not being accounted for. A different perspective of workforce training has been presented by Senese et al. (2015) which proposed a MILP model to determine the optimal assignment of medical grants across medical specialties.

Existing studies also differ in terms of the nature of the objectives pursued for planning purposes. Although they recognize that health workforce planning potentially involves multiple stakeholders with multiple and often conflicting concerns and objectives (Malgieri et al., 2015), most of the existing studies do not account for this diversity. For instance, the models from Lavieri and Puterman (2009), Hu et al. (2016), Schell et al. (2015) and Schell et al. (2016) rely on the minimization of costs (including costs related with the recruitment, training, wages, promotions and retirements), whereas the model from Senese et al. (2015) relies on the minimization of the overall training gaps (i.e., the minimization of shortages and oversupply across specialties). To the best of our knowledge, only Li et al. (2007) considered multiple objectives by combining cost- and quality-related objectives, with quality being measured in different ways, through the minimization of number of employees externally recruited and also through the minimization of the number of higher-level employees doing lower-level jobs. Also, although not considered in this workforce literature, other objectives typically considered in many NHS-based country (Baker, 2000; Simões et al., 2017), such as equity-related objectives, may be relevant for health workforce planning.

Fig. 1 summarizes the conclusions from this review, showing that there is no study proposing a planning approach to support health workforce training that simultaneously: i) starts from a comprehensive problem structuring method to support the development of planning models that closely represent the multiplicity of concerns of stakeholders in the area; ii) develops a planning model that considers the specificities of physicians' training pathway; and iii) accounts for multiple policy objectives. The *Multi-PiHLOT* proposed in this article aims at filling this gap in the literature. Behind the *Multi-PiHLOT* multi-methodology is the recognition that different stakeholders tend to hold tacit knowledge on different parts of the health system that need to be taken into account for an adequate planning (Brailsford & Vissers, 2011). It is also based on the recognition that there are training specificities that may be modelled with problem structuring tools; and that combining soft OR methods with hard OR methods (such as optimization models) is a step for better model acceptance (Lagergren, 1998).

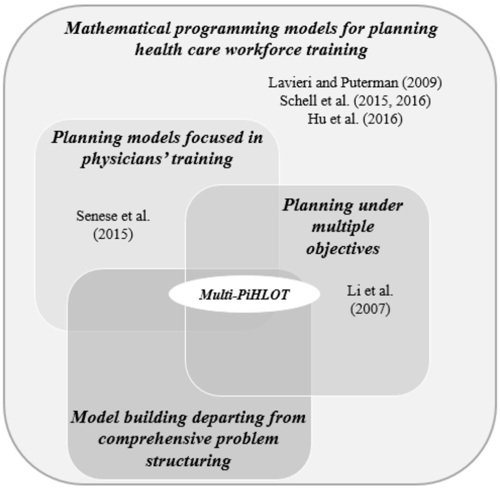

Figure 1. Key features for planning medical training

3. Multi-methodology

This study proposes a multi-methodology: following the definition by Mingers and Brocklesby (1997), the *Multi-PiHLOT* combines an optimization model based on mathematical programming (a typical hard OR approach) with Soft Systems Methodology (SSM) to support the planning of medical training at a strategic level. SSM is a learning system that accepts that real-world actions are messy and uses '*models to structure a debate in which different conflicting objectives, needs, purposes, interests and values can be teased out and discussed*' (Checkland, 1989) (p. 67). SSM thus provides a structured process to characterise the medical training problem, while mathematical programming models are then built so as to reflect such problem structuring. The proposed multi-methodology thus comprises two stages: a problem structuring stage based on a SSM; and a planning modelling stage based on optimization. An overview of this multi-methodology is presented in Figure 2.

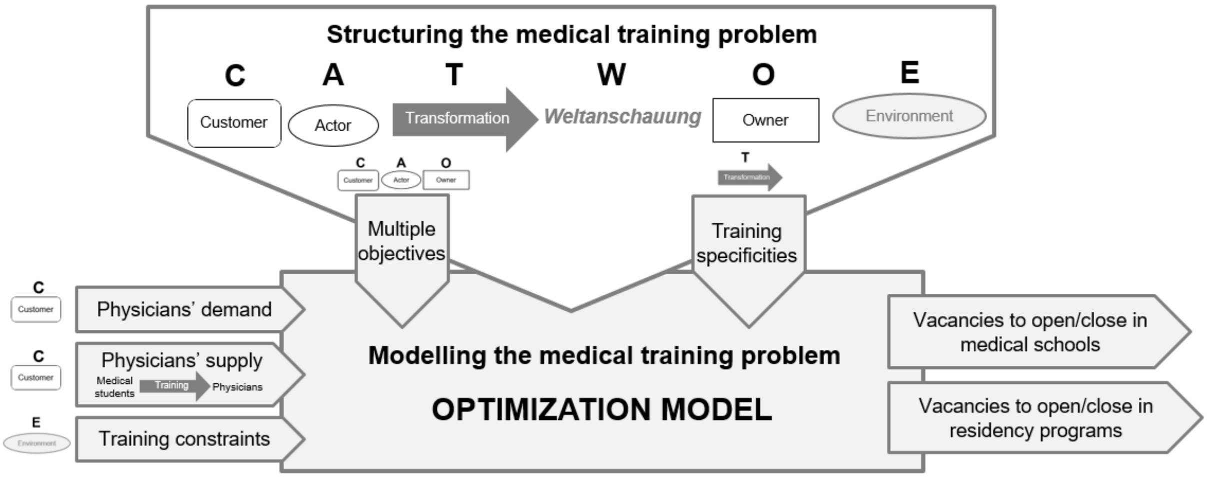

Figure 2. Overview of the *Multi-PiHLOT*

In the problem structuring stage a CATWOE analysis is performed. A CATWOE analysis was chosen because it enables characterizing the training process itself, mapping key stakeholders, as well as defining a common understanding of what is key in medical training. Specifically, the CATWOE approach, which will be later detailed, helps identifying and categorising all the stakeholders, processes and external factors involved in the system under analysis, with CATWOE standing for all the components to be analysed to frame the decision problem (Mingers & Rosenhead, 2004): **C**ustomer (people who are the recipients of the systems output); **A**ctor (people who perform the activities of the system); **T**ransformation (the change that the system brings about); ***W**eltanschauung* (worldview – the viewpoint that justifies the activities of the system); **O**wner (person or system who can create, change or destroy the system and who supply the *Weltanschauung*); and **E**nvironment (external systems or constraints that must be taken as given). Although variants of CATWOE analysis are available (Pownall, 2012) – such as the BATWOVE analysis, in which C is replaced with B for Beneficiaries and V is added for Victims; or the TASCOI analysis, in which there is an identification of Actors, Customers and Owners together with Suppliers and Interveners –, CATWOE was shown to provide the appropriate detail for characterising the human resources (training) planning context. In fact, several studies have shown that the CATWOE analysis is the most commonly used approach and that is simple and effective in implementing SSM (Wang et al., 2015).

Based on the CATWOE analysis, the aim and objectives for workforce planning can be clearly set. Starting from such a structured view of the training problem, an optimization approach based on a MILP model is proposed. This model thus reflects the diversity of objectives of the stakeholders, with key stakeholders including the ones identified by customers, actors and owners of the system (C, A and O from CATWOE) and with the different objectives arising according to the stakeholders to whom the planning model is supposed to inform. Moreover, the model reflects training specificities that characterize the training process itself (according to the transformation process T from CATWOE), as well as its key constraints (according to the environmental constraints associated to the medical education system (E from CATWOE)). A MILP model is suggested because planning the medical training process involves decisions that should be modelled by both integer and continuous variables (with inherence to the opening of specialty vacancies and to increases/decreases in vacancies, respectively). This choice is in line with the literature earlier presented in which MILP models have been the preferred approach in strategic planning contexts.

Accordingly, the following steps should be followed so as to apply the proposed multi-methodology:

- Step I: Problem structuring stage, which includes the following sub-steps:
    - Step I.A: Perform the CATWOE analysis, by identifying a) the weltanschauung, b) the transformation process, detailing the key inputs and outputs of the system, c) the key stakeholders, distinguishing between customers, actors and owners, and d) the environment, detailing the environmental constraints that are likely to affect the transformation process identified before;
    - Step I.B: Afterwards, using as a basis the information from the CATWOE analysis on who are the customers, actors and owners of the system, identify which is(are) the group(s) of stakeholders to whom the planning model is supposed to inform. Following this, identify which planning objectives traduce the viewpoints and concerns of that(those) group(s);
    - Step I.C: To conclude the problem structuring stage, the results from performing the previous sub-steps should be presented and discussed with key stakeholders in the area. This sub-step enables validating the first version of the CATWOE analysis obtained under Step I.A and to confirm upon policy objectives identified under Step I.B.
- Step II: Planning Modelling stage, in which the MILP model should be applied, reflecting the following results from the problem structuring stage:
    - Result 1: The diversity of objectives of stakeholders (according to the concerns identified for the selected group(s) of stakeholders, as mapped in Step I.B);
    - Result 2: The training specificities that characterize the training process itself (according to the transformation process identified in Step I.A);
    - Result 3: The key constraints of the training process (according to the environmental constraints associated to the medical education system, as identified in Step I.A).

The use of a structured method to describe the problem ensures the optimization model is both a faithful representation of reality and that the stakeholders' concerns are effectively being taken into consideration in the approach. All the different objectives captured under the CATWOE analyses are then mapped into objective functions and constraints that the optimization model needs to abide to.

The details of applying the two stages of the proposed multi-methodology displayed in Figure 2 are described in the next sub-sections.

3.1 Structuring the medical training problem

This section describes how the CATWOE analysis was implemented. This structuring of the medical training problem followed the three-step approach above identified:

i. Following Step I.A, key stakeholders involved in the medical training process (through the identification of customers, actors and owners) and the specificities of the medical training process (resulting in the detailed characterization of the transformation process and of the environment) were identified. Policy statements and official documents were used in this step;

ii. Applying Step I.B, key objectives (both tacit and explicit) relevant by stakeholders were identified. Similarly to Step I.A, policy statements and official documents were used as the information basis;

iii. Following Step I.C, the conclusions resulting from the previous steps were discussed and validated with members of the Portuguese Ministry of Health, and adjustments were made to the generated information.

Although the structuring was made with reference to the Portuguese NHS-based system, this system shares many features with NHS systems operating in many countries, and so key results and features can be generically seen as representing those of NHS systems. However one should note that the proposed multi-methodology can be easily adapted for distinct contexts.

3.1.1 CATWOE analysis

The medical training problem structuring is based on a CATWOE analysis, as depicted in Figure 3. This visual representation shows all the elements involved in the system (customers, actors, transformation, *weltanschauung*, owners and environment), thus enabling an understanding on how they relate with each other.

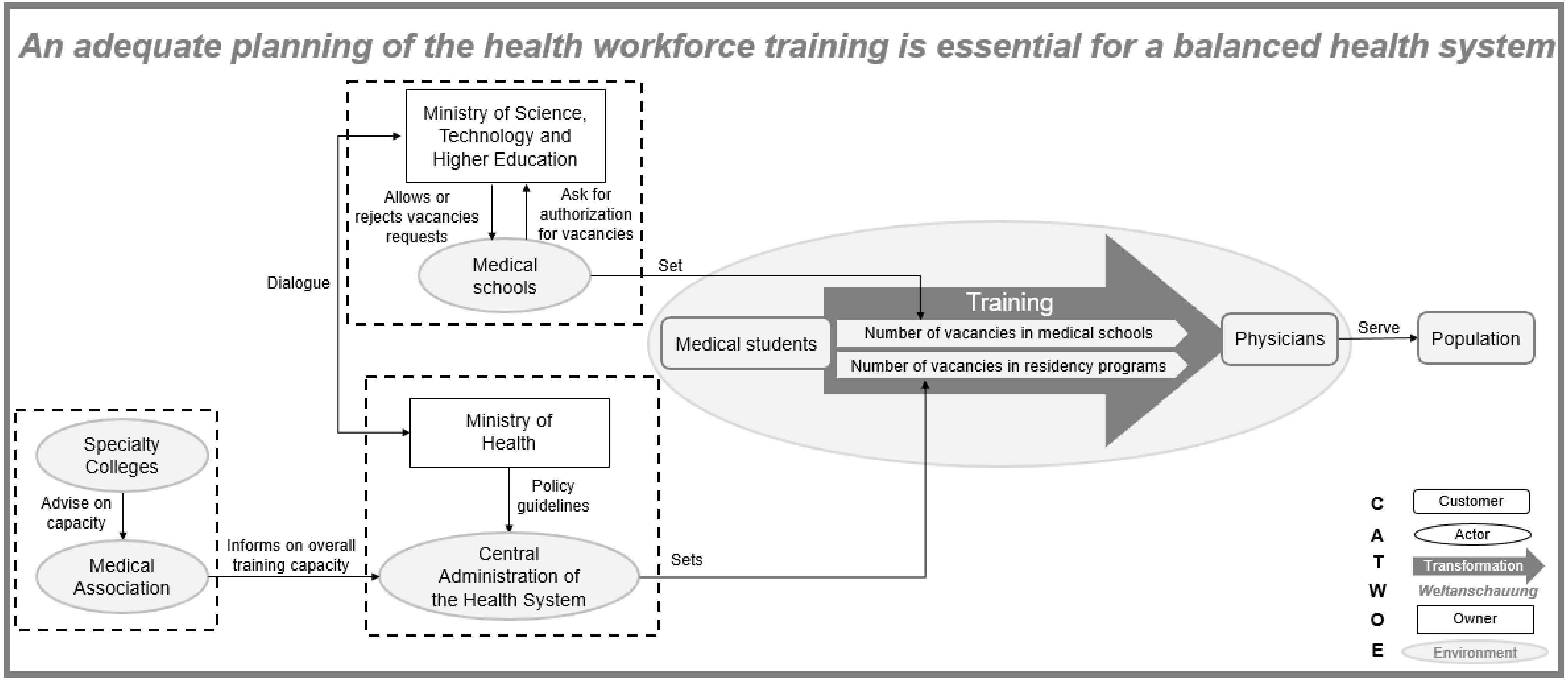

Figure 3. CATWOE analysis to structure the medical training problem (the title stands for the *Weltanschauung*)

Weltanschauung

As mentioned in Section 1, maintaining a balanced health workforce to satisfy the population health needs is a key policy issue, and requires an adequate planning of the health workforce training (Perfilieva & Buchan, 2015). Therefore, for European countries with medical training planning issues, the worldview is as follows: *An adequate planning of the medical training is essential for a balanced health system*. A single worldview was generated in our case, similarly to what has been found in several studies (see, for instance, Kotiadis et al. (2013) and Wang et al. (2015)).

Transformation

Medical training constitutes the transformation process of interest to this study. It involves medical students as inputs and physicians (with or without specialty) as outputs, and it is represented by a grey arrow in Figure 3. Along the process the number of vacancies that are required in medical schools and in residency programs is defined. The medical training pathway depicted in Figure 4 was built with information from Simões et al. (2017).

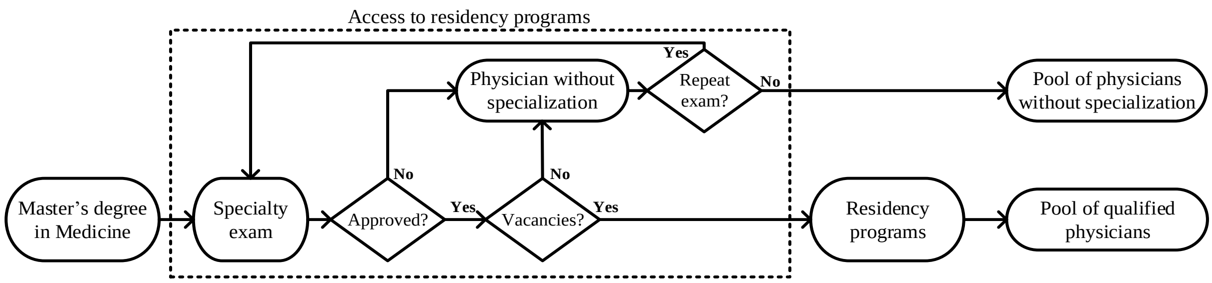

Figure 4. Flowchart depicting the medical training pathway

Medical training starts in the medical schools once students graduate in medicine (leading to a Master degree), with graduation taking at least 6 years to complete (DGES, 2017). Once graduation is completed, the recent graduates must pass an exam (called specialty exam), which when successfully taken enables them to apply to a residency program (Ministry of Health, 2015a). The acceptance on a given specialty is subject to the grade obtained in the specialty exam and to the number of available vacancies per specialty (Amorim-Lopes, Soares, et al., 2016). Whenever a student fails the specialty exam or is unable to join a residency program, two options are available: he/she can repeat the exam the following year; or he/she can practice as a general practitioner. Students accepted in residency programs enrol into specialized training which, depending on the specialty chosen, can take between 4 and 6 years. Once this training is concluded, physicians can practice in the public and/or private systems.

Different situations can affect the expected duration and the medical training sequence. During the Master's degree, students may quit the course or fail. It is also possible to fail the specialty exam or delay the conclusion of the medical training (for instance, by taking a gap year). During the residency programs, students may also quit, fail a step in the program or even switch specialty. Furthermore, they may also study abroad, start a PhD and/or take maternity/paternity leaves.

Customers, Actors and Owners

Multiple stakeholders are usually involved in the medical training process, with the following customers, actors and owners from CATWOE being identified:

i. Customers: Medical students; Physicians; and Population;

ii. Actors: Medical schools; Central Administration of the Health System; Medical Association; and Specialty Colleges;

iii. Owners: Ministry of Science, Technology and Higher Education; and Ministry of Health.

These stakeholders can be grouped into two main groups: a first group related with awarding the Master's degree in Medicine, and a second group related with the residency programs and vacancies. Concerning the first, the Ministry of Science, Technology and Higher Education is the entity responsible for this degree, thus being an owner of the system, and being in charge of fixing the *numerus clausus* in Medical Schools. The *numerus clausus* are defined together with Medical Schools, which, as actors, inform on the number of vacancies that are required; and medical students, as customers, are enrolled in the Master's degree in view of becoming physicians after graduation. Regarding the Ministry of Health, it is responsible for the medical specialized training and is also a system owner. Nevertheless, other stakeholders (actors) have also other responsibilities at this level. The Central Administration of the Health System sets the number of residency vacancies based on the information of the overall training capacity provided by the Medical Association (previously advised by the Specialty Colleges), while also taking into account the policy guidelines provided by the Ministry of Health. Moreover, these institutions are involved in defining the programs and periods of training for each specialty, the vacancies per specialty and the maximum capacity of each institution suitable for medical education (Ministry of Health, 2015a). The physicians (customers), after completing the specialty, integrate the health workforce available to satisfy the population (customers) health needs.

Since medical training is a continuous process that combines medical and specialized training, the two entities responsible for the Master's degree and for the residency programs – the Ministries of Science, Technology and Higher Education and of Health – need to dialogue so as to promote a balanced health workforce.

Environment

Medical training is subject to several environmental constraints mainly related to the education system. These include: a) the budget available for medical training; b) the number of professionals available to train medical students and physicians; c) the number of qualified institutions for residency programs; and d) the maximum number of vacancies that can be opened/closed in medical schools and for each specialty. These constraints are imposed by Medical Schools, Central Administration of the Health System, Medical Association and Specialty Colleges. Therefore, these stakeholders are also categorized within the training environment as they provide the resources necessary to achieve the transformation. In fact, according to Wang et al. (2015), within a CATWOE analysis they are considered suppliers of resources, thus belonging to an extra category of stakeholders (environmental groups). The population health needs influence directly the physicians' demand, and consequently the physicians' supply. Accordingly, the population is also considered as part of the environment.

So, based on the CATWOE analysis, it is clear that a key policy issue for any NHS-based system includes maintaining a balanced health workforce that fulfils the population health needs, which is captured by the single *Weltanschauung* that is somewhat defended by all stakeholders. And achieving this balanced workforce requires an adequate planning for the vacancies to open in medical schools and in residency programs, which are decisions that typically involve different groups of stakeholders with different views and perspectives. These different views and perspectives can translate into (often conflicting) objectives to be considered in planning.

3.1.2 Defining the focus of the study

Following the identification of the multiplicity of stakeholders involved in the planning of medical training in the CATWOE analysis, stakeholders' views are then translated into different planning objectives. Accordingly, this article takes the point of developing meaningful models to assist planning for the following stakeholders: the owners of the system – the Ministry of Science, Technology and Higher Education and the Ministry of Health. A summary of the plurality of objectives relevant for our medical training problem owners is depicted in Figure 5, representing these explicit and tacit objectives that were identified based on the analysis of several documents (as detailed below). One should however note that other objectives may be considered if the aim is to develop a planning model that meets the interest of other stakeholder groups.

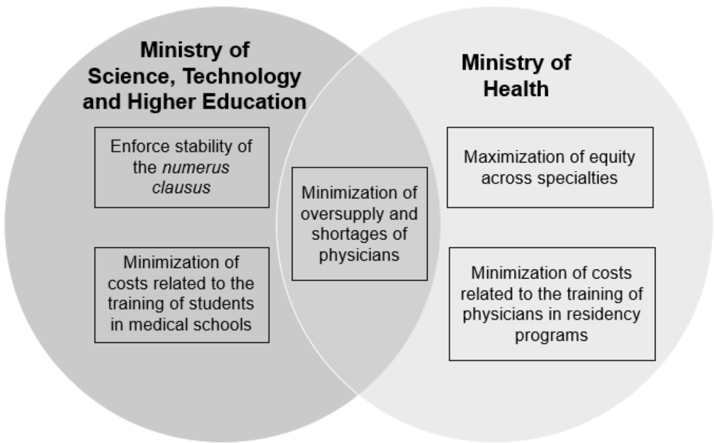

Figure 5. Key objectives and perspectives from owners regarding medical training planning

Within the context of NHS based-systems and of a planned workforce, an equitable and balanced health care workforce at the minimum cost is to be pursued (Baker, 2000; Ministry of Health, 2015b; Senese et al., 2015; Simões et al., 2017). Accordingly, three objectives were defined:

i. **Minimization of oversupply and shortages of physicians** – In order to achieve a balanced health care workforce that meets the population health needs, it is necessary to minimize the imbalances (either in excess and deficit) between the supply and demand of physicians today and also in the future;

ii. **Minimization of costs** – In a context of limited resources, health systems cannot disregard cost considerations as a priority, being thus relevant to consider the minimization of training costs, including the costs of opening and closing vacancies of medical universities and of residency programs;

iii. **Maximization of equity** – For the Ministry of Health, it is not only important to promote an overall match between the supply and demand for physicians, but also to ensure an equitable (and not necessarily equal) distribution of physicians across specialties. This issue is particularly relevant when it is not possible to train all the necessary physicians, and under such circumstances, addressing the prioritization of medical training by specialty is required (with equalization of shortages in absolute or relative terms or the minimization of the worst gap being possible strategies).

For the Ministry of Science, Technology and Higher Education in particular, concerns are not only related with achieving a balanced health workforce and with training costs. A key concern is also to promote a stability in terms of *numerus clausus* (Smits et al., 2010). The main reason being that such a highly specialized teaching should be supported by qualified teachers, as well as by specific facilities and materials, which are expensive resources. Accordingly, this makes it difficult and expensive to enforce quick vacancy adjustments in short periods of time.

Summing up, these are publicly acknowledged objectives that should be taken into account in any state-controlled health system, in which imbalances in the health labour market are to be avoided, resources should be managed efficiently, and also where changes to the *numerus clausus* do not disrupt the Universities' budget.

3.2 Building a mathematical programming model to assist the planning of medical training

After characterizing the medical training context with CATWOE, a multi-objective mixed integer linear programming (MILP) model is proposed to support the strategic planning of medical training and to inform decision-makers on how many vacancies should be opened/closed per year in medical schools and in residency programs. The following subsections present the mathematical details of the proposed model. First, the notation in use is presented. Then, the planning objectives are described, followed by the model's constraints.

The proposed model is generic but can be easily adjusted so as to better reflect other contexts. Particularly, adjustments may be performed on: model constraints, so that these reflect the training specificities of the medical workforce training process, as characterized by the transformation process and the environment within the CATWOE analysis (as identified under Step I.A); and model objectives, so that these reflect the concerns of the stakeholders (in this case, the owners) considered for planning purposes (as identified under Step I.B).

Also, the model is built upon the assumption that the demand with regard to medical school vacancies is inelastic, i.e. the number of students wishing to study medicine far outweigh the number of vacancies. There is ample evidence to support this stance. First, the minimum entry requirements are very high (avg. 19 points out of 20), suggesting a high demand for the course. Second, a large number of students enrol on other courses, waiting for the next opportunity to be transferred to medical schools. Third, many Portuguese students emigrate to foreign countries to pursue their medical studies.

3.2.1 Notation

The notation used for the model formulation is presented below. It should be noted that several model input parameters related with the demand and supply of physicians should reflect population ageing and other demographic changes, as well as medical innovation issues.

Indices

t, k	Time periods, in years
p	Sub-periods of the planning period
y	Year of the Master's degree in medicine
s	Medical specialties

Sets

$T = T^i \cup T^f \cup T^r = \{t_1, t_2, \dots, t_{\lvert T \rvert}\}$	Set of time periods, divided into subsets T^i (time periods representing the first year of each sub-period, $t \in T^i \subseteq T$), T^f (time periods representing the final year of each sub-period, $t \in T^f \subseteq T$) and T^r (remaining time periods, $t \in T^r \subseteq T$)
$P = \{p_1, p_2, \dots, p_{\lvert P \rvert}\}$	Set of sub-periods of the planning period
$Y = \{y_1, y_2, \dots, y_{\lvert Y \rvert}\}$	Set of years of the master's degree in medicine
$S = \{s_1, s_2, \dots, s_{\lvert S \rvert}\}$	Set of medical specialties
$R = \{(t, s): t \in T, p \in P\}$	Set of time periods t belonging to sub-periods p
$M = \{(t, s): t \in T^i, p \in P\}$	Set of time periods t representing the first year of each sub-period p
$N = \{(t, s): t \in T^f, p \in P\}$	Set of time periods t representing the last year of each sub-period p

Parameters

$\Phi_{s,t}/\Psi_{s,t}$	Number of physicians of specialty s available/required at t
$\Omega_{s,p}$	Number of physicians of specialty s required per sub-period p
$\Delta_{s,t}$	Difference between the demand and the supply of physicians per specialty s and per year t ($\Psi_{s,t} - \Phi_{s,t}$)
$\Gamma_{s,p}$	Difference between the demand and the supply of physicians per specialty s and per sub-period p
$\overline{\Delta}_t$	Average difference between the demand and the supply of physicians across specialties at t
ϕ	Duration of the Master's degree in medicine (in years)
φ_s	Duration of specialty s (in years)
α_t	Probability of a Master's degree student finishing the course in due time at t
$\beta_{y,t}$	Probability of a Master's degree student giving up the course in the y^{th} year of the course at t (dropout rates)
$\gamma_{y,t}$	Probability of a Master's degree student failing the y^{th} year of the course at t (attrition rates)
$\delta_{s,t}$	Probability of a physician giving up specialty s at t (dropout rates)
$\varepsilon_{s,t}$	Probability of emigration of a physician with the specialty s at t (emigration rates)
$pi_{s,t}$	Number of physicians with the specialty s deciding to immigrate at t (migration rates)

$nh_{s,t}$	Number of hours of training required per physician of specialty s per t
$v_{s,t}^0$	Number of opened residency vacancies for specialty s at t
v_t	Maximum number of vacancies available for all specialties at t (capacity of the education system)
$v_{s,t}^{open}/v_{s,t}^{close}$	Maximum number of vacancies that can be opened/closed for specialty s at t
$\theta_{s,p}$	Maximum number of vacancies to be opened for specialty s per year of the sub-period p
ω_t	Cost (in €) per Master's degree student per t, which includes components of cost associated with opening a vacancy (such as costs related with salaries and materials)
$\mu_{s,t}$	Cost (in €) of the salary paid to a physician doing the specialty s per t
$\lambda_{s,t}$	Cost (in €) of the salary paid to a physician giving training of specialty s per t
$\chi_{s,t}$	Cost (in €) per vacancy closed per specialty s per t

Variables

$P_{s,t}^-/P_{s,t}^+$	Shortage/Oversupply of physicians of specialty s at t
$Q_{s,p}^-/Q_{s,p}^+$	Shortage/Oversupply of physicians of specialty s per sub-period p
$VO_{s,t}/VC_{s,t}$	Number of additional vacancies to be opened/closed for specialty s at t
$\widetilde{VO}_{s,t}/\widetilde{VC}_{s,t}$	Number of additional vacancies to be opened/closed for specialty s at t, considering the attrition, dropout and emigration rates
$VS_{s,t}$	Total number of vacancies to be opened per specialty s at t
VM_t	Total number of vacancies to be opened in the Master's degree in medicine at t
C_t	Maximum capacity to be added to the medical internship at t
$C_{s,t}$	Maximum capacity to be added per specialty s at t
NS_y	Number of students in the y^{th} of the Master's degree in medicine
NS_t	Number of students doing the Master's degree in medicine at t
$NP_{s,t}$	Number of physicians of specialty s at t
W_t^-/W_t^+	Relative shortage/oversupply of physicians for the worst-off specialty (i.e., specialty with the higher shortage/oversupply of physicians at t)
W_t^w/W_t^b	Relative shortage/oversupply of physicians for the worst-off/better-off specialty at t

Figure 6 visually displays the relation between time-related indices and sets, i.e. for indices $t \in T$ and $p \in P$. Accordingly, P represents the set of sub-periods ($\{p_1, p_2, \dots, p_{|P|}\}$) of the complete planning period T, with T ranging from t_1 to $t_{|T|}$. T further includes three different subsets: T^i (first year of each sub-period,$\{t_1^i, t_2^i, \dots, t_{|P|}^i\}$, with $t_1 = t_1^i$); T^f (final year of each sub-period,$\{t_1^f, t_2^f, \dots, t_{|P|}^f\}$, with $t_{|T|} = t_{|P|}^f$); and T^r (remaining years, $\{t_1^{i+1}, t_1^{i+2}, \dots, t_1^{f-1}, \dots, t_{|P|}^{i+1}, t_{|P|}^{i+2}, \dots, t_{|P|}^{f-1}\}$).

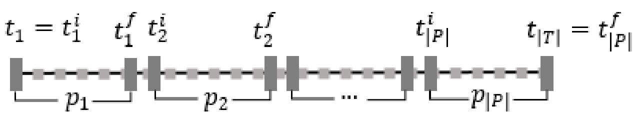

Figure 6. Graphical representation of time-related indices and sets

3.2.2 Objectives

3.2.2.1 Defining the multiple objectives

Following the CATWOE analysis, the multiplicity of policy objectives above described is modelled (see Figure 5). These multiple objectives may be jointly considered for planning or may also be considered independently, depending on the planning circumstances. Accordingly, the following measures operationalise these objectives:

- **Minimization of oversupply and shortages of physicians**: Minimizing the relative shortages and oversupply of physicians across specialties over time, such that the imbalances found between the supply and demand of physicians are minimized. Two alternatives of this measure can be considered:
  a) $\boldsymbol{Min\ f_{1a}}$**:** Minimizing the relative shortages and surpluses of physicians across specialties on a yearly basis. These shortages ($P^-_{s,t}$; Eq. 2) and surpluses ($P^+_{s,t}$; Eq. 3) depend on the annual difference between the demand and supply of physicians (hereafter called gap, indicating the need to open [$\Delta_{s,t} > 0$] or close [$\Delta_{s,t} < 0$] vacancies per specialty s and per year t; Eq. 4), as well as on the number of vacancies that were opened ($VO_{s,t}$) or closed ($VC_{s,t}$) beforehand.

$$Min\ f_{1a} = \sum_{s\in S}\sum_{t\in T}\left(\frac{P^-_{s,t}}{\Psi_{s,t}} + \frac{P^+_{s,t}}{\Psi_{s,t}}\right) \tag{1}$$

$$P^-_{s,t} = \Delta_{s,t} - \sum_{k=t_1}^{t-\varphi_s}(VO_{s,k} - VC_{s,k}) \qquad t \geq (t_1 + \varphi_s), \forall s \in S, \Delta_{s,t} \geq 1 \tag{2}$$

$$P^+_{s,t} = -\,\Delta_{s,t} + \sum_{k=t_1}^{t-\varphi_s}(VO_{s,k} - VC_{s,k}) \qquad t \geq (t_1 + \varphi_s), \forall s \in S, \Delta_{s,t} \leq -1 \tag{3}$$

$$\Delta_{s,t} = \Psi_{s,t} - \Phi_{s,t} \quad \forall s \in S, \forall t \in T \tag{4}$$

  b) $\boldsymbol{Min\ f_{1b}}$**:** So as to allow for a smooth opening/closure of vacancies over time (in line with the concerns of teaching institutions), minimizing the relative shortages and oversupply of physicians across specialties over multi-year sub-periods can be considered (Eqs. 5-7). Accordingly, the planning period turns to be divided into several sub-periods p (with the gap of each sub-period, $\Gamma_{s,p}$, being defined as the average of all the gaps belonging to that sub-period, as given in Eq. 8), thus allowing minimizing the gap at the end of each sub-period (rather than on a yearly basis). This minimization will thus avoid large variations in the number of vacancies over time.

$$Min\ f_{1b} = \sum_{s\in S}\sum_{p\in P}\left(\frac{Q^-_{s,p}}{\Omega_{s,p}} + \frac{Q^+_{s,p}}{\Omega_{s,p}}\right) \tag{5}$$

$$Q^-_{s,p} = \Gamma_{s,p} - \sum_{k=t_1}^{\substack{t-\varphi_s \\ t:(t,p)\in N}} (VO_{s,k} - VC_{s,k}) \qquad \forall s \in S, \forall p \in P, \Gamma_{s,p} \geq 1 \tag{6}$$

$$Q^+_{s,p} = -\,\Gamma_{s,p} + \sum_{k=t_1}^{\substack{t-\varphi_s \\ t:(t,p)\in N}} (VO_{s,k} - VC_{s,k}) \qquad \forall s \in S, \forall p \in P, \Gamma_{s,p} \leq -1 \tag{7}$$

$$\Gamma_{s,p} = \frac{\sum_{t:(t,p)\in R}\Delta_{s,t}}{|P|} \qquad \forall s \in S, \forall p \in P \tag{8}$$

- **Maximization of Equity**: Minimizing shortages and oversupply of physicians across specialties may by itself result in an inequitable distribution of vacancies across specialties. Whenever ensuring such an equitable distribution is a concern, it is relevant to account for the minimization of these differences in the provision of care across specialties, which leads to:
    a) $\boldsymbol{Min\ f_{2a}}$**:** Minimizing the relative shortage/oversupply of physicians for the worst-off specialty (Eq. 9), i.e., for the medical specialty with the highest shortage (Eq. 10) or oversupply (Eq. 11) of physicians throughout the planning horizon. This measure ensures an equitable distribution of vacancies across medical specialties by targeting the specialties with higher absolute gaps – i.e., by opening a higher number of vacancies in specialties with higher levels of demand and lower levels of supply.

$$Min\ f_{2a} = \sum_{t \in T} (W_t^- + W_t^+) \tag{9}$$

$$W_t^- \geq \frac{P_{s,t}^-}{\Psi_{s,t}} \qquad \forall s \in S, \forall t \in T \tag{10}$$

$$W_t^+ \geq \frac{P_{s,t}^+}{\Psi_{s,t}} \qquad \forall s \in S, \forall t \in T \tag{11}$$

    b) $\boldsymbol{Min\ f_{2b}}$: Minimizing absolute deviations between the relative shortage/oversupply of physicians and the mean gap across specialties throughout the planning horizon (Eqs. 12-13). This measure will result in increasing the vacancies for specialties whose gap is higher than the average gap, as well as in diminishing it for specialties whose gap is below the average gap.

$$Min\ f_{2b} = \sum_{t \in T} \sum_{s \in S} \left| \frac{P_{s,t}^-}{\Psi_{s,t}} - \frac{P_{s,t}^+}{\Psi_{s,t}} - \overline{\Delta}_t \right| \tag{12}$$

$$\overline{\Delta}_t = \frac{\sum_{s \in S} \frac{\Delta_{s,t}}{\Psi_{s,t}}}{|S|} \qquad \forall t \in T \tag{13}$$

    c) $\boldsymbol{Min\ f_{2c}}$: Minimizing the difference between the relative shortage/oversupply of physicians for the worst-off specialty (i.e., specialty with the highest shortage/oversupply throughout the planning horizon; Eq. 15) and the relative shortage/oversupply of physicians for the better-off specialty (i.e., specialty with the lowest shortage/oversupply throughout the planning horizon; Eq. 16) (Eq. 14).

$$Min\ f_{2c} = \sum_{t \in T} (W_t^w - W_t^b) \tag{14}$$

$$W_t^w \geq \left(\frac{P_{s,t}^-}{\Psi_{s,t}} + \frac{P_{s,t}^+}{\Psi_{s,t}} \right) \tag{15}$$

$$W_t^b \leq \left(\frac{P_{s,t}^-}{\Psi_{s,t}} + \frac{P_{s,t}^+}{\Psi_{s,t}} \right) \tag{16}$$

- **Minimization of cost - $Min\, f_3$**: Minimization of the total cost associated with the medical training process throughout the planning horizon (Eq. 17). This total cost includes costs supported by the educational sector (in line with the concerns of the Ministry of Science, Technology and Higher Education) and costs supported by the health sector (in line with the concerns of the Ministry of Health). Concerning the cost supported by the educational sector, this represents the cost per Master's degree student in medicine (first term, Eq. 17). On the other hand, the costs supported by the health sector include the salaries paid to physicians doing a specialty (second term, Eq. 17), the salaries paid to the physicians giving medical training (third term, Eq. 17), and the cost of closing vacancies (fourth term, Eq. 17). All costs used as parameters in Eq. 17 are adjusted to inflation effects.

$$Min\, f_3 = \left(\sum_{t\in T} (NS_t \times \omega_t) + \sum_{t\in T} \sum_{s\in S} \left(NP_{s,t} \times \mu_{s,t} + NP_{s,t} \times nh_{s,t} \times \lambda_{s,t} + VC_{s,t} \times \chi_{s,t} \right) \right) \quad (17)$$

$$NS_t = \begin{cases} \sum_{t\prime=t^0}^{t} VM_t & t < \phi \\ \sum_{t'=t-\phi}^{t} VM_t & t > \phi \end{cases} \quad (18)$$

$$NP_{s,t} = \begin{cases} \sum_{t\prime=t^0}^{t} VS_{s,t} & t < \varphi_s, \forall s \in S \\ \sum_{t'=t-\varphi_s}^{t} VS_{s,t} & t > \varphi_s, \forall s \in S \end{cases} \quad (19)$$

The number of students enrolled in the Master's degree in medicine per year (NS_t) is given by Eq. 18 and the number of physicians doing each medical specialty per year ($NP_{s,t}$) is given by Eq. 19. These variables are calculated based on the total number of vacancies opened in the Master's degree (VM_t) and the total number of vacancies opened in medical specialties ($VS_{s,t}$), as it will be detailed further in Section 3.2.3 (Eqs. 25 and 31). The details on the number of vacancies to be closed at t ($VC_{s,t}$) are also presented in Section 3.2.3 (Eq. 24).

3.2.2.2 Dealing with the multiple objectives

As shown in the literature, when there is need to address multiple objectives, in general there is no single optimal solution that simultaneously optimizes all the objectives. Under these circumstances, one can identify the so-called Pareto optimal or non-dominated solutions, or alternatively model preferences from key stakeholders and policy-makers in the area so as to prioritize these multiple objectives and identify the optimal solution (Cohon, 1978). Given the objective of building models with a potential to address concerns of several stakeholders, the option was to explore Pareto optimal or non-dominated solutions. This approach makes it possible to evaluate alternative solutions without making explicit or modelling stakeholders' specific preferences, which turns to be particularly relevant in cases in which stakeholders may not be interested in making explicit their structure of preferences. Accordingly, the first of these approaches is adopted in this study – whenever more than one objective needs to be accounted for, a subset of the Pareto-optimal set will be determined by applying the augmented ε-constraint method proposed by Mavrotas (2009).

This method is a novel version of the conventional ε-constraint method that solves its well-known pitfalls, namely: i) it calculates the range of each objective function over the efficient set; ii) it guarantees the efficiency of the obtained solution; and iii) it allows for a faster resolution for problems with more than two objective functions. In practice this method encompasses the selection of one objective to be optimized, while the remaining are imposed as constraints (more details on this method can be found in Mavrotas (2009)). Based upon this, the following procedure is adopted to define the Pareto frontier:

a. A payoff table composed by the optimal planning values for each individual objective is calculated;
b. Based on the payoff table, the range of planning values for each objective (between the best and the worst) is determined;
c. The grid points for the different objectives are defined by dividing the ranges of planning values into equal intervals, with the number of intervals depending on the number of solutions found adequate for the Pareto frontier;
d. The selected objective function is maximised or minimised while imposing as minimum/maximum value each one of the grid points obtained for the remaining objectives (the ones used as constraints).

The following example illustrates how this method should be applied. This example assumes that policy-makers consider as key objectives the minimization of costs and the maximization of equity for the worst-off specialties. Accordingly, the cost is selected as the objective to be minimized, and so the objective function should now be given by Eq. 20, whereas the equity objective should be written as a constraint, according to Eq. 21. Note that different versions of Eqs. (20-21) are thus possible depending on the objectives considered by policy-makers. In these equations *eps* stands for a small number, usually between 10^{-3} and 10^{-6}, s_{2a} represents slack or surplus variables and e_{2a} depends on the minimum and maximum values of the equity objective and on the number of grids points selected for building the Pareto frontier.

$$min(f_3 + eps \times s_{2a}) \quad (20)$$

$$f_{2a} - s_{2a} = e_{2a} \quad (21)$$

3.2.3 Constraints

The model makes use of the following set of constraints. These constraints depict the medical training pathway and are mostly related to limitations to medical training imposed by the educational and the health system.

3.2.3.1 Opening and closure of vacancies per specialty

The opening and closure of vacancies per specialty is limited by $\theta_{s,p}$, as defined in Eq. 22. This parameter determines the maximum number of vacancies that should be opened ($\theta_{s,p} > 0$) or closed ($\theta_{s,p} < 0$) on a yearly basis in order to close the gap per specialty s and per sub-period p:

i. For the first sub-period, $\theta_{s,p}$ is given by the average gap of the first sub-period divided by the number of years in which it is possible to open or close vacancies, corrected by the duration of each medical specialty (if a specialty takes 4 years to be concluded, the final 4 years of the sub-period should not be considered – no physicians will be concluding specialty during that period) (first branch, Eq. 22);

ii. For the following sub-periods, $\theta_{s,p}$ is given by the difference between average gaps of two consecutive sub-periods divided by the number of years in which it is possible to open or close vacancies (second branch, Eq. 22).

$$\theta_{s,p} = \begin{cases} \dfrac{\Gamma_{s,p}}{|P| - \varphi_s} & p = p_1, \forall s \in S \\ \dfrac{\Gamma_{s,p} - \Gamma_{s,(p-1)}}{|P|} & p > p_1, \forall s \in S \end{cases} \quad (22)$$

Opening of vacancies

The opening of vacancies should only occur when $\theta_{s,p} > 0$, i.e., when the number of existing vacancies is not enough to meet the demand for physicians. Under such circumstances, the number of vacancies to open per specialty and per year cannot exceed $\theta_{s,p}$ (Eq. 23):

i. For the first sub-period, vacancies can be opened from the first year until the last year minus the duration of the specialty – if a specialty takes 3 years to be concluded, closing the gap in *t* implies that vacancies should be opened before *t-3* (first branch, Eq. 23);

ii. For the following sub-periods, the opening of vacancies can occur in the period between the first year of the sub-period minus the duration of the specialty and the last year of the sub-period minus the duration of the specialty (second branch, Eq. 23).

$$VO_{s,t} \leq \begin{cases} \theta_{s,p} & \forall s \in S, p = p_1 : (s,p) \in R, p_1 \leq t \leq p_{|P|} - \varphi_s, \theta_{s,p} > 0 \\ \theta_{s,p} & \forall s \in S, p > p_1 : (s,p) \in R, p_1 - \varphi_s \leq t \leq p_{|P|} - \varphi_s, \theta_{s,p} > 0 \\ 0, otherwise & \end{cases} \quad (23)$$

In this formulation, the number of vacancies is not bounded by any operational constraint such as by the availability of specialists to supervise medical residents. Accordingly, the solution provided may far exceed the available training capacity. However, such constraints can be added if they are deemed as appropriate and data is available (note that currently there is no objective evidence on the current availability of specialists to supervise medical students).

Closure of vacancies

The closure of vacancies should only occur when $\theta_{s,p} < 0$, i.e. when the number of existing vacancies is higher than required to meet the demand for physicians. In such case, the number of vacancies to close per specialty and per year cannot exceed $\theta_{s,p}$ (Eq. 24):

i. For the first sub-period, vacancies can be closed from the first year until the last year minus the duration of the specialty (first branch, Eq. 24);

ii. For the following sub-periods, the closure of vacancies can occur in the period between the first year of the sub-period minus the duration of the specialty and the last year of the sub-period minus the duration of the specialty (second branch, Eq. 24).

$$VC_{s,t} \leq \begin{cases} -\theta_{s,p} & \forall s \in S, p = p_1 : (s,p) \in R, p_1 \leq t \leq p_{|P|} - \varphi_s, \theta_{s,p} > 0 \\ -\theta_{s,p} & \forall s \in S, p > p_1 : (s,p) \in R, p_1 - \varphi_s \leq t \leq p_{|P|} - \varphi_s, \theta_{s,p} > 0 \\ 0 \; otherwise & \end{cases} \quad (24)$$

One should note that vacancies to be closed are distinguished by vacancies to be opened by having a negative value.

Total number of vacancies to open per specialty

Eq. 25 determines the total number of vacancies to be opened per specialty s at t, which is given by summing the number of opened base vacancies and the number of additional vacancies to be opened and subtracting the number of vacancies to be closed on an yearly basis. This is only valid for $t < t_{|T|} - ds_s$, since from this point onwards no new vacancies will result in new physicians concluding the specialty before $t_{|T|}$.

$$VS_{s,t} = \begin{cases} v_{s,t}^0 + VO_{s,t} - VC_{s,t} & t < t_{|T|} - \varphi_s, \forall s \in S \\ v_{s,t}^0 & t \geq t_{|T|} - \varphi_s, \forall s \in S \end{cases} \quad (25)$$

Number of vacancies to open and close considering the dropout, immigration and emigration rates

The number of vacancies to be opened/closed may need to be adjusted due to several situations that need to be accounted for. First, students admitted to the training programs may choose to emigrate to practice in a foreign country (with an associated probability $\varepsilon_{s,t}$) or even give up the specialty (with an associated probability $\delta_{s,t}$). Secondly, physicians studying or practicing abroad may decide to immigrate ($pi_{s,t}$), thus diminishing the need for new vacancies. Under these circumstances, the number of vacancies to be opened/closed given in Eqs. (23-24) should be substituted by Eqs. (26-27).

$$\widetilde{VO}_{s,t} = VO_{s,t} \times \left(1 + \delta_{s,t} + \varepsilon_{s,t}\right) - pi_{s,t+\varphi_s} \; \forall s \in S, \forall t \in T \quad (26)$$

$$\widetilde{VC}_{s,t} = VC_{s,t} \times \left(1 - \delta_{s,t} - \varepsilon_{s,t}\right) + pi_{s,t+\varphi_s} \; \forall s \in S, \forall t \in T \quad (27)$$

3.2.3.2 Capacity of the medical training system

The planning of medical training must consider the capacity of the physicians' training system – including maximum capacities for both the opening and closure of vacancies.

Maximum limit of vacancies to open

Eq. 28 ensures that the number of additional vacancies to be opened per specialty s at t cannot exceed the sum between the maximum number of vacancies and the number of vacancies to be closed per specialty s subtracted by the number of base vacancies. Eq. 29 imposes a similar limit, but considering that a capacity limit is imposed for all the specialties.

$$\widetilde{VO}_{s,t} \leq v_{s,t}^{open} + \widetilde{VC}_{s,t} - v_{s,t}^{0} \quad \forall s \in S, \forall t \in T \tag{28}$$

$$\sum_{s \in S} \widetilde{VO}_{s,t} \leq v_t + \sum_{s \in S} \widetilde{VC}_{s,t} - \sum_{s \in S} v_{s,t}^{0} \quad \forall t \in T \tag{29}$$

Maximum limit of vacancies to close

Eq. 30 defines that the maximum number of vacancies to be closed per specialty s at t cannot exceed a maximum value ($v_{s,t}^{close}$). This maximum number of closures may be defined by planners in the area.

$$\widetilde{VC}_{s,t} \leq v_{s,t}^{close} \quad \forall s \in S, \forall t \in T \tag{30}$$

3.2.3.3 Medical training pathway

Medical training involves two distinct stages: the first corresponds to the Master's degree in medicine; and the second to the residency programs, whose duration depends on the type of specialty. Eqs. 31 and 32 model the progression of students throughout these two stages.

Eq. 31 relates the vacancies to be opened in each specialty ($VS_{s,t}$) with the vacancies to be opened in the Master's degree of medicine (VM_t), considering the different training periods and the probability of students finishing the Master's degree in due time (α_t):

i. For periods $t < t_{|T|} - \phi$, all the students beginning the Master's degree at *t* and concluding it in due time should have a vacancy in a specialty (independently of the specialty) as soon as they conclude the degree (i.e., after ϕ years) (first branch, Eq. 31);

ii. After $t_{|T|} - \phi$, only the base vacancies will be opened for specialties, and no additional vacancies are opened because students starting the Master's degree after $t_{|T|} - \phi$ will not conclude the degree before $t_{|T|}$ (second branch, Eq. 31).

One should note that students not concluding the Master's degree in due time are no longer followed by the model.

$$VM_t = \begin{cases} \dfrac{1}{\alpha_t} \times \sum_{s \in S} VS_{s,(t+\phi)} & t < t_{|T|} - \phi \\ \dfrac{1}{\alpha_t} \times \sum_{s \in S} v_{s,t}^{0} & t \geq t_{|T|} - \phi \end{cases} \tag{31}$$

On the other hand, Eq. 32 determines the number of students per year y of the Master's degree (NS_y), taking into account the probability of students failing (attrition rates) or giving up (dropout rates) the degree in the y^{th} year at time t:

i. In the first year of the Master's degree ($y = y_1$), the number of students is equal to the total number of open vacancies (first branch, Eq. 32).

ii. Considering the possibility of failing and/or drooping the Master's degree, the number of students in y^{th} year is equal to the number of students in the $(y+1)^{th}$ year corrected by attrition and dropout rates (second branch, Eq. 32).

iii. Finally, the number of students on the last year of the Master's degree ($y = y_{|T|}$) is equal to the number of students starting the specialty (third branch, Eq. 32).

$$NS_y = \begin{cases} VM_{t-\phi} & y = y_1, \forall s \in S, \forall t \in T \\ \dfrac{NS_{(y+1)}}{1 - \beta_{y,t} - \alpha_t} & y_1 < y < y_{|T|}, \forall s \in S, \forall t \in T \\ \sum_{s \in S} \left(v^0_{s,t} + \widetilde{VO}_{s,t} - \widetilde{VC}_{s,t} \right) & y = y_{|T|}, \forall s \in S, \forall t \in T \end{cases} \quad (32)$$

4. Case Study

This section presents the results from applying the *Multi-PiHLOT* at the national level in Portugal for the 2017-2050 period, thus illustrating how it can assist planning decisions related to medical training. The selection of such a long planning period took into account the specificities of medical training. On the one hand, it accommodates a minimum training period (10-12 years) that is required to complete the medical training process – 6 years for the Master's degree in medicine, followed by the residency program, which can long 4 to 6 years, depending on the medical specialty. On the other hand, it also includes an extra 18-20 years period to ensure the *numerus clausus* stability, since this aspect was mentioned as being of the utmost importance for the Ministry of Science, Technology and Higher Education. Nevertheless, one is aware that planning rarely takes into account such a long horizon and that this period may be adapted according to policy-makers' views.

The policy questions considered relevant for analysis are first described in this section, followed by the dataset used, and finally the results obtained for each policy question are analysed.

The MILP model was implemented in the General Algebraic Modelling System (GAMS) 23.7 with CPLEX 12 on a Two Intel Xeon X5680, 3.33 GHz computer with 12 GB RAM.

4.1 Selected policy questions

Decisions related to the medical training in Portugal involve the Ministry of Science, Technology and Higher Education and the Ministry of Health, which are interested in the following policy questions (as discussed in the workshop with members of the Portuguese Ministry of Health):

- **Policy Question A:** How much would it cost to ensure a full demand satisfaction, i.e., how many vacancies should be opened such that all the required physicians are trained, while simultaneously minimizing medical training costs? For this analysis it is imposed that the oversupply/shortage of physicians is null for all specialties (Eq. 33).

$$W_t^-, W_t^+ = 0 \quad \forall t \in T \tag{33}$$

- **Policy Question B:** Given the available capacity of the training system, how should existing vacancies be redistributed so as to minimize the oversupply and shortages of physicians, while simultaneously ensuring the minimization of medical training costs?
- **Policy Question C:** Given the available capacity of the training system, how should existing vacancies be redistributed so as to maximize equity in the distribution of specialties, while simultaneously ensuring the minimization of medical training costs?

Table 1 presents a summary on these policy questions, showing that a different set of values for the objective functions is used when running the model under each case.

Table 1. Policy questions under study

<table>
<tr><th>Policy Question</th><th>Model's objectives</th><th colspan="2">Observations</th></tr>
<tr><td>A</td><td>Minimization of total cost (Eqs. 17-19)</td><td colspan="2">Total cost is minimized and a full demand satisfaction is imposed.</td></tr>
<tr><td>B</td><td>Minimization of total cost (Eqs. 17-19) & Minimization of oversupply and shortages of physicians (Eqs. 5-8)</td><td rowspan="2">The two objectives are accounted for through the augmented ε-constraint method, with the total cost being taken as the objective to be minimized.</td><td>Relative shortages and oversupply of physicians across specialties is minimized according to Eqs. 5-8, thus also ensuring a smooth opening/closure of vacancies over time.</td></tr>
<tr><td>C</td><td>Minimization of total cost (Eqs. 17-19) & Maximization of equity (Eqs. 9-11)</td><td>Equity is maximized according to Eqs. 9-11, thus ensuring the minimization of the relative shortage/oversupply of physicians for the worst-off specialty.</td></tr>
</table>

4.2 Dataset

The dataset used includes a wide range of information, namely:

- Demand and supply for physicians between 2017 and 2050, disaggregated by specialty (Amorim-Lopes, Almeida, et al., 2016);
- Duration of the Master's degree in medicine (DGES, 2017) and of specialties (data provided by the Central Administration of the Health System [ACSS]);
- Number of residency vacancies per specialty at the beginning of 2017 (data provided by the ACSS);
- Maximum number of vacancies available per specialty (data provided by the ACSS);
- Costs incurred with the medical training (data provided by the ACSS and the Ministry of Science, Technology and Higher Education [MSTHE]): cost per Master's degree student; salary

paid to physicians doing the specialty; salary paid to physicians giving training; and cost of closing existing vacancies;

- Approval rate in the Master's degree in medicine (data provided by the MSTHE);
- Attrition and dropout rates in residency programs (Amorim-Lopes, Almeida, et al., 2016);
- Emigration rates of physicians (data provided by the Medical Association).

Also, whenever a smooth opening/closure of vacancies over time is preferred, it is considered that the planning period is divided into tree sub-periods (see Figure 7) – from 2017 to 2030; from 2031 to 2040; and from 2041 to 2050 (i.e., 34 years divided into 3 sub-periods). A total of 43 specialties are considered in the case study (see list in Appendix B).

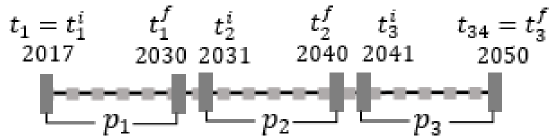

Figure 7. Planning period and sub-periods considered in the case study

Regarding the demand and supply for physicians input parameters, it should be considered that these were generated by an agent-based simulation model that factored workforce (physicians grow old, retire and die) and population demographic changes. Such input estimates (as described in Amorim-Lopes, Almeida, et al. (2016)) not only took into consideration three demographic projections developed by the Portuguese National Statistics Office, but also that health care consumption is not constant throughout life and that healthcare spending patterns maintain. Such input estimates project that despite it is expected a population decrease, ageing plays a much bigger effect in the demand for health care, being a key driver for the planning of vacancies in the optimization model. Given that the planning horizon runs until 2050, the model does not consider the ageing of new physicians as it is unlikely that they die or retire in the adopted planning period.

More details about the dataset used in this case study is available in Appendix C.

4.3 Results

The model is run for each one of the policy questions above presented, and key results are presented below. The computational results obtained for all these policy questions are presented in Appendix A, showing that the model performs efficiently in computational terms for all the cases.

4.3.1 Policy Question A

Fully satisfying the demand involves a total cost of 20240 million Euros for the entire planning period (Table 2) – assuming the current budget available for physicians' training, this cost is 18% higher than the total budget available for the same period (17300 million Euros). Results also show that salaries paid to physicians giving medical training (9 089 million Euros) and to physicians on a specialty residency (7612 million euros) correspond to 83% of this total cost.

Table 2. Costs incurred with the medical training process (in million Euros)

Master's degree	3538
Salaries paid to physicians doing specialties	7612
Salaries paid to physicians giving medical training	9089
Closing existing vacancies	1.5
Total Cost	**20240**

Fig. 8 depicts the total cost incurred with medical training over time, along with the capacity needed to ensure a full demand satisfaction (in terms of the *numerus clausus* and the total number of residency vacancies). These results are presented for the 2020-2050 period because changes in the *numerus clausus* in 2016 are only reflected in residency vacancies from 2020 onwards. Results show that there is a need to have an increase in both *numerus clausus* and residency vacancies in the short term. For instance, an increase of 908 residency vacancies is required by 2020 (representing an increase of 56%, when compared to the 1620 vacancies currently available), which corresponds to a cost of approximately 650 million euros in 2020. On the other hand, from 2020 onwards a decrease should be observed in the number of vacancies needed over time – this happens because as soon as physicians are trained, the shortage of physicians will be reduced in the following years. These results clearly indicate that the current supply of physicians is far from being able to satisfy all the demand in Portugal, and so the medical training capacity should increase significantly so as to ensure a full demand satisfaction.

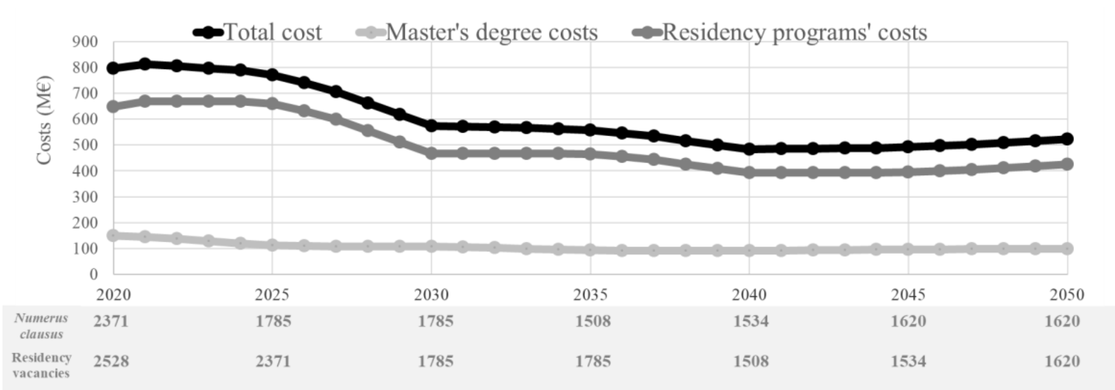

Figure 8. Total cost incurred with medical training (in million Euros) and capacity of the training system (in terms of the *numerus clausus* and the total number of residency vacancies) over the 2017-2050 period

4.3.2 Policy Question B

Figure 9 presents the results obtained for the relative shortage/oversupply of physicians across specialties over the 2017-2050 period, with each objective function value being obtained by summing 43 ratios, each ratio being the number of physicians in excess/deficit divided by the number of physicians required for full demand satisfaction per specialty. These results are relevant when the aim is to analyse how to redistribute existing vacancies across specialties when considering cost- and shortages/oversupply-related concerns. Particularly, Figure 9 depicts the Pareto frontier obtained when running the multi-objective model when simultaneously considering the minimization of costs (f_3) and the minimization of oversupply and shortages of physicians across specialties (f_{1b}). Solution A represents the solution with the minimum total cost (approximately 16864 million Euros) and with the highest relative level of shortage/oversupply of physicians over time - this solution is therefore characterized by a reorganization of residency vacancies that results in the lowest level of demand coverage. As we move from solution A to G, the investment in medical training increases, along with the total cost incurred in this process, and the relative level of shortage/oversupply of physicians is simultaneously decreased. Accordingly, solution G corresponds to the solution with the highest coverage of demand for physicians – this solution presents substantial adjustments on the *numerus clausus* and on the residency vacancies when compared to the current distribution, thus being associated with the highest total cost for the training system (approximately 17300 million Euros).

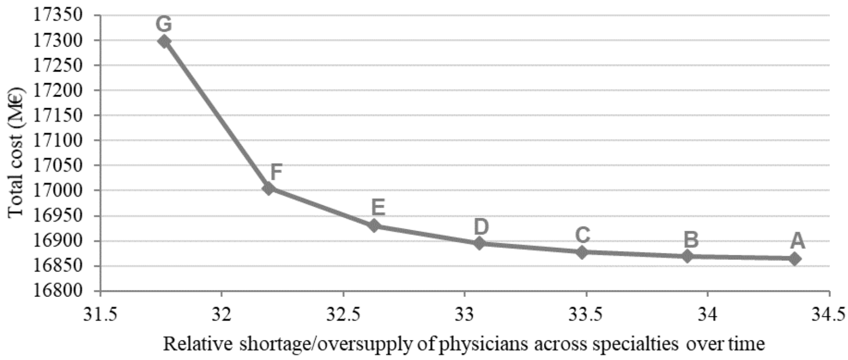

Figure 9. Pareto frontier obtained when running the multi-objective model when simultaneously considering the minimization of costs (f_3) and the minimization of shortages and oversupply of physicians across specialties (f_{1b}) over the 2017-2050 period

As one moves from solution A to G, vacancies are redistributed from specialties with higher durations to specialties with lower ones. This redistribution will result in improvements to the level of demand satisfaction, since it allows for training a higher number of specialized physicians within the same time frame. Nevertheless, it will also result in increasing training costs, mainly due to the increase in the *numerus clausus* in medical schools (note that in this case specialty vacancies remain constant and equal to the current values, i.e., 1620). Based on these results, it is possible to conclude that the minimization of shortages/oversupply of physicians across specialties may result in an inequitable distribution of vacancies across specialties, with a lower investment taking place for specialties with higher durations.

4.3.3 Policy Question C

Figure 10 presents the results obtained by minimizing the relative shortage/oversupply of physicians for the worst-off specialties over the 2017-2050 period, in which each solution represents the (single) ratio between the number of physicians in excess/deficit and the number of physicians that are required for full demand satisfaction for the worst-off specialty. These results are obtained when the aim is to explore how to redistribute existing vacancies (i.e., 1620 specialty vacancies per year) across specialties while considering cost- and equity-related concerns. Particularly, Figure 10 depicts the Pareto frontier obtained when running the multi-objective model when simultaneously considering the minimization of costs (f_3) and the maximization of equity, with equity being measured by the relative shortage/oversupply of physicians for the worst-off specialties throughout the planning horizon (f_{2a}). Solution A represents the solution with the minimum total cost and worst level of equity (i.e., with the highest level of relative shortage/oversupply of physicians for the worst-off specialties); and as one moves from solution A to G, the training cost increases and the equity improves, with solution G being characterized by the maximum cost and best equity level.

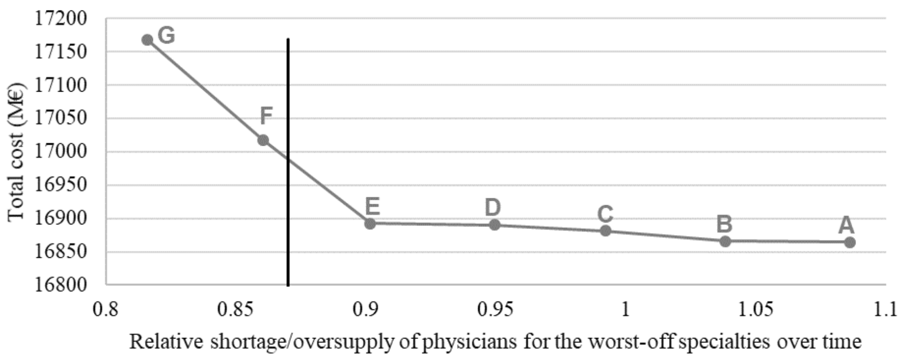

Figure 10. Pareto Frontier obtained when running the proposed multi-objective model when simultaneously considering the minimization of costs (f_3) and the maximization of equity (f_{2a}) over the 2017-2050 period

First, as we move from solution A to E, the equity level decreases (i.e., improves) steadily whereas the total cost increases slightly (from approximately 16864 to 16892 million Euros). In particular, from solution A to B and from solution D to E, the total cost remains approximately constant, meaning that it is possible to reorganize the residency vacancies so as to improve the equity in the distribution of vacancies across specialties without increasing medical training costs. Therefore, these solutions correspond to different allocations of vacancies across specialties, with vacancies being redistributed to specialties with higher levels of shortage/oversupply of physicians. One should note that it is possible to improve the equity as one moves from solution A to E by opening 4 (A to B), 43 (B to C), 29 (C to D) and 5 (D to E) additional vacancies for the entire planning period (see Table 3). Since the maximum capacity per year is 1620 vacancies, it is possible to observe that none of these solutions uses all the available vacancies in each sub-period (22680 for the first one and 16200 for the others).

Table 3. Vacancies to open in residency programs for the different sub-periods and the entire planning period

Legend: *V*- number of vacancies to open, *ΔV* - difference between *V* of consecutive solutions

Solution	**1st sub-period (2017-2030)**		**2nd sub-period (2031-2040)**		**3rd sub-period (2041-2050)**		**Planning period (2017-2050)**	
	V	***ΔV***	***V***	***ΔV***	***V***	***ΔV***	**Total *V***	***ΔV***
A	22629	-	15606	-	15565	-	53800	-
B	22633	4	15606	0	15565	0	53804	4
C	22631	-2	15631	25	15585	20	53847	43
D	22632	1	15641	10	15603	18	53876	29
E	22643	11	15644	3	15594	-9	53881	5
F	22658	15	15799	155	15769	175	54226	345
G	22673	15	16008	209	16003	234	54684	458

On the other hand, as we move from solution E to F and from F to G, the total cost increases significantly, meaning that further improvements on equity require higher investments in medical training. Accordingly, Table 3 shows that from solution E to F and from solution F to G, 345 and 458 additional vacancies need to be opened for the entire planning period, respectively – a considerable difference when compared to the previous 4, 43, 29 and 5 additional vacancies opened as we move from solution A to E. This difference is mainly related to the high increase in the ΔV for the second and third sub-periods, 155 and 175 from E to F and 209 and 234 from F to G. One should bear in mind that vacancies opened in the first sub-period contribute to reduce the shortage/oversupply of physicians in every sub-period, and so vacancies opened in the second or third sub-periods have a lower impact on the level of shortage/oversupply of physicians. For this reason, for solutions F and G there is a need to open a higher number of vacancies in the second/third sub-periods in order to continue to improve the equity level by the same amount. And this because the number of vacancies opened in the first sub-period is very close to the maximum capacity (24300 vacancies), which consequently increases significantly the training cost.

The maximum total cost (approximately 17168 million Euros for the 2017-2050 period) and best equity level (0.82) that is possible to achieve while redistributing the residency vacancies is achieved under solution G. However, it is worth noting that the total cost obtained under solution G is lower than the total cost currently incurred with medical training (17300 million Euros), which makes clear that an adequate planning and a wisely investment in medical training may be achieved when considering equity concerns.

In order to illustrate which type of results can be further analysed by applying the *Multi-PiHLOT*, Solution F in Figure 10 is selected for further analysis. This is the solution that allows to improve the current equity (level of equity characterizing the current distribution of vacancies; equal to 0.87, represented by the black vertical line in Figure 10) at the minimum additional costs. Key results include: *numerus clausus* per year (Figure 11); and total number of residency vacancies per specialty groups (Table 4).

The number of vacancies that should be opened in medical schools (*numerus clausus*) for each year is represented in Figure 11. According to this figure, throughout the 2017-2029 and 2043-2050 periods, the *numerus clausus* should be kept in line with the existing ones; whereas between 2030 and 2042 the number of vacancies to open should be reduced (up to 86 vacancies).

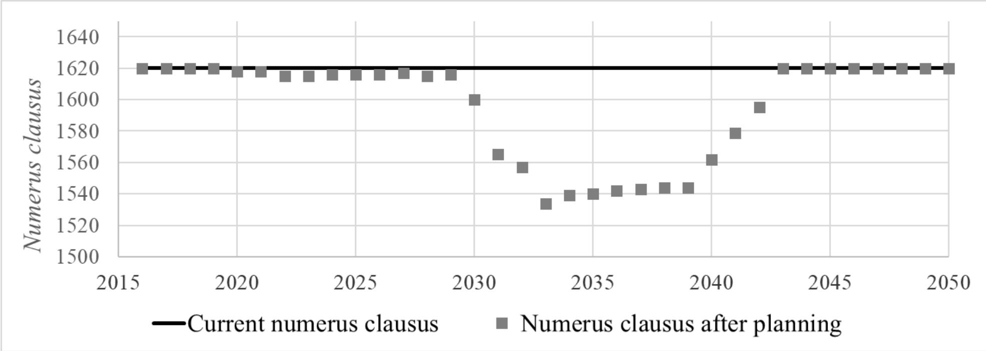

Figure 11. *Numerus clausus* in medical schools over time for solution F - current vacancies and vacancies obtained through the application of the *Multi-PiHLOT*

Table 4 presents the number of additional vacancies to be open (with a plus sign) and the number of vacancies to close (with a minus sign), illustrating the redistribution of residency vacancies across specialties for a smaller sub-period – in this case the 2025-2035 period. The balance between the adjustments performed (openings and closures) corresponds to the difference between the current 1620 residency vacancies and the total number of vacancies to open/close for residency programs after planning.

Table 4. Number of additional residency vacancies to open (+), vacancies to close (-) and total number of vacancies to open for residency programs for solution F. Legend: *CCT - Cirurgia Cárdio-torácica, CG – Cirurgia Geral, CPR - Cirurgia Plástica Reconstrutiva, CV – Cirurgia Vascular, E – Estomatologia, GM - Genética Médica, GO – Ginecologia Obstetrícia, I – Imunohemoterapia, MN - Medicina Nuclear, MT - Medicina Trabalho, R – Radiologia*

Specialty	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**	**2034**	**2035**
CCT			+1								
CG	-3	-3	-3	-3	-2	-2	-2	-3	-2	-2	-2
CPR		+1									-1
CV											-1
E	+1	+1	+1	+1					+1		
GM								+2			
GO	-2	-3	-2	-2	-2	-2	-2	-2	-2	-2	-2
I				+2							
MN		+2	+1		+1						+1
MT	+2										
R			-3	-3	-1				-2		
Balance	**-2**	**-2**	**-5**	**-5**	**-4**	**-4**	**-4**	**-3**	**-5**	**-4**	**-5**
	Total vacancies to open										
	1618	**1618**	**1615**	**1615**	**1616**	**1616**	**1616**	**1617**	**1615**	**1616**	**1615**

To sum up, the obtained results of the *Multi-PiHLOT* suggest that substantial adjustments need to be performed both on the *numerus clausus* and on the residency vacancies to anticipate future health system changes in Portugal.

5 Conclusion

The planning of HHR training is a policy priority in the health policy agenda of many European countries (including Portugal), with this planning being a very complex process, given that the interests and policy objectives of the multiple stakeholders involved need to be considered. Hence, models with a potential to assist the planning of HHR training need to address this complexity, and should thus be informed by a comprehensive structuring of the planning problem.

Addressing this concern, this study proposes a multi-methodological framework (the *Multi-PiHLOT*) to assist the planning of medical training in NHS-based countries with a public medical education system. The proposed framework starts with a comprehensive structuring of the problem by applying a Soft Systems Methodology (SSM) - the CATWOE approach. This structuring of the training planning problem shows how all the elements involved in the system (customers, actors, transformation, *weltanschauung*, owners and environment) relate with each other, and generates useful information to feed the development of a Mixed Integer Linear Programming model meaningful to health decision-makers and planners. The *Multi-PiHLOT* is focused on the training of physicians, informing on the number of vacancies that should be opened/closed per year in medical schools and in each specialty so as to avoid imbalances. Multiple objectives relevant to this sector are considered, including the minimization of oversupply and shortages in the provision of care, the maximization of equity in the distribution of physicians across specialties and the minimization of costs incurred in the training process. It also ensures the stability in the *numerus clausus* in order to avoid disruptive changes. The model depicts what is important according to policy-makers of the Portuguese Ministry of Health.

The present work contributes to the literature in the area in several ways. First, it combines an optimization approach with problem structuring methods to support the planning of physicians training, with problem structuring methods being key to ensure that the diversity of concerns of the multiple stakeholders in the area are reflected in the planning model. Second, it proposes a generic framework that can be easily adapted so as to support the planning of training for other health care professions. Third, it models the training of physicians process and the specificities of physicians' training pathway, which has not been modelled in the reported literature.

So as to illustrate the usefulness of the proposed framework, this was applied to a Portuguese case study for the 2017-2050 period, making use of real data from the Portuguese health system. Key results confirm that the current supply of physicians is far from being able to satisfy all the health care demand in Portugal. The model results also show the impact of considering different policy objectives while planning the reorganization of the *numerus clausus* and residency vacancies. In fact, results show that considering only costs- and shortage/oversupply-related concerns may result in an inequitable distribution of vacancies across specialties. Results also suggest that it is possible to achieve a more equitable distribution of physicians across specialties by adjusting the distribution of residency vacancies across specialities, and this can be done without significant increases in costs. Running the model under different policy questions has also shown that the model performs efficiently in computational terms, which supports its use in real decision-making processes.

It should also be noted that, although applied to the Portuguese case, the developed framework can be easily adapted to assist planners in other NHS-based countries with similar concerns. Also, one should be aware that model building should aim to be requisite (as defined by Phillips (1984)), i.e. to be as simple as possible, but proving to consider all aspects relevant for decision-makers using the model for decision support.

As future work, several research topics are worth pursuing. First, it is relevant to explore approaches to deal with uncertainty, given that HHR planning is strategic, and several sources of uncertainty significantly impact on the planning of physicians training (such as the demand for and supply of physicians, as well as attrition, dropout and migration rates, among others). Evaluating the impact of larger or shorter planning horizons in planning decisions may also be pursued. Also, the impact of different HHR-policy decisions (including HHR-related production policies; policies to address inflows and outflows of physicians; policies to address maldistribution and inefficiencies; and policies to regulate the private sector) that change behaviour and incentives needs to be analysed. Further work could also be developed to model stakeholders' preferences (which may conflict across stakeholders), so as to explore which training plan would specifically meet those preferences, as well as differences in the plans desired by distinct stakeholders. A first step to model these preferences may be the adoption of the protocol suggested by Cardoso et al. (2016) to understand which relative (objectives) weights capture the preferences of each stakeholder. Moreover, the framework can be adapted to model the training of other health professions, such as nurses. In addition, further work may include the development of easy-to-use tools, integrating the *Multi-PiHLOT* with a decision support system, to potentiate the interactive use of the framework in real decision-making processes.

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References

Amorim-Lopes, M., Almeida, Á. S., & Almada-Lobo, B. (2015). Handling healthcare workforce planning with care: where do we stand? *Human Resources for Health, 13*(1), 38.

Amorim-Lopes, M., Almeida, Á. S., & Almada-Lobo, B. (2016). Forecasting the medical workforce: a stochastic agent-based simulation approach. *Health Care Management Science*, 1-24.

Amorim-Lopes, M., Soares, C., Almeida, A., & Almada-Lobo, B. (2016). Comparing comparables: an approach to accurate cross-country comparisons of health systems for effective healthcare planning and policy guidance. *FEP Working Papers nº 563*.

Baker, M. (2000). *Making sense of the NHS White Papers* (2nd ed.). Oxon: Radcliffe Medical Press.

Barber, P., & López-Valcárcel, B. G. (2010). Forecasting the need for medical specialists in Spain: application of a system dynamics model. *Human Resources for Health, 8*(1), 24.

Brailsford, S., & Vissers, J. (2011). OR in healthcare: A European perspective. *European Journal of Operational Research, 212*(2), 223-234.

Cardoso, T., Oliveira, M. D., Barbosa-Póvoa, A., & Nickel, S. (2016). Moving towards an equitable long-term care network: A multi-objective and multi-period planning approach. *Omega, 58*, 69-85.

Checkland, P. (1989). Soft Systems Methodology. In J. Rosenhead (Ed.), *Rational Analysis for a Problematic World: Problem Structuring Methods for Complexity, Uncertainty and Conflict* (pp. 71-100). Chichester: Wiley.

Checkland, P. (1999). *Systems thinking systems practice*. Chichester. UK: Wiley.

Cohon, J. L. (1978). *Multiobjective programming and planning*. London: Academic Press, Inc.

de la Torre, R., Lusa, A., & Mateo, M. (2016). A MILP model for the long term academic staff size and composition planning in public universities. *Omega, 63*, 1-11.

DGES. General Directorate of Higher Education (2017). http://www.dges.mctes.pt/DGES/pt/ Accessed 2017, July.

Dreesch, N., Dolea, C., Dal Poz, M. R., Goubarev, A., Adams, O., Aregawi, M., et al. (2005). An approach to estimating human resource requirements to achieve the Millennium Development Goals. *Health Policy and Planning, 20*(5), 267-276.

EHMA. HEALTH PROMeTHEUS – Health Professional Mobility in the European Union Study (2017). http://ehma.org/projects/past-projects/health-prometheus/ Accessed July, 2017.

Ernst, A. T., Jiang, H., Krishnamoorthy, M., Owens, B., & Sier, D. (2004). An Annotated Bibliography of Personnel Scheduling and Rostering. *Annals of Operations Research, 127*(1), 21-144.

European Commission. (2008). Green Paper on the European Workforce of Health. Brussels: Commission of the European Communities.

European Union. EU Joint Action on Health Workforce Planning & Forecasting (2017). http://healthworkforce.eu/ Accessed July, 2017.

Fritzen, S. A. (2007). Strategic management of the health workforce in developing countries: what have we learned? *Human Resources for Health, 5*, 4-4.

Horn, M. E. T., Elgindy, T., & Gomez-Iglesias, A. (2016). Strategic workforce planning for the Australian Defence Force. *Journal of the Operational Research Society, 67*(4), 664-675.

Hu, W., Lavieri, M. S., Toriello, A., & Liu, X. (2016). Strategic health workforce planning. *IIE Transactions, 48*(12), 1127-1138.

Kotiadis, K., Tako, A. A., Rouwette, E. A. J. A., Vasilakis, C., Brennan, J., Gandhi, P., et al. (2013). Using a model of the performance measures in Soft Systems Methodology (SSM) to take action: a case study in health care. *Journal of the Operational Research Society, 64*(1), 125-137.

Lagarde, M., & Cairns, J. (2012). Modelling human resources policies with Markov models: an illustration with the South African nursing labour market. *Health Care Management Science, 15*(3), 270-282.

Lagergren, M. (1998). What is the role and contribution of models to management and research in the health services? A view from Europe. *European Journal of Operational Research, 105*(2), 257-266.

Lavieri, M. S., & Puterman, M. L. (2009). Optimizing nursing human resource planning in British Columbia. *Health Care Management Science, 12*(2), 119-128.

Li, Y., Chen, J., & Cai, X. (2007). An integrated staff-sizing approach considering feasibility of scheduling decision. *Annals of Operations Research, 155*(1), 361-390.

Lodi, A., Tubertini, P., Grilli, R., Mazzocchetti, A., Ruozi, C., & Senese, F. (2016). Needs forecast and fund allocation of medical specialty positions in Emilia-Romagna (Italy) by system dynamics and integer programming. *Health Systems, 5*(3), 213-236.

Malgieri, A., Michelutti, P., & Hoegaerden, M. V. (2015). *Handbook on Health Workforce Planning Methodologies Across EU Countries - Joint Action Health Workforce Planning and Forecasting*. Bratislava: Ministry of Health of the Slovak Republic.

Mavrotas, G. (2009). Effective implementation of the ε-constraint method in Multi-Objective Mathematical Programming problems. *Applied Mathematics and Computation, 213*(2), 455-465.

Mingers, J., & Brocklesby, J. (1997). Multimethodology: Towards a framework for mixing methodologies. *Omega, 25*(5), 489-509.

Mingers, J., & Rosenhead, J. (2004). Problem structuring methods in action. *European Journal of Operational Research, 152*(3), 530-554.

Ministry of Health. (2010). WHO Evaluation of the National Health Plan of Portugal (2004–2010). Geneva: World Health Organization.

Ministry of Health. (2015a). Decree-Law n^{er} 86/2015: Legal regime of specialized medical training [Decreto-lei nº 101/2006: Regime jurídico da formação médica especializada]: Diário da República: Série I, 98, 21st May.

Ministry of Health. (2015b). Health National Plan - Review and Extension up until 2020 [*Plano Nacional de Saúde - Revisão e extensão a 2020*]. Lisbon: Ministry of Health, Government of Portugal.

Monks, T. (2016). Operational research as implementation science: definitions, challenges and research priorities. *Implementation Science, 11*(1), 81.

Perfilieva, G., & Buchan, J. (2015). Making progress towards health workforce sustainability in the WHO European region. Denmark: World Health Organization.

Pessôa, L. A. M., Lins, M. P. E., da Silva, A. C. M., & Fiszman, R. (2015). Integrating soft and hard operational research to improve surgical centre management at a university hospital. *European Journal of Operational Research, 245*(3), 851-861.

Phillips, L. D. (1984). A theory of requisite decision models. *Acta Psychologica, 56*(1), 29-48.

Pownall, I. (2012). *Effective Management Decision Making: An Introduction* (1st ed.): Ian Pownall & Ventus Publishing ApS.

Sachdeva, R., Williams, T., & Quigley, J. (2007). Mixing Methodologies to Enhance the Implementation of Healthcare Operational Research. *The Journal of the Operational Research Society, 58*(2), 159-167.

Santana, P., Peixoto, H., & Duarte, N. (2014). Demography of Physicians in Portugal: Prospective Analysis [Demografia Médica em Portugal: Análise Prospetiva]. *Acta Med Port, 27*(2), 246-251.

Scheffler, R. M., Liu, J. X., Kinfu, Y., & Dal Poz, M. R. (2008). Forecasting the global shortage of physicians: an economic- and needs-based approach. *Bulletin of the World Health Organization, 86*, 516-523B.

Schell, G. J., Lavieri, M. S., Jankovic, F., Li, X., Toriello, A., Martyn, K. K., et al. (2016). Strategic modeling of the neonatal nurse practitioner workforce. *Nursing Outlook, 64*(4), 385-394.

Schell, G. J., Lavieri, M. S., Li, X., Toriello, A., Martyn, K. K., & Freed, G. L. (2015). Strategic Modeling of the Pediatric Nurse Practitioner Workforce. *Pediatrics, 135*(2), 298-306.

Senese, F., Tubertini, P., Mazzocchetti, A., Lodi, A., Ruozi, C., & Grilli, R. (2015). Forecasting future needs and optimal allocation of medical residency positions: the Emilia-Romagna Region case study. *Human Resources for Health, 13*(1), 7.

Simões, J. d. A., Augusto, G. F., Fronteira, I., & Hernández-Quevedo, C. (2017). Portugal: Health system review. *Health systems in transition, 19*(2), 1-184.

Smits, M., Slenter, V., & Geurts, J. (2010). Improving Manpower Planning in Health Care. *BLED 2010 proceedings, 31*.

Srour, I. M., Haas, C. T., & Morton, D. P. (2006). Linear Programming Approach to Optimize Strategic Investment in the Construction Workforce. *Journal of Construction Engineering and Management, 132*(11), 1158-1166.

Tako, A. A., & Kotiadis, K. (2015). PartiSim: A multi-methodology framework to support facilitated simulation modelling in healthcare. *European Journal of Operational Research, 244*(2), 555-564.

Tomblin Murphy, G., Birch, S., MacKenzie, A., & Rigby, J. (2016). Simulating future supply of and requirements for human resources for health in high-income OECD countries. *Human Resources for Health, 14*(1), 77.

Van den Bergh, J., Beliën, J., De Bruecker, P., Demeulemeester, E., & De Boeck, L. (2013). Personnel scheduling: A literature review. *European Journal of Operational Research, 226*(3), 367-385.

Wang, W., Liu, W., & Mingers, J. (2015). A systemic method for organisational stakeholder identification and analysis using Soft Systems Methodology (SSM). *European Journal of Operational Research, 246*(2), 562-574.

Wishon, C., Villalobos, J. R., Mason, N., Flores, H., & Lujan, G. (2015). Use of MIP for planning temporary immigrant farm labor force. *International Journal of Production Economics, 170*, 25-33.

Yu, G., Pachon, J., Thengvall, B., Chandler, D., & Wilson, A. (2004). Optimizing Pilot Planning and Training for Continental Airlines. *Interfaces, 34*(4), 253-264.

Appendix A

Computational results

Table A1 displays the computational results obtained when applying the *Multi-PiHLOT* for all the policy questions under analysis, showing that the model performs efficiently in computational terms for all the cases. In particular, for policy questions B and C, each solution was obtained by imposing a 240 minutes limit for the computation time, and optimality gaps were below 0.5% for all the solutions. The computational results presented in the table for these two policy questions refer to the solution G.

Table A1. Computational results

Policy question	Total equations	Total variables	Integer variables	Iterations	CPU (sec)	Gap (%)
A	22718	19778	6020	76	0.02	0
B	28776	21356	6020	27	0.09	0
C	28776	21356	6020	290	0.11	0.36

Appendix B

Table B1. Medical specialties

Medical specialty (*s*)	**Medical specialty name**
s_1	*Anatomia Patológica*
s_2	*Anestesia*
s_3	*Cardiologia*
s_4	*Cardiologia Pediátrica*
s_5	*Cirurgia Cardio-Torácica*
s_6	*Cirurgia Geral*
s_7	*Cirurgia Maxilo-Facial*
s_8	*Cirurgia Pediátrica*
s_9	*Cirurgia Plástica Reconstrutiva*
s_{10}	*Cirurgia Vascular*
s_{11}	*Dermatovenerologia*
s_{12}	*Endocronologia*
s_{13}	*Estomatologia*
s_{14}	*Gastrenterologia*
s_{15}	*Genética Médica*
s_{16}	*Ginecologia Obstetrícia*
s_{17}	*Hematologia Clínica*
s_{18}	*Imunoalergologia*
s_{19}	*Imunohemoterapia*
s_{20}	*Infecciologia*
s_{21}	*Medicina Física e Reabilitação*
s_{22}	*Medicina Geral e Familiar*
s_{23}	*Medicina Interna*
s_{24}	*Medicina Nuclear*
s_{25}	*Medicina Trabalho*
s_{26}	*Nefrologia*
s_{27}	*Neurocirurgia*
s_{28}	*Neurologia*
s_{29}	*Neurorradiologia*
s_{30}	*Oftalmologia*
s_{31}	*Oncologia Médica*
s_{32}	*Ortopedia*
s_{33}	*Otorrinolaringologia*
s_{34}	*Patologia Clínica*
s_{35}	*Pediatria Médica*
s_{36}	*Pedopsiquiatria*
s_{37}	*Pneumologia*
s_{38}	*Psiquiatria*
s_{39}	*Radiologia*
s_{40}	*Radioterapia*
s_{41}	*Reumatologia*
s_{42}	*Saúde Pública*
s_{43}	*Urologia*

Appendix C

Table C1. Number of physicians of specialty s required at t ($\Psi_{s,t}$)

	2016	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**	**2034**	**2035**
s_1	311	322	327	345	340	344	343	344	343	359	365	369	377	384	380	377	380	381	385	394
s_2	2319	2367	2393	2395	2436	2466	2481	2468	2463	2462	2482	2477	2463	2445	2460	2470	2461	2453	2451	2459
s_3	863	852	860	871	880	895	895	905	906	906	900	908	893	886	898	891	892	880	876	879
s_4	122	108	92	87	83	92	94	98	100	103	105	107	111	111	114	114	108	106	107	105
s_5	181	173	170	170	166	158	151	155	155	158	157	163	156	154	156	155	160	158	160	164
s_6	2115	2115	2096	2056	2048	2016	1989	1961	1945	1911	1906	1857	1830	1800	1757	1721	1707	1670	1628	1589
s_7	55	63	74	76	82	86	92	97	95	96	96	101	105	109	113	111	110	110	111	112
s_8	190	171	149	128	129	128	118	117	122	124	120	122	119	111	114	117	117	120	121	121
s_9	215	203	195	202	211	214	213	217	220	220	224	220	229	223	233	232	230	232	236	234
s_{10}	210	199	199	198	199	197	198	195	199	197	205	216	214	213	206	205	204	196	194	194
s_{11}	283	283	293	296	294	298	302	299	299	298	294	292	300	299	297	297	300	301	303	304
s_{12}	243	243	259	268	268	283	286	298	316	333	336	337	348	362	383	389	395	407	420	427
s_{13}	305	291	275	260	248	242	234	229	234	231	234	234	234	235	246	248	255	259	256	257
s_{14}	521	521	532	543	564	571	581	587	593	602	589	592	592	610	602	589	589	591	589	596
s_{15}	29	34	41	48	50	53	56	55	59	58	61	64	63	63	62	66	66	64	65	69
s_{16}	1863	1808	1748	1669	1643	1652	1653	1623	1595	1550	1529	1499	1462	1449	1426	1388	1369	1341	1311	1293
s_{17}	350	317	312	312	314	317	335	335	338	345	351	354	364	381	384	391	399	407	404	409
s_{18}	163	177	188	201	207	209	214	222	228	241	250	255	259	264	266	274	267	271	272	280
s_{19}	302	311	325	336	346	335	329	331	335	340	344	346	358	366	369	384	389	405	408	423
s_{20}	247	260	270	296	310	315	323	339	339	341	350	358	366	378	384	380	381	379	379	384
s_{21}	447	481	489	502	525	541	557	573	580	584	586	585	600	596	608	604	605	618	630	638
s_{22}	13760	13573	13324	13105	12995	12921	12855	12845	12811	12818	12807	12823	12894	12895	12959	13056	13156	13254	13323	13430
s_{23}	3584	3757	3929	4124	4228	4329	4408	4467	4533	4616	4704	4769	4794	4875	4908	4949	5012	5075	5115	5144
s_{24}	54	59	61	63	66	69	71	77	73	76	79	81	82	85	85	91	96	94	102	109
s_{25}	84	88	92	93	95	94	98	98	107	116	123	126	130	144	145	155	167	171	179	187
s_{26}	388	398	411	424	429	432	444	445	452	458	467	475	484	491	500	501	499	505	515	507
s_{27}	209	213	215	217	215	211	217	213	226	234	228	231	226	234	242	249	249	252	252	246
s_{28}	617	619	613	607	616	612	620	640	644	663	673	682	685	695	688	692	694	703	700	702
s_{29}	213	229	247	248	260	263	267	272	281	283	284	287	274	263	266	271	271	278	286	280
s_{30}	812	823	831	820	820	825	814	822	811	807	804	791	782	772	765	765	748	745	737	737
s_{31}	305	325	365	401	426	459	481	516	549	597	623	661	689	720	760	778	799	810	840	853
s_{32}	1329	1305	1265	1265	1282	1286	1268	1247	1236	1241	1256	1262	1273	1273	1291	1276	1275	1284	1285	1291
s_{33}	566	570	572	568	566	565	567	571	577	572	578	590	588	588	588	596	588	584	590	591

s_{34}	654	642	659	669	666	661	666	678	688	708	709	736	755	772	776	791	803	819	831	842
s_{35}	2113	2242	2345	2431	2469	2508	2547	2576	2582	2582	2571	2572	2581	2607	2620	2626	2627	2637	2631	2619
s_{36}	169	178	191	198	200	207	221	230	239	249	261	265	270	274	290	300	297	295	305	311
s_{37}	702	705	707	710	711	711	720	733	750	760	780	787	804	820	829	842	844	842	857	863
s_{38}	1094	1103	1122	1149	1178	1184	1207	1235	1265	1277	1293	1333	1370	1388	1400	1452	1481	1507	1533	1550
s_{39}	792	823	858	888	885	888	878	862	849	817	791	765	755	735	717	690	670	651	646	619
s_{40}	109	122	136	149	156	154	156	158	158	158	153	153	158	160	156	154	159	153	153	149
s_{41}	141	158	176	194	208	213	219	226	230	239	250	257	260	270	279	281	284	288	291	303
s_{42}	646	614	626	626	605	602	613	616	635	662	677	682	708	731	749	773	799	820	842	860
s_{43}	512	476	449	437	425	415	416	411	418	413	415	429	419	414	409	408	399	389	383	381

Table C2. Number of physicians of specialty s required at t ($\Psi_{s,t}$) (continued)

	2036	**2037**	**2038**	**2039**	**2040**	**2041**	**2042**	**2043**	**2044**	**2045**	**2046**	**2047**	**2048**	**2049**	**2050**
s_1	404	415	415	423	435	442	453	460	467	472	480	484	491	484	485
s_2	2447	2450	2428	2415	2388	2379	2361	2355	2359	2337	2308	2308	2300	2298	2327
s_3	876	866	857	857	854	837	852	842	844	845	824	820	818	820	819
s_4	107	104	104	102	104	101	98	97	102	104	99	95	98	99	98
s_5	167	161	160	163	163	166	167	168	166	167	159	158	158	151	146
s_6	1575	1556	1551	1546	1532	1528	1515	1485	1474	1449	1432	1425	1378	1367	1355
s_7	114	112	114	114	123	118	115	112	114	112	112	111	111	117	113
s_8	123	124	130	126	125	128	124	124	122	123	126	118	113	112	115
s_9	238	239	242	248	239	246	240	246	250	240	239	239	238	241	235
s_{10}	190	186	183	186	181	175	176	173	174	178	180	181	177	174	174
s_{11}	297	291	290	299	294	292	293	293	289	287	284	288	283	284	285
s_{12}	439	438	442	445	452	453	459	467	470	470	477	487	500	513	525
s_{13}	269	275	286	290	294	301	306	307	306	317	329	334	329	338	341
s_{14}	594	587	582	571	573	572	574	562	561	555	562	557	549	551	543
s_{15}	71	73	78	79	80	79	78	74	71	73	73	75	77	84	87
s_{16}	1275	1249	1237	1223	1220	1209	1196	1170	1157	1132	1093	1075	1070	1056	1059
s_{17}	405	408	414	429	423	418	423	430	430	427	420	428	433	433	429
s_{18}	281	283	287	294	294	301	304	310	309	309	314	319	321	333	328
s_{19}	408	412	418	415	410	420	421	427	431	438	438	437	463	465	466
s_{20}	389	407	406	407	415	411	416	419	412	423	433	430	425	412	408
s_{21}	647	649	652	665	669	669	662	668	661	673	674	664	670	662	662
s_{22}	13532	13598	13650	13717	13796	13912	14015	14072	14115	14119	14159	14209	14251	14219	14174

s_{23}	5140	5140	5146	5146	5178	5167	5142	5140	5107	5105	5117	5086	5066	5070	5023
s_{24}	108	114	110	113	115	122	122	126	126	129	131	125	125	124	131
s_{25}	187	196	201	210	214	221	222	228	233	237	246	248	258	262	266
s_{26}	514	529	530	534	534	525	539	537	540	542	547	558	555	558	556
s_{27}	250	258	255	257	262	256	250	246	247	249	249	247	247	248	247
s_{28}	712	712	710	723	726	728	723	735	723	726	719	720	718	711	720
s_{29}	273	269	267	268	264	258	256	250	256	254	250	252	245	248	258
s_{30}	720	712	709	698	685	673	653	635	630	628	620	601	571	560	555
s_{31}	879	894	914	941	956	973	986	994	1001	1022	1020	1037	1061	1083	1103
s_{32}	1310	1312	1315	1306	1317	1315	1311	1299	1301	1299	1303	1280	1276	1252	1249
s_{33}	593	600	602	592	590	581	577	579	583	581	581	574	571	567	577
s_{34}	858	883	913	917	934	957	972	997	1014	1023	1042	1071	1085	1113	1136
s_{35}	2594	2599	2560	2561	2559	2535	2535	2520	2497	2479	2472	2443	2408	2371	2350
s_{36}	317	317	319	318	316	318	321	330	325	323	326	327	326	333	341
s_{37}	870	867	878	889	894	891	884	885	889	882	873	868	889	890	886
s_{38}	1593	1639	1681	1690	1691	1734	1753	1755	1779	1804	1812	1803	1785	1780	1756
s_{39}	609	585	574	539	514	489	476	460	453	446	435	431	421	399	393
s_{40}	151	149	152	148	144	139	142	146	144	140	139	141	137	130	132
s_{41}	302	299	309	308	302	292	286	280	282	287	290	288	280	285	286
s_{42}	880	914	951	974	1000	1025	1042	1078	1096	1124	1144	1161	1177	1198	1197
s_{43}	384	383	372	366	370	367	359	355	352	350	345	349	347	336	329

Table C3. Number of physicians of specialty s available at t ($\Phi_{s,t}$)

	2016	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**	**2034**	**2035**
s_1	305	284	266	267	258	259	257	255	254	264	267	270	275	279	276	274	277	279	284	293
s_2	2271	2088	1947	1851	1847	1854	1856	1832	1822	1812	1818	1813	1798	1777	1787	1796	1795	1798	1810	1827
s_3	845	752	700	673	667	673	670	672	670	667	659	665	652	644	652	648	651	645	647	653
s_4	119	95	75	67	63	69	70	73	74	76	77	78	81	81	83	83	79	78	79	78
s_5	177	153	138	131	126	119	113	115	115	116	115	119	114	112	113	113	117	116	118	122
s_6	2071	1866	1705	1589	1553	1516	1488	1456	1439	1406	1396	1359	1336	1308	1276	1251	1245	1224	1202	1181
s_7	54	56	60	59	62	65	69	72	70	71	70	74	77	79	82	81	80	81	82	83
s_8	186	151	121	99	98	96	88	87	90	91	88	89	87	81	83	85	85	88	89	90
s_9	211	179	159	156	160	161	159	161	163	162	164	161	167	162	169	169	168	170	174	174
s_{10}	206	176	162	153	151	148	148	145	147	145	150	158	156	155	150	149	149	144	143	144
s_{11}	277	250	238	229	223	224	226	222	221	219	215	214	219	217	216	216	219	221	224	226

s_{12}	238	214	211	207	203	213	214	221	234	245	246	247	254	263	278	283	288	298	310	317
s_{13}	299	257	224	201	188	182	175	170	173	170	171	171	171	171	179	180	186	190	189	191
s_{14}	510	460	433	420	428	429	435	436	439	443	431	433	432	443	437	428	430	433	435	443
s_{15}	28	30	33	37	38	40	42	41	44	43	45	47	46	46	45	48	48	47	48	51
s_{16}	1825	1595	1422	1290	1246	1242	1237	1205	1180	1141	1120	1097	1067	1053	1036	1009	999	983	968	961
s_{17}	343	280	254	241	238	238	251	249	250	254	257	259	266	277	279	284	291	298	298	304
s_{18}	160	156	153	155	157	157	160	165	169	177	183	187	189	192	193	199	195	199	201	208
s_{19}	296	274	264	260	262	252	246	246	248	250	252	253	261	266	268	279	284	297	301	314
s_{20}	242	229	220	229	235	237	242	252	251	251	256	262	267	275	279	276	278	278	280	285
s_{21}	438	424	398	388	398	407	417	425	429	430	429	428	438	433	442	439	441	453	465	474
s_{22}	13477	11975	10839	10128	9853	9716	9618	9535	9476	9433	9379	9387	9412	9372	9413	9493	9597	9716	9839	9979
s_{23}	3510	3315	3196	3187	3206	3255	3298	3316	3353	3397	3445	3491	3499	3543	3565	3598	3656	3720	3777	3822
s_{24}	53	52	50	49	50	52	53	57	54	56	58	59	60	62	62	66	70	69	75	81
s_{25}	82	78	75	72	72	71	73	73	79	85	90	92	95	105	105	113	122	125	132	139
s_{26}	380	351	334	328	325	325	332	330	334	337	342	348	353	357	363	364	364	370	380	377
s_{27}	205	188	175	168	163	159	162	158	167	172	167	169	165	170	176	181	182	185	186	183
s_{28}	604	546	499	469	467	460	464	475	476	488	493	499	500	505	500	503	506	515	517	522
s_{29}	209	202	201	192	197	198	200	202	208	208	208	210	200	191	193	197	198	204	211	208
s_{30}	795	726	676	634	622	620	609	610	600	594	589	579	571	561	556	556	546	546	544	548
s_{31}	299	287	297	310	323	345	360	383	406	439	456	484	503	523	552	566	583	594	620	634
s_{32}	1302	1151	1029	978	972	967	949	926	914	913	920	924	929	925	938	928	930	941	949	959
s_{33}	554	503	465	439	429	425	424	424	427	421	423	432	429	427	427	433	429	428	436	439
s_{34}	641	566	536	517	505	497	498	503	509	521	519	539	551	561	564	575	586	600	614	626
s_{35}	2069	1978	1908	1879	1872	1886	1906	1912	1910	1900	1883	1883	1884	1895	1903	1909	1916	1933	1943	1946
s_{36}	166	157	155	153	152	156	165	171	177	183	191	194	197	199	211	218	217	216	225	231
s_{37}	688	622	575	549	539	535	539	544	555	559	571	576	587	596	602	612	616	617	633	641
s_{38}	1071	973	913	888	893	890	903	917	936	940	947	976	1000	1009	1017	1056	1080	1105	1132	1152
s_{39}	776	726	698	686	671	668	657	640	628	601	579	560	551	534	521	502	489	477	477	460
s_{40}	107	108	111	115	118	116	117	117	117	116	112	112	115	116	113	112	116	112	113	111
s_{41}	138	139	143	150	158	160	164	168	170	176	183	188	190	196	203	204	207	211	215	225
s_{42}	633	542	509	484	459	453	459	457	470	487	496	499	517	531	544	562	583	601	622	639
s_{43}	501	420	365	338	322	312	311	305	309	304	304	314	306	301	297	297	291	285	283	283

Table C4. Number of physicians of specialty s available at t ($\Phi_{s,t}$) (continued)

	2036	**2037**	**2038**	**2039**	**2040**	**2041**	**2042**	**2043**	**2044**	**2045**	**2046**	**2047**	**2048**	**2049**	**2050**
s_1	302	312	314	322	333	341	350	357	363	368	374	376	381	375	374
s_2	1829	1842	1836	1840	1829	1834	1826	1826	1834	1823	1798	1795	1785	1780	1796
s_3	655	651	648	653	654	645	659	653	656	659	642	638	635	635	632
s_4	80	78	79	78	80	78	76	75	79	81	77	74	76	77	76
s_5	125	121	121	124	125	128	129	130	129	130	124	123	123	117	113
s_6	1177	1170	1173	1178	1173	1178	1172	1151	1146	1130	1116	1108	1069	1059	1046
s_7	85	84	86	87	94	91	89	87	89	87	87	86	86	91	87
s_8	92	93	98	96	96	99	96	96	95	96	98	92	88	87	89
s_9	178	180	183	189	183	190	186	191	194	187	186	186	185	187	181
s_{10}	142	140	138	142	139	135	136	134	135	139	140	141	137	135	134
s_{11}	222	219	219	228	225	225	227	227	225	224	221	224	220	220	220
s_{12}	328	329	334	339	346	349	355	362	365	367	372	379	388	397	405
s_{13}	201	207	216	221	225	232	237	238	238	247	256	260	255	262	263
s_{14}	444	441	440	435	439	441	444	436	436	433	438	433	426	427	419
s_{15}	53	55	59	60	61	61	60	57	55	57	57	58	60	65	67
s_{16}	953	939	935	932	934	932	925	907	899	883	852	836	830	818	817
s_{17}	303	307	313	327	324	322	327	333	334	333	327	333	336	335	331
s_{18}	210	213	217	224	225	232	235	240	240	241	245	248	249	258	253
s_{19}	305	310	316	316	314	324	326	331	335	342	341	340	359	360	360
s_{20}	291	306	307	310	318	317	322	325	320	330	337	334	330	319	315
s_{21}	484	488	493	507	512	516	512	518	514	525	525	516	520	513	511
s_{22}	10115	10222	10321	10452	10566	10724	10840	10910	10972	11013	11032	11049	11059	11013	10940
s_{23}	3842	3864	3891	3921	3966	3983	3977	3985	3970	3982	3987	3955	3931	3927	3877
s_{24}	81	86	83	86	88	94	94	98	98	101	102	97	97	96	101
s_{25}	140	147	152	160	164	170	172	177	181	185	192	193	200	203	205
s_{26}	384	398	401	407	409	405	417	416	420	423	426	434	431	432	429
s_{27}	187	194	193	196	201	197	193	191	192	194	194	192	192	192	191
s_{28}	532	535	537	551	556	561	559	570	562	566	560	560	557	551	556
s_{29}	204	202	202	204	202	199	198	194	199	198	195	196	190	192	199
s_{30}	538	535	536	532	525	519	505	492	490	490	483	467	443	434	428
s_{31}	657	672	691	717	732	750	763	771	778	797	795	806	823	839	851
s_{32}	979	986	994	995	1009	1014	1014	1007	1011	1013	1015	995	990	970	964
s_{33}	443	451	455	451	452	448	446	449	453	453	453	446	443	439	445
s_{34}	641	664	690	699	715	738	752	773	788	798	812	833	842	862	877

s_{35}	1939	1954	1936	1951	1960	1954	1961	1954	1941	1934	1926	1900	1869	1836	1814
s_{36}	237	238	241	242	242	245	248	256	253	252	254	254	253	258	263
s_{37}	650	652	664	677	685	687	684	686	691	688	680	675	690	689	684
s_{38}	1191	1232	1271	1288	1295	1337	1356	1361	1383	1407	1412	1402	1385	1379	1355
s_{39}	455	440	434	411	394	377	368	357	352	348	339	335	327	309	303
s_{40}	113	112	115	113	110	107	110	113	112	109	108	110	106	101	102
s_{41}	226	225	234	235	231	225	221	217	219	224	226	224	217	221	221
s_{42}	658	687	719	742	766	790	806	836	852	877	891	903	913	928	924
s_{43}	287	288	281	279	283	283	278	275	274	273	269	271	269	260	254

Table C5. Medical specialties, number of opened vacancies per specialty and duration of specialty

Medical specialty (s)	**Medical specialty name**	**Number of opened vacancies ($v_{s,t}^{0}$)**	**Duration of specialty (in years) (φ_s)**
s_1	*Anatomia Patológica*	19	5
s_2	*Anestesia*	80	4
s_3	*Cardiologia*	32	5
s_4	*Cardiologia Pediátrica*	3	5
s_5	*Cirurgia Cardio-Torácica*	5	6
s_6	*Cirurgia Geral*	49	6
s_7	*Cirurgia Maxilo-Facial*	4	6
s_8	*Cirurgia Pediátrica*	4	6
s_9	*Cirurgia Plástica Reconstrutiva*	9	6
s_{10}	*Cirurgia Vascular*	7	6
s_{11}	*Dermatovenerologia*	11	5
s_{12}	*Endocronologia*	20	5
s_{13}	*Estomatologia*	14	4
s_{14}	*Gastrenterologia*	21	5
s_{15}	*Genética Médica*	3	5
s_{16}	*Ginecologia Obstetrícia*	40	6
s_{17}	*Hematologia Clínica*	18	5
s_{18}	*Imunoalergologia*	13	5
s_{19}	*Imunohemoterapia*	20	5
s_{20}	*Infecciologia*	16	5
s_{21}	*Medicina Física e Reabilitação*	25	5
s_{22}	*Medicina Geral e Familiar*	469	3
s_{23}	*Medicina Interna*	185	5
s_{24}	*Medicina Nuclear*	4	4
s_{25}	*Medicina Trabalho*	10	5
s_{26}	*Nefrologia*	20	5
s_{27}	*Neurocirurgia*	10	6
s_{28}	*Neurologia*	28	5
s_{29}	*Neurorradiologia*	10	5
s_{30}	*Oftalmologia*	18	4
s_{31}	*Oncologia Médica*	43	5
s_{32}	*Ortopedia*	47	6
s_{33}	*Otorrinolaringologia*	23	5
s_{34}	*Patologia Clínica*	45	4
s_{35}	*Pediatria Médica*	84	5
s_{36}	*Pedopsiquiatria*	14	5
s_{37}	*Pneumologia*	35	5
s_{38}	*Psiquiatria*	72	5
s_{39}	*Radiologia*	10	5
s_{40}	*Radioterapia*	4	4
s_{41}	*Reumatologia*	11	5
s_{42}	*Saúde Pública*	51	4
s_{43}	*Urologia*	14	6

Table C6. Duration of the Master's degree in medicine and capacity of the education system

Duration of the Master's degree in medicine (in years) (ϕ)	5
Maximum number of vacancies available for all specialties at t (v_t)	1620

Table C7. Probabilities

Probability of a Master's degree student finishing the course in due time at t (α_s)	0.91
Probability of a physician giving up specialty s at t ($\delta_{s,t}$)	0.01
Probability of emigration of a physician with the specialty s at t ($\varepsilon_{s,t}$)	0.01

Table C8. Costs (in €) associated with the medical training process and number of hours of training required per physicians doing the specialty

Cost per Master's degree student per t (ω_t)	12000
Cost of the salary paid to a physician doing the specialty s per t ($\mu_{s,t}$)	27132
Cost of the salary paid to a physician giving training of specialty s per t ($\lambda_{s,t}$)	135
Cost per vacancy closed per specialty s per t ($\chi_{s,t}$)	1200
Number of hours of training required per physician of specialty s per t ($nh_{s,t}$)	240