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“Fair cost sharing: big tech vs telcos”

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Fair cost sharing: big tech vs telcos*

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Abstract

We study a cost-sharing mechanism where a content provider contributes to covering the costs incurred by a network operator when delivering content to consumers. The cost-share not only boosts the content provider's incentives to moderate traffic but also affects the price composition for consumers buying access and content. We show the overall effect on consumer welfare depends on the content provider's ability to monetize users. When that ability is high, introducing a cost-share can lead to lower overall prices and higher consumer welfare. We study the robustness of this result to long-term investments in cost reduction by the operator and to heterogeneity in consumers' taste for content. In extensions with multiple contents and multiple operators, contractual externalities arise that suggest a role for regulation.

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1 Introduction

This paper revisits the question of cost-sharing between a telecommunication network operator and content providers distributing content over the network in light of the evolution of the telecom industry and the digital economy.

This was intensely debated a decade ago when the key question was the issue of prioritization and innovation on the content side (see Greenstein et al. (2016)). The situation has evolved fast due to three factors that call for a new appraisal. First, while telcos used to have market power stemming from their bottleneck position, Internet giant tech companies have emerged with substantially more bargaining power than any telecom operator. Second, the innovation boom that followed the development of the Internet is creating an explosion of the demand for data. This is already apparent for video streaming, online game, and teleworking but the emergence of cloud services, AI and IoT will generate even more demand leading to exponential growth in the future.[1] Such an explosion of traffic justifies focusing more on investment in the network than a decade ago when the capacity issue was not as critical. Third, environmental policy is reaching all sectors and telecom regulators start including carbon footprint considerations in their regulations.

These forces create a context where telcos need to consider large investment challenges to expand and upgrade capacity while facing powerful content platforms with deep financial and technical resources. Moreover, large content providers can also invest to optimize traffic and currently lack proper incentives to do so. This raises the broad question whether network costs should be shared with the content layer of the digital economy. If that is the case, we also need guidance on which contents should contribute and to which extent.

To address this issue, we build a stylized model with two layers in a vertical chain: access and content. A critical feature of the model is that access and content are complements: users need to buy access from a network operator to consume content. When consuming content, users generate traffic that is costly for the network operator and that the content provider could contain at the cost of lower content quality. An inefficiency can arise in this setting as the content provider may not properly internalize the traffic cost its content generates for the network operator. This provides a rationale for a transfer of some of this cost from the

[1] Global mobile traffic was 67 exabytes (EB) per month by the end of 2021, it is projected to reach 282 EB per month by 2027 (See Ericsson Mobility Report (June 2022) at `https://www.ericsson.com/49d3a0/assets/local/reports-papers/mobility-report/documents/2022/ericsson-mobility-report-june-2022.pdf`).

network operator to the content provider even though the operator already charges consumers for access.

However, introducing a cost-share has further-reaching effects: it impacts the equilibrium pricing of both the network operator and the content provider. We show the overall effect of a cost-share critically depends on content providers' business models. Indeed, content providers may sell content for a subscription price or may provide the content for free. In the latter case, the content provider is financed by advertising, and we follow Anderson and Coate (2005) in assuming an attention cost for consumers. Hence consumers support a nuisance (measured in monetary equivalent) proportional to the volume of ads, which constitutes the implicit price of free content. A key parameter in our model will then be the "return to ads," that is, the ad revenue generated per unit of nuisance which captures content providers' efficiency at monetizing users. In the baseline model where the operator's cost per unit of traffic is fixed, we show that a positive level of cost-sharing raises profit and consumer surplus if the content is paid or if it is free but more efficient than a paid service (i.e., if given the choice at no cost between a paid and an ad-financed business model, the content provider prefers ad-financing).[2] The same holds for medium return to ads, but above some level of cost-sharing welfare declines. Finally, if the content provider has a low return to ads, cost-sharing reduces consumer and total welfare.

The reason why content providers' business model matters is because the return to ads introduces a spread between the attention cost that advertising imposes on consumers and the revenue these ads generate. When part of the cost of traffic is transferred from the network operator to the content provider, it leads to a higher content price/nuisance and to a lower access price. The effect on the overall price of access and content depends on the content provider's propensity to pass that cost increase through to consumers. The key intuition is that this pass-through decreases with the return to ads. Intuitively, when the content provider is more efficient at monetizing users, its opportunity cost of losing demand is higher. In that case, a cost-share can lead to a lower total price and to higher consumption, in addition to higher effort by the content provider to contain traffic. For lower returns to ads, the total price for access and content increases with cost-sharing, which can overcome the positive impact of cost-sharing on the content provider's containment efforts.

[2] Our model considers the possibility of ad-financed services that are less or more efficient in monetization than paid services. In the former case we assume that a paid service is not feasible or too costly (due to fixed cost or consumers' mistrust).

We then examine several extensions of this baseline model. First, we account for the possibility that in the long run, the network operator may respond to the introduction of a cost-share by adjusting its cost structure. Specifically, we allow the operator to make ex-ante investments that reduce its cost per unit of traffic. The impact of the cost-share on the operator's incentive to invest in cost reduction is ambiguous. On the one hand, there is a substitution effect: the cost-share incentivizes traffic moderation by the content provider, which lowers the operator's benefit from reducing the per-unit cost of traffic. On the other hand, there is a demand effect: when the content provider is efficient at monetizing users, the cost-share boosts the demand for content and for the corresponding traffic, which increases the operator's benefit from reducing the per-unit cost. We then turn to welfare where our analysis suggests the intuition in the baseline model carries over if cost reduction by the operator is costly enough at the margin. In that case, a positive cost-share is more likely to increase welfare for contents with high return to ads.

Second, we study the case where access and content are imperfect complements. We introduce a second type of user who values access to the operator's network but does not consume the content provider's offering (or consumes another content with no associated cost-share and a rigid price). This extension delivers two additional insights. First, users who are not interested in the content benefit from cost-sharing because the access price decreases. Second, the size of the demand for content matters: the larger that demand, the more likely it is that a cost-share raises consumer and total welfare. Indeed, when the fraction of consumers who buy from the content provider is higher, cost-sharing has a larger impact on the network operator, which triggers a stronger downward adjustment of the price for access.

Third, we introduce competition between differentiated network operators, while retaining a monopoly content provider. Competition between operators benefits both users and the content provider as prices for access are lower and aggregate equilibrium demand expands. In this competitive setting, the conclusions of the baseline model readily extend, i.e., a cost-share raises consumer welfare if the content provider's return to ads is high enough. More precisely, competition intensity magnifies the effect of the cost share on welfare (whether positive or negative) but not the direction of this effect. In addition, competition introduces a wedge between network operators' individual incentives to introduce a cost-share and their collective interest. Intuitively, when operator n introduces a cost-share, it causes the content provider to increase its price, which affects demand for all other network operators. This negative impact on other operators is an effect that n not only fails to internalize but in fact benefits from as the overall price for access and content with other operators rises, which relaxes competitive pressure on n. Overall, the analysis suggests that competing operators may benefit from a coordinated approach to setting the level of their respective cost-shares.

Fourth, we allow multiple contents to be delivered by a single operator. This gives rise to a different type of externality operating now across contents: introducing a cost-share for one content leads to a drop in the operator's fee which benefits not only the consumers of that content but also the consumers of all other contents. While this unambiguously increases surplus for the latter consumers, the net benefit for the consumers of the content with the attached cost-share is lower than in the baseline model with only one content because the price of the content rises in the same proportion as in the baseline but the drop in the operator's fee is lower. However, it still is the case that the effect of introducing a cost-share for one content on overall consumer welfare is higher for contents with a higher advertising efficiency, in line with the original intuition.

We close the analysis with a robustness section that allows for marginal advertising efficiency to be decreasing with per-user advertising volume, and for a direct cost for the content provider to moderate traffic (in addition to the indirect of cost of reducing quality and therefore demand). There also, our results remain consistent with the key intuition in the baseline model that introducing a cost-share for contents that are more efficient at monetizing users creates welfare gains for consumers.

Related literature

Our contribution builds on previous literature on net neutrality, although this literature focused more on second-degree discrimination and prioritization, analyzing the implications of allowing multiple layers of quality (e.g., see Hermalin and Katz, 2007, Choi and Kim, 2010, Economides and Hermalin, 2012, Bourreau et al., 2015, Reggiani and Valletti, 2016). We do not consider product differentiation in this work.

Surprisingly, there are few contributions on the optimal level and targets for cost-sharing, in particular for ad-financed services. Economides and Tåg (2012) examine this issue from a two-sided market perspective but they do not allow transfers between content providers and consumers (whether monetary or non-monetary). Peitz and Schuett (2016) points that cost-sharing can alleviate traffic inflation by content providers and reduce congestion on the network. We build on this insight but we allow content providers to affect demand by choosing both the content quality and the advertising intensity, emphasizing the heterogeneity of content in size and ad-revenue generated.

The literature on zero rating (Jullien and Sand-Zantman, 2018, Krämer and Peitz, 2018,) considers content discrimination at the level of consumer prices, hence on the consumer side of the market. By contrast our analysis considers the differential treatment of content providers on the content side of the market.

As is already known (see Rochet and Tirole, 2003, and the discussion in Greenstein et al., 2016), with a fixed technology, allowing for direct monetary transfers between the participants on both sides of a two-sided platform results in the neutrality of the access price paid by one side for the equilibrium allocation. We reinvigorate this literature by considering the attention cost for free services and investment, and showing that such neutrality does not hold in general.

In doing so, we build on the literature on ad-financed media with endogenous choice of ad-intensity, initiated by Anderson and Coate (2005). A review of this approach is provided by Anderson and Jullien (2015). We abstract however from considerations of competition between ad-financed services[3] to focus on the interaction between network operators and content providers.

2 The baseline model

We consider a firm O (the network operator) that sells a good to users, referred to as access. The good can be combined with another complementary good sold by another firm CP (a large distributor), referred to as content (indexed by C). In our leading example, O is a telecom operator running the physical network along with network intelligence, while CP is a content provider, sending content through interconnected networks.

Users have a unit demand for basic access to the network and they have a unit demand for the content provided by CP. Consuming CP's content requires access to the network from the operator so that the content can be viewed as an ancillary service. Users value basic access at $\bar{P}_0$, minus a convenience cost $\overline{\varepsilon}$ uniform over $[0, \overline{\varepsilon}_{max}]$. They value CP's content at u.

[3] Contributions along these lines include among others Gabszewicz et al. (2004), Peitz and Valletti (2008), Crampes et al. (2009), De Corniere and Taylor (2014), Ambrus et al. (2016), Anderson and Peitz (2020).

Access to the network of the operator O generates traffic which costs c per unit of traffic. At this stage, we do not distinguish between the cost of traffic and the cost of building capacity because capacity will be exhausted by traffic. Hence c can be interpreted as a long-run traffic-sensitive cost that is exogenous in this baseline model but will later be endogenized with an ex-ante investment stage for O (section 4).

Each user generates basic traffic $\overline{\alpha}$, independent of the traffic generated by consuming CP's content. In addition, each unit of CP's content generates a quantity of traffic β, which we refer to as the "load factor." The load factor is chosen by CP within an interval $[\underline{\beta}, \overline{\beta}]$ and affects users' utility according to

$$u = v(\beta).$$

The function $v(.)$ is increasing capturing the idea that higher content quality $v(\beta)$ requires a higher traffic load β. Hence, in this baseline specification, the cost of moderating traffic is lower content quality and therefore lower demand (everything else equal). Note that there could also be direct costs for CP to contain traffic, which we will examine in a robustness section (subsection 7.2).

We assume the operator O offers a single contract that provides access at a fee T_O. In particular, O is not allowed to discriminate between different types of consumption or usage intensity. We provide in Appendix a micro-foundation for the optimality of a zero-usage price for data based on ambiguity aversion of consumers and the risk of a large demand shock (see Appendix A).

To simplify notation we normalize the network price by assuming that the fee T is a markup over the cost $c\overline{\alpha}$ of base traffic. In other words, the total fee paid by any consumer to the network operator is

$$T_O = T + c\overline{\alpha}.$$

We then redefine users' maximal willingness to pay as net of $c\overline{\alpha}$:

$$\overline{P}(c) = \overline{P}_0 - c\overline{\alpha},$$

that we will shorten to $\overline{P}$, unless the argument is needed.

Importantly, the operator may share the cost of traffic with CP. In this case, it charges a price a per unit of traffic to CP for delivery of the content. This price is referred to as the *cost-share*. It is non-negative and less than the cost:[4]

$$0 \leq a \leq c.$$

On the content side we want to consider both the case of free ad-financed content and the case of paid content with no ad. For ad-financed content, we follow Anderson and Coate (2005) in assuming that the "implicit" cost borne by the consumer is the disutility from advertising. In other words, we assume that attention is scarce and that advertising imposes a cost in terms of attention. The seller of the content can vary this cost by varying the volume of advertising. We represent the choice of advertising by the monetary equivalent λ of the disutility to the consumer, the *nuisance*. We wish to emphasize that different content distributors have different abilities to monetize the attention of their consumers. For a given nuisance λ, we thus assume that the content provider CP receives a revenue $R = r.\lambda$ per consumer, where r is referred to as the *return to ads*. Assuming a linear relationship between nuisance and revenue simplifies the analysis and is sufficient to capture the main difference between the free and the paid model, i.e., the possibility that the (monetary equivalent) cost of the consumer does not coincide with the revenue of the seller.[5] We explore the case where the revenue R is a non-linear function of λ in subsection 7.1.

The case where $r = 1$ corresponds to the situation of a paid service, and in this case we will interpret λ as a monetary price charged to consumers. We then say the service is free if $r \neq 1$ while the service is paid when $r = 1$. With an abuse of language we will sometime refer to λ as the "price" of content.

For an ad-financed service, the return to ads r varies with the efficiency of the ad technology (such as targeting for instance). In the case $r \geq 1$, the ad-financed business model is more profitable than a paid business model–that is the revenue is higher under ad-financing than under paid service for a given demand of content. In this case, CP prefers to adopt this business model even if it can opt for a paid service. For lower efficiency of ads, there may be

[4]Negative prices are not allowed. One reason is that it would lead CP to artificially traffic inflation. Conversely, a charge a above c would induce O to artificially inflate the traffic.

[5]Implicitly we assume that advertisers are non-competing, homogeneous with unit demand of ad per consumer and constant benefit across consumers. Then if b is the profit per consumer of an advertiser, m is the volume of advertisers per consumer, and d is the disutility per unit of ads, we have $\lambda = m.d$ and $r = b/d$. In this model, advertisers have no utility in equilibrium as long as CP is a bottleneck and the social benefit per consumer from advertising is $\lambda(r - 1)$.

technical or behavioral reasons that prevent the adoption of a paid business model that are not represented in our model–for instance a fixed cost of micro payments or consumers' lack of trust, in which case CP has to rely on ad-financing. To capture this possibility, we allow r to be less or larger than one.

Given the fee T and nuisance λ, a user is willing to buy access and consume the content if $u \geq \lambda$ and $\overline{P} + u - \overline{\varepsilon} - \lambda \geq T$. She then pays a total hedonic price

$$P \equiv T + \lambda.$$

Suppose there is a mass $\overline{m}$ of users, their aggregate demand is

$$\overline{D}(P) = \frac{\overline{m}}{\overline{\varepsilon}_{max}}[\overline{P} + u - P] \equiv \overline{\gamma}[\overline{P} + v(\beta) - P].$$

Assumptions To streamline the analysis we make the following assumptions:

(A1) The "choke price" $\overline{P}$ is large relative to the cost c.

(A2) Users' utility from content quality $v(.)$ is concave, quadratic and increasing on $[\underline{\beta}, \overline{\beta}]$, with $v'(\overline{\beta}) = 0$ and $v'\left(\underline{\beta}\right) > c$. Moreover $v(\underline{\beta})$ is large enough that $\lambda < v(\beta)$ in any equilibrium.

These assumptions will ensure that the operator finds it always optimal to serve users and that equilibrium prices are interior solutions. As for the linearity of the advertising revenue in λ, the choice of a quadratic value function v is done to simplify the exposition. The function can then be written as

$$v(\beta) = v_0 - \frac{v_2}{2}\left(\overline{\beta} - \beta\right)^2 \text{ with } v_2\left(\overline{\beta} - \underline{\beta}\right) > c.$$

Timing To conclude the model, suppose that the timing is the following

1. The cost-share a is set,

2. O sets T and CP sets λ and β simultaneously,

3. Users consume $\overline{D}(P)$ and traffic $\overline{D}(P)(\overline{\alpha} + \beta)$ is generated.

Note that the cost-share a is negotiated or regulated at stage 1, and assumed to be rigid throughout the stage 2 and 3.

3 Analysis

3.1 Equilibrium

Consider stage-2 decisions. Given the demand $\overline{D}(P)$ the operator chooses price T to maximize

$$\Pi_O \equiv (T + \beta(a-c))\,\overline{D}(P). \tag{1}$$

Therefore O's fee is characterized by the standard first-order condition:

$$\overline{\gamma}(T - \beta(c-a)) = \overline{D}(P). \tag{2}$$

Moreover, CP chooses the nuisance and the load factor that maximize the profit

$$\Pi_{CP} \equiv (r\lambda - a\beta)\,\overline{D}(P). \tag{3}$$

Notice that CP's margin per unit of content can be expressed as $r\left(\lambda - \frac{a\beta}{r}\right)$. Intuitively, this means that increasing the return to ad should have a similar effect as lowering the cost for content. Indeed CP's first-order condition for λ is

$$(r\lambda - a\beta)\,\overline{\gamma} = r\overline{D}(P) \Leftrightarrow \left(\lambda - \frac{a\beta}{r}\right)\overline{\gamma} = \overline{D}(P). \tag{4}$$

When choosing the load β, CP faces the following trade-off. Decreasing the load reduces the cost when the cost-share is positive but it degrades the quality of the content and as a result reduces demand. With zero cost-sharing, CP would therefore choose the highest quality and load factor $\overline{\beta}$. But with a positive cost-share a, it will trade off the impact of a against quality according to the first-order condition:

$$(r\lambda - a\beta)\overline{\gamma}v'(\beta) \leq a\overline{D}(P), \tag{5}$$

where (5) holds with equality if $\beta > \underline{\beta}$.

This first-order condition can be further simplified by using the optimality of the nuisance to derive the equilibrium margin. Combining (4) and (5) then allows to write the new optimality condition for the load factor:

$$v'(\beta) = \frac{a}{r} \text{ if } a \leq rv'(\underline{\beta}), \text{ and } \beta = \underline{\beta} \text{ otherwise.} \tag{6}$$

This defines a unique relationship between the cost-share and the load factor:

$$\beta = \beta^*(a).$$

Interestingly the optimal load factor $\beta^*(a)$ is only affected by the cost-share and the return to ads. This is a key difference between our model and models with exogenous ad revenue. Indeed if the nuisance were exogenous, the load factor β would depend both on λ and T according to condition (5).

An increase in the cost-share a has the intuitive effect of inducing content moderation by CP. However, this effect is weaker when r is large. Intuitively, when CP is very efficient at monetizing users, a marginal reduction in demand has a higher opportunity cost. Hence CP is more reluctant to degrade quality in order to reduce traffic when r is high. Then assumption (A2) implies $\beta > \underline{\beta}$ for any $a \in (0, c]$ if $r \geq 1$.

Combining (2) and (4) gives the equilibrium fee and nuisance:

$$T = \frac{1}{3}\left(\overline{P} + v(\beta) - \beta\frac{a}{r} - 2\beta a + 2\beta c\right), \tag{7}$$

$$\lambda = \frac{1}{3}\left(\overline{P} + v(\beta) + 2\beta\frac{a}{r} + \beta a - \beta c\right). \tag{8}$$

We are interested in the effect of the cost-share a on the equilibrium outcome:[6]

- What is the effect of cost-sharing on prices and quantities?
- When is it socially optimal to impose a positive cost-share?
- How does the optimal cost-share varies with the characteristics of the content?

3.2 The impact of cost sharing

To better understand the effect of the cost-share a, it is useful to identify existing distortions. To this end, consider the sum of first-order conditions (2) and (4):

$$T + r\lambda - \beta c = (1 + r)\left(\overline{P} + v(\beta) - P\right). \tag{9}$$

Conditions (2) and (4) exhibit two departures from total profit maximization. The first distortion is the usual double-marginalization for complementary products–the Cournot effect. It is related to the term $1 + r$ in the right hand side, that is too high and due to the fact that each firm does not internalize the effect of its price on the profit of the other firm, and thus charges an excessive price.

[6]Note that assumptions (A1) and (A2) ensure that prices are positive and that the equilibrium demand by both types of users is strictly positive (see Equation 27 below).

The second distortion is novel and specific to ad-financed services. When r differs from 1, a given change in demand does not generate the same revenue when it is due to change in the operator's fee T than when it is due to a change in the ad-nuisance λ. Hence, when O and CP do not coordinate, the mix between price and nuisance is suboptimal. In our model, maximizing the joint profit of O and CP would require setting either T or λ to zero and focus on the most efficient financing mode. For $r < 1$, an integrated monopoly would set $\lambda = 0$ and

$$T - \beta c = \overline{P} + v(\beta) - T.$$

Symmetrically, for $r > 1$, raising revenues through advertising is more efficient than charging consumers for traffic so that an integrated monopoly could increase its profit by lowering the access charge to $T = 0$ and choosing the ad level so that

$$r\lambda - \beta c = r\left(\overline{P} + v(\beta) - \lambda\right).$$

Hence the total price for users is too high and its composition in terms of access fee and ad nuisance is inefficient. The latter observation suggests that the cost-share a could play a meaningful role: by shifting costs between O and CP, a affects relative prices and therefore the magnitude of the composition distortion.

To build intuition on the impact of a on prices, consider the unilateral pass-through rate for each firm. By unilateral we mean the change in price induced by a change in the cost-share a holding the load factor β and the other firm's price (T or λ) constant. Differentiating O's first-order condition (2), we get the pass-through rate

$$\left.\frac{dT}{da}\right|_{\beta,\lambda=cst} = -\frac{\beta}{2}. \tag{10}$$

Similarly, using CP's first-order condition (4) we get the pass-through rate

$$\left.\frac{d\lambda}{da}\right|_{\beta,T=cst} = \frac{\beta}{2r}. \tag{11}$$

Holding constant the load factor, increasing a amounts to a transfer of cost of traffic from O to CP. When $r = 1$ (paid service) the pass-through rates are the same and at constant load factor, the increase of one price is compensated by the decrease of the other price so that the total price $P = T + \lambda$ remains the same. However, this neutrality result no longer holds for ad-financed services. In particular, when $r > 1$ the pass-through rate is smaller for CP than for O, so we expect the total price of content to decrease with a. We formalize this intuition in the next result.

Lemma 1 *Suppose β is fixed. Then for paid content ($r = 1$), the equilibrium levels of content consumption, and O's and CP's margins are independent from the cost-share. For ad-financed content, if $r > 1$ (resp. $r < 1$), a marginal increase in the cost-share raises (resp. lowers) consumption.*

Proof.

From equations (7) and (8),

i) if $r = 1$, the equilibrium price $P = T + \lambda$ is independent from a, and so are O's profit margin $T + \beta a - \beta c$, and CP's profit margin $\lambda - \beta a$.

ii) if $r > 1$, the total price P is decreasing in a, therefore demand $\overline{D}(P)$ is increasing in a,

iii) the reasoning is symmetric if $r < 1$.

■

Notice that content consumer surplus is

$$CS = \frac{1}{2\overline{\gamma}}\overline{D}(P)^2 \tag{12}$$

while profits are

$$\Pi_O = \frac{1}{\overline{\gamma}}\overline{D}(P)^2 \text{ and } \Pi_{CP} = \frac{r}{\overline{\gamma}}\overline{D}(P)^2. \tag{13}$$

Hence total welfare is proportional to the square of demand. Under a fixed load factor β, when CP's return to ads is greater than 1, increasing the cost-share a raises consumer surplus and profits. In particular, the decline of the operator's fee T and the induced increase in demand compensate for the extra wholesale cost for CP.

Lemma 1 ignores the incentive effect of the cost-share a on CP's incentives to moderate traffic. Cost-sharing induces CP to internalize part of the impact of the load β on the traffic cost, which can lead to additional efficiency gains. Accounting for this incentive effect, we obtain our main baseline result:

Proposition 1 *There exists $\underline{r} < 1$ such that:*
- if $r \geq 1$, an increase in the cost-share raises consumption, consumer surplus and total welfare.
- if $\underline{r} < r < 1$, consumption, consumer surplus and total welfare are maximal at some interior cost-share a^*.
- if $r \leq \underline{r}$, then consumption, consumer surplus and total welfare are maximal at $a = 0$.

Proof.

From (12) and (13), maximizing consumer welfare and O's and CP's profits is akin to maximizing demand. From (7) and (8),

$$\overline{D}(P) = \frac{\overline{\gamma}}{3}\left[\overline{P} + v(\beta) - \beta\frac{a}{r}(1-r) - \beta c\right] \tag{14}$$

From (6), for $a < rv'(\underline{\beta})$:

$$\frac{d\beta}{da} = -\frac{1}{rv_2} < 0 \text{ and } \frac{d\beta}{dr} = \frac{a}{r^2 v_2} > 0.$$

It follows that

$$\begin{aligned}\frac{d\overline{D}(P)}{da} &= -\frac{\overline{\gamma}}{3}\frac{\beta}{r}(1-r) + \frac{\overline{\gamma}}{3}\left[v'(\beta) - \frac{a}{r}(1-r) - c\right]\frac{d\beta}{da} \\ &= \frac{\overline{\gamma}}{3}\left[\frac{\beta}{r}(r-1) + (a-c)\frac{d\beta}{da}\right] \end{aligned} \tag{15}$$

If $r \geq 1$, (15) is always positive. If $r < 1$, we have

$$\frac{d\left(\frac{\beta}{r}(r-1) + (a-c)\frac{d\beta}{da}\right)}{da} = \frac{2r-1}{r}\frac{d\beta}{da}$$

which has a constant sign if $a < v'(\underline{\beta})r$. (15) is negative at $a = c$. The slope is positive at $a = 0$ if

$$r > \underline{r} = 1 - \frac{c}{\overline{\beta}v_2} \tag{16}$$

Hence for $\underline{r} < r < 1$, the equilibrium quantity is maximal at some interior a. For smaller values of r the slope is negative at $a = 0$ and $a = c$. Hence the slope is negative for all a. ■

When the return to ads is larger than one, the two effects of the cost-share work in the direction of increasing efficiency and consumer welfare. Through the cost-share, CP better internalizes the cost its traffic imposes on O. CP ultimately benefits from higher traffic moderation through a lower access fee T which boosts demand despite content being of lower quality. In addition, the price composition effect in Lemma 1 also leads to a lower overall price for access and content which further stimulates demand. Overall, the cost-share not only benefits consumers but also the network operator and the content provider (as from (13) O's and CP's profits increase with demand).[7]

[7] The conclusion on profits follows from the assumption in the base model that content and access are pure complements and may not extend to the general case with heterogenous demand for content–see Section 5 for the case of partial complementarity.

For returns to ads lower than one, a trade-off emerges between traffic moderation and the price effect. The traffic moderation induced by the cost-share is still efficient but for very low levels of r, it is dominated by an overall price increase due to inefficient price composition. In that case, the introduction of a cost share leads to lower demand and lower profits for both firms.

In the intermediate range between $\underline{r}$ and 1 there is a balance between the two extreme cases, with an initial positive effect of the cost-share that reaches its maximum before full cost-sharing. At a low level of cost-sharing, the main effect is the reduction of the price P triggered by CP's moderation of traffic, which dominates the direct effect of a on T and λ. At larger level of cost-sharing, the quality reduction $v'(\beta)\,(d\beta/da)$ becomes larger and the direct effect on price becomes stronger so that the overall effect on demand becomes negative.

To conclude this section, we notice that from equation (16), the threshold $\underline{r}$ is decreasing in the per-unit cost of traffic c. Hence a larger c broadens the range of content that should contribute to the cost. Moreover the threshold $\underline{r}$ is increasing in v_2 which measures the impact of traffic moderation on the quality of the content. Intuitively, a larger impact reduces the benefit of cost moderation.

4 Technology choice by the network operator

The direct effect of a higher cost-share is to raise CP's incentives to moderate traffic. However, the total effect on cost moderation is a priori unclear if the operator's cost c endogenously responds to a change in the equilibrium level of traffic. For instance, O may have weaker incentives to contain the per-unit cost c if the traffic generated by CP's content is lower. This may in turn affect the impact of a on consumer demand and welfare.

To pin down O's endogenous response, we allow the operator to invest in cost reduction: in this section, O can choose $c \in [\underline{c}, \overline{c}]$ at cost $\phi(c)$, where $\phi(.)$ is decreasing and convex. O's investment decision is observable, made after the cost-share a is determined but before O and CP set prices T and λ. That is, we think of c a long-term structural choice that affects O's cost structure throughout the subsequent stages 2 and 3 of the model.

Relative to (13), O's equilibrium profit is modified to account for the investment:

$$\Pi_O = \frac{1}{\gamma}\overline{D}(P)^2 - \phi(c).$$

Therefore the marginal benefit of *lowering* c (of investing in cost reduction) is

$$-\frac{2}{\overline{\gamma}}\overline{D}(P)\frac{dD(P)}{dc} = \frac{2}{3}\overline{D}(P)(\overline{\alpha}+\beta). \tag{17}$$

That is, a lower c benefits O because it lowers both the cost $\overline{\alpha}c$ of delivering basic access and the cost βc of delivering content to consumers.[8]

To establish some regularity conditions, let us first consider an exogenous increase in demand $\overline{P}_0$. O's first-order condition with respect to c is

$$-\frac{2}{3}\overline{D}(P)(\overline{\alpha}+\beta) - \phi'(c) = 0. \tag{18}$$

Differentiating (18) with respect to $\overline{P}_0$ and using (14) yields

$$\left(\phi''(c) - \frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)^2\right)\frac{dc}{d\overline{P}_0} = -\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta).$$

Hence we find that an exogenous increase in demand raises incentives to invest in cost reduction if the bracketed term is positive, that is if ϕ is convex enough. We assume this holds in the rest of this section:

(A3) An exogenous upward shift in demand ($d\overline{P}_0 > 0$) induces the operator to choose a lower marginal cost of traffic, that is $\phi''(c) > 2\overline{\gamma}(\overline{\alpha}+\overline{\beta})^2/9$.

Now consider how this marginal benefit changes with the cost-share a. Differentiating (18) with respect to a, we obtain:

$$\frac{d\left(\frac{2}{3}\overline{D}(P)(\overline{\alpha}+\beta)\right)}{da} = \frac{2}{3}\left(\left.\frac{d\overline{D}(P)}{da}\right|_{c=cst}(\overline{\alpha}+\beta) + \frac{d\beta}{da}\overline{D}(P)\right). \tag{19}$$

Suppose the cost-share a is in the region where a marginal increase raises consumer welfare for a given c (see Proposition 1). Then this increase in a has two opposing effects on O's incentives to invest in cost reduction. The first term between brackets in (19) captures the demand effect of the cost-share: as a increases, demand increases and therefore O's marginal benefit of reducing the per-user cost goes up. The second term captures the cost effect of the cost-share: when a increases, CP reduces the load factor β (i.e., $\frac{d\beta}{da} < 0$), which lowers O's cost of delivering content and therefore diminishes its incentives to invest in further cost reduction.

[8] Recall that the normalization $\overline{\alpha} = 0$ implies that $\frac{d\overline{P}(c)}{dc} = -\overline{\alpha}$.

To see that this second effect can dominate, suppose $r \in (\underline{r}, 1)$ and a is close to the cost-share a^* that maximizes consumer welfare. In that case, $\frac{d\overline{D}(P)}{da}\Big|_{c=cst}$ is close to 0 and therefore (19) is negative, i.e., a marginal increase in a lowers O's investment in cost reduction. Conversely for r large enough, the effect of a on β is small: CP is reluctant to reduce demand by lowering quality when the return to ads is very high. As a result, the second effect vanishes and the positive effect of a on demand induces O to invest more in cost reduction.

Let us turn now to the effect on welfare. That is, consider the total differentiation of demand with respect to the cost-share a:

$$\frac{d\overline{D}(P)}{da} = \frac{\overline{\gamma}}{3}\left[\frac{\beta}{r}(r-1) + (a-c)\frac{d\beta}{da} - (\overline{\alpha}+\beta)\frac{dc}{da}\right] \tag{20}$$

Relative to subsection 3.2 where c is exogenous, the third term between brackets in (20) can alter the conclusions in Proposition 1. To see this, consider again the case where $r \in (\underline{r}, 1)$ and a tends to the optimal cost-share a^* defined in Proposition 1 from below. In that case, $\frac{\beta}{r}(r-1) + (a-c)\frac{d\beta}{da}$ is close 0 (i.e., the effect of the cost-share on demand for a given c is small). Furthermore from the previous paragraph, $\frac{dc}{da} > 0$, i.e., an increase in the cost-share depresses O's incentives to invest in cost reduction. The overall effect of a marginal increase in a on demand is therefore negative for a close enough to a^*.

This discussion suggests the results in Proposition 1 do not readily extend to the case where c is endogenous. However, qualitatively similar insights obtain. First, the impact of a on the profit of the operator is as in Proposition 1. This is just a consequence of O optimizing on c so that the variation O's profit with respect to a only depends on the effect of a on demand for a fixed c:

$$\frac{d\Pi_O}{da} = \frac{d\left(\frac{1}{\overline{\gamma}}\overline{D}(P)^2\right)}{da}\Bigg|_{c=cst} = \frac{2}{\overline{\gamma}}\overline{D}(P)\ \frac{d\overline{D}(P)}{da}\Bigg|_{c=cst}.$$

As for the impact of the cost-share a on consumer welfare, it should intuitively remain similar to Proposition 1 as long as the cost c is not too responsive to a change in a. Formally, this corresponds to ϕ convex enough:

$$\phi''(.) > \frac{2\overline{\gamma}^2(\overline{\alpha}+\overline{\beta})(\overline{P}(\underline{c}) + v(\overline{\beta}))}{27\underline{c}}. \tag{21}$$

Proposition 2 *If c is endogenous and (21) holds, there exists $\underline{r}^c < 1$ such that for any $r > \underline{r}^c$ consumer welfare is maximized for $a > 0$.*

Proof.

Differentiating (18) with respect to a and using (14) yields

$$-\frac{2}{3}\left[\left.\frac{dD(P)}{da}\right|_{c=cst}(\overline{\alpha}+\beta)+\overline{D}(P)\frac{d\beta}{da}\right]-\left(\phi''(c)-\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)^2\right)\frac{dc}{da}=0,$$

where

$$\left.\frac{dD(P)}{da}\right|_{c=cst}=\frac{\overline{\gamma}}{3}\left[\frac{\beta}{r}(r-1)+(a-c)\frac{d\beta}{da}\right]$$

(see (15)). Total differentiation of demand is

$$\begin{aligned}\frac{d\overline{D}(P)}{da} &= \left.\frac{dD(P)}{da}\right|_{c=cst}-\frac{\overline{\gamma}}{3}(\overline{\alpha}+\beta)\frac{dc}{da}\\ &= \left.\frac{dD(P)}{da}\right|_{c=cst}+\frac{2\overline{\gamma}}{9}\frac{\overline{\alpha}+\beta}{\phi''(c)-\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)^2}\left[\left.\frac{dD(P)}{da}\right|_{c=cst}(\overline{\alpha}+\beta)+\overline{D}(P)\frac{d\beta}{da}\right]\\ &= \frac{\phi''(c)\left.\frac{dD(P)}{da}\right|_{c=cst}+\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)\overline{D}(P)\frac{d\beta}{da}}{\phi''(c)-\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)^2}.\end{aligned}$$

This is strictly positive for $r\geq 1$ and $a=0$ if $\phi''(c)>\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)^2$ and

$$\phi''(c)>\frac{2\overline{\gamma}}{9}(\overline{\alpha}+\beta)\frac{\overline{D}(P)}{c} \tag{22}$$

By assumption $\overline{P}$ is high enough so that $\overline{D}(P)>(\overline{\alpha}+\beta)c$. Therefore condition (22) is sufficient and always verified if condition (21) holds. The results then follow because consumer welfare is an increasing function of demand and by continuity for r smaller but close to 1.

■

5 Cost sharing with heterogeneous demand for content

The previous sections analyzed a case where traffic and content are pure complements in that users consume traffic only if they also want to consume content. In this section we extend the analysis to partial complementarity between the demand for access and the demand for CP's content.

Formally, we keep the users already introduced in section 2 unchanged, but refer to them as *content* users in this section. This label distinguishes them from a second category called *basic* users, which we introduce now. Basic users only have a unit demand for access to the network and no demand for the content. This is interpreted as access to basic services and to other content with a fixed exogenous price. Basic users value this access at $\hat{P}_0$ minus a convenience cost $\hat{\varepsilon}$ uniform over $[0, \hat{\varepsilon}_{max}]$. Basic consumers generate a traffic $\overline{\alpha} + \hat{\alpha}$, where $\hat{\alpha}$ is the traffic differential between basic and content consumers, which can be positive, zero or negative.

To keep notation consistent with the basic model, we define the maximal willingness to pay of basic users net of $c\overline{\alpha}$ as,

$$\hat{P}(c) \equiv \hat{P}_0 - c\overline{\alpha},$$

which we assume to be large relative to c, an assumption that mirrors (A1). Again we omit the argument unless it is needed for clarity.

Since basic users only pay the fee T, their total demand given a mass $\hat{m}$ is

$$\hat{D}(T) \equiv \frac{\hat{m}}{\hat{\varepsilon}_{max}}[\hat{P} - T] = \hat{\gamma}[\hat{P} - T].$$

The addition of basic users leads to a generalized profit function for the operator,

$$\Pi_O \equiv (T + \beta(a - c))\, \overline{D}(P) + (T - \hat{c})\, \hat{D}(T),$$

where $\hat{c} = c\hat{\alpha}$ is the cost differential of traffic unrelated to CP's content, between basic and content consumers. Therefore O's first-order condition with respect to T becomes

$$(\hat{\gamma} + \overline{\gamma})T - \overline{\gamma}\beta(c - a) - \hat{\gamma}\hat{c} = \overline{D}(P) + \hat{D}(T). \tag{23}$$

Note that CP's optimization problem in λ and β remains unchanged, i.e., CP's profit function and the corresponding first-order conditions (4) and (5) are as in the baseline model. Combining these conditions with (23) yields equilibrium prices that generalize (7) and (8):

$$T = \frac{2\hat{\gamma}(\hat{P} + \hat{c}) + \overline{\gamma}\left(\overline{P} + v(\beta) - \beta\frac{a}{r} - 2\beta a + 2\beta c\right)}{4\hat{\gamma} + 3\overline{\gamma}}, \tag{24}$$

$$\lambda = \frac{\hat{\gamma}(2\overline{P} + 2v(\beta) - \hat{P} + 2\beta\frac{a}{r} - \hat{c}) + \overline{\gamma}\left(\overline{P} + v(\beta) + 2\beta\frac{a}{r} + \beta a - \beta c\right)}{4\hat{\gamma} + 3\overline{\gamma}}. \tag{25}$$

The presence of basic consumers mitigates the response of O's price to cost-sharing, as only part of the cost is reduced. This can be seen again by evaluating the pass-through at constant nuisance λ and constant load β:

$$\left.\frac{dT}{da}\right|_{\lambda,\beta=cst} = -\frac{\overline{\gamma}}{\overline{\gamma} + \hat{\gamma}}\frac{\beta}{2}, \tag{26}$$

which decreases in the fraction $\hat{\gamma}/\overline{\gamma}$ of basic consumers. This attenuated effect reduces the overall impact of cost-sharing on the price and demand of content users.

Then from (24) and (25) we get

$$\overline{P} - P = \frac{\hat{\gamma}\left(2\overline{P} + 2v(\beta) - \hat{P} - \hat{c} - 2\beta\frac{a}{r}\right) + \overline{\gamma}\left(\overline{P} + v(\beta) - (1-r)\beta\frac{a}{r} - \beta c\right)}{4\hat{\gamma} + 3\overline{\gamma}}, \tag{27}$$

where we see that the sign of the effect of the cost-share a on the total price P is the sign of $\overline{\gamma}(r-1) - 2\hat{\gamma}$. In particular, for a fixed value of the load factor β, the demand is not affected by cost-sharing if $r = 1 + 2\hat{\gamma}/\overline{\gamma}$.

The previous conclusions for pure complements then extend as follows to imperfect complements.

Proposition 3 *Assume that $\hat{\gamma} > 0$. An increase in the cost-share a always raises basic consumers' consumption and surplus. In addition, defining*

$$\underline{r} \equiv 1 + 2\frac{\hat{\gamma}}{\overline{\gamma}} - \frac{c}{\overline{\beta} v_2} \quad \textit{and} \quad \hat{r} \equiv 1 + 2\frac{\hat{\gamma}}{\overline{\gamma}} :$$

- if $r \geq \hat{r}$, an increase in the cost-share raises content consumers' consumption surplus.
- if $\underline{r} < r < \hat{r}$, content consumers' consumption and surplus increase with a up to a maximum at a^* interior.
- if $r \leq \underline{r}$, an increase in the cost-share always decreases content consumers' consumption and surplus.

Proof. From (7), $\frac{dT}{da}$ has the sign of

$$-\beta\left(\frac{1}{r} + 2\right) + (c-a)\frac{d\beta}{da} < 0,$$

therefore basic consumers' demand increase in a. Turn to content consumers' demand:

$$\begin{aligned}
\frac{d\overline{D}(P)}{da} &= \frac{\overline{\gamma}\hat{\gamma}}{4\hat{\gamma} + 3\overline{\gamma}}\left[-2\frac{\beta}{r} + 2\left(v'(\beta) - \frac{a}{r}\right)\frac{d\beta}{da}\right] \\
&\quad + \frac{\overline{\gamma}^2}{4\hat{\gamma} + 3\overline{\gamma}}\left[-\frac{\beta}{r}(1-r) + \left(v'(\beta) - \frac{a}{r}(1-r) - c\right)\frac{d\beta}{da}\right] \\
&= \frac{\overline{\gamma}^2 \beta}{r(4\hat{\gamma} + 3\overline{\gamma})}\left[-2\frac{\hat{\gamma}}{\overline{\gamma}} + r - 1 + \frac{a-c}{\beta} r \frac{d\beta}{da}\right]
\end{aligned} \tag{28}$$

If $r \geq 1 + 2\frac{\hat{\gamma}}{\overline{\gamma}}$, then (28) is always positive.

If $r < 1 + 2\frac{\hat{\gamma}}{\overline{\gamma}}$, (28) is negative at $a = c$. The slope is positive at $a = 0$ if

$$r > \underline{r} \equiv 1 + 2\frac{\hat{\gamma}}{\overline{\gamma}} - \frac{c}{\overline{\beta} v_2}.$$

Furthermore, as in the proof of Proposition 1, $\frac{d^2\overline{D}(P)}{da^2}$ has a constant sign on the range $a < v'(\underline{\beta})r$, hence for $\underline{r} < r < 1$, $\overline{D}(P)$ is increasing for a small enough and decreasing afterwards.

Finally if $r < \underline{r}$, $\overline{D}(P)$ is decreasing in a.

The result for content consumer surplus follows from the fact that it is proportional to $\overline{D}(P)^2$. ■

The first immediate effect of cost-sharing is a reduction of the operator's fee T that benefits all users, including those not consuming CP's content. Thus cost-sharing between CP and O generates a positive externality for basic consumers that can be interpreted as a reduction in the market power exercised by O.

For consumers of CP's content, the decline in T comes with an increase in λ, as in the baseline model with pure complements (section 3), albeit adjusted for the lower pass-through on O's price. As a result, the level of the cost-share a that maximizes content consumers' welfare increases with the weight of content consumers relative to basic consumers, captured by $\frac{\overline{\gamma}}{\hat{\gamma}}$. We formalize this intuition in the following corollary.

Corollary 1 *The cost-share a^* that maximizes content consumers' welfare is (weakly) increasing in the relative weight of content consumers $\frac{\overline{\gamma}}{\hat{\gamma}}$.*

Proof. From Proposition 3, we know that the regions of r where $a^* = 0$ and $a^* = c$ are shifted upwards by a factor $2\hat{\gamma}/\overline{\gamma}$. Therefore if $a^* = c$ then for any increase in $\frac{\overline{\gamma}}{\hat{\gamma}}$, we will still have $a^* = c$. Conversely and if $a^* = 0$, for any increase in $\frac{\overline{\gamma}}{\hat{\gamma}}$ we will have $a^* \geq 0$.

Next, suppose a^* is interior, i.e., $r \in (\underline{r}, 1 + 2\hat{\gamma}/\overline{\gamma})$. Then from the proof of Proposition 3 (Equation 28), we have

$$-2\frac{\hat{\gamma}}{\overline{\gamma}} + r - 1 - \frac{a^* - c}{\beta v_2} = 0,$$

where β is independent from $\frac{\hat{\gamma}}{\overline{\gamma}}$. It follows that

$$\frac{da^*}{d\frac{\overline{\gamma}}{\hat{\gamma}}} = 2\left(\frac{\hat{\gamma}}{\overline{\gamma}}\right)^2 \beta v_2 > 0.$$

■

As discussed above, the intuition for this corollary is driven by O cutting T more aggressively in response to an increase in the cost-share when the relative weight of content consumer is high. Note that the marginal benefit of an increase in the cost-share a is also higher for basic consumers when the weight of content consumer $\frac{\overline{\gamma}}{\hat{\gamma}}$ is higher. Overall, this result suggests that a cost-share is more likely to be welfare-improving when the content provider is larger.

Regarding profits, we notice that CP's profit is proportional to the content consumption. Thus the conclusions for CP's profit are the same as for content consumer surplus. For the profit of the operator O, we need to account for the change in profit on the basic segment. There are two main effects of a on the total profit Π_O. First, there is the effect on profit from the content consumer segment. As in the case of pure complements, this is positive for high return to ads and negative for very low return to ads.

Secondly, there is the indirect effect on the basic segment. In this segment, profit would be maximal at the monopoly price

$$T^b = \frac{\hat{P} + \hat{c}}{2}.$$

As T decreases with a, the effect on the profit generated by the basic segment depends on whether the price T is above or below T^b. More precisely

Proposition 4 *The effect of cost-sharing on the profit Π_O^C generated by the content consumers is as follows:*

- *If $r > \frac{\overline{\gamma}+\hat{\gamma}}{\overline{\gamma}+2\hat{\gamma}}$ then Π_O^C increases with a,*

- *If r is below $\frac{\overline{\gamma}+\hat{\gamma}}{\overline{\gamma}+2\hat{\gamma}}$ but not too small, then Π_O^C is maximal at some interior value of a,*

- *If r is small, then Π_O^C decreases with a.*

The profit from the basic segment increases with a if and only if $T > T^b$.

Proof. See Appendix.

Overall studying heterogeneous demand for content delivers two main insights. First, the results in the current section show that the intuitions from the pure-complement case are robust. It still is the case that a cost-share can improve overall consumer welfare for two reasons, first because it leads the content provider to better internalize the cost βc of delivering higher-quality content and second because it shifts the overall charge for content consumers from O to CP which is an efficient price re-composition when CP can better monetize users. Therefore as in the pure-complement case, the cost-share is all the more efficient as r is large. Proposition 4 also shows that an increase in the cost-share can raise O's profits provided r is large enough.

In addition, this section shows the impact of the share of content consumers relative to basic consumers. In particular, the welfare benefit of the cost-share for content consumers increases when content consumers represent a higher relative share. As for basic consumers, they always benefit from an increase in the cost-share, and this benefit is magnified by the relative share of content consumers: when this share is higher, O has a stronger price response to an increase in the cost-share, which benefits basic consumers.

6 Extensions

6.1 Competition between operators

In this section, we consider competition between N identical network operators indexed by n. There still is a single content provider who sets a nuisance λ and a load factor β common to all operators. Each operator sets a subscription fee T_n for traffic and collects a cost-share a_n from CP. The total price for content with operator n is $P_n = T_n + \lambda$. We let the demand for content on operator n be

$$D_N\left(P_n, P_{-n}\right) \equiv \frac{\gamma_N}{N}\left(\bar{P} + v\left(\beta\right) - \frac{P_n - \sigma_N P_{-n}}{1-\sigma_N}\right),$$

where

$$P_{-n} = \frac{\sum_{j \neq n} P_j}{N-1},$$

γ_N is non-decreasing in N and σ_N is increasing in N.

This specification extends the analysis of the pure-complement case in subsection 3.2 to competing operators, where σ_N increases substitutability between operators but keeps aggregate demand constant for symmetric prices. If all total prices are equal to P, the total demand for content is

$$Q = \sum D_N\left(P, P\right) = \gamma_N\left(\bar{P} + v\left(\beta\right) - P\right).$$

In addition, the case where γ_N strictly increases in N captures how network variety may increase total demand.

Turn to the analysis. For a given cost-share a_n, the profit of operator n is

$$\Pi_N = (T_n + (a_n - c)\,\beta)\, D_N\,(P_n, P_{-n})\,.$$

It follows that the operator chooses a price such that

$$\frac{T_n + (a_n - c)\,\beta}{1-\sigma_N} = \bar{P} + v\,(\beta) - \frac{P_n - \sigma_N P_{-n}}{1-\sigma_N}. \tag{29}$$

CP's profit is then

$$\Pi_{CP} = \frac{\gamma_N}{N} \sum_{n=1,N} (r\lambda - a_n\beta)\left(\bar{P} + v\,(\beta) - \frac{T_n - \sigma_N T_{-n}}{1-\sigma_N} - \lambda\right)$$

where

$$T_{-n} = \frac{1}{N-1}\sum_{j\neq n} T_j.$$

For the price λ of content we find

$$r\lambda - \frac{\sum_{n=1,N} a_n}{N}\beta = r\left(\bar{P} + v\,(\beta) - \frac{\sum_{n=1,N}(T_n - \sigma_N T_{-n})}{N\,(1-\sigma_N)} - \lambda\right). \tag{30}$$

Finally the choice of traffic moderation is

$$\sum_{n=1,N} \frac{a_n}{N}\left(\bar{P} + v\,(\beta) - \frac{T_n - \sigma_N T_{-n}}{1-\sigma_N} - \lambda\right) = \left(r\lambda - \frac{\sum_{n=1,N} a_n}{N}\beta\right) v'\,(\beta)\,. \tag{31}$$

Symmetric cost-shares

We focus here on the case where $a_n = a$ for all n. Then for all n, $T_n = T$ and $P_n = P$ such that

$$T + (a-c)\,\beta = (1-\sigma_N)\left(\bar{P} + v\,(\beta) - P\right) \tag{32}$$

$$r\lambda - a\beta = r\left(\bar{P} + v\,(\beta) - P\right) \tag{33}$$

$$v'\,(\beta) = \frac{a}{r}\ if\ v'\left(\underline{\beta}\right) \geq \frac{a}{r},\ or\ \beta = \underline{\beta} \text{ otherwise} \tag{34}$$

Hence the analysis in subsection 3.2 readily extends to the case of symmetric multiple operator. In particular, CP's best reply is not affected by the number of operators, because aggregate demand is independent from σ_N. However, σ_N affects the perceived elasticity of the residual demand curve of each operator, leading to more aggressive pricing as competition intensifies (N grows larger). Lower fees from operators stimulate demand and allow CP to raise prices and increase profits. We formalize this intuition in the following proposition

Proposition 5 *Holding constant the market size γ_N, increasing in the number N of operators raises total demand and CP's profit.*

Proof.

We have

$$P = \frac{(2-\sigma_N)\left(\bar{P}+v(\beta)\right)-(a-c)\beta+\frac{a\beta}{r}}{3-\sigma_N} \tag{35}$$

$$\lambda = \frac{\left(\bar{P}+v(\beta)\right)+(a-c)\beta+(2-\sigma_N)\frac{a\beta}{r}}{3-\sigma_N} \tag{36}$$

The choice of traffic moderation is not affected by N. Total demand is

$$Q = \gamma_N\left(\frac{\bar{P}+v(\beta)+(a-c)\beta-\frac{a\beta}{r}}{3-\sigma_N}\right), \tag{37}$$

which is increasing in σ_N. The CP price λ is also increasing in σ_N, hence CP's profit, $(r\lambda-a)Q$ increases in σ_N. ■

The analysis of the effect of the cost-share a is similar to the monopoly case. As earlier, consumer surplus,

$$CS = \frac{\gamma_N\left(\bar{P}+v(\beta)-P\right)^2}{2},$$

the profits of the operators,

$$\Pi_O = (T+(a-c)\beta)\,Q = (1-\sigma_N)\,\gamma_N\left(\bar{P}+v(\beta)-P\right)^2,$$

and the profit of content provider

$$\Pi_{CP} = r\left(\lambda-\frac{a}{r}\beta\right)D(P) = r\gamma_N\left(\bar{P}+v(\beta)-P\right)^2$$

increase with the aggregate quantity $Q=\gamma_N(\bar{P}+v(\beta)-P)$. From (37), we get that in equilibrium,

$$\frac{dQ}{da} = \gamma_N\left(\frac{\frac{\beta}{r}(r-1)+(a-c)\frac{d\beta}{da}}{3-\sigma_N}\right). \tag{38}$$

Keeping in mind that content moderation β is independent from N (from (34)), it follows that the sign of (38) does not depend on N.

Proposition 6 *The conclusions of Proposition 1 are valid for any number N of operators. Moreover the values of $\underline{r}$ and a^* are independent of N.*

Note however that the magnitude of the effect of the cost-share on consumer welfare and profit will depend on competition intensity. For instance, consider the case where the market size γ_N is constant and $r > \underline{r}$. Then

$$\frac{dCS}{da} = \gamma_N Q \frac{dQ}{da},$$

is increasing in N. This is because not only aggregate quantity Q but also the sensitivity of Q to the cost-share a, $\frac{dQ}{da}$, is increasing in N. Intuitively, the pass-through of the access sector increases as competition intensifies:

$$\left.\frac{dT}{da}\right|_{\lambda,\beta=cst} = -\frac{\beta}{2-\sigma_N}.$$

That is, each operator reacts not only to the direct effect of receiving the cost-share a from the content provider, but also to other operators lowering their prices, which amplifies the decrease in T.

Decentralized cost-shares

Suppose now each operator can set its own cost-share a_n. A higher cost-share leads the content provider to increase its price λ and reduce the load factor β. When the operator is a monopoly, it perfectly internalizes how this price increase affects its demand. Hence, a monopoly operator sets a positive cost-share when it is profit-maximizing. When multiple operators compete, an externality arises: following an increase in a_n by operator n, the increase in λ and the reduction in β affect not only operator n but also the demand of all other operators.

To formalize this intuition, we study under which condition operators, when allowed to set their own cost-share a_n, choose a strictly positive contribution. Specifically, consider the incentive of operator n to deviate to a strictly positive a_n when all other operators choose a zero cost-share. Using the first-order condition in (32), the profit of operator n is

$$(1-\sigma_N)\frac{\gamma_N}{N}\left(\bar{P} + v(\beta) - \frac{P_n - \sigma_N P_{-n}}{1-\sigma_N}\right)^2,$$

therefore the slope in a_n of Π_n when evaluated at $a_j = 0$ for all j is

$$\left.\frac{\partial \Pi_n}{\partial a_n}\right|_{\forall j, a_j=0} = \frac{\gamma_N}{N}\left[(1-\sigma_N)\frac{\partial(v(\beta)-P)}{\partial \beta}\frac{\partial \beta}{\partial a_n} - \frac{\partial(P_n - \sigma_N P_{-n})}{\partial a_n}\right] \tag{39}$$

The first term in (39) captures the effect of a_n on content moderation by CP. Using CP's first-order condition for content moderation in (31) and the expression for P in (35) we obtain that the marginal effect of a_n on the profit of operator n operating through content moderation (evaluated at $a_j = 0$ for all j) is

$$\left.\frac{\partial \Pi_n}{\partial a_n}\right|_{\lambda=cst,\{T_j\}_j=cst} = \frac{\gamma_N}{N}\frac{1-\sigma_N}{3-\sigma_N}\frac{c}{Nrv_2}, \tag{40}$$

which is strictly positive as traffic moderation by CP reduces operator n's cost of delivering content. Note that increasing a_n has the exact same marginal effect on the profit of all other operators $j \neq n$. While operator n does not internalize these effects, its incentives to choose a positive cost-share are aligned with the interests of the other operators in that dimension.

However, a positive cost-share a_n also affects per-unit income for operator n, as well as pricing for all operators and for CP. The effect of this change on CP's profit is captured in the second term in (39). We show in the Appendix:

$$\left.\frac{\partial \Pi_n}{\partial a_n}\right|_{\beta=cst} = \frac{\gamma_N}{N}\frac{1-\sigma_N}{3-\sigma_N}\left[\frac{(N-1+\sigma_N)(3-\sigma_N)}{(2N-2+\sigma_N)(1-\sigma_N)}(N-1)+\frac{r-1}{r}\right]\beta. \tag{41}$$

For operator n, the impact of an increase in a_n first depends on the efficiency of CP's content monetization r, as in the monopoly case in subsection 3.2: everything else equal, a higher r makes a marginal increase in a_n more profitable for n. This effect is magnified here relative to the monopoly case because CP, when adjusting λ, is less reactive to a unilateral raise in a_n given that all other operators keep $a_j = 0$. That is, even if $\sigma_N = 0$ and competition between operators is therefore muted, the first term in (41) still is strictly positive and depends on N.

In addition, an increase in a_n has a competitive effect: it leads CP to increase the price for content λ, which depresses the associated demand for traffic for all other operators $j \neq n$ and therefore boosts the demand for operator n. In particular, we show

$$\frac{\partial(P_n - P_j)}{\partial a_j} = -\frac{N-1}{2N-2-\sigma_N}\beta.$$

That is, in equilibrium, a unilateral marginal increase of the cost-share from 0 by operator n results in users facing a lower overall price for traffic and content when using operator n than any other operator j.

Overall, the price externality across operators that operates through the content provider implies that operators will set a strictly positive cost-share more often (for lower returns to ads r) under competition than a monopoly would. We formalize this intuition in the next proposition where $\underline{r}$ is defined in Proposition 1 as the minimum r above which a strictly positive cost-share raises the profit of a monopoly operator.

Proposition 7 *There exist $\underline{r}^c \leq \underline{r}$ such that some operators choose a strictly positive cost-share when $r > \underline{r}^c$. Operators choose a strictly positive cost-share more often when competition intensifies: $\underline{r}^c$ is decreasing in N and in σ_N.*

Proof. See Appendix.

We know from Proposition 6 that the threshold $\underline{r}$ above which setting a positive cost-share for every operator improves their joint profits is the same as for a monopoly operator. Then it follows from Proposition 7 that operators will set a strictly positive cost-share too often from the perspective of their joint profit. That is, there is a range of values for r below $\underline{r}$ where operators choose a strictly positive cost-share while if they could commit to keep it at zero they would each be better off.

Corollary 2 *If $r \in (\underline{r}^c, \underline{r})$, operators choose strictly positive cost-shares in a symmetric equilibrium whereas setting $a_j = 0$ for all j would raise their profits.*

Corollary 2 suggests there is a benefit for operators to coordinate. Because the introduction of a cost-share by one operator affects other operators through the reaction of the content provider, incentives for each operator to unilaterally share costs may not perfectly align with the incentives to introduce a coordinated cost-share for all operators.

6.2 Multiple contents

We now consider the case where one network operator delivers multiple contents. There is a continuum of content providers of mass 1 indexed by θ uniformly distributed over $[0, 1]$. The network operator chooses the access fee T and content provider θ chooses an advertising nuisance λ_θ and a traffic load β_θ. The demand for content θ is then

$$D_\theta = \gamma_\theta(\overline{P}_\theta + v_\theta(\beta_\theta) - \lambda_\theta - T). \tag{42}$$

Finally, content provider θ transfers a cost-share a_θ to the network operator.

The profit of the network operator is

$$\Pi_O = \int_0^1 (T - \beta_\theta (c - a_\theta)) \gamma_\theta \left(\bar{P}_\theta + v_\theta (\beta_\theta) - \lambda_\theta - T\right) d\theta,$$

and the profit of content provider θ is

$$\Pi_\theta = (r_\theta \lambda - a_\theta \beta_\theta)\gamma_\theta(\overline{P}_\theta + v_\theta(\beta_\theta) - \lambda_\theta - T).$$

Using that content provider's first-order condition with respect to λ, we get

$$\lambda_\theta = \frac{\bar{P}_\theta + v_\theta\left(\beta_\theta\right) + \frac{a_\theta}{r_\theta}\beta_\theta - T}{2},$$

which, from Equation 42 implies

$$D_\theta = \gamma_\theta \left(\frac{\bar{P}_\theta + v_\theta(\beta_\theta) - \frac{a_\theta}{r_\theta}\beta_\theta - T}{2} \right), \tag{43}$$

and therefore consumer surplus is

$$CS = \int_0^1 \frac{\gamma_\theta}{2} \left(\frac{\bar{P}_\theta + v_\theta(\beta_\theta) - \frac{a_\theta}{r_\theta}\beta_\theta - T}{2} \right)^2 d\theta. \tag{44}$$

We are interested in the variation of this equilibrium consumer surplus with respect to the cost-share a_θ of content provider j. In what follows we assume that $\beta_\theta > \underline{\beta}$ for all θ for conciseness. Using (44) and the fact that from content provider θ's first-order condition with respect to β_θ,

$$v'_\theta(\beta_\theta) = \frac{a_\theta}{r_\theta}, \tag{45}$$

we get

$$\frac{dCS}{da_\theta} = -\frac{D_\theta}{2}\frac{\beta_\theta}{r_\theta} - \frac{D}{2}\frac{dT}{da_\theta}, \tag{46}$$

where D is the total equilibrium demand.

The first term in (46) captures the effect of a_θ that only affects consumers of content θ. This effect runs through content provider j's response to the cost-share in terms of pricing λ_θ and traffic moderation β_θ. The second term in (46) corresponds to the effect of the cost-share a_θ for consumers of *all* content providers as the operator lowers its access fee T. This positive externality implies that the effect of the cost-share a_θ on overall consumer welfare (the sign of (46)) depends on the size of the average demand D relative to the size of the demand D_θ for content θ. This dependence on the relative size of equilibrium demands makes it difficult to analytically characterize an optimal cost-share profile from a consumer welfare standpoint. However, we can derive the conditions under which starting from a regime with no cost-shares, the introduction of a cost-share for one content raises consumer surplus.

Proposition 8 *Suppose $a_\theta = 0$ for all $\theta \in [0,1]$, a marginal increase of a_θ raises consumer surplus if and only if*

$$r_\theta > \underline{r}_\theta = \frac{3\gamma}{2\gamma_\theta}\frac{\bar{D}_\theta}{\bar{D}} - \frac{1}{2} - \frac{c}{\overline{\beta}_\theta v_2^\theta} \tag{47}$$

where $\gamma \equiv \int_\theta \gamma_\theta d\theta$ is the average size,

$$\bar{D}_\theta = \frac{\gamma_\theta}{2}\left(\bar{P}_\theta + v_j(\overline{\beta}_\theta) - \frac{\int_0^1 \gamma_j\left(\bar{P}_\theta + \overline{v}_\theta + 2\overline{\beta}_\theta c\right)d\theta}{3\gamma}\right)$$

and

$$\bar{D} = \frac{\int_0^1 \gamma_\theta\left(\bar{P}_\theta + \overline{v}_\theta - \overline{\beta}_\theta c\right)d\theta}{3}.$$

Proof. See Appendix.

Proposition 8 suggests that the mechanism identified in the baseline model is still at work in this multi-content setup: introducing a cost-share is welfare-enhancing for users provided content providers are sufficiently efficient at monetizing users. The parallel with the baseline case is even more apparent if we limit the dimensions along which contents may differ: in what follows, we will assume that content providers only differ in the size of their demand γ_θ and their return to ads r_θ.

Corollary 3 *Suppose all contents are identical except for γ_θ and r_θ. Then evaluated at $a_\theta = 0$ for all $\theta \in [0,1]$, a marginal increase of a_θ raises consumer surplus if and only if*

$$r_\theta > \underline{r} = 1 - \frac{c}{\overline{\beta} v_2}. \tag{48}$$

Proof. Suppose for all $\theta \in [0,1]$, $\overline{P}_\theta = \overline{P}$, $v_\theta(.) = v(.)$. Using (43) with $a_\theta = 0$ for all $\theta \in [0,1]$ we get

$$\frac{D_\theta}{D} = \frac{\gamma_\theta}{\gamma}.$$

Substituting into (47) yields (48). ■

The key observation in Corollary 3 is that despite the externality from one cost-share a_θ on other contents and despite the heterogeneity across different groups of users and content providers, the threshold $\underline{r}$ over which a positive cost-share increases aggregate consumer surplus is the same for every content and the same as in the baseline model with only one content (Proposition 1). Taken together, these results suggest a policy where contents with r_θ above a threshold contribute a cost-share and other contents are exempted.[9]

Decentralized negotiations of cost sharing

As in the case of competition between networks (subsection 6.1), we can consider the incentives of the content providers to accept or not a cost-share. Suppose each content provider negotiates the costs-share with the operator and these negotiations are not coordinated. Then from the perspective of a content provider, the access fee T is taken as given because with multiple contents, each content provider considers that his contribution has a negligible effect on the fee charged by the operator. Hence, under decentralized negotiations, each content provider will resist any increase in the cost-share. Provided content providers have some bargaining power, decentralized negotiations will result in insufficient cost sharing.

By contrast, a collective agreement would factor in that the introduction of a coordinated cost share across multiple contents leads to a decrease in the access fee.

Which content should contribute?

We provide two additional results in line with the intuition that contents with r_θ above a threshold should contribute a cost-share. First, we show that if some contents are already contributing a cost-share (albeit not too high), then introducing a cost-share for a content with $r_\theta < \underline{r}$ decreases consumer welfare at the margin.

Corollary 4 *Suppose all contents are identical except for γ_θ and r_θ. If the cost-shares paid by contents other than content provider θ are not too high and $r_\theta < \underline{r}$, then $\left.\frac{\partial CS}{\partial a_\theta}\right|_{a_\theta=0} < 0$.*

Proof. See Appendix.

A second avenue to characterize a cost-sharing policy that improves welfare for users is to reduce the dimensionality of the problem by restricting the space a feasible cost-shares. Specifically, we consider here a "uniform cost-share policy with exemptions," such that content providers contribute the *same* cost-share a except for a subset Θ that is exempted and

[9] There is no effect of size on the threshold under the assumptions of Corollary 3 because all contents have the same demand elasticity, hence the same pass-through.

contributes nothing. Consumer surplus is then

$$CS = \int_{\theta\in\Theta} \frac{\gamma_\theta}{2} d\theta \left(\frac{\bar{P} + \bar{v} - T}{2}\right)^2 + \int_{\theta\notin\Theta} \frac{\gamma_\theta}{2} \left(\frac{\bar{P} + v_\theta - \frac{a}{r_\theta}\beta_\theta - T}{2}\right)^2 d\theta, \tag{49}$$

and optimizing within this set of policies implies deriving the set Θ that maximizes (49) for a given a. We show this set includes all content providers with a return to ads r_θ above a threshold.

Proposition 9 *Suppose all contents are identical except for γ_θ and r_θ. For uniform cost-share policy a with exemptions, it is optimal to impose the cost-share on content providers with r_θ above a threshold $\underline{r}(a)$.*

Proof. Consider moving a mass ε around θ from no cost-share to cost-share zone. Then T decreases by $\frac{\gamma_\theta}{3\gamma}\left(\bar{v} + 2\bar{\beta}c - v_\theta + \frac{a}{r_\theta}\beta_\theta - 2\beta_\theta (c-a)\right) > 0$. On the other hand the direct effect of the move is to decrease CS by $\frac{\gamma_\theta}{2}\left(\frac{\bar{P}+\bar{v}-T}{2}\right)^2 - \frac{\gamma_\theta}{2}\left(\frac{\bar{P}+v_\theta-\frac{a}{r_\theta}\beta_\theta-T}{2}\right)$. The overall effect is then

$$\varepsilon\gamma_\theta \begin{bmatrix} \frac{1}{2}\left(\frac{\bar{P}+v_\theta-\frac{a}{r_\theta}\beta_\theta-T}{2}\right)^2 - \frac{1}{2}\left(\frac{\bar{P}+\bar{v}-T}{2}\right)^2 \\ -\frac{\partial CS}{\partial T}\left(\bar{v} - v_\theta + \frac{a}{r_\theta}\beta_\theta + 2\bar{\beta}c - 2\beta_\theta (c-a)\right) \end{bmatrix}. \tag{50}$$

A content providers should contribute if the effect is positive. The slope of the bracket term in θ is

$$\frac{a\beta_\theta}{(r_\theta)^2}\frac{dr_\theta}{d\theta}\left[\frac{\bar{P}+v_\theta-\frac{a}{r_\theta}\beta_\theta-T}{4} - \frac{1}{3\gamma}\frac{\partial CS}{\partial T}\left(-1-2\frac{c-a}{b\beta_\theta}\right)\right] \tag{51}$$

But the bracket term is increasing in θ implying that (50) is quasi convex. Moreover, for very large values of r_θ, (50) is positive because $v_\theta - \frac{a}{r_\theta}\beta_\theta$ goes to $\bar{v}$. Hence (50) is positive above a threshold $r(a)$. It is negative below the threshold at least if r_θ is not too small relative to the threshold.

■

Overall, this extension to multiple contents points to a policy where more efficient content providers contribute a cost-share as improving welfare for users.

7 Robustness

In this section we examine the robustness of our results by relaxing some assumptions that were assumed for conciseness.

7.1 Decreasing return to ads

We consider here a generalized functional form for the efficiency of users' monetization by the content provider. Specifically, we allow for decreasing marginal returns to advertising: CP's revenue per user is $R(\lambda)$ with $R'(\lambda) \equiv r(\lambda) > 0$ and $r'(\lambda) \leq 0$.

The profit function of the content provider becomes

$$\Pi_{CP} = \left(R\left(\lambda\right) - a\beta\right)\bar{\gamma}\left(\bar{P} + v\left(\beta\right) - T - \lambda\right), \tag{52}$$

while the profit function of the operator is as in the baseline model (Equation 1). This yields the following first-order conditions, respectively for T, λ and β.

$$T - \beta\left(c - a\right) = \bar{P} + v\left(\beta\right) - T - \lambda, \tag{53}$$

$$R\left(\lambda\right) - a\beta = r\left(\lambda\right)\left(\bar{P} + v\left(\beta\right) - T - \lambda\right), \tag{54}$$

$$v'\left(\beta\right) = \frac{a}{r\left(\lambda\right)}. \tag{55}$$

We then ask under which condition a positive cost-share raises consumer surplus. To study the marginal impact of the cost-share on consumer surplus around $a = 0$, we define λ_0 as the solution to equations (53), (54) and (55) for $a = 0$. Using this notation, we obtain the following result.

Proposition 10 *The marginal impact of the cost-share on consumer surplus at $a = 0$ is strictly positive if and only if*

$$r\left(\lambda_0\right) > \frac{1}{1 - \frac{R(\lambda_0)r'(\lambda_0)}{r(\lambda_0)^2}} - \frac{c}{\bar{\beta}v_2}. \tag{56}$$

Condition (56) provides a condition under which a strictly positive cost-share improves consumer surplus and captures an intuition similar to Proposition 1. The second term on the right-hand side of (56) accounts for the direct effect of the cost-share on CP's incentives to moderate traffic. That term makes it more likely that condition (56) is satisfied because the cost-share leads CP to better internalize the cost βc borne by O to deliver content. The rest of (56) captures the price effect of the cost-share, i.e., the condition under which the cost-share would be welfare-improving for consumers if the traffic load β were exogenously fixed.

Note that the right-hand side of (56) is a function of CP's equilibrium decisions λ_0 and $\overline{\beta}$, unlike in Proposition 1 where $\underline{r}$ only depends on the primitives. However, since $r'(.) \leq 0$, we have

$$\frac{1}{1 - \frac{R(\lambda_0) r'(\lambda_0)}{r(\lambda_0)^2}} - \frac{c}{\overline{\beta} b} \leq 1 - \frac{c}{\overline{\beta} v_2} = \underline{r}.$$

In words, under strictly decreasing return to advertising ($r'(.) < 0$), a positive cost-share benefits consumers for a wider range of marginal returns to advertising (at $a = 0$) than under the constant return to advertising we considered in the baseline model. Intuitively, this is because CP's incentive to increase advertising λ in response to an increase of the cost-share a is lower under decreasing return to advertising.

7.2 Costly traffic moderation

We have assumed in the baseline model that the cost of traffic moderation by CP was lower quality for users, and therefore lower demand. There could also be direct costs for CP to reduce traffic (e.g., investment in compression technologies). We show here that the baseline model is robust to the introduction of a cost

$$\Phi(\beta) \equiv \phi(\overline{\beta} - \beta)$$

borne by CP when setting the traffic load at β, where $\phi < c$.

The profit function of the content provider becomes

$$\Pi_{CP} = (r\lambda - a\beta - \Phi(\beta))\, \bar{\gamma} \left(\bar{P} + v(\beta) - T - \lambda \right), \tag{57}$$

while the profit function of the operator is as in the baseline model (Equation 1). This yields the following first-order conditions, respectively for T, λ and β.

$$T - \beta (c - a) = \bar{P} + v(\beta) - T - \lambda, \tag{58}$$

$$r\lambda - a\beta - \Phi(\beta) = r \left(\bar{P} + v(\beta) - T - \lambda \right), \tag{59}$$

Combining (58) and (59) yields the equilibrium demand:

$$\overline{D}(P) = \frac{\overline{\gamma}}{3} \left(\overline{P} + v(\beta) - \beta(c - a) - \frac{a\beta + \Phi(\beta)}{r} \right) \tag{60}$$

Finally, combining CP's first-order condition with respect to β and (59) delivers CP's traffic moderation:

$$\begin{cases} \beta = \overline{\beta} & \text{if } a < \phi, \\ v'(\beta) = \frac{a-\phi}{r} & \text{if } \phi \leq a \leq \phi + rv'(\underline{\beta}), \\ \beta = \underline{\beta} \text{ otherwise.} \end{cases} \tag{61}$$

Combining with (60), we get that the total derivative of equilibrium demand with respect to the cost-share is

$$\frac{\partial \overline{D}(P)}{\partial a} = \frac{\overline{\gamma}}{3}\left[\beta\left(1 - \frac{1}{r}\right) + (a-c)\frac{d\beta}{da}\right]. \tag{62}$$

From this expression, we obtain the following result

Proposition 11 *With CP bearing cost of traffic moderation $\Phi(\beta)$,*

- *if $r \geq 1$, an increase in the cost-share raises consumer surplus,*
- *there exists if $\underline{r}^{\phi} < 1$, such that if $\underline{r}^{\phi} < r < 1$ consumer surplus is maximal at some interior cost-share $a^*_{\phi} \in (\phi, c)$,*
- *the set of r such that a strictly positive cost-share raises consumer surplus is strictly smaller than in Proposition 1 (no direct cost of moderating traffic).*

Proof. If $r \geq 1$, (62) is always positive. For $r = 1$, we know $a = c$ maximizes consumer surplus. Therefore (60) evaluated at $r = 1$ and $a = c$ (and the corresponding equilibrium β) is strictly larger than the same demand evaluated at $r = 1$ and $a = 0$. By continuity, this property holds in a region $(\underline{r}^{\phi}, 1)$ where a strictly positive cost-share is therefore optimal for users.

Suppose $r \in (\underline{r}^{\phi}, 1)$, then (62) is negative at $a = c$, therefore the cost-share that maximizes consumer surplus, a^*_{ϕ}, is strictly smaller than c. In addition, if $a \leq \phi$, we have

$$\frac{d\beta}{da} = 0$$

and therefore

$$\frac{\partial \overline{D}(P)}{\partial a} = \frac{\overline{\gamma}}{3}\overline{\beta}\left(1 - \frac{1}{r}\right) < 0,$$

which implies $a^*_{\phi} > \phi$.

Finally, a necessary condition for the optimal cost-share to be strictly positive is that (62) is strictly positive at $a = \phi$, which is equivalent to

$$r > 1 - \frac{c - \phi}{\overline{\beta} v_2} > \underline{r},$$

where $\underline{r}$ is defined in Proposition 1.

■

The intuition for Proposition 11 can be traced to (61). It states that obtaining the same level of traffic moderation from CP as in the baseline model requires a higher cost-share: the cost $\Phi(\beta)$ dampens CP's incentives to lower β. But for $r < 1$, increasing the cost-share lowers consumer surplus for a fixed β through its impact on CP's and O's pricing. In other words, obtaining a given level of traffic moderation through a cost-share comes at a higher cost in terms of price distortion than in the baseline model where $\phi(.) = 0$. This makes the cost-share less effective, even though a strictly positive one still is optimal for returns to ad greater than one and for some returns to ad smaller than one.

8 Conclusion

Our analyses of cost-sharing emphasizes the complementarity between the network and the content delivered on the network. Considering that network operators and content providers both have market power, an efficient and fair cost-sharing policy must account for the price externality between them and for their ability to reduce the cost of traffic. Our baseline analysis brings several insights on the trade-offs involved and on the nature of an efficient policy.

First, our model highlights the critical role of content providers' business models. A positive cost-share will be at least partially passed-through to consumers through a higher price of content or a higher volume of advertising. A key consideration is then the magnitude of this content provider's pass-through relative to the network operators' pass-through, which may raise an empirical challenge for free content.

Our main insight is that cost-sharing can be efficient when it is restricted to large and efficient content providers, where efficiency refers to the ability to monetize demand through prices, advertising or other means. In this case, cost-sharing raises demand as well as the incentives of the content providers to contain traffic and the incentives of the network to reduce the cost. By contrast, imposing a positive contribution to content providers with low ability to monetize users is inefficient in our model.

Those insights generalize to imperfect competition between networks. In fact, increasing competition magnifies the effect (positive or negative) of cost-sharing on social welfare, but does not affect the sign of this effect.

We also point that decentralized negotiations of cost-sharing involve contractual externalities between the stake-holders, as a contribution negotiated between one network operator and one content provider affects the terms of trade for all network operators and all content providers. Such externalities suggest a rationale for a regulatory intervention coordinating the contributions to cost.

While our work clarifies the policy debate, there are still many aspects that remain to be investigated.

On the network side, we have assumed simple contracting reflecting the current state of the market, where data allowances are rarely binding. However, rapidly growing traffic and the pressure it is likely to impose on network infrastructures may lead to more complex contracting with consumers that remains to be investigated.

On the content providers' side, we have focused on the trade-off between traffic moderation and the quality of content. Some investment may reduce traffic but raise quality. In this case we conjecture that our analysis still applies as investment should still be sub-optimal and boosted by cost-sharing of traffic.

As a final point, our analysis ignores externalities between the network activities and the rest of society. However, the regulation of telecommunications and of the Internet should incorporate the growing concern for the environmental impact of human activity.[10] Accounting for this wider impact may change the perspective on the relative contributions of the various actors.

[10] See Poudou and Sand-Zantman (2022) for a first contribution along this line

References

Ambrus, Attila, Emilio Calvano, and Markus Reisinger, "Either or both competition: A" two-sided" theory of advertising with overlapping viewerships," *American Economic Journal: Microeconomics*, 2016, *8* (3), 189–222.

Anderson, Simon P and Bruno Jullien, "The advertising-financed business model in two-sided media markets," in "Handbook of media economics," Vol. 1, Elsevier, 2015, pp. 41–90.

_ **and Martin Peitz**, "Media see-saws: Winners and losers in platform markets," *Journal of Economic Theory*, 2020, *186*, 104990.

_ **and Stephen Coate**, "Market provision of broadcasting: A welfare analysis," *The Review of Economic Studies*, 2005, *72* (4), 947–972.

Bourreau, Marc, Frago Kourandi, and Tommaso Valletti, "Net neutrality with competing internet platforms," *The Journal of Industrial Economics*, 2015, *63* (1), 30–73.

Choi, Jay Pil and Byung-Cheol Kim, "Net neutrality and investment incentives," *The RAND Journal of Economics*, 2010, *41* (3), 446–471.

Corniere, Alexandre De and Greg Taylor, "Integration and search engine bias," *The RAND Journal of Economics*, 2014, *45* (3), 576–597.

Crampes, Claude, Carole Haritchabalet, and Bruno Jullien, "Advertising, competition and entry in media industries," *The Journal of Industrial Economics*, 2009, *57* (1), 7–31.

Economides, Nicholas and Benjamin E. Hermalin, "The economics of network neutrality," *The RAND Journal of Economics*, 2012, *43* (4), 602–629.

_ **and Joacim Tåg**, "Network neutrality on the Internet: A two-sided market analysis," *Information Economics and Policy*, 2012, *24* (2), 91–104.

Gabszewicz, Jean J, Didier Laussel, and Nathalie Sonnac, "Programming and advertising competition in the broadcasting industry," *Journal of Economics & Management Strategy*, 2004, *13* (4), 657–669.

Gilboa, Itzhak and David Schmeidler, "Maxmin expected utility with non-unique prior," in "Uncertainty in economic theory," Routledge, 2004, pp. 141–151.

Greenstein, Shane, Martin Peitz, and Tommaso Valletti, "Net neutrality: A fast lane to understanding the trade-offs," *Journal of Economic Perspectives*, 2016, *30* (2), 127–50.

Hermalin, Benjamin E. and Michael L. Katz, "The economics of product-line restrictions with an application to the network neutrality debate," *Information Economics and Policy*, 2007, *19* (2), 215–248.

Jullien, Bruno and Wilfried Sand-Zantman, "Internet regulation, two-sided pricing, and sponsored data," *International Journal of Industrial Organization*, 2018, *58*, 31–62.

Krämer, Jan and Martin Peitz, "A fresh look at zero-rating," *Telecommunications Policy*, 2018, *42* (7), 501–513.

Peitz, Martin and Florian Schuett, "Net neutrality and inflation of traffic," *International Journal of Industrial Organization*, 2016, *46*, 16–62.

_ **and Tommaso Valletti**, "Content and advertising in the media: Pay-tv versus free-to-air," *international Journal of industrial organization*, 2008, *26* (4), 949–965.

Poudou, Jean-Christophe and Wilfried Sand-Zantman, "The Environmental Impact of Internet Regulation," 2022.

Reggiani, Carlo and Tommaso Valletti, "Net neutrality and innovation at the core and at the edge," *International Journal of Industrial Organization*, 2016, *45*, 16–27.

Rochet, Jean-Charles and Jean Tirole, "Platform competition in two-sided markets," *Journal of the european economic association*, 2003, *1* (4), 990–1029.

Appendices

A A model for unlimited data

In this section, we present a simple model that explains why an operator may offer unlimited access to data service despite a positive cost. The basic assumption is that consumers are uncertain about future consumption and are ambiguity averse. We model ambiguity aversion with Maxmin Expected Utility (Gilboa and Schmeidler, 2004).

Consider an operator that offers a linear contract to a consumer, with tariff $T + pq$ for consumption q and usage cost cq. The consumer faces ex-post uncertain needs. More precisely his consumption utility is $U(q - z)$ where $z \in Z$ is a random variable, that is realized only ex-post and take value between $[\underline{z}, \overline{z}]$. The consumer is uncertain over the distribution ω of z and assumes it belong to a set Ω. To simplify matters assume that the distribution with $\Pr\{z = \overline{z}\} = 1$ belongs to Ω.

The operator has belief μ over Z, with $\mathbb{E}_\mu(z) < \overline{z}$, and maximizes expected profits. The consumer evaluates the contract using Maxmin Expected Utility Criterion, that is she assigns a value equal to the minimal expected utility over all distributions in Ω.

Assuming Inada conditions $U'(0) = +\infty$ and $U'(\overline{q}) = 0$, the consumer's realized consumption is $d(p)+z$ and consumers utility can be written as $w(p)-zp$ where $w(p) = max_x U(x)-px$.

If the consumer was an expected utility maximizer and shared common beliefs with the operator, then it is straightforward to see that the optimal contract would set $p = c$ and capture the whole consumer surplus with $T = w(p)$. But with ambiguity aversion, there is a form of disagreement over the distribution of z. The consumer evaluates the contract under the worst case $z = \overline{z}$ so that the maximal subscription fee that the operator can charge is $T = w(p) - \overline{z}p$ with total profit

$$\Pi = w(p) + (p - c)d(p) + (\mathbb{E}_\mu(z) - \overline{z})p$$

Hence the slope is

$$\frac{d\Pi}{dp} = (p - c)d'(p) + \mathbb{E}_\mu(z) - \overline{z}.$$

It follows that it is optimal to set $p = 0$ if

$$-c\overline{q} < \overline{z} - \mathbb{E}_\mu(z).$$

To summarize when a consumer faces the risk of a large need for consumption and is ambiguity averse, the operator charges only for access and not for traffic.

B Proofs

Proof of Proposition 4

$$\begin{aligned}\Pi_O &= (T + \beta (a - c)) \overline{\gamma} \left(\overline{P} + v(\beta) - T - \lambda\right) + (T - \hat{c}) \hat{\gamma} \left(\hat{P} - T\right) \\ &= (T + \beta (a - c)) \overline{D}(P) + (T - \hat{c}) \hat{D}(T).\end{aligned}$$

Therefore from the envelop theorem:

$$\frac{d\Pi_O}{da} = \left(\beta + (a - c) \frac{d\beta}{da}\right) \bar{D}(P) + (T + \beta (a - c)) \bar{\gamma} \left(v'(\beta) \frac{d\beta}{da} - \frac{d\lambda}{da}\right).$$

From (8),

$$\begin{aligned}\frac{d\lambda}{da} &= \frac{\hat{\gamma}\left(4\frac{a}{r}\frac{d\beta}{da} + 2\beta\frac{1}{r}\right) + \bar{\gamma}\left(3\frac{a}{r}\frac{d\beta}{da} + (a - c)\frac{d\beta}{da} + \frac{2\beta}{r} + \beta\right)}{4\hat{\gamma} + 3\bar{\gamma}} \\ &= \frac{a}{r}\frac{d\beta}{da} + \frac{\hat{\gamma}\left(2\beta\frac{1}{r}\right) + \bar{\gamma}\left((a - c)\frac{d\beta}{da} + \frac{2\beta}{r} + \beta\right)}{4\hat{\gamma} + 3\bar{\gamma}}\end{aligned}$$

Using the O's first-order condition (2),

$$\bar{\gamma}(T + \beta(a - c)) + \hat{\gamma}(T - \hat{c}) = \overline{D}(P) + \hat{D}(T),$$

we get

$$\frac{d\Pi_O}{da} = \left(\beta + (a-c)\frac{d\beta}{da}\right)\overline{D}(P) - \overline{D}(P)\frac{\hat{\gamma}\left(2\beta\frac{1}{r}\right) + \overline{\gamma}\left((a-c)\frac{d\beta}{da} + \frac{2\beta}{r} + \beta\right)}{4\hat{\gamma} + 3\overline{\gamma}}$$
$$+ \left(\frac{a}{r}\frac{d\beta}{da} - \frac{d\lambda}{da}\right)\left(\hat{D}(T) - \hat{\gamma}(T - \hat{c})\right)$$
$$= \left(\beta\left(1 - \frac{\hat{\gamma}2\frac{1}{r} + \overline{\gamma}\left(\frac{2}{r}+1\right)}{4\hat{\gamma}+3\overline{\gamma}}\right) + (a-c)\frac{d\beta}{da}\left(1 - \frac{\overline{\gamma}}{4\hat{\gamma}+3\overline{\gamma}}\right)\right)\overline{D}(P)$$
$$+ \left(\frac{a}{r}\frac{d\beta}{da} - \frac{d\lambda}{da}\right)\left(\hat{D}(T) - \hat{\gamma}(T - \hat{c})\right)$$
$$= \left(2\beta\left(\frac{2\hat{\gamma}+\overline{\gamma}}{4\hat{\gamma}+3\overline{\gamma}}\left(1 - \left(\frac{\hat{\gamma}+\overline{\gamma}}{2\hat{\gamma}+\overline{\gamma}}\right)\frac{1}{r}\right)\right) + \underbrace{(a-c)\frac{d\beta}{da}\left(\frac{4\hat{\gamma}+2\overline{\gamma}}{4\hat{\gamma}+3\overline{\gamma}}\right)}_{>0}\right)\overline{D}(P)$$
$$- \underbrace{\frac{\hat{\gamma}\left(2\beta\frac{1}{r}\right) + \overline{\gamma}\left((a-c)\frac{d\beta}{da} + \frac{2\beta}{r} + \beta\right)}{4\hat{\gamma}+3\overline{\gamma}}}_{>0}\left(\hat{D}(T) - \hat{\gamma}(T - \hat{c})\right) \tag{63}$$

The first term in (63) is the derivative of Π_O^C with respect to a. If

$$r > \frac{\hat{\gamma}+\overline{\gamma}}{2\hat{\gamma}+\overline{\gamma}},$$

that derivative is always strictly positive. Otherwise the derivative is negative at $a = c$ while it is positive at $a = 0$ if r is not too small.

The second term is the effect on the profit from basic consumers and it is positive if

$$\hat{P} - T > T - \hat{c}.$$

■

Proof of Proposition 7

We start with an intermediate result

Lemma 2 *The total price $P_n = T_n + \lambda$ for operator n is given by*

$$(3 - \sigma_N)(\bar{P} + v(\beta) - P_n) =$$
$$\bar{P} + v(\beta) + \left(\frac{a_n(3-\sigma_N) - \left(1 - \sigma_N\frac{N}{N-1}\right)\left(\frac{\sum_n a_n}{N}\right)}{2 + \frac{\sigma_N}{N-1}} - c\right)\beta - \frac{\sum_n a_n \beta}{Nr}$$

Proof.

Summing the first-order conditions for T_n in (29) we obtain

$$\frac{\sum_n T_n}{N} + \left(\frac{\sum_n a_n}{N} - c\right)\beta = (1-\sigma_N)\left(\bar{P} + v(\beta) - \frac{\sum_n P_n}{N}\right),$$

which gives

$$\begin{aligned}\lambda &= \frac{\sum_n P_n}{N} - \frac{\sum_n T_n}{N} \\ &= \frac{\sum_n P_n}{N} - (1-\sigma_N)\left(\bar{P} + v(\beta) - \frac{\sum_n P_n}{N}\right) + \left(\frac{\sum_n a_n}{N} - c\right)\beta. \end{aligned} \tag{64}$$

From the first-order condition for λ in (30) we also have

$$\lambda - \frac{\sum_n a_n \beta}{Nr} = \bar{P} + v(\beta) - \frac{\sum_n (P_n - \sigma_N P_{-n})}{N(1-\sigma_N)} = \bar{P} + v(\beta) - \frac{\sum_n P_n}{N} \tag{65}$$

Combining (64) and (65):

$$\frac{\sum_n P_n}{N} + \left(\frac{\sum_n a_n}{N} - c\right)\beta - \frac{\sum_n a_n \beta}{Nr} = (2-\sigma_N)\left(\bar{P} + v(\beta) - \frac{\sum_n P_n}{N}\right)$$

which gives

$$\begin{aligned}&\bar{P} + v(\beta) - \frac{\sum_n P_n}{N} = \\ &\frac{1}{3-\sigma_N}\left(\bar{P} + v(\beta) + \left(\frac{\sum_n a_n}{N} - c\right)\beta - \frac{\sum_n a_n \beta}{Nr}\right)\end{aligned} \tag{66}$$

Therefore using (64) again, we get

$$\lambda = \frac{1}{3-\sigma_N}\left(\bar{P} + v(\beta) + \left(\frac{\sum_n a_n}{N} - c\right)\beta + (2-\sigma_N)\frac{\sum_n a_n \beta}{Nr}\right) \tag{67}$$

Using the first-order condition for T_n in (29), we have

$$P_n - \lambda + (a_n - c)\beta = (1-\sigma_N)\left(\bar{P} + v(\beta) - \frac{P_n - \sigma_N P_{-n}}{1-\sigma_N}\right) \tag{68}$$

Using

$$P_{-n} = \frac{1}{N-1}\left(\sum_\theta P_\theta - P_n\right)$$

we obtain after rearranging

$$\begin{aligned}&\left(\bar{P} + v(\beta) - P_n\right)\left(2 + \frac{\sigma_N}{N-1}\right) = \\ &\bar{P} + v(\beta) - \lambda + \sigma_N \frac{N}{N-1}\left(\bar{P} + v(\beta) - \frac{\sum_\theta P_\theta}{N}\right) + (a_n - c)\beta\end{aligned}$$

Finally, using (66) and (67) to substitute $\bar{P} + v(\beta) - \frac{\sum_j P_j}{N}$ and λ, and rearranging, we get the equation in Lemma 2. ∎

Using Lemma 2, we get

$$\left.\frac{\partial P_n}{\partial a_n}\right|_\beta = \frac{1}{3-\sigma_N}\left(\frac{-(3-\sigma_N)+\left(1-\sigma_N\frac{N}{N-1}\right)\frac{1}{N}}{2+\frac{\sigma_N}{N-1}}\beta + \frac{\beta}{Nr}\right)$$

$$= \frac{1}{3-\sigma_N}\left(\frac{(3-\sigma_N)}{2+\frac{\sigma_N}{N-1}}\left(\frac{1-N}{N}\right)\beta + \frac{\beta}{N}\left(\frac{1-r}{r}\right)\right)$$

$$\left.\frac{\partial P_j}{\partial a_n}\right|_\beta = \frac{1}{3-\sigma_N}\left(\frac{(3-\sigma_N)}{2+\frac{\sigma_N}{N-1}}\frac{\beta}{N} + \frac{\beta}{N}\left(\frac{1-r}{r}\right)\right)$$

Therefore

$$-\left.\frac{\partial(P_n - \sigma_N P_{-n})}{\partial a_n}\right|_\beta = \frac{N+\sigma_N-1}{2+\frac{\sigma_N}{N-1}}\frac{\beta}{N} + \left(\frac{1-\sigma_N}{3-\sigma_N}\right)\frac{\beta}{N}\left(\frac{r-1}{r}\right)$$

and using Equation 39 and Equation 40,

$$\frac{\partial \Pi_n}{\partial a_n} = \frac{\gamma_N}{N}\frac{1-\sigma_N}{3-\sigma_N}\left[\frac{-c}{r\beta v''} + \frac{(N-1+\sigma_N)(3-\sigma_N)}{(2N-2+\sigma_N)(1-\sigma_N)}(N-1) + \frac{r-1}{r}\right]\beta.$$

From (1) and (6), operators' profit strictly increases for a marginal increase of all cost-shares $\{a_\theta\}_\theta$ from 0 if

$$\frac{c}{r\beta v_2} + \frac{r-1}{r} > 0.$$

Suppose first $\frac{c}{\beta v_2} > 1$, then both $\frac{\partial \Pi_n}{\partial a_n}$ and $\frac{\partial \Pi_n}{\partial a}$ are strictly positive for any positive r, i.e., $\underline{r} = \underline{r}^c = 0$.

Suppose now $\frac{c}{\beta v_2} \leq 1$, then

$$\underline{r}^c = \frac{1-\frac{c}{\beta v_2}}{1+\frac{(N-1+\sigma_N)(3-\sigma_N)}{(2N-2+\sigma_N)(1-\sigma_N)}(N-1)} < 1 - \frac{c}{\beta v_2} = \underline{r}$$

Finally,

$$\frac{(N-1+\sigma_N)(3-\sigma_N)}{(2N-2+\sigma_N)(1-\sigma_N)}(N-1) = \frac{(N-1+\sigma_N)(3-\sigma_N)}{\left(2+\frac{\sigma_N}{N-1}\right)(1-\sigma_N)}$$

is strictly increasing in both N and σ_N which implies $\underline{r}^c$ is strictly decreasing in N and σ_N.

Proof of Proposition 8

From CP θ first-order condition with respect to λ_θ, we have

$$\lambda_\theta = \frac{\bar{P}_\theta + v_\theta\left(\beta_\theta\right) + \frac{a_\theta}{r_\theta}\beta_\theta - T}{2}.$$

Combine with O's first-order condition with respect to T to get

$$T = \frac{\int_0^1 \gamma_\theta \left(\bar{P}_\theta + v_\theta(\beta_\theta) - \frac{a_\theta}{r_\theta}\beta_\theta + 2\beta_\theta\left(c - a_\theta\right)d\theta\right)}{3\gamma},$$

which implies (using CP j first-order condition with respect to a_θ in (45))

$$\frac{dT}{da_\theta} = \frac{\gamma_\theta\left(-\frac{\beta_\theta}{r_\theta} + 2(c - a_\theta)\frac{d\beta_\theta}{da_\theta} - 2\beta_\theta\right)}{3\gamma}. \tag{69}$$

Plugging (69) into (46), we get

$$\frac{dCS}{da_\theta} = \frac{\gamma_\theta}{3\gamma}\frac{D}{r_\theta}\beta_\theta\left[r_\theta - \frac{3\gamma}{2\gamma_\theta}\frac{D_\theta}{D} + \frac{1}{2} + \frac{1}{\beta_\theta v_2^\theta}\left(c - a_\theta\right)\right], \tag{70}$$

if a_θ is in the region where $\frac{d\beta_\theta}{da_\theta} < 0$, which holds for a_θ in a neighborhood of 0. Finally, if $a_\theta = 0$ for all $\theta \in [0, 1]$, then

$$D = \int_0^1 \gamma_\theta\left(\frac{\bar{P}_\theta + v_\theta(\overline{\beta}_\theta) - \overline{\beta}_\theta c}{3}\right)d\theta \equiv \overline{D},$$

and

$$D_\theta = \frac{\gamma_\theta}{2}\left(\bar{P}_\theta + v_j(\overline{\beta}_\theta) - \frac{\int_0^1 \gamma_\theta\left(\bar{P}_\theta + \overline{v}_\theta + 2\overline{\beta}_j c\right)d\theta}{3\gamma}\right) \equiv \overline{D}_\theta.$$

Using Equation 70, we get

$$\left.\frac{dCS}{da_\theta}\right|_{\forall\theta, a_\theta = 0} > 0 \Leftrightarrow r_\theta > \frac{3\gamma}{2\gamma_\theta}\frac{\overline{D}_\theta}{\overline{D}} - \frac{1}{2} - \frac{1}{\overline{\beta}_\theta v_2^\theta}c.$$

∎

Proof of Corollary 4

We have at $a_j = 0$:

$$\frac{D_i}{D_j} = \frac{\gamma_i\left(\frac{\bar{P}_i + v_i(\beta_i) - \frac{a_i}{r_i}\beta_i - T}{2}\right)}{\gamma_j\left(\frac{\bar{P}_j + v_j(\overline{\beta}_j) - T}{2}\right)}$$

$$\frac{d}{da_i}\frac{D_i}{D_j} = \frac{\gamma_i}{\gamma_j\left(\frac{\bar{P}_j+\bar{v}_j-T}{2}\right)}\left(-\frac{\beta_i}{r_i}-\frac{dT}{da_i}+\frac{\left(\bar{P}_i+v_i-\frac{a_i}{r_i}\beta_i-T\right)}{\left(\bar{P}_j+\bar{v}_j-T\right)}\frac{dT}{da_i}\right)$$
$$= \frac{\gamma_i}{\gamma_j\left(\frac{\bar{P}_j+v_j(\overline{\beta}_j)-T}{2}\right)}\left(-\frac{\beta_i}{r_i}+\frac{\bar{P}_i+v_i(\beta_i)-\frac{a_i}{r_i}\beta_i-\bar{P}_j-v_j(\overline{\beta}_j)}{\bar{P}_j+v_j(\overline{\beta}_j)-T}\frac{dT}{da_i}\right)$$

Now suppose that contents are identical except for the return r_i and $\underline{r} > r_j$. Then for a_i small $\bar{P}_i + v_i(\beta_i) - \frac{a_i}{r_i}\beta_i - \bar{P}_j - v_j(\overline{\beta}_j) = v_i(\beta_i) - \frac{a_i}{r_i}\beta_i - v_j(\overline{\beta}_j)$ is close to $v_i(\overline{\beta}_i) - v_j(\overline{\beta}_j) = 0$. As β_i is positive, $\frac{D_i}{D_j}$ is decreasing in a_i on some interval $(0, \bar{\varepsilon})$. Moreover, for the same reason the marginal impact of a_i on $\frac{D_i}{D_j}$ at $\mathbf{a} = (0, 0.., 0)$ is zero.

$$\frac{d}{da_i}\frac{D_i}{D_j} = \frac{\gamma_i}{\gamma_j\left(\frac{\bar{P}_j+v_j(\overline{\beta}_j)-T}{2}\right)}\left(\frac{\bar{P}_i+v_i(\beta_i)-\frac{a_i}{r_i}\beta_i-\bar{P}_j-v_j(\overline{\beta}_j)}{\bar{P}_j+v_j(\overline{\beta}_j)-T}\right)\frac{dT}{da_i}$$

Thus we have for $\mathbf{a} = \mathbf{0}$
$$\left.\frac{\partial}{\partial a_i}\frac{D_j}{D}\right|_{\mathbf{a}=0} > 0$$

This implies that $\frac{3\gamma}{2\gamma_j}\frac{\bar{D}_j}{\bar{D}} - \frac{1}{2} - \frac{c}{\bar{\beta}_j v_2^j}$ is increasing in a_i if all a_θ are not too large. Therefore if condition (47) is violated at $a_j = 0$ and $a_\theta = 0$ for all θ ($r_j < \underline{r}_j$), then it is also violated at $a_j = 0$ and a_θ small enough for all θ. ∎