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THE INTERACTION BETWEEN ROUTE GUIDANCE AND SIGNAL CONTROL: MODEL RESULTS

Tom van Vuren

FUNDAMENTAL REQUIREMENTS OF FULL-SCALE DYNAMIC ROUTE GUIDANCE SYSTEMS

The Interaction Between Route Guidance and Signal Control: Model Results

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Institute for Transport Studies, University of Leeds, Leeds, LS2 9JT. England. Tel: Leeds 335325

Transportation Research Group, University of Southampton, Southampton, SO9 5NH, England. Tel: Southampton 592192.

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ABSTRACT

A multiple user class assignment model is developed to simulate the interaction between a route guidance system and an urban traffic control system. For two real-life networks of Leeds and Southampton four basic scenarios are investigated:

- current situation;
- expected situation with route guidance but without interaction with the traffic control system;
- expected situation with route guidance and with interaction with the traffic control system;
- optimum future situation with 100% take-up and compliance with system optimal guidance and delay minimising signal control.

The computed extra travel time savings of an integration of route guidance and signal control are of a similar magnitude as those of a route guidance system alone. With static demand delay minimisation performs best, but when demand is dynamically increasing Smith's P_0 policy gives rise to lowest network travel times. The degrading effect of imperfect advice or non-compliance is small at realistic levels of take-up of up to 25%. Along the same line, the benefits of system optimal guidance are also modest (in the order of less than 1%) when take-up of guidance is less than 25%.

0. Introduction

In this report a multiple user class assignment model will be developed to simulate an operational route guidance system; my main interest will be in the interaction between such a route guidance system and a signal control system. I will test a number of scenarios for such an integrated traffic control system, related to, for example, the level of take-up of guidance, the level of congestion, the guidance criterion and the level of interaction between the guidance and signal control system. In Chapter 1 I will describe the model development in more detail, in Chapter 2 I will discuss the scenarios assumed, and in Chapters 3 and 4 I will present model results for two networks: Weetwood (Leeds) and Southampton. Note that these are SATURN buffer networks, which are conventional, link-based; no junction simulation takes place.

1 Model development

In Van Vuren (1990d) I established a family of polynomial link cost functions that guarantees a unique equilibrium solution to the MUC assignment problem with UE and SO drivers. Such link cost functions, however, are inappropriate for the representation of traffic signals, with start-stop behaviour and queueing (see Van Vuren, 1990b). Rather than imposing such strict conditions on the cost functions, I have chosen here to apply the most realistic cost assumption available: the sheared delay curve. This function does not satisfy condition (14) in Van Vuren (1990d) and therefore uniqueness of the equilibrium is uncertain for combinations of SO and UE drivers.

A MUC assignment of UE and SUE drivers **would** have a unique equilibrium solution if for each link a the class-dependent link error terms ε_{ai} were mutually independent between classes, and independent of t_{ai} (Daganzo, 1983). Sheffi (1985) suggests therefore to base the error term in the perceived link cost functions on the free flow travel cost $t_{a i}$; I disagree with that, as in my view perception errors and variability in travel time clearly increase with congestion. Sheffi's suggestion would counter-intuitively result in large perception errors for, say, uncongested freeways that have relatively large free-flow travel costs, and relatively small perception errors for city streets.

As a remedy Van Vuren & Watling (1990) relate link error terms to t_a^{UE}, which is the resulting link cost from a UE assignment **of all drivers**. This, however, requires a full separate user equilibrium assignment to be run for most scenario options. In the test runs described here I have chosen to relate perception errors to current link costs t_a; this also means that for combinations of UE and SUE drivers the resulting MUC equilibrium is not necessarily unique, as conditions for uniqueness are not satisfied.

Another important consideration concerns the actual form of the probability distribution used for drivers' perception errors. When the stochastic loading procedure as employed in SUE assignment was introduced by Burrell (1968) he suggested the use of a uniform distribution with the observed link cost as the mean. On theoretical grounds a Normal distribution may be more appropriate, but this would be computationally much more cumbersome (Burrell, 1976). Daganzo & Sheffi (1977) even suggest a Gamma distribution, or any non-negative distribution with a long tail to the right. Because of computational efficiency I employ Burrell's original uniform distribution with mean t_a and class-specific spread $\Theta_i t_a$, so that perceived link costs will be sampled from the distribution $[t_a - \Theta_i t_a , t_a + \Theta_i t_a]$.

The model consists of four classes of drivers, following

- system optimal routes,
- user equilibrium routes,
- stochastic user equilibrium routes (with perception errors ε_1),
- stochastic user equilibrium routes (with larger perception errors ε_2).

The perception errors ε_1 and ε_2 are sampled from two uniform distributions $[-\Theta_1 t_a , \Theta_1 t_a]$ and $[-\Theta_2 t_a , \Theta_2 t_a]$ respectively, where $\Theta_2 > \Theta_1$.
Combinations of these four classes can represent a route guidance system with different routing assumptions for the guided and unguided drivers, as discussed earlier in Van Vuren (1990d). What is of particular importance here is the calculation of the value of the spread parameter Θ for the two SUE classes, as with these model assumptions this value directly determines the potential benefits of a route guidance system.

For the determination of an appropriate value for Θ for the unguided drivers an idea due to Van Vliet (1976) and, later, Breheret et al. (1990) is used. For a number of values of Θ the average network inefficiency $I(\Theta)$ is calculated, given by

$$I(\Theta) = 100\,(TTT(\Theta) - 1)\% \qquad (1)$$

where

$$TTT(\Theta) = \frac{\text{total travel time under SUE } (\Theta)}{\text{total travel time under UE}} \qquad (2)$$

and where SUE (Θ) means a SUE assignment with spread parameter Θ. $I(\Theta)$, or the network inefficiency, is a measure of the average excess travel time due to drivers' perception errors. Jeffery (1987) found by analyzing times and distances from a sample of journeys made in the UK that the average inefficiency of drivers was about 6%. Therefore, I have chosen a parameter value that gives rise - under current conditions - to approximately a 6% inefficiency.

Figure 1 shows the calculated inefficiency for Weetwood, observed demand, and various values for Θ; a value of the spread parameter of 0.6 looks appropriate. Note that this means sampling perceived link costs from the distribution $[0.4\ t_a,\ 1.6\ t_a]$. I personally think it is very unlikely that drivers will make such large perception errors, but we have to remember that the SUE assignment model is not a behavioural model, but a rather crude tool to introduce route choice errors and consequent network inefficiencies.
For the SUE representation of drivers following imperfect guidance the above calculations cannot be set up, as this class of drivers does not exist as yet; I have chosen an arbitrary value $\Theta = 0.3$ for this class. $I(\Theta)$ is also congestion-related, as found by Van Vuren and Watling (1990). As the development of the magnitude of perception errors with increasing congestion is unclear, I have chosen to apply the same Θ value with all congestion levels.

Each of the classes as a fixed part of the OD-matrix represents either guided or unguided drivers; in Chapter 2 on scenarios the combinations tested will be discussed.
The assignment algorithm employed is the method of successive averages (as introduced by Sheffi and Powell, 1982). Daganzo (1983) proved that this algorithm almost surely converges for the MUC assignment problem (under the above-mentioned conditions on the link error terms ε_{ai}). The actual form in a MUC environment is as follows:

1. Initialisation: Assign all classes according to an A-o-N assignment, yielding $f_{ai}^{(1)}$, $F_a^{(1)}$ and $t_{ai}^{(1)}$; iteration counter $n = 1$;
2. Re-calculate all class-specific cost functions, based on current flows; $n + n+1$.
3. For all classes $i = 1,m$ in turn:
   - 3.1 calculate A-o-N routes for this class, leaving all other class flows unchanged, and find auxiliary class link flows $h_{ai}^{(n)}$
   - 3.2 update the flows for this user class according to $f_{ai}^{(n+1)} = (1\text{-}1/n)\, f_{ai}^{(n)} + 1/n\, h_{ai}^{(n)}$
   - 3.3 update total link flows: $F_a = F_a + f_{ai}^{(n+1)} - f_{ai}^{(n)}$
4. Unless convergence, go to 2.

To calculate SO routes marginal link costs $\tilde{t}_a = t_a + f_a\, \partial t_a/\partial f_a$ must be determined; their derivation for the case of sheared delays is presented in Appendix 1. Secondly, if only a single user class is assigned with deterministic costs (SO or UE), the more efficient Frank-Wolfe algorithm may be employed. Then, for checking the convergence of the UE assignment the objective function

$$Z = \sum_a \int_0^{f_a} t_a(x)\, dx \tag{3}$$

must be evaluated; this is described for the sheared delay function in Appendix 2.

Note that the model as developed assigns all user class flows in such a way that flows of one class affect the costs, and hence the route choice, of all other user classes. Therefore, unguided drivers are allowed to re-route to routes with minimum perceived costs as a result of changed routing by guided drivers.
In this form the model is most appropriate for the long-term simulation of route guidance under recurring congestion, and therefore well suited for the investigation of an interaction with signal control as is the case here. I am well aware that route guidance systems may offer the greatest scope for travel time savings in situations of non-recurring congestion (incidents); different assumptions for the behaviour of, particularly, the unguided drivers are then needed. This, however, is outside the scope of this research project, and incidents are not considered.

Also, the model assumes that several routes can be advised to guided drivers on identical OD-relations. Current systems, like AUTOGUIDE and ALI-SCOUT, are based on single route strategies, and may be expected to be only effective when the percentage of equipped vehicles is small. A discussion of the possible feedback problems in such a situation when the level of take up increases is given in Van Vuren (1990a); Breheret et al. (1990) present numerical results that indicate the rapid deterioration of a single-route guidance system under conditions of increased take-up. Multiple route strategies, such as the equilibrium-based strategies discussed here, should be able to cope much better.

Finally, four control policies are incorporated in the model: Webster's policy, delay minimisation P_0 and the power policy, as introduced in Van Vuren (1990c). As the sheared delay curve is employed the power policy needs to be adapted for over-capacity conditions, as described in Appendix 3. Based on findings in Van Vuren (1990c), where the iterative assignment control procedure was employed with a single user class, the streamlined algorithm is implemented, so that signal settings are adjusted after each assignment iteration, instead of converging the assignment process before updating the signals. Now, an assignment iteration consists of the determination of a single new route per OD pair for each of the user classes, plus a move of length 1/n in that direction. The complete algorithm looks as follows:

1. Initialisation: Assign all classes according to an A-o-N assignment, yielding $f_{ai}^{(1)}$ and $t_{ai}^{(1)}$; iteration counter n = 1;
2. Re-calculate all class-specific cost functions, based on current flows and green splits. n = n+1.
3. For all classes i = 1,m in turn:
   - 3.1 calculate A-o-N routes for this class, leaving all other class flows unchanged, and find auxiliary class link flows $h_{ai}^{(n)}$
   - 3.2 update the flows for this user class according to $f_{ai}^{(n+1)} = (1-1/n)\, f_{ai}^{(n)} + 1/n\; h_{ai}^{(n)}$
   - 3.3 update total link flows: $F_a = F_a + f_{ai}^{(n+1)} - f_{ai}^{(n)}$
4. Optimise green splits for current flow pattern so as to equalise stage pressures. Unless convergence, goto 2.

2 Scenarios

Nobody can as yet know what exact form an operational route guidance system will take. Considerable research effort has been initiated into expectations and needs that people have with respect to route guidance (Bonsall & Parry, 1990), but it is still unclear how they will react to guidance, and if they will actually buy the equipment. Further, travel time forecasting algorithms and routing algorithms are still being developed, and the performance of the hardware can only be tested in real-life.

In circumstances such as these, we will need to model a number of scenarios that represent possible forms that the final system might take. Not only will that help us determine the influence of various undecided factors on ultimate system performance, it will also help identify those subject areas that deserve extra research most.
Van Vuren & Hounsell (1990) distinguish 11 scenario parameters for route guidance systems; see Appendix 4. Of these, network size is descriptive and of no concern. Traffic incidents are not considered, as the main interest in this research project is equilibrium related; therefore, only multiple route guidance systems will be considered. The signal control system is assumed to be traffic responsive on isolated intersections: the model cannot take account of the influence of offsets and linked signals. Of the remaining parameters beacon density, beacon update time, the quality of the forecasting algorithms and the driver response can be incorporated in a link cost error term ε_{ai} and a SUE guidance objective for the system. Finally, the effect of exclusion of certain links from the guidance network is not considered here either.

The following parameters remain, and the different values have been tested in the scenarios; Table 1.

Table 1: Scenario parameters and values investigated

Routing objective guided drivers Routing objective unguided drivers	SO + UE	SO + SUE	UE + SUE	SUE + SUE
Interaction signal control-route guidance	none / full			
Congestion level	as observed / increasing			
Level of take up	0% (current situation) 10% (expected situation) 25% 50% 75% 100% (optimum situation)			

Four basic scenarios have been investigated, as indicated in Table 2:

- current situation (A),
- expected situation under guidance without interaction (B),
- expected situation under guidance with interaction (C),
- optimum situation (D).

Table 2: Basic scenarios

	A	B	C	D
Routing (guided)	-	UE	UE	SO
Objectives (unguided)	SUE	SUE	SUE	-
Interaction	none	none	full	full
Congestion	as observed	as observed	as observed	as observed
Take-up	-	10%	10%	100%

Scenario A consists of a full SUE assignment, with signals calculated via the iterative assignment control procedure and 4 policies. Note that this may be too optimistic an assumption for the current flow/green time pattern, as at the moment green splits are not determined with the interaction with drivers' route choice in mind. A basic scenario as this, however, removes the dependence of travel times in the modelled current situation on initial green times; I expect, though, that this approach underestimates the potential benefits of an integrated system of route guidance and signal control.

In scenario B 10% of all drivers (the guided ones) are re-routed via UE optimum routes, with unguided drivers also reacting, but via a SUE assignment; signals, however, are not re-calculated and stay as in scenario A.
In scenario C the interaction between a route guidance system and the traffic control system is represented by an iterative re-calculation of the green times (according to the four policies) and of the flow patterns for both guided and unguided drivers. During this iterative assignment control process guided drivers are assumed to be advised of UE routes, whilst unguided drivers are assumed to follow SUE routes consistent with the calculated green times in each step.
The objective function for the system optimum situation D is:

$$\text{Min } \check{Z} = \text{Min}_{f,\lambda} \ \Sigma_a f_a t_a \qquad (4)$$

The iterative assignment control procedure with system optimal assignment and delay minimising signal control is guaranteed to find an optimum for this objective function. This can be checked as follows:

The system optimal assignment objective:

$$\text{Min}_f \ \Sigma_a f_a t_a \qquad (5)$$

determines a flow pattern for fixed green splits λ, which gives rise to minimum network costs for the current green splits.

The delay minimising signal control objective:

$$\text{Min}_\lambda \ \Sigma_a f_a d_a \qquad (6)$$

determines green splits that **locally** minimise junction delays at signalised intersections for fixed flow. As the assignment model used has no interactions between junctions, this locally optimal green time pattern is also globally optimal: the delay minimising sub-step determines green splits which give rise to minimum total network costs $\Sigma_a f_a t_a$ for the current flow pattern.

As each of the two sub-steps minimises the overall objective function with respect to the dependent variable, the iterative assignment control procedure is bound to find an overall optimum. This optimum, however, may be a local optimum.

Global optimality depends on convexity properties of the objective function. Convexity can be checked in the same way as was done in Smith and Van Vuren (1990): now the cost function used in the assignment is the marginal cost function. In Appendix 5 it is shown that objective function (4) is convex with the BPR cost function, but non-convex with Webster's cost assumptions. Because of its complexity the sheared delay curve has not been tested in this context; it is conceivable, therefore, that the optimum situations calculated throughout this report are local minima and thus sub-optimal.

Apart from these scenarios a number of other combinations of parameter values have been tested, to investigate their influence on the system performance. Such influences include:

- the influence of routing assumptions for both guided and unguided drivers,
- benefits of an interaction with signal control,
- the influence of increasing congestion,
- the influence of the level of take-up,
- system deterioration because of imperfect advice.

3 Results for the Weetwood network

The Weetwood network consists of 70 zones, 105 nodes and 442 directional links. Of the nodes, 17 are signal controlled with 42 stages in total. The network is depicted in Figure 10 in Van Vuren (1990c); the modelled situation is the AM Peak with strong North-South flows.

3.1 The four basic scenarios

Here I will discuss the four basic scenarios introduced in Chapter 2, specifically to determine

- expected benefits of a route guidance system;
- the relative advantage of a link between the guidance system and traffic control;
- maximum attainable benefits in the future.

Figure 2 contains the relevant information. It can be seen that a route guidance system on its own (with 10% take-up, and given all other assumptions mentioned before) will reduce total network costs for Weetwood by only about 0.5%, equivalent to 12-13 PCUH per day in the AM peak hour, independent of the control policy used. Interaction with a signal control system increases advantages only slightly. Delay minimisation, Webster's policy and the power policy all perform very alike, but P_0's resulting travel times are some 1.5% higher. More interesting is the extra benefit of 170 PCUH that can be attained by a fully implemented (and obeyed) situation with system optimal guidance and full interaction with delay minimising signal control, which is an extra 7% to the benefits found for scenario C.

3.2 Increased Congestion

The original situation in scenario A gives rise to network speeds of about 36 km/h; here I want to investigate the effect of an increase in network congestion on possible benefits of a route guidance system, and on the advantages of an interaction with signal control. To achieve this the observed OD-matrix has been multiplied by 1.5, resulting in network speeds of about 19 km/h without guidance. Figure 3 shows network travel times for the same four basic scenarios with such increased congestion.

The expected benefits of an isolated route guidance system have increased to about 1%, which is as expected, though still very small. Also the relative advantage of an interaction with signal control has increased; extra benefits over an isolated route guidance system are none with the power policy, 0.4% with delay minimisation, 0.8% with P_0 and 1.1% with Webster's policy. Note how the comparative performance of each of the policies has changed. Delay minimisation still performs best but the performance of P_0 has considerably improved, and the performances of Webster's policy and the power policy, in particular, have deteriorated. Finally, a fully implemented system as in scenario D with perfect compliance to system optimum advice and full interaction with delay minimising signal control offers travel time reductions of 13-23% compared with the do-nothing situation.

3.3 Influence of the control policy used

Assessing potential benefits of route guidance systems is not the main aim of this research. I am well aware that the developed MUC model is quite restrictive, and that some of the assumptions, particularly related to equilibrium behaviour, can be easily challenged. It is the interaction with signal control that I am most interested in, and as stated before, an equilibrium-based model is very appropriate there.

In a way, therefore, this subsection on the comparative performance of the four policies in a route guidance context is the most important of all described test results. All discussions will be based on Figures 4 to 11, which show for each of the routing assumptions and both congestion levels the performance of the four policies in the iterative assignment control procedure, expressed in total network travel time.

In all graphs delay minimisation performs best, giving rise to lowest travel times. The difference with other policies may be quite small (less than 1%), certainly in the 1.0M case, but with higher congestion differences of well over 10% may occur. In addition, delay minimisation's behaviour is most plausible and more stable with respect to take-up levels than the other policies, irrespective of the routing strategy applied.

Webster's policy performs well when congestion is low, but is generally outperformed by P_0 when congestion increases. Its behaviour is plausible in most cases, apart from the 1.5M situation with guided vehicles following system optimal routes and guided one pursuing a user equilibrium (Figure 11). The first part of the curve shows an increase in total travel time with increased acceptance of guidance. This instability is certainly undesirable, and rather implausible, though not necessarily wrong.

The same instability is experienced by P_0; this policy further follows the pattern of a rather poor performance when congestion is low and an improved performance when congestion increases, as observed in previous work, irrespective of the routing assumptions made for guided and unguided drivers.

Finally, the power policy performs well at low congestion, though never as well as Delmin. In the 1.5M case, however, large instabilities with respect to the level of take-up appear in total travel times. Under those conditions its comparative performance is always worst, and often implausible.

Summarising, in these tests delay minimisation tends to interact best with route guidance strategies. Resulting total travel times are plausible, in that they are decreasing with a rise in take-up of guidance. The three other policies give rise to higher total travel times and, for some guidance strategies, instabilities, so that travel times increase with increased take-up of guidance. The power policy, in particular, suffers from this at the higher congestion levels. In the rest of this Chapter I will therefore concentrate on the performance of delay minimisation in interaction with a route guidance system.

3.4 The effect of imperfect advice

The basic scenarios were all based on assumed routing errors by unguided drivers, which would be completely removed by the route guidance system. It is to be expected, however, that the advice given will contain errors, because of time lags and forecasting errors; also, drivers may lose their way when following advice, giving rise to inefficiencies; finally, a driver information system (as opposed to a route guidance system) may not advise routes, but display information about congestion and incidents, based on which drivers must decide their own routes. All these factors can be represented by stochastic user equilibrium routing for the guided drivers, with a smaller error than that for the unguided drivers.

Here I have assumed a spread parameter $\Theta = 0.6$ for the unguided drivers and $\Theta = 0.3$ for the guided drivers - see also Chapter 1. In Table 3 the increase in total network travel times as a result of such imperfect advice is shown for the situation with full interaction with delay minimising signal control. I compare the base situation with unguided drivers following SUE (0.6) routes and guided drivers following UE routes with the case of imperfect information, in which the route choice of unguided drivers is still according to a SUE (0.6), but in which the guided drivers follow a SUE (0.3).

Table 3: Percentage increase in total network travel time as a result of imperfect advice; SUE (0.3) - route advice assumed. Delay minimising control.

% guided	1.0M	1.5M
0	-	-
10	+ 0.08%	+ 0.34%
25	+ 0.17%	+ 0.68%
50	+ 0.25%	+ 1.44%
75	+ 0.67%	+ 2.05%
100	+ 0.63%	+ 2.33%

The first observation in Table 3 is that a larger increase in travel time is suffered with an increased take-up of guidance. This was expected, as in such circumstances more drivers actually suffer from the erroneous information. Secondly, the increase in travel times is considerably higher in congested conditions (the 1.5M situation). This confirms that good advice is necessary in congested conditions, and that the expected benefits of route guidance in such conditions will be greatest. Remember in this context the observation in Van Vuren and Watling (1990) that errors in the route guidance system may be expected to reduce with an increase in take-up.

3.5 The advantage of system optimal guidance

One of the advantages of route guidance systems that I have always been most interested in is the possibility to influence drivers' route choice to the system's advantage. The system performance will be borne in mind when a (political) decision is taken to leave certain environmentally sensitive roads out of the guidance network. Also, it is generally hoped and believed that the network system would implicitly benefit from a user-oriented route guidance system because of improved routing decisions by equipped drivers, which would benefit unguided drivers too; and indeed, various of the model studies described in Van Vuren and Watling (1990) calculated system-wide travel time reductions of up to 21%, dependent on routing assumptions for, particularly, unguided drivers.

What I am interested in here is an explicit routing objective for the guidance system, which will minimise network travel times. Such system optimal guidance will advise routes that are not always in the individual's interest, and acceptance of the advice is an issue of importance here. For the moment I concentrate on the possible travel time savings of SO guidance over UE guidance, without reference to such compliance problems. Table 4 shows macroscopic network travel time savings from SO route guidance. We can make two observations.

Table 4: Percentage benefits in total network travel time as a result of system optimum advice; delay minimising control.

% guided	1.0M	1.5M
0	-	-
10	0.38%	0.20%
25	1.21%	0.51%
50	2.33%	1.37%
75	2.75%	2.20%
100	3.38%	2.72%

Firstly, travel time savings by SO guidance increase as the level of take-up increases. This was to be expected, as in those circumstances more and more drivers follow SO routes, thus reducing total network travel times more and more (though not necessarily their personal travel times!). Whereas the benefits are negligible at low proportions of take-up of under 25%, with higher take-up levels the difference between SO and UE travel times can be more substantial. It is unlikely that levels of take-up of well over 25% will be achieved in the short term, and thus system optimal guidance may be a non-starter.
The second observation is that the benefits of SO guidance tend to reduce with increased congestion. This is quite counter-intuitive, and may be network-related.

An important element here is the travel time advantage that can be obtained by each of the classes, guided or unguided. Figure 12 shows class-dependent benefits for both UE-based guidance systems and SO-based guidance systems for the integrated system with delay minimising signal control, and two congestion levels. Here three observations can be made. In the first place, drivers guided via system optimal routes are only worse off (in comparison with the do-nothing situation) in the situation with 10% guidance and observed demand. Apart from that **all** classes of drivers always win as a result of the SO guidance system.
A second observation is that for higher levels of take-up SO guided drivers are better off than the unguided drivers: apparently routes with minimum marginal costs are preferable to routes with perception errors. Clearly the assumed magnitude

of the perception errors is critical here. In fact the class of system optimum drivers benefits even more than their counterparts under UE guidance assumptions when the level of take-up is high (over 75%), although drivers on certain OD-relations may still lose out.

Finally, drivers following UE guidance obtain their largest decrease in travel time at the very low levels of take-up; this had been observed by other researchers, too. In the 1.0M case the individual benefits remain at that level, and the benefits to the unguided drivers are very small indeed. Under higher congestion (1.5M) the individual benefits of guided drivers keep slightly increasing with increased take-up, and so do the travel time savings for the unguided drivers; their savings are considerable too!

3.6 The influence of routing assumptions

Here I want to devote some attention to the influence that my assumptions with regard to route choice of guided and unguided drivers have on the model outcomes. Again I will refer to Figure 4 to 11.
It is clear that the benefits of a route guidance system are highest when we route equipped drivers via a system optimum (SO) and the unguided drivers via a stochastic user equilibrium (SUE) with a large spread parameter Θ; UE routing will produce second-best results.
If we acknowledge that a guidance system is bound to make errors, we lose some benefits for the guided drivers, resulting in an increase in travel times of up to 4%; again the assumed magnitude of the route choice error term is of importance here.

A system which assumes SO routes for guided drivers and UE routes for unguided drivers gives rise to modest system-wide benefits; also instabilities with respect to the level of take-up are introduced into the combined assignment/signal control problem, as discussed before. Class-dependent benefits are vital here too, and illustrated in Figure 13, again for the integrated case with delay minimising control. System optimising guidance under such circumstances brings disbenefits to the equipped drivers up to a quite high level of take-up of 50-70%. These disbenefits are greater if the congestion is high. Even though the class of guided drivers may benefit at the highest levels of take-up, they are always worse off than the unguided drivers, who follow a user equilibrium. This observation had already been made in the theoretical analysis in Van Vuren (1990d).

All calculations here have been carried out in an equilibrium context, where unguided drivers can react to routing changes by those following advice; also the guided drivers can follow several routes between a single OD-pair. We could on the other hand assume that unguided drivers do not change their routes; and that only one route per OD-pair can be advised. Clearly, results would differ then; Watling (1990) describes some test results following this approach with virtually the same network, but quite different cost assumptions.

3.7. Convergence characteristics

As before in Van Vuren (1990c) I have chosen to run the iterative assignment control loop a fixed large number of times, instead of checking for convergence. The reasons for this are:

1. I employ the method of successive averages in the assignment substep. Convergence of this method is not very well-behaved; a check for convergence is not as straightforward as with the Frank-Wolfe algorithm.
2. Differences between the various scenarios are very small indeed. Convergence errors would have large influences on the results.
3. Finally, Van Vuren and Watling (1990) report convergence problems of SO assignment that cause implausible outcomes.

Nevertheless, in Figures 14 to 17 the convergence behaviour for scenarios A and C, delay minimising control and the two congestion levels is illustrated. In Figures 14 and 15 a convergence indicator S_n due to Sheffi and Powell (1982) is shown. This indicator is calculated as follows:

$$S_n = \sum_a (1/(I-1) \; (\sum_{i=n-I+1}^{n} (f_a^{(i)})^2 - I(\bar{f}_a^{(n)})^2))^{1/2} \; / \; \sum_a \bar{f}_a^{(n)} \qquad (7)$$

$$\text{where} \quad \bar{f}_a^{(n)} = 1/I \sum_{i=n-I+1}^{n} f_a^{(i)} \qquad (8)$$

The indicator smooths the random fluctuations in link flows by combining data over I iterations; I = 5 was used in my tests. Note that the Y-axes in Figures 14 to 17 are logarithmic.

In Figures 16 and 17 I show the average absolute difference (AAD) in green times between subsequent iterations in the iterative assignment control procedure, also as a convergence indicator.

The Figures speak for themselves. Convergence is fast and very good: the indicator S_n reaches a value of about 10^{-5} within 50 iterations, but keeps reducing slowly during the subsequent iterations to a value of approximately 10^{-6}. The convergence with respect to average absolute differences in green times reaches a value of less than 0.1 sec within 50 iterations; after 300 iterations the AAD has stabilised around 0.01 sec. The fluctuations in the later iterations are most likely caused by the stochastic character of the SUE flows: remember that for this indicator no smoothing was applied. Convergence according to these convergence indicators appears to be independent of the level of take-up and the level of congestion.

4 Results for the Southampton network

The network of (part of) Southampton was obtained from the Transportation Research Group of the University of Southampton. The network has been used for CONTRAM-based studies of route guidance, such as reported by Breheret et al. (1990). It was transformed into a SATURN-compatible format, and is depicted in Figure 18.

The network of Southampton is considerably smaller than Weetwood: only 14 nodes (13 of which are signal-controlled), 28 links and 9 zones. The advantage of the network is, though, that it will allow a future comparison between two different models of route guidance, based on different behavioural assumptions and modelling techniques. I will describe the model results in an identical way as for Weetwood in Chapter 3. Again, two levels of congestion are investigated, by multiplying the observed OD-matrix by 1.0 and 1.5, giving rise to average network speeds of 27 km/h and 19 km/h respectively.

4.1 The four basic scenarios

As before I will first discuss the four reference scenarios as introduced in Table 2:

- current situation
- expected situation without interaction with signal control
- expected situation with interaction with signal control
- optimum situation

Figure 19 shows these scenarios. As for Weetwood a route guidance system with perfect information and 10% take-up will only produce very small travel time benefits to the system of about 0.3 to 1.1%. Integration of the guidance system with signal control does not reduce travel times further when Webster's control policy or P_0 is employed; with delay minimisation and the power policy an additional reduction of about 0.7% is achieved. As before delay minimisation performs best, and P_0 in particular behaves inefficiently, giving rise to 15% higher network travel times than Delmin. System optimal guidance, 100% take-up, full integration and total compliance (scenario D) improves network performance with a further 4% compared with the original situation A.

4.2 Increased congestion

In the 1.5M case, as Figure 20 shows, the benefits of an isolated route guidance system with 10% take-up are about 0.7-1.0%; considerably less than other model studies have found. The advantage of integration with the signal control system is now 0.2 to 2.7%, dependent on the control policy. Of all four policies delay minimisation performs best again, and as observed before, P_0's performance has improved, though travel times with this policy are still 7% higher than with Delmin. As in the Weetwood case the performance of the power policy deteriorates in increased congestion with resulting travel times about 11% up on Delmin. Particularly striking here is the extra possible advantage of an optimum system, as in scenario D, offering an extra saving of 15% over the best performing policy in scenario C.

4.3 Influence of the control policy used

The performance of each of the four signal control policies in the iterative assignment control procedure with various routing assumptions for guided and unguided drivers is shown in Figures 21 to 28.

As observed in Weetwood, the delay minimising policy performs best of all, independent of the routing assumptions made for each of the classes. Although the power policy performs roughly as well as Delmin in the lower congestion case, the differences in resulting travel times between delay minimisation and the other three policies in the 1.5M case are considerable, amounting up to some 17%.

Webster's policy also shows a similar picture to the Weetwood findings, performing reasonably at the lower level of congestion, but suffering from instability in the 1.5M case. An increase in take-up may then result in increased average network travel times, which is counter-intuitive and undesirable.

P_0 performs very poorly in the 1.0M situation giving rise to network travel times that are 9 - 14% higher than the worst of the other 3 policies. Although its relative performance improves when congestion rises, it does not even approach delay minimisation in resulting travel times.

The power policy performs rather inefficiently when demand is high. It performs like delay minimisation when congestion is low, but some instabilities appear in the 1.5M case. Its performance is particularly poor at low levels of take-up and with SUE routing assumptions for unguided drivers.

These findings confirm the Weetwood results; delay minimisation is the best signal control policy in interaction with route guidance. Like in Chapter 3, I will therefore concentrate on delay minimisation throughout the rest of this Chapter.

4.4 The effect of imperfect advice

Instead of assuming user equilibrium routes for guided drivers, we can model the influence of erroneous advice or driver errors by a stochastic user equilibrium routing pattern for the guided drivers, with smaller errors than for the unguided ones.

In Table 5 I compare resulting network travel times for the two cases of UE and SUE (0.3) routing by guided drivers, whilst unguided drivers always follow SUE (0.6) routes.

Table 5 Percentage increase in total network travel times as a result of imperfect advice; SUE (0.3) route advice assumed. Delay minimising control

% guided	1.0M	1.5M
0	-	-
10	+0.38%	-0.18%
25	+0.38%	-0.36%
50	+0.77%	0%
75	-1.15%	+0.55%
100	+0.77%	+1.50%

The results in Table 5 are less clear-cut than for Weetwood in Table 3. In some cases imperfect advice gives rise to an actual decrease in total travel times. This is against the expectations, but not impossible if the erring SUE drivers choose routes with low marginal costs. It is clear, though, that the benefits of a route guidance system in such circumstances will be limited, or even negative! Note that a different modelling framework might have found a rather different behaviour.

The influence of increased congestion is also rather unclear for this Southampton network, related to the apparent closeness of the SUE and UE flow patterns. It is impossible, therefore, to draw any conclusions from this Table.

4.5 The advantage of system optimal guidance

Instead of a comparison between UE and SUE routing for guided drivers, I am here interested in a comparison between travel times that result from UE and SO routes for equipped drivers. This should show possible advantages of a route guidance system that explicitly strives to improve system-wide network performance, and should also pinpoint potential problems related to such an advice system, particularly with regard to acceptability of that advice. Total network travel time savings through SO advice are shown in Table 6.

Table 6 Percentage benefits in total network travel time as a result of system optimum advice; delay minimising control.

% guided	1.0M	1.5M
0	-	-
10	0.14%	2.54%
25	0.25%	6.17%
50	1.15%	10.56%
75	2.67%	12.18%
100	2.32%	12.90%

As before, benefits increase with take-up, as then more and more drivers follow SO routes with beneficial effects on the system's performance. Again, the benefits of SO guidance are low with realistic levels of take-up of up to 25%. An exception is the 1.5M case with delay minimising control: benefits of SO guidance are always substantial in that situation, caused by the large difference between the travel times resulting from a full UE and a full SO route pattern, as already mentioned in Section 4.2. This may be a network-specific anomaly.

The class-dependent benefits of SO and UE guidance are rather different to those for Weetwood; see Figure 29. Again, guided drivers following system optimal advice suffer an increase in travel time at a level of take-up of 10%, but now with observed demand (1.0M).

Further, it can be seen that guided drivers following UE advice may experience reducing benefits when the level of take-up increases; this performance goes together with a rise in travel times for unequipped drivers. This is a very implausible result, most likely related to the poor quality of the user equilibrium, in comparison with a stochastic user equilibrium, which was already observed in Section 4.4.

4.6 The influence of routing assumptions

The assumptions we make about the route pattern for guided and unguided drivers determine likely benefits of a route guidance system. SO routing for guided drivers with unguided drivers following a SUE pattern gives rise to greatest benefits. The savings of a UE based guidance system, and of an imperfect system that will result in a SUE pattern for guided drivers, are very much the same for Southampton.

The relative quality of the full user equilibrium very strongly determines the likely system benefits under SO assumptions for equipped drivers, and UE assumptions for the unequipped drivers. Thus, the benefits are virtually non-existent in the 1.0M case, and they are considerably larger in the 1.5M case. But the disbenefits in all cases at lower levels of take-up to guided drivers, as shown in Figure 30, are the main problem of a model framework based on SO and UE routing for guided and unguided drivers respectively. In such situations advice will not be accepted, which will result in a deterioration of the system.

The results from the network of Southampton are more diffuse than those using Weetwood; some of the results confirm earlier findings, and some of them are contradictory. A specific problem is the poor quality with respect to total travel time of the user equilibrium in comparison with a stochastic user equilibrium. Under the model assumptions made here, route guidance may actually result in system disbenefits.

5 Increasing take-up and congestion

It is not expected that the level of take-up of route guidance will stay fixed over time; certainly the manufacturers do not hope so! In this Chapter I investigate the dynamic case of an increase in the level of take-up over time in interaction with signal control. Clearly, travel demand will also rise over time, so this has been taken into account in the tests, too. The simulation procedure followed is very similar to that described in Section 9.3 of Van Vuren (1990c), but an important. difference is that now sheared delay is employed as the cost function, so that infeasibility will not occur.

The situation investigated consists of:

- an annual increase in travel demand of 2.8% over 25 years to twice the original demand; plus
- an increase in the level of take-up of guidance over the same period from 1% to 10% of the total driver population.

Traffic signals are updated every 1.5 month, to fit the observed flows (both guided and unguided) according to the four control policies. The route choice of unguided drivers is continuously adjusted to changing flow levels and green times via the usual SUE(0.6) assignment, whilst the guidance system keeps advising updated routes based on the UE objective.

Resulting network travel times for Weetwood and Southampton are shown in Figures 31 and 32. The fluctuations on the left hand sides of the graphs a starting-up effect of the iterative assignment control procedure, when signals and flows are relatively far out of balance; they are of no concern here. My main interest lies in the relative performance of the four policies in this dynamic environment. In the Weetwood network delay minimisation and P_0 perform virtually identically, with Webster's policy giving slightly higher travel times. The power policy gives rise to very high network travel times. A comparison with Figure 4.13 (in which Delmin performs notably worse than P_0) gives another indication of the influence of cost assumptions on the performance of each of the four signal control policies.
For Southampton in Figure 32 the picture is rather different: at lower demand levels Webster's policy, delay minimisation and the power policy outperform P_0. As the travel demand increases, however, P_0's re-distributing properties result in lowest overall travel times. Travel times with delay minimisation are up to 10% higher, followed by the power policy, and finally Webster's.

This dynamic analysis throws a different light on the relative performance of the four control policies in interaction with route guidance. P_0, in particular, performs well with slowly increasing demand: apparently its capacity maximising settings then achieve their objective.

6 Conclusions from route guidance model tests

It is impossible to draw any hard conclusions from tests on just two networks. In that light these conclusions are not meant to be accepted as the final truth, but merely meant to summarise the main findings from the previous Chapters. It also must be borne in mind that the model used has some strong underlying assumptions, which may have a considerable influence on these test results.

The most important finding from the model simulations is probably that the expected benefits of proper accounting of route choice in signal control (via e.g. the iterative assignment control procedure) may lead to extra travel time savings, which are comparable to the savings of a route guidance system alone. In that light an integration of any route guidance system with the traffic control centre is of definite importance. The test results also indicate that expected benefits of such an integrated system rise with an increase in congestion. Of the four signal control policies tested in the iterative assignment control procedure delay minimisation performs best in the static demand case, both with respect to resulting network travel times and plausibility and stability of the resulting network performance. In a model framework of slowly increasing demand and take-up P_0 performs best.

The effect of imperfect advice, non-compliance and in-vehicle information systems (as opposed to route **guidance** systems) has been investigated via stochastic user equilibrium for equipped drivers, with a smaller error term than that for the unequipped drivers. Resulting network deterioration tends to be small, particularly at more realistic levels of take-up of up to 25%. The effect of imperfect advice increases with a growth in congestion.

In the same way, the benefit of system optimal advice is modest with realistic levels of take-up. An additional problem is that such SO guidance disbenefits the equipped drivers, particularly at those lower levels of take up. At first sight the related problems with non-compliance and the noted computational problems would indicate that an explicit attempt to reduce network travel times by system optimal guidance is not worth the effort.

As stated before, however, any of these tentative conclusions must be verified by

a) more simulations on a multitude of test networks;

b) simulations with identical networks, but different model assumptions.

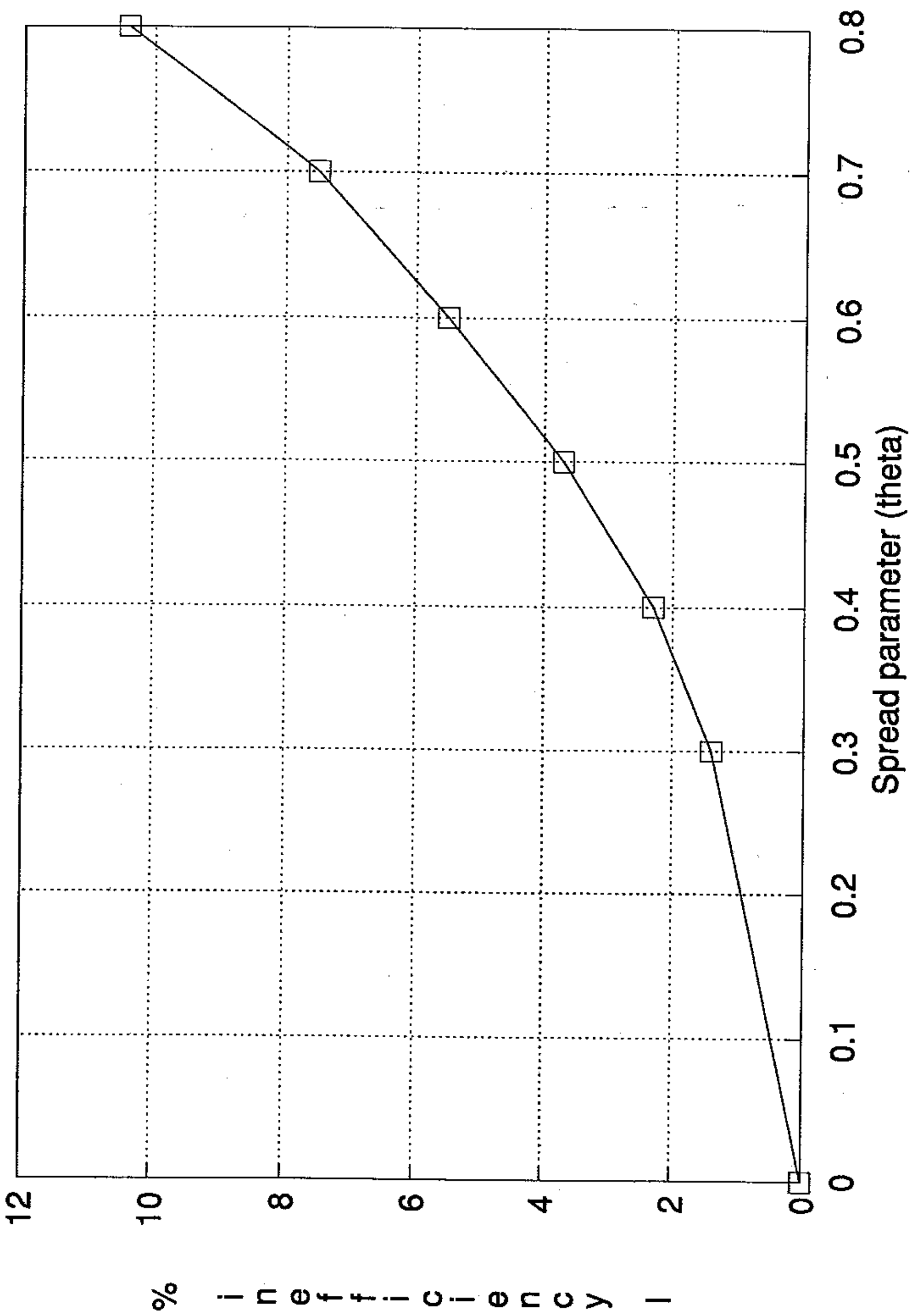


Figure 1 Inefficiency for Weetwood as a function of Θ; observed demand

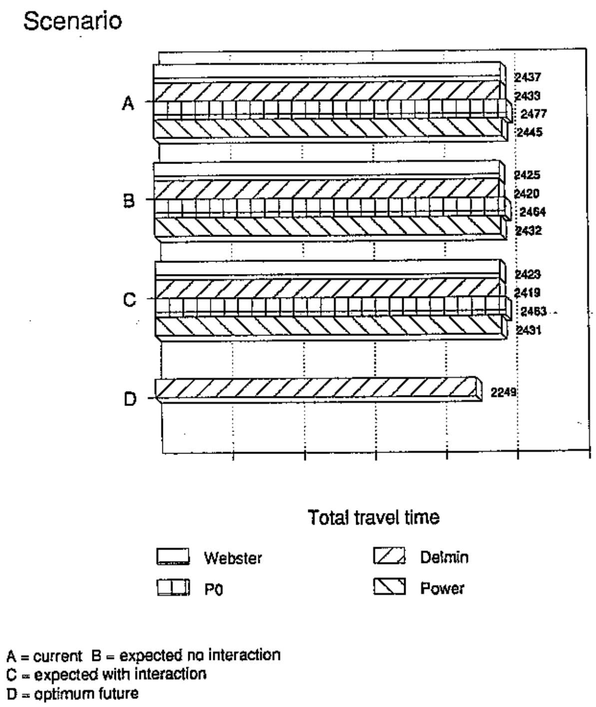


Figure 2 Total network travel time for four basic scenarios; Weetwood; observed demand

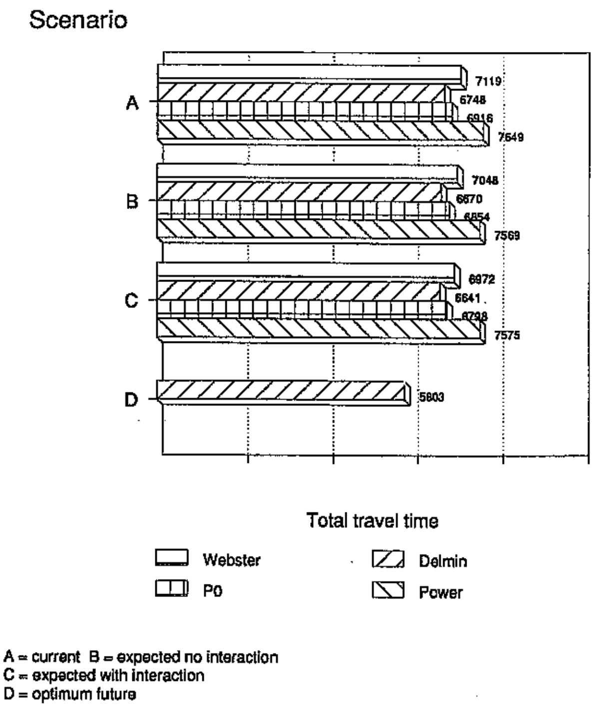


Figure 3 Total network travel time for four basic scenarios; Weetwood; demand x 1.5

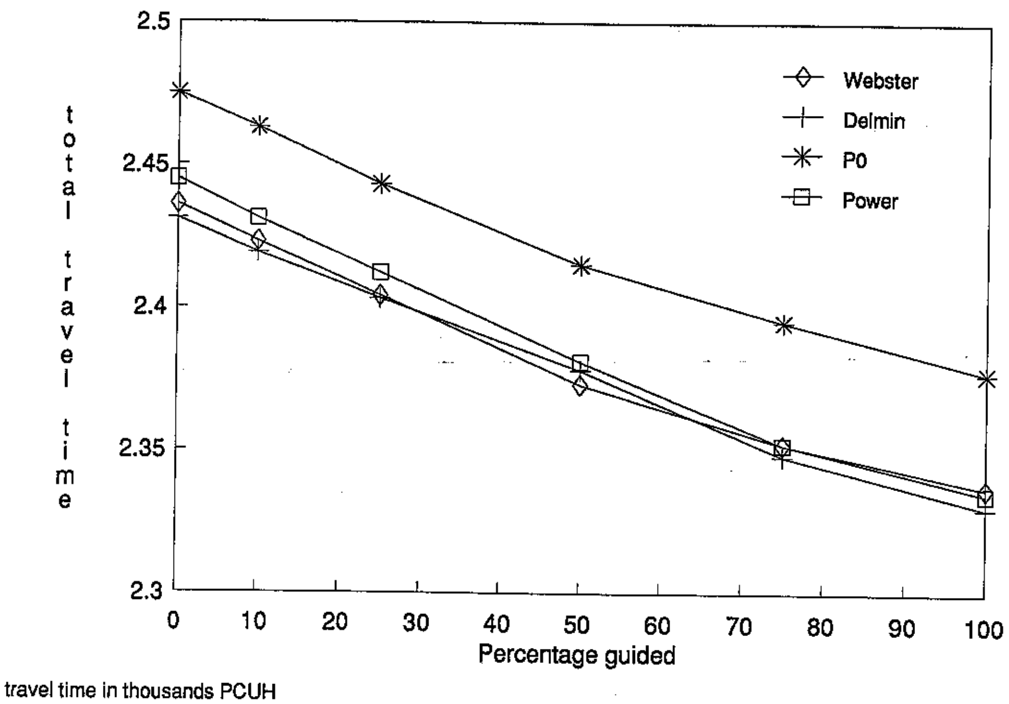


Figure 4 Network travel time as a function of signal control strategy and level of take-up; Weetwood; observed demand
guided = UE
unguided = SUE(0.6)

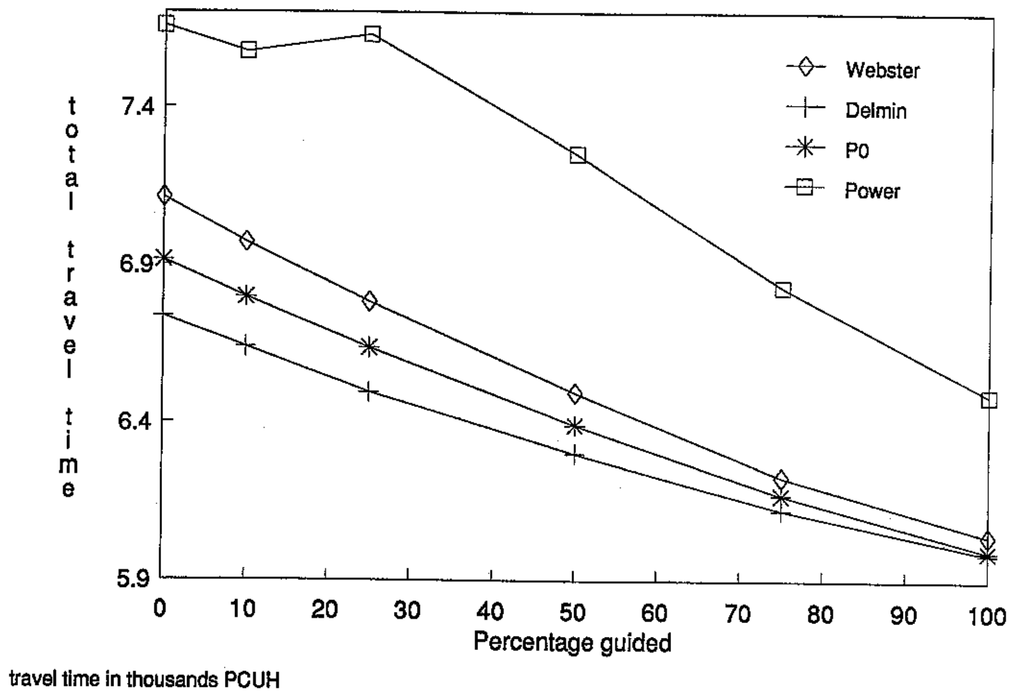


Figure 5 Network travel time as a function of signal control strategy and level of take-up; Weetwood; demand x 1.5
guided = UE
unguided = SUE(0.6)

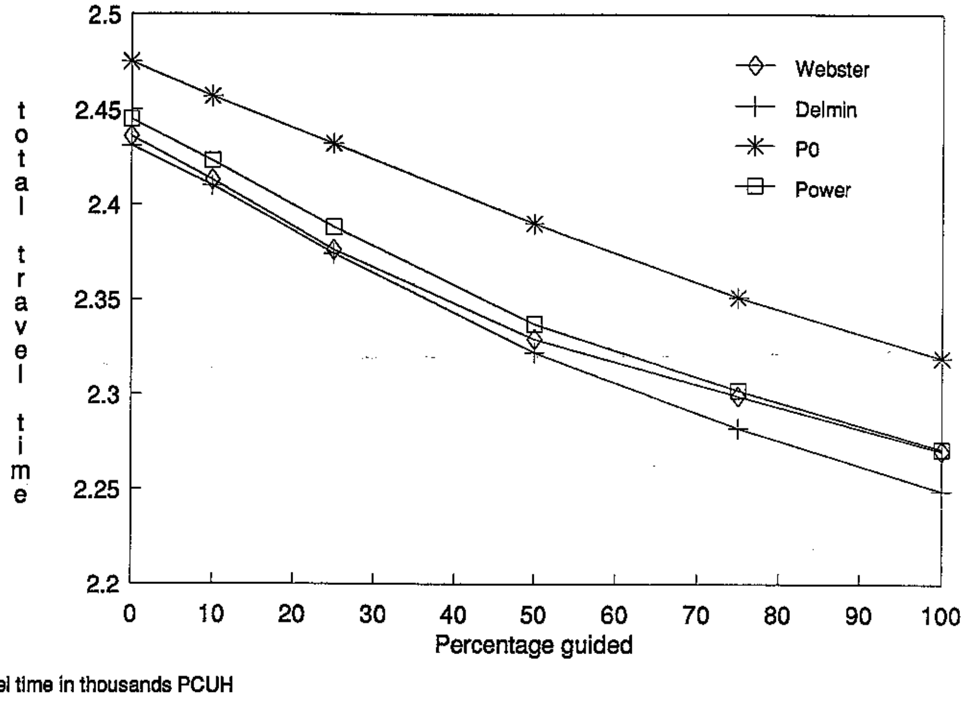


Figure 6 Network travel time as a function of signal control strategy and level of take-up; Weetwood; observed demand
guided = SO
unguided = SUE(0.6)

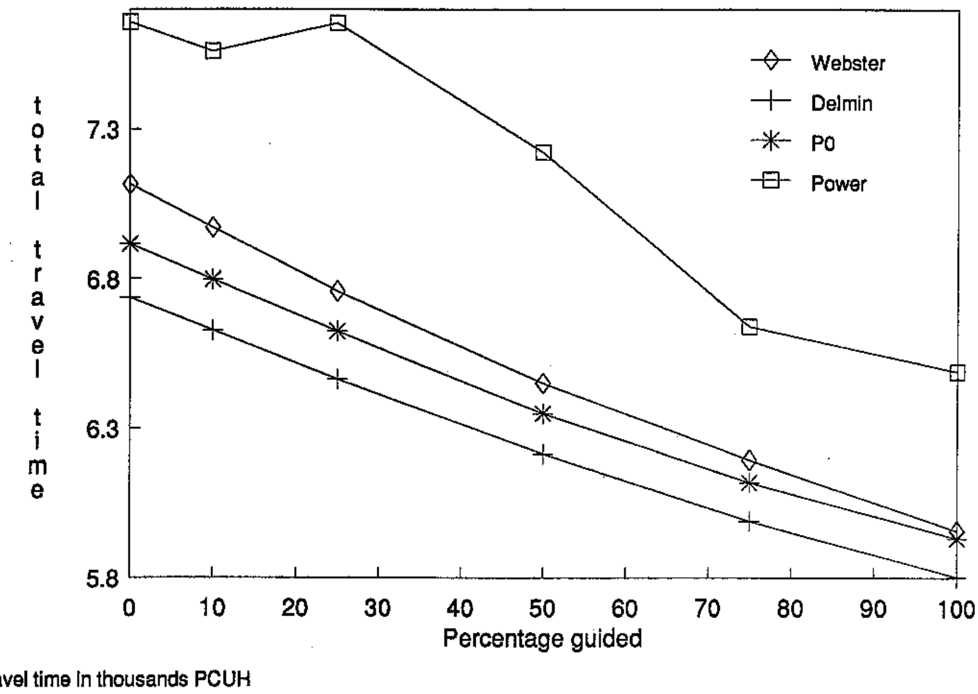


Figure 7 Network travel time as a function of signal control strategy and level of take-up; Weetwood; demand x 1.5
guided = SO
unguided = SUE(0.6)

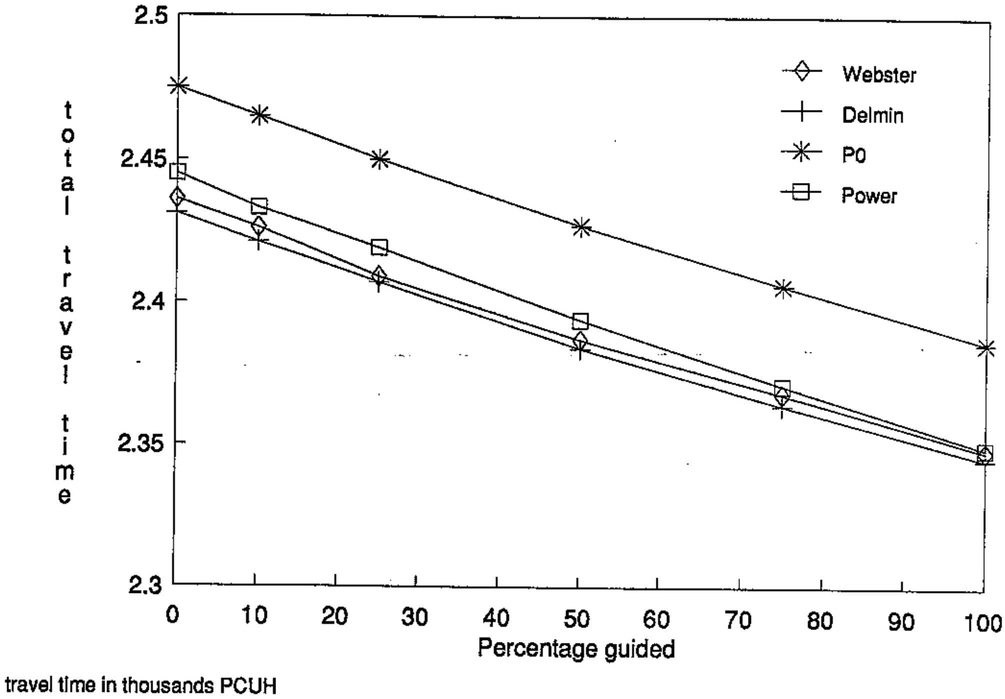


Figure 8 Network travel time as a function of signal control strategy and level of take-up; Weetwood; observed demand
guided = SUE(0.3)
unguided = SUE(0.6)

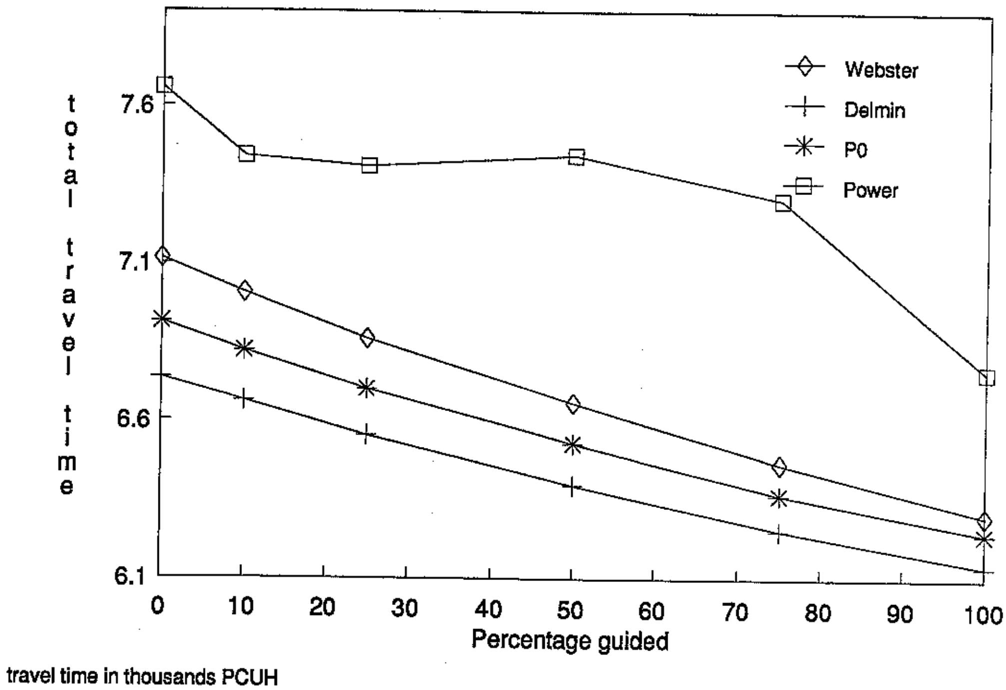


Figure 9 Network travel time as a function of signal control strategy and level of take-up; Weetwood; demand x 1.5
guided = SUE(0.3)
unguided = SUE(0.6)

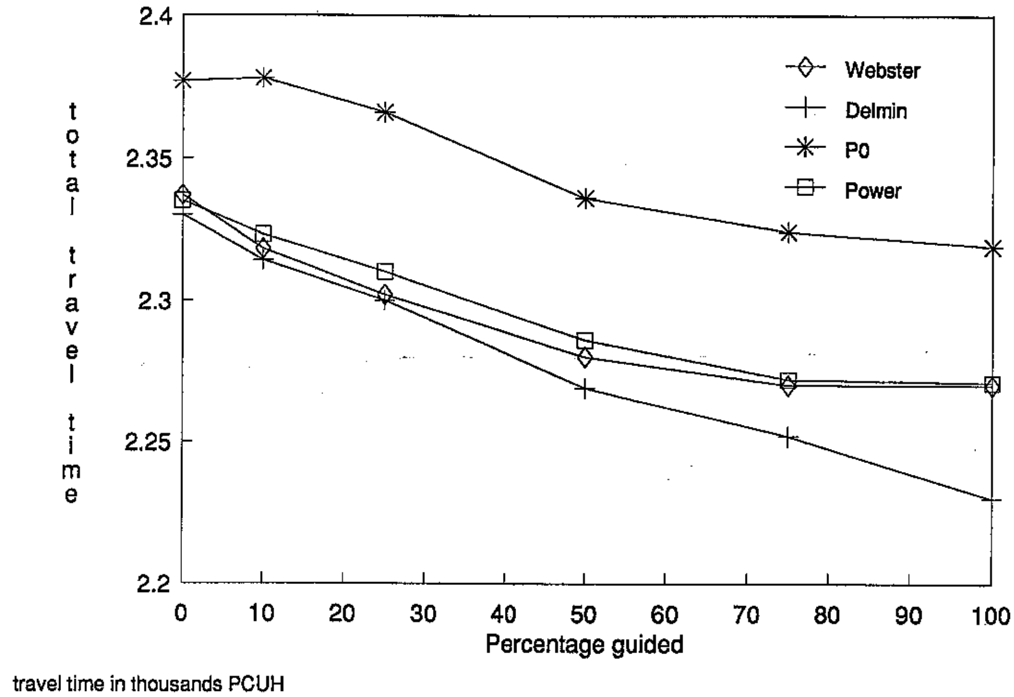


Figure 10 Network travel time as a function of signal control strategy and level of take-up; Weetwood; observed demand
guided = SO
unguided = UE

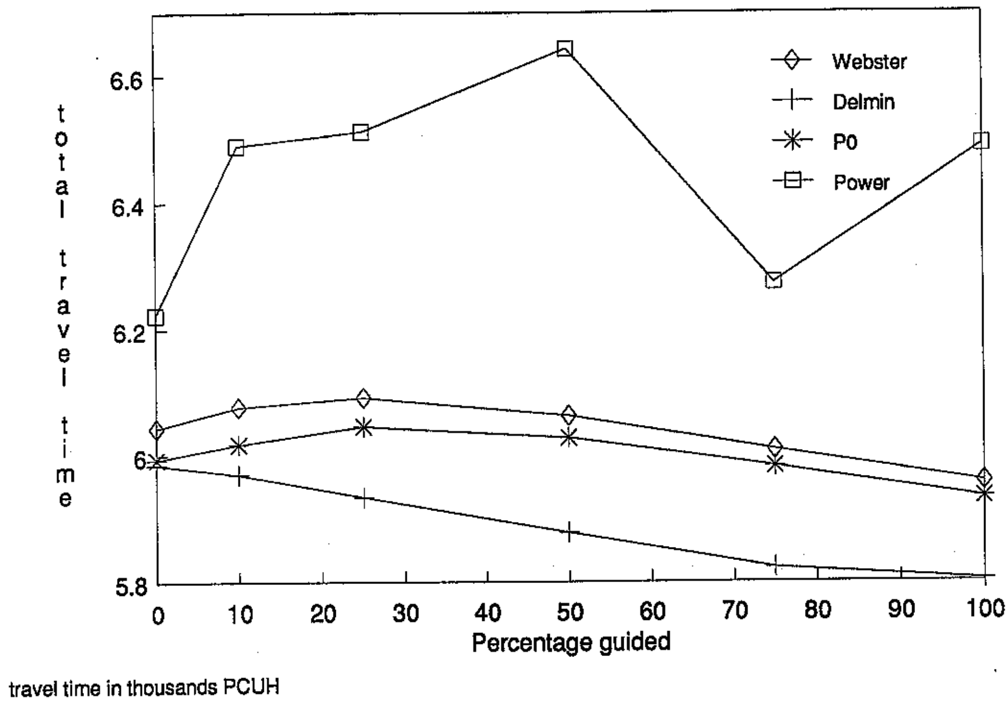
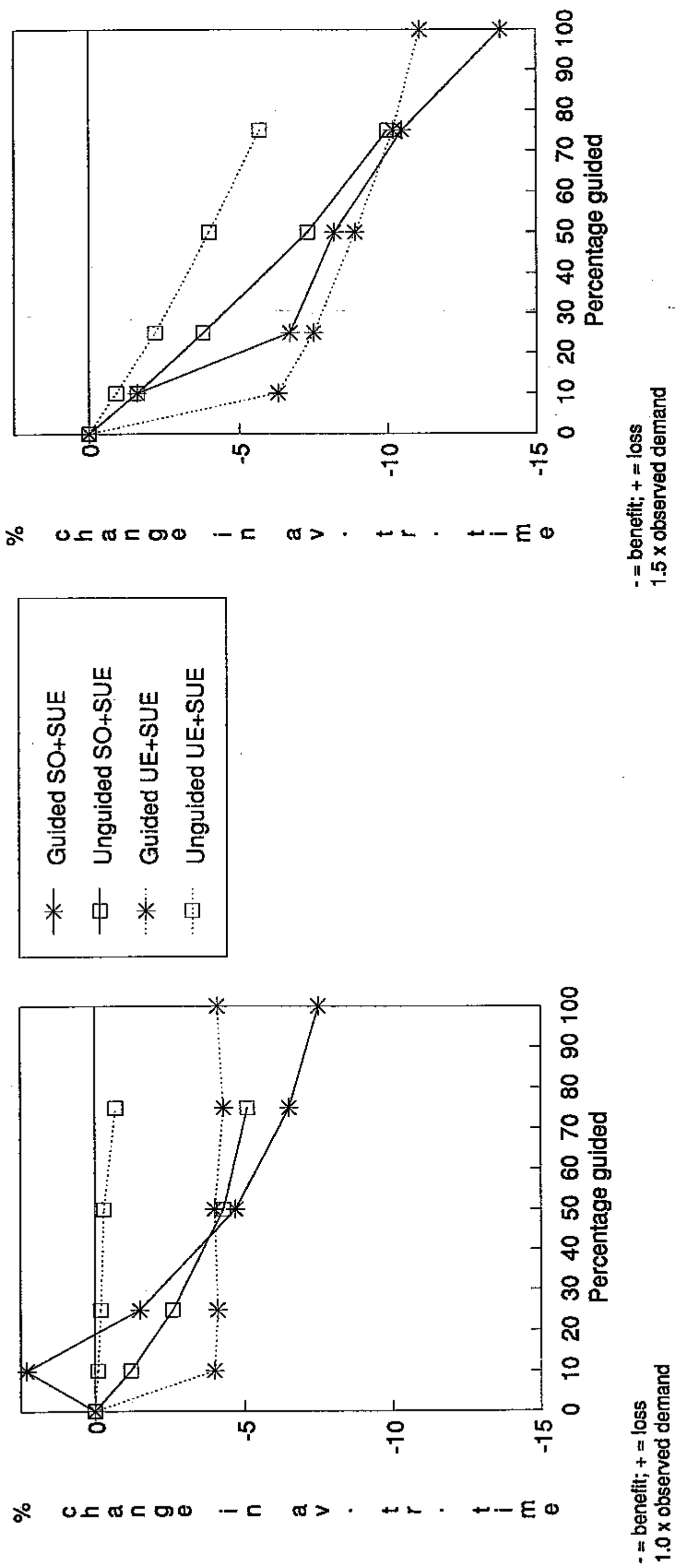
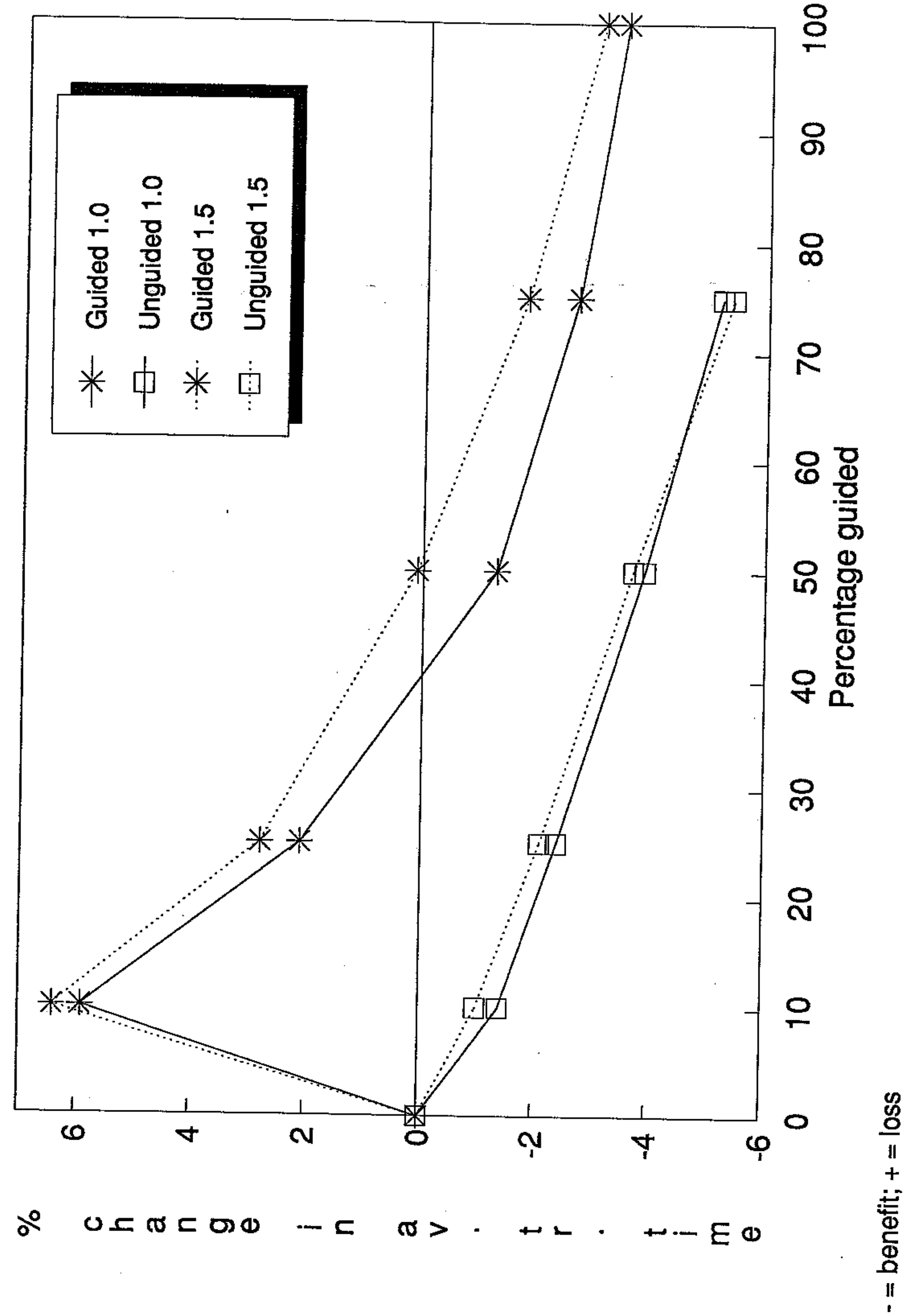


Figure 11 Network travel time as a function of signal control strategy and level of take-up; Weetwood; demand x 1.5
guided = SO
unguided = UE



<u>Figure 12</u> Class-dependent benefits of system optimum guidance for integrated situation with delay minimisation; Weetwood; both demand levels



<u>Figure 13</u> Class-dependent benefits of a route guidance system with guided drivers following SO routes and unguided drivers following UE routes; integrated delay minimising signal control; Weetwood; both demand levels

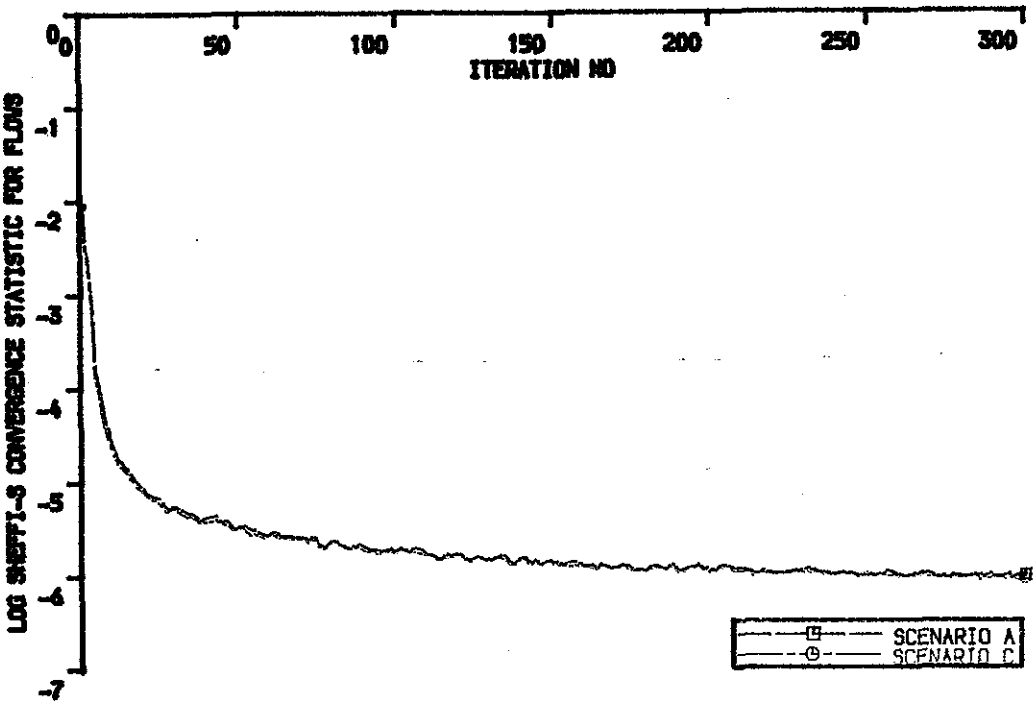


Figure 14 Convergence expressed in $\log(S_n)$ with respect to flows; Weetwood; observed demand. Scenarios A and C; integrated system with delay minimising control.

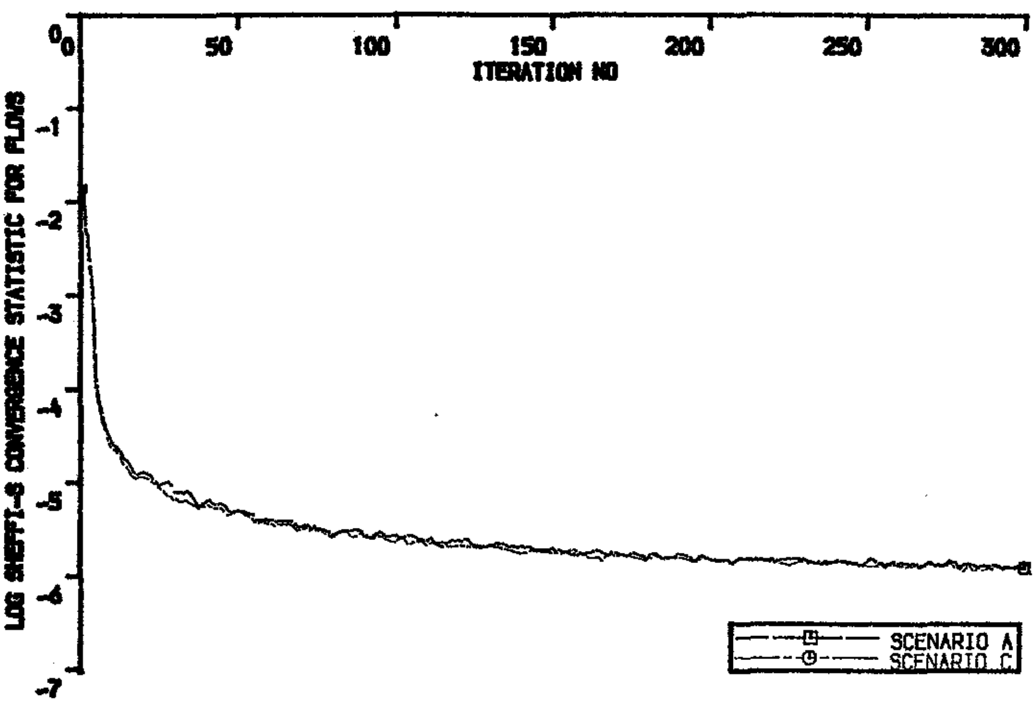


Figure 15 Convergence expressed in $\log(S_n)$ with respect to flows; Weetwood; demand x 1.5. Scenarios A and C; integrated system with delay minimising control.

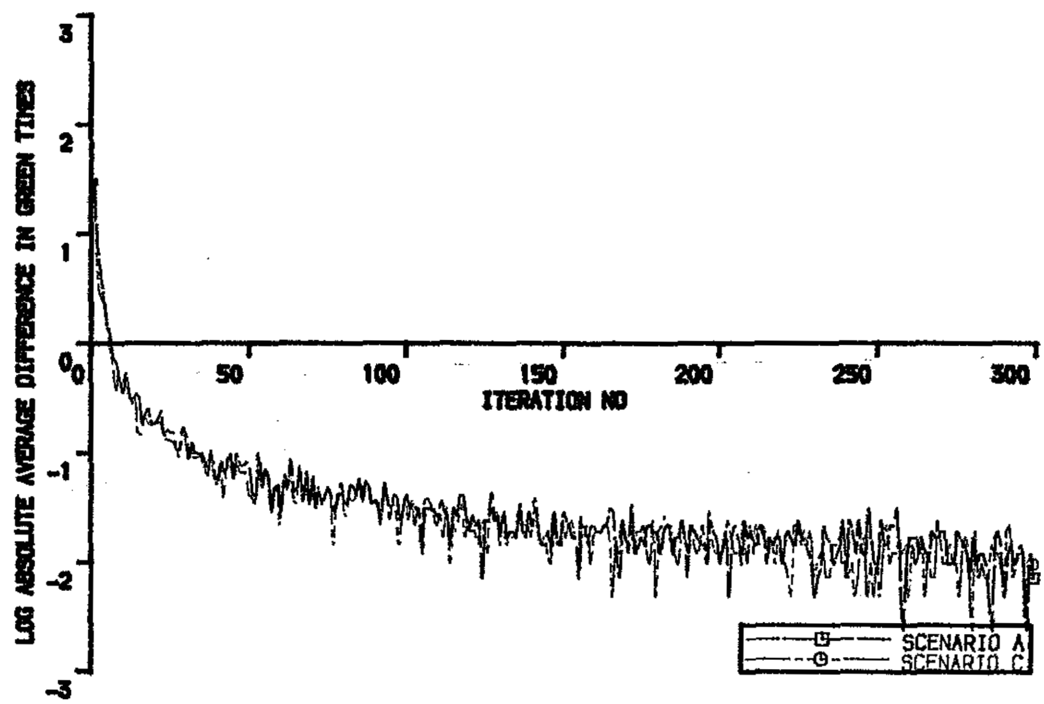


Figure 16 Convergence expressed in log(AAD) with respect to green times; Weetwood; observed demand. Scenarios A and C; integrated system with delay minimising control.

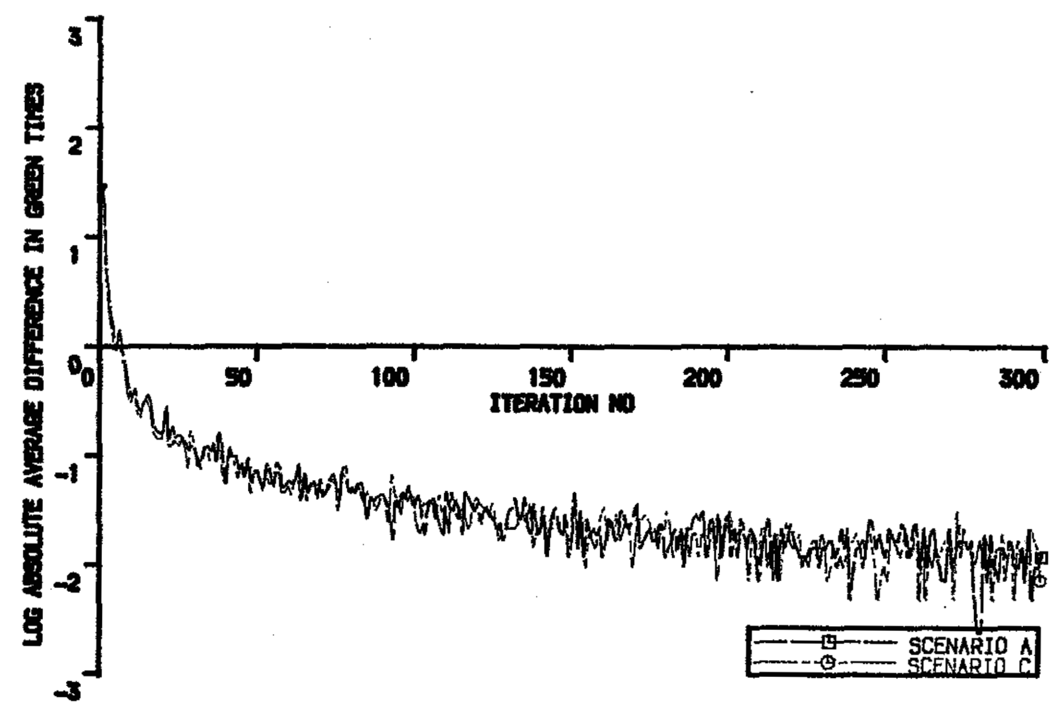


Figure 17 Convergence expressed in log(AAD) with respect to green times; Weetwood; demand x 1.5. Scenarios A and C; integrated system with delay minimising control.

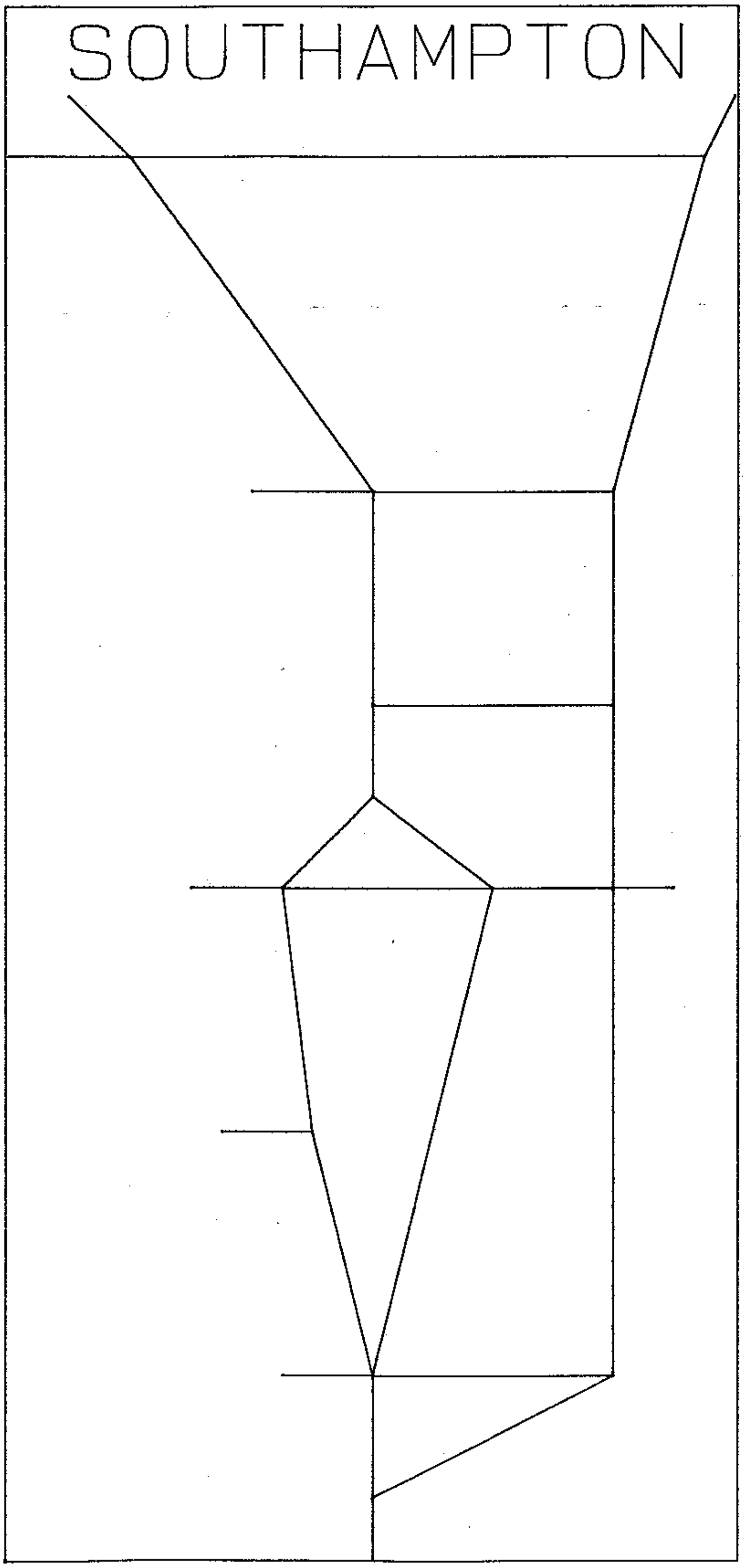


Figure 18 Southampton network

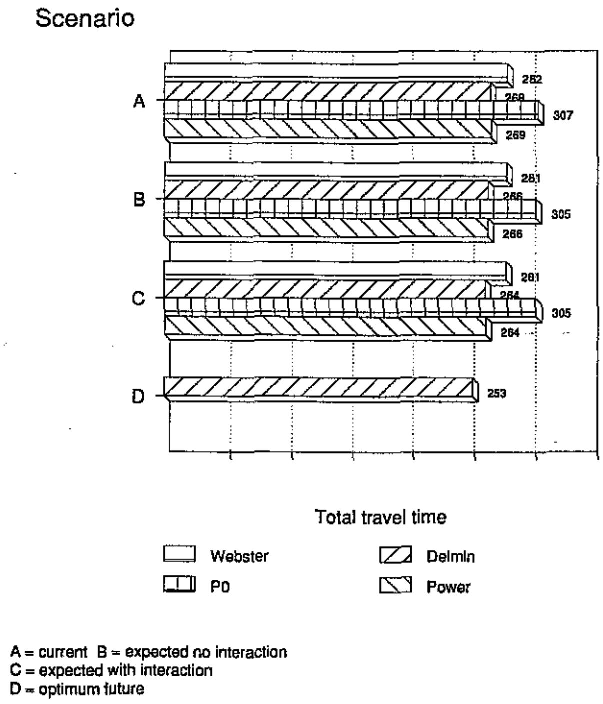


Figure 19 Total network travel time for four basic scenarios; Southampton; observed demand

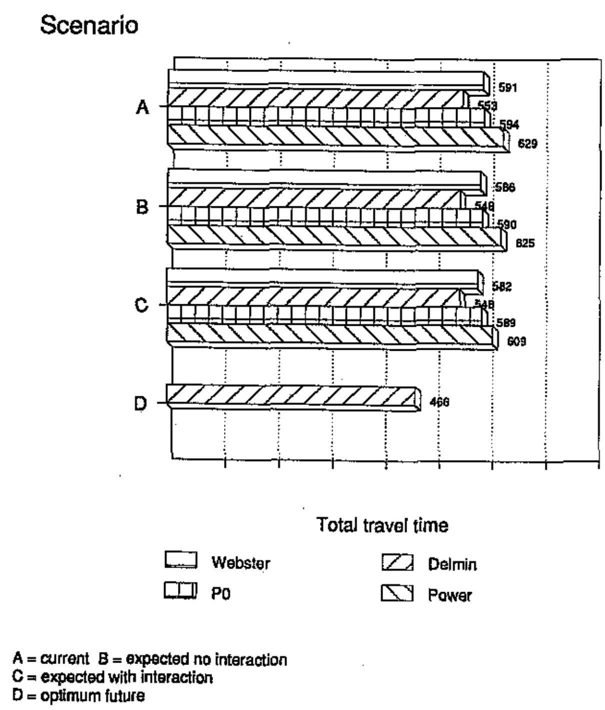


Figure 20 Total network travel time for four basic scenarios; Southampton; demand x 1.5

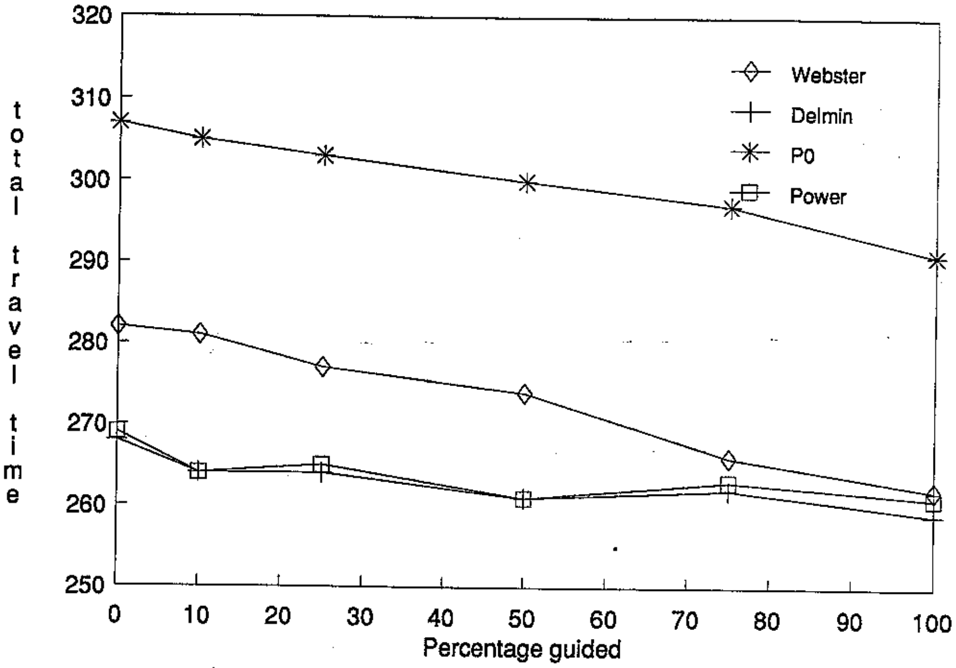


Figure 21 Network travel time as a function of signal control strategy and level of take-up; Southampton; observed demand
guided = UE
unguided = SUE(0.6)

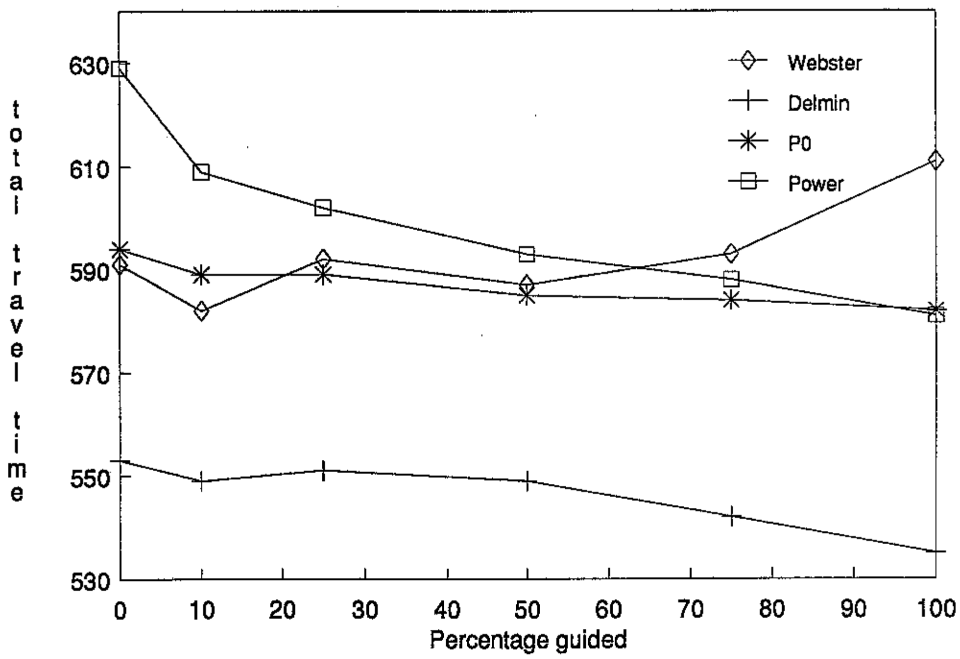


Figure 22 Network travel time as a function of signal control strategy and level of take-up; Southampton; demand x 1.5
guided = UE
unguided = SUE(0.6)

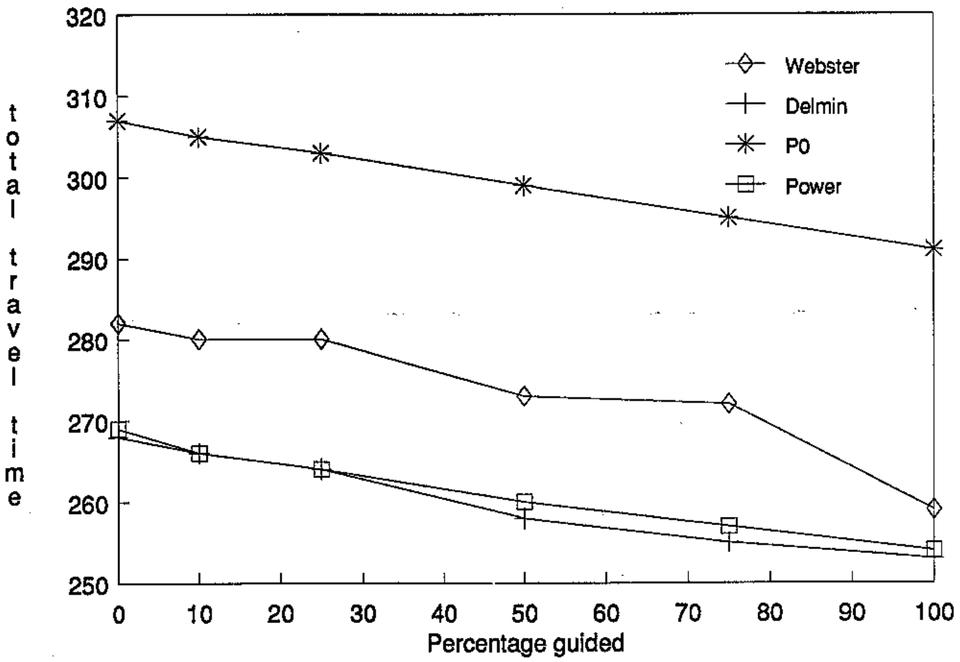


Figure 23 Network travel time as a function of signal control strategy and level of take-up; Southampton; observed demand
guided = SO
unguided = SUE(0.6)

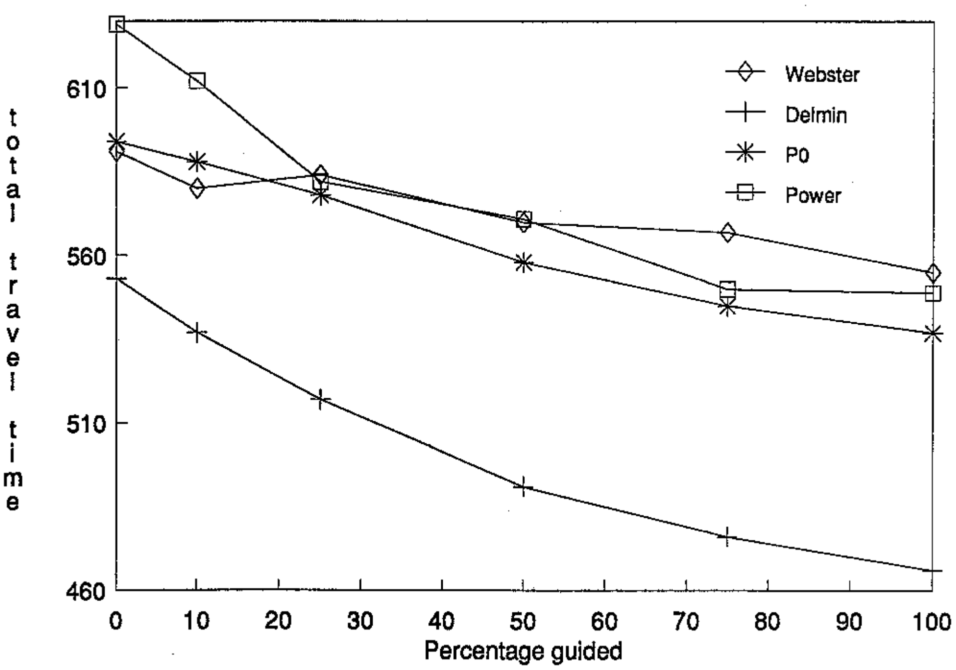


Figure 24 Network travel time as a function of signal control strategy and level of take-up; Southampton; demand x 1.5
guided = SO
unguided = SUE(0.6)

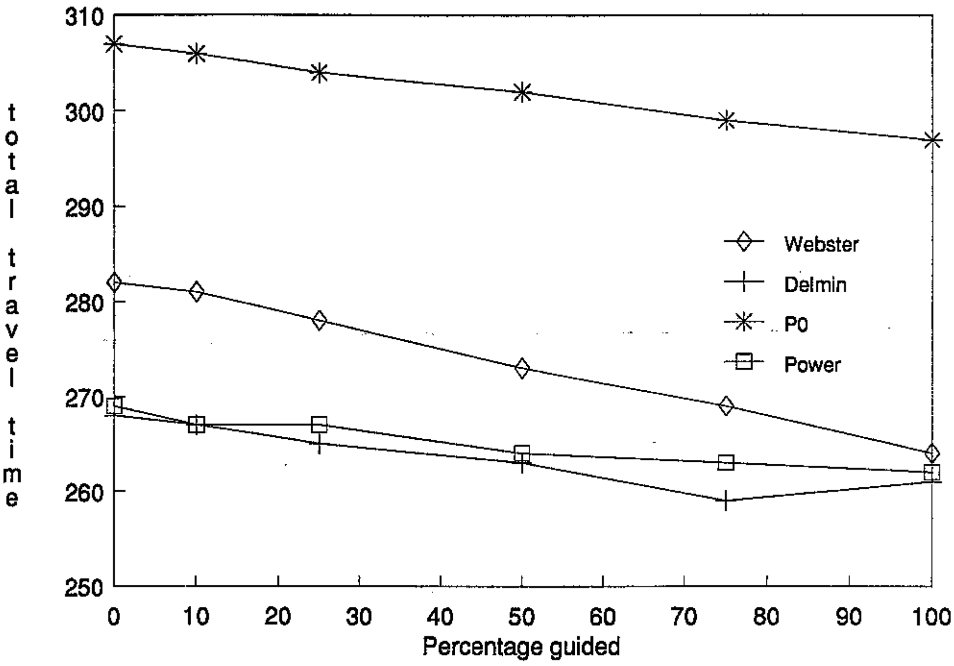


Figure 25 Network travel time as a function of signal control strategy and level of take-up; Southampton; observed demand
guided = SUE(0.3)
unguided = SUE(0.6)

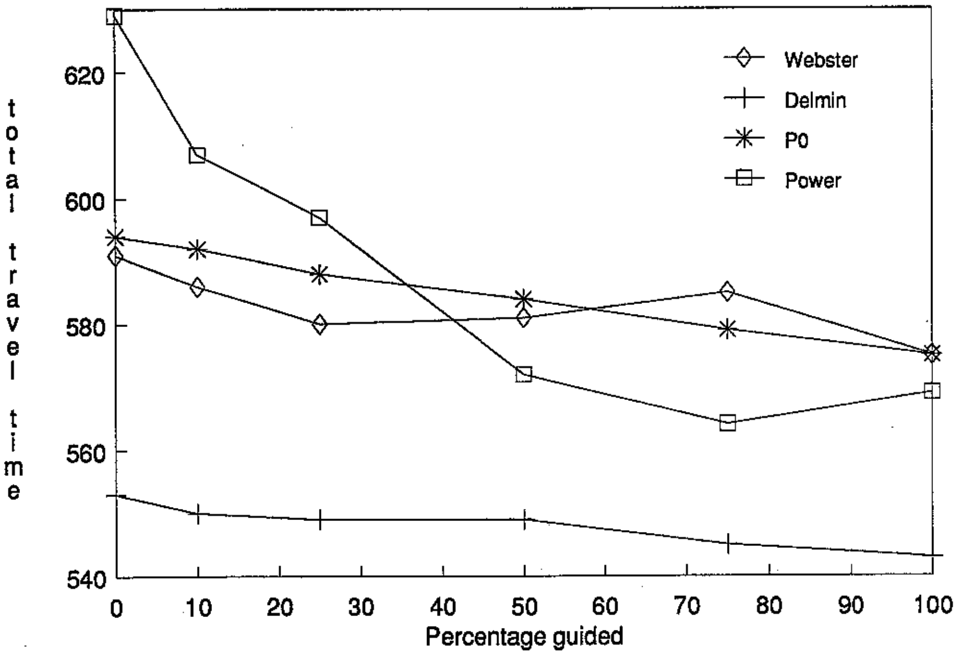


Figure 26 Network travel time as a function of signal control strategy and level of take-up; Southampton; demand x 1.5
guided = SUE(0.3)
unguided = SUE(0.6)

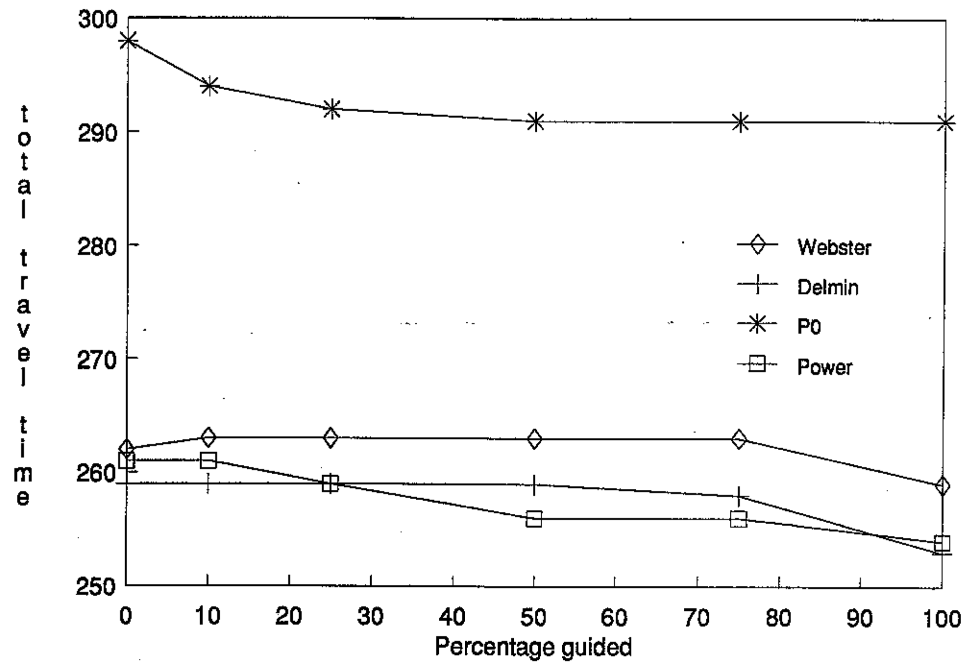


Figure 27 Network travel time as a function of signal control strategy and level of take-up; Southampton; observed demand
guided = SO
unguided = UE

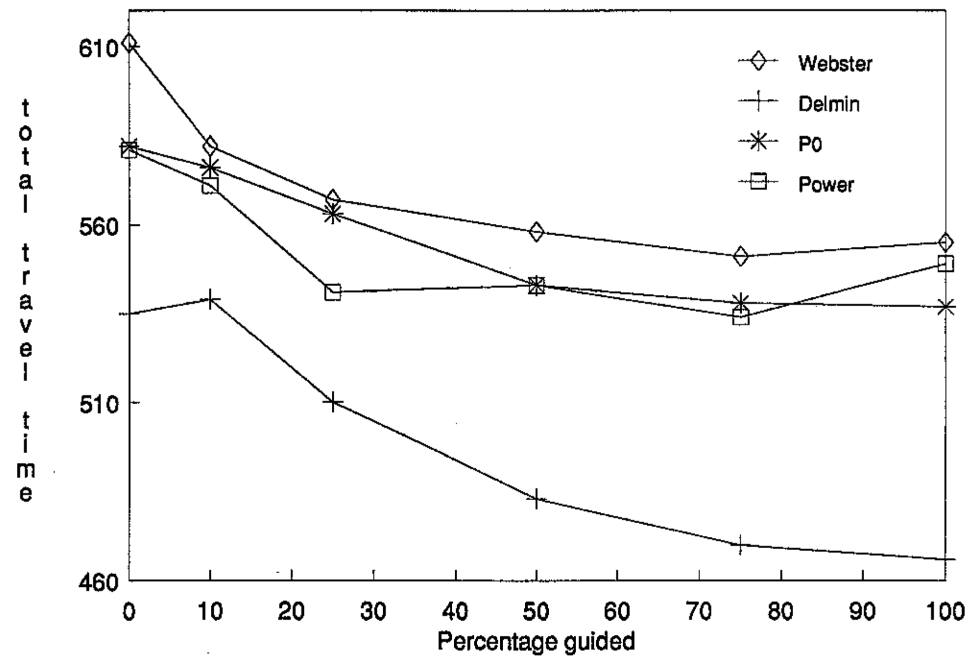


Figure 28 Network travel time as a function of signal control strategy and level of take-up; Southampton; demand x 1.5
guided = SO
unguided = UE

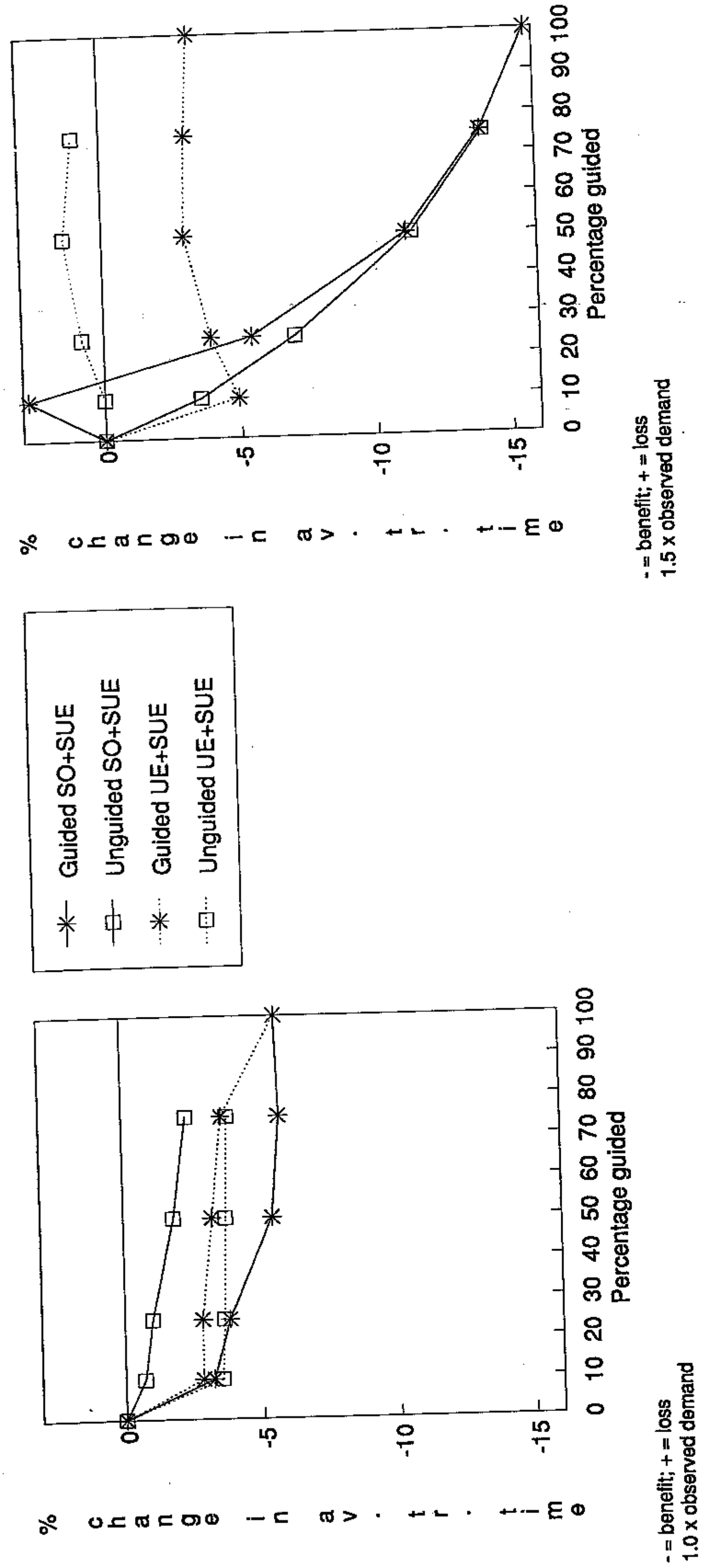
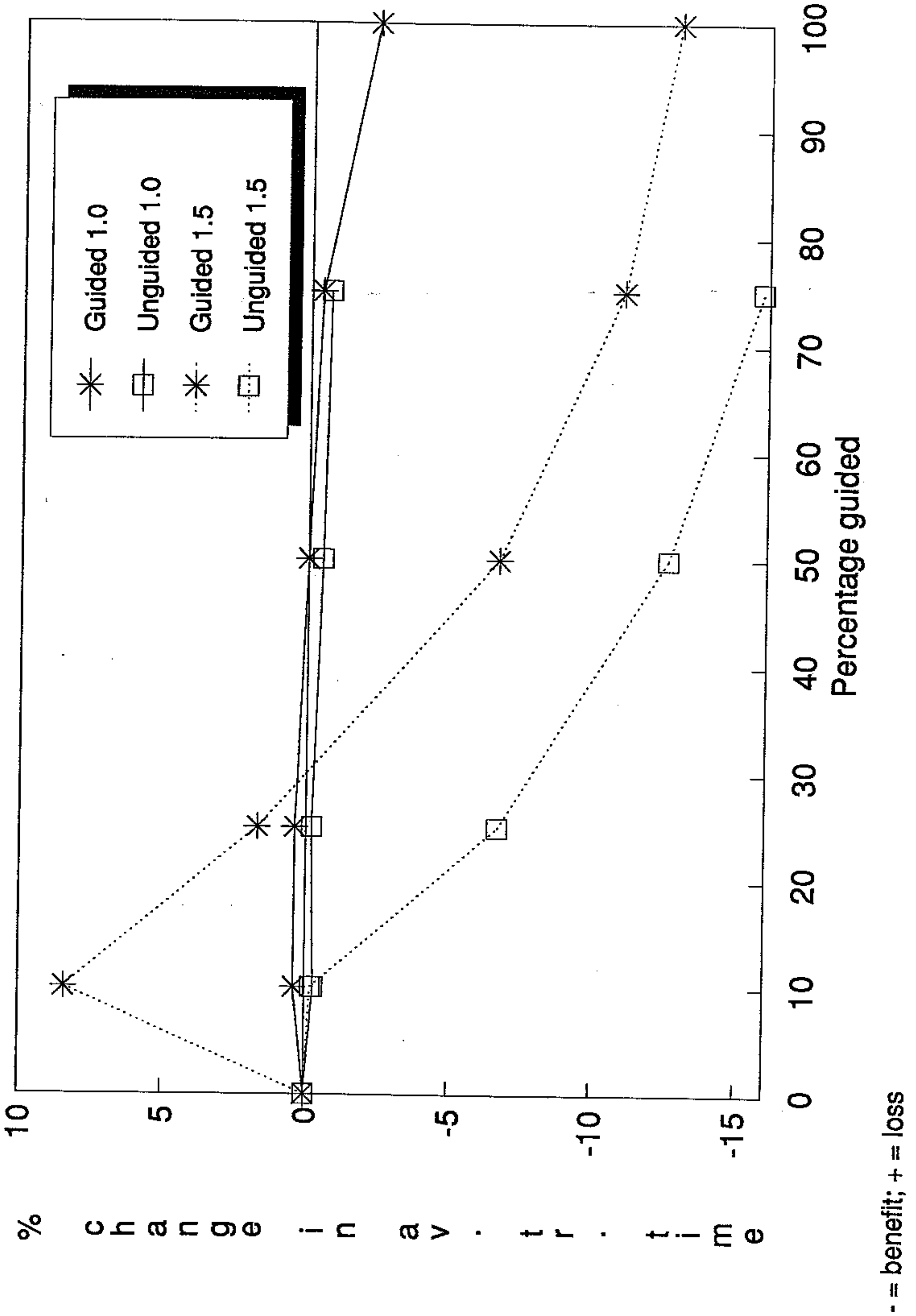


Figure 29 Class-dependent benefits of system optimum guidance for integrated situation with delay minimisation; Southampton; both demand levels



<u>Figure 30</u> Class-dependent benefits of a route guidance system with guided drivers following SO routes and unguided drivers following UE routes; integrated delay minimising signal control; Southampton; both demand levels

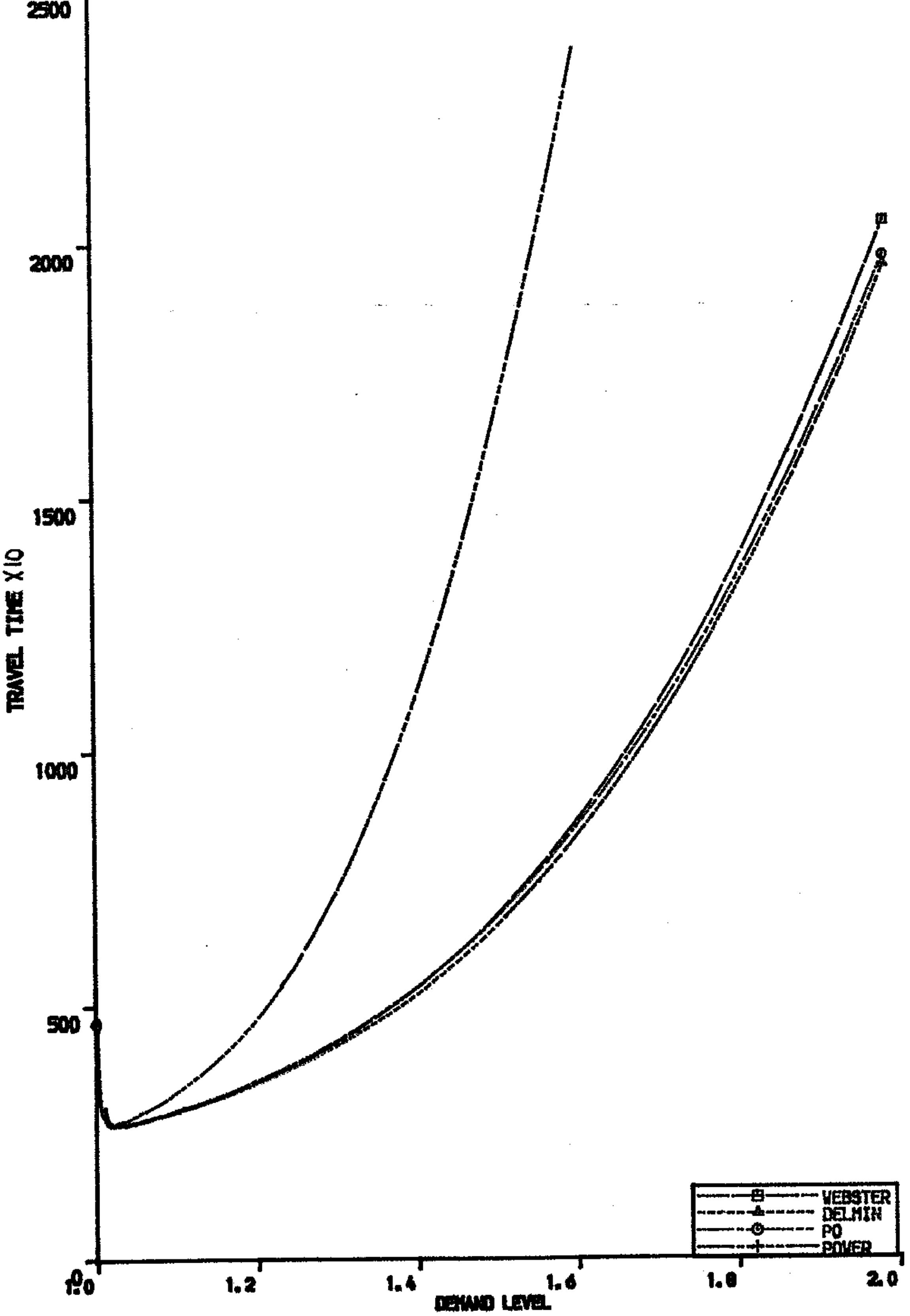


Figure 31 Resulting network travel times as a result of application of the four policies in a route guidance situation with slowly increasing travel demand and take-up. Weetwood.

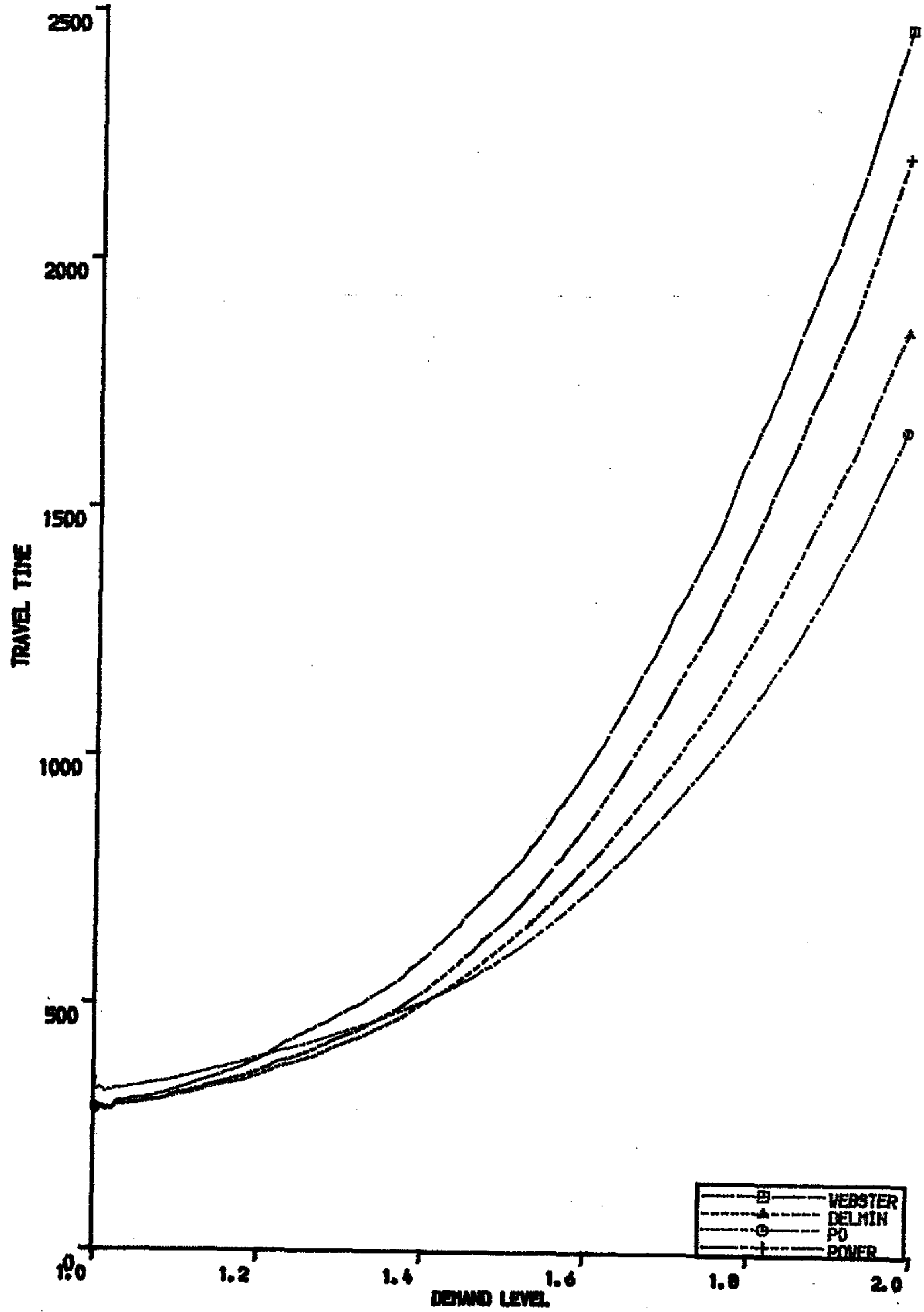


Figure 32 Resulting network travel times as a result of application of the four policies in a route guidance situation with slowly increasing travel demand and take-up. Southampton.

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APPENDIX 1: Marginal cost definition with sheared delay curve

In the case of separable cost functions marginal costs are defined by:

$$\check{t}_a = t_a + f_a\, \partial t_a/\partial f_a$$

$$= t_{0a} + d_a + f_a\, \partial d_a/\partial f_a$$

The general form of the sheared delay curve is

$$d_a = T/2\ \left(\sqrt{A^2 + B}\ - A\right)$$

where $A = \dfrac{c_a - f_a}{2c_a}$ and $B = \dfrac{f_a}{Tc_a^{\,2}}$

Omitting subscripts:

$$A + f\, \partial A/\partial f = \frac{c - 2f}{2c}$$

$$\partial/\partial f\ (A^2 + B)^{1/2} = 1/2\ [\ ((c-f)/2c)^2 + f/Tc^2\ ]^{-1/2} \times [\ (f-c)/2c^2 + 1/Tc^2\ ]$$

$$\Rightarrow \quad \check{t}_a = t_{0a} + T/2\ [\ \sqrt{((c-f)/2c)^2 + f/Tc^2} - (c-2f)/2c\ ] +$$

$$+\ Tf/4\ [\ (f-c)/2c^2 + 1/Tc^2\ ]\ \ /\ \ \sqrt{((c-f)/2c)^2 + f/Tc^2}$$

APPENDIX 2: **User equilibrium objective function with sheared delay curve**

$$Z = \sum_a \int_0^{f_a} t_a(x)\,dx$$

$$= \sum_a \int_0^{f_a} t_a^0 + d_a(x)\,dx$$

$$= \sum_a f_a t_a^0 + \int_0^{f_a} d_a(x)\,dx$$

Omitting subscripts:

$$d = T/2 \left[\sqrt{((c-f)/2c)^2 + f/Tc^2} - (c-f)/2c \right]$$

$$= T/2 \left[\sqrt{(c^2 - 2fc + f^2)/4c^2 + f/Tc^2} - (c-f)/2c \right]$$

$$= T/2 \left[\sqrt{1/4 - f/2c + f^2/4c^2 + f/Tc^2} - (c-f)/2c \right]$$

$$= T/2 \left[\sqrt{(1/4c^2)f^2 + 2(1/2Tc^2 - 1/4c)f + 1/4} - (c-f)/2c \right]$$

$a = 1/4c^2$

$b = (1/2Tc^2 - 1/4c)$

$c = 1/4$

$$\int_0^{f} -T/2\,(c-x)/2c\,dx = -T/2 \int_0^{f} 1/2 - x/2c\,dx = T/2\,(f^2 - 2fc)/4c$$

$$\int_0^{f} \sqrt{ax^2 + 2bx + c}\,dx =$$

$$= (f/2 + b/2a)\sqrt{af^2+2bf+c} + [(ac-b^2)/2a\sqrt{a}] \ln k_1[((af+b)/\sqrt{a})\sqrt{af^2+2bf+c}]$$

$$f = 0 \Rightarrow \quad b\sqrt{\ }/2a + [(ac-b^2)/2a\sqrt{a}] \ln k_1 (b/\sqrt{a} + 1/2) = 0$$

$$\Rightarrow \qquad \ln k_1 = -[b\sqrt{a}/2(ac-b^2) + \ln(b/\sqrt{a} + 1/2)]$$

$$\Rightarrow \qquad Z = \sum_a \int_0^{f_a} t_a\,(x)\,dx$$

$$= T/2\ \Sigma_a\ [\ (f^2 - 2fc)/4c + (f/2 + b/2a)\sqrt{af^2+2bf+c} +$$

$$+ ((ac - b^2)/2a\sqrt{a})[\ln k_1 + \ln\{(af + b)/\sqrt{a} + \sqrt{af^2+2bf+c}\}]\ ]$$

with a, b, c and $\ln k_1$ as defined above.

APPENDIX 3: Implementation of the power policy

The power policy, as introduced in Van Vuren (1990c), can in its original form only be applied in under-capacity situations, as the pressure definition

$$f^k s^{1-k}/(\lambda s-f)$$

has no proper interpretation when flows exceed capacity. The promising behaviour in the tests described in Van Vuren (1990b), however, warrant application and further testing in more realistic circumstances.

Application in the SATURN context with sheared delay assumptions allows links to be oversaturated. In such circumstances the term $(\lambda s - f)$ in the denominator is replaced by $(0.001f)$, ensuring huge pressures, though still positive and existing, in over-capacity situations. I am aware that this is a rather crude way of dealing with infeasibility, but the tests show that it actually works!
Stage pressures are generalised as follows:

$$\text{stage pressure} = \sum_a f_a^k s_a^{1-k}/(\lambda_a s_a - f_a)$$

for all links a that have green during that stage. The power k is calculated as:

$$k = \sum_a (\lambda_a s_a - f_a)/s_a$$

summed over all incoming turns at the junction, with a minimum value of 0, and an upper limit of 1.

APPENDIX 4: Scenario parameters

Source: Van Vuren and Hounsell (1990)

Item	Descriptor	Possible Categories
1. Network size (i)	No of nodes	(i) small (<50), medium (50-200) large (>200)
	type	(ii) grid, radial, other
2. DRG network	% total road length	low, medium and high, related to classification and/or traffic flow
3. DRG system	Beacon density (no/km²)	low, medium, high
	Beacon update (mins)	frequent (5), infrequent (30/60)
	Route options	1, multiple
4. Traffic control system	'density' 'response'	isolated, UTC fixed time, traffic responsive
5. DRG/TCS		Nil, DRG→TCS, TCS→DRG, TCS↔DRG
6. Objective function		User optimal, system optimal, minimum time, minimum cost, other
7. Forecasting algorithms		To be determined
8. Traffic incidents	Duration (mins)	Short (<10), medium (10-60), long (>60)
	Severity (% cap loss)	Slight (<20), moderate (50), severe (>80)
	Location	Various
	System response delay (min)	Short (<5), long (>10)
9. Congestion	Ave. network speed (kph)	Low (>35), medium (25-35) high (<25)
10. Subscription level	% equipped drivers	Low (<10), moderate (10-30) high (>30)
11. Driver response	Extent of response to guidance	To be determined

APPENDIX 5: Convexity of system optimal assignment and delay minimising signal control.

Convexity is checked for via an identical approach as in Appendices 4.1 to 4.3. See there for more details.

BPR cost function

$$t = t_o \,(1 + \alpha(f/\lambda s)^{\beta})$$

$$V = \Sigma\, f\, t = \Sigma\, f\, t_o \,(1 + \alpha(f/\lambda s)^{\beta})$$

$$\partial V/\partial f = t_o + \alpha(\beta+1)\, f^{\beta}/\lambda^{\beta}s^{\beta}$$

$$\partial V/\partial \lambda = -\alpha\beta\, f^{\beta+1}/\lambda^{\beta+1}s^{\beta}$$

$$\partial^2 V/\partial f^2 = \alpha\beta(\beta+1)\, f^{\beta-1}/\lambda^{\beta}s^{\beta}$$

$$\partial^2 V/\partial f\partial\lambda = -\alpha\beta(\beta+1)\, f^{\beta}/\lambda^{\beta+1}s^{\beta}$$

$$\partial^2 V/\partial\lambda\partial f = -\alpha\beta(\beta+1)\, f^{\beta}/\lambda^{\beta+1}s^{\beta}$$

$$\partial^2 V/\partial\lambda^2 = \alpha\beta(\beta+1)\, f^{\beta+1}/\lambda^{\beta+2}s^{\beta}$$

$$J = \alpha\beta(\beta+1)(f/\lambda s)^{\beta} \begin{pmatrix} \frac{1}{f} & -\frac{1}{\lambda} \\ -\frac{1}{\lambda} & \frac{f}{\lambda^2} \end{pmatrix}$$

$$\|J\| = 1/\lambda^2 - 1/\lambda^2 = 0$$

and thus V is convex.

Webster's cost function

$$t = t_1 + t_2$$

$$= \frac{1}{2}\ \frac{C\,(1-\lambda)^2}{(1-f/s)} + \frac{1}{\lambda s - f} - \frac{1}{\lambda s}$$

$$V = \Sigma f t = \Sigma f t_1 + \Sigma f t_2$$

Omitting the factor $^1/_2$ the two terms will be investigated separately.

First term (f t_1)

$$V_1 = \frac{fC\,(1-\lambda)^2}{(1-f/s)} = \frac{C\,(1-\lambda)^2}{(1/f - 1/s)}$$

$$\partial V_1/\partial f = C\,(1-\lambda)^2 / (1/f - 1/s)^2$$

$$\partial V_1/\partial\lambda = -2C\,(1-\lambda)/(1/f - 1/s)$$

$$\partial^2 V_1/\partial f^2 = 2C\,(1-\lambda)^2 / (1/f - 1/s)^3$$

$$\partial^2 V_1/\partial f\partial\lambda = -2C\,(1-\lambda) / (1/f - 1/s)^2$$

$$\partial^2 V_1/\partial\lambda\partial f = -2C\,(1-\lambda) / (1/f - 1/s)^2$$

$$\partial^2 V_1/\partial\lambda^2 = 2C / (1/f - 1/s)$$

$$\|J\| = \frac{4C^2\,(1-\lambda)^2}{(1/f - 1/s)^4} - \frac{4C^2\,(1-\lambda)^2}{(1/f - 1/s)^4} = 0$$

thus V_1 is convex.

Second term f t_2

$$V_2 = f/(\lambda s - f) - f/\lambda s$$

$$\partial V_2/\partial f = 1/(\lambda s - f) + f/(\lambda s - f)^2 - 1/\lambda s$$

$$\partial V_2/\partial\lambda = -fs/(\lambda s - f)^2 + f/\lambda^2 s$$

$$\partial^2V_2/\partial f^2 = 1/(\lambda s\text{-}f)^2 + 1/(\lambda s\text{-}f)^2 + 2f/(\lambda s\text{-}f)^3$$

$$\partial^2V_2/\partial f\partial\lambda = -s/(\lambda s\text{-}f)^2 - 2fs/(\lambda s\text{-}f)^3 + 1/\lambda^2 s$$

$$\partial^2V_2/\partial\lambda\partial f = -s/(\lambda s\text{-}f)^2 - 2fs/(\lambda s\text{-}f)^3 1 + 1/\lambda^2 s$$

$$\partial^2V_2/\partial\lambda^2 = 2fs^2/(\lambda s\text{-}f)^3 - 2f/\lambda^3 s$$

$$J = \begin{pmatrix} \partial^2V/\partial f^2 & \partial^2V/\partial f\partial\lambda \\ \partial^2V/\partial\lambda\partial f & \partial^2V/\partial\lambda^2 \end{pmatrix}$$

$$\|J\| = -\{s/(\lambda s\text{-}f)^2 - 1/\lambda^2 s\}^2 < 0$$

Thus J is not positive semi-definite and V_2 is non-convex.