On the Nonclassical Behaviour of Unipartite and Bipartite Systems

A summary of the PhD Dissertation of

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I. The subject of the dissertation and the goals of the research

The results presented in this thesis can be divided into two groups. Concerning unipartite quantum systems (in the sense that no tensor product structure of the state space is considered) we have presented an alternative view of quantum state discrimination, while for bipartite systems we have considered the connection between nonlocal correlations and game theory. Our approach examines thos aspects of the statistical behaviour of bipartite systems that are determined by general principles (like no communications faster than the speed of light) not the particular physical design of the device. These can have significance for example in device independent cryptography.

In the long since and extensively studied field of quantum state discrimination we have introduced a new approach by using ROC (Receiver Operating Characteristic) curve which is used in classical statiscics for characterizing an ensemble of discriminators. We have introduced its quantum variant and examined its properties. As a result we could relate the curve

to several quantum properties and quantitative features, gaining an expressive and intuitive picture about their correspondence.

Additionally, we have found a quantity whose definition naturally follows from the shape of the ROC curve: the quantum-Bhattacharyya coefficient. This can serve as a similarity measure for quantum states. The most important open question in this part is our conjecture that two density matrices commute if and only if their square root fidelity and quantum-Bhattacharyya coefficient are equal.

The study of noclassical behaviour of bipartite systems is a central topic of quantum informatics. Both from theoretical (e.g. Tsirelson conjecture) and practical (e.g. practical quantum encription devices) side have remarkable significance. In the second part of the dissertation we have investigated a relatively new research topic: the correspondence between two-party nonlocal correlations and Bayesian games.

We have introduced a systematic method for generating games with nonlocal advantage that covers most of the cases found in the literature. We have analysed in detail the struc-

ture of the local and no-signaling polytopes in case of two-input-three-output Bell situations, including the illustration of our method. Usig different notions of game theoretical equilibrium we have uncovered certain structural properties of Bell inequalities.

Meanwhile, an exception is also found: the correlation discovered originally by Vértesi and Bene. The authors did not investigate this correlation from the game theorethic point of view, their intention was to demontrate that if the dimension of the Hilbert space is bounded, general POVM-s may have advantage over projective measurement with respect to violating Bell inequalities. We have found that their inequality is a basis for a game which has unusual equilibrium properties.

II. New scientific results

1. I introduced the notion of quantum ROC curve. It is a generalization of the classical ROC curve which gives a detailed picture about the discriminability of two quantum states.

   On the classical ROC diagram every binary discrimina-

tor corresponds to one point determined by the true positive and false negative rates. When discriminating probability distributions, the area determined by all theoretically possible discriminators, or its upper boundary carries important imformation about the probability distributions. We get the quantum ROC curve if we examine the discriminators of density matrices instead.

The upper boundary of the then covered area is the quantum ROC curve. From this curve we can easily read off the trace distance, and its shape determines the possibility or impossibility of unambigous quantum state discrimination. (publication IV.)

2. I defined the *quantum Bhattacharyya-coefficient* which is a similarity measure like, e.g., the fidelity. It was inspired by the fact, that the (classical) Bhattacharyya-coefficient can be obtained from the ROC curve by it's line integral according to the Minkowski metric. However in the quantum case the integral does not reproduce the square root of fidelity rather gives another (generally smaller) value.

I proved that this quantity is zero if and only if the two states are perfectly distighuisable, i.e. their density matrices have disjoint support, while it is one if and only if the two states are identical. Additionally, the quantity is monotone increasing under completely positive maps. (publication IV.)

3. I gave a constructive method for generating games with nonlocal advantage that covers all games of the kind known to me from the literature. (publications I., II., III.)

4. By analysing in detail the structure of the two-input-three-output Bell situation, a new Bayesian game with nonlocal advantage is introduced (Secretary game). In this game, both players have two types and three actions. If the types are identical, then Alice must choose the action that is one larger than Bob's action (modulo 3), if they have different types, then they must choose identical actions in order to win. This game has both quantum and nonlocal advantage. (publications I., II., III.)

5. I showed that the game derived from the Vértesi-Bene Bell inequality has different *ex-ante* and *ex-post* equilibra. For the other games known from the literature these two equilibra coincide. (publications I., II., III.)

III. List of related publications

I. Koniorczyk Mátyás, Bodor András and Pintér Miklós Péter. Ex ante versus ex post equilibria in classical Bayesian games with a nonlocal resource. *PHYSICAL REVIEW A*, **101**(2020). ISSN 1050-2947. doi:10.1103/PhysRevA.101.062115.

II. Koniorczyk Mátyás, Bodor András and Pintér Miklós Péter. Nemklasszikus korrelációk Bayes-i játékokban. In *XXXIII. MAGYAR OPERÁCIÓKUTATÁSI KONFERENCIA: Program és előadáskivonatok*. 2019.

III. Koniorczyk Mátyás and Bodor András. No-Signaling in Quantum Mechanics. *JOURNAL OF RUSSIAN LASER RESEARCH*, **39**(2018):376–381. ISSN 1071-2836. doi:10.1007/s1094
018-9731-3.

IV. Bodor András and Koniorczyk Mátyás. Receiver Oper-

ation Characteristics of Quantum State Discrimination. *JOURNAL OF RUSSIAN LASER RESEARCH*, **38**(2017):150–163. ISSN 1071-2836. doi:10.1007/s10946-017-9629-5:.

IV. Other publications

[1] Kalmár, Alexandra, Nagy, Zsófia Brigitta, Udvardyné Galamb, Orsolya, Csabai, István, Bodor, András, Wichmann, Barnabás, Valcz, Gábor, Barták, Barbara Kinga, Tulassay, Zsolt, Igaz, Péter and Molnár, Béla. Genome-wide expression profiling in colorectal cancer focusing on lncRNAs in the adenoma-carcinoma transition. *BMC CANCER*, **19**(2019). ISSN 1471-2407. doi:10.1186/s12885-019-6180-5.

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