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Laissez-faire or full redistribution?

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ABSTRACT

We explore the implications of three basic and intuitive axioms for income redistribution problems: *equal treatment of equals*, *additivity* and *stability*. We show that the combination of the three axioms characterizes two focal and polar rules: *laissez-faire* and *full redistribution*.

1. Introduction

The maxim *laissez-nous faire* is traditionally attributed to the merchant Legendre, addressing Colbert some time towards the end of the seventeenth century, and many others (including, most notably, Bentham and Franklin) fixed *laissez-faire* in the popular mind ever since (e.g., Keynes, 2010). It precludes, among other things, the existence of income taxes. Its polar counterpart is *full redistribution*, which advocates to share equally the aggregate income in the population. We show in this note that, somewhat surprisingly, the two polar principles share a solid normative ground, when formalized as rules to solve income redistribution problems.

More precisely, we study income redistribution problems axiomatically. To do so, we consider a stylized model in which an income profile reflects the taxable and observable income of a group of agents. The issue is to construct rules that transform the given income profile into another income profile, with the requirement that nothing is wasted in the redistribution process, and that no agent ends up with a negative post-tax income. *Laissez-faire* leaves the income profile untouched. *Full redistribution* associates to each agent an equal amount of the aggregate income. We show that the two rules are jointly characterized when we combine three basic and intuitive axioms for this model. The first axiom (*equal treatment of equals*) states that equal pre-tax incomes guarantee equal post-tax incomes. The second axiom (*additivity*) states that redistribution is additive on incomes. The third one (*stability*) states that no further redistribution takes place once a solution is obtained.

2. The result

Let $N = \{1, \ldots, n\}$ be a set of **agents**. For each $i \in N$, let $y_i \in \mathbb{R}_+$ be i's **income**. We denote by $y \equiv (y_i)_{i \in N} \in \mathbb{R}^n_+$ the profile of incomes. The aggregate income is $Y \equiv \sum_{i \in N} y_i$. A (redistribution) **rule** is a mapping $R : \mathbb{R}^n_+ \longrightarrow \mathbb{R}^n_+$ such that $\sum_{i=1}^n R_i(y) = Y$, where $R_i(y)$ is the income of agent i after the redistribution. Two rules are focal (and polar). One corresponds to the identity function; the other imposes a uniform distribution of the aggregate income. That is,

Laissez-faire. For each $y \in \mathbb{R}^n_+$, and each $i \in N$,

$$R_i^{LF}(y) = y_i.$$

Full redistribution. For each $y \in \mathbb{R}^n_+$, and each $i \in N$,

$$R_i^{FR}(y) = \frac{Y}{n}.$$

We now present three axioms for rules, which reflect ethical or operational principles that are natural for this framework.

The first axiom is a standard formulation of the principle of impartiality, a basic requirement in the theory of justice (e.g.,

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Moreno-Ternero and Roemer, 2006). It states that if two agents have equal income, then they receive the same amounts in the redistribution.

Equal treatment of equals. For each $y \in \mathbb{R}^n_+$ and each $\{i,j\} \subseteq N$ such that $y_i = y_j$, $R_i(y) = R_j(y)$.

The second one is also standard in axiomatic work and it can be traced back to Shapley (1953). It states that redistribution must be additive with respect to incomes, which precludes some externalities, while conveying a form of simplicity.

Additivity. For each pair $y, y' \in \mathbb{R}^n_+$, $R(y+y') = R(y) + R(y')$.

The last one was introduced by Chambers and Moreno-Ternero (2021). It states that after a rule redistributes income for a given income profile, the rule proposes no further redistribution for the resulting income profile.

Stability. For each $y \in \mathbb{R}^n_+$, $R(R(y)) = R(y)$.

The next result states that, among all possible options to redistribute income, only the two introduced above meet the three axioms just presented.

Theorem 1. *A rule satisfies equal treatment of equals, additivity, and stability if and only if it is either laissez-faire or full redistribution.*

Proof. It is straightforward to show that both *laissez-faire* and *full redistribution* satisfy the three axioms. Conversely, let R be a rule that satisfies *equal treatment of equals*, *additivity*, and *stability*. Let $y \in \mathbb{R}^n_+$, and $i \in N$. By *additivity*,[1]

$$R_i(y) = \sum_{k=1}^{n} R_i(y_k, 0_{-k}) = \sum_{k=1}^{n} y_k R_i(1_k, 0_{-k}).$$

By *equal treatment of equals*, there exists $\alpha_k \in [0,1]$ such that

$$R_i(1_k, 0_{-k}) = \begin{cases} \alpha_k & \text{if } i = k, \\ \frac{1-\alpha_k}{n-1} & \text{otherwise.} \end{cases}$$

We now show that $\alpha_1 = \cdots = \alpha_n$. To do so, let $k \in N\backslash\{1\}$. By *additivity*, and the above,

$$R_1(1_{\{1,k\}}, 0_{-\{1,k\}}) = R_1(1_1, 0_{-1}) + R_1(1_k, 0_{-k}) = \alpha_1 + \frac{1-\alpha_k}{n-1},$$

and

$$R_k(1_{\{1,k\}}, 0_{-\{1,k\}}) = R_k(1_1, 0_{-1}) + R_k(1_k, 0_{-k}) = \frac{1-\alpha_1}{n-1} + \alpha_k.$$

By *equal treatment of equals*,

$$\alpha_1 + \frac{1-\alpha_k}{n-1} = \frac{1-\alpha_1}{n-1} + \alpha_k.$$

Or, equivalently, $\alpha_k = \alpha_1$, as claimed.

Let $\bar{\alpha} = \alpha_1 = \cdots = \alpha_n$ and $\tau = \frac{n\bar{\alpha}-1}{n-1}$. As $\bar{\alpha} \in [0,1]$, it follows that $\tau \in [\frac{-1}{n-1}, 1]$. Furthermore,

$$\begin{aligned} R_i(y) &= \bar{\alpha} y_i + \sum_{k \neq i} \frac{1-\bar{\alpha}}{n-1} y_k \\ &= \bar{\alpha} y_i + \frac{1-\bar{\alpha}}{n-1}(Y - y_i) \\ &= \frac{n\bar{\alpha}-1}{n-1} y_i + \frac{n(1-\bar{\alpha})}{n-1}\frac{Y}{n} \\ &= \tau y_i + (1-\tau)\frac{Y}{n}. \end{aligned}$$

Thus,

$$R_i(R(y)) = \tau\left[\tau y_i + (1-\tau)\frac{Y}{n}\right] + (1-\tau)\frac{Y}{n} = \tau^2 y_i + (1-\tau^2)\frac{Y}{n}.$$

By *stability*, $\tau y_i + (1-\tau)\frac{Y}{n} = \tau^2 y_i + (1-\tau^2)\frac{Y}{n}$. As $y \in \mathbb{R}^n_+$ was arbitrary, it follows that $\tau^2 = \tau$ and, therefore, $\tau \in \{0,1\}$. Now, if $\tau = 0$, it follows that $R \equiv R^{FR}$, whereas if $\tau = 1$, it follows that $R \equiv R^{LF}$, which concludes the proof. □

Theorem 1 provides normative foundations for both *laissez-faire* and *full redistribution*. It follows from inspection of its proof that the following family of rules, compromising between them, is precisely characterized by the first two axioms in its statement (*equal treatment of equals* and *additivity*).

Compromise rules. For each $\tau \in [-\frac{1}{n-1}, 1]$, each $y \in \mathbb{R}^n_+$, and each $i \in N$,

$$R_i^{\tau}(y) = \tau y_i + (1-\tau)\frac{Y}{n}.$$

When $\tau \in (0,1)$, we obtain the (non-degenerate) convex compromises. When $\tau \in [-\frac{1}{n-1}, 0)$, we obtain non-convex compromises, which are less likely for large populations (the larger n, the lower the number of non-convex compromises). In particular, the compromise rule obtained for $\tau = -\frac{1}{n-1}$ is an intriguing rule assigning each agent $i \in N$ an equal share of the overall remaining income, i.e., $\frac{Y-y_i}{n-1}$.

Note that compromise rules impose a flat tax, together with an equal reallocation (of the overall tax revenue) among all individuals. This implies that some agents are taxed, whereas others are subsidized, but all at an equal rate. More precisely, for each $\tau \in [0,1]$, each $y \in \mathbb{R}^n_+$, and each $i \in N$, $y_i - R_i^{\tau}(y) = (1-\tau)\left(y_i - \frac{Y}{n}\right)$. Thus, agents above the average (pre-tax) income are taxed (at an equal rate $1-\tau$), whereas agents below the average (pre-tax) income are subsidized (at the same equal rate).[2] If $\tau \in [-\frac{1}{n-1}, 0)$, then the (equal) rate $1-\tau$ is higher than 1.

3. Final remarks

We have shown that the two basic and polar rules of *laissez-faire* and *full redistribution* are jointly characterized when we combine three intuitive axioms: *equal treatment of equals*, *additivity* and *stability*. We acknowledge that our definition of rules involves two notions that might be considered as additional axioms too. On the one hand, the *efficiency* condition, which says that total income does not change. That is, $\sum_{i=1}^{n} R_i(y) = Y$. On the other hand, the *non-negativity* condition, which imposes non-negative incomes after redistribution. That is, $R(y) \geq 0$. Thus, one can restate our result as follows: A rule $R: \mathbb{R}^n_+ \longrightarrow \mathbb{R}^n$ satisfies *non-negativity*, *efficiency*, *equal treatment of equals*, *additivity* and *stability* if and only if it is either *laissez-faire* and *full redistribution*.

The model we consider in this note was first studied by Ju et al. (2007), as an instance of their generalized claims problems. They focus on the notion of *reallocation-proofness* (a group of agents cannot manipulate the outcome of the rule upon reallocating the incomes within the group) and show that this axiom, together with *no transfer paradox* (transferring some income before redistribution takes place does not increase income after redistribution), characterizes the family of income-tax schedules with a flat tax rate and personalized lump-sum transfers. That is,

$$R_i^{FG}(y) = (1 - F(Y))\, y_i + G_i(Y),$$

[1] See Aczel (2006, page 34).

[2] The general principle of comparing individual performance with average performance is often used in many contexts (e.g., Allen et al., 2017; Bergantiños and Moreno-Ternero, 2020; Ju et al., 2021).

where $\sum_{i\in N} G_i(Y) = F(Y)Y$. Thus, F determines the flat tax rate as a function of the aggregate income while G determines the reallocation scheme $(G_i(Y))_{i\in N}$ as a function of agents' identities subject to the budget balance. If $F(Y) = 1 - \tau$ and $G_i(Y) = (1-\tau)\frac{Y}{n}$, we precisely obtain the family of compromise rules R^{τ} mentioned above.

More recently, Chambers and Moreno-Ternero (2021) have studied the bilateral case of this model and have showed that *continuity*, *no transfer paradox* and *stability* characterize a large family (dubbed *threshold* rules), that guarantee partial redistribution for unequal incomes. The family is wide enough to encompass the two rules we characterize in Theorem 1.

Casajus (2015a,b, 2016) and Yokote and Casajus (2017) also studied variants of this same model and characterized several rules. The closest result to our work is the characterization of the family of convex compromise rules by means of the so-called *differential monotonicity* axiom. This axiom states that non-decreasing income differentials (for any pair of agents) translate into non-decreasing differentials of their post-redistribution rewards. In our context, *differential monotonicity* is essentially equivalent to the combination of *additivity* and *order preservation* (a natural strengthening of *equal treatment of equals*). Thus, it follows that the family can also be characterized by the combination of *additivity* and *order preservation*. As mentioned above, replacing the latter by the weaker axiom of *equal treatment of equals*, we characterize a larger family of compromise rules involving some non-convex ones. Similarly, *laissez-faire* and *full redistribution* are jointly characterized by *differential monotonicity* and *stability*.

To conclude, our model is also reminiscent of the taxation model considered by Young (1987, 1988, 1990).[3] We also deal with the same taxation problems here, with an important proviso: a zero tax revenue to be raised. Thus, negative taxes are possible in our model.

Data availability

No data was used for the research described in the article.

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[3] See also Ju and Moreno-Ternero (2011) and Chambers and Moreno-Ternero (2017), among others. Such a model is just a reinterpretation of the seminal model to analyze claims problems, introduced by O'Neill (1982) and recently surveyed by Thomson (2019).