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# Throughput Analysis For Layout Optimisation Of Modular Conveyor Systems

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**Abstract**

In this paper, objective functions for the optimisation of modular conveyor systems will be introduced. Modular conveyor systems consist of conventional as well as modular conveyor hardware, which are arranged in form of matrix-like layouts. The aim of an ongoing research project is to provide small and medium-sized enterprises with a user-friendly decision support for the selection and planning of modular conveyor systems. For this purpose, the conveyor systems should be evaluated according to the objectives throughput and space requirement. Therefore, mathematical equations have been developed, which enable a fast and precise evaluation of layouts. The paper focuses mainly on the efficient calculation of the throughput. The result quality of the evaluation equations regarding the throughput was proven by a simulation of example systems.

**Keywords**

modular conveyor; conveyor system evaluation; throughput analysis; layout optimisation; logistics

## 1. Introduction

Conveyor systems are defined as technical devices that are used to automatically transport goods between two or more locations. Until now, conventional conveyor technology, such as belt, roller or chain conveyors, has been used in intralogistics systems such as warehouses, distribution centres or production. These conveyors are characterised by a high handling capacity. However, in case of complex material flows with many curves, junctions or intersections, they often require many additional routes, which are similar to the intersections on large motorways. This routing leads to an increased space requirement. At the same time, conventional conveyors are inflexible with regard to modifications, for example when adding new inputs and outputs or changing transport quantities.

In recent years, various modular conveyor systems were developed. Modular conveyor technology consists of functional components or modules that can quickly and flexibly be connected to each other via defined hardware and software interfaces. In this way, it is possible to adapt a conveyor system to changed requirements with minimal effort. In general, conveyor modules are characterised by a uniform aspect ratio. In addition, they can convey goods in multiple directions, in contrast to conventional conveyor technology, which can usually only convey in one dimension (forwards and backwards). In this way, different intralogistics functions can be realised, such as conveying, sequencing, buffering as well as infeed and outfeed. Another advantage is that the conveyor modules can be decentrally controlled. Examples of modular conveyor technology are the Celluveyor from cellumation [1] as well as the FlexConveyor and the GridSorter from flexlog [2]. In addition to these commercially available systems, there are also other modular conveyor systems such as the conveyor matrix from the research projects CogniLog and netkoPs (Figure 1), which are still being developed [3,4].

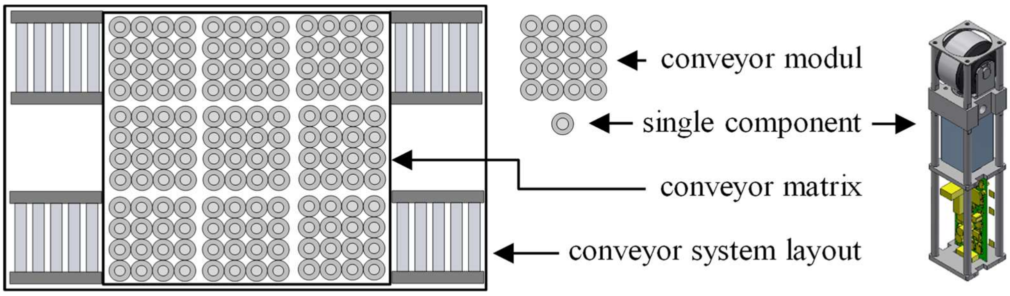


Figure 1: components of a modular conveyor system layout

Due to the two-dimensional transport, modular conveyor technology makes it possible to realise complex material flows in a very confined space. Due to the novelty of the systems, however, companies and logistics planners do not have substantial experience in planning and setting them up. For example, the question arises how many conveyor modules are necessary for the transport of a defined quantity of goods and how these must be positioned considering given inputs and outputs. To support this planning problem, a method for layout optimisation of modular conveyor systems is developed within the research project OptiLay – "Automated creation of optimised conveyor system layouts for modular conveyor systems" [6,5]. Optimisation algorithms are used for the placement of the conveyor modules. To evaluate the computer-generated layouts, a quantitative calculation or measurement of objective criteria is necessary. The evaluation must also be as fast and automated as possible in order not to slow down the optimisation. This paper presents an approach for the evaluation of modular conveyor systems for the objective criteria throughput and space requirement.

## 2. Related Research

The calculation and evaluation of the objective criteria space requirement can be performed with simple analytical formulas. Preliminary work on this is provided by SHCHEKUTIN [7]. The throughput of a conveyor system with only one material flow can also be calculated easily. If there are several material flows between different sources and sinks, complex intersection situations arise depending on the layout, for those no simple analytical approach exists to determine the total throughput. DALLERY AND GERSHWIN [8] as well as LI [9] provide exact solution approaches for simple intralogistics conveyor systems and machines with a steady-state distribution of the transport goods. These approaches are not suitable for complex conveyor systems with many conveyor modules, as the individual intersection situations have to be modelled in a complex process [10]. There are various approximation methods for the throughput calculation, which are based on the decomposition of the system into subsystems. For each subsystem, a throughput is calculated. The subsystems are then reconnected and the throughput of the entire system is estimated. ARNOLD provides a basic framework for the calculation of various subsystems [11]. SCHMIDT and JACKMANN developed a decomposition approach for recirculating conveyor systems with blocking before service. GAO ET AL. introduced a decomposition approach for multiple material flows [10]. They developed an algorithm for the decomposition into subsystems. The throughput analysis is based on their previous research [12]. The approach was applied to three conveyor systems in which only simple intersections and merges occur. The analysed systems were simulated for comparison, and a calculation correctness of 90 % could be confirmed. SHECHKUTIN also developed a layout optimisation approach for modular conveyor systems [7,13]. The study is mainly focused on the results of the research project netkoPs. The throughput calculation is based on the

detached calculation of the throughput time per material flow. Here, the transport times for the used conveyors are accumulated. If the material flow runs through an intersection point or a modular conveyor matrix, the transport time of this transport route is offset by a factor. The factor is calculated for each intersection situation based on the size of the modular conveyor matrix. It is assumed that a larger conveyor matrix leads to a simplification of the conflicts. The conflicts themselves are not calculated concretely.
In addition to analytical modelling, simulation is used to evaluate the throughput of conveyor systems. Extensive simulation software is available for this purpose, such as AnyLogic or Plant Simulation. There are many throughput analyses in the literature that have been carried out using simulation. With the help of simulation, almost any complex conveyor system can be analysed. However, complex and time-consuming modelling is necessary, which has so far been done manually for the most part, since complex control procedures have to be implemented (routing, blocking prevention). Due to the modelling effort, simulation is not suitable for a quick calculation of the throughput in a layout optimisation. MAYER developed a routing method for the FlexConveyor [14]. Within the study, a throughput analysis was carried out by simulation for different layouts of the FlexConveyor (e.g. straight conveyor, line sorter, circles and circles with intersections). The aim was to check the routing approach. SEIBOLD developed and simulation-based validated a routing method for the GridSorter, focusing on avoiding deadlocks [15]. The GridSorter consists mainly of the components of the FlexConveyor, but is characterised by a uniform transport direction. With the help of the GridSorter, goods can be sorted between different lanes. KRÜHN [16] and SOHRT [17] also developed routing methods but for the CogniLog respectively netkoPs conveyor matrix. The routing methods were also validated with simulation. KRÜHN used a reservation logic to avoid blockages or deadlocks. SOHRT, on the other hand, developed a time-window-based approach.
In summary, none of the existing approaches meets the previously described requirements for direct application in the context of an optimisation (computing time and automated modelling). Therefore, the approach presented below was developed. Of course, existing approaches such as decomposition were adopted in the design process.

## 3. Representation of the optimisation problem

The optimisation problem is formulated as an extended quadratic assignment problem. Accordingly, the area in which the conveyor modules will be arranged is covered with a grid of uniform squares. This results in a discrete coordinate system of cells. The conveyor modules also have a square shape. The cells of the base area and the conveyor modules must be of equal size. Accordingly, only conveyor modules with the same dimensions can be combined within a layout. The sources and sinks of the conveyor system are adjacent to the footprint. Each material flow is defined by a type of specific goods, a transport quantity, a source and a sink. Each type of good has dimensional attributes in form of a horizontal and a vertical length. Whereby the horizontal length always reflects the longer side length of a good. This means that if a good is transported in a horizontal direction (e.g. from east to west in plan view), the longer edge is parallel to the direction of flow. Accordingly, the shorter edge is parallel to the flow direction when a good is transported in a vertical direction. If several material flows run parallel respectively together in one path section, they are combined with regard to their attributes. The transport quantity is summed up. The dimensions of the goods are converted into quantity-weighted average lengths.
Based on the arrangement of the conveyor modules, a graph of the conveyor system is also generated. This can be used to calculate the transport routes. For this purpose, the conveyor modules have a rotation attribute in addition to a position attribute. In this way, it can be checked whether the inputs and outputs of the conveyor modules are adjacent to each other and thus if a transport is possible. If this is the case, the nodes of the conveyor modules are connected by an edge. The transport path of a material flow can be determined by common path-finding algorithms. Figure 2 a) shows an exemplary conveyor system layout. Figure 2 b) illustrates the graph derived from it. Here it is important to consider that a conventional conveyor is

represented by three nodes and a modular conveyor by up to five nodes. Each conveyor has a node for its centre point and nodes for the transitions to other conveyors. The adjacent transitions are aggregated into one node.

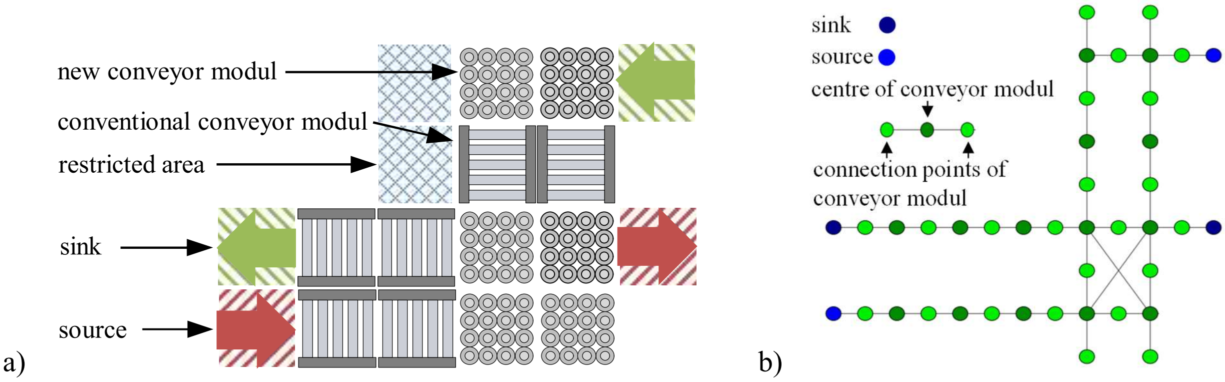


Figure 2: a) conveyor system layout, b) graph representation

## 4. Evaluation of conveyor systems

In the following section, the equations for the evaluation of conveyor systems are presented. In addition to calculating the objective values for each objective $Z$, it is also important to normalise them. This is necessary because the values of the objectives do not have the same scaling. For example, the summation of a throughput with 1,000 pieces per hour and a space requirement of 20 square metres would mean, that a change of the required space would have almost no influence on the sum of the objective values. There are several approaches to the normalisation of objective values. HARMONOSKY and TOTHERO developed a method in which each sub-value of an objective (e.g. the throughput of a single material flow) is divided by the sum of all sub-values for that objective [18]. SINGH and SINGH developed a procedure in which the normalisation is performed with the help of a multi-stage calculation process that includes standard deviations and mean values of sub-values [19].

Within this research project, the evaluation formulas of the objective criteria were designed in such a way that a percentage value between 0 and 100 is returned. Thus, the individual objective values neither have to be scaled nor normalised in order to enable a comparison. In most cases, the percentage value is computed by calculating the ratio of the best-known value to the current value of an objective. The disadvantage of the method is that when a new best objective value is found, all previously calculated values of the same objective must be updated. If these values are not directly needed to control the optimisation method, the update can also be done at the end of the optimisation. The weighting of the objectives is possible without restrictions.

### 4.1 Throughput

In order to calculate the throughput $\lambda^k$ of a conveyor system $k$, the decomposition approach is also used. For this, the transport paths of the material flows are first checked for intersections in the graph. A conveyor module $f$ is an intersection $u$ if two or more material flows do not use the same adjacent conveyor modules. Accordingly, a parallel transport on a straight line or in a curve is not considered as an intersection unless the transport direction is opposite. This definition results in the intersections shown in Figure 3. Case B represents only one possible instance of an intersection over several conveyor modules, which is described below. There are no other intersection cases beyond the ones shown. A deviating number of material flows is mapped via a virtualisation of material flows. This means that material flows that pass through an intersection completely in parallel are combined into a virtual material flow based on their attributes.

In the throughput calculation, only the intersections are considered afterwards. In general, the bottleneck in a linked system determines the throughput of the system. In the case of the conveyor system, an intersection with the lowest throughput determines the throughput of the entire conveyor system (equation 2). This only applies as long as all material flows within a conveyor system intersect each other. If there are several independent material flows or material flow systems in a conveyor system, the bottleneck must be determined for each subsystem. In this case, the total throughput is the sum of the bottleneck throughputs of the subsystems.

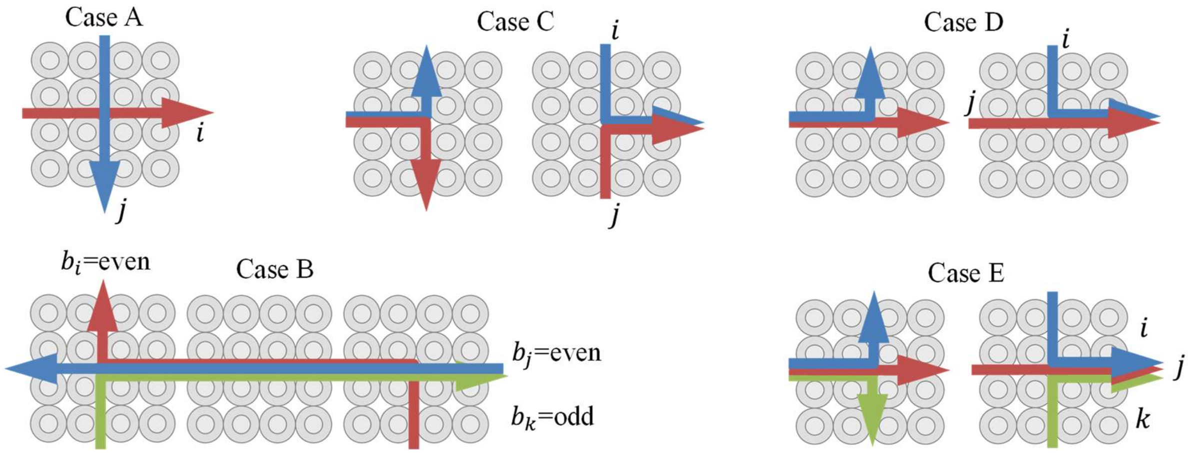


Figure 3: types of intersection scenarios

$f$: conveyor module, with $f \in F$

$u$: Conveyor module on which material flows intersect, with $u \in U$ und $U \subseteq F$

$\lambda_u$: throughput of an intersection $u$

$\lambda^q$: throughput of a conveyor system $q$

$\lambda^{best}$: best-known throughput

$$\lambda_u = \frac{t_u}{3600} \tag{1}$$

$$\lambda^q = \min(\lambda_u)\ \forall\, u \tag{2}$$

$$Z_\lambda = \frac{\lambda^q}{\lambda^{best}} * 100 \tag{3}$$

In the following subsections, evaluation formulas for all relevant intersection situations are presented. Basically, these are based on the calculation of the cycle time $t_u$. For this purpose, the quantities of goods $a_i$ of the material flows $i$ are first set in relation to each other in order to calculate a batch size $n_i$. For example, two material flows with 400 and 600 pieces per hour correspond to batches of 2 and 3 goods per load cycle. A load cycle represents the sequence of transport movements on a conveyor that are necessary to process the batches. The number of load cycles results from the greatest common divisor of the quantities of the material flows. Dividing the quantity of a material flow by the number of load cycles gives the batch size. Then the transport time $t_u$ of each load cycle of the intersection is calculated. This calculation is based on the accumulation of transport distances. Distances are, for example, the conveyor module length $s_u$ or the respective length of the goods to be transported $l_{iH}$ and $l_{iV}$. The sum of the distances is then divided by the conveying speed $v_u$. The following parameters are used for the calculations:

$i; j; k$: material flow with specific good ($i \neq j \neq k$)

$a_i$: target quantity of goods [parts/hour]

$n_i$: batch size of material flow $i$

$l_{iH}$: dimension of a good of material flow $i$ in horizontal direction ($H$)

$l_{iV}$: dimension of a good of material flow $i$ in vertical direction ($V$)

$i'$: virtual material flow resulting from combination of several material flows with same transport direction

$\overline{l_{i'H}}$: quantity-weighted dimension of a virtual good from the combination of several material flows $i$ in horizontal direction ($H$)

$\overline{l_{i'V}}$: quantity-weighted dimension of a virtual good from the combination of several material flows $i$ in vertical direction ($V$)

$\overline{n_{i'}}$: batch size of the virtual material flow

$v_u$: transport velocity of a conveyor module $u$

$s_u$: length of a conveyor module $u$

$t_u$: time for a load cycle of all material flows on a conveyor module $\sum t_{ui}$

$t_{ui}$: time in which a batch $n_i$ of material flow $i$ is transported on a conveyor module $u$

#### 4.1.1 Basic intersection of two material flows (Case A)

The most basic form of intersection occurs when two material flows pass a conveyor module where the transport direction is offset by 90°. See case A Figure 3. The time of the load cycle is composed as follows. First all goods of the lot of material flow $i$ pass the intersection. For this purpose, the corresponding lengths are added up. The lot size is multiplied by the length of the goods, and the distance across the intersection is added. This results in the total length that must be moved so that all goods of the batch pass the conveyor. This is then also done for the second material flow $j$, which flows in a 90° rotated direction.

$$t_u = \frac{(s_u + n_i * l_{iH}) + (s_u + n_i * l_{jV})}{v_u} \qquad (4)$$

#### 4.1.2 Intersection over several conveyor modules (Case B)

The extension of the basic intersection is already the most difficult intersection situation in terms of controlling a conveyor system. It is an intersection of material flows with opposite directions. To avoid colliding in the intersection, the goods must be stopped before entering. Like in other intersections, the goods of the material flows must wait for each other. Accordingly, all conveyor modules are blocked in the intersection situation. In order to calculate the load cycle, it must also be determined whether the goods change direction in the intersection, as this has an influence on the travel distance. If goods change direction with an even number, then they flow out of the crossing area in the same direction as they came in (equation 5a). If the number of direction changes is odd, the horizontal and vertical length of the goods must be considered once when calculating the transport distance (formula 5b).

$h$: number of conveyor modules $f$ in an intersection situation in opposite directions

$b_i$: number of direction changes of a material flow in the intersection situation

$$t_{ui} = \frac{(h * s_u + n_i * l_{iV})}{v_u} \forall\, b_i \text{ or } t_{uj} = \frac{(h * s_u + n_j * l_{jH})}{v_u} \forall\, b_j\text{, with } b_i, b_j = \text{even number} \qquad (5a)$$

$$t_{uk} = \frac{\left(h * s_u + n_k * \left(\frac{l_{kH}}{2} + \frac{l_{kV}}{2}\right)\right)}{v_u} \forall\, b_k = \text{odd number} \qquad (5b)$$

$$t_u = \frac{\sum t_{ui}}{v_u} \tag{6}$$

4.1.3 Merging of two opposing material flows (Case C)

Previously, intersections were described where the material flows are independent of each other before and after passing the intersection. In addition, there are intersections where material flows are merged or separated. In the first of these cases, two material flows moving in opposite directions are merged and both make a turning movement. In the second case, two material flows come from the same direction and are separated by a turn. The equation for the separation is identical to the one for the merging.

$$t_u = \frac{n_i * \left(\frac{s_u}{2} + \frac{l_{iV}}{2} + \frac{s_u}{2} + \frac{l_{iH}}{2}\right) + n_j * \left(\frac{s_u}{2} + \frac{l_{jV}}{2} + \frac{s_u}{2} + \frac{l_{jH}}{2}\right)}{v_u} \tag{7}$$

4.1.4 Merging of two material flows by integrating $i$ into $j$ (Case D)

In the second case of merging or separating, one material flow $j$ passes the intersection in a straight line. The second material flow $i$ merges into it. After the second material flow changes direction, a joint movement can take place, which significantly increases the throughput. For the intersection situation, two subcases arise depending on the batch size of the material flows. The following evaluation formulas regarding junctions apply to the case where the straight material flow passes the intersection horizontally. By exchanging $l_h$ and $l_v$, an intersection situation rotated by 90° can be modelled.
In the first case, there are more horizontal than vertical goods (equation 8). The first term of the numerator describes the vertical movement of a good from material flow $i$ to the middle of the intersection. Then the transport direction of the conveyor is changed. The second term describes the joint horizontal movement of one good from $i$ and one good from $j$. For this, only the length of the conveyor and that of the good of $j$ must be taken into account. Since good $i$ is moved automatically. The two terms and the corresponding movements are executed as often as there are goods of $i$ in the batch. Then another horizontal transport is carried out (term 3). All other goods from $j$ are transported. Since this movement follows seamlessly after the last execution of term 2, the length of the conveyor does not have to be considered again.

$$t_u = \frac{n_i * \left(\frac{s_u}{2} + \frac{l_{iV}}{2}\right) + n_i * (l_{jH} + s_u) + \left(l_{jH} * (n_j - n_i)\right)}{v_u} \ \forall\, n_i \leq n_j \tag{8}$$

In the second case, there are fewer horizontal goods than vertical goods. The first term again describes the vertical movement of a good from $i$ to the centre of the conveyor. This movement must be carried out as many times as the batch size requires. The second term describes the horizontal movement of a good from $i$. This must be executed individually if no good from $j$ is available for joint transport. The third term again describes a joint movement of a good from $i$ and a good from $j$. The joint movement can be executed as often as the lot size of $j$ requires.

$$t_u = \frac{n_i * \left(\frac{s_u}{2} + \frac{l_{iV}}{2}\right) + (n_i - n_j) * \left(\frac{l_{jH}}{2} + \frac{s_u}{2}\right) + n_j * (l_{jH} + s_u)}{v_u} \ \forall\, n_i > n_j \tag{9}$$

4.1.5 Merging of three material flows by integrating $i$ and $k$ in $j$ (Case E)

The evaluation equations presented in this section are based on those described above, but for the case where one straight material flow and two opposing turning material flows exist. The evaluation equations again apply to a horizontal case, with $n_i \geq n_k$ for the turning material flows. For the intersection situation, three subcases arise depending on the batch size of the material flows. Both equations 10 and 11 are similar to equation 9, considering how often goods from one of the two material inflows make a joint movement with goods from $j$. The first term in equation 10 describes a separate movement across the conveyor of goods from one of the material inflows. The second term in equations 11 and 12 adds the vertical movement of the goods of the additional material flow $k$. In equation 11, the additional last term describes the horizontal movement of the goods of the additional material inflow $k$, which is not carried out as a joint movement because there are not enough suitable goods from the horizontal direction. Equation 12 is the adaptation of equation 8, in addition to the extension described above, only the changed number of joint and independent movements is taken into account in the last terms.

$$t_u = \frac{n_i*\left(\frac{s_u}{2}+\frac{l_{iV}}{2}+\frac{s_u}{2}+\frac{l_{iH}}{2}\right)+n_k*\left(\frac{s_u}{2}+\frac{l_{kV}}{2}\right)+n_j*(l_{jH}+s_u)+(n_k-n_j)*\left(\frac{s_u}{2}+\frac{l_{kH}}{2}\right)}{v_u} \quad \forall\, n_i \geq n_k \geq n_j \tag{10}$$

$$t_u = \frac{n_i*\left(\frac{s_u}{2}+\frac{l_{iV}}{2}\right)+n_k*\left(\frac{s_u}{2}+\frac{l_{kV}}{2}\right)+n_j*(l_{jH}+s_u)+((n_i+n_k)-n_j)*\left(\frac{s_u}{2}+\frac{l_{kH}}{2}\right)}{v_u} \quad \forall\, n_j > n_i > n_k;\, n_j < (n_i+n_k) \tag{11}$$

$$t_u = \frac{n_i*\left(\frac{s_u}{2}+\frac{l_{iV}}{2}\right)+n_k*\left(\frac{s_u}{2}+\frac{l_{kV}}{2}\right)+(n_i+n_k)*(l_{jH}+s_u)+\left(n_j-(n_i+n_k)\right)*(l_{jH})}{v_u} \quad \forall\, n_j \geq (n_i+n_k) \tag{12}$$

### 4.1.6 Validation of the equations by simulation

In order to validate the equations for the intersection situations described above, they were simulated. The intersections were modelled and simulated with discrete event simulation via Plant Simulation. For the simulation, a continuous good flow was assumed and stochastic influence were not considered. First, individual intersections were simulated and second, conveyor systems were simulated. The latter was done to test the hypothesis that the throughput of the bottleneck is also the maximum throughput of the conveyor system. Table 1 shows the result of the throughput calculation compared to the simulation results. Basically, the throughput of the bottleneck is slightly overestimated by the equations. The previously mentioned hypothesis could be confirmed, because the bottleneck defines the maximum throughput of the system. This can result, for example, from insufficient control of the simulation. This problem can also occur in the control of real systems. An example of this is the control of the previously described joint movement of goods in junction situations. To enable this, the systems must continuously track the exact position of the goods in order to be able to calculate the start time and the duration or length of the joint path. However, the validation shows that the deviations are very small (less than 1 %), so the procedure for throughput calculation can be used in the context of optimisation.

Table 1: Comparison of evaluation equations and simulation

| Case | Parameter | Evaluation equations [goods/hour] | Simulation [goods/hour] |
|---|---|---|---|
| A | $n_1 = n_2 = 1$ | $\lambda = 2{,}440$ | $\lambda = 2{,}427$ |
| | $n_1 = 1;\ n_2 = 3$ | $\lambda_1 = 867;\ \lambda_2 = 2{,}601$ | $\lambda_1 = 865;\ \lambda_2 = 2{,}593$ |
| B | $n_1 = n_2 = n_3 = 1; h = 3$ | $\lambda = 1{,}058$ | $\lambda = 1{,}059$ |
| | $n_1 = 3;\ n_2 = 3;\ h = 2$ | $\lambda_1 = 1{,}270;\ \lambda_2 = 846$ | $\lambda_1 = 1269;\ \lambda_2 = 847$ |
| C | $n_1 = n_2 = 1$ | $\lambda = 2{,}618$ | $\lambda = 2{,}603$ |
| | $n_1 = 1;\ n_2 = 3$ | $\lambda_1 = 867;\ \lambda_2 = 2{,}601$ | $\lambda_1 = 865;\ \lambda_2 = 2{,}593$ |
| D | $n_1 = n_2 = 1$ | $\lambda = 3164$ | $\lambda = 3147$ |
| | $n_1 = 3;\ n_2 = 2$ | $\lambda_1 = 1{,}845;\ \lambda_2 = 1{,}230$ | $\lambda_1 = 1{,}835;\ \lambda_2 = 1{,}224$ |
| | $n_1 = 2;\ n_2 = 3$ | $\lambda_1 = 1{,}398;\ \lambda_2 = 2{,}097$ | $\lambda_1 = 1{,}390;\ \lambda_2 = 2{,}085$ |
| E | $n_1 = n_2 = n_3 = 1$ | $\lambda = 3{,}063$ | $\lambda = 3{,}046$ |
| | $n_1 = 4;\ n_2 = 5; n_3 = 2$ | $\lambda_1 = 1{,}142;\ \lambda_2 = 1{,}428; \lambda_3 = 571$ | $\lambda_1 = 1{,}136;\ \lambda_2 = 1{,}420; \lambda_3 = 569$ |
| | $n_1 = 2;\ n_2 = 3; n_3 = 1$ | $\lambda_1 = 1{,}059;\ \lambda_2 = 1{,}588; \lambda_3 = 529$ | $\lambda_1 = 1{,}053;\ \lambda_2 = 1{,}579; \lambda_3 = 527$ |

### 4.2 Space requirement

The space requirement of a conveyor layout can be calculated statically or dynamically. When applying the optimisation method being developed, a maximum permissible area requirement must be defined by the user.

This is done by specifying a horizontal length $l_H$ and a vertical length $l_V$. When using the static method, the permissible area requirement is set in relation to the sum of the areas of all conveyor modules.

$l_f$: edge length respectively dimension of a conveyor $f$

$$Z_{Space} = \frac{\sum_{f=1}^{F} l_f^{\ 2}}{l_H * l_V} * 100 \tag{13}$$

The dynamic method uses the same equation, but the denominator is the area spanned by the conveyor modules. For this, the minima and maxima of position coordinates form the conveyor modules must be determined and subtracted from each other. The set of position coordinates in $x$-direction is $X$ with $x_c \in X$.

$$l_H = \max(X) - \min(X) \tag{14}$$

$$l_V = \max(Y) - \min(Y) \tag{15}$$

## 5. Conclusion

In this paper, evaluation equations respectively analysis methods for conveyor systems have been presented. In the focus of the paper is the evaluation of the throughput of conveyor systems in complex intersection situations and with several materials. For this purpose, a decomposition approach was used, with which the critical intersection situations respectively the bottlenecks in the conveyor system can be considered. With the help of simulation, the evaluation equations could be validated and it could be shown that the error rate is less than 1 %. Accordingly, the analysis method can be used to evaluate conveyor systems in the context of layout optimisation. The further evaluation equation for space requirements is based on simple mathematical principles. In the next steps of the research project, a software will be developed with which users can plan their individual conveyor systems. In addition to the optimisation method, the evaluation equations presented must also be implemented in such a way that they can be calculated automatically. This requirement was of course taken into consideration in the development of the latter. The evaluation equations presented can be further detailed. Regarding the throughput calculation, for example, failure rates of the conveyors or transport processes of goods with very small dimensions ($l_{iH} < s_u$ or $l_{iV} < s_u$) could still be taken into account. Furthermore, objective functions for buffer capacity and costs will be developed.

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