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Abstract

This research project describes how mathematical and EFL literacy processes unfolded in a bilingual and private institution in Bogota when the Singapore method was implemented in elementary grades. The method intends to position learners at the center of the math-knowledge construction giving them the opportunity to display and develop different language skills; therefore, mathematics lessons became the convergent point for English language and mathematics. Accordingly, the decisions made for the instructional implementation focused on encouraging learners to make use of English as a second language to read stories, discuss them,, support their decisions and compose texts in order to solve math tasks regarding the concepts of fractions, whole numbers, decimal numbers, area and perimeter. This particular context led to posing the research question for this project: How do EFL and mathematical literacy unfold when students are engaged in mathematical tasks? The objective was to describe the EFL and mathematical literacy development when children engaged in mathematical tasks. Classroom observations and students artifacts served as research instruments to collect data in this qualitative, descriptive study.

Findings revealed that students developed the two literacies simultaneously. Social and cognitive factors played a meaningful role in their development as well as the interactions between the teacher and the students and among them. Interaction between learners offered them the opportunity to apply their math and language background knowledge, discover and implement different alternatives to solve problems. The use of students' first language became an important element in order to make sense of texts, expand their knowledge in the second language and in math understanding. Finally, teacher questioning prompted students to support their strategies, and the discussions held between teacher and students led to build math language.

The study points at the need to promote pedagogies that help students' to build knowledge based on their social backgrounds. It also highlights the idea that different knowledge representations such as drawing and diagrams produced by learners are valuable resources and genuine attempts to project their internal cognitive processes.

Key words: EFL literacy, mathematical literacy, collaboration, interaction

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Chapter I

Introduction

There is an increasing need to think schooling as a collaborative construction and set up classrooms as educational sites where students can be co-constructors of their own knowledge. For this need to be fulfilled the curriculum should be oriented towards developing their potentialities and skills that allow them to reach their learning goals and prepare them to be active informed citizens in the current century. Fandiño (2013) states that twenty first century students are required to develop skills that favor their access to information; synthetize and communicate it. Parallel to those skills students need to manage how to work collaboratively across differences and be able to solve problems together.

This educational challenge positions schools and teachers as directly responsible to rethink the curriculum. Therefore, students learning experiences and pedagogies used with them should foster thinking and meaningful learning. Literacy is an important dimension in the education of children, it permeates all subject areas and enables individuals to understand and be understood by the world and everyone involved in it. Hence, as literacy permeates different areas of knowledge, it is of great importance to examine what is happening inside non-content language classrooms and their relationship with language construction.

In the light of this situation, this research project aims to identify EFL literacies and mathematic literacies in the mathematics classroom with fourth graders. The theoretical framework presents socio-constructivism as the educational perspective from which I propose to develop literacy and mathematical literacy. Socio-constructivism takes as main theorist Vygotsky (1978). The concept of literacy is based on the theory of Kern (2000), UNESCO (2004), OECD (2017), Short, Harste, Burke and Harste (1996) and Smagorinsky (2011) Finally,

the concept of mathematical literacy is developed by the definitions of UNESCO (2004), Martin (2007) OECD (2013), Ali, Karim and Yusof (2016).

This project was framed in a qualitative perspective and involved an intervention plan based on a pedagogical proposal. As a researcher, I set up an instructional design and objectives that could be implemented in an academic semester and that could yield observable data that could be later analyzed. The instruments that provided data were classroom observations (video and audio recordings) and students' artifacts.

Statement of the Problem

The needs analysis I carried out in the school aimed to identify the relationship between teaching, learning, language, and mathematics class. The instruments used to establish the needs analysis were two questionnaires, a revision of official documents regarding students' performance in international tests and class observations. The questionnaires had two main purposes, to reveal the strengths and difficulties teachers and students experienced in the mathematics class and to have a general vision of how mathematics classes at school are developed. The review of students' results in international tests aimed to gather evidence on how students performed in tests that require them to solve different situations implementing what they have learned during their schooling experience. A third valuable element was the observation of my math classes because they evidenced how language and math are interwoven and the difficulties I have found during my experience as math and English teacher.

The questionnaires were designed in Spanish in order to gather the opinion of different participants, elementary school and high school teachers and students from fourth to eleven grade (See annex 1 and annex 2). Thus, one questionnaire was administered to math teachers and

another questionnaire was administered to students. Below is a summary of the responses gathered and the implications of those for the starting point of this research.

Teachers' responses present their experience time as math teachers, how lessons were arranged and the difficulties they found during their experience as math teachers. Most of teachers had more than three years of experience teaching math in different grades to diverse population from kids to adults. The classes began with an introduction and explanation of a topic, then a practical part and its application to real-life situations; and finally, an assessment part. Teachers' responses also revealed that reading comprehension plays a crucial role in solving mathematical problems; moreover, it was stated that the learning process is clearly affected by students' lack of comprehension on mathematical language and its application to different real life situations.

Similarly, students revealed aspects related to the development of math class, teachers' teaching practices and their own learning process. Students expressed that the math classes had the same organization as the teachers described them; teachers introduce and explain a topic, then students practice solving the exercises according to the teacher explanation and finally teachers assess and check students' progress. Most students stated that teachers' lack of communication skills, and language they used made mathematics difficult to understand. Another aspect students highlighted had to do with class development. Mainly that the class was teacher-centered. Students stated that teachers did not allow students to find their own way to solve mathematical situations. Students claimed that when they understand a topic their attitude in class changes, and consequently their learning increases. Some students on the contrary, stated that they feel confused due to teaching approaches in which teachers do not take into account their learning styles or needs.

Given my position at the school as English language and mathematics teacher, I focused my attention in students' responses from fourth grade and the teachers' responses that are part of the bilingual project and use the Singapore method. Both, teachers and students gave positive responses in terms of their teaching and learning processes. Students felt comfortable in the math class due to their active participation, which apparently was linked to their understanding of the concepts prompted by the method. The same perspective was expressed by teachers, who also added that sometimes the English language became a difficulty for students to solve mathematical situations.

The answers from teachers and students highlighted three main points that were of my interest to start the development of this research project. First, that language should not be taught and learned exclusively in the language class (mother tongue or second language) but should be a bridge that links knowledge in all grades and in every subject at school. Second, teaching approaches should be aimed to allow students discover and apply knowledge to solve real life situations. Finally, classroom environment should be directed for students to feel they are active contributors to the creation of knowledge.

In order to expand the needs analysis, I reviewed official documents regarding Colombian students' performance on PISA test on 2012 and 2015. The situation the school is facing is directly linked to Colombia's current situation in terms of schooling practices. The Organization for Economic Cooperation and Development (OECD), together with several countries around the world, tries to understand and solve by means of shared experiences, countries' common problematic situations. Colombia is one of the countries that is looking forward to join this organization. Our interest as well as the organization, lies in improving social, economic and environmental situations. The objective of joining the OECD has to do with learning from other countries' experiences and consequently formulate policies that stimulate necessary changes. One

of the topics in which the OECD and Colombian government are interested in is the educational field; based on this interest, the organization has developed a test called PISA (Program for International Student Assessment). This test assesses 15-year-old students that are finishing their secondary school cycle and are about to enter tertiary education, or are about to be part of the labor force. The information that it provides works as a source to make decisions and formulate policies that can improve education at a local level (OECD, 2017).

PISA is designed by experts in education and is directed to find out what students have learnt in school. The test assesses three main areas: mathematics, literacy and science; its objective has to do with knowing the skills and aptitudes students use to analyze and solve problems, managing information and knowing what skills students will use when facing situations in adult life. According to the results, it places students in six different levels. Colombia's performance during the past years has shown an improvement although the results are not entirely desirable. (ICFES, 2016) In this regard, I would like to turn the attention to the results obtained by Colombian students in 2012 and 2015 in order to have a wide perspective of what has been happening for the last years in Colombian primary and secondary educational field. In 2012, in mathematics students obtained 376 points, in literacy 403 and in Science 399. Similarly, in 2015 the results show that students obtained in mathematics 390 points, in literacy 425 and in science 416. According to the results from both years, most of Colombian students are placed in the level one and level two revealing a poor performance in all three areas, which according to ICFES (2016), refer to students who reached the lowest scores and were unable to complete basic tasks in each area. The chart below provides information about scores and capacities displayed by Colombian students in the PISA test in 2015.

Table 1

Descriptors of proficiency levels in science, literacy and mathematics.

Level	**Lower Score Limit**	**Descriptor**
Mathematics	Level 1 357.8	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli. (OECD, 2009, p. 122)
Science	Level 1 334.9 Level 2 409.5	At Level 1, students have such a limited scientific knowledge that it can only be applied to a few, familiar situations. They can present scientific explanations that are obvious and follow explicitly from given evidence. At level 2 students have adequate scientific knowledge to provide possible explanations in familiar contexts or draw conclusions based on simple investigations. They are capable of direct reasoning and making literal interpretations of the results of scientific inquiry or technological problem solving. (OECD, 2009, p. 144)
Literacy		At level 1 some tasks require the reader to locate one or more pieces of information, which may need to be inferred and may

	Level 1 335	need to meet several conditions. Others require recognizing the main idea in a text, understanding relationships, or construing meaning within a limited part of the text when the information is not prominent and the reader must make low level inferences. Tasks at this level may involve comparisons or contrasts based on a single feature in the text. Typical reflective tasks at this level require readers to make a comparison or several connections between the text and outside knowledge, by drawing on personal experience and attitudes.
	Level 2 407	At level 2 Some tasks at this level require the reader to locate one or more pieces of information, which may need to be inferred and may need to meet several conditions. Others require recognizing the main idea in a text, understanding relationships, or construing meaning within a limited part of the text when the information is not prominent and the reader must make low level inferences. Tasks at this level may involve comparisons or contrasts based on a single feature in the text. Typical reflective tasks at this level require readers to make a comparison or several connections between the text and outside knowledge, by drawing on personal experience and attitudes. (OECD, 2012, p 82)

Note. Adapted from "*Pisa 2009 assessment framework – key competencies in reading, mathematics and science"*OECD, 2009, p. 122 & 144. And, "*Literacy, Numeracy and Problem*

Solving in Technology-Rich Environments. Framework for the OECD Survey of Adult Skills" OECD, 2012, p. 82.

The results show that Colombian students are unable to support ideas based on different information sources and argument their decisions or to extrapolate concepts to solve different and unfamiliar situations. Without the development of these skills students are in a disadvantageous position towards the social, economic, technologic, scientific, and various other challenges that this century brings with it.

Based on the information provided by the OECD (2016), official Colombian institutions such as the Instituto Colombiano para el Fomento de la Educación Superior (ICFES) provides recommendations in order to rethink and improve students' learning processes and teachers' educational processes. Some of those recommendations have to do with learners' development of study habits and proper learning strategies; and with teachers' practices that embrace strengthening pedagogical knowledge. They also point at the design and implementation of learning strategies for students that show poor performance to increase their motivation, and pleasant classroom environments to enhance better attitudes towards school and learning. According to the ICFES (2016), combining all these elements will result in the development of students' skills and competences and in better academic achievements.

According to UNESCO (2011) and Turbay (2016), everybody should have access to high quality education. Colombian students, as others in different parts of the world, should be given an opportunity to develop different skills to enhance their literacy processes and therefore fully participate in their local and global communities. In that sense, this project intended to enrich students' development of math and language skills. From a research perspective, it sought to understand language and mathematical literacy processes and the relation between them when using the Singapore method. The method for teaching math becomes a fundamental part of this

project because it intends to position students as the main constructors of knowledge by turning classrooms in spaces where they can discover and apply their knowledge in different situations. Thus, this project challenges traditional math classes in which memory, decontextualized and repetitive exercises are the core of the learning process. Additionally, it focus on the solution of mathematical situations through the development of math concepts and language skills as well, consequently language becomes a crucial component in development of the math class. The question that guided this project was:

Research Question

How do math and EFL literacy processes unfold when using the Singapore method?

Research Objectives

1. Determine students' EFL literacy development when engaged in mathematical tasks proposed by the Singapore method.
2. Determine students' mathematical literacy development when working on problem solving activities proposed by the method

Rationale

Since 1994 the National Ministry of Education included English as mandatory subject in schools' curricula. English is a subject of great importance and all teachers' effort should be directed at students mastering the English language. However, the importance of teaching and learning English is now beyond the development of communication competences. It has to do now with the development of what the global competences (Ministry of National Education [MEN], n.d).

These competences embrace the abilities people have to access different opportunities that today's world offers.

The school where I work is going through the process of becoming a bilingual institution. Subjects such as arts, science and math are now being taught in English. The bilingual project brought a shift in instruction paradigms, in the mathematics subject specially, which implied changes in the way we, English teachers, teach and on our vision of learning. Therefore, this project was an opportunity for teachers in this educational context to systematize our teaching practices by interpreting and reflecting on using English as the language of instruction in the mathematics class, the actions we took and the effects those actions provoked in learners.

This systematization was based on a method called Singapore. The Singapore methodology for teaching mathematics aims to develop math concepts and skills for everyday use. Those skills include thinking, reasoning, communicating, collaborating, and developing metacognition, among others. They are linked to what it is expected from people in order to make contributions the current 21st century, an era whose main assets are information and knowledge. Consequently, schools and classrooms should be places in which teachers and learners work collaboratively to be successful in today's world.

This project was also a contribution to the research line *Literacy Processes* of the Master of Arts in Applied Linguistics to TEFL at Universidad Distrital Francisco José de Caldas. It yielded insights about learner's learning processes when combining math and English through a method that stimulates students' cognitive capacities, particularly problem-solving skills. The project provided useful information for institutions and teachers interested in improving their teaching practices.

Chapter II

Theoretical Framework

This chapter presents the theoretical constructs that serve as a framework to support the development and findings in my research study. The nature of this research project involves the understanding of four main concepts, socio-constructivism, collaboration, literacy and mathematical literacy. The first part of this chapter presents the theoretical foundations; the second part includes an overview of studies that were developed drawing on the similar theoretical basis.

Socio-Constructivism

How people relate and understand the world depends on their perspective and beliefs about how individuals construct knowledge. The socio-constructivism perspective defines knowledge construction, individuals and the world from a particular perspective. This term can be understood by examining the two words that compose it, “socio” and “constructivism”. The first one refers to the interaction between individuals and the second to the construction of knowledge that is based on their social exchanges. World and reality are understood considering the historical and social situations that surround people, who are immersed in that reality to produce social exchanges which result in the generation of meaning and understandings about their reality. How people, jointly, perceive and understand their reality is of great importance to socio-constructivists. Au (1995) and Amineh and Davatgari (2015) state that social interaction is the basis to construct knowledge, and that such interaction is generated from language, which allows the development of thought (Vygostky, 1986; Oldfather, West, White & Wilmarth 1999). Below,

I present a review on the principles of this theoretical construct and the relation between language and learning.

Lev Vygotsky was one of the most important theorists whose studies on the development of children provided enormous contributions to the theory known as socio-constructivism. Based on Vygotsky's principles of knowledge construction through social interaction, Steiner and Mahn (1996) highlighted three tenets of his theory. First, the mental functions and the development of individuals occur in social interactions. Mental functions such as cognition and language are constructed as individuals are linked with other individuals. The second, semiotic mediation is understood as the use of conventional signs that comprise language, for instance by which people construct, support and transform their mental activities. It is through the mediation of semiotics that people appropriate and internalize knowledge. The third tenet has to do with learning and development. They are shaped in social and cultural specific contexts. This cognitive process, or the functional systems of an individual, depends on his or her internal mechanisms to face new learning challenges and perform tasks. Internally, individuals modify their cognitive strategies which then are translated into external devices (talking, charts, writing, and so on); they are representations of how individuals mediate and make meaning of their world. Ergo, when these three functions are interwoven, learning processes develop and knowledge is constructed.

The learning concept that grows from this perspective is one of its big contributions to the educational field in recent years. Carwile (2007) and Amineh and Davatgari (2015) understand learning as the interaction of an individual with other individuals, in which they relate to each other and reorganize their mental functioning. Interaction is therefore a social event in which language takes up an important role. Popkewitz (1998), regarding language, Vygotsky states that thought and language are linked and through language, people can expand and construct knowledge and make sense of their world. Accordingly, in order to illustrate socio-constructivism

inside classrooms, teachers should be in charge of setting up engaging environments that enable students to make use of the cognitive and communicative dimensions of language to interact with each other and perform active roles as producers of knowledge.

Collaborative Learning

The concepts of internalization and the Zone of Proximal Development (ZPD) developed by Vygotsky (1978) represent learning processes and the role of social interactions within it. They serve as a channel to build the formulation of what collaboration is. “Learning and development are interrelated from the child’s very first day of life” (Vygostky, 1978, p. 84). This premise accounts for the origins of cognition, the beginning of individuals’ social life means also the beginning of their cognitive development. It is at this point that Vygotsky (1978) consolidates the concept of internalization which is defined as the internal process of how children reconstruct external situations and social exchanges. To this regard, Forman and Cazden (2013) state that internalization, as result of interaction and by means of speech, allows individuals “to create new mental formations and develop higher processes of mental life” (p.189).

Although cognition and general social interactions are linked, I want to ground the latter to school settings where interaction is inevitable and internalization process occurs. Social exchanges between teacher - students, and student -student contribute to the development of learning processes (Forman & Cazden, 2013), they involve what Vygotsky (1978) called Zone of Proximal Development (ZPD)“[it] is the distance between the actual developmental level as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 38). Internalization and the ZPD converge in the idea that learning is a social mediated process in which individuals are able to complete tasks in collaboration with others, leading them to reach specific learning goals.

The ideas presented above allow us to understand learning processes from a social position. The concept of collaborative learning is of great importance to the development of this research project, and it is directly linked to the socio-constructivism theory and how knowledge is socially constructed. Despite of the fact that socio-constructivism positions learning as a social activity, it is necessary to highlight that it also occurs at an individual dimension. According to Dillenbourg (1999), in both cases different mechanisms are activated. Individually, we can develop skills such as induction, deduction; make compilations, and many others. When we read for example, we build predictions; likewise, when we interact with others, we use explaining, supporting ideas, disagreeing and negotiating. These activities allow us to develop learning mechanisms such as internalization; knowledge elicitation, that refers to students' ability to demonstrate and communicate to others the knowledge possessed; reduction of cognitive load, which has to do with the distribution of tasks between the members of a group, so all have the opportunity to participate and contribute to the development of a major task; among other mechanisms. For instance, members of a group can be in charge of different tasks according to what they know about it; then, through displaying and discussing the processes followed, all members in the group can have access to that knowledge and learn from their peers.

It is my personal belief that both language and math literacy are socially mediated and therefore, this vision of learning from a collaborative perspective is adequate to answer the questions I posed at the beginning of this research project. "Collaboration invites students to be decision makers. As they discuss and make plans, students practice not only their linguistic and cognitive skills but their social skills as well" (Gilles & VanDover, 1988, p. 30). However, collaboration itself may not guarantee learning. Dillenbourg (1999) specifies actions for teachers to take inside their classrooms that can increase the probabilities of success when working collaboratively. First, it is of great importance to establish concrete initial conditions such as

number of students per group, which students to group, kind of task to be developed and objectives of the tasks. Secondly, teachers need to specify the roles of each member of the group; state the interaction rules and finally, during tasks facilitate and monitor students' interactions and progress.

Based on a review of various authors, Shahamat and Mede (2016) conclude that collaborative learning has been defined from multiple angles, but that it generally depicts activities carried out by individuals to accomplish a goal. It implies individuals providing assistance to one another, exchanging information, coordinating attempts to solve a problem or building interdependence and attempting to construct knowledge together. The authors mention that Johnson, Johnson, and Stanne (2000, as cited in Shahamat & Mede, 2016) proposed that collaborative learning required accomplishing these five aspects:

(1) Positive interdependence: Team members are obliged to rely on one another to achieve the goal. If any team members fail to do their part, everyone suffers consequences.

(2) Individual accountability: All students in a group are held accountable for doing their share of the work and for mastery of all materials to be learned.

(3) Face-to-face promotive interaction: Although some of the group work may be parceled out and done individually, some must be done interactively, with group members providing one another with feedback, challenging reasoning and conclusions, and perhaps most importantly, teaching and encouraging one another.

(4) Appropriate use of collaborative skills: Students are encouraged and helped to develop and practice trust building, leadership, decision-making, communication and conflict management skills.

> (5) Group processing: Team members set group goals, periodically assess what they are doing well as a team and identify changes they will make to function more effectively in the future (p. 683).

In tune with the socio-cultural perspective that underlines this study, this conception of collaboration is adopted. It is of great importance to highlight that when individuals get involved in social encounters, by means of speech, mental processes are transformed (Forman & Cazde, 2013), supporting Vygotsky's idea of how learning occurs socially and then personally (1978). On the educational field, this concept is essential due to by working collaboratively students are able to develop skills such as inter-independence, active participation on tasks, communication, coaching and leadership, assessment, negotiation, decision-making, among several skills. (Dillenbourg, 1999; Johnson, Johnson, & Stanne, 2000, as cited in Shahamat & Mede, 2016). Giving these points, it is a teachers calling to create collaborative class environments that contribute to the students' cognitive as well as social potentialities.

Literacy

The discussion presented below deals with an understanding of literacy as a cultural and historical situated practice. The construction of this concept aimed to have a clear definition in current times and it served as conceptual frame to analyse the data gathered during the implementation of the instructional design.

As it was stated, literacy is embedded in history, society and culture. It implies that it is a dynamic concept that changes according to social circumstances. Smagorinsky (2011) states that literacy is not purely a cognitive phenomenon but a cultural one, society ways of thinking, its values and beliefs permeate how literacy is developed. For instance, Graff and Duffy (2008) historical review on literacy support the idea of how literacy shapes social contexts and people's

needs. It was not until the end of the eighteen century that literacy took an important position within educational contexts. Although, during this time it was used to maintain social control by training people to be disciplined and obedient to authority; later with the development of the Enlightenment and the promotion of the concept of human being as the axis of a new time, literacy started to take a different position. It had to be imparted to masses aiming at the promotion of personal and social growth.

Regarding literacy as a factor that contributes to the improvement of human welfare, UNESCO (2004) proposed that education was a fundamental right to all human beings. This new vision of it provoked that skills such as reading, and writing needed to be spread, so social and economic concerns in nations could be solved. The most recent definition of literacy given by UNESCO (2017) can be interpreted as a new time for societies and education. It positions literacy as a social construct that is locally embedded; therefore its practices such as reading and writing should respond to specific contexts. Currently, authors have converged in the understanding of literacy beyond reading and writing as passive and isolated skills but a "cultural tool imbued with ideology and employed toward particular social ends" (Smagorinsky, 2011, p. 119).

The OECD (2017) reduces the term literacy to reading literacy, they consider that reading has to do with the development of skills and strategies that are constructed in different contexts and interactions within communities. Reading requires not only decoding words but the usage of linguistic and textual structures to have a deeper understanding of what the world is. They link reading as a competence that implies the economic force of a nation, the participation of people in a community and an economic world depends on their development of reading skills.

These two visions of literacy acknowledge the fact that literacy brings a social welfare. It is also noticed that both assure that literacy goes beyond decoding words, they expand their

definition of literacy to the development of cognitive skills that bring social and economic benefits and that allow the understanding of how the world has been consolidated. It is necessary to take into account these definitions due to the great influence the sources have on the political decisions of governments around the world. Educational landscape is affected by the visions about literacy they promote. I believe it is of great importance to go deeper in to what really literacy means in current times. The definitions presented below embrace some elements described by the UNESCO and the OECD and adds essential elements to a broader and more complex understanding of what literacy is.

Thus, literacy is not positioned as simply learning how to read or how to write; rather it is a concept that embraces both skills plus speaking skill, which makes it a context-bound communicative practice, therefore a social phenomenon (Cook-Gumperz, 2006). The idea of the development of several actions when literacy is supported by Gee (1999); multiple abilities or multiple literacies rise when facing a text, those skills draw on our social background and how meaning construction depends on it. For instance, Gee (1999) explains that when interacting with a text we are not only decoding words, but we are bringing past interactions, talks, thoughts, values and beliefs that make possible the creation of personal and particular meanings.

According to the intention of this research project I will draw upon the definition of literacy given by Kern (2000)

> literacy is more than a set of academic skills, more than inscribing and decoding words, and more than prescribed patterns of thinking. It involves an awareness of how acts of reading, writing and conversation *mediate* and *transform* meanings, not merely transfer them from one individual or group to another. Literacy is neither natural, nor universal, nor ideologically neutral, but culturally constructed. It is precisely because literacy is variable

> and intimately tied to the sociocultural practices of language use in a given society that it is of central importance in our teaching of language and culture (p. 23).

Additionally, Kern (2000) establishes seven principles which support at the same time his vision of reading and writing as acts of communication. Literacy involves interpretation, collaboration, conventions, cultural knowledge, problem solving, reflection and self-reflection and language use. All of them are embraced in the word communication; he believes that reading and writing are socially-constructed-acts of communication who aim is the creation of meaning.

Literacy Development: Communication

Who we are and the actions we take within our community determine our literacy practices. Reading and writing are communicative practices that enable the activation of socially-learnt values, beliefs, ideas, behaviours, conventions, along with others (Short et al., 1996; Ali, Karim and Yusof, 2016). The world and our knowledge about it is transformed when we read and write, reading and writing imply to fill with meaning our representation of the world and the social exchanges we constantly provoke. Accordingly, Vygotsky (1978) states that the cultural and cognitive development of a person is regulated by stages: first, social and then individual. When a child interacts with members of a community and shares different social activities, children begin to master tools and signs that later they will internalize. Then, these children, by accounts of their social exchanges, make meaning of their thinking and experiences.

As literacy involves communication, we constantly produce and interpret meaning of the world around us through texts; people by means of texts, understand the functionality of language to convey a message. Short et al. (1996) expands the notion of literacy to a multimodal event. This conception of literacy allows us to think about a variety of sign systems (language, mathematics, art, music, dance, etc.) that serves us to mediate the world. For example, in

academic contexts, the interaction with texts by reading or writing and by using different sign systems provoke in students learning processes, thus our knowledge structures and meaning resources as continuously modified.

Literacy also involves the interaction with texts. I have subscribed to the social construction of knowledge, which implied adopting literacy and therefore reading and writing as social practices. Kern (2000) proposes reading and writing as communicative processes characterized by a cognitive and a social dimension. The cognitive dimension refers to how the link between textual forms and knowledge is created; the social dimension has to do with how these acts of communication are built on learners' social exchanges and communicative practices. Accordingly, Short et al. (1996) state that reading and writing as social acts cannot be separated from other skills, "writing always involves some amount of reading and reading always involves speaking and listening" (p. 25).

Kern (2000) proposes a model to understand how communication, reading and writing are developed and how their relationship with literacy connects to social and cultural set of practices. He describes the role of the three factors when trying to understand and make meaning of a text. Learners draw upon a set of resources (available designs) that serve as potentials and constraints to communicate. These initial resources are built from our knowledge of L1 and L2. To this regard, we make use of vocabulary, grammar structures, style, genres, writing systems, stories, among others, to make meaning of texts. He states that although these resources are necessary they are not sufficient. Short et al. (1996) conception of reading and writing involves the same elements presented by Kern (2000) and adds a perspective on the social dimension of the two skills. The authors present learners as active language users, who are not just receiving rules and structures about how language works, but who use intuitively their language knowledge, social and historical background to read and write texts of personal meaning to them. .

Therefore, reading and writing are nurtured by our knowledge about language in terms of structures but also social exchanges and the contexts we move into are of great importance and essential when making meaning of ourselves and the world around us. For instance, in our educational context, Clavijo, Guerrero, Ramirez, Torres and Torres (2004) concluded that literacy practices should not be exclusively part of language classes, but they should be extended to all areas of curriculum because students are able to develop the same skills in different subject areas. This idea allows understanding how students, by means of literacy, can develop an extended vision of their reality. Furthermore, Riquelme and Quintero (2017) concluded that literacy as a socio cultural construct that promotes acquisition and growth of knowledge and that has an enormous impact on people's lives. Thus, by means of texts, readers and writers share different elements that allow the creation of meaning: conventions, values, assumptions, beliefs, cultural patterns, purposes, interpretations, representations of the world, reflections about the world and about us, the understanding of pragmatics and the sociocultural practices immersed of different contexts.

Mathematical Literacy

This type of literacy has to do with the development of skills that emerge in specific situations that require the application of the language of mathematics to solve problems. Applying mathematical concepts and skills into situations brings to the classroom the creation of strategies among one self and the interaction with others. They allow making sense of reality through the system of symbols the mathematics language provides, the integration of personal background and the possibility of communicating and listening to others' ideas so informed decisions can be made.

Mathematical literacy serves individuals as a medium to make sense of the world. Some aspects that involve mathematics have a deep relation with the understanding of real life. In this regard the OECD (2002, cited in Martin, 2007), states that mathematics embraces quantity, space and shape, change and relationships, and uncertainty. Quantity refers to understanding the meaning of numbers and how learners can demonstrate it when communicating by means of mathematical language and symbols. Space and shape refer to geometry concepts and their applicability to other spheres of knowledge such as geography, arts, architecture, among others. Change and relationship involves the application of mathematics in all-natural phenomena, it enables the understanding of how the world functions the way it does. Finally, uncertainty has to do with data collection and analysis and all it can reveal and inform.

The elements mentioned by the OECD (2002, cited in Martin 2007) have to be understood in relation with the definition of mathematical literacy. Additionally, the OECD (2017) presents a definition constructed from a specific social and historical context. Thus, the understanding of mathematics is essential for students because it will serve as a tool to mediate or confront possible issues or challenges in their personal and professional lives. It states:

> Mathematical literacy is an individuals' capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (p. 67).

This definition acknowledges a set of actions an individual takes on in order to make sense of the world. They encompass, formulation of situations through mathematics; secondly, employment of concepts, procedures, and reasoning to solve problems; finally interpretation

which refers to a reflection of mathematical solutions or conclusions to the situation posed. Moreover, the OECD (2017) states that those processes which are embedded in the definition of mathematical literacy, involve the development of several capabilities: communication, mathematising, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, and using mathematical tools. A brief explanation of each is presented below.

Communication has to do with understanding, clarifying, formulating, summarizing, presenting, justifying or explaining mathematical texts or situations. Mathematizing, involves how individuals translate a situation into mathematical terms. Representation refers to how students interact and capture a situation. It includes the use of pictures, tables, graphs, diagrams. Reasoning and argument involve thought processes that allow making inferences and justify solutions of problems. Devising strategies for problem solving refer to planning a mathematical strategy to solve problems. Using symbolic, formal and technical language and operations, has to do mainly with mathematical language (symbols, rules, systems) needed for mathematical tasks or situations. Finally, mathematical tools, this capability encompasses the use of physical tools (measuring instruments, technological devices, and so on) for students to complete or account conclusions for a task.

It is noted that mathematical literacy from the OECD (2012, in Martin 2007) perspective involves a complex interwoven of processes and a detailed description of several skills to be develop by individuals in order to solve situations. Similarly, The Livingston and Washtenaw Mathematics Steering Committee (2008) states that mathematical literacy as a concept embraces the understanding of mathematical concepts to be applied to real life situations. This conceptualization aims to understand this type of literacy as a way to meet people's needs. According to the committee, the use of mathematics should embrace more than the development

of fixed skills. It takes into account factors that go beyond concepts and skills, these factors embrace the use of mathematics and the way people integrates it to communication, reasoning, analysis, and formulation and resolution of problems. They proposed five principles on what mathematical literacy is:

> coherent, integrated and functional understanding of concepts, operations and relations. The ability to carry out procedures flexibly, accurately, efficiently and appropriately. The capacity for logical thought, reflection, explanation and justification. The ability to use mathematics to meet a person's life needs. Finally, to see mathematics as an integral part of a global society (p. 3).

The OECD (2017) and The Livingston and Washtenaw Steering Committee (2008) present important aspects about individuals' skills and procedures embraced in mathematical literacy. In addition to those two definitions Martin (2007) introduces aspects that are directly directed to students' contexts. Mathematical literacy poses solving real life problematic situations as its trigger. The author states that this literacy is developed when learners are connected to their realities; those situations allow them to make links with different disciplines and at the same time, through communication, they can construct strategies to solve problems. Henceforth, such strategies are translated in the creation of learners' own models, without the memorization of meaningless procedures of rules, to bridge abstract mathematical concepts to real life.

It has been noted that mathematical literacy is posed as a set of principles to follow and to teach inside classrooms, also that it is made up of different components and their development inside classrooms should be directed to promote solution of problems and the construction of knowledge in different areas of knowledge. Accordingly, the OECD (2012) proposed an additional dimension when referring to mathematical literacy. This organization stated that this literacy has to do with an essential skill for the 21st century. Citizens of this new century are

about to face challenges in which they will have to make use of mathematics, therefore the capacities for formulating, applying and interpreting mathematics in several contexts have to be a must in classrooms. Mathematics plays an important role in today's society, learners taken as engaged and reflective citizens, should be encouraged from early ages (Fandiño, 2013) to develop mathematical concepts, procedures, in order to make substantiated decisions that bring welfare to all members of their communities.

Given these points of what mathematical literacy is and what it embraces, Solomon (2009) and Hemphill (2010) highlighted the social dimension of the concept. Mathematical knowledge is socially directed. This characteristic understood as learning occurs when we interact, allows us to be aware of the learning we do in terms of mathematical ideas (Solomon, 2009). Meanwhile, Hemphill (2010) relates the social dimension of literacy to mathematics literacy. She highlights the richness people bring when making meaning of texts, interpretations of them are permeated by personal background and experiences. Giving these circumstances, when people interact with the objective of solving a mathematical situation, personal strategies emerge and are shared provoking knowledge awakening to the parts involved.

These ideas on mathematics as socially constructed literacy clearly show a link between the construct of literacy and the vision of Vygotsky (1978) about learning. The two concepts also rely on the principle of literacy as a socially situated practice. In the light of these ideas, the implications for classroom are enormous. For instance, Janzen (2008) states that language fosters mathematical literacy. This link is strengthened within good teaching and learning practices. Janzen (2008) claims that everyday language serves as the base for math language and that teachers should be aware of their second language learners' linguistic characteristics because they are fundamental to effective teaching.

Taking into account sociocultural and cognitive issues that enhance language and math, Janzen (2008) and Yore, Pimm and Tuan (2007) suggest that collaborative work, students' oral interventions, writing productions, peer discussion, interaction and awareness of students' interests and backgrounds provoke and improvement in math as well as second language learning processes.

Becoming literate therefore implies reasoning through mathematics language and its representations, developing language and literacies to express how individuals understand and represent the world and solve real life situations through language. Therefore, classroom settings should encourage learners to have the opportunity to use language as a means to interact and to develop different abilities. One of them is to participate in discussions where students can support ideas based on their knowledge and demonstrate concept domain and to interpret situations and provide solutions based on their personal experiences. They can also make connections between their contexts, new knowledge and the content developed through lessons to reach the core of what mathematical literacy intends: solving problems.

State of the Art

This study intends to understand how foreign language literacy and mathematics literacy develop when learners use the Singapore method. It is important to examine research has been done regarding this topic, what methods have been used, what population has been studied and what results have those studies produced. This part of the document summarizes and compares the research I found. The following lines account for what has been researched so far in relation to my research topic; mainly, the relationship between language and mathematics in the mathematics classroom are approached in this chapter.

The studies that related to the topic of my interest are described considering two moments. First, I began with three research projects which had a qualitative nature; they intent to display realities of bilingual mathematics classrooms from teachers and students' perspectives and the complexity that second language adds when learning mathematics. Tavares (2015) conducted the first study I refer to here. She carried out a study in which they aimed to examine how bilingual teachers' use L1 and how they mobilize communicative resources to help mediate, scaffold and facilitate learners' comprehension of mathematical concepts, learning and internalization of the language of mathematics. The participants were high school students from Hong Kong that receive math lessons with English language instruction.

By means of interviewing students and the teacher and classroom observations, the study revealed that the teacher used different strategies to enhance learning. She created a safe and supportive classroom atmosphere where learners were constantly engaged in interaction and collaboration, within this environment they were able to become more confident and feel more empowered. Moreover, she enhanced the comprehension of mathematical concepts through activating 'math talk' via questioning and peer interaction and through skillfully handling mistakes at the conceptual level. The teacher promoted students' interaction on problems, mathematical operations or questions that she posed; they varied in difficulty, and were aimed to recall learners understanding, application analysis, synthesis and evaluation. Lastly, regarding learning and internalization of the language of mathematics researchers found six characteristics: One, inside the classroom the L1 allowed learners to consolidate concepts in L2; concepts were understandable if the L1 was ensured. Two, students mentioned that they paid attention to important aspect of language in the mathematics class with the highlight strategy, taught by the teacher. Three, teacher directed students to understand mathematics as a spiral area of knowledge, going back to old topics helped students to have a better understanding of new ones

and to recycle vocabulary. Four, the teacher taught students how prefixes can help them to understand the meaning of words and how to relate that knowledge to different subject areas. Five, students acknowledge the gently way the teacher correct them. Six, the staged questions teacher posed during lessons encouraged learners to activate, revisit, recycle, consolidate and communicate using mathematical language, hence, questioning lead learners to embrace both language and concepts simultaneously.

The second study was the work of Yeh (2017). The objective in this 3-year study was to explore the teaching and learning experiences of elementary school mathematics teachers for their emergent bilingual students. The findings in terms of first and second language and classroom environment of this study are closely related to the findings of Tavares (2015); Yeh (2017) concludes that teacher' views of learning shape the manner they organize learning experiences in language and in mathematics lessons. It was evidenced that in language and mathematics discussions learners made use of their L1 (Spanish) and also the mixed the L1 and the L2 (English), however teachers modeled language and presented new vocabulary so it was connected to the mathematics lessons. Additionally, teachers encouraged students to interact in order to hear and be heard about reasoning and problem solving. This type of interaction allowed students to build their own meanings and definitions. The study also revealed that students made use of language in multimodal and hybrid manners. Students continually discussed using Spanish, English and codeswitching between the two languages. The researcher observed that while the students were trying to reason and make sense of math problems, in their interaction they attempted to use mathematical concepts and the discourse they usually heard during class time. Regarding the classroom environment, it was noticeable that how students and teachers were positioned in terms of power and status affected students learning. Classrooms in which the

power was distributed students were invited to demonstrate their learning by means of questioning.

The third study presents an interwoven relation between second language learning and mathematics development. The study was conducted by Banse, Palacios, Merrit and Rimm-Kaufman (2017) at a school located in the south-Atlantic region of the United States. Their objective had to do with examining discussion practices occurring in classrooms in which a high number of students were characterized as English Language Learners (ELLs). A curricular program proposed by the school required teachers to develop a Calendar math that intended to strengthen students' mathematics understandings and to encourage them to have discussion activities at the beginning of math lessons. Thus, this context served researchers to pose two research questions; how do teachers support mathematical discussions within that context? And how do teachers incorporate discussion into a regular element of their lessons taking into account the high number of ELLs in their classrooms? The study was described as a comparative case study. The researchers followed the principles of grounded theory, which served to answer the questions. In order to gather the data, they used video tapes of the two classrooms.

The study generated important conclusions concerning teachers' teaching practices and the relation between language and math development. First, there was a difference in time and in the number of activities, each teacher proposed for the calendar math implementation. This situation revealed that in one of the classrooms observed, students had more time to get involved in discussions. Another aspect that emerged had to do with the type of questions teachers asked to students. Generally, both teachers used close-ended questions limiting students' opportunities to develop language and cognitive skills such as constructing arguments, critique or agree with others' reasoning, learn from discussions, be engaged in mathematical reasoning or practice using mathematical language. Although closed-ended questions emerged as a factor that could have

decreased ELLs learning, researchers acknowledged that when students were required to answer multiple-option questions (a type of closed-ended question), they were forced to provide a rationale. It was understood as a mean to scaffold ELLs towards answering more complex questions in the future.

Repetition, reformulation and elaboration of responses were important findings in the research. Researchers defined repetition as the action to affirm and broadcast students' responses. Reformulation refers to reframe students' contribution by the use of academic language and elaboration has to do with the extension or explanation of a student reasoning. The teacher or another student does it. According to the findings, the researcher noted that, as teachers did not ask open-ended questions (referential questions) students were not able to present or demonstrate their mathematical reasoning. Consequently, teachers did not reformulate students ideas, thus they were not able to hear their own ideas in academic language or mathematical terms. Elaboration and repetition were present simultaneously, when students were allowed to elaborate their ideas they extended their responses; similarly, when teachers elaborated responses, it meant the extension of students' ideas. These types of practices allowed ELLs to understand language use or to expose them to new language use. Teacher-talk was also part of the study findings. It refers to the moments in which the teachers modeled problem-solving activities. As students listened to teachers' work through problems or answers to questions, they were able to reinforce their understanding of problem solving approaches and the language linked to it.

The studies presented before, highlight the importance of language in a non-language subject area in schools. Language allows the understanding and development of core concepts of any area of knowledge; ergo, teaching practices and lessons design should offer learners the opportunity to be in constant contact with context-appropriate language use and to communicate

whether in L1 or L2; in other words, to be engaged in discussions where they can express ideas and convey arguments or strategies used in order to support their learning processes.

According to the ideas presented before, the next studies aimed to give an account on the literacy and mathematics relationship. The studies that follow were carried out from a quantitative perspective. Most of the studies tried to present the reality of a greater number of students: they presented and analyzed data from entire schools and cities. Another significant similarity among these studies was the use of standardized tests to gather data. However, it is of great interest the variety of participants and the contexts where the studies were carried out. They ranged from preschoolers to university students. The first two studies I report take into account literacy and mathematics and their relationship with collaboration as knowledge building medium. The other studies only report literacy and mathematics by the means of tests.

Firstly, I describe the work of Genlott and Grönlund (2016). This study emerged with the intention to expand a previous pilot study developed in 2013 that included the analysis of students within two groups; one group called traditional in which neither the students nor teacher used the ICT and another in which ICT was used without any specific method for learning, and for fostering social interaction or formative feedback. This project expanded the previous one in terms of the sample, and use of standard national tests for third grade, analyzing three groups: writing to learn (WTL), traditional and individual technology use (ITU), finally the inclusion of mathematics tests. According to the characteristics of the project, in 2016 the authors conducted a quantitative study with the participation of 503 third-grade students in Switzerland. The main objective was to test if the method Writing to Learn (WTL, -that was supported by ICT and that includes elements from socio-cultural learning theory and notably feedback and formative assessment, was better than two other methods: traditional, without ICT use, and method using ICT different from WTL method.

After analyzing the students' scores, the researchers found the WTL method reached better results in literacy and mathematics than the other two groups. They highlighted the success of the method mainly over the results obtained in the ITU group. Researchers claimed that although both groups used ICT and the same gadgets, the richness of the WTL had to do with the implementation of the socio-cultural theory. WLT encouraged teachers and students to integrate ICT as the mean to create classroom environments in which social interaction, collaboration, formative feedback and assessment were integrated. Another important finding in this study were the mathematics results.

As I mentioned before, the WTL aims teachers and students to communicate therefore when they are engaged in mathematics tasks in which they discussed about number logic, thinking and reasoning or how to solve mathematical problems. Hence, the WTL fostered discussion of math issues and pushed students to formulate math problems, pushed others to seek solutions and fostered collective reasoning in the form of formative feedback.

Similarly, Sarama, Lange, Clements and Wolfe (2011) conducted a large study in two elementary schools in the United States. It aimed to investigate the effects of an intensive pre-kindergarten mathematics intervention on the oral language and letter recognition silks of preschool children. The participants were 2064 children from 4 to 5 years old. Researchers used four instruments in order to reach their objective, a test that measured letter recognition and an assessment that included children retelling stories based on pictures and inferential questions. This assessment aimed to characterized children's oral language. Then, a test called TEAM (tools for early assessment of mathematics) that measures children's mathematical knowledge and skills. It included two interviews for each child. Finally, classroom observations, based on the COEMET (Classroom Observation of Early Mathematics – Environment and Teaching)

which measures quality of mathematics environment and activities, it comprises three elements: classroom elements, classroom culture and specific mathematics activities.

The results revealed that there were great differences between the two groups of children. In terms of mathematics, researchers claimed that a mathematics curriculum could benefit literacy as math lessons emphasize on reasoning, problem solving and communication. According to the test results, children linked to the *bulding blocks* overpassed children of the other group in oral language. The test measured information, complexity, independence and inferential questions. According to the researchers, the oral test was "a far-transferred task" the mathematical curriculum that was implemented allowed children to translate skills developed in the mathematics classes to the development of language skills.

Another important finding was the benefit that interaction between students brought. As children interacted, they were willing to share their ideas, reason, represent and make connections. All actions that were performed during the mathematics class engaged students in the development of language and additionally built confidence, which could be transferred to other domains in which they would use the same skills.

The study conducted by Molly (2017) in Washington wanted to explore the relationship between language literacy and mathematical scores on state performance assessment. The project was approached from a quantitative perspective and used regression analysis in order to investigate the correlation of limited language proficiency and the performance in English Language Learners (ELL) students on two standardized math tests. The participants chosen for this study were eight grade students, enrolled in the bilingual project fostered by the school; all of them were considered limited English students.

The research found that evidently, language literacy and mathematics are closely related. Molly (2017) claims that language proficiency can definitively predict mathematics success. She

highlighted two main aspects, ELL students find tests administered in English challenging due to their lack of reading, comprehending, writing skills and therefore their lack of understanding of mathematical concepts. Secondly, she found that ELL students scored lower in mathematics when their English language proficiency is also low.

Recently, Dahm and De Angelis (2018) carried out a study in secondary schools in France. A total of 607 students with an average age of 15 were selected; most participants were multilingual with knowledge of French as L1 or as non-native language. The objective of the project was to examine the multilingual benefit in relation to mother tongue literacy and two different types of learning: language learning and mathematical learning. To reach the objective, the participants were divided into three different groups: school multilinguals, multilinguals with literacy in the home language and multilinguals without literacy in the home language.

Mathematics and English tests along with questionnaires for students and parents served as data. The math tests and questionnaires were modified from PISA tests 2010; and A2 and B1 Cambridge tests were used to measure English language proficiency. The research revealed five important aspects. Results indicated that language background has a beneficial effect on learning, particularly on language learning. Secondly, mother tongue literacy helps multilinguals to reach native peers faster and perform similarly in mathematics tests; simultaneously, multilinguals with home language literacy overpass multilinguals with no home language literacy.

Results indicated that language background benefits greatly on learning mathematics and in a greater manner learning another language. School multilinguals (natives) and multilinguals with home language literacy demonstrated similar results in both English language and mathematics tests; multilinguals without home language literacy were lagged behind. Researchers refer to multilingualism not only as beneficial for learning but as a greatly important cognitive

activity. They also claimed that mother tongue literacy is of great value to students because it places them in advantage when learning another language and when this knowledge is extended to other subject areas such as mathematics.

With similar findings O'Donoghue and Ríodáin (2009) presented a study related to mathematical word problems and language proficiency in Ireland. This quantitative-type study had three different groups as participants. First , there was a group of English – speakers only, Gaeilge-speakers only, and university, college and institutes of technology students who had learnt mathematics through the medium of English during school years and mathematics was part of their degree courses. Thus, the study included a total of 70 participants. The objective of the research was to examine the influence of language proficiency performance on mathematical word problems for Gaeilgeoirí in the transition to English-medium mathematics at second and third – level education. Researchers used two instruments to measure language and mathematics performance; a mathematics word problem test in Gaeilge (bilingual students only) and in English; and a language proficiency test in English and in Gaeilge (bilingual students only).

They found that Gaeilgeoirrí's performance on mathematical word problems is related to their linguistic proficiency in both languages. Students who have a strong academic language proficiency in L1 are most likely to transfer those strong skills to the learning of L2. This was evident when students transferred their language abilities to mathematical tasks using English. Improving language proficiency in English may improve students' performance in mathematics with English. The researchers invite teachers of mathematics to assess first students' L1 and gradually assess tasks in L2 until they can be very adapted to it.

From these studies it is of great importance to highlight three aspects. Collaboration, social interactions, classroom discussions, formative assessment and formative feedback provokes in learners a better understanding and a genuine engagement in the development of mathematics.

Literacy as a classroom practice positively affects learners' development in mathematical skills such as reasoning, problem solving and communication. Last, L1 is a valuable resource for students when learning another language and home literacy makes learning processes easier as learners are able to transfer their background knowledge to their academic contexts.

After having examined different research based on the relationship between second language and mathematics, I found relevant connections between my research project and the investigations that approached the two literacies from a socio-constructivism perspective. Studies converge in aspects of great importance for pedagogy: collaboration, students' background, students' first language, role of teacher inside classrooms, teacher-students interaction, language influence in the development of mathematical reasoning, engagement in communication, among others.

Regarding the Latin American context, I present three different research projects that relate to mine. The first two illustrate the connection between language and non-language content areas from students and teachers' perspectives. The third research relates to an experience from the countryside of Colombia that involves mathematics and its application to social purposes. Firstly, Pistorio (2010) developed a research whose objective was to create a socially constructed learning environment through the implementation of Content Language Integrated Learning-CLIL, an approach that integrates language and content instruction. The project was conducted based on the principles of action research and the participants were fifth graders from a private school located in Argentina. In order to measure students' language performance two tests were applied: one at the beginning and the other at the end of the project. Then teacher assessed students in terms of contributions, cooperative behavior, responsiveness to feedback and time and task management.

The findings of this project showed that reading, writing and oral skills improved in a 20% difference in comparison to the tests applied at the beginning of the project. The researcher found that students had a deeper understanding of texts; oral speeches showed great fluency and rich content management and vocabulary. Another important aspect was students' improvement in their critical thinking, which may be connected to the constant invitations they had to express their opinions and differ from others' opinions by giving clear and supported responses. According to classroom roles, students became more active participants when constructing their own knowledge and the teacher adopted a mediator and facilitator role by helping students construct knowledge instead of memorizing or reproducing facts. Finally, journals showed positive attitudes from students towards the class. They recalled feeling motivated because they were encouraged to perform tasks that were meaningful and demanding despite of their linguistic level.

In Colombia, McDougald (2015) conducted a research that aimed to understand the current state of the implementation of CLIL in Latin America Using questionnaires from 140 Colombian teachers that taught in levels from preschool to university, the research revealed that teachers believe that the approach develops both subject knowledge skills as well as language skills. They also reported that training and more knowledge about this methodology is necessary. Furthermore, course materials available need to be adapted or adjusted so they meet students' needs. Another aspect reported by the teachers had to do with the support from schools. They stated that cooperation between peers, subject-area teachers and administrators is of great importance when implementing CLIL. Accordingly, researchers recalled that it is imperative for universities to review their curricula programs so they are consistent with the reality teachers' face when becoming part of schools.

Contrasting with the two previous studies, Cadavid (2013) presents a report of an educational experience in the countryside of Colombia. Her experience is based on the work done regarding mathematics and the collaboration between teachers, parents and students. The project wanted to promote collaborative work and strengthen academic relationships among teachers in order to adjust curricular projects that involve local issues. It also intended to engage students in curricular projects in which they could link math knowledge to the issues of their communities. The axis of the experience referred to the water and the value it had for the community. Teachers, parents and students began visiting different places including the water spring that supplies water to the aqueduct and the water treatment plant. Those visits helped to pose different situations for the math class.

Cadavid (2013) stated that the experience not only strengthened bonds among teachers but among students, parents and community. Additionally, students were able to acknowledge the role of mathematics to understand social situations. The researcher also highlights that although such curricular initiatives subtract class time to deliver several contents that have been traditionally taught for years in math and in other content areas, they should be implemented more regularly because they demonstrate that learning occurs outside classrooms, in real-life contexts and most important, their development can have a great social impact.

I found those three studies of great interest for my project. They illustrate that language literacy and other types of knowledge can be developed simultaneously and at the same time bring diverse advantages for students learning processes. Finally, although the third study (Cadavid, 2013) did not take into account language literacy development, it shows that mathematical literacy can be transferred to different scenarios and that a school community benefits when real life situations are examined.

Chapter III

Research Design

This chapter presents the research framework aimed to answer the research questions: how do Math and EFL literacy processes unfold when using the Singapore method? Within this chapter I present the type of study, the context and participants, instruments applied to gather the data and finally, the role of the researcher and ethical issues.

Type of Study

The research project was developed following the principles of qualitative research. This type of research allows researchers to explain social processes; as people construct their reality by means of interaction and having experiences with others, they are constructing meanings of their world. Qualitative research therefore aims at understanding diverse human systems, and explaining how people make sense of their world (Merriam, 2009; Yin, 2011). This research is framed in the qualitative research approach because I examined my students' experiences, actions and behaviors as they unfolded in the particular context where they are involved. I wanted to observe my students, and how they made sense of the math task in the naturally occurring context of my class.

Concerning the research methodology, I drew upon the qualitative descriptive approach. This type of study is supported by the idea of studying something in its natural state, "it is a comprehensive summarization, in everyday terms, of specific events experienced by individuals or group of individuals" (Lambert & Lambert, p. 255). In this regard, the process consists on a constant comparative analysis of the data, a research feature of grounded theory. Thus, although this type of research adopts phases from grounded theory, its main product is not the creation of

theory grounded in data; rather the description of a particular reality through the analysis of data (Lambert &Lambert, 2012). The process to describe how this research project was carried out will be explained later in order for the reader to recognize how I conducted the qualitative descriptive phase.

Context

The school in which this research project was carried out is Colegio Nuestra Señora del Rosario Bogotá, located in Puente Aranda, a neighborhood in the southwest of Bogotá. It is a large private and Catholic school that serve 1700 students. Its mission is to offer its students a holistic education that encompasses different educational principles, physical, affective, cognitive, spiritual, communicative, social, and work. These principles represent the human potentialities that need to be developed for an integral education. Additionally, the pedagogical practices are also based on the development of values such as truth, love, justice, liberty and transcendence. Since the year 2013, the school decided to implement a bilingual curriculum fostering English and Spanish as the languages to use. This situation led to change teaching and learning processes in classrooms. Subjects such as science and mathematics were chosen to be taught in English. In the midst of this situation, and in order to increase students' contact with the foreign language, students take six hours per week of English as well as mathematics. Before the beginning of the implementation of the bilingual project, English classes focused mostly on grammar and vocabulary and the development of communicative skills was not taken into thoughtful consideration. Math classes for elementary grades were directed to teach and learn concepts by memory and exercises were repetitive, without any context and topics were detached from one another.

Currently, English teachers are active participants in the implementation of the English and math curricula. For the English subject, they are in charge of engaging and improving students' development of critical thinking, communicative and citizenship competences. Regarding the mathematics curriculum, the school adopted the Singapore Method, which changed drastically the way classes were conducted. This curriculum accounts for strengthening links among topics, manipulating objects aiming the comprehension and development of basic mathematical operations and their application in solving problems. It also aims at improving students' communicative competence so they can explain and use arguments that show their understandings about different concepts. For the implementation of the mathematics curriculum, the school requires English teachers to have several and rigorous training sessions in which they learn about mathematical pedagogical practices to reach curriculum objectives.

Participants

The participants selected for this project were 23 fourth grade students,16 girls and 7 boys whose ages ranged from 9 to 10 years old. Participants can be described as a homogeneous group since most of have gone through the bilingual process in the school; they belong to 3 and 4 socioeconomic strata, and most of their parents reached a professional degree. The participants were chosen based on the criteria given by Patton (1990), which was purposeful, convenient sampling. Purposeful sampling is used to identify information-rich cases related the object of study. It was a convenient sample because the students were in my assigned classes, and most of them had been involved in the bilingual project since first grade. Therefore, they were in the process of learning mathematics through English instruction. The needs analysis carried out revealed that language and learners' roles are important components to the development of math literacy. To this regard the method implemented in elementary grades for teaching and learning

math, the Singapore method, engages students in the development of math knowledge and furthermore second language development.

A very important aspect of research is to maintain an ethical conduct. I followed the guidelines for research in the social studies that aims at promoting benefits for the community involved in the study and protecting individuals' rights. I requested permission from the school administration to carry out this study. I informed the school principal, as well as the coordinators about the purpose of the study and its benefits for the institution. The principal supported my initiative and signed a consent form (Annex 4). Equally important, I obtained permission from the parents or family of the children who were my sample. I wrote a letter (consent form) explaining the intention of the study and described the ways I protected my students' identity and well-being. I used numbers to identify each student in the transcription of data from my observations for example, and deleted names in their written production. Parents and family signed a consent form (Annex 3). I also had a conversation with my students about what we were going to do because it is my belief that children need to be informed as well.

Instruments

The instruments to gather data for this research project were selected following the principles of qualitative research approach.

Classroom Observations

Observation is defined as the description of a social context and what occurs around it: events, behaviors, situations (Kawulich, 2005). One of the purposes of this project was to understand the role of collaboration when learners were engaged in the mathematics class. Having in mind this objective, I chose participant observation as the main data collection method. Kawulich (2005)

understands participant observations as "the process enabling researchers to learn about the activities of the people under study in the natural setting through observing and participating in those activities" (p.2). Additionally, Robson and McCartan (2016) present the advantages of using participant observation as data collection method. This type of observation involves the researcher actively into the situation under study and offers him or her the opportunity to observe a situation or phenomenon closely and analyze a wide range of situations. Another advantage has to do with the generation of rich qualitative data which allows the researcher to understand complex realities and relationships.

I used audio and video recording as techniques for gathering data on my observations. These techniques provide the researcher with valuable details about the interaction among participants and their behaviors. They allow "researchers to reflect on the implicit beliefs, classroom scripts or mental schemata which are brought to the classroom" (Burns, 1999, p. 94). As I adhered to the idea of literacy as a social and historical situated practice, these techniques helped me understand how learners, when interacting, developed EFL literacy and mathematics literacy.

Students' artifacts

In terms of literacy development, which is one of the specific objectives of this research project, I collected learners' artifacts. McGreal, Broderick and Jones (1984) state that artifacts are tools or ornaments referring to human workmanship. How they are built, the meanings people assign to their creations how they select them or modify them are of great importance due to the idea that they encompass a direct use and meaning people assign to them. In educational terms, they have a great impact because they provide important insights about learners' learning processes. Students' written, visual and oral production are artifacts. Burns (1999) states that

students' documents support and complement researcher's observations. They contribute valuable information when understanding the setting of research. To this research project, students' construction of documents helped to the understanding of the relationship between EFL and mathematical literacy. I collected all the written production of my students when working with the Singapore method.

With respect to the validation of these instruments, my peers at the M.A. program in Applied Linguistics to TEFL at Universidad Distrital, and I analyzed in different sessions the quality of the data I was gathering when conducting my observations. These discussions, along with the feedback from my advisor helped to validate the instrument and make sure it was being used in a consistent way to gather rich data.

Role of the Researcher

My role in this project was as participant observer. Burns (1999) explains that participant observation "involves entering the research context and observing oneself as well as others in that context" (p. 82) Teachers as active participants in the research process become part of students' activities. This role is suitable to this study due to I am the teacher assigned for the group of students which it made easier for me to gather data while applying each one of the lessons I designed for the research project.

Ethical issues

As stated before, research that involves people requires guaranteeing participants several aspects. Behi and Nolan (1995), proposed researchers to consider anonymity, confidentiality, and informed consent. Research subjects have the right to privacy. Thus, according to the type of research and the conflicts that they can generate, it is necessary to make identity anonymous and

subject information confidential. The informed consent has to do with the right of participants to have truthful and sufficient information to decide whether or not they want to be part of a research. As the subjects for this research project were children, it was mandatory to ask permission from their parents or an adult. As previously stated, in order to obtain it, a consent form was administered. Copies can be found at the end of the document in the annex section.

Next, I will explain the pedagogical intervention that was a central part in this research. I will explain its foundations and draft the general components and key factors in its implementation.

Chapter IV

Instructional Design

This chapter describes the pedagogical intervention that served as a source to reach the objectives of the research project. The pedagogical intervention was planned and developed taking into account the findings of the needs analysis stage. The contents include a description of the school context, the curricular platform and its articulation with the vision of curriculum, language, learning and teaching that I used; and finally, an account of the instructional activities and learning experiences developed in this phase of the research.

This pedagogical intervention was implemented at Nuestra Señora del Rosario School, a private bilingual school with a population of 1700 students approximately. The participants selected for this project were students from fourth grade between nine and ten years old. They have been part of the school's bilingual project since 2014. Their schedule involves mathematics, science and arts classes to be taught in English. English teachers therefore were trained to follow a specific method for teaching math, the Singapore method. This method requires the integration of different elements to reach its objective, which include problem solving and collaboration.

Curricular Platform

The objective of this pedagogical intervention was to engage learners in mathematical situations that could foster EFL and mathematical literacy simultaneously. Consequently, the curricular basis of this intervention is the theoretical foundation of the Singapore Method for mathematics that includes five main components: problem-solving, concepts, skills, procedures, metacognition and developing positive attitudes. The method also serves as a guide to support the curriculum, language, learning and teaching visions.

The Singapore method pays special attention to the development of skills and solution of everyday problems by focusing on three pillars called ‘concrete, pictorial and abstract’ (CPA). It promotes learners’ mathematical thinking and understanding of concepts first by the manipulating objects (concrete), representing concepts with drawings (pictorial) and applying abstract forms of the mathematics (symbols) (Calderón, 2012). The method works on the following five aspects (Ministry of Education of Singapore [MES], 2012):

Concepts. Concepts are the group of numerical, algebraic, geometric, statistical, probabilistic and analytical concepts that are connected and interdependent. These concepts should be understood and applied by students through a variety of learning experiences that link abstract mathematical concepts with concrete experiences.

Skills. Skills are the mathematical principles that embrace numerical calculation, algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematical tools and estimation.

Processes. Processes refer to the development of three types of skills learners develop in the acquisition and application of mathematics. The first group integrates reasoning, communication and connections. These refer to learners’ abilities to construct logical arguments regarding mathematical situations, the mathematical language to express ideas and shape mathematical thinking, and finally, the ability to create connections between mathematics and the real world.

The second group of skills encompasses applications and modelling. Application skills have to do with the ability of learners to link their knowledge on concepts to real world

situations. The modelling skills allow students to develop appropriate processes, concepts, make informed decisions based on given or collected data to solve real-life situations. The last group refers to thinking skills and heuristics which relate to the multiple strategies learners are able to use in order to solve a problem.

Metacognition. Metacognition refers to the ability learners develop when becoming aware of the mental processes they construct when solving problems: in other words monitoring and regulating their own thinking and learning.

Attitudes. Attitudes refer to affective aspects that influence the learning of mathematics. Those attitudes connect to the personal experiences, interests, beliefs, and awareness of the usefulness of mathematics, appreciation for the subject, and confidence and perseverance in solving problems. The Mathematics class should be aimed to the creation of environments in which learners can construct positive attitudes towards the subject.

According to the objectives of the pedagogical intervention, the components of the method for teaching math are articulated to the vision of curriculum, teaching, learning and language. They are presented below.

Vision of curriculum

The vision of curriculum I used encompassed the principles of the learner-centered curriculum presented by Cleveland and Emes (2005). This kind of curriculum allows learners to be fully active in their own learning processes and aims to achieve two purposes: one, learners should reach meta-learning, or the ability to be aware of their own learning processes, strategies and

methods that work for them. Second, learners become active agents in the design and construction of meaningful learning activities.

Given the nature of the pedagogical intervention and the vision of curriculum, the curriculum should foster learners' opportunity to work together, communicate ideas through language and mathematical concepts, discover solutions by themselves and monitor their thinking process, as they are involved in learning experiences (Cleveland & Emes, 2005).

Vision of teaching

The mathematics class is supported by two approaches: activity-based learning and teacher-directed inquiry. The first approach draws upon the principles of socio-constructivism. Its main interest is to position students at the center of the learning process through engaging them in interactive activities where they have the opportunity to manipulate appropriate material and consequently build understanding about concepts while working with their peers (Padmavathi, 2013). The teacher-directed inquiry approach. This approach is about learning through guiding inquiry. Instead of giving the answers, teachers lead students to explore, investigate and find answers on their own. Students learn to focus on specific questions and ideas and are engaged in communicating, explaining, and reflecting on their answers. They also learn to pose questions, process information and data and seek appropriate methods and solutions. (MES, 2012, p.24)

In my pedagogical plan learners were guided to explore, understand, make-meaning, and uncover abstract mathematical concepts first by manipulating objects and tools. Second, learners, by means of discussions, questioning and communicating through mathematical language, were invited to wonder about a situation, a math problem and work out strategies to solve them.

Amineh and Davatgari (2015), based on the social constructivism principles, state that teachers should be facilitators by helping learners to get to their own understandings. In this

regard, the skills they should foster when teaching have to do with posing questions, supporting students' attempts to solve a problem, providing guidelines, and creating appropriate environments to foster dialogue among learners. This role allows learners to position themselves at the center of the class development. They become knowledge constructors. When learners are provided with chances to pose questions and work collaboratively, they are simultaneously given the opportunity to articulate their mathematical knowledge and personal experiences to interpret, analyze and hence, reach the solution for tasks.

Vision of learning

Learning is an ongoing process that involves transmission, construction, transaction and transformation. These characteristics of learning are stimulated when individuals are immersed in interactions (John-Steiner & Mahn, 1996). The Singapore method proposes that language and math learning occur when individuals interact and experience real life situations. To that respect the world becomes the laboratory in which learners can take advantage to make-sense, gain understandings, construct insights, in other words, learn from the world. Experimenting with the world becomes the core of curriculum. Learners should be exposed to a variety of activities that could become meaningful for them. The curricular unit that I proposed aimed to provide students with experiences that were related to the principles of the Singapore method and also emphasized the idea that learning is stimulated when people share and interact through language their own personal experiences.

Vision of Language

The pedagogical intervention drew on Amineh and Dsvatgari (2015) definition of language. They state that language is positioned as the most important tool learners make use of to make

sense of reality. Likewise, Wells (cited by Smagorinsky, 2000), claims that language is understood as the main tool through which learning and meaning negotiation occur; therefore, language in the classroom should be described as a tool to communicate ideas and construct meaning. Through this pedagogical intervention, I intended to create interaction environments between learners and between them and the teacher so they can be involved in situations that fostered negotiation of meaning and thus, knowledge construction.

Pedagogical Intervention

This section presents the goals, the development of lessons designed and the learning experiences of the pedagogical intervention.

Objectives

The objective of this pedagogical intervention was to engage learners in several mathematical situations to foster mathematical literacy through EFL instruction while implementing the Singapore method for mathematics. My main objectives were:

1. To foster students' collaborative work to solve mathematical situations
2. To engage students in group discussions to enact mathematical situations
3. To learn to solve math problems
4. To foster the use of EFL

The construction of this intervention implied the understanding of two complex learning processes; namely, English language and mathematics. Teaching math in English presents diverse challenges for teachers as well as for learners. Those challenges imply learning specialized vocabulary, reading comprehension, writing, using specific content-language,

presenting ideas and expressing concerns, among others. These difficulties interfere with students reaching a full comprehension of contents. Teaching practices and classroom environments should allow language to be the bridge for learners to develop concepts, solve problems, be aware of metacognitive process and so forth. To this respect, Janzen (2008) states that math and second language literacies are enriched when "teachers ask students to articulate their thinking process, share ideas in groups, or thinking through new ideas verbally or in writing, students are extending their engagement with an understanding of new information" (p. 1030).

According to the ideas presented above, the objectives and the requirements of the school curriculum for mathematics, I designed a pedagogical intervention based on the Singapore method for mathematics and its problem solving approach. The approach is illustrated below and includes the 5 components of the method; namely, attitudes, skills, concepts, metacognition and processes

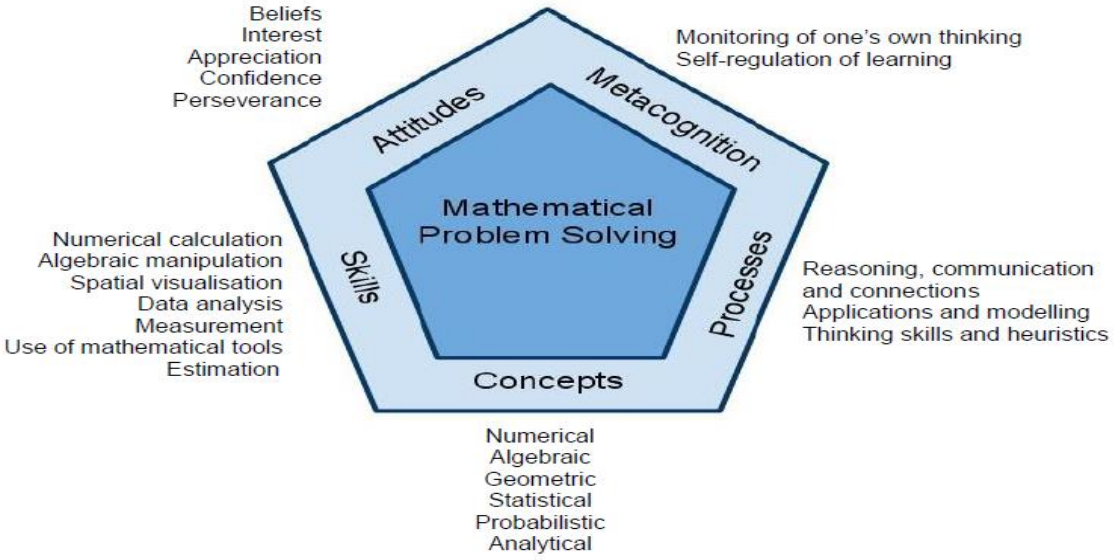

Figure 1. Visual display. This figure illustrates the problem solving approach (MES, 2012, p 14)

Methodology

The pedagogical intervention was implemented from August to October 2017. Its design was subject to the school's academic schedule for fourth grade. A total of seven lessons were implemented and students were invited to solve different mathematical situations while working collaboratively with the Singapore method. The lessons were developed in three stages. First, I engaged students in situations that were familiar to them through the reading of stories. During this stage, I fostered communication of students' personal experiences. After that, I posed problem-solving situations to seek for solutions and gave students instructions to work in groups.

I acted as a monitor and a guide, listening, questioning students' work, and helping them in using problem solving strategies. Finally, I encouraged groups to present their outcomes and I asked students several questions about their processes to solve the problems. I highlighted their strategies to solve mathematical situations. This last stage was directed to promote discussions among students in which they could support the ideas that allowed them to choose one strategy over others; additionally, students could recognize which strategy was more accurate than other to solve the same situation, so their decision-making capacity was enhanced. The following pictures show the stages I have described:

Figure 2. This picture illustrates the presentation of a mathematical situation based on a story. Lesson 2.

Figure 3. Students working collaboratively. Preparation of skewers. Lesson 1.

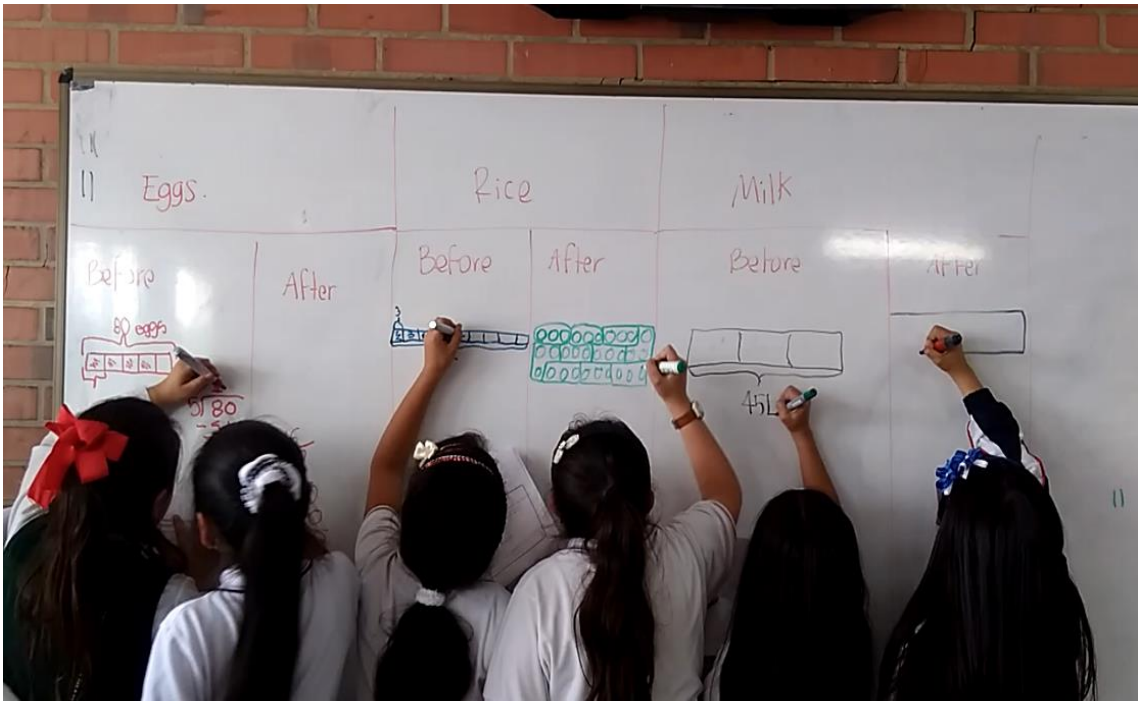

Figure 4. Students presenting their strategies to solve a mathematical situation. Lesson 2.

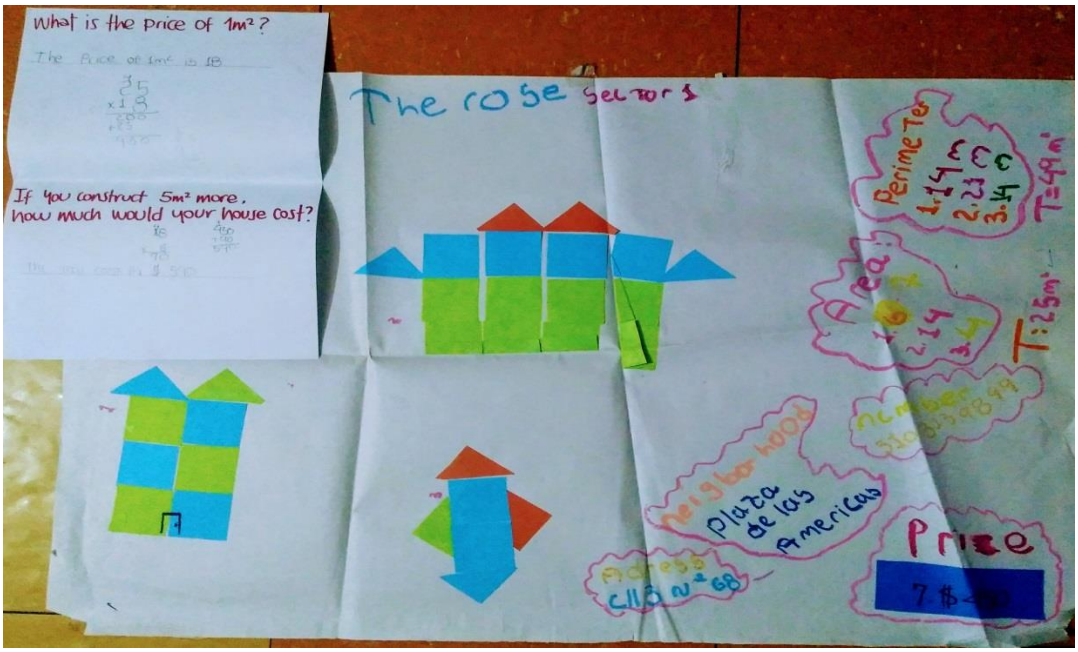

Figure 5. Students' artifact: posters. Lesson 4

Pedagogical Intervention Plan

The pedagogical intervention was implemented during the months of August, September and October in 2017. I designed seven lesson plans that consisted of solving mathematical problems regarding different topics, fraction of whole numbers, area and perimeter, and decimal numbers. The materials designed encompassed booklets, workshops, realia, and concrete material (counters: blocks, beads, pictures, cuttings). All materials designed require students to read and interpret information so they can propose solutions to the math problems posed. A sample lesson and the material used are included in the annex (See annex 5 and annex 6). The following chart presents a summary of the lessons implemented for the pedagogical intervention.

Table 2

Lessons implemented

Lesson's Topic	**Objective**	**Materials**	**Situation to be solved**	**Students' outcome**
Lesson 1: Fraction of a whole number	To understand a fraction of a set of objects. To understand the relation between fractions and whole numbers	Workshop with instructions to create a fruit and candy skewer	Prepare candy and fruit skewers taking into account that the number of ingredients was dictated by a set of mathematic exercises	Candy and fruit skewers presentation supported by guidance on the process of preparation
Lesson 2: Fraction of a whole number	To understand a fraction of a set of objects. To solve word problems involving more than one operation	Booklet with a story about a convenience store and a supermarket	Read a story about the comparison between a convenience store and a supermarket and solve a situation regarding the	Presentation of what strategy was used to solve the mathematical problem (Making drawings, representing the situation with bar models, doing

			income of the sale of diverse products.	mathematical operations that suit the topic)
Lesson 3: perimeter	To infer the concept of perimeter To use different measuring tools in order to find the perimeter of objects and images. To construct the definition of perimeter by using mathematics language	Poster with pictures and wool strings	Find the perimeter of different images using wool strings. Create different shapes and find its the perimeter. Write the definition of perimeter.	Posters in which students show the figures they created using wool. Presentation of the strategies they used to find the perimeter of the shapes. Definition of perimeter based on the lesson experience.
Lesson 4: Perimeter and area	To use mathematics language to talk about comparing prices of houses and units of measure To apply the concept of perimeter and area in every day problem situation To solve word problems involving more than one operation.	Newspaper advertising. Paper, rectangles, squares and triangles.	Create a house using paper and triangles squares. According to a given price of that house, students had to find the value of each square meter. Also, they had to find the area and the perimeter for their creation.	Newspaper Flyer showing the house created and the cost of the house as well as the cost of each meter square, other details such as location and facilities re displayed. Presentation of the process and the strategies used to solve the situation.
Lesson 5: Perimeter and area of compound figures.	To construct instructional texts by means of mathematical language.	Paper rectangles and a Composition worksheet.	Find the perimeter and area of a compound figure they create using two rectangles.	Instructional texts describing the steps they follow to solve the situation.

	To recognize the concept of compound figures To find the length and breadth of a figure by its given perimeter and area		Write the steps they followed to find the perimeter and area of their figure	
Lesson 6: Perimeter and area.	To apply the concept of area and perimeter to diverse situations with different complexity.	Workshop with different mathematical situations to be solved.	Students make groups to solve a workshop that encompasses the concepts of area and perimeter.	Workshop that included situations in which students needed to extrapolate the concepts of area and perimeter.
Lesson 7: Decimal numbers.	To represent different decimal numbers To solve situations using addition and subtraction (regrouping) of decimal numbers To assess students reading comprehension referring to mathematics texts.	Learning stations; each station had situations to be solved, the materials varied from concrete material, pictures and math problems.	By manipulating different objects, images and texts students solve different situations located indifferent learning stations (7).	A workshop in which students show how they solve the situations of the learning stations. Socialization of the answers and the strategies used to solve the situations.

Source: Own elaboration.

Chapter V

Data Analysis and Findings

This chapter contains the description of the stages and processes I followed to organize and analyze the data collected. The corpus of data for analysis includes students' interaction recordings, classroom observations and students' artifacts. The results I present along this section respond to the research question: how do math and EFL literacy processes unfold when using the Singapore Method?

In order to explain the analysis process, first, I describe how the data was collected, then I mention how I managed the data; finally, I present the emerging categories as result of analyzing the data. Engaging in educational research has brought enormous growth, professionally and personally. I have realized that research implies dedication, discipline and reflection, those values have nurtured my life as teacher and human being.

Data Management

The data for this qualitative study was gathered through the implementation of seven lessons during the months of August, September and October in 2017. The procedure followed to collect data involved the video recording of lessons and the audio recording of students' interaction when working together. In addition the pedagogical intervention involved students to solve different mathematical situations; after each lesson students' productions were handed in, for instance they were required to prepare skewers by solving math exercises, solve questions regarding stories, design posters and newspaper flyers, write instructional texts, and complete workshops based on learning stations I set up.

Audio and video recording were my primary sources of information. I also collected students' artifacts which provided me with insights about students' understanding of the tasks and the dialogues that emerged inside their groups of work. They served as evidence of the strategies used by students to solve the tasks proposed in the intervention.

Framework of Analysis

The analysis of the data followed the principles proposed by Glaser and Strauss (1967) who considered that immersion in the data from an inductive point of view serves as the foundation to understand a given phenomenon or social reality. The authors indicated that it is possible to establish categories derived from the data itself through what they called *the constant comparative method*. This process implies identifying emerging concepts, comparing, and contrasting them with the aim of identifying broader categories or concepts. An important characteristic of this process is the attention to details in the data and to what it means in a given context. The process implies three main stages. The first one is called open coding, which refers to reading the data and labeling it with names that indicate key concepts. In my case, I read the transcriptions of the observations and codified them line by line keeping in mind the objectives I had proposed. I assigned the name that was more logical for me to indicate a development in math or English literacy.

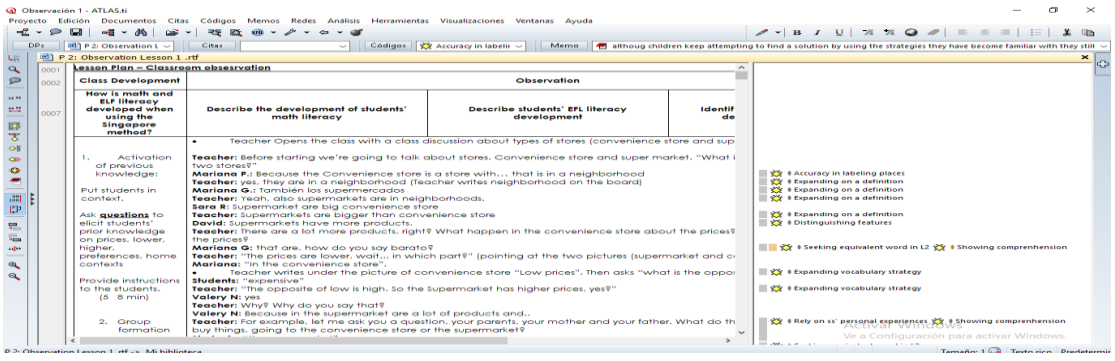

Figure 6. Open coding stage. This figure illustrates the open coding process.

The second stage is axial coding, which refers to making a selection of codes that are more significant and appear repeatedly. This phase allows for the classification of large amounts of data initially fragmented in the open coding stage. Axial coding implies grouping codes and assigning a name to the groups formed. The researcher is attentive to the characteristics of the groups. Then, I moved to the third stage, which is called focused coding. Strauss and Corbin (as cited by Charmaz, 2010), define this phase as forming a whole by going back to the data to find commonalities and logically gather them. In this point, I began to review the codes that emerged from reviewing the data from the instruments and, at the same time, I used memos (an analytical strategy used to reflect upon the meaning of the data, in order to keep a record of theoretical ideas that could support the characterization of the categories). Thus, codes grouping and memo writing helped the consolidation of categories and subcategories that responded to the research question.

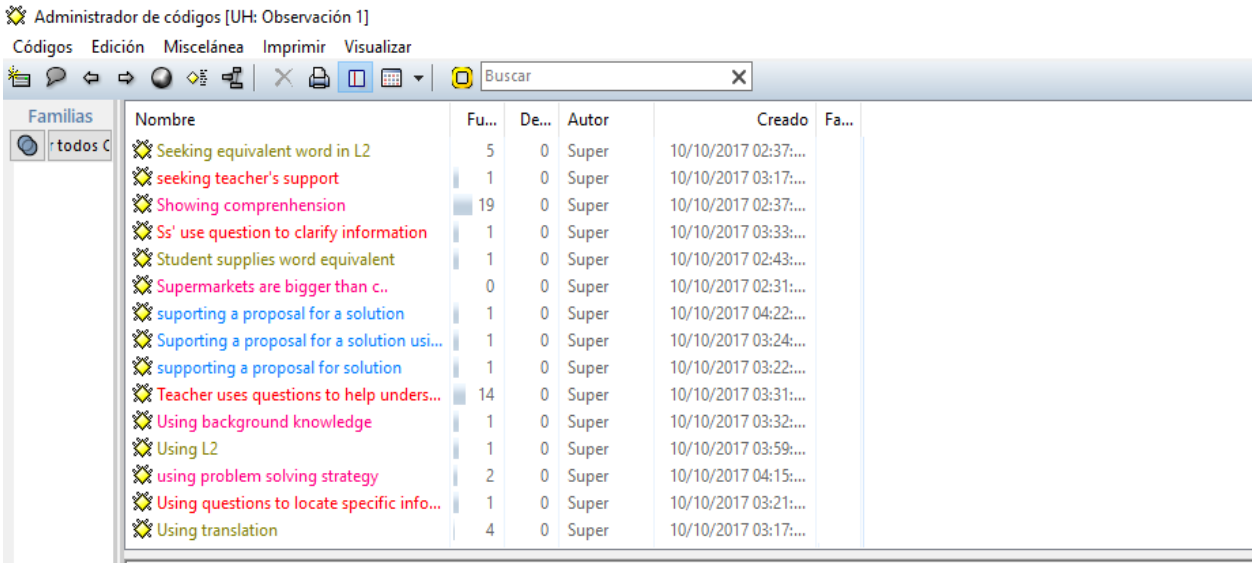

Figure 7. Focus Coding. This figure illustrates the focus coding process.

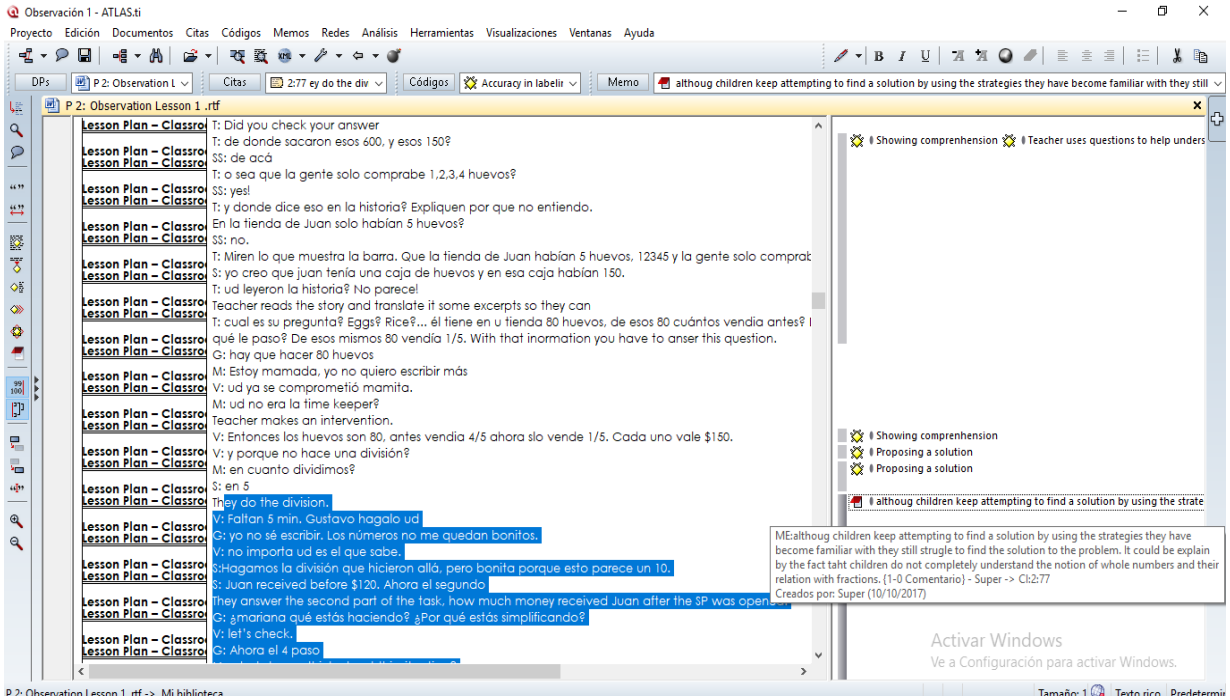

Figure 8. Memo writing. This figure illustrates an example of memo writing.

A very important aspect of analysis in qualitative data is to show that the data has robustness and transparency. In order to make sure that my analysis was free of bias and to confirm the analysis I was conducting, I used methodological triangulation. It refers to the use of at least two different sources of information on the same issue or phenomenon (Merriam, 2009). In this project, I used transcriptions from the sessions and students' artifacts. I began the analysis of the data using the transcriptions first and then I compared and contrasted my codification with students artifacts that also served to complement the description of the process followed by students and described in this chapter.

Findings

The following table presents the categories and subcategories that resulted from analyzing the data gathered.

Table 3

Categories and subcategories

Research Question	**Categories**	**Subcategories**
How do math and EFL literacy processes unfold when using the Singapore method?	Students' cognitive paths to the development of literacy	Background knowledge as a means to construct knowledge and expand understandings
		Students' knowledge representations, a window to their internal processes
	Students' social paths to the development of literacy	Characterization of L1 and L2 when interacting in the math class
		Collaboration as social construction of knowledge.
		Teacher as mediator and facilitator of literacy processes.

Source: Elaboration done by the researcher

Cognitive and social paths to develop literacy

Two categories emerged from the data gathered. Students' cognitive and students' social paths to develop literacy. EFL and mathematical literacy development unfolded from a combination of cognitive and social factors They were intertwined, and played a major role when students were

engaged in using English language to solve different mathematical situations. The cognitive aspects involved using background knowledge, using different knowledge representation forms and experimenting with several strategies to approach the solution of a problem. The social aspects embraced permanent interaction, collaboration and using L1 and L2 to gain understanding. There was an interesting aspect that also emerged from the data, which was the role of the teacher as a mediator and facilitator of the process. Each category will be described next.

Learners' cognitive paths to the development of EFL and math literacies.

This category is divided into two subcategories; one is math and language background knowledge as means to construct knowledge and expand understandings and the second students' knowledge representations, a window to their internal processes. These subcategories implied cognitive processes learners followed when working in mathematical situations

Math and language background knowledge as means to construct knowledge and expand understandings. Many aspects played a significant role in students' possibilities to understand and approach the solution of different math problems proposed in this project. One of them was using their background knowledge in math and English language when discussing the problems. Literature on children's development highlights the importance of social exchanges to facilitate learning (Vygotsky, 1978). When individuals share their personal background when interacting, such knowledge allows them to perform different actions such as to hear and to be heard, to present ideas, to reject or support ideas, to construct or deconstruct knowledge. Thus, interaction allows us to widen our perspective of the world in relation to others' perspectives (Short et al., 1996).

The interaction students had when solving mathematical situations in this project, allowed them to illustrate the way they understood reality based on their personal background. For instance, at the beginning of the second session, students gave their opinion on convenience stores and supermarkets. They recalled what their parents bought in the two types of stores and determined why the profit of stores varied according to their size or products. Similarly, during session four students by means of discussion could relate that the area of an apartment varies according to its price, the number of people that might live there and its location in the city.

The data gathered when learners interacted in the mathematics class showed that they used their prior knowledge to propose solutions. Learners' prior knowledge, according to the data, consisted on knowledge they had acquired in their previous schooling years and in their everyday life experiences outside the class. During first, second and third grade students were introduced to the concepts of geometrical shapes and area, later in fourth grade this knowledge is recycled and linked to arithmetic operations in order to find the area of surfaces with different shapes. A similar situation occurs with numbers. Before fourth grade, students understand that numbers are bonded, which is a principle that is extrapolated to understand that fractions bond to make a whole and decimal numbers. Drawings and diagrams serve to support the comprehension of the reading material and to support their decisions when conducted any arithmetic operation. Hence, skills, concepts, procedures, understandings, different ways of representing knowledge and so on, allowed learners the possibility to visualize the problems and to propose and support logically different paths to solve math problems.

The following excerpt illustrates how teachers and students' interaction allowed the understanding of the situations depicted in the math problems. It is an example of the way students used their background knowledge to show comprehension of a piece of advertising for selling apartments.

Excerpt 1 from students and teacher discussion, September 19th, 2017.

Teacher shows a piece of advertising to the students.

Teacher: A calm apartment (nodding her head). What do we have here? Can you see? (pointing at the inferior part of the picture)

Student 1: The things that are in the, the things that are in this place.

Student 1: (Reading the information) Zona Verde central, lobby y piscina.

Teacher: So, what is it?

Student 1: A conjunto.

Teacher: A block of apartments. So, what is the intention with this publicity?

Student 2: Make that the person live in this place.

Teacher: Make people live here, yes?

Student 2: How do you say vender?

Teacher: (She asks all students) how do you say vender?

Students: Sell

Student 2: Sell the apartment

Teacher: Sell apartments, very good. So, in this publicity, I know that you can't see, but I'm going to read it. It says: Apartamentos de 34 m^2. Hasta 154 m^2. So, we have, what are they selling?

Student 1: Que el conjunto es muy grande.

Student 3: That the apartments are small and big

Student 4: That are for families of for one people

Teacher: Right!

The previous sample shows that students distinguished housing and areas that are usually included in the places where they live. They attempted to use words in English to respond to the questions formulated. Using authentic material, the teacher helped to build a class discussion. Through questioning, she set the environment so that learners could read and interpret several aspects involved in a piece of advertising. The pedagogical purpose of the teacher was to show the kind of information the ad contained and that students could visualize that mathematics are also part of real world and that through discussions second language and math concepts blossom.

Figure 9. Dialogue between teacher and students about newspaper flyer. Lesson 4.

The excerpt shows that students understood the situation, they answered the question about the advertisement intention (student 2: *make that the person live in this place*.) and attempted to use the L2 to construct their answers in English (the translation of the verb "vender").

The questions posed by the teacher intended to help learners recall their prior knowledge. Kern (2000) states that the knowledge we have gained through our personal experience "influence everything we do as readers and writers" (p. 32). Thus, the resources for children to make decisions and propose solutions to solve mathematical problems are available in their previous personal experiences and in the knowledge acquired in their mathematics classes during their

school years. It is the teacher's role to trigger students' knowledge to help them advance to acquire a new one. The discussion engaged learners to discuss about what they know about houses or residential complexes, during the discussion the students recalled the words 'big' and 'small' when this information was presented: Apartamentos de 34 m^2. Hasta 154 m^2; therefore they are relating size to the concept of area. It indicates that this type of interaction serves as starting point for understanding future tasks (Martin, 2007) and also it fosters conversational language and consequently mathematical understandings (Kern, 2000; Janzen, 2008)

In the following excerpt, learners were counting material given by the teacher (squares, triangles and rectangles). The task requested that students found the area of a house they created. It illustrates how children, when manipulating objects, raised their awareness of mathematical concepts. As they were required to find the area of a shape, the material available challenged them to extrapolate basic concepts of geometry to the concept of area. It is important to highlight the fact that the use of concrete materials allow students to experiment and to reach conclusions about mathematical concepts (MES, 2012; Angulo, Castillo & Pérez, 2016).

Excerpt 2 from students' conversation when working together, September 19th, 2017.

Student 1: ¿Contamos los rectángulos o los cuadrados?

Student 2: Mira, dos rectángulos son un cuadrado, dos triángulos son un cuadrado

Student 1: ¿Cómo hacemos el área?

Student 2: Por eso, dos rectángulos son un cuadrado

Student 1: Ahh por eso solo contamos los cuadrados

Student 1: Hacemos una cajita

Student 2: Sí, hacemos como una cajita. Ustedes vayan haciendo el resto que esto yo lo hago rápido.

(They began constructing their house with the triangles, squares and rectangles)

Student 3: El perímetro es 45

Student 1: Área son los cuadritos. El perímetro son los bordes, ¿si? Los bordes

Student 3: A ver, 1, 2, 3 (she continues counting the squares)

Student 2: ¿Están contando el perímetro? ¿cuál es el perímetro? (He began counting the triangles). Andrea, díctame.

The sample illustrates that students knew that the area of shape implies counting squares, as student 1 stated. In collaboration with student 2, student 1 realized that by joining two triangles or two rectangles the resulting shape is a square. This excerpt also shows that even without the mediation or questioning of the teacher, learners, by themselves, are taking advantage of their prior knowledge: two different shapes can be put together to form a new one. In order for learners to reach this conclusion, they must have had successful learning experiences in the past to help them ground the new information and make new conceptual elaborations. Implicitly, students' dialogue indicates that they are making sense of the situation by asking questions, proposing alternatives, checking with their peers and confirming their solutions. Accordingly, Bill and Jamar (2010) proposed that the creation of mathematical ideas depends on learners experiencing with "investigating patterns, forming and testing conjectures, debating convincing arguments that both prove and explain" (p. 65). Like many other fragments from the lessons I gathered, I observed that the sessions conducted revealed that collaboration, along with the previous knowledge learners can bring to their teamwork, becomes a matter of great importance when talking about the development or application of concepts.

Literacy is how humans make sense of our world and learn from it. The way we mediate what the world is has to do with managing and understanding different sign systems (Short et al., 1996). According to this definition of literacy, and what this category reveals, I can affirm that for learners, numbers are sign systems to mediate their world, and should not be understood as

just numbers in isolation. Numbers that learners are in contact with outside the context of the mathematics class refer to a socially-constructed, evolved and changeable reality. Students can build their understanding of numbers and symbols inside and outside the classroom, which accounts for their understanding of the world.

S**tudents' knowledge representations, a window to their internal processes.** The analysis of the data also showed that when learners collaborated to make sense of texts, they used different ways to represent the knowledge they possessed or were constructing. They used graphic representations to comprehend and provide a solution to mathematical problems. How students represent mathematical problems is exemplified in the following two excerpts:

Excerpt 3 from students' conversation when working together, August 30th, 2017.

Student 1: We have one bar and divide in three

Student 3: Why you divide in three?

Student 5: Because before the supermarket the cost of the milk he buy 2/3 and the bar, because we do a bar divide in three because the 3 in the fraction the down number (the denominator) is divided.

Student 2: No entendí ni pío.

After reading the story, student 1 and student 5 agreed about drawing a bar divided into three equal parts. Student 3 got involved in the conversation by asking the reason why the bar should be divided into three parts. The explanation that student 5 provided unveils two aspects; first, that by drawing the bars she was showing understanding of what occurred in the story; and second that she was demonstrating her understanding of the function of the denominator in the text. Despite her willingness to transmit her understanding, student 2 stated that what student 5 said was not clear at all '*no entendí ni pio'*. What probably caused this failure to understand was

that the explanation was provided in English, and the level of proficiency of this particular student was low. The following excerpt is the extension of the conversation showed in excerpt 3.

Excerpt 4 from students' conversation when working together, August 30th, 2017.

As the learners continued working, student 5 led the task, as she was the one who proposed drawing a bar.

Student 5: The pencil please. Do this, this is the milk. Señalar que esta es la leche que siempre compra.

Student 1: In English!

Student 2: The milk of Juan 2/3.

Student 5: This is the milk.

Student 5: Then we do the operation. Ok Luisa what operation?

Student 1: Tenemos que averiguar what value are these..

Student 5: What is the question?

Student 1: How much money did Juan receive before and after

Student 5: Ok.

Student 5 asked her partners to show the amount of milk stated in the story. She used L1 as a way to ensure her partners do as she told. After doing so, student 4 and student 5 confirmed what was represented in the drawing. As the representation of the story was clear for some members of the group, they seemed to be ready to answer the question posed for the mathematical problem. The corresponding artifact (figure 7) shows the pictorial representation they used.

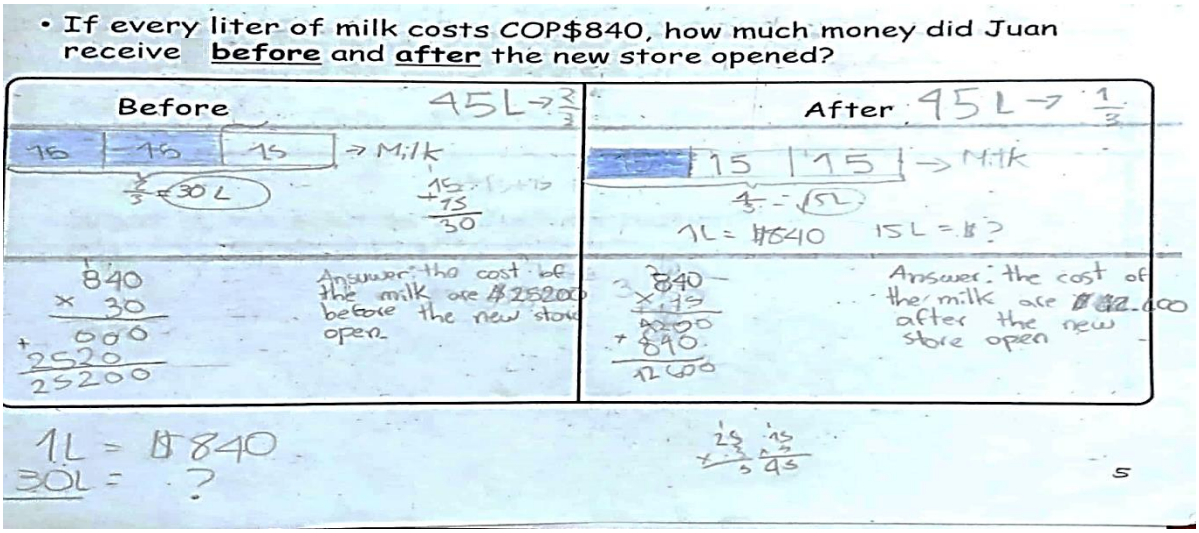

Figure 10. Students' artifact. Finding the fraction of a whole number based on a story. Lesson 2.

In figure 10, students represented the sales of Juan by drawing a bar; as he sold 2/3 of 45 liters of milk, they divided the bar into 3 parts. At the bottom of the page they did two multiplications, 25 times 3 and 15 times 3; these two operations show students' attempts to find the value of each part of the bar; they learned that *15* was the number they were looking for. Then, they colored with blue two parts of the bar to demonstrate that 2/3 of 45 is equal to 30. In order to answer the first part of the question, *how much money did Juan receive before the new store opened?* They multiplied 840 (The value of each liter of milk) times 30. Additionally, they answered the question by writing the amount of money that Juan received for the milk sold. Then, they repeated the same process to answer the second part of the question: *how much money did Juan receive after the new store opened?*

During the same session, another group of students decided to use an alternative way to represent their understanding. In this case, they resorted to drawings in order to show their interpretation of the mathematics text.

Excerpt 5 from students' conversation when working together, August 30th, 2017.

Student 1: A ver yo propongo hacer esto. Propongo hacer 30 círculos (she began drawing the circles) y entonces hacemos 10 grupos.

Students 2: Que cada uno tenga de a tres, mire.

Student 3: Que cada uno tenga de a 3. Porque 3X10 es 30

Student 1: Aquí hay un, do, tres, cuatro…

(They counted and circled all the groups until 10 got 10 groups).

Student 1: Los grupos son de a tres y ahora coloreamos 8.

Members of the group proposed to draw circles in order to make an accurate representation of the text. Student 1 proposed drawing ten groups with 30 circles. Student 2 agreed and added that each group should have had 3 circles. Accordingly, student 3 summed up what had been said by her partners, she stated that three times ten equals thirty. The conversation held by the learners showed that besides using pictorial symbols to represent their understanding of the problem, they applied their previous knowledge gained through the development of similar exercises during their school years and that through questioning and experimenting with different alternatives they proposed a solution. The role of questions and answers is an indicator that they were building knowledge while supporting each other.

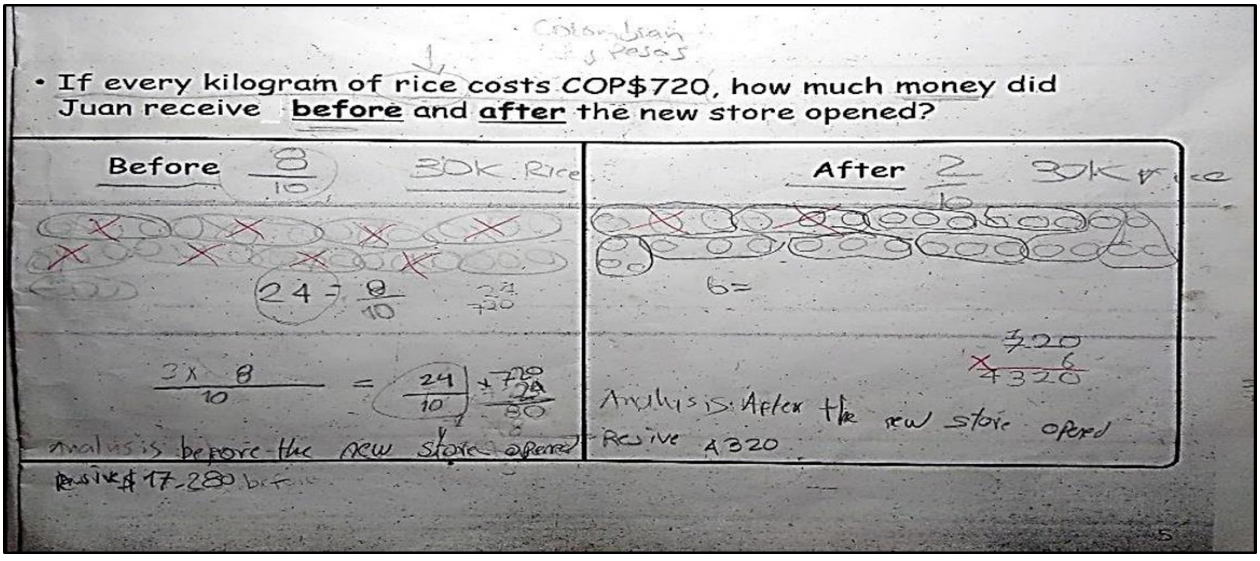

Figure 11. Students' artifact. Finding the fraction of a whole number based on a story. Lesson 2.

Contrary to what was presented in figure 10, the students in figure 11 used circles to represent and make sense of the story and the question posed. As it was noticed in the discussion, at excerpt 5, students drew 30 circles to represent the 30 kilograms of rice and circled them into 10 groups, each group had 3 circles; in order to represent 8/10, they crossed 8 groups. The pictorial representation allowed the students to find the answer which was *24*, however they applied and operation to show how to get to the same answer by the use of math; it is noticed that the operation process has some mistakes; conveniently, students arranged numbers in the operation to get 24. Then, they multiplied 24 times 720 to find the answer and finally they wrote the amount of money Juan received for selling 24 kg of rice. Finally, they followed the same process to find the amount of money Juan received after the new store opened.

The two excerpts and figures 7 and 8 illustrate that learners approach mathematical texts differently. Learners used their background knowledge, applied different ways of representing knowledge (bars and circles), used their conceptual knowledge (understanding the function of denominators in fractions) and expressed themselves in L1 to ensure understanding between members of the group and finally were able to solve the math situation. This shows, as Kern (2000) proposed, that reading and understanding transcends codification of words and implies a multiplicity of factors that make comprehension a complex process. Dialogue and collaboration between all members of a group is crucial to enhance learning (Chapetón, 2007). As learners are engaged in conversation they present ideas, listen to others' proposals and build upon those ideas. The excerpts also illustrated that for conversation to occur, members of the group should develop basic social skills such as listening attentively so they can reflect about what has being said. The samples presented relate to what others have found in terms of the value of using different ways to represent knowledge.

In a recent research about math and school engagement conducted in Taiwan, Lin, Wang and Yan (2018) emphasized the importance of providing students with different ways to represent their knowledge in order to support their cognitive development. The authors mention that there are varied ways to do so: enactive, iconic, symbolic, real scripts, manipulative objects, spoken and written language, and that all of them enable students to advance from concrete to abstract concepts. The authors indicate that independently of the type of system used to represent knowledge "a common belief shared by many researchers was the use of concrete and nonlinguistic representations, such as, manipulatives, physical movements, and pictographs at the start of teaching mathematical ideas to enhance student understanding" (p. 180). This comment is aligned with the findings of my project. When manipulating objects students could comprehend basic math and geometry concepts that later on played a role in helping them solve problems.

Learners' social paths to the development of EFL and math literacies.

This category encompasses three subcategories, characterization of L1 and L2 when interacting in the math class, collaboration as social construction of knowledge and the teacher as mediator and facilitator of literacy processes. These subcategories illustrate students' literacy development processes under a social lens.

Characterization of L1 and L2 when interacting in the math class. Many aspects played a significant role in students' possibilities to understand and approach the solution of the math problems proposed in this project. One of them was collaborative work and interaction. When learners worked together and collectively tried to address the tasks proposed, their efforts significantly contributed to solving the math problems proposed in the math class. The continuous use of L1 and L2 was clearly involved in their interaction and it served different

purposes such as clarifying, seeking understanding, proposing alternative solutions, making sure the problem was understood, questioning partners' ideas and building L2 vocabulary.

The school where this project was carried out is categorized as bilingual (Spanish – English), and the development of the lessons in this project was characterized by the continuous use of L1 and L2. For learners, as well as for teacher, the L1 and L2 remained resources to reach learners' comprehension and construction of conceptual knowledge. Spanish for this group of learners was an essential resource when interacting. Resorting to the students' mother tongue, which in this case is Spanish, served various purposes as mentioned before. One of the most frequent uses was to make content of the math problems and process understandable for all members of the group. In the following excerpt, teacher and learners are having a conversation about types of stores and their characteristics.

Excerpt 6 from students and teacher conversation, August 30th, 2017.

Teacher: Before starting, we're going to talk about stores. Convenience store and super market. What is the difference between those two stores?

Student 1: Because the convenience store is a store with… that is in a neighborhood

Teacher: Yes, they are in a neighborhood.

(Teacher writes neighborhood on the board)

Student 2: También los supermercados.

Teacher: Yeah, also supermarkets are in neighborhoods.

Student 3: Supermarket are big convenience store.

Teacher: Supermarkets are bigger than convenience store.

Student 4: Supermarkets have more products.

Teacher: There are a lot more products, right? What happens in the convenience store about the prices? What are the differences about the prices?

Student 2: That are, how do you say *barato*?

Teacher: The prices are lower, wait... in which part? (Pointing at two pictures: supermarket and convenience store)

Student 2: In the convenience store.

Teacher: (She writes under the picture of convenience store "Low prices") what is the opposite of low?"

Students: "Expensive"

Teacher: The opposite of low is high. So, the Supermarket has higher prices, yes?

Student 4: Yes.

Teacher: Why? Why do you say that?

Student 4: Because in the supermarket are a lot of products and…

Teacher: For example, let me ask you a question, your parents, your mother and your father. What do they prefer when they have to buy things, going to the convenience store or the supermarket?

Students: "The supermarket"

Student 5: "Why in the Supermarket…" (Teacher corrects the word why for because. She continues: "Because in the supermarket, how do you say calidad?"

Teacher: (Answers to student 5) Better quality.

Student 5: Better quality.

Student 2: The convenience store because is low the prices but is better the supermarket because when the parents do the shopping is all.

In the previous excerpt, the use of L2 was noticed and it was activated through the interaction with the teacher. Learners were more engaged in the conversation as the teacher formulated questions and responded to students' queries. Students began to expand their

vocabulary in English while ensuring comprehension of the material presented. For instance, student 2 and student 5 wanted to characterize the two types stores and used the question: "*how do you say…*"? , to find the equivalent of the word they wanted to use to characterize each store. When word equivalence in L2 was obtained, the learners could effectively send the message they intended to transmit. The role of L2, on the other hand, was a means to be engaged in the conversation and to guarantee its flow. Student 2 concluded the conversation by presenting arguments to support why supermarkets were better than convenience stores. This student's final piece of speech shows a strong attempt to use the L2 and that she is engaged in the conversation while simultaneously expressing ideas about her personal context.

Interaction between learner – learner and learner – teacher also revealed that the L1 serves learners and teacher to reach understanding of texts. While discussing, learners made use of false cognates, codeswitching and translation; three strategies that helped them make sense of texts. In the following excerpts, students were reading a story about two stores. The aim of the two groups of students was to understand the text in order to start planning how to solve a problem posed in the story.

Excerpt 7 from students' conversation when working together, August 30th, 2017.

Student 1: Ok the *anteretion* part is what? Ok…

Student 2: We have one bar and divide in three.

Student 3: Why you divide in three?

Student 5: The pencil please. Do this, this is the milk. Señalar que esta es la leche que siempre compra.

Student 3: In English.

Student 4: The milk of Juan 2/3.

Student 5: This is the milk.

Student 4: Then we do the operation. Ok Luisa what operation? Tenemos que averiguar what value are these.

Student 5: What is the question?

Student: How much money did Juan receive before and after.

Student 5: Ahh ok.

Student 2: Pong attention.

The excerpt 7 reveals the use of codeswitching and the use of false cognates; student 5 and student 4 interventions illustrate codeswitching: "*then we do the operation. Ok Luisa what operation. Tenemos que averiguar what value are these."* and *"The pencil please. Do this, this is the milk. Señalar que esta es la leche que siempre compra*. While students 1 and 2 made use of false cognates: "*ok the anteretion part is what? Ok"* and "*Pong attention*"

This short interaction among learners reveal that L1, in the form of codeswitching and false cognates, has the role of supporting L2 when learners' vocabulary is not enough and there is a necessity to be heard. The two strategies allow learners to fill gaps in communication hence, be more precise when interacting. According to Turnbull and Dailey-O'Cain (2009), code switching and false cognates "aid comprehension and increase and improve students' target language production" (p. 34). Thus, language takes a different shape from an instrumental perspective. Solomon (2009) and Hemphill (2010) agree on social exchanges allows people to broaden understandings as well as experiences, we can take advantage of others' knowledge to comprehend collaboratively a mathematical situation. In the light of these ideas, language is perceived to be a mean to comprehend, learners are not reproducing language, and rather they are producing and using language (L1 and L2) genuinely in order to understand and to be understood.

The following excerpt demonstrates how in learner – learner and teacher – learner interactions translation is used for understanding.

Excerpt 8 from students and teacher conversation, August 30th, 2017.

Student 1: What is selling?

Student 1: (Asks again) what's worried?

Student 2: Selling es como precio.

Student 1: What is worried?

Student 3: Preguntémosle a la teacher.

Student 1: (She reads the question) If every kg of rice cost 720, how much money did Juan receive before and after the store opened? ¿Entendieron?

Student 4: Sí, ¡entendimos!

Student 1: ¿Qué entendieron?

Student 4: We understand that Juan… he has…

(Teacher comes to the group to check if there are questions)

Teacher: Did you understand the story?

Student 1: Yes! What is 'sells'?

Teacher: So, tell me, Sara has a question. Can you help Sara with that question? She doesn't know what is 'sells'. What is 'sells'? So, you didn't understand the story because she has this question.

Student 3: Vender.

Teacher: ¿Es vender? ¿Según la historia, es vender? Read the story.

Student 2: Sí teacher. Él vende 4/5 de huevos y 8/10 de arroz y 2/3 de leche.

Teacher: What do you have to find?

Student 3: The cost the… of the rice.

Teacher: How much money did Juan receive before and after?

Student 3: ¿Qué plata antes tenía y ahora qué tiene?

Teacher: Yes! But only with the rice.

Translation was one of the strategies that appeared when learners wanted to approach a text in L2. Learners used this strategy with two purposes: First, it allowed them to recall important information from written texts and second, when they translate in collaboration with the teacher, were able to display their understanding of the text. In the sample, Student 1 sought for the word equivalent of "*selling"* in Spanish. Although her peers provided the word equivalent in L1, they seemed to doubt the meaning of the word: "*Selling es como precio"*. In that moment the interaction with the teacher became crucial. She became the mediator between the learners and the text, as they were able to recall the meaning of the word and the role it played inside the story. Additionally, teacher-learner interaction was produced in L2 and in L1; and the use of both linguistic codes was done to ensure that the students understood the material.

It seems that by making a first reading in the L2 learners might overlook relevant details they need to solve a situation or to propose strategies. This circumstance provokes to choose another strategy: translating parts of the text. The words *sell and buy* in the story were of great importance for learners to understand it; consequently, ignoring the definition of these words could have provoked difficulties in the selection of a strategy to solve the problem. To this regard, Kern (1994) states that translation becomes merely functional. After a research study conducted with English speakers learning French and the role of mental translation, he concluded that translation brings different benefits and its use becomes strategic. Translation allows learners to develop the semantic understanding of words and it has a positive impact on memory; these benefits contribute to the L2 reading comprehension process. When learners make use of translation then they are able to interpret the information they gather; it allows readers to create

links between diverse symbols, in this case linguistic elements (Kern, 2000). As readers make connections, they are transforming their knowledge not only in L2 but also in mathematics. Thus, in order to make sense of the text, learners had to use a strategy that allowed them not only find equivalent words in their L1 but also interpret that information, modify their knowledge with the purpose of finding solutions to the task proposed.

Collaboration as social construction of knowledge. Vygotsky (1986) understood the link between thought and language as a relationship that is transformed when we use means to represent it. Hence, it is possible to understand writing under this idea as an act of thought representation. This project shows that when learners are asked to write in the math classroom, they are stimulated to interact and consequently that facilitates reaching consensus to materialize their knowledge and thinking processes in both, mathematics and the English language. The following excerpt shows the interaction a group of learners had while completing a writing task: write the steps you followed to find the area and the perimeter of the shape built.

Excerpt 9 from students' conversation when working together, September 28th, 2017.

Student 1: Addition of the area of two is 70 plus 35, yes?

Student 2: No, the answer of the area is… Ashly, ahsly (she asks her partner to write what she is saying)

Student 1: No, and the answer is 105

Student 3: Because

Student 2: Because is the total of the two shapes

Student 1: Because the… multiplication and the addition… o sea, porque la multiplicación y la suma nos dio

Student 2: Pues sumamos

Student 1: Porque la multiplicación y la suma nos dio 105. Si porque el perímetro…

Student 4: El resultado de la multiplicación

Student 3: Más el área que ya la teníamos

Student 1: And this answer of the area of all shape

Student 4: Yes, the answer is all the shape

Student 5: And the perimeter is 52 cm^2

Student 3: The area is 105 cm^2

Student 1: ¿El área no se dice con squares?

Student 3: Yes! Es el perímetro el que no se dice con cuadrados

Student 4: Excuse me, the perimeter is 52 cm

Student 3: And the area is 105 cm^2 Sí porque el área es cuando se cuentan los cuadrados. Para que nosotros consiguiéramos el perímetro sumamos: 28, 15, 7 and 2.

Excerpt 9 shows how the writing process is developed from a socio-constructivist perspective. it is noticeable that all learners were actively engaged in the construction of the text, they supported their ideas and complemented other partners' ideas based on the experience of having participated actively and achieved the first task of the lesson. The process of writing collaboratively required discussion among the students involved, at the beginning of the excerpt students 1 and 2 were recalling the processes they followed to find the total area of the shape they built. As they reached to a consensus, student 1 enunciated in English what she was going to write. At the end of the excerpt it is noticed that discussion served as a means to sharp the text; student 3 stated: *the area is 105 cm^2,* the response to the statement was a question made by student 1, *¿el área no se dice con squares?.* This exchange of ideas allowed students to review their text in order to make it more precise in the use of mathematical language. Additionally, it

served to evoke what they learned about the concept of area and its difference with the concept of perimeter.

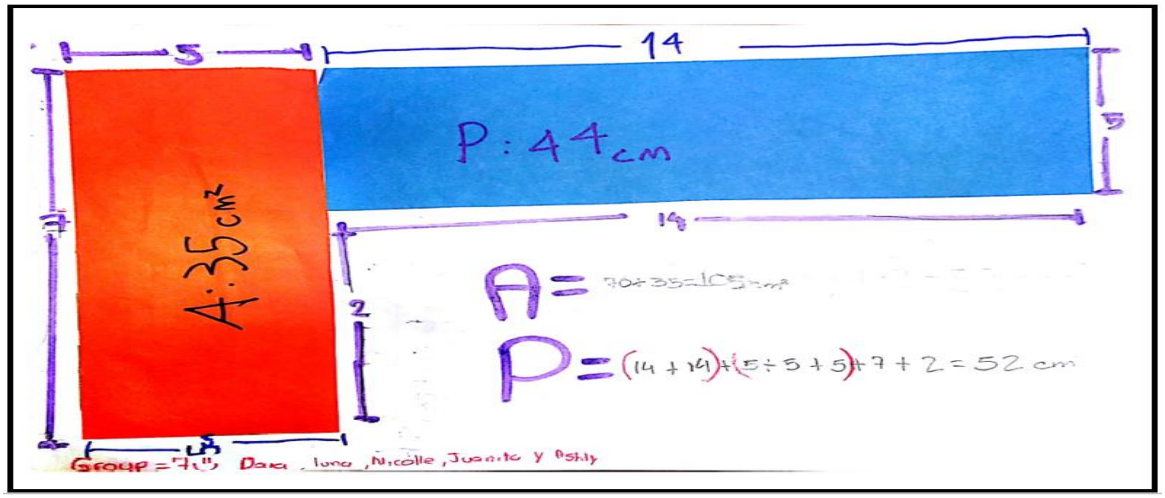

Figure 12. Students' artifact. Finding the area and perimeter of a compound figure. Lesson 5.

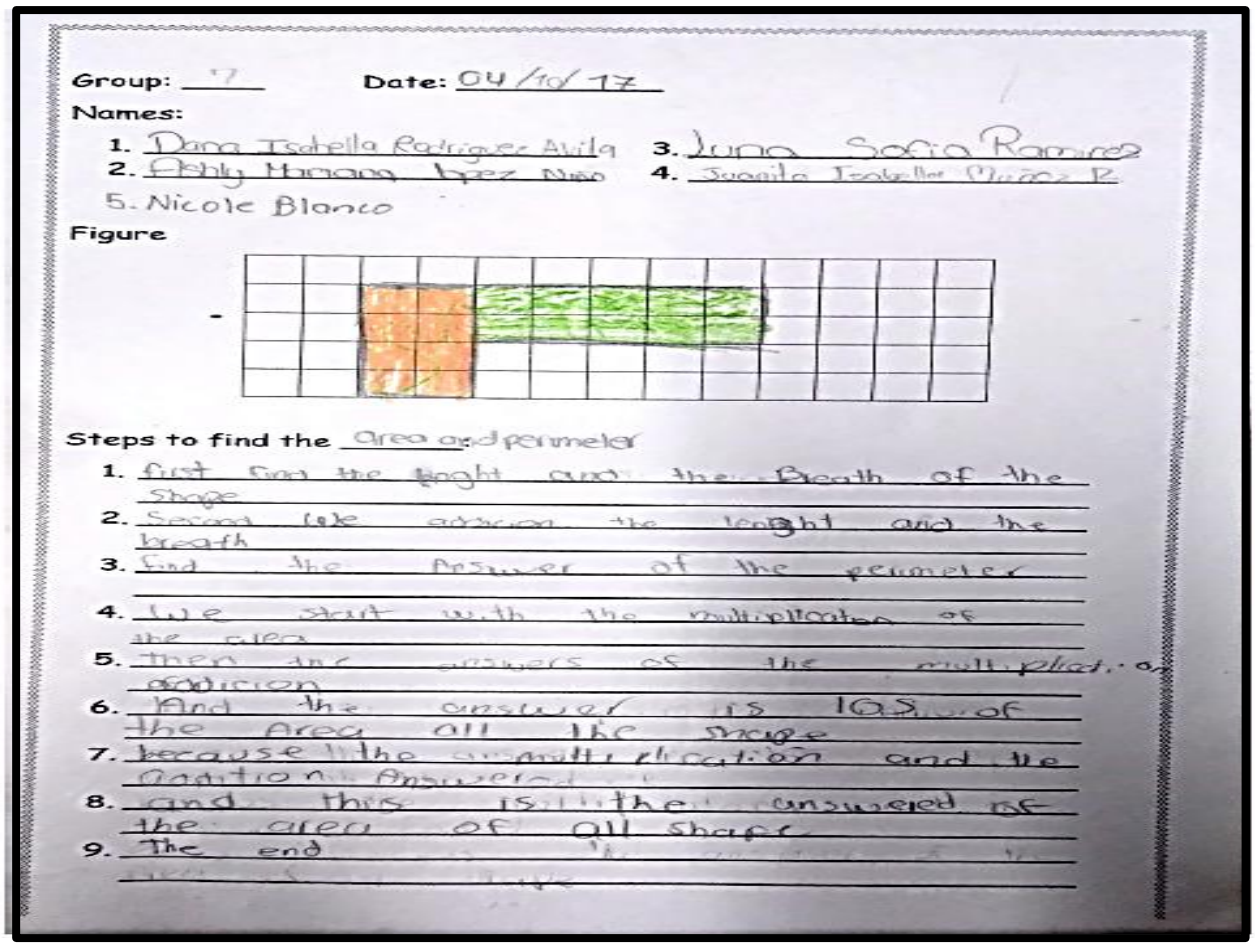

Figure 13. Students' artifact. Finding the area and perimeter of a compound figure. Lesson 5.

The text developed by students (figure 13) presents a summary of the processes they followed to find the area and perimeter of a figure. The text shows that the students followed a coherent process, in order to find the perimeter and area of a figure composed by rectangles, students had to find first its length and its breadth. The text also shows that students know the procedure to find the area of the two rectangles '*and the answer is 105 of the area all the shape because the multiplication and addition answer'*. One aspect that is not explicit in the text is how they found the perimeter. However it is explicit in the conversation they held (excerpt 9) and the addition shown in the figure 12.

The excerpt and the figures presented in this subcategory are greatly valuable for this project; they demonstrate the link of ELF and math literacy development. The process of how students constructed the text involved several aspects described by Kern (2000) and Short et al (1996): social exchanges, L1 and L2 knowledge, vocabulary, grammar structures, knowledge background, writing styles, among other elements. Moreover, students gain several benefits when the writing process takes part in a content-subject area. They become aware of their own learning processes by recalling the actions they performed to solve a problem; they can also create connections, identify difficulties or advantages when facing mathematical tasks (Borasi & Rose, 1989; Burns, 2005). Additionally, Pugalee (1999) states that "writing has also been shown to create an environment that supports the type of metacognitive thinking that, in turn, supports mathematical reasoning" (p. 21).

The teacher as mediator and facilitator of literacy processes. Another aspect that emerged from the data collected was the role the teacher played when learners were engaged in collaborative work. Teachers must act as mediators in order to support and assure understanding

between members of groups. The following two events show the interaction between teacher and the learners and the use of questioning as a strategy to guide the learning of mathematics.

The questions formulated by the teacher and the complexity of those questions were key factors during the lessons. The information requested by the teacher became an important strategy to guide learners in their actions upon the text. Excerpt 10 exemplifies these ideas:

Excerpt 10 from students and teacher conversation, August 31st, 2017.

Student 1: Profe yo no estoy entendiendo nada.

Student 2: Estamos multiplicando 3 que fue por cada grupo lo que lo dividimos y después cogimos 8 /10 y no entendimos si toca simplificarlo o solamente da 24.

Teacher: Explícame este dibujo.

Student 2: 30 kilogramos

Teacher: Explícame qué es esto.

Student 2: Los kg de arroz.

Student 3: Es lo que él compraba.

Teacher: ¿Lo que él compraba para qué?

Student 3: Para vender.

Teacher: Para la tienda, sí. ¿Cuánto vendía? ¿De esos 30 cuánto vendía?

Student 2: 8/10

Teacher: ¿Cada bolita qué representa?

Student 3: 1 kg

Teacher: ¿Entonces cuánto vendía de eso?

Student 3: ¿24?

Teacher: Pues muéstramelo, show me show me

Student 3: 24 kg

Teacher: Pero muéstramelos ¿en dónde está? Si ustedes planearon hacer un dibujo, muéstramelos ¿En dónde están? ¿De dónde sacaron esos 24? Ahí hay 24, ¿sí?, ¿Cómo me muestran eso?

Student 1: Porque de los 8 grupos que nosotros cogimos empezamos a sumar así: 1, 2, 3, 4; bueno así y entre esos grupos de a tres hay 24.

Teacher: Entonces, ¿qué es ese 24?

Student 3: The kilograms.

Student 2: Los kilograms que él vendía antes

Teacher: Muy bien. ¿Respondieron la pregunta?

Student 1: No.

Teacher: What is the question?

Student 2: ¿Cuántos vendía antes y después?

Teacher: ¿Cuánto?

Student 2: ¿Cuánto el vendía?

Student 3: ¿Cuántos kilogramos de arroz?

Student 2: ¿Cuánto ganaba antes y después de que la tienda abrió?

Student 3: ¿Cuánto dinero él recibía antes y después que la tienda abriera?

Teacher: This information, this is the kilograms that he sold, right? With this information then you have to find how much money he received.

Student 3: 24 times 720

Student 4: Tocaría multiplicar 24 por 720

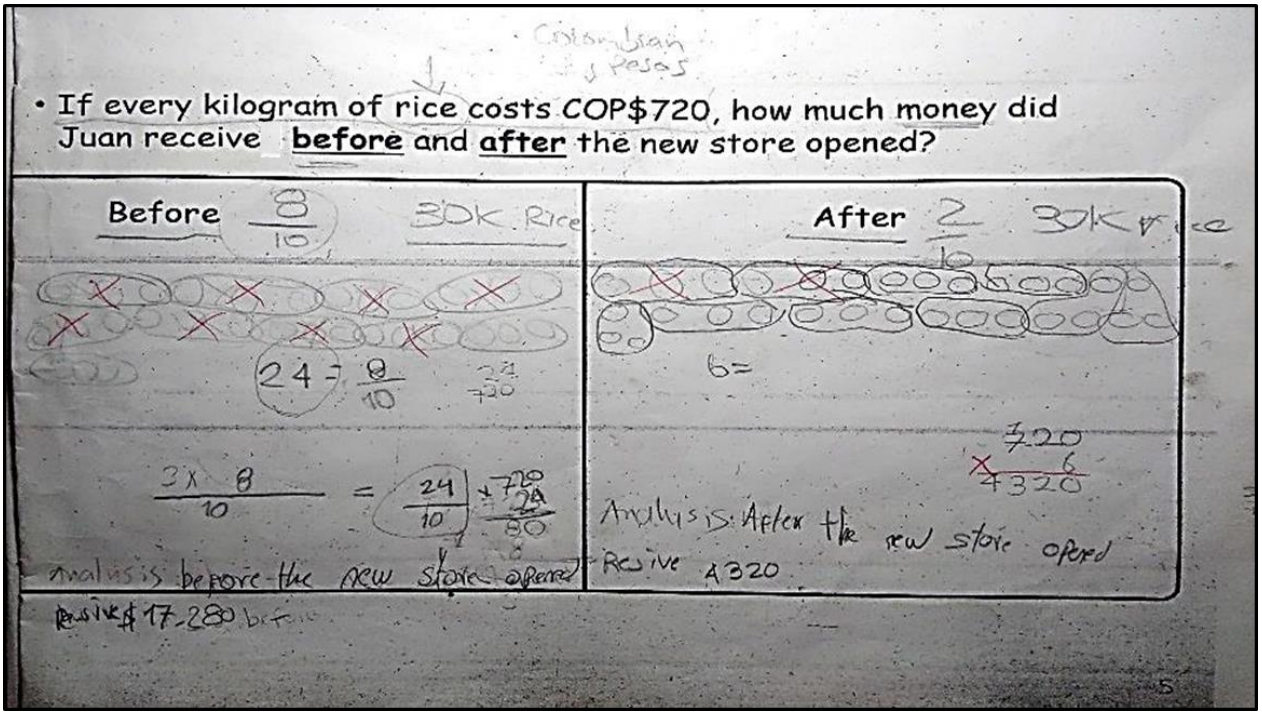

Figure 11. Students' artifact. Finding the fraction of a whole number based on a story. Lesson 2.

The excerpt 10 illustrates the way teacher's questioning required the learners to provide information about the manner in which they interpreted the story using drawings. At the beginning of the sample student 1 showed concern for her lack of understanding about the math problem. Student 2 tried to explain the process they followed and reiterated the lack of understanding. The teacher instead of providing the answer guided the students to reflect upon it by asking them multiple questions about what they were doing. At the end of the sample, it is noticed that students also began inquiring in order to make sense of the problem; then the teacher made a summary of their process and rephrased the question. Teacher intervention through questioning, led students to understand the operation they have to do in order to answer the question, as it is shown in figure 8.

This type of interaction benefits students by consolidating different learning and communicative skills. The conversation between teacher and students revealed that students were

able to support and demonstrate their actions and decisions, they became more reflective and aware of their own learning processes. In other words their metacognitive processes were improved, they were able to review their strategies and interpretation processes that helped them to make sense of the text. The quality of interactions and questions teachers posed determines the quality of students' engagement and learning processes (Banse et al., 2017).

The last excerpts serves as evidence of how the teacher, when interacting with students, assumes a role of facilitator and mediator; that position clearly fits Vygotsky's (1970) ideas about learning as a socially mediated process. Despite the fact, that students' learning processes are benefited when interacting with the teacher, data gathered revealed that in different occasions that intervention might cause a decrease of the possibilities for learners to propose strategies based on their own interpretations. The next excerpt exemplifies this situation.

Excerpt 12 from students and teacher conversation, October 6th, 2017.

Student 1: Nata, no es así, mira. Es 9.0 menos 6.4.

Student 2: Ahh, no Luna espera.

Student 1: ¿Profe cierto que es 9.0 menos 6.4? ¿Cierto Andrea que es así?

Teacher: What happened? Show me, show me.

Student 1: ¿Pero por qué 7?

Student 2: Es que estos dos se suman para poder hacer una resta.

Teacher: Ohh you can do that.

Student 1: Pero pues…

Teacher: Look she bought a packet of cookies and an eraser, so you have 9 ones. With this money you bought these two, she says that she can do the cookies and the eraser, and make the total, right? the cookies is 6.4 and the eraser is 0.08.

Student 1: Pero profe acá faltaría un número.

Teacher: These two things cost 6.48. The money that she has is this, and she spent this.

Student 2: Entonces se hace una resta, me equivoque.

Student 3: Lo restamos.

Student 1: Nata, 9.

(They did the subtraction)

Student 3: 2.56.

The reading strategy used by the teacher for students to make an interpretation of the text was retelling the story and then summing up main aspects to enlighten learners with the path to follow. After reading a mathematical problem student 1 proposed a solution based on her own interpretation; however, her proposal was not considered by her peers or the teacher. The teacher when listening to student 2 proposal supported it right away. The intervention of student 1 indicates that she was reluctant to carry out a different proposal; however, with the teacher's intervention she gave up, assuming that what the teacher said was the most valid strategy to follow. Although collaboration inside classrooms demands participation from all parties (Gilles & Vandover, 1988), the excerpt shows that the figure of the teacher biased some students to ignore a different proposal revealing that the role assumed by a teacher can either expand or diminish learning opportunities (Yeh, 2017)

To sum up, this chapter presented the main categories that responded to the question research project: How do math and ELF literacy processes unfold when using the Singapore method? The categories show that students developed their comprehension of math concepts and strategies to solve math problems when both cognitive and social factors were combined. The cognitive factors implied using their background knowledge about math and about the world to

understand the situations described in the problems and the use of different ways to represent knowledge. Those cognitive factors were also strengthened EFL literacy through students' interactions and conversations with the teacher. The dialogue between all these members helped to clarify doubts and to experiment with different alternatives. Making sense of math implies collaboration and interaction between students and teacher. Students and the teacher also used English to communicate with one another and used different strategies to help themselves make sense of the texts which was explicit with the excerpts and figures presented along this chapter.

Chapter VI

Conclusions and Discussion

The main purpose of this qualitative research was to determine students' EFL and mathematical literacy development when engaged in mathematical tasks. The project was shaped through the application of different lesson plans in which the Singapore Method for teaching math was used. Data was gathered through students' interactions, classroom observations and students' artifacts to answer the question: How are math and ELF literacy developed when using the mathematics Singapore method? This chapter presents the conclusions of the research, its pedagogical implications, limitations and directions for possible further research.

During the data analysis process of the research, two main categories emerged: EFL literacy and mathematical literacy are developed through learners' cognitive and the second was the relevance of social processes. Regarding learners' cognitive processes, data highlighted the relevance of three main aspects; namely, background knowledge, varied ways to represent knowledge and experimenting with different strategies to solve a problem.

Learners' background knowledge represented a possibility for students to recall specific experiences from their context that served to understand the math problems posed. During the sessions they were encouraged to talk about their own social experiences, and through them find a relation with mathematical concepts. Connections between mathematics and students' contexts allow them to make sense of what they are learning in mathematics (Ministry of Education Singapore [MES], 2012). Therefore, using students' background knowledge is indispensable to ground the new knowledge they encounter and

to help them make sense of the math problems. The use of contextual clues, as suggested by Chan (2015) is necessary to provide guidance to facilitate the understanding of math problems.

Another aspect linked to cognitive processes was students' knowledge representation and their strategies to solve mathematical problems. During the sessions, besides examining diverse situations that were close to their reality, students made use of different representations such as drawings, concrete material, mathematic operations, suppositions, walking through processes, and so on to make sense of those situations. These different ways to represent knowledge were modelled and practiced in class, as suggested by the MES (2012) and Solomon (2009). Short et al. (1996) and Hernandez (2016) state that classroom environments must provide students with the opportunity to experiment with different human expressions that can allow the construction of meaning of the world around them. This study also shows that students' varied knowledge representations are valuable because they are genuine expressions to make sense of the diversity of concepts they enter in contact with during the lessons. It is therefore important for teachers not only to take advantage of what students bring to classrooms, but also to foster opportunities for them to recreate their knowledge. "Mathematics draws on multiple semiotic (meaning-creating) systems to construct knowledge: symbols, oral language, written language, and visual representations such as graphs and diagrams" (Schleppegrell, 2007, p. 141) and it is the teacher's responsibility to ensure that students use those semiotic systems. This also implies that as teachers, we conceive the classroom as democratic environments in which students can have the opportunity to raise their voice, be heard, and demonstrate what they know and how they extrapolate learning strategies, knowledge or concepts to make sense of their own world.

Regarding the second category, or the centrality of learners' social processes when developing literacy, three aspects emerged Collaboration and interaction, the role of L1 and L2 and the teacher as a mediator. During the development of the lessons proposed, students worked groups. They, based on their experiences as learners, discussed several approaches to solve different mathematical situations. Through interaction, they negotiated, proposed, experimented, questioned others' ideas, reached consensus, and supported ideas. In other words, they developed their social skills in order to make sense of diverse texts.

This research project relies on the definition of EFL and mathematical literacy as social constructions; therefore, when students are given the opportunity of communicate in the mathematics class, they have the possibility to make logical and consistent arguments. This helps them to sharpen their thinking and therefore improve their mathematical processes to solve situations (Chan, 2015; MES, 2012). When students communicate and interact, they are developing the idea of literacy as a social construct (Hemphill, 2010; Kern, 2000; Short et al., 1996; Smagorinski, 2011;; Solomon 2009); the interaction and collaboration between students and between teacher –and students allowed me to conclude that the confrontation of different visions and strategies helped all parties in the development of social skills. This in turn, helped in the transformation, expansion and understanding of math concepts and in the development of L2.

It is also important to highlight the role of L1 and L2 in students' interactions to help them comprehend the problems and tackle them. They are closely linked to the above-mentioned aspects about interaction and collaboration. Students and teacher interacted by means of L1 and L2. The L1 revealed itself as a bridge for students to make texts content and messages understandable. It also helped students to be engaged in conversations. Through it, students were able to convey ideas and personal experiences; additionally, one

of the most interesting findings had to do with how L1 allowed students to expand their L2 usage. Students used code-switching and translation to gain linguistic knowledge of the L2, which is a necessary condition for confronting more complex math problems (Chan, 2015). Additionally, the L2 during the implementation of the lessons mostly appeared when teacher and students interacted. The teacher, by means of questioning students' actions, encouraged students to demonstrate and to support their decisions in L2. Therefore, the role of L2 went further than a simply repetition of L2 structures but a genuine and personal production of language that had as objective to understand and to be understood.

The third aspect from the second category is related to my role as facilitator and mediator. During my experience as math and English teacher, I have had the opportunity to rethink my language teaching practices and to reflect about my experience as math learner back in my schooling years. In this project, questioning students' actions and processes emerged as one driving force to understand how math and EFL literacy developed. This practice allowed me to act as facilitator and mediator; as I held dialogues with students, they were able to strengthen different cognitive and social skills, think about their thinking processes, and support their ideas and strategies. Although, there was a huge benefit from this approach, it is important to highlight that during various moments, my interventions while students discussed when working together, limited students' possibilities to reformulate their processes or strategies. Teaching practices should be focused on giving students opportunities to present their own thinking or understandings, share and listen to others ideas. By not having this possibility we are limiting not only students' cognitive processes but social processes as well.

Given the points that rose from the second category, I think that the implications for the educational field are of great importance. First, I strongly believe that classroom

environments have to be re-arranged. Silent classrooms do not guarantee students reaching learning objectives, and noisy classrooms do not mean lack of teacher classroom management, or students' lack of interest in learning. In order to construct knowledge, students require to communicate and interact, the cognitive and social skills that they develop are countless and most important, will probably help them cope with future math and language challenges. Therefore, classes and tasks must be arranged so that students have the opportunity to interact and collaborate. Cooperation is an important aspect to include in EFL classes in which math is a component. Many studies (as demonstrated by Chan, 2015) have shown that when students asks questions, provide feedback, translate, question each other, they are providing opportunities to re-accommodate their knowledge. Secondly, L1 and L2 teachers require constant reflection on their pedagogical practices. This is indispensable to help students develop skills that result in the expansion of knowledge and necessary literacies in different subject areas. Finally, teachers should become aware of the type of interactions and dialogues we have with our students; constant guidance and appropriate mediation are of enormous importance due to they help avoiding misconceptions and incomplete or disorganized knowledge (Kirschner, Sweller & Clarck, 2006).

To summarize, this study served as a window to understand that EFL literacy and mathematics literacy are developed simultaneously and as a combined action that involved different aspects. Students' social backgrounds and knowledge representations are valuable resources and genuine attempts to project their internal cognitive processes. L1, L2, interaction, collaboration and good and conscious pedagogical practices allow the transformation and expansion of knowledge, and consolidation of social skills, which in the long term will serve to the constitution of integral-twenty- first-century citizens.

Limitations and Further Research

During the development of this research I found research and pedagogical limitations. The pedagogical challenges I found had to do with time. Although, the lessons I designed were aligned to the math curricular program, some of them became extensive because students had some difficulties completing the tasks. For example, at the beginning of the implementation, the type of mathematics situations, I proposed required students to go through long processes that made them and me take more time than expected. I realized that my mediation was necessary for almost all groups to clarify concepts, strategies or to help students overcome disagreements or arguments. Thus, the sessions had to be resumed later. The fraction of lessons affected students' engagement and continuity of processes; additionally, I lagged behind with the implementation of the instructional design as well as the school's mathematics curriculum.

The research limitations had to do with the research questions, they required me to pay close attention to students' interactions; hence during the implementation of the lessons I asked students to record their conversations and in few occasions I realized that some recordings did not have the quality I expected, so I was not able to listen to all my students. For instance, this issue with recording gadgets made impossible for me to listen to particular students that could have provided me with detailed information about cognitive and social dimension of literacy development.

Another research difficulty was related to finding studies in which EFL literacy and mathematics literacy were related. Although, these types of studies were common separately, the data bases I had access to did not show results at Colombian contexts. This situation led me to reflect on language literacy and its relation with the development of different literacies of subject

areas in schools. Thus, the involvement of teachers in research emerges as an urgent commitment to understand classroom reality and the objectives students are attached to.

Further Research Directions

Research might have an interest on the ideas and conclusions that this study raise. Future research can encourage teachers from bilingual institutions and diverse disciplines to propose consistent and rigorous plans that result in the reflection of their beliefs about literacy practices and their relation to students' learning processes, concept acquisition and development that belong to each discipline.

Furthermore, I believe that research about literacy should be extended to how its development and pedagogical practices can contribute to understand different disciplines from a critical perspective and its integration to understand social realities that are related to students' everyday life.

Finally, I think that it would be interesting that we teachers take into account students' voices in order to reveal what are those literacy practices in L1 and L2 that allow them to make connections that make sense of their school life from a critical perspective.

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Annex 1

Cuestionario - Docentes

Las siguientes preguntas tienen como objetivo determinar las dificultades y fortalezas que los docentes del Colegio Nuestra señora del Rosario Bogotá encuentran en la clase de matemáticas. Las respuestas a las preguntas servirán de apoyo para el desarrollo del proyecto de investigación que la docente Paola Andrea Rodríguez viene adelantando en la maestría de lingüística aplicada a la enseñanza del inglés de la Universidad Distrital Francisco José de Caldas.

1. ¿Cuántos años de experiencia lleva enseñando matemáticas?

   _____

2. ¿A qué población ha enseñado matemáticas?

   _____

3. ¿Qué pasos se desarrollan generalmente en una clase suya de matemáticas? Descríbalos brevemente.

   _____
   _____
   _____
   _____

4. Si es el caso, responda brevemente: ¿qué dificultades y fortalezas ha encontrado como docente de matemáticas a lo largo de su experiencia?

   _____
   _____
   _____
   _____
   _____

5. Si es el caso, responda brevemente: ¿qué dificultades y fortalezas ha encontrado en cuanto a los procesos de aprendizaje de sus estudiantes a lo largo de su experiencia?

   _____
   _____
   _____

_____
_____

Annex 2

Cuestionario - Estudiantes

Las siguientes preguntas tienen como objetivo determinar las dificultades y fortalezas que las estudiantes del Colegio Nuestra señora del Rosario Bogotá encuentran en la clase de matemáticas. Las respuestas a las preguntas servirán de apoyo para el desarrollo del proyecto de investigación que la docente Paola Andrea Rodríguez viene adelantando en la maestría de lingüística aplicada a la enseñanza del inglés de la Universidad Distrital Francisco José de Caldas.

1. ¿En qué grado se encuentra actualmente?

   _____

2. ¿Cuáles son los pasos en lo que se desarrolla generalmente una clase de matemáticas? Descríbalos brevemente.

   _____
   _____
   _____
   _____
   _____

3. Si es el caso, responda brevemente: ¿qué dificultades y fortalezas que ha encontrado en los docentes de matemáticas a lo largo de su experiencia como estudiante?

   _____
   _____
   _____
   _____
   _____

4. Si es el caso, responda brevemente: ¿qué dificultades y fortalezas ha encontrado como estudiante de matemáticas?

   _____
   _____
   _____

_____
_____

Annex 3

CARTA DE AUTORIZACIÓN
PARTICIPACIÓN EN UNA TESIS DE MAESTRÍA EN LINGÜÍSTICA APLICADA A LA ENSEÑANZA DEL INGLÉS COMO LENGUA EXTRANJERA

YO,_____, identificad@ con cédula de ciudadanía número _____ de _____, en mi calidad de representante legal del estudiante _____ identificado con tarjeta de identidad No. _____. **Autorizo SI_____ NO____** al Colegio Nuestra Señora del Rosario Bogotá, el uso de la imagen de mi representado, mediante la reproducción o la comunicación pública de sus escritos e ideas frente a observaciones de clase, con la finalidad de ser incluidos dentro del reporte de los hallazgos de un proyecto de tesis para la maestría en Lingüística Aplicada a la Enseñanza del Inglés de la Universidad Distrital, estrictamente con fines educativos y para la enseñanza del idioma.

Por virtud de este documento, el suscrito declara que es legalmente titular de la patria potestad del menor, y en consecuencia garantiza que puede otorgar la presente autorización y cesión, sin limitación alguna, de conformidad con el Código de Infancia y Adolescencia vigente.

Cordialmente,

Nombre:_____ Firma:_____

C.C. No._____

Docente: _____
Paola Andrea Rodríguez G.

Jefe Departamento de Inglés _____
Eimy Arango

Coordinadora Académica: _____
XXXXXXXXXX

Annex 4

Formato consentimiento informado para Proyecto de investigación

Bogotá

Junio de 2017

Estimada Rectora

Cordial saludo,

Actualmente estoy cursando cuarto semestre de la maestría ofrecida por la universidad Distrital, Lingüística aplicada a la enseñanza del inglés como lengua extranjera. Para optar al título de Magíster estoy desarrollando una investigación donde los estudiantes son los participantes.

La investigación tiene como propósito determinar cómo se desarrollan los procesos de alfabetización en matemáticas y lengua inglesa mientras que los estudiantes trabajan en la solución de problemas matemáticos. Con este proyecto los estudiantes participarán en el desarrollo de varias sesiones de clase en donde trabajarán en grupo y tendrán la oportunidad de desarrollar habilidades tanto sociales como cognitivas.

Los datos para dicha investigación serán recolectados durante el segundo semestre del presente año. De acuerdo al propósito de la investigación, los estudiantes serán grabados en video y en audio mientras trabajan en solución de talleres, realización de afiches, lectura de historias, recorridos por estaciones de aprendizaje, escritura de textos, entre otros.

La información recolectada será usada con absoluta confidencialidad.

El nombre del colegio será dicho en caso de ser aceptado por usted y con el adecuado uso de tal, dejando en alto el nombre la institución.

Agradezco su participación en este proyecto. Para hacer formal su aceptación debe firmar la presente carta y devolverla lo más pronto posible. Si tiene alguna duda al respecto puede hablar directamente conmigo.

Cordialmente, Paola Andrea Rodríguez González Docente –investigadora
Nombre: _____

Firma: _____
Permito el uso del nombre de la institución: SI_____ No_____

Annex 5

Leeson Plan 3

Teacher: • Andrea Rodríguez
Date: • September 15th
Topic: • Perimeter

Objective (Math): • To infer the concept of perimeter
• To use different measuring tools in order to find the perimeter of objects and images.

Objective (Language): • To construct the definition of perimeter by using mathematics language

Time: 1 hour (60 min)

Class Stages	Class Development	Materials
1. modeling	Teacher displays two objects to the students: a school-uniform jacket and a painting. She asks students how one can measure all of those objects sides. Students provide different measurement tools (as they mention them teacher shows a ruler, a measuring tape, a row made up with paper clips and a wool string). Teacher projects a chart on the board for students to complete with the data from measuring the two objects (Measurement of the jacket's sides in clips and in centimeters and measurement of the box' sides in centimeters). Next, she asks two students to use the row of clips to measure the jacket and other two students to use	Object to be measured: jacket and painting. Measuring tools: rulers, measuring tape, paper clips and measuring tape. Power point slide: Chart with objects' measuring data

	the string of wool to measure the painting. All students discuss the results. Teacher says that what they just did was finding the perimeter of an object.	
2. Group Formation and measuring objects using different measuring tools.	Students are divided in groups of four students. Teacher gives them the instructions for the class: 1. to measure the sides of an image using strings; 2. to create two figures using wool (glue and paste); 3. to name and write the perimeter of each figure in centimeters; 4. Finally, to discuss and write the definition of perimeter. Each group chooses a presenter to display their work to the class. Next, teacher and students discuss the definition of perimeter so all students can come to an agreement.	Big piece of paper Markers Wool strings
3.	In groups, students solve exercises from the book. Teacher projects the exercises on the board so the results can be discussed and students' doubts and opinions can be heard.	Text book

Annex 6

Mathematical Story (material) Lesson 2

1.

• Juan has a grocery store. Every month he goes to the market and buys eighty eggs, thirty kilograms of rice and forty-five liters of milk.

2.

• His costumers love going to Juan's store. They say he is very kind and the, prices are fair. Every month, he sells $\frac{4}{5}$ of eggs, $\frac{8}{10}$ of rice and $\frac{2}{3}$ of milk.

3.

• One day, a bigger store opened next to Juan's store. Quickly, people began going to the other store because the prices were lower.

4.

• Juan was very worried, his sellings every month were:

Eggs $\frac{1}{5}$

Rice $\frac{2}{10}$

Milk $\frac{1}{3}$

5.

• If every egg costs COP$150, how much money did Juan receive **before** and **after** the new store opened?

Before	After