

LOCATION OF BURST AND REPEATED BURST ERROR IN SINGLE AND ADJACENT SUB-BLOCKS

PANKAJ KUMAR DAS¹, SUBODH KUMAR², §

ABSTRACT. The paper gives necessary and sufficient conditions for the existence of linear codes capable of identifying burst/repeated burst errors whether it is confined to one sub-block or spread over two adjacent sub-blocks. Examples of such codes are also provided. We also provide two methods one using tensor product and other using cyclic code to construct such codes. Finally, comparisons on the number of check digits of such codes with the corresponding error detecting and correcting codes are also provided.

Keywords: Syndromes, Bounds, Bursts, Repeated bursts, Error locating codes.

AMS Subject Classification: 94B05, 94B65.

1. INTRODUCTION

To improve the efficiency of the communication channel, it is very important to know the nature of the channel. Once it is known that a particular type of error occurs in a channel, codes are constructed accordingly. In view of this, the codes that deal with only burst error and repeated burst are studied in [1, 5, 6]. It is observed by Berardi, Dass and Verma that in busy communication channels, burst errors repeat themselves and they called the errors as “2-repeated burst error” in [1]. Further, they observed in [5] that burst error repeat more than two times in more busy communication channels and they considered “ m -repeated burst error”. Such type of errors are found in channels like lutamate-injured networks, glutamate-injured networks [12].

Definition 1.1. [6] *A burst of length b is an n -tuple whose only nonzero components are confined to some b successive positions, the first and the last of which is nonzero.*

Definition 1.2. [5] *An m -repeated burst of length b is an n -tuple whose only nonzero components are confined to m distinct b successive positions, the first and the last component of each being nonzero.*

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§ Manuscript received: May 05, 2020; accepted: August 04, 2020.

TWMS Journal of Applied and Engineering Mathematics, Vol.12, No.3 © Işık University, Department of Mathematics, 2022; all rights reserved.

In [3, 4], the authors obtain necessary and sufficient conditions for the existence of burst and repeated burst error locating codes. This concept of error locating (EL) codes was introduced by Wolf and Elspas [13]. In [3, 4], the authors consider the situation when error is confined to one sub-block only. This work was done keeping in view of channels where the error or fault occurring in one sub-block (like error in data of RAM chips) does not affect its adjacent sub-block [7]. But in case of data recorded on a continuous surface (medium), the error may not be confined to one sub-block, it may affect two adjacent sub-blocks (written as 2-adjacent sub-blocks) also. This motivates us to work on burst/repeated burst error which may spread over to adjacent sub-blocks. This type of errors falls in the category of B1 type errors [8] with the restriction that the burst/repeated error occurs within 2 consecutive sub-blocks.

Consider an $(n = ft, k)$ linear code over $GF(q)$, subdivided into f sub-blocks, each of length t and H its parity check matrix. Let $E_b (E_{m,b})$ be the set of all burst (m -repeated burst) errors of length at most b which may spread over to its adjacent sub-block ($b \leq t$). In order to locate such errors, the following three conditions need to be satisfied.

- (i) $eH^T \neq 0 \quad \forall e \in E_b (E_{m,b})$.
- (ii) $e_i H^T \neq e_j H^T \quad \forall e_i, e_j \in E_b (E_{m,b})$ such that e_i and e_j represent the errors occurring in two distinct single sub-blocks.
- (iii) $e'_i H^T \neq e'_j H^T \quad \forall e'_i, e'_j \in E_b (E_{m,b})$ such that e'_i represents the error spreading over any 2-adjacent sub-blocks and e'_j represents the error spreading over in any other 2-adjacent sub-blocks or confined to any one single sub-block.

We denote such an $(n = ft, k)$ linear code over $GF(q)$ that detects and locates any error from the set E_b by a q -ary $(n = ft, k)$ E_bL -code and from the set $E_{m,b}$ by a q -ary $(n = ft, k)$ $E_{m,b}L$ -code.

Rest of the paper is organized as follows. In Section 2, we obtain necessary and sufficient conditions for a q -ary $(n = ft, k)$ E_bL -code followed by an example. Two methods, one using tensor product and the other using cyclic code, to construct such codes are also given. In Section 3, we obtain similar conditions for a q -ary $(n = ft, k)$ $E_{m,b}L$ -code, followed by an example and the analogous two methods. Section 4 gives some comparisons of check digits of these codes with the corresponding error detecting and correcting codes.

2. LOCATION OF BURST ERROR IN ADJACENT SUB-BLOCKS

In this section, we derive necessary and sufficient conditions needed to exist a q -ary $(n = ft, k)$ E_bL -code. The following is the necessary condition.

Theorem 2.1. *A q -ary $(n = ft, k)$ E_bL -code satisfies*

$$q^{n-k} \geq 1 + f(q^b - 1) + (f - 1)\lfloor b/2 \rfloor (q - 1). \tag{1}$$

Proof. According to the conditions (i) and (ii), there are $1 + f(q^b - 1)$ distinct syndromes, including the zero syndrome (refer Theorem 1, [3]).

In order to satisfy the condition (iii), let X be the set of n -tuples such that in one 2-adjacent sub-blocks, the $(t - i + 1)^{th}$ position of the first sub-block and i^{th} position of the second sub-block, where $i = 1, 2, \dots, \lfloor b/2 \rfloor$, are both occupied by same non-zero component out of the $q - 1$ nonzero components. The elements of X will be in different cosets due to condition (iii). The number of elements of X is $\lfloor b/2 \rfloor (q - 1)$ and so, the total number of distinct syndromes satisfying conditions (i) – (iii) is $1 + f(q^b - 1) + (f - 1)\lfloor b/2 \rfloor (q - 1)$. The result follows as the maximum number distinct syndromes is q^{n-k} . \square

Now, we give the sufficient condition for existence of a q -ary $(n = ft, k)$ E_bL -code.

Theorem 2.2. *The existence of a q -ary $(n = ft, k)$ E_bL -code can be ensured provided*

$$q^{n-k} > q^{b-1} \left[1 + (f-1)q^{b-1}(q-1)t \right]. \tag{2}$$

Proof. For the existence of the required code, we follow the same technique used in the proof of Theorem 4.7 of [10] (also refer Sacks [11], Theorem 2, Dass [3]) by constructing suitably the parity check matrix $H_{(n-k) \times n}$ of the required code.

Let the first $f-1$ sub-blocks of $H_{(n-k) \times n}$ and the first $\rho-1$ columns of the f^{th} sub-block are suitably added satisfying the conditions (i) – (iii). Then, to add the ρ^{th} column h_ρ of the f^{th} sub-block to H satisfying the conditions (i) – (iii), we proceed as follows.

The total number of linear combinations satisfying conditions (i) – (ii) that h_ρ should not be equal to, and this number is (by Theorem 2, [3])

$$q^{b-1} + (f-1)q^{b-1} \left[q^{b-1} \left((q-1)(t-b+1) + 1 \right) - 1 \right].$$

Now according to condition (iii), we can add the column h_ρ of f^{th} sub-block provided

$$h_\rho \neq (\alpha_1 h_{\rho-1} + \alpha_2 h_{\rho-2} + \dots + \alpha_{b-1} h_{\rho-(b-1)}) + (\beta_1 h_i + \beta_2 h_{i+1} + \dots + \beta_b h_{i+(p-1)}), \tag{3}$$

where $\alpha_i, \beta_i \in GF(q)$, $2 \leq p \leq b$ and β_i 's are such that the first and the last of the p consecutive columns h_i 's lie in both sub-blocks in any 2-adjacent sub-blocks.

In the expression (3), α_i 's can be chosen by q^{b-1} ways and β_i 's can be chosen by $\sum_{p=2}^b (p-1)q^{p-2}(q-1)^2 = bq^{b-1}(q-1) - q^b + 1$. Therefore the total number of linear combinations, according to the condition (iii), is given by $q^{b-1}(f-1) \left[bq^{b-1}(q-1) - q^b + 1 \right]$. Therefore, by conditions (i) – (iii), addition of the column h_ρ is possible provided

$$q^{n-k} > q^{b-1} + q^{b-1}(f-1) \left[q^{b-1} \left((q-1)(t-b+1) + 1 \right) - 1 \right] + q^{b-1}(f-1) \left[bq^{b-1}(q-1) - q^b + 1 \right].$$

On simplification, we get the sufficient condition (2). □

Example 2.1. The following is a parity check matrix of a 2-ary (20, 12) E_3L -code, where $t = 5, b = 3, f = 4, q = 2$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Now, we give two methods, one using tensor product (\otimes) and another using cyclic code, to construct our E_bL -code. The first one, being similar as in [14], is without proof.

Theorem 2.3. *Let H be a parity check matrix of an $(n_1 = mt, k)$ linear code that detects burst errors of length at most b within a sub-block of length t and P be a parity check matrix of an $(n_2 = ms, \rho)$ linear code that corrects any burst of length at most 2. Then, the $(n_1 n_2, k\rho)$ code obtained from the parity check matrix $P \otimes H$ is an E_bL -code.*

Theorem 2.4. *If $C(n, k)$ is a cyclic code with the irreducible polynomial $g(x)$ as the generator polynomial, the order of roots of $g(x)$ is p and if the code C corrects all bursts of length at most b , then there exists an E_bL -code of length $n = p(p+4)$.*

Proof. Let α be a root of $g(x)$. Then, the cyclic code C , generated by $g(x)$, is the null space of the check matrix $A = [1 \ \alpha \ \alpha^2 \ \dots \ \alpha^{p-1}]$, where each entry α^i is to be regarded as a binary $(n - k)$ -tuple. Then, the matrix A is of size $n - k$ by p . Consider the matrix H

$$H = \begin{bmatrix} A & 0 & 0 & 0 & A & A & 0 & A & 0 & A & \dots & X \\ 0 & A & 0 & 0 & A & \alpha A & 0 & \alpha^3 A & 0 & \alpha^5 A & \dots & Y \\ 0 & 0 & A & 0 & A & 0 & A & 0 & A & 0 & \dots & Z \\ 0 & 0 & 0 & A & A & 0 & \alpha^2 A & 0 & \alpha^4 A & 0 & \dots & W \end{bmatrix},$$

where $\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} A \\ \alpha^{p-1} A \\ 0 \\ 0 \end{bmatrix}$, when p is even and $\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ A \\ \alpha^{p-1} A \end{bmatrix}$, when p is odd. We can verify that the columns of H satisfy the conditions (i) – (iii). Therefore, the code obtained from the parity check matrix H is an E_bL -code and its length is $p(p + 4)$. \square

3. LOCATION OF REPEATED BURST ERROR IN ADJACENT SUB-BLOCKS

In this section, we derive necessary and sufficient conditions required for existence of such a q -ary $(n = ft, k)$ $E_{m,b}L$ -code.

Theorem 3.1. *A q -ary $(n = ft, k)$ $E_{m,b}L$ -code satisfies*

$$q^{n-k} \geq 1 + f(q^{mb} - 1) + (f - 1)\lfloor (mb)/2 \rfloor (q - 1). \tag{4}$$

Proof. By Theorem 2.1 [4], the number of distinct syndromes (including the zero syndrome) satisfying the conditions (iv) – (v) is $1 + f(q^{mb} - 1)$. For the condition (vi) to be satisfied, let X be a set of the collection of all n -tuples in which the $(t - i + 1)^{th}$ position of the first sub-block and i^{th} position of the second sub-block of any one 2-adjacent sub-blocks are same nonzero component, where $i = 1, 2, \dots, \lfloor (mb)/2 \rfloor$. The syndromes of elements of X have to be distinct among themselves and from syndromes computed following conditions (iv) – (v). So, the total number of distinct syndromes satisfying conditions (iv) – (vi) is $1 + f(q^{mb} - 1) + (f - 1)\lfloor (mb)/2 \rfloor (q - 1)$. The proof is complete. \square

Corollary 3.1. *A q -ary $(n = ft, k)$ $E_{2,b}L$ -code satisfies*

$$q^{n-k} \geq 1 + f(q^{2b} - 1) + (f - 1)b(q - 1).$$

Remark 3.1. *For $m = 1$, Theorem 3.1 coincides with Theorem 2.1.*

Theorem 3.2. *The existence of a q -ary $(n = ft, k)$ $E_{m,b}L$ -code $(t > mb)$ is ensured if*

$$q^{n-k} > q^{m(b-1)} \left[\binom{t - mb + (m - 1)}{m - 1} (q - 1)^{m-1} + \sum_{i=0}^{m-2} \binom{t - mb + i}{i} (q - 1)^i q^{m-2-i} \right] \times \\ \left[1 + \left\{ (f - 2) \binom{2t - mb + m}{m} - (f - 2) \binom{t - mb + m}{m} + \binom{2t - 1 - mb + m}{m} \right. \right. \\ \left. \left. - \binom{t - 1 - mb + m}{m} \right\} (q - 1)^m q^{m(b-1)} + \sum_{i=0}^{m-1} \left\{ (f - 2) \binom{2t - mb + i}{i} \right. \right. \\ \left. \left. - (f - 2) \binom{t - mb + i}{i} + \binom{2t - 1 - mb + i}{i} - \binom{t - 1 - mb + i}{i} \right\} (q - 1)^i q^{mb-1-i} \right]. \tag{5}$$

Proof. For proof, we follow the same procedure as of Theorem 2.2. Suppose the first $f - 1$ sub-blocks of $H_{(n-k) \times n}$ and the first $\rho - 1$ columns of the f^{th} sub-block have been suitably added. The ρ^{th} column h_ρ of the f^{th} sub-block can be added to H provided the conditions (iv) – (vi) are satisfied.

The number of the linear combinations satisfying the conditions $(iv) - (v)$ that h_ρ should not be equal to, is (refer Theorem 2.3, [4])

$$\left[\binom{\rho - mb + (m - 1)}{m - 1} (q - 1)^{m-1} q^{m(b-1)} + \sum_{i=0}^{m-2} \binom{\rho - mb + i}{i} (q - 1)^i q^{mb-2-i} \right] \left[1 + (f - 1) \times \left[\binom{t - mb + m}{m} (q - 1)^m q^{m(b-1)} + \sum_{i=0}^{m-1} \binom{t - mb + i}{i} (q - 1)^i q^{mb-1-i} - 1 \right] \right]. \tag{6}$$

For condition (vi) , the column h_ρ of f^{th} sub-block can be added provided it is not be a linear combination of

- (A) immediately preceding at most $b - 1$ columns, together with linear combinations of at most b consecutive columns out of the first $\rho - b$ columns of the f^{th} sub-block, and together with
- (B) linear combinations of any m sets of at most b consecutive columns which are spread over any 2-adjacent sub-blocks previously chosen.

The number of linear combinations in (A) and (B) is

$$\left[\binom{\rho - mb + (m - 1)}{m - 1} (q - 1)^{m-1} q^{m(b-1)} + \sum_{i=0}^{m-2} \binom{\rho - mb + i}{i} (q - 1)^i q^{mb-2-i} \right] \times \left[1 + \left\{ \binom{t + \rho - 1 - mb + m}{m} - \binom{t - mb + m}{m} - \binom{\rho - 1 - mb + m}{m} \right\} (q - 1)^m q^{m(b-1)} + \sum_{i=0}^{m-1} \left\{ \binom{t + \rho - 1 - mb + i}{i} - \binom{t - mb + i}{i} - \binom{\rho - 1 - mb + i}{i} \right\} (q - 1)^i q^{mb-1-i} + (f - 2) + (f - 2) \left[\left\{ \binom{2t - mb + m}{m} - 2 \binom{t - mb + m}{m} \right\} (q - 1)^m q^{m(b-1)} + \sum_{i=0}^{m-1} \left\{ \binom{2t - mb + i}{i} - 2 \binom{t - mb + i}{i} \right\} (q - 1)^i q^{mb-1-i} \right] \right]. \tag{7}$$

Therefore, the column h_ρ can be added to H provided

$$q^{n-k} > Expr.(6) + Expr.(7). \tag{8}$$

On replacing ρ by t , (8) reduces to the required result. □

Corollary 3.2. *The existence of a q -ary $(n = ft, k)$ $E_{2,b}L$ -code $(t > 2b)$ is ensured provided*

$$q^{n-k} > q^{2(b-1)} \left[(t - 2b + 1)(q - 1) + 1 \right] \left[1 + \left\{ (f - 2) \left(\frac{(t - 2b + 2)(3t - 2b + 1)}{2} \right) + \frac{(t - 2b + 1)(3t - 2b)}{2} \right\} (q - 1)^2 q^{2(b-1)} + (f - 1)t(q - 1)q^{2(b-1)} \right].$$

Remark 3.2. *For $m = 1$, Theorem 3.2 coincides with Theorem 2.2.*

Now, we give an example of an $E_{m,b}L$ -code and the extension of Theorem 2.3 and Theorem 2.4 for m -repeated burst errors.

Example 3.1. The following matrix is a parity check matrix of a 2-ary $(n = 20, 7)$ $E_{2,b}L$ -code, where $t = 5, b = 2, f = 4, q = 2$.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Theorem 3.3. Let H be a parity check matrix of an $(n_1 = mt, k)$ linear code that detects m -repeated burst errors of length at most b within a sub-block of length t and P be a parity check matrix of an $(n_2 = ms, \rho)$ linear code that corrects any burst of length at most 2. Then, the $(n_1n_2, k\rho)$ code obtained from parity check matrix $P \otimes H$ is an $E_{m,b}L$ -code.

Theorem 3.4. If $C(n, k)$ is a cyclic code with the irreducible polynomial $g(x)$ as the generator polynomial, the order of roots of $g(x)$ is p and if the code C corrects all m -repeated bursts of length at most b , then there exists an $E_{m,b}L$ -code of length $n = p(p + 4)$.

4. COMPARISON OF NECESSARY AND SUFFICIENT NUMBER OF CHECK DIGITS

In this section, we compare the necessary and sufficient number of check digits needed for the codes of this paper with the burst error detecting and correcting code $([10, 1, 5])$.

First, we give comparison among the necessary sufficient number of check digits needed for a E_bL -code (Theorem 2.1–2.2) with burst error detecting and correcting codes (Theorem 4.13 and Theorem 4.16; and Theorem 4.14 and Theorem 4.17 of [10]). The following Table 1-2 and Figure 1-2 show that number of check digits of a E_bL -code lies in between burst error detecting and correcting code.

Comparison on necessary check digits for codes detecting, correcting burst errors with our E_bL -codes for $q = 2$

f	t	b	n	$n - k$ Codes in Theorem 4.13 [10]	$n - k$ Our codes in Theorem 2.1	$n - k$ Codes in Theorem 4.16 [10]
5	9	2	45	2	5	7
6	9	2	54	2	5	7
7	9	2	63	2	5	7
8	9	2	72	2	5	8
9	9	2	81	2	6	8
10	9	2	90	2	6	8
11	9	2	99	2	6	8
12	9	2	108	2	6	8
13	9	2	117	2	6	8
14	9	2	126	2	6	8
15	9	2	135	2	6	9

Table 1

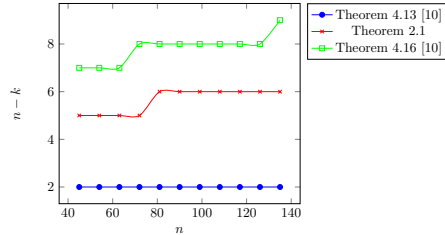


Figure 1

Comparison on sufficient check digits for codes detecting, correcting burst errors with our E_bL -codes for $q = 2$

f	t	b	n	$n - k$ Codes in Theorem 4.14 [10]	$n - k$ Our codes in Theorem 2.2	$n - k$ Codes in Theorem 4.17 [10]
5	9	2	45	2	8	8
6	9	2	54	2	8	8
7	9	2	63	2	8	8
8	9	2	72	2	8	9
9	9	2	81	2	9	9
10	9	2	90	2	9	9
11	9	2	99	2	9	9
12	9	2	108	2	9	9
13	9	2	117	2	9	9
14	9	2	126	2	9	9
15	9	2	135	2	9	10

Table 2

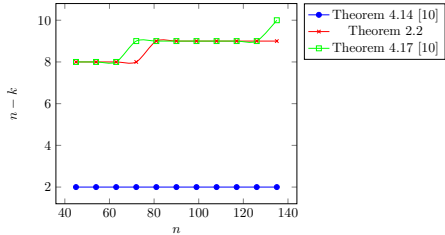


Figure 2

Now, we give the comparisons among the necessary and sufficient number of check digits required for a q -ary $E_{2,b}L$ -code (Theorem 3.1 – 3.2) with that of 2-repeated burst error detecting and correcting codes (Theorem 2.1–2.2 of [1]; Theorem 2.1–2.2 of [5]). The following tables and figures show that the number of check digits for our codes lies between repeated burst error detecting and correcting codes.

Comparison on necessary check digits for our $E_{2,b}L$ -code with the 2-repeated detecting and correcting codes for $q = 2$

f	t	b	n	$n - k$ Codes in Theorem 2.1 [1]	$n - k$ Our codes in Theorem 3.1	$n - k$ Codes in Theorem 2.1 [5]
5	9	2	45	4	7	12
6	9	2	54	4	7	13
7	9	2	63	4	7	13
8	9	2	72	4	8	14
9	9	2	81	4	8	14
10	9	2	90	4	8	14
11	9	2	99	4	8	15
12	9	2	108	4	8	15
13	9	2	117	4	8	15
14	9	2	126	4	8	15
15	9	2	135	4	8	16

Table 3

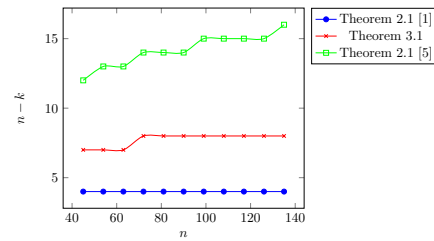


Figure 3

Comparison on sufficient check digits for our $E_{2,b}L$ -code with the 2-repeated detecting and correcting codes for $q = 2$

f	t	b	n	$n - k$ Codes in Theorem 2.2 [1]	$n - k$ Our codes in Theorem 3.2	$n - k$ Codes in Theorem 2.2 [5]
5	9	2	45	8	16	18
6	9	2	54	8	16	19
7	9	2	63	8	17	19
8	9	2	72	9	17	20
9	9	2	81	9	17	21
10	9	2	90	9	17	21
11	9	2	99	9	17	22
12	9	2	108	9	18	22
13	9	2	117	9	18	22
14	9	2	126	9	18	23
16	9	2	135	10	18	23

Table 4

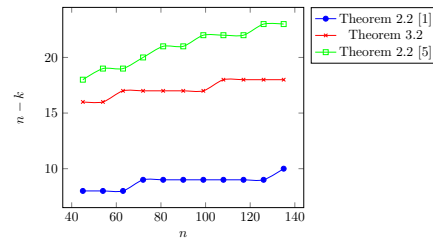


Figure 4

5. CONCLUSIONS

In this paper, we present linear codes that are capable of locating burst/repeated burst errors occurring beyond single sub-block. These codes have the capability of locating burst/repeated burst errors which are spread over two adjacent sub-blocks. We can also extend this work for other types of errors like CT-burst error [2], cyclic burst error [9], low density burst error [15] etc.

Acknowledgement. The authors would like to extend their gratitude to the anonymous reviewer(s) for their valuable comments.

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