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ON THE LYAPUNOV FUNCTION FOR THE ROTATING BÉNARD PROBLEM

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In this paper we study the nonlinear Lyapunov stability of the conduction-diffusion solution in a layer of a rotating Newtonian fluid, heated and salted from below.

If we reformulate the nonlinear stability problem, projecting the initial perturbation evolution equations on some suitable orthogonal subspaces, we preserve the contribution of the Coriolis term, and jointly all the nonlinear terms vanish.

We prove that, if the principle of exchange of stabilities holds, the linear and nonlinear stability bounds are equal. We find that the nonlinear stability bound is nothing else but the critical Rayleigh number obtained solving the linear instability problem.

Key words: Stability - Energy Method.

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1 Introduction

The classical Bénard problem, have been largely studied, [1], [2], [3], [4], [5], also including the influence, of effects such as a rotation field, a magnetic field, chemical reactions of reactive fluids.

This problem, of a big importance in astrophisics, geophysics, oceanography, meteorology has been largely investigated in the Oberbeck- Boussinesq approximation [3]-[11].

The point of loss of linear stability is usually also a bifurcation point, at which convective motions set in [1] -[21], and subcritical instabilities may occur explaining unusual phenomena..

In [10] [11] the problem of the coincidence of the critical and nonlinear stability bounds is studied, and in [10] the coincidence of linear and nonlinear stability parameters is deduced under some restriction on initial data.

In the magnetohydrodynamic case, in [12], for free boundaries, is deduced the coincidence of linear and nonlinear stability parameters, for a fully ionized fluid , i.e., if the conduction diffusion solution is linearly stable, it is conditionally nonlinearly asymptotically stable.

The mechanical equilibrium of a Newtonian thermoanisotropic fluid mixture in a horizontal layer heated from below was studied in [22], [23], [24], [25], where the given problem was changed in an equivalent one, with better symmetry properties, and a system equivalent to the perturbation evolution equations was derived.

The nonlinear stability analysis of the conduction-diffusion solution of the Be'nard problem, in presence of chemical surface reactions, was performed in [26] [27] [28] and, in the case of coincidence of Prandtl and Schmidt numbers, the equality between linear and nonlinear stability bounds was proved.

The nonlinear stability of the thermodiffusive equilibrium for a homogeneous fluid in a horizontal rotating layer with free boundaries was studied in [29], i.e. the rotating Bénard problem, and the equality of linear and nonlinear stability bounds was obtained, in the region of stationary convection of linear instability theory, without any restriction on initial data.

In [30] the nonlinear Lyapunov stability of the mechanical equilibrium for a fluid mixture in a plane layer, in presence of linear skewsymmetric effects, such as the Coriolis term in the rotating Bénard problem, was studied.

In this paper we reformulate the nonlinear stability problem of the thermodiffusive equilibrium for a mixture in a plane layer. Projecting the initial perturbation evolution equations on some suitable orthogonal subspaces, we preserve the contribution of the skewsymmetric term, and jointly all the nonlinear terms vanish.

We derive (Section 2) the perturbation evolution equations in terms of poloidal and toroidal fields, and then we project them on some orthogonal subspaces, obtaining (Section 3) an energy relation where all nonlinear terms vanish, whatever the boundary conditions are, and, jointly, the contribution of the skewsymmetric term is preserved.

We recover, in a simpler way, using the L^2 norm for the perturbation energy, the same result obtained in [29], i.e., in the region of the parameter space where the principle of exchange of stabilities holds, the coincidence of the nonlinear stability parameter with the critical Rayleigh number of the linear instability, without any restriction on initial data.

2 The initial/boundary value problem for perturbation

Let us consider a fluid mixture, in the framework of physics of continua and in the Oberbeck-Boussinesq approximation, in a horizontal layer $S,$ bounded by the surfaces $z=0$ and $z=d$ in a frame of reference $\{O,\vec{i},\vec{j},\vec{k}\}$, with $\vec{k}$ unit vector in the vertical upwards direction, in rotation around the fixed vertical axis z with a constant angular velocity $\vec{\Omega}=\Omega\vec{k}.$

Let us now perturb the zero solution corresponding to a motionless state, $\{\vec{0},\overline{T},\overline{C},\overline{P}\}$, where $\vec{0},\overline{T},\overline{C},\overline{P},$ represent, respectively, the velocity, the temperature, the concentration and the pressure fields.

The perturbation $(\vec{u},\theta,\gamma,p')$ of velocity, temperature, concentration and pressure fields satisfy the following nondimensional equations

$$\frac{\partial}{\partial t}\vec{u}=-(\vec{u}\cdot\nabla)\vec{u}-\nabla p'+R\theta\vec{k}-R_c\gamma\vec{k}+2\vec{u}\times\vec{\Omega}+\Delta\vec{u}, \tag{1}$$

$$P_r(\frac{\partial}{\partial t}\theta+\vec{u}\cdot\nabla\theta)=\Delta\theta+Rw, \qquad (t,\vec{x})\in(0,\infty)\times V \tag{2}$$

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$$S_c(\frac{\partial}{\partial t}\gamma + \vec{u}\cdot\nabla\gamma) = \Delta\gamma - R_c w, \qquad (t,\vec{x}) \in (0,\infty)\times V \tag{3}$$

$$\nabla\cdot\vec{u} = 0, \tag{4}$$

in the following subset of the Sobolev space $W^{2,2}(V)$,

$$N = \{(\vec{u}, p, \theta, \gamma) \in W^{2,2}(V) \,|\, \frac{\partial}{\partial z}u = \frac{\partial}{\partial z}v = w = \theta = \gamma = 0 on \partial V\}. \tag{5}$$

Let us suppose that each term of (1), as function of the space variable $\vec{x}$, belongs to the Sobolev space $W^{2,2}(V).$

In (1)-(3) $\vec{u} = (u,v,w)$, $V = V\times[0,1]$ denotes the three dimensional box over the rectangle V, periodic in the x, y directions and its boundary is denoted by ∂V, after assuming the perturbation fields, depending on the time t and space $\vec{x} = (x, y, z),$ doubly periodic functions in x and y, of period $2\pi/k_1$ and $2\pi/k_2.$

R^2, R_c^2, are the Rayleigh and solute Rayleigh numbers, P_r and S_c are the Prandtl and Schmidt numbers, respectively.

If we multiply (1) by $\vec{u},$ the contribute of the skewsymmetric term is lost, namely

$$(\vec{u}\times\vec{\Omega}, \vec{u}) = 0,$$

whence, to avoid a weaker resulting stability criterion, we modify the energy relation projecting the perturbation evolution equations on some suitable orthogonal subspaces of $W^{2,2}(V).$

Alternatively, we are forced to consider more complicated Lyapunov functions to evaluate the contribution of the skewsymmetric term [7].

Because of the representation theorem of solenoidal vectors [3] in a plane layer, if the mean values of u, v, w vanish over V [31], that is if the conditions

$$\int_V u(x,y,z)dxdy = \int_V v(x,y,z)dxdy = \int_V w(x,y,z)dxdy = 0, \qquad z\in[0,1],$$

hold, the velocity perturbation $\vec{u}$ has the unique decomposition [3]

$$\vec{u} = \vec{u}_1 + \vec{u}_2, \tag{6}$$

with

$$\nabla\cdot\vec{u}_1 = \nabla\cdot\vec{u}_2 = \vec{k}\cdot\nabla\times\vec{u}_1 = \vec{k}\cdot\vec{u}_2 = 0, \tag{7}$$

$$\vec{u}_1 = \nabla\frac{\partial}{\partial z}\chi - \vec{k}\Delta\chi \equiv \nabla\times\nabla\times(\chi\vec{k}), \vec{u}_2 = \vec{k}\times\nabla\psi = -\nabla\times(\vec{k}\psi), \tag{8}$$

where $\nabla\times$ is the curl operator, the poloidal and toroidal potentials χ and ψ are doubly periodic satisfying the equations [3]

$$\Delta_1\chi \equiv \frac{\partial^2}{\partial x^2}\chi + \frac{\partial^2}{\partial y^2}\chi = -\vec{k}\vec{u}, \quad \Delta_1\psi = \vec{k}\cdot\nabla\times\vec{u}. \tag{9}$$

From now going on, we denote $\frac{\partial}{\partial x}f \equiv f_x$, where f is an arbitrary function. The boundary conditions for χ and ψ, for free planar surfaces, are [3]:

$$\chi = \chi_{zz} = \psi_z = 0 \quad z = 0,1. \tag{10}$$

From (6)-(7) it follows that

$$\vec{u}\cdot\vec{k}=\vec{u}_1\cdot\vec{k}=-\Delta_1\chi. \tag{11}$$

In order to project the perturbation equation (1) on some suitable subspaces of $L^2(V)$ we observe that

$$\vec{u}=\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}), \tag{12}$$

because of $\nabla\cdot\vec{u}=0,$

$$\Delta\vec{u}=-\nabla\times\nabla\times\vec{u}. \tag{13}$$

Let us recall the Weyl decomposition theorem [4], [33]

$$L^2(V)=G(V)\oplus N(V), \tag{14}$$

with $G(V)$ and $N(V)$ spaces of generalized solenoidal and potential vectors respectively.

So, the advective term in (1) can be uniquely obtained as

$$(\vec{u}\cdot\nabla)(\vec{u}_1+\vec{u}_2)=\nabla U+\nabla\times\vec{A}, \tag{15}$$

where U is a scalar function and $\vec{A}$ a vector field we specify later.

If we define the scalar and vector fields

$$\Phi=\nabla\cdot(\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2)),\quad \vec{W}=\nabla\times(\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2)), \tag{16}$$

the imbedding Sobolev theorems of $W^{2,2}(V)$ in the space of continuous functions $C(\overline{V})$ [32] allows us to prove the following identity

$$\nabla\times(\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2))\equiv\nabla\times(\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2)-\nabla U). \tag{17}$$

Let us define

$$\vec{B}=\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2)-\nabla U, \tag{18}$$

by choosing $\nabla\cdot\vec{B}=0$, the scalar function U is (up to a constant) the solution of the interior Neumann problem [33] in the periodicity cell V

$$\Delta U=\Phi \tag{19}$$

$$\frac{\partial}{\partial\vec{n}}U=\Gamma, \tag{20}$$

where $\frac{\partial}{\partial\vec{n}}U$ is the normal derivative of U on the boundary ∂V of the periodicity cell V and $\Gamma=-\vec{B}\cdot\vec{n}.$

It is well known that the interior Newmann problem in the general case has no solution, only if the relation

$$\int_V\Phi dv-\int_{\partial V}\Gamma dv=\int_{\partial V}(\vec{u}\cdot\nabla(\vec{u}_1+\vec{u}_2))\cdot\vec{n}d\sigma+\int_V\nabla\cdot\vec{B}dv=0, \tag{21}$$

is fulfilled, a solution of (19), (20) can exist.

Taking into account the solenoidality of $\vec{B}$, it follows that exists a vector field $\vec{A}$ such that $\vec{B}=\nabla\times\vec{A}$, i.e. (15).

The perturbation equation (1), taking into account (12), (13) and (14), becomes

$$\begin{aligned}
&\frac{\partial}{\partial t}(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))+\nabla U+\nabla\times\vec{A}=-\nabla p'+2P[(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))\times\vec{\Omega}]+\\
&\quad-\nabla\times\nabla\times(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))+2(I-P)[(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))\times\vec{\Omega}]+\\
&\quad+P(R\theta\vec{k}-R_c\gamma\vec{k})+(I-P)(R\theta\vec{k}-R_c\gamma\vec{k})
\end{aligned} \tag{22}$$

where I and P are, respectively, the identity operator and the projection on the subspace $G(V)$ of $L^2(V)$.

We can write again the linear skewsymmetric term as follows [30]

$$\vec{\Omega}\times(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))=$$

$$\nabla(\vec{\Omega}\cdot\nabla\times(\chi\vec{k}))+\nabla\times(\vec{\Omega}\times\nabla\times(\chi\vec{k}))-\nabla\times(\vec{\Omega}\times\psi\vec{k})-$$

$$\nabla(\psi\Omega_3)+\vec{\Omega}\nabla\cdot(\psi\vec{k}). \quad (23)$$

We evaluate now the last term $\vec{\Omega}\nabla\cdot(\psi\vec{k})$ in the previous equality.

$$\vec{\Omega}\nabla\cdot(\psi\vec{k})=\Omega_3\partial_3\psi\vec{k}.$$

Therefore we can write again the last two terms of (23) as follows

$$-\nabla(\psi\Omega_3)+\vec{\Omega}\nabla\cdot(\psi\vec{k})=-\nabla_1(\psi\Omega_3).$$

Whence the projection of Coriolis term on the subspace $G(V)$ is given by

$$P[\vec{\Omega}\times(\nabla\times\nabla\times(\chi\vec{k})-\nabla\times(\psi\vec{k}))]=-\nabla_1(\psi\Omega_3). \quad (24)$$

If we project the equation (22) on the subspaces of solenoidal and potential vectors, taking into account that the only vector belonging to both previous subspaces is null [4], from the imbedding Sobolev theorems of $W^{2,2}(V)$ in the space of continuous functions $C(\overline{V})$ [32], it follows that

$$\nabla U=-\nabla p'+2\nabla_1(\psi\Omega_3)+P(R\theta\vec{k}-R_c\gamma\vec{k}). \quad (25)$$

If we consider the scalar product of (25) by $\vec{u}_1$, which is solenoidal, we obtain

$$(\nabla_1(\psi\Omega_3),\vec{u}_1)=0, \quad (26)$$

that, in terms of poloidal and toroidal fields, becomes

$$(\psi_x,\chi_{xz})+(\psi_y,\chi_{yz})=0. \quad (27)$$

This is the equation that allows us to preserve the contribution of the skewsymmetric term in nonlinear Lyapunov stability, as we shall see later.

3 Lyapunov stability

If we multiply (1) by $\vec{u}$, (2) by $\frac{b}{P_r}\theta$ and (3) by $\frac{d}{S_c}\gamma$, where b,d are some positive parameters, adding the resulted equations and integrating over $V,$ we obtain

$$\frac{d}{dt}\frac{1}{2}\int_V\{(\vec{u}^2+b\theta^2+d\gamma^2\}dV=R(1+\frac{b}{P_r})(\theta,w)-R_c(1+\frac{d}{S_c})(\gamma,w)+$$

$$+\frac{b}{P_r}(\theta,\Delta\theta)+\frac{d}{S_c}(\gamma,\Delta\gamma)+(\vec{u},\Delta\vec{u}). \quad (28)$$

Let us introduce the function

$$E_L(t)=\frac{1}{2}\{|\chi_{xz}|^2+|\chi_{yz}|^2+|\Delta_1\chi|^2+|\psi_y|^2+|\psi_x|^2+b|\theta|^2+d|\gamma|^2\}. \quad (29)$$

Taking into account (27), (28) and (29), we can write again the previous energy relation as follows

$$\frac{d}{dt}E_L = RI - E, \tag{30}$$

where

$$I \equiv -(1+\frac{b}{P_r})(\theta,\Delta_1\chi)+\frac{R_c}{R}(1+\frac{d}{S_c})(\gamma,\Delta_1\chi)+\frac{f(\vec{\Omega})}{R}((\psi_x,\chi_{xz})+(\psi_y,\chi_{yz})) \tag{31}$$

$$E \equiv \frac{b}{P_r}|\nabla\theta|^2+\frac{d}{S_c}|\nabla\gamma|^2+|\nabla\chi_{xz}|^2+|\nabla\chi_{yz}|^2+|\nabla\Delta_1\chi|^2+$$

$$+|\nabla\psi_x|^2+|\nabla\psi_y|^2, \tag{32}$$

where f is an arbitrary function of $\vec{\Omega}$.

Now we determine a condition ensuring that

$$\frac{d}{dt}E_L \le 0, \quad \forall t \ge 0. \tag{33}$$

The equation (30) becomes:

$$\frac{d}{dt}E_L = RI - E = -E(1-R\frac{I}{E}). \tag{34}$$

If $E_L > 0,\ E > 0,$ from the the inequality

$$R < \sqrt{Ra_*}, \tag{35}$$

where

$$\frac{1}{\sqrt{R_{a*}}} = \max\frac{I}{E}, \tag{36}$$

in the class of admissible functions, we deduce

$$\frac{d}{dt}E_L \le -(1-\frac{R}{\sqrt{R_{a*}}})E. \tag{37}$$

Hence, in this case, if (35) is satisfied, the functional E_L is a decreasing function of t. The inequality (33) represents a stability uniqueness criterion [3], [4].

4 The maximum problem and the stability bound

We will study the variational problem (36) and later determine the parameters b,d in terms of the physical quantities, such that $\sqrt{R_{a*}}$ will be maximal.

The Euler Lagrange equations associated with the maximum problem (36) are:

$$-(1+\frac{b}{P_r})\Delta_1\theta+\frac{R_c}{R}(1+\frac{d}{S_c})\Delta_1\gamma+\frac{f(\vec{\Omega})}{R}\Delta_1\psi_z+\frac{2}{\sqrt{R_{a*}}}\Delta\Delta_1\chi=0,$$

$$-(1+\frac{b}{P_r})\Delta_1\chi+\frac{b}{P_r}\frac{2}{\sqrt{R_{a*}}}\Delta\theta=0, \tag{38}$$

$$\frac{R_c}{R}(1+\frac{d}{S_c})\Delta_1\chi+\frac{d}{S_c}\frac{2}{\sqrt{R_{a*}}}\Delta\gamma=0, \tag{39}$$

$$\frac{f(\vec{\Omega})}{R}\Delta_1\chi_z+\frac{2}{\sqrt{R_{a*}}}\Delta\Delta_1\psi=0.$$

Taking into account (9), the system of Euler equations equivalently read

$$-(1+\frac{b}{P_r})\Delta_1\theta+\frac{R_c}{R}(1+\frac{d}{S_c})\Delta_1\gamma+\frac{f(\vec{\Omega})}{R}\zeta_z-\frac{2}{\sqrt{R_{a*}}}\Delta\Delta w=0,$$

$$(1+\frac{b}{P_r})w+\frac{b}{P_r}\frac{2}{\sqrt{R_{a*}}}\Delta\theta=0, \tag{40}$$

$$-\frac{R_c}{R}(1+\frac{d}{S_c})w+\frac{d}{S_c}\frac{2}{\sqrt{R_{a*}}}\Delta\gamma=0,$$

$$-\frac{f(\vec{\Omega})}{R}w_z+\frac{2}{\sqrt{R_{a*}}}\Delta\zeta=0,$$

where $\zeta\equiv\Delta_1\psi$.

We shall suppose that the principle of exchange of stabilities holds, i.e. we assume that the instability occurs as a stationary convection.

In the class of normal mode perturbations

$$w(\vec{x})=W(z)exp[i(k_1x_1+k_2x_2)],\quad \zeta(\vec{x})=Z(z)exp[i(k_1x_1+k_2x_2)]$$

$$\theta(\vec{x})=\Theta(z)exp[i(k_1x_1+k_2x_2)],\quad \gamma(\vec{x})=\Gamma(z)exp[i(k_1x_1+k_2x_2)],$$

the equations (40) become

$$k^2(1+\frac{b}{P_r})\Theta-\frac{R_c}{R}k^2(1+\frac{d}{S_c})\Gamma+\frac{f(\vec{\Omega})}{R}DZ-\frac{2}{\sqrt{R_{a*}}}(D^2-k^2)^2W=0,$$

$$(1+\frac{b}{P_r})W+\frac{b}{P_r}\frac{2}{\sqrt{R_{a*}}}(D^2-k^2)\Theta=0, \tag{41}$$

$$-\frac{R_c}{R}(1+\frac{d}{S_c})W+\frac{d}{S_c}\frac{2}{\sqrt{R_{a*}}}(D^2-k^2)\Gamma=0,$$

$$-\frac{f(\vec{\Omega})}{R}DW+\frac{2}{\sqrt{R_{a*}}}(D^2-k^2)Z=0,$$

where $k^2=k_1^2+k_2^2$ is the wave number.

To (41) we add the following boundary conditions:

$$W=D^2W=\Theta=D^2\Theta=\Gamma=D^2\Gamma=DZ=0. \tag{42}$$

Owing to (9), and (10), if we assume, [5] $W(z)=\sum_{n=1}^{\infty}W_n\sin(n\pi z)$, from (41) we have

$$R_{a*}(R^2,R_c^2,k^2,n^2\pi^2,b,d)= \tag{43}$$

$$\frac{4R^2(n^2\pi^2+k^2)^3}{-[f(\vec{\Omega})]^2n^2\pi^2+k^2R^2\frac{P_r}{b}(1+\frac{b}{P_r})^2-k^2R_c^2\frac{S_c}{d}(1+\frac{d}{S_c})^2},$$

on the subset of the parameter space where the denominator is positive definite.

Differentiating (43) with respect to the parameters b , and d we obtain

$$\frac{\partial}{\partial b}R_{a*}=0\Leftrightarrow\frac{b}{P_r}=1,\quad \frac{\partial}{\partial d}R_{a*}=0\Leftrightarrow\frac{d}{S_c}=1. \tag{44}$$

Substituting (44) in (43) we obtain R_{a*} as a function of R^2 and R_c^2

$$R_{a*}(R^2,R_c^2,k^2,n^2\pi^2)=\frac{R^2(n^2\pi^2+k^2)^3}{\frac{-[f(\vec{\Omega})]^2n^2\pi^2}{4}+k^2R^2-k^2R_c^2}, \tag{45}$$

defined on the subset $\frac{-[f(\vec{\Omega})]^2n^2\pi^2}{4}+k^2R^2-k^2R_c^2>0$.

The critical Rayleigh function of the linear instability theory, that is

$$R_{cr}^2=R_c^2+\frac{(n^2\pi^2+k^2)^3+4\Omega^2n^2\pi^2}{k^2}, \tag{46}$$

belongs to the subset where the denominator of (45) is definite positive, if $f(\vec{\Omega})\equiv 4\Omega.$

Evaluating (45) for $R^2=R_{cr}^2$, we obtain

$$R_{cr}^2(k^2,n^2\pi^2)=R_{a*}(k^2,n^2\pi^2). \tag{47}$$

Hence we proved the following theorem

Theorem 4.1 If the principle of exchange of stabilities holds, the zero solution of (1)-(5), corresponding to the basic conduction state is nonlinearly globally stable if

$$R^2<R_{a*},$$

where $R_{cr}^2=R_{a*}(R_{cr}^2,k^2,n^2)$, attains its minimum where $n=1$. Whence the linear and non linear stability bounds, obtained for $n=1$

$$R_{cr}^2(k^2)=R_{a*}(R_{cr}^2,k^2).$$

coincide.

5 Conclusions

In this paper we have studied the nonlinear Lyapunov stability in a rotating Bénard problem, considering the proiection of perturbation evolution equations, written in terms of toroidal and poloidal fields, in some orthogonal subspaces of $L^2(V)$.

We have reduced the number of scalar fields, applying no more differential operators to perturbation evolution equations, and after we have obtained an energy relation for the Lyapunov function in which all the nonlinear terms disappear and the skewsymmetric term is preserved.

We have studied the nonlinear Lyapunov stability. After solving the Euler-Lagrange equations associated with the maximum problem, we maximize the stability domain with respect to the parameters introduced in the Lyapunov functional and, if the principle of exchange of stabilities holds, we recover the equality between the linear and nonlinear critical parameters for the global stability, we obtain a sufficient condition of nonlinear stability and a necessary an sufficent condition of linear stability.

We observe that, anyhow, in this paper we applied an idea similar to used in [22] [23] [24] [26] [27], where the given problem governing the perturbation evolution was changed in order to obtain an optimum energy relation, and we modified the evolution equations obtaining equations with better symmetries, which incorporate the given equations. In this way, [22] we preserve the contribution of some skewsymmetric terms, otherwise, if the initial evolution equations were used, such a contribution would be null and, correspondingly, the stability criterion weaker. Alternatively, we are forced to use some more complicated energies, to preserve the contribution of the skewsymmetric term [7] .

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