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ON REGULAR FUZZY RESOLVING SET

R. SHANMUGAPRIYA[1], M. JINY D.[2*], §

Abstract. In a fuzzy graph G, if the degree of each vertex is the same, then it is called a regular fuzzy graph. The representation of $\sigma - H$ with respect to the subset H of σ are all distinct then H is called the resolving set of the fuzzy graph $G(V, \sigma, \mu)$. In this article, we define a regular fuzzy resolving set, regular fuzzy resolving number and the properties of a regular fuzzy resolving set in a fuzzy graph whose crisp graph is a cycle, even or odd. And also we prove that, if G be a regular fuzzy graph with G^* is a cycle, then any minimum fuzzy resolving set of G is a regular fuzzy resolving set of G.

Keywords: Fuzzy resolving set, Cyclic graph, Vertex degree, Fuzzy resolving number, Regular fuzzy graph.

AMS Subject Classification: 05C72.

1. Introduction

Lotfi Asker Zadeh is the one who explained the Fuzzy Mathematics in 1965, thereafter in the year 1975, Fuzzy Graph was introduced by Rosenfield. It is a young research area in mathematics and therefore a lot of new theory is proved and the growth of the subject is commendable and also it is applied in various fields like networking, fuzzy cluster analysis, coloring, etc..

Slater in the year 1975, introduced resolving number of a graph. It is used to locate robots position differently in a graph-structured framework in terms of distance. Shanmugapriya and Jiny introduced the fuzzy resolving set, fuzzy resolving number, fuzzy super resolving number in the year 2019 by considering that the relation between two vertices indicates the safety level or temperature etc. in terms of a fuzzy number. Nagoor Gani and Radha introdued regular fuzzy graph in the year 2008. In this paper, we have defined the regular fuzzy resolving set and regular fuzzy resolving number of a fuzzy graph and few properties of regular fuzzy resolving set, the relation between regular fuzzy graph and regular fuzzy resolving set in a cyclic graph is explained.

[1] Department of Mathematics, Vel Tech Rangarajan Dr., Sagunthala R&D Institute of Science and Technology, Avadi, Chennai-600062, Tamil Nadu, India.
e-mail: spriyasathish11@gmail.com; ORCID: https://orcid.org/0000-0002-8668-0101.

[2] Department of Mathematics, Saveetha School of Engineering, SIMATS, Chennai, 602105, India.
e-mail: maryjinymaths@gmail.com; ORCID: https://orcid.org/0000-0002-2501-6442.

[*] Corresponding author.

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2. Definitions

Definition 2.1. The ordered triple $G(V,\sigma,\mu)$, where V is a non-empty set of vertices σ is the function from set of all vertices V to $[0,1]$ and μ is a function from $V\times V$ to $[0,1]$ such that for all $a,b\in V$, $\mu(a,b)\leq\sigma(a)\wedge\sigma(b)$ is called a fuzzy graph. The support of μ and σ are represented as $\mu^*=\{(u,v)/\mu(a,b)>0\}$ and $\sigma^*=\{u/\sigma(u)>0\}$. [2]

Definition 2.2. The weakest edge of the path $P=v_1,v_2,...v_n$ is the edge having lowest membership value in the path. And strength or weight of the path P is defined as the weight or the membership value of the weakest edge in the path P.

Definition 2.3. The weight of connectedness between v_1 to v_2 is the maximum of strength or weight of all paths connecting v_1 and v_2, it is denoted by $\mu^\infty(v_1,v_n)$[3], for simplicity we represent it as $w(v_1,v_n)$.

Definition 2.4. The fuzzy path $P=v_1,v_2,...v_n$ is called a fuzzy cycle if $v_1=v_n$ and $n\geq 3$.

Definition 2.5. The degree of a vertex $x\in V$ in a fuzzy graph $G(V,\sigma,\mu)$ is the sum of all the membership values of the edges incident on x [10]. That is,

$$d_G(x)=\sum_{xy\in\mu^*}\mu(xy)$$

and total degree of a vertex x is

$$d_G(x)=\sum_{xy\in\mu^*}\mu(xy)+\sigma(x)$$

Definition 2.6. If all the vertices of the fuzzy graph $G(V,\sigma,\mu)$ has same degree m then G is called m-regular fuzzy graph. And all the vertices of G has the same total degree m, then G is called the m-totally regular fuzzy graph. If G^*, the crisp graph corresponding to G is regular then G is called partially regular. If G and G^* is regular then G is called full regular fuzzy graph.

Definition 2.7. Let $G(V,\sigma,\mu)$ be a fuzzy graph with $|V|\geq 3$. A subset H where $|H|\geq 2$ of σ is called the fuzzy resolving set of G if the representation of $\sigma-H$ with respect to σ are all distinct.The cardinality of the minimum resolving set of G is called the resolving number of G denoted as $Fr(G)$.[1]

Note 1: Through out this paper we consider a fuzzy graph G with $|G|\leq 3$.
Note 2: If H be the resolving set of the graph G with n vertices then, $2\leq|H|\leq n-1$.
Note 3: We consider the fuzzy set $\sigma=\{(x_i,\sigma(x_i)\}\quad i=1,2,..n$ as a crisp set $\sigma=\{\sigma_1,\sigma_2,...\sigma_n\}$ where $(x_i,\sigma(x_i))=\sigma_i$ to find the resolving set of the fuzzy graph so that the subset of σ will be the power set of σ.
Result 1 Let G be a fuzzy regular graph with G^* is an odd cycle then μ is a constant function.[7]
Result 2 Let G be a fuzzy regular graph with G^* is an even cycle then either μ is constant or alternate edges have the same membership values.[7]
Result 3 Let G be a totally regular fuzzy graph with σ is a constant function then G is a regular fuzzy graph.[7]
Result 4 If G be a fuzzy graph with μ is a constant function, then G is partially regular iff G is regular.[8]

Example 2.1.

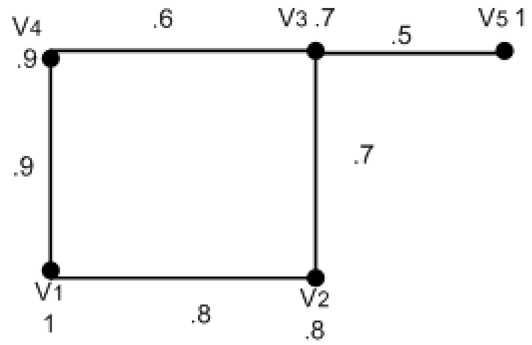

Fig1. Fuzzy Graph $G(V, \sigma, \mu)$

$V = \{v_1, v_2, v_3, v_4, v_5\}$; $\sigma = \{(v_1, 1), (v_2, .8), (v_3, .7), (v_4, .9), (v_5, 1)\}$
$\mu = \{(v_1v_2, .8), (v_2v_3, .7), (v_3v_4, .6), (v_4v_1, .9), (v_3v_5, .5)\}$
Let $H = \{\sigma_1, \sigma_2\}$; $\sigma - H = \{\sigma_3, \sigma_4, \sigma_5\}$ where $\sigma_1 = (v_1, \sigma(v_1))$
The representation of the elements of $\sigma - H$ with respect to H are as follows:
$\sigma_3/H = (w(v_3, v_1), w(v_3, v_2)) = (.7, .7)$
$\sigma_4/H = (w(v_4, v_1), w(v_4, v_2)) = (.9, .8)$
$\sigma_5/H = (w(v_5, v_1), w(v_5, v_2)) = (.5, .5)$
these represents are all distinct, therefore H is the resolving set of G and $Fr(G) = 2$.

3. Regular Fuzzy Resolving Set

In this section, we introduce the regular fuzzy resolving set and the relation between the regular fuzzy graph and the regular fuzzy resolving set.

Definition 3.1. *Let H be the fuzzy resolving set of a fuzzy graph $G(V, \sigma, \mu)$. If for all $v \in (\sigma - H)^*$, $\sum_{u_i \in H^*} w(v, u_i) = k$, then H is called a **regular fuzzy resolving set**. The regular fuzzy resolving set of minimum cardinality is called the **regular fuzzy resolving number** denoted as $F_{rr}(G)$.*

Theorem 3.1. *Any resolving set of the fuzzy connected graph G with $|V| = 3$ is a regular resolving set.*

Proof. Any resolving set H of fuzzy connected graph G with $|G| = 3$ has cardinality two [by definition 2.7], therefore there exist only one representation of $\sigma - H$ with respect to σ and hence the proof. □

Example 3.1. Example of a regular resolving set

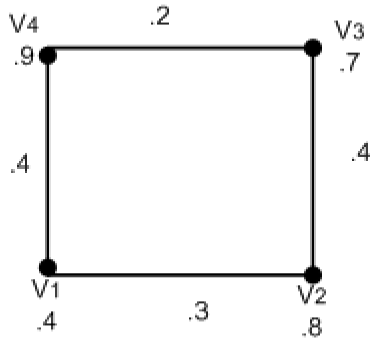

Fig.2

$\sigma = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$
Let $H = \{\sigma_1, \sigma_3\}$, $\sigma - H = \{\sigma_2, \sigma_4\}$ then,
$\sigma_2/H = (w(v_2, v_1), w(v_2, v_3)) = (.3, .4)$
$\sigma_4/H = (w(v_4, v_1), w(v_4, v_3)) = (.4, .3)$
$\sum_{u_i \in H^*} w(v, u_i) = .7$ for all $v \in (\sigma - H)^*$
Therefore H is a regular fuzzy resolving set of G.

Theorem 3.2. *If $G(V, \sigma, \mu)$ be a regular fuzzy graph where G^* is an odd $(2n-1)$-cycle then $Fr(G) = 2n - 2$.*

Proof. Let $G(V, \sigma, \mu)$ be a regular fuzzy graph where G^* is a (2n-1) odd cycle, then μ is a constant function.[by result 1]. Now it is enough to prove that there does not exist a resolving set H with $|H| < 2n - 2$.
Now let us assume that H is a resolving set of G with $|H| < 2n-2$ say $|H| = 2n-3$.That is $H = \{\sigma_1, \sigma_2, ...\sigma_{2n-3}\}$, and $\sigma - H = \{\sigma_{2n-2}, \sigma_{2n-1}\}$. Then the representation of $\sigma - H$ with respect to H, σ_{2n-2}/H and σ_{2n-1}/H should be the same, since μ is a constant function. Therefore H is not a resolving set of G. Similary, for any subset H with $|H| < 2n-2$, the representation of $\sigma - H$ with respect to H will be the same. Hence, there does not exist a resolving set of G with $|H| < 2n - 2$. Which will imply that the resolving number of a regular fuzzy graph where G^* is an odd $(2n-1)$-cycle is $2n - 2$. □

Theorem 3.3. *If $G(V, \sigma, \mu)$ be a regular fuzzy graph where G^* is an even $2n$-cycle then either $Fr(G) = n$ or $Fr(G) = 2n - 1$.*

Proof. : If G be a fuzzy regular graph with G^* is an even $2n$-cycle then either μ is constant or alternate edges have the same membership values.
case(i) If μ is a constant function
By the proof of theorem 3.2, $Fr(G) = 2n - 1$.
case(ii) If the alternate edges have the same membership values
Let $x_1, x_2, ...x_{2n}$ is an even cycle and $\mu(x_{2i-1}x_{2i}) = r$ and $\mu(x_{2i}x_{2i+1}) = s$ $(r < s)$ where $i = 1, 2, ...n$, and $x_{2n+1} = x_1$. Now let us consider a subset $H = \{\sigma_1, \sigma_3, ...\sigma_{2n-1}\}$ of σ.
The representation of $\sigma - H$ with respect to H,

$$\sigma_{2i}/H = \{r, s(i^{th} term), r, ...r(n^{th} term)\}, i = \{1, 2, ...n\} \tag{1}$$

are all distinct. Therefore H is a resolving set of G, Since we have a resolving set of order n. now, it is enough to prove that there does not exist a resolving set of order less than n. If there exist a resolving set H of G with $|H| < n$, say $n - 1$.
Assume that $H = \{\sigma_1, \sigma_3, ...\sigma_{2n-3}\}$ then the representation of $\sigma - H$ with respect to H has $n+1$ vectors with $n-1$ entries which are either r or s $[r < s]$. Since any two adjacent edges of the cycle has different membership value, there will be n different representation similar to equation (1) and the remaining one will be similar to any one of these n representation. Therefore H will not be a resolving set of G, That is, there does not exist a resolving set H of G with $|H| < n$. And hence, the resolving number of of G is n.
From case (i) and (ii) we conclude that, if G is a regular fuzzy graph with G^* is an even $2n$-cycle then $Fr(G) = n$ or $Fr(G) = 2n - 1$. □

Example 3.2. Example of a fuzzy regular odd and even cycle

$Fr(G) = 4, d_G(x) = 1$ for all x in the first odd 5-cycle and $Fr(G) = 3, d_G(x) = 1.1$ in the second even 6-cycle of fig.3.

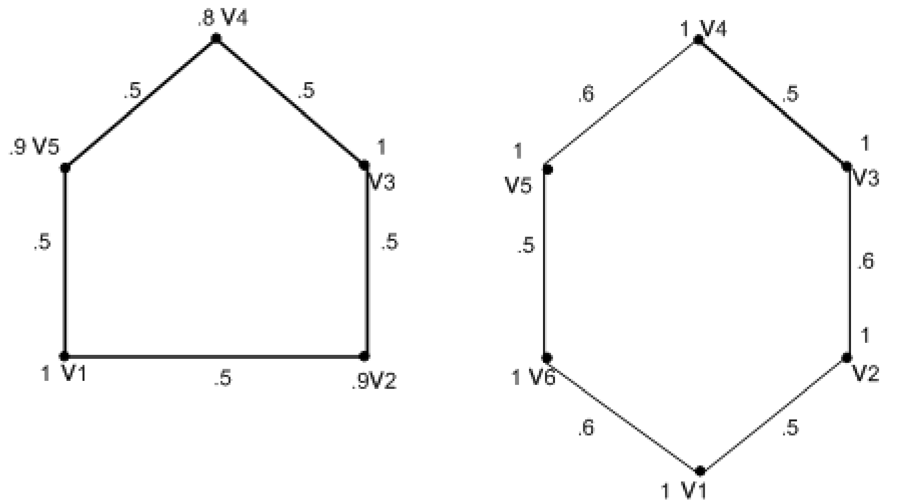

Fig.3

Theorem 3.4. *If $G(V, \sigma, \mu)$ be a regular fuzzy graph with G^* is a cycle, then any resolving set of G with minimum cardinality is a regular resolving set of G.*

Proof. : If $G(V, \sigma, \mu)$ be a regular fuzzy graph with G^* is a cycle.
case(i) G^* is an odd n-cycle
If G^* is an odd cycle then μ is a constant function by result [1]. Therefore $Fr(G) = n - 1$, any minimum resolving set H of G has cardinality $n - 1$ and there exist only one representation of $\sigma - H$ with respect to H. Hence any minimum resolving set of G is a regular resolving set.
case(ii) G^* is an even n-cycle
If G^* is an even n-cycle then, either μ is a constant function or alternate edges have same membership value.
(a) If μ is a constant function then by case(i) any minimum resolving set is a regular resolving set.
(b)Alternate edges have same membership value say r and s with $r < s$. Let $x_1, x_2, ...x_{2n}$ is an even cycle and $\mu(x_i x_{i+1}) = r$ and $\mu(x_{i+1} x_{i+2}) = s$ $(r < s)$ where $i = 1, 2, ...2n-1$,and $x_{2n+1} = x_1$. By theorem [3.3], $Fr(G) = n$ and any minimum resolving set of G will be of the form equation (1). Hence for all $v \in (\sigma - H)^*$, $\sum_{u_i \in H^*} w(v, u_i) = (n - 1)r + s$. That is, any minimum resolving set of G is a regular resolving set of G by definition [3.1].
hence from case(i) and (ii), if $G(V, \sigma, \mu)$ be a regular fuzzy graph with G^* is a cycle, then any resolving set of G with minimum cardinality is a regular resolving set of G. □

Theorem 3.5. *If $G(V, \sigma, \mu)$ be a partially regular fuzzy graph with G^* is a cycle and μ is a constant function, then any resolving set of G with minimum cardinality is a regular resolving set of G.*

Proof. : Let $G(V, \sigma, \mu)$ be a partially regular fuzzy graph with G^* is a cycle and μ is a constant function. By result [4], If G be a fuzzy graph with μ is a constant function, then G is partially regular iff G is regular. Hence G is a regular fuzzy graph with G^* is a cycle, then by theorem [3.4], any resolving set of G with minimum cardinality is a regular resolving set of G. □

Corollary 3.1. If $G(V, \sigma, \mu)$ be a totally regular fuzzy graph with G^* is an odd $(2n-1)$-cycle and σ is a constant function then $Fr(G) = 2n - 2$.
Proof: If G be a totally regular fuzzy graph with σ is a constant function then, G is a regular fuzzy graph[Result 3]. Hence G be a regular fuzzy graph with G^* is an odd $(2n-1)$-cycle. Therefore, by theorem [3.2] $Fr(G) = 2n - 2$.

Corollary 3.2. If $G(V, \sigma, \mu)$ be a totally regular fuzzy graph where G^* is an even $2n$-cycle and σ is a constant function then $Fr(G) = n$ or $Fr(G) = 2n - 1$.

Corollary 3.3. If $G(V, \sigma, \mu)$ be a totally regular fuzzy graph with G^* is a cycle and σ is a constant function, then any resolving set H of G with minimum cardinality is a regular resolving set of G.

Corollary 3.4. *If $G(V, \sigma, \mu)$ be a full regular fuzzy graph with G^* is a cycle, then any resolving set H of G with minimum cardinality is a regular resolving set of G.*

Proof. : If $G(V, \sigma, \mu)$ be a full regular fuzzy graph then, G is a regular fuzzy graph. Hence, the proof is obvious from theorem [3.4]. □

4. Conclusions

The resolving number of the fuzzy graph and super resolving number of the fuzzy graph is introduced by Shanmugapriya and Jiny. As an extension, in this paper, we have introduced the regular fuzzy resolving set and regular fuzzy resolving number. We have discussed some properties of regular fuzzy resolving set and we have proved that, if G is a regular fuzzy graph with G^* is a cycle, then any minimum fuzzy resolving set of G is a regular fuzzy resolving set of G. The further research can be done in studying the various properties of fuzzy resolving set.

References

[1] Shanmugapriya, R., Jiny, D. M., (2019), Fuzzy Super Resolving Number of Fuzzy Labelling Graphs, Journal of Advanced Research in Dynamical and Control Systems, pp. 606-611.

[2] Gani, A. N., (2012), Properties of Fuzzy Labeling Graph, Journal of Mathematics, pp. 1-6.

[3] Mordeson, J. N. et al., (2018), Fuzzy Graph Theorey with Appplications to Human Trafficking, Springer Publication, ISSN: 1434-9922, pp. 57-84.

[4] Rajan, B., Sonia, K. T., Monica, M. C., (2011), Conditional resolvability of Honeycomb and Hexagonal networks, Math. Comput.Sci., 5, (1), pp. 89-90.

[5] Mathew, S., Mordeson J. N., Malik, D. S., (2018), Fuzzy Graph Theory, Springer publication, ISSN:1434-9922, pp. 14-41.

[6] Bondy J. A., Murty, U. S. R., (2008), Graduate Texts in Mathematics series, Springer publication, ISSN: 0072-5285, pp. 1-39, pp. 361-362.

[7] Gani, A. N., Radha, K., (2008), On Regular Fuzzy Graphs, Journal of Physical Sciences, pp. 33-40.

[8] Gani, A. N., Radha, K., (2008), Regular Properties of Fuzzy Graphs, Bulletin of Pure and Applied Science, pp. 415-423.

R. Shanmugapriya for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.11, N.2.

Mary Jiny D. for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.11, N.2.