A Transfer Function Method to Calculate the Outcome of Radiation Measurements of a Buck Converter

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1   Introduction

Radiation measurements are performed in fully or semi-anechoic chambers or GTEM cells and are usually costly and time-consuming. In addition, most manufacturers of electronic devices do not have the capability to perform such measurements quickly by themselves. The transfer function approach aims at simplifying this problem: once the characteristics of the measurement setup are captured in a transfer function, the conducted characterization of the device under test (DUT) is sufficient to calculate the results of a radiation measurement. This is justified by the assumption that the radiation is mainly generated by the currents on the supply cables and the DUT is small compared to the smallest wavelength considered. Additionally, the DUT should behave linearly. In previous work [2] it was shown that a characterization of the DUT as a Norton equivalent source together with the scattering parameters of the measurement setup are sufficient to calculate the voltage at the received antenna in an anechoic chamber. In this work, the method is applied to a buck converter for both a GTEM cell and a fully-anechoic chamber setup.

2   Fundamentals

In Image 1, a typical radiation measurement setup for a GTEM cell with a spectrum analyzer (SA) as receiver at the connector of the cell is shown. The power supply lines for the DUT are connected via artificial mains networks (AMNs), which are placed outside the cell. If the DUT is shielded or if its radiation is negligible compared to the radiation generated by the currents on the supply cables, the following transfer function approach can be applied.
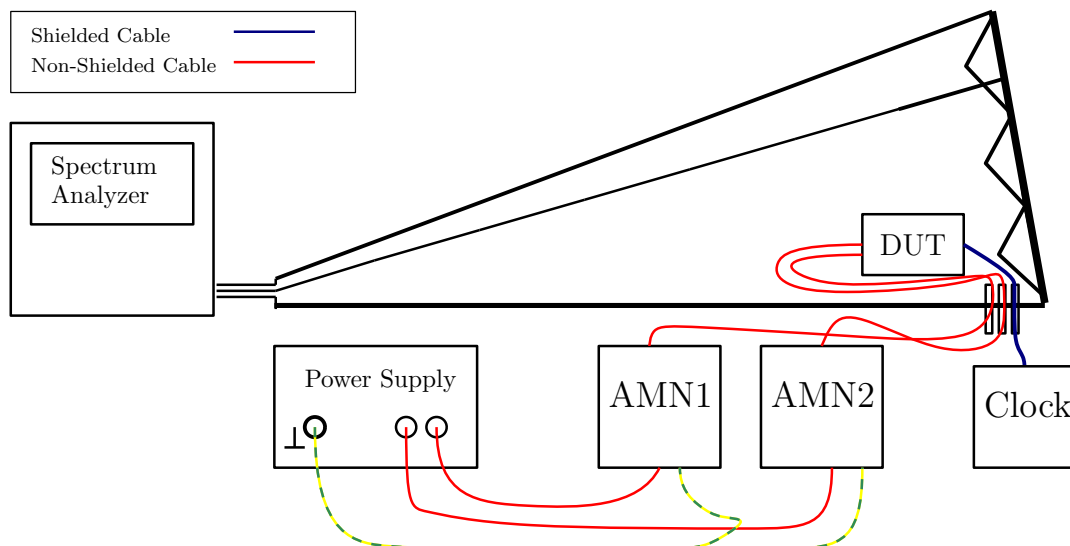

Image 1: Setup for a radiation measurement in a GTEM cell. In this example, the DUT has two power ports and one clock input.
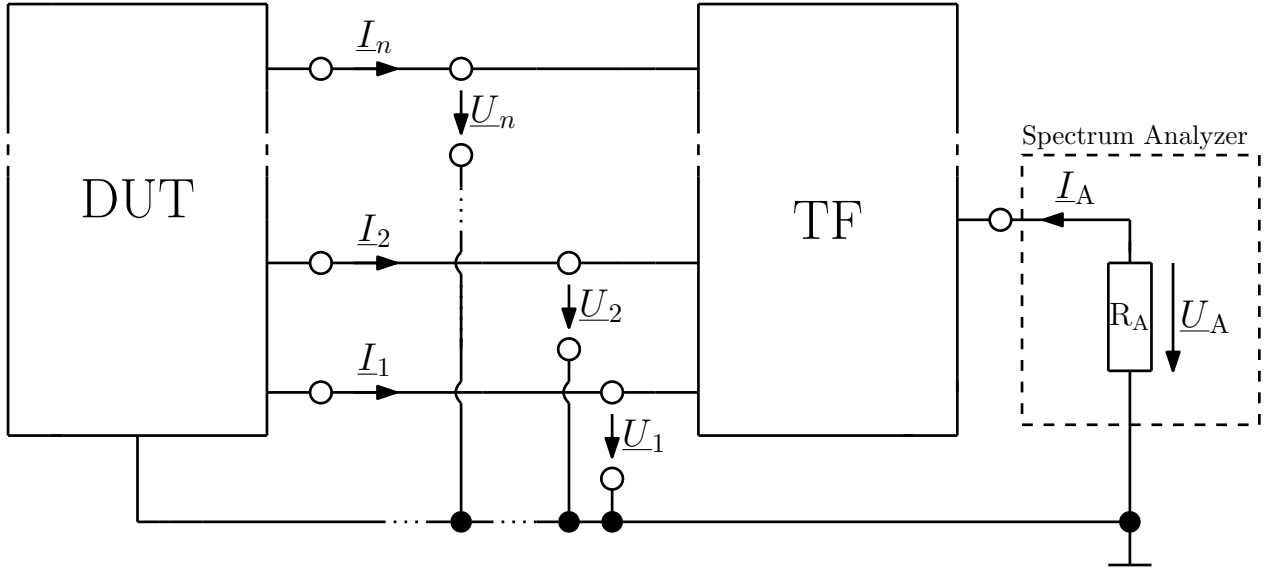
Image 2: Schematic model of the transfer function approach. Anything between the DUT and the spectrum analyzer is characterized as an unknown multiport circuit.

2.1 Transfer Function Approach

The setup is split at the point where the DUT is connected to the wiring harness of the supply lines. The DUT is characterized as an equivalent source, whose interference signals are transmitted to the SA by a transfer function. This transfer function should include every component of the setup as the cables, the properties of the surrounding and termination impedances of the AMNs (TF, see Image 2). In the case of a GTEM cell, the SA is connected to the coax connector at the top of the cell. In the case of a chamber it is connected to an antenna. Consider the case of a n port DUT with one additional common ground. The TF can be written in form of an $(n+1) \times (n+1)$ scattering matrix $\underline{\mathbf{S}}_{\mathrm{TF}}$, relating the n ports, where the DUT would be connected, and the SA as port $n + 1$. The scattering parameters are converted into admittance parameters [6]

$$\underline{\mathbf{Y}}_{\mathrm{TF}} \cdot Z_0 = (\mathbf{E} + \underline{\mathbf{S}}_{\mathrm{TF}})^{-1} \cdot (\mathbf{E} - \underline{\mathbf{S}}_{\mathrm{TF}}) \tag{1}$$

with the characteristic impedance $Z_0 = 50\,\Omega$ and the unit matrix $\mathbf{E}$. The admittance parameters relate the voltages and currents at the spectrum analyzer $(\underline{U}_{\mathrm{A}}, \underline{I}_{\mathrm{A}})$ to the port voltages and currents [4]

$$\underline{\mathbf{Y}}_{\mathrm{TF}} \cdot \begin{bmatrix} \underline{U}_{\mathrm{A}} \\ \underline{U}_1 \\ \vdots \\ \underline{U}_n \end{bmatrix} = \begin{bmatrix} \underline{I}_{\mathrm{A}} \\ \underline{I}_1 \\ \vdots \\ \underline{I}_n \end{bmatrix} . \tag{2}$$

2.2 Norton Equivalent Source

The DUT is described by a Norton equivalent source (see Image 3). It is assumed that the DUT behaves linearly at the frequencies of interest, which is the case for most devices with input capacitors as the buck converter used in this work. The model includes n current sources and $n \cdot \frac{n+1}{2}$

admittances [5]. The internal admittance $\underline{\mathbf{Y}}_{\mathrm{DUT}}$ is a symmetric $n \times n$ matrix

$$
\begin{bmatrix} \underline{I}_{0,1} \\ \vdots \\ \underline{I}_{0,n} \end{bmatrix} - \begin{bmatrix} \underline{I}_1 \\ \vdots \\ \underline{I}_n \end{bmatrix} = \underline{\mathbf{Y}}_{\mathrm{DUT}} \cdot \begin{bmatrix} \underline{U}_1 \\ \vdots \\ \underline{U}_n \end{bmatrix}
$$

$$
= \begin{bmatrix} \underline{Y}_1 + \sum\limits_{i \neq 1}^{n} \underline{Y}_{1i} & -\underline{Y}_{12} & \cdots & -\underline{Y}_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ -\underline{Y}_{n1} & -\underline{Y}_{n2} & \cdots & \underline{Y}_n + \sum\limits_{i \neq n}^{n} \underline{Y}_{ni} \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_1 \\ \vdots \\ \underline{U}_n \end{bmatrix} \tag{3}
$$

and the voltages and currents are vectors. The nomenclature of the matrix elements corresponds to Image 3.

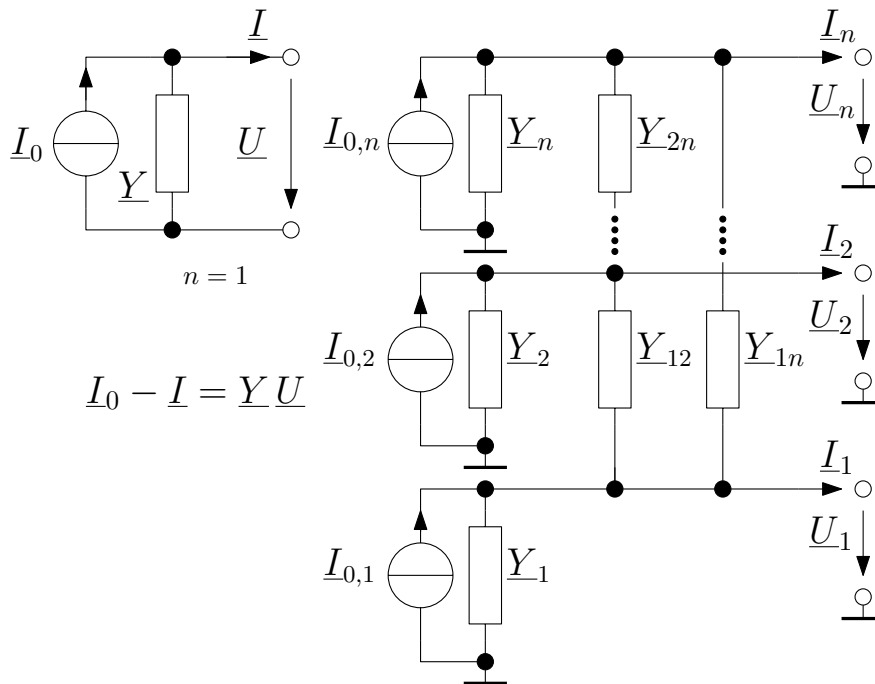$$ \underline{I}_0 - \underline{I} = \underline{Y}\,\underline{U} $$

Image 3: On the left side, a Norton equivalent source with one port is shown together with the fundamental equation relating the source parameters to the port voltage $\underline{U}$ and current $\underline{I}$. The general model with n ports is shown on the right side. Reciprocity implies that $\underline{Y}_{ki} = \underline{Y}_{ik}$ [2].

2.3   Solving for the Spectrum Analyzer Voltage

Ohm's law relates the current to the voltage at the spectrum analyzer

$$ \underline{U}_{\mathrm{A}} = -R_{\mathrm{A}} \cdot \underline{I}_{\mathrm{A}} \tag{4} $$

with $R_{\mathrm{A}} = 50\,\Omega$ being the SA input impedance. Equation (3) is solved for the currents $\underline{I}_{1..n}$ and inserted into (2). The current $\underline{I}_{\mathrm{A}}$ is replaced according to (4) giving

$$
\left(\underline{\mathbf{Y}}_{\mathrm{TF}} + \begin{bmatrix} \frac{1}{R_{\mathrm{A}}} & 0 \\ 0 & \underline{\mathbf{Y}}_{\mathrm{DUT}} \end{bmatrix} \right) \cdot \begin{bmatrix} \underline{U}_{\mathrm{A}} \\ \underline{U}_1 \\ \vdots \\ \underline{U}_n \end{bmatrix} = \begin{bmatrix} 0 \\ \underline{I}_{0,1} \\ \vdots \\ \underline{I}_{0,n} \end{bmatrix} . \tag{5}
$$

as relation between the equivalent source currents, the port and spectrum analyzer voltage, solving this system of equations yields the SA input voltage $\underline{U}_\mathrm{A}$.

3 Measurements

The DUT is a buck converter with an input voltage of $48\,\mathrm{V}$ switched at a clock rate of $130\,\mathrm{kHz}$ with a duty cycle of $25\,\%$. The converter is placed in a metallic enclosure with three SMA ports for the input voltage, the driver supply of $14\,\mathrm{V}$ and the clock. This ensures that the radiation is generated exclusively by the cables. In addition, the cables can be positioned reproducibly due to the fixed adapter position. The output voltage of $12\,\mathrm{V}$ is applied to a resistor, a power of $3\,\mathrm{W}$ is delivered. The resistor is located inside the enclosure to simplify the model to the case of two external radiating cables. The current return path is given by a metallic ground plane. Now, the determination of all unknowns from equation 5 will be described. The port of the input voltage is considered to be port number one and the second port is the port where the driver voltage is applied. The clock signal is provided by a shielded cable and therefore not considered as part of the transfer function.

3.1 Reference Measurement

First, a reference measurement is made with a spectrum analyzer (SA, Keysight PXE EMI Receiver N9048B). A bandwidth (RBW) of $9\,\mathrm{kHz}$ is selected, which is lower than required in e. g. the CISPR 25 standard [1] to increase the signal to noise ratio. The buck converter is put into operation both in a GTEM cell and a fully-anechoic chamber in a setup shown in Image 4. In the latter case, a Rohde&Schwarz HK116 biconical antenna is used, limiting the frequency range from $30\,\mathrm{MHz}$ to $300\,\mathrm{MHz}$. In the case of the GTEM cell, the frequency range of interest ranges from $30\,\mathrm{MHz}$ to $1\,\mathrm{GHz}$. The DUT is positioned outside of the cell in this case since a small GTEM cell (TESEQ GTEM 250A SAE) is used, which dimensions would not allow to put the big casing of the buck converter inside.

3.2 DUT Admittance

Since the buck converter has smoothing capacitors on both input and driver lines, its terminal behavior can be considered linear in the frequency range of interest from $30\,\mathrm{MHz}$ to $1\,\mathrm{GHz}$, which is much higher than the switching frequency. A vector network analyzer (VNA, Rohde & Schwarz ZVA8) is used to determine its scattering parameters, which are converted into admittance parameters $\underline{\mathbf{Y}}_\mathrm{DUT}$. It is important to calibrate to the ports where the supply lines are connected to the DUT. At higher frequencies, even small errors in the scattering parameter phase lead to a large absolute error in the admittance. Therefore, phase shifts generated by adapters which are not covered by the calibration are figured out by time domain reflectometry and corrected. The admittances labeled according to Image 3 are shown in Image 5. It is evident that the off-diagonal elements of the admittance matrix are negligible. The periodic variations of the diagonal elements result from the attached cables.

3.3 Equivalent current sources

The equivalent current sources are determined from an oscilloscope measurement (Keysight MXR608A) of the interference voltage at the AMNs, if the DUT is directly attached to AMNs. A highpass with a $3\,\mathrm{dB}$ pass frequency of $27.5\,\mathrm{MHz}$ is used to reduce spectral leakage due to low frequency variations of the signal. The signals are fourier transformed to get $\underline{U}_{\mathrm{AMN},1...n}$. Since the AMNs are not specified for such high frequencies, the measured voltages are not equal to the voltages $\underline{U}_{\mathrm{DUT},1...n}$ actually present at the output of the DUT.
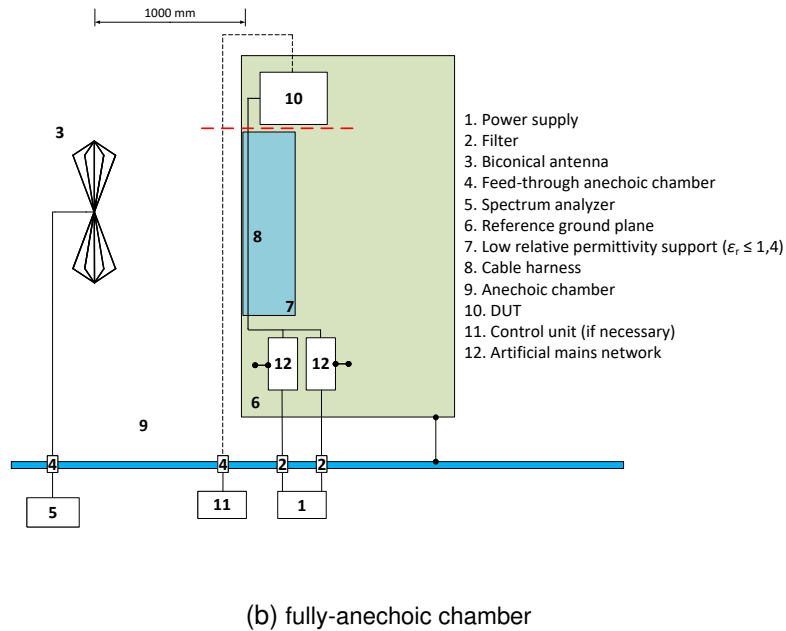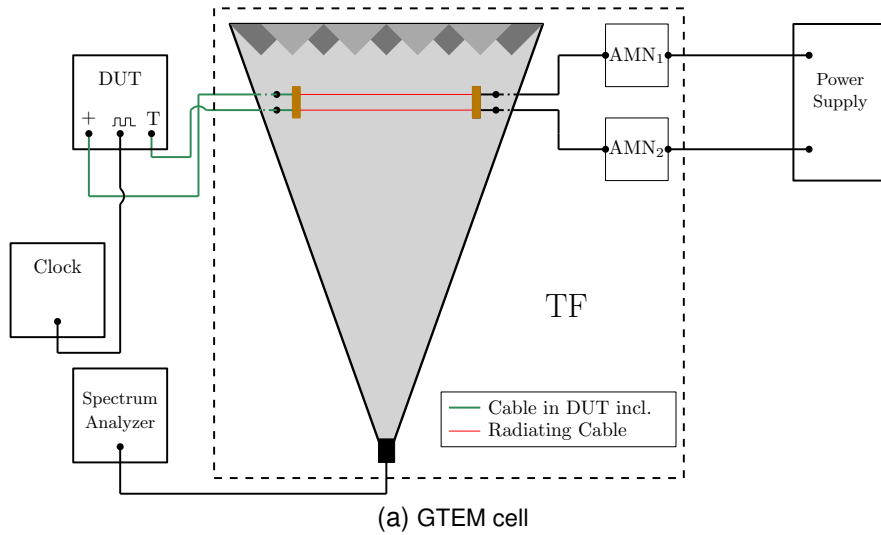
(a) GTEM cell

1. Power supply
2. Filter
3. Biconical antenna
4. Feed-through anechoic chamber
5. Spectrum analyzer
6. Reference ground plane
7. Low relative permittivity support ($\varepsilon_r \leq 1{,}4$)
8. Cable harness
9. Anechoic chamber
10. DUT
11. Control unit (if necessary)
12. Artificial mains network

(b) fully-anechoic chamber

Image 4: Setup of a radiation measurement. In (a) the cables that belong to the DUT are green-colored, in (b) the split is marked by a red dashed line.

For this purpose, the scattering parameters of the AMNs are determined and converted into chain parameters **A** with the definition [6]

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} \underline{U}_2 \\ -\underline{I}_2 \end{bmatrix} \tag{6}$$

for currents $\underline{I}_1$, $\underline{I}_2$ with the current direction arrows towards the two-port network. The voltage at the DUT

$$\underline{U}_{\mathrm{DUT},k} = \left(A_{11} + \frac{A_{12}}{R_{\mathrm{A}}}\right) \cdot \underline{U}_{\mathrm{AMN},k} \tag{7}$$
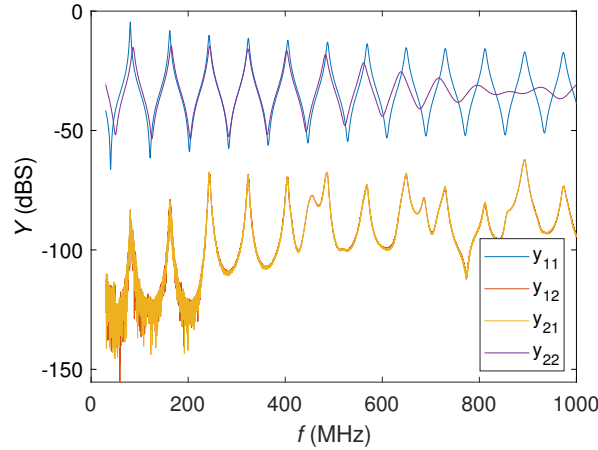
is usually larger than the measured one.

Image 5: Admittance matrix elements of the buck converter including the connection cables.

The equivalent current sources depicted in Image 3

$$
\begin{bmatrix} \underline{I}_{0,1} \\ \vdots \\ \underline{I}_{0,n} \end{bmatrix} = (\underline{\mathbf{Y}}_{\mathrm{DUT}} + \underline{\mathbf{Y}}_{\mathrm{AMN}}) \cdot \begin{bmatrix} \underline{U}_{\mathrm{DUT},1} \\ \vdots \\ \underline{U}_{\mathrm{DUT},n} \end{bmatrix} \tag{8}
$$

can be obtained directly by nodal analysis.

The settings of the oscilloscope depend on the settings of the spectrum analyzer in the reference measurement. The sampling rate should be more than twice the maximum frequency one wants to observe. The frequency spacing of the fourier transform should be in the order of the RBW of the SA. In this case a duration of $100\,\mu s$ is chosen.

3.4   Transfer Function

The scattering parameters of the transfer function are determined by a vector network analyzer (VNA). The DUT and the spectrum analyzer are removed and the VNA is connected according to 2.1 - the first port is connected to the antenna or the connector of the GTEM cell. The other ports are connected where the DUT is located in the reference measurement. A correction for additional adapters is required.

3.5   Spectrum Analyzer Influence

The fourier transform of the signal arriving at the input of the SA is not exactly the spectrum it displays. The influence of the resolution bandwidth and the detector need to be considered. The filter of the SA is modelled as a Gaussian filter curve [3, 7]

$$
G_{f_{\mathrm{C}}}(f) = \mathrm{e}^{-(f-f_{\mathrm{C}})^2/c^2}
$$
$$
c = \frac{\mathrm{RBW}}{2\sqrt{\ln(2)}} \tag{9}
$$

with RBW being the $-6\,\mathrm{dB}$ bandwidth and f_{C} as its center frequency. The RBW filter is tuned to that center frequency and multiplied with the input spectrum. The time dependent signal at the envelope detector is obtained by an inverse fourier transform.

In Image 6, an example with $f_{\mathrm{C}} = 150\,\mathrm{MHz}$ and a bandwidth of $120\,\mathrm{kHz}$ is shown. The procedure is repeated for every frequency point of the input spectrum as center frequency. The number of
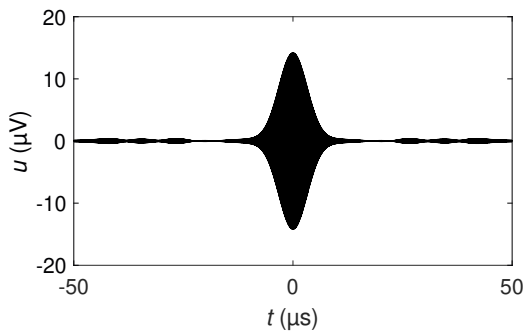
points displayed are usually far less than the number of points of the input spectrum. These display points are the maxima of a group of envelope time signals in the case of a peak detector.
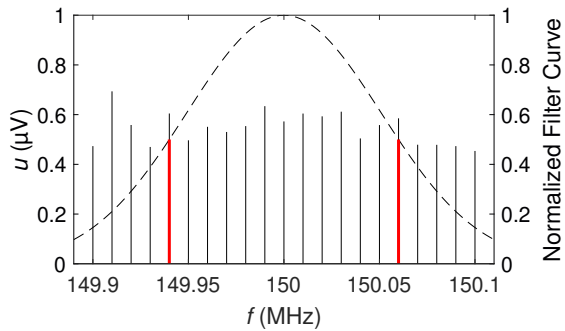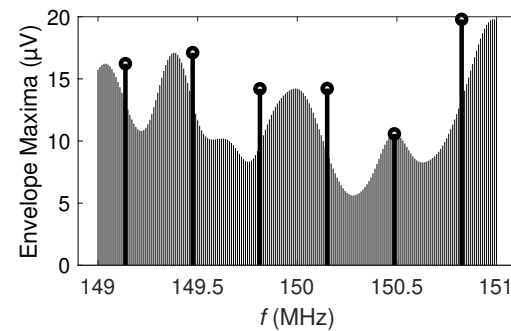

(a) Spectrum of an example input signal and normalized gaussian filter curve with red marked resolution bandwidth.
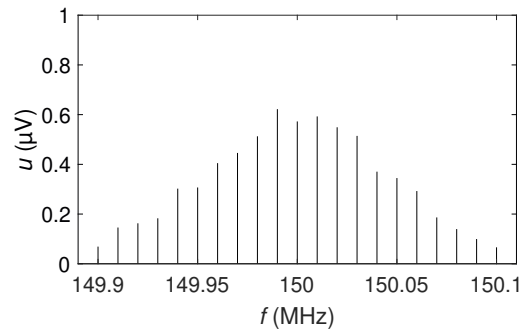
(b) Example spectrum multiplied with the filter curve.

(c) Time domain signal of the multiplication result in (b).

(d) Maxima of the time signal envelopes for several center frequencies. The marked ones are the displayed points in the case of a peak detector.

Image 6: Schematic procedure of the spectrum analyzer model.

4 Results

In Image 7 the comparison of the calculated result to the measured spectrum analyzer voltage is shown. In the case of the GTEM cell there are some peaks at about $90\,\text{MHz}$ and $900\,\text{MHz}$ that are not covered by the calculation. They originate from radio and mobile communication which couple into the supply lines outside the cell. The internal noise of the SA is captured and set as minimum output for the calculation routine. Additionally, the frequencies are restricted to values below $800\,\text{MHz}$ because the high pass used in the oscilloscope measurement is not specified for higher frequencies.

5 Conclusion and Outlook

A buck converter is characterized as a Norton equivalent circuit. The radiation generated by the currents on its supply lines is calculated applying the transfer function approach. An $(n+1)$-port is placed where the DUT is connected to its n supply lines and the SA as last port. The multi-port transfer function is characterized by a scattering parameter measurement. Combining both characterizations leads to the voltage at the SA, whose properties are considered as well. It is shown that the method is able to calculate the outcome of a radiation measurement within an accuracy of a few dB even if the signal level is as low as $-110\,\text{dBV}$. Once the transfer function of a standardized measurement setup is determined, manufacturers of small devices could estimate

the outcome of radiation measurements with low effort. The conditions are that the devices can be characterized as linear n-ports, are small compared to the measurement setup and do mainly radiate on their supply cables. In further work the method should be applied to non-linear devices or devices with radiating load circuits.
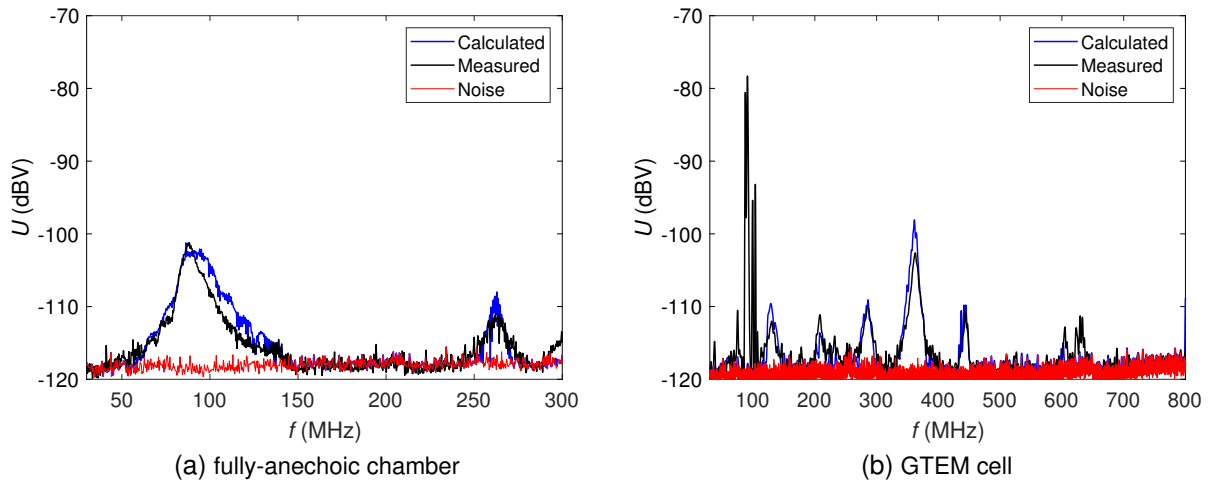

(a) fully-anechoic chamber

(b) GTEM cell

Image 7: Comparison of the spectrum analyzer output with the calculated result.

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