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The aggregate "portfolio": Econometrics of economic rates of return with a Portuguese illustration

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Abstract. This research forwards estimation procedures – applications of weighted and generalized least squares techniques - designed to infer expected values, variances and covariances of rates of return in the presence of variously correlated sample observations but uncorrelated "sample waves" (strata of) and availability of already aggregate data – under which inference must rely on averages (means) of averages of averages... The same principles are extended to the method of order statistics, appropriate for univariate inference of a truncated distribution parameters. Simple tests of portfolio – market - efficiency based on correlations (or special rank correlations) between actual and estimated optimal shares are also proposed. Illustrative estimates for Portuguese economic sectors are provided – relying on yearly, semi-aggregate information for firms with 20 or more employees, covering the period 1996-2002: on the one hand, sector means, variances and covariances of economic returns to unitary (tangible and intangible asset) applications are presented and reduced by principal components. On the other, optimal ("unrestricted") portfolios for nested subsets are reported, having been generated by a stepwise elimination procedure. Industries' betas are approximated and market efficiency tested. Finally, parameter MOS estimates under (univariate) truncated normal assumptions are obtained.

Keywords. Industry economic rates of return; Firm size; Optimal portfolio; Mean – Variance; CAPM; Market efficiency; Weighted least squares; "Weighted" SUR; Weighted method of order statistics; Weighted principal components; Dummy variables. Index numbers; Aggregation.

JEL. G11; G12; G30. C39; C43; C51; C61. C24. L16; L25..

1. Introduction

Finance is full of examples where weighting by some relevant budget share is required to generate appropriate test statistics. With respect to means, as is well documented in statistical textbooks, weighting unitary dimensions – prices, rates of return - is of common usage, usually generating aggregate indices. Less explored is the appropriate refinement to weight and extract variances and co-variances – in particular, from published series on aggregate or already averaged information -, either to be assessed *per se* or used as input in other empirical research. It is the purpose of this note to digress over the subject, illustrating with some basic financial econometric applications.

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Firstly, we note that appropriately weighted least squares of a regression of profits on the capital stock can produce an adequate estimate of the average rate of return. Then, for a multiple and interconnected assets pooled sample, the use of an equation system of such single regressions provides a convenient device to proceed to multivariate (assets) inference. Such weighting is also valid under univariate treatment: if a particular distribution function is assumed for the unit rates of return, one can use the same principles to estimate, using the method of order statistics (Martins, 2005b), the parameters of a potentially truncated distribution, with a lower truncation point having an obvious meaning in finance applications.

Inquiry of factors composing observed returns can also be subject to weighting. Principal component techniques are also modified in order to account for different representativeness of each variable's observations – the method involves intermediate regressions and decomposes an uncentered cross-moment matrix.

Finally, inference of assets' betas – measuring each asset's contribution to the overall variance of a portfolio - obviously face the same concern, as well as market efficiency (see Sharpe, 1964; Lintner, 1965; Black, 1972) testing.

The several issues are exemplified with Portuguese data on trading profits and tangible plus intangible assets for the period 1996 to 2002, and interaction with average firm size and aggregate importance of the industry briefly inspected. With an overall average economic return of around 11%, empirical traits of the pursued inquiries apparently generated different clustering – or ranking - of the sectors – and different from that implied by the industries' relative importance in the total existing stock. Simulated optimal portfolios do not seem to generate the later; yet, the actual "market" efficiency does not seem to be overall rejected.

In the empirical applications, the weighting methods are used by force of available data, based on aggregate yearly information by industry; yet, they have general applicability whenever observations pertain to elements of different sizes themselves. Naturally, international comparisons, as well as with micro data estimates – where the time length of the investments also arises as another weighting dimension -, suggest themselves for further developments.

The analysis proceeds as follows: Section 2, justifies econometric estimation procedures based on regressions without constant term and with multiplicative dummy variables on a theoretical level – special cases of SUR (and GLS) estimation; conformable estimates are forwarded in section 3, where data is introduced and statistically synthesized. Section 4 presents the implied efficient portfolios, contrasting the sectors' optimal with the actual asset shares in the economy. Aggregate betas towards the effective portfolio are provided along with some tests in section 5. Section 6 forwards parameter estimates for the unit rates of return distribution function under a truncated normal assumption. Concluding appraisals are summarized in this introduction.

2. (Weighted) Least squares rate of return estimates

The appropriate concept of the average return in an industry depends on its purposes. As an overall – descriptive - performance indicator, it should take the form of an aggregate index. As is well-known, that involves a weighted average by the amounts invested. Denote by R_{it} aggregate profits generated in year t by sector i and K_{it} the value of the assets allocated to that sector in the same year; the average return on the monetary unit invested in the sector in year t is $r_{it} = \frac{R_{it}}{K_{it}}$. Assume that the annual return of a "monetary" unit investment in sector i is a stable aggregate over the years, a random variable with fixed expected value, r_i, and variance, σ_i^2 (= σ_{ii}); the contemporaneous covariance structure (σ_{ij}'s) between – the joint pdf of - the unit returns in any two (and of all…) sectors is also stable. Across time, all such returns are statistically independent [2]. Being our objective to infer the first and second centered moments (and cross-moments) of the relevant distributions, the appropriate yearly estimator may differ from the previous aggregate ratio; an estimator derived from multiple-year samples may not have a direct correspondence with it. On the other hand, available information may condition our choice.

We may take different views of the statistical properties of the K_{it} observations sampled in year t – t = 1, 2, …, T - for sector i – i = 1, 2, …, n -, each of which indeed yields a unit return r_{itl}, l =1,2,..., K_{it}, unobserved by the researcher that only has access to $R_{it} = \sum_{l=1}^{K_{it}} r_{itl}$ - and thus, for each year and sector, to the yearly mean $r_{it} = \frac{R_{it}}{K_{it}}$ over the K_{it} units.

i) Let the observations be independent within each sector i.

Then, a natural estimator of the expected value r_i is the weighted (by $\frac{K_{it}}{\sum_{t=1}^{T} K_{it}}$) average of the T yearly means, the r_{it}'s, i.e.:

$$\bar{r}_i = \sum_{t=1}^{T} \frac{R_{it}}{K_{it}} \frac{K_{it}}{\sum_{t=1}^{T} K_{it}} = \frac{\sum_{t=1}^{T} R_{it}}{\sum_{t=1}^{T} K_{it}}, \; i=1, 2,...,n \qquad (1)$$

[2] This may, of course, be an unlikely assumption, even if we are not in the presence of a panel. Nevertheless, given the small number of time observations in the sample, it will be maintained throughout this research.

As $\bar{r}_i$ is identical to the standard mean over the $\sum_{t=1}^{T} K_{it}$ observations, $Var(\bar{r}_i) = \frac{\sigma_{ii}}{\sum_{t=1}^{T} K_{it}}$. Theoretically, $Var(r_{it}) = \frac{\sigma_{ii}}{K_{it}}$ - which is not constant over t – and therefore $Var(\bar{r}_i) = \sum_{t=1}^{T} \frac{K_{it}}{\sum_{t=1}^{T} K_{it}} \frac{Var(r_{it})}{T}$: being $\bar{r}_i$ the average over T of the r_{it}'s, an estimator for $Var(\bar{r}_i)$, $\widehat{Var}(\bar{r}_i)$, would relate to the estimators of $Var(r_{it})$ according to:

$$\widehat{Var}(\bar{r}_i) = \frac{\overline{Var(r_{it})}}{T} \qquad (2)$$

Of course, an estimator of $Var(r_{it})$ is $(\frac{R_{it}}{K_{it}} - \bar{r}_i)^2$:

$$\overline{Var(r_{it})} = \sum_{t=1}^{T} (\frac{R_{it}}{K_{it}} - \bar{r}_i)^2 \frac{K_{it}}{\sum_{t=1}^{T} K_{it}} = \sum_{t=1}^{T} (\frac{R_{it}}{K_{it}})^2 \frac{K_{it}}{\sum_{t=1}^{T} K_{it}} - \bar{r}_i^{\,2}, \quad i = 1, 2, ..., n \qquad (3)$$

In $\bar{r}_i$, all observations of sector i share the same status, no matter from which year they come from. Then, the variance of the monetary unit return in sector i, if we assumed each to be an independent draw, would be approximated by:

$$Var(r_{itl}) = \sum_{t=1}^{T} K_{it} \; Var(\bar{r}_i) = (\sum_{t=1}^{T} \frac{R_{it}^{\,2}}{K_{it}} - \sum_{t=1}^{T} K_{it} \; \bar{r}_i^{\,2})/T, \; i = 1, 2..., n \qquad (4)$$

An alternative estimation approach – and another indirect justification of the proposed estimators -would rely on the estimator of the parameter of the regression on the sum of the sector's yearly returns:

$$R_{it} = r_i K_{it} + ◎_{it}, \quad t = 1, 2, \ldots, T; \; i = 1, 2, ..., n \qquad (5)$$

$◎_{it}$ is an null mean error term. It is reasonable to assume it heteroscedastic, with variance proportional to the number of invested units – once they are assumed independent and summed to form each equation i,t, i.e.:

$$Var(◎_{it}) = ◎_i^2 K_{it}, \quad t = 1, 2, \ldots, T; \; i = 1, 2, ..., n \qquad (6)$$

If the error terms are uncorrelated, in time as across sectors, single equation estimates by weighted least squares, using $\frac{1}{K_{it}}$ as weights [3], or applying ordinary least squares to the transformed system:

$$\frac{R_{it}}{\sqrt{K_{it}}} = r_i \sqrt{K_{it}} + ◎^*_{it} \quad , \quad t = 1, 2, \ldots, T \; ; \; i = 1, 2, \ldots, n \qquad (7)$$

will provide as estimate of r_i:

$$\hat{r}_i = \frac{\sum_{t=1}^{T} R_{it}}{\sum_{t=1}^{T} K_{it}} = \bar{r}_i \quad , \quad i = 1, 2, \ldots, n \qquad (8)$$

The estimated variance will be:

$$Var(\hat{r}_i) = \sigma_i^2 \frac{1}{\sum_{t=1}^{T} K_{it}} \quad , \quad i = 1, 2, \ldots, n \qquad (9)$$

where:

$$\sigma_i^2 = \frac{\sum_{t=1}^{T} e_{it}^{*2}}{T-1} = \frac{1}{T-1} \left(\sum_{t=1}^{T} \frac{R_{it}^2}{K_{it}} - \sum_{t=1}^{T} K_{it} \bar{r}_i^2 \right) = \frac{T}{T-1} Var(r_{itl}) \; , \; i = 1, 2, \ldots, n \qquad (10)$$

with e_{it}^* denoting the estimated residuals from (7).

Hence:

$$Var(\hat{r}_i) = \frac{T}{T-1} Var(\bar{r}_i), \; i = 1, 2, \ldots, n \qquad (11)$$

Admit we also want to estimate the covariances of unit returns across sectors. Under independence, $Cov(r_{it}, r_{jt}) = Cov(r_{itl}, r_{jtl})$ = $◎_{ij}$: estimators of both covariances should coincide.

If we admit independence of sector observations, Cov($\frac{R_{it}}{K_{it}}, \frac{R_{jt}}{K_{jt}}$) = $◎_{ij}$: $\overline{Cov(r_{it}, r_{jt})}$ would reproduce $Cov(r_{itl}, r_{jtl})$ and we can propose a weighted (by K_{it} K_{jt}) estimator:

[3] Standard weighted least squares as commonly presented in econometric textbooks– see Greene (2003), for example.

$$Cov(r_{itl}, r_{jtl}) = \sum_{t=1}^{T} \frac{(\frac{R_{it}}{K_{it}} - \bar{r}_i)(\frac{R_{jt}}{K_{jt}} - \bar{r}_j)K_{it}K_{jt}}{\sum_{t=1}^{T} K_{it}K_{jt}} = \frac{\sum_{t=1}^{T}(R_{it} - \bar{r}_i K_{it})(R_{jt} - \bar{r}_j K_{jt})}{\sum_{t=1}^{T} K_{it}K_{jt}} \quad (12)$$

and – as we can show that $Cov(\bar{r}_i, \bar{r}_j) = \frac{\sum_{t=1}^{T} K_{it}K_{jt}}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{it}}$ ◎$_{ij}$ - it is reasonable to infer:

$$Cov(\bar{r}_i, \bar{r}_j) = \frac{\sum_{t=1}^{T} K_{it}K_{jt}}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{it}} Cov(r_{itl}, r_{jtl}) = \sum_{t=1}^{T} \frac{(\frac{R_{it}}{K_{it}} - \bar{r}_i)(\frac{R_{jt}}{K_{jt}} - \bar{r}_j)K_{it}K_{jt}}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}} =$$

$$\frac{\sum_{t=1}^{T}(R_{it} - \bar{r}_i K_{it})(R_{jt} - \bar{r}_j K_{jt})}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}} \quad (13)$$

Consistently, under independence, the errors of the distribution of the yearly sum of returns of (5) would be correlated in such a way that:

Cov(◎$_{it}$, ◎$_{jt}$) = ◎$_{ij}$ $K_{it}\, K_{jt}$, t = 1, 2, …, T ; i◎j, i, j= 1, 2, ..., n (14)

Then, referring to (7), Var(◎*$_{it}$) = ◎$_{ii}$ = ◎$_i^2$. Yet:

Cov(◎*$_{it}$, ◎*$_{jt}$) = ◎$_{ij}$ $\sqrt{K_{it}K_{jt}}$, t = 1, 2, …, T ; i◎j, i,j = 1, 2, ..., n (15)

Let ◎* = (◎*$_1$, ◎*$_2$, …, ◎*$_T$)′ represent the error term(s) of system (7) in vector form with ◎*$_t$ denoting the error of the t-th period observations, i.e., ◎*$_t$ = (◎*$_{1t}$, ◎*$_{2t}$, …, ◎*$_{nt}$)′ - ordering observations by sector and forming yearly blocks. Then:

Cov(◎◎) = V = (16)

$$
\begin{bmatrix}
\sigma_{11} & \sigma_{12}\sqrt{K_{11}K_{21}} \ldots \sigma_{1n}\sqrt{K_{11}K_{n1}} & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\
\sigma_{21}\sqrt{K_{21}K_{11}} & \sigma_{22} \quad \ldots \sigma_{2n}\sqrt{K_{21}K_{n1}} & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\
 & \ldots & & & & & & & & & \\
\sigma_{n1}\sqrt{K_{n1}K_{11}} & \sigma_{n2}\sqrt{K_{n1}K_{21}} \quad \ldots \quad \sigma_{nn} & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \sigma_{11} & \sigma_{12}\sqrt{K_{12}K_{22}} \ldots \sigma_{1n}\sqrt{K_{12}K_{n2}} & \ldots & & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \sigma_{21}\sqrt{K_{22}K_{12}} & \sigma_{22} \quad \ldots \sigma_{2n}\sqrt{K_{22}K_{n2}} & \ldots & & 0 & 0 & \ldots & 0 \\
 & & & & \ldots & & & & & & \\
0 & 0 & 0 & \sigma_{n1}\sqrt{K_{n2}K_{12}} & \sigma_{n2}\sqrt{K_{n2}K_{22}} \quad \ldots \quad \sigma_{nn} & \ldots & & 0 & 0 & \ldots & 0 \\
 & & & & \ldots & & & & & & \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & \sigma_{11} & \sigma_{12}\sqrt{K_{1T}K_{2T}} \ldots \sigma_{1n}\sqrt{K_{1T}K_{nT}} & \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & \sigma_{21}\sqrt{K_{2T}K_{1T}} & \sigma_{22} \quad \ldots \sigma_{2n}\sqrt{K_{2T}K_{nT}} & \\
 & & & & \ldots & & & & & & \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & \sigma_{n1}\sqrt{K_{nT}K_{1T}} & \sigma_{n2}\sqrt{K_{nT}K_{2T}} \quad \sigma_{nn} &
\end{bmatrix}
$$

Take the single equation OLS estimates – by sector – and obtain:

$$\sigma_{ii} = \frac{\sum_{t=1}^{T} e_{it}^{*2}}{T-1}, \quad i = 1, 2, \ldots, n \tag{17}$$

For ◎$_{ij}$ we could propose:

$$\sigma_{ij} = \frac{\sum_{t=1}^{T} \frac{e_{it}^{*} e_{jt}^{*}}{\sqrt{K_{it}K_{jt}}}}{T-1} = Cov(r_{itl}, r_{jtl}), \quad i◎j,\ i, j = 1, 2, \ldots, n \tag{18}$$

Alternatively – consistently with (12) – we can infer it from the OLS estimator of ◎$_{ij}$ of the equation: $e_{it}^{*} e_{jt}^{*}$ = ◎$_{ij}$ $\sqrt{K_{it}K_{jt}}$ + error$_t$, t =1,2,…T. Then we get:

$$\sigma_{ij} = \frac{\sum_{t=1}^{T} e_{it}^{*} e_{jt}^{*} \sqrt{K_{it}K_{jt}}}{\sum_{t=1}^{T} K_{it}K_{jt}} \tag{19}$$

On the one hand, the appropriate covariance matrix of the OLS estimator $\hat{r} = \bar{r} = (\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n)'$ may be obtained from:

$$Cov(\bar{r}) = (X'X)^{-1} X' V X (X'X)^{-1} \tag{20}$$

where, in V, ◎$_{ij}$ is replaced by σ_{ij}. Then, one can show that for i◎j, i, j= 1, 2, …, n:

$$Cov(\bar{r}_i, \bar{r}_j) = \frac{\sum_{t=1}^{T} K_{it} K_{jt}}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}} \frac{\sum_{t=1}^{T} (\frac{R_{it}}{K_{it}} - \bar{r}_i)(\frac{R_{jt}}{K_{jt}} - \bar{r}_j)}{T-1}$$

or $$\frac{\sum_{t=1}^{T} (R_{it} - \bar{r}_i K_{it})(R_{jt} - \bar{r}_j K_{jt})}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}} \tag{21}$$

according to which estimation approach we followed and we expect to approximate $Cov(r_{itl}, r_{jtl})$ from:

$$Cov(r_{itl}, r_{jtl}) = \frac{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}}{\sum_{t=1}^{T} K_{it} K_{jt}} Cov(\bar{r}_i, \bar{r}_j) \tag{22}$$

On the other, SUR estimation of equation system (7) is naturally suggested [4]: a second step would involve applying GLS to it with that same replacement. Denoting by Y the vector containing $\frac{R_{it}}{\sqrt{K_{it}}}$ stacked by period, and by X that of the corresponding $\sqrt{K_{it}}$, containing n multiplicative sector dummy variables: DI_{it} multiplied by $\sqrt{K_{it}}$, where DI_{it} = 1 for i =I, 0 for i ◎ I, I = 1,2,…,n[5]:

$$\hat{r}_{GLS} = (X' V^{-1} X)^{-1} X' V^{-1} Y \tag{23}$$

and

$$Cov(\hat{r}_{GLS}) = (X' V^{-1} X)^{-1} \tag{24}$$

Through Cholesky decomposition, one can obtain P such that V = P P' and estimate by OLS, as is well-known, of the transformed model:

$$P^{-1} Y = P^{-1} X + v \tag{25}$$

[4] For identification, we would need an adequately sized T – possibly not smaller than n + 1 (or n + 2 - see Gibbons, Ross & Shanken (1989), footnote 3 -, once we are estimating n parameters and n(n+1)/2 different elements of V with only nT observations. Given the special structure, a smaller sample could perform the task.

[5] Standard generalized least squares – see Dhrymes (1978), for example.

is legitimate, originating much the same results. In terms of relative order of magnitude, we would expect that matrix (24) would approximate:

1) in the diagonal,

$$\frac{T}{T-1}\ Var(\bar{r}_i),$$

$$\text{and}\ \frac{T}{T-1}\ \frac{1}{\sum_{t=1}^{T} K_{it}}\ Var(r_{itl}) \qquad (26)$$

2) off the diagonal

$$Cov(\bar{r}_i, \bar{r}_j),$$

$$\text{and}\ \frac{\sum_{t=1}^{T} K_{it} K_{jt}}{\sum_{t=1}^{T} K_{it} \sum_{t=1}^{T} K_{jt}}\ Cov(r_{itl}, r_{jtl}) \qquad (27)$$

Of course, new – more efficient – estimates $\hat{r}_i$ will also be produced – as well as σ_{ij}'s.

ii) Independence between observations of the same class or category is not the logic behind the portfolio variance estimation: rather, perfect (and positive) correlation between observations of the same type/asset is assumed – variances equating covariances for applications on the same title – rather than independence. That has important implications: consider a vector of observations X and define vector L = (1 , 1, …, 1)′, so that $\bar{X}$ = L′X / n. Then Var($\bar{X}$) = L′ Cov(X) L / n^2. If the observations in X are independent and share a common variance ◎^{2}, Cov(X) = ◎^{2} I_n, and, in fact, Var($\bar{X}$) = ◎^{2} / n. But if they are perfectly (positively) correlated, Cov(X) = ◎^{2} L L′ and Var($\bar{X}$) = ◎^{2}. The same would not be true for weighted means.

. Admit that, yearly, we are in the presence of a "portfolio" and that within each sector and year the correlation is perfect between the unit application returns (which is hardly the case for aggregate data like ours). Then the ◎$_{it}$ of (5) will more adequately be such that:

Var(◎$_{it}$) = ◎$_i^2$ K_{it}^2 = ◎$_{ii}$ K_{it}^2 , t = 1, 2, …, T ; i = 1, 2, ..., n (28) and (14) will still hold. If the error terms are uncorrelated, single equation estimates by weighted least squares, using $\frac{1}{K_{it}^2}$ as weights, or applying ordinary least squares to the transformed system:

$$\frac{R_{it}}{K_{it}} = r_i + ◎^*_{it} \quad , \quad t = 1, 2, \ldots, T\,; \; i = 1, 2, \ldots, n \qquad (29)$$

will provide a first step to obtain $\sigma_{ij} = \frac{\sum_{t=1}^{T} e_{it}^* e_{jt}^*}{T-1}$ or $\frac{\sum_{t=1}^{T} e_{it}^* e_{jt}^*}{T}$, for all i, j = 1, 2, ..., n, elements of the n by n covariance matrix ◎ of any t-th year's residuals. Notice that now, the first step estimator of r_i is the unweighted yearly mean of the weighted yearly means, i.e.: .

$$\hat{r}_i = \frac{\sum_{t=1}^{T} \frac{R_{it}}{K_{it}}}{T} \quad , \quad i = 1, 2, \ldots, n \qquad (30)$$

One can show that

$$Var(r_i) = \frac{\sigma_{ii}}{T}, \; i = 1, 2, \ldots, n; \text{ and } Cov(r_i, r_j) = \frac{\sigma_{ij}}{T}, \; i, j = 1, 2, \ldots, n \qquad (31)$$

As now

$$Cov(◎^*) = V = I_T ◎ ◎ \qquad (32)$$

(◎ denotes the Kronecker product of matrices) we could apply GLS to the system formed by the n equations of type (29) to obtain estimates of variances, covariances and of the estimators of r_i. However, under the new format, the right hand-side variables of the system are the same for all equation groups (a single constant term, always equal to 1) – rendering single-equation-OLS and GLS estimates identical. That is, $Cov(\hat{r}_{GLS})$ = $Cov(\hat{r}_{OLS})$ - obtainable as in (20) but with the new V - and we would approximate:

1) in the diagonal,

$$Var(r_i), \text{ and } \frac{1}{T} Var(r_{itl}) \qquad (33)$$

2) off the diagonal

$$Cov(r_i, r_j), \text{ and } \frac{1}{T} Cov(r_{itl}, r_{jtl}) \qquad (34)$$

One can show that under the assumption of independence between the observations of each sector, $Var(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}}{T}) = \frac{1}{T^2}\sum_{t=1}^{T}\frac{1}{K_{it}}$ σ_{ii} > $Var(\frac{\sum_{t=1}^{T}R_{it}}{\sum_{t=1}^{T}K_{it}}) = \frac{1}{\sum_{t=1}^{T}K_{it}}$ σ_{ii} (the arithmetic mean is larger than the harmonic mean) - hence, the aggregate estimator is more efficient [6]. Under perfect correlation of within sector observations, $Var(\frac{\sum_{t=1}^{T}R_{it}}{\sum_{t=1}^{T}K_{it}}) = \frac{\sum_{t=1}^{T}K_{it}^{2}}{(\sum_{t=1}^{T}K_{it})^2}$ σ_{ii} > $Var(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}}{T}) = \frac{\sigma_{ii}}{T}$ (because a variance – of K_{it} - is positive) – then, the simple mean of yearly rates is more efficient than the aggregate ratio.

iii) Finally, admit that within each year t and for sector i, we have n_{it} – say, the existing firms in the industry - sets each with $\frac{K_{it}}{n_{it}}$ perfectly correlated observations among themselves, yet, independent across sets of the same sector. One can show that then Var($\frac{R_{it}}{K_{it}}$) = $\frac{1}{n_{it}}$ σ_{ii} and Cov($\frac{R_{it}}{K_{it}}, \frac{R_{jt}}{K_{jt}}$) = σ_{ij}.

One can advance another weighted estimator for r_i based on n_{it} and available $\frac{R_{it}}{K_{it}}$:

$$\hat{r}_i = \frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}n_{it}}{\sum_{t=1}^{T}n_{it}} \quad , \quad i = 1, 2, ..., n \tag{35}$$

Its variance is now given by:

$$Var(\hat{r}_i) = \mathrm{Var}(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}n_{it}}{\sum_{t=1}^{T}n_{it}}) = \frac{1}{\sum_{t=1}^{T}n_{it}}\sigma_{ii} \tag{36}$$

[6] This is not surprising, once the LS regressions are weighted to provide conditions for increased efficiency of the estimators and we proved equivalence between the weighted means and the estimators of the first-step under independence, of the simple mean under perfect correlation…

The average variance of the yearly means $Var(r_{it})$ would be estimated by:

$$\overline{Var(r_{it})} = \sum_{t=1}^{T}\left(\frac{R_{it}}{K_{it}} - r_i\right)^2 \frac{n_{it}}{\sum_{t=1}^{T} n_{it}} = \sum_{t=1}^{T}\left(\frac{R_{it}}{K_{it}}\right)^2 \frac{n_{it}}{\sum_{t=1}^{T} n_{it}} - \hat{r}_i^{\,2} = \mathrm{T}\, Var(\hat{r}_i), \quad i = 1, 2, \ldots, n \qquad (37)$$

Hence, an estimator for ◎ii is:

$$Var(r_{itl}) = \sum_{t=1}^{T} n_{it} \quad \left(\sum_{t=1}^{T}\left(\frac{R_{it}}{K_{it}}\right)^2 \frac{n_{it}}{\sum_{t=1}^{T} n_{it}} - \hat{r}_i^{\,2}\right) / \mathrm{T}, \quad i = 1, 2, \ldots, n \qquad (38)$$

We would propose:

$$Cov(\hat{r}_i, \hat{r}_j) = \sum_{t=1}^{T}\left(\frac{R_{it}}{K_{it}} - \bar{r}_i\right)\left(\frac{R_{jt}}{K_{jt}} - \bar{r}_j\right) \frac{n_{it} n_{jt}}{\sum_{t=1}^{T} n_{it} \sum_{t=1}^{T} n_{jt}}, \quad i◎j,\ i, j = 1, 2, \ldots, n \qquad (39)$$

and – as under the assumptions, $Cov(\hat{r}_i, \hat{r}_j) = \frac{\sum_{t=1}^{T} n_{it} n_{jt}}{\sum_{t=1}^{T} n_{it} \sum_{t=1}^{T} n_{it}}$ ◎ij – for i◎j, i, j= 1, 2, ..., n:

$$Cov(r_{itl}, r_{jtl}) = \frac{\sum_{t=1}^{T} n_{it} \sum_{t=1}^{T} n_{it}}{\sum_{t=1}^{T} n_{it} n_{jt}} \; Cov(\hat{r}_i, \hat{r}_j) = \frac{\sum_{t=1}^{T}\left(\frac{R_{it}}{K_{it}} - \hat{r}_i\right)\left(\frac{R_{jt}}{K_{jt}} - \hat{r}_j\right) n_{it}\, n_{jt}}{\sum_{t=1}^{T} n_{it} n_{jt}} \qquad (40)$$

Under the assumptions, the error of (5) would be such that:

$$\mathrm{Var}(◎_{it}) = ◎_i^2 \frac{K_{it}^2}{n_{it}} = ◎_{ii} \frac{K_{it}^2}{n_{it}}, \quad t = 1, 2, \ldots, T; \; i = 1, 2, \ldots, n \qquad (41)$$

The transformed system:

$$\frac{R_{it}}{K_{it}} \sqrt{n_{it}} = r_i \sqrt{n_{it}} + ◎^*_{it} \quad t = 1, 2, \ldots, T; \; i = 1, 2, \ldots, n \qquad (42)$$

with ◎*it homoscedastic, i.e., Var(◎*it) = ◎ii, but with Cov(◎*it, ◎*jt) = $\sqrt{n_{it} n_{jt}}$ ◎ij for i◎j, i, j= 1, 2, ..., n. The single-equation estimators coincide with (35). The application of the GLS procedures of i) to the new format is straightforward. We would expect that the inferred $Cov(\hat{r}_{GLS})$ would approximate:

1) in the diagonal,

$$\frac{T}{T-1}\,Var(\hat{r}_i),\quad \text{and}\quad \frac{T}{T-1}\,\frac{1}{\sum_{t=1}^{T} n_{it}}\,Var(r_{itl}) \tag{43}$$

2) off the diagonal

$$Cov(\hat{r}_i,\hat{r}_j),\ \text{and}\ \frac{\sum_{t=1}^{T} n_{it}n_{jt}}{\sum_{t=1}^{T} n_{it}\sum_{t=1}^{T} n_{jt}}\,Cov(r_{itl},r_{jtl}) \tag{44}$$

Obviously, for $n_{it} = K_{it}$, expressions resume to those derived for case i). For $n_{it} = 1$, to case ii).

Under the new assumptions, $Var\left(\frac{\sum_{t=1}^{T} R_{it}}{\sum_{t=1}^{T} K_{it}}\right) = \frac{\sum_{t=1}^{T}\frac{K_{it}}{n_{it}}K_{it}}{(\sum_{t=1}^{T} K_{it})^2}\,\sigma_{ii}$ and $Var\left(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}}{T}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\frac{1}{n_{it}}\,\sigma_{ii}$, the latter larger than (36) – the current weighted average is more efficient.

If each unit within each set was in fact independent and not perfectly correlated as assumed, $Var\left(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}n_{it}}{\sum_{t=1}^{T} n_{it}}\right) = \frac{\sum_{t=1}^{T}\frac{n_{it}}{K_{it}}n_{it}}{(\sum_{t=1}^{T} n_{it})^2}\,\sigma_{ii}$ and expected to be larger than $Var\left(\frac{\sum_{t=1}^{T} R_{it}}{\sum_{t=1}^{T} K_{it}}\right) = \frac{1}{\sum_{t=1}^{T} K_{it}}\,\sigma_{ii}$. (Of course we are always assuming that K_{it} and n_{it} – and $\frac{K_{it}}{n_{it}}$ - are deterministic, otherwise, we would stumble into the fact that the variance of the product of even two statistically independent variables is related to their second moments in a different way that the expected value of their product…). If all the K_{it} observations are perfectly correlated, $Var\left(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}n_{it}}{\sum_{t=1}^{T} n_{it}}\right) = \frac{\sum_{t=1}^{T} n_{it}^{\,2}}{(\sum_{t=1}^{T} n_{it})^2}\,\sigma_{ii} > Var\left(\frac{\sum_{t=1}^{T}\frac{R_{it}}{K_{it}}}{T}\right) = \frac{\sigma_{ii}}{T}$: the simple yearly mean is a more efficient estimator.

Some crude heteroscedasticity tests were performed on residuals of equation (29) in the hope to validate one of the three cases. Unfortunately results – summarized in the Appendix - are very vague.

Possibly, we would like to have information at firm level – rather than by sector - and apply model ii) to the corresponding data set; arrangement iii) would somehow capture that pattern. Also, iii) would be preferable to i) due to arbitrariness of the relevant "unit" in i) – the one in which the K_{it}'s are measured: the estimated variance of the "unit" return is proportional to the sum of K_{it}'s and depends on its measurement unit – the units within 1 unit would be those we would be assuming to be able to replicate to, say, form a portfolio, which would not make much sense. Therefore, in the empirical research below, we only report results for cases ii) and iii). Still, i) has the intuitive advantage of producing for the OLS estimate of the expected value of a sector's unit return the general weighted mean of all the observed returns. Moreover, it provides the adequate procedures to produce variances and covariances of individuals' attributes from averaged (or aggregate) data.

Nevertheless, in the formation of a portfolio from assets of either type, one should only use the estimated σ_{ij}'s rather than $Cov(\hat{r})$ if one admits that only one firm can be picked to represent the i-th title – which would be rather unusual.

A final comment can be made with respect to environment iii): it maybe not be reasonable to assume that some units within a sector are mutually independent and yet consistently correlated to other sectors'… One could justify that theoretically only if there are links between sectors' – say, through intermediate product exchange – but not so much between units of the same sector. On the other extreme – and favoring now i) -, one could argue that at a given point in time a firm's assets are the result of different waves of investments, which indeed give a different return. Yet, we are left with the unknown measure of the relevant unit…

3. Sector economic rates of return: Source data and estimation results

Aggregate information on (gross, end-of-the-year) "Tangible Assets" and "Intangible Assets" of firms' balance sheets by industry and the corresponding yearly Trading Profits ("Excedente Bruto de Exploração"), along with the number of sampled units, is published by the official Portuguese statistical institution, I.N.E; we collected it from the statistical periodical: "Sistema de Contas Integradas das Empresas", using information from the issues 1996-1997, 1997-1998, 1998-1999, (from all of which only the oldest year was used in the research), 1999-2000 and 2001-2002 [7]. We constructed a pooled data set with information by industry (totaling n = 23

[7] Each (annual) volume has information for two consecutive years and the number of covered enterprises of each sector and category – for representativeness purposes, we presume, which is convenient under our estimation logic - change even in the same issue. The periodical is quite recent, starting in 1994-1995 and data for firms with 20 to 99 people are only available from the 1996-1997 issue onwards; aggregates are reported in euros since the 1998-1999 issue.

sectors, excluding Agriculture and most Financial Services and Institutions, with different accounting rules) and year (1996 to 2002 – T = 7) pertaining to firms with 20 or more employees [8]. Measurement units of aggregate data were all converted to 10^6 PTE – at the rate 200.482 PTE per Euro. The coding used for sectors is reported below:

0. Total
3. Mining Industries
40. Manufacturing Industries: 4 to 17
4. Food, Beverages and Tobacco
5. Textiles and Clothing
6. Leather and Leather Articles
7. Woodwork and Cork Manufacturing
8. Paper, Graphical Arts and Publishing
910. Chemical Industries from Oil and Coke
11. Rubber and Plastic Articles
12. Other non-Metallic Minerals
13. Heavy Metallurgy and Metallic Products
14. Machinery and Equipment
15. Electric and Optical Equipment
16. Transportation Material
17. Non-Specified Manufacturing Industries
18. Electricity, Water and Gas
19. Construction and Public Infrastructure
20. Commerce
21. Restoration and Lodging
22. Transportation, Storage and Communications
24. Real Estate and Service to Firms
26. Education
27. Health and Social Service
28. Other Collective and Personal Services

Proceeding to the estimates of section 2, we inferred the sector rates of return, estimates of variances and covariances of both the rates as their estimators. Results are reported in Tables 1, 1.A and 1.B below. The first column exhibits the inferred sector average rate of return. In the last seven rows of the Tables, we report the simple correlation coefficients between each series and the estimates of the rates of return ($\hat{r}_i$), the variance of the rates ($Var(r_i)$), of the estimates ($Var(\hat{r}_i)$ - given the proportionality between the two variances under hypothesis ii), only one of them is reported), the sector's (yearly average) assets' share on the total ($\bar{s}_{Ki}$), the sector's profits

[8] Published information offers disaggregation for each sector and year by two firm size classes – 20 to 99 people and 100 or more. Refinements of the procedures to benefit from it could be developed and are left for future research.

share ($\bar{s}_{Ri}$), firms' (sampled units) share ($\bar{s}_{ni}$), the average firm's asset size ($\overline{\frac{K_i}{n_i}}$), the average firm's profit size ($\overline{\frac{R_i}{n_i}}$), and average – from a regression of the logarithm of the sector series on a time trend – annual growth rates of K_i, R_i, n_i, $\frac{K_i}{n_i}$, $\frac{R_i}{n_i}$ and r_i. (For 23 observations, correlations are significant at the 5% level for $| r |$ larger than 0.4142985; at the 10% level, for $| r |$ larger than 0.3522931.)

For hypothesis ii) of section 2 - results are depicted in Table 1 -, the elements of the covariance matrix of the residuals was derived according to (29). The average rate of return in the economy is around 11%. By industry – one can inspect the (descending) order/rank of the reported rates and variances by industry in the first columns -, the highest returns are found in sectors 20, 19, 27, 15, 11, 17 and 12; lowest, in 910, 3, 5, 22, 18, 21 and 28. The highest volatility occurs in sectors 6, 28, 27, 16, 26, 24 and 910; sectors 17, 14, 22, 21, 13, 18 and 4 exhibit low variances. The correlation between the average sector rates of return and their variance was -0.042122, with a p-value of .985. Hence, non-significant.

Tables 1.A and 1.B present the corresponding weighted estimates under assumption iii) of section 2. The correlation between the variances of these two tables is 0.53811. The correlation between the mean returns and the estimated variances of the unit application [9] – Table 1.A - is 0.304706 and almost significant (which could be almost expected: higher unit variances would require higher returns – if we disregard the covariances effect); that with the variance of the mean returns – Table 1.B - is –0.043765.

The ordering of the variances of the mean returns – Table 1.B – coincides with the ones inferred in Table 1. The same is true for the estimated mean returns – even if under weighting, estimates are slightly larger [10].

With respect to the correlations with the other variables, we note a consistent positive and significant relation between the firms share – and hence the number of firms in the sector – and the average returns, suggesting that more profitable sectors would attract entry, less profitable would push exit. And a negative relation between the sector's capital and profit share (hence total assets and profits importance) and the variance of the estimator of the rate of return – not present for the variance of the rate of return itself for case iii).

[9] We report approximations inferred after (43)-(44), indeed close to the direct (first-step GLS) calculations of σ_{ij}.

[10] Average yearly economic rates of return by sampled firm – an average of which would be similar to what we capture in our weighted estimates of type iii - were reported in the statistical publications of 1998-1999 and 1999-2000 only. In general, such average rates were larger than the average rates calculated from the ratios of the total sector's aggregate profits by the aggregate assets.

The correlation of the sectors' estimates covariance series with the variances is high, sometimes positive, sometimes negative – but not the covariances of the rates themselves (Table (1.A)). Other variables seem to be independent of those covariances.

Table 1. *Industry Economic Return Rates: Means, Variances and Covariances (ii)*

				$\hat{Cov}(r_{itl}, r_{jtl})$ *1000000																						
Sector	$\hat{r}_i$	Order of R: Magnit.	Ord. of V: Magnit.	3	4	5	6	7	8	910	11	12	13	14	15	16	17	18	19	20	21	22	24	26	25	28
Total	0.11023																									
3	0.09919	18	12	170.98	-33.44	103.18	357.15	130.40	-2.63	172.30	78.72	42.37	-5.57	-60.58	48.30	163.80	-0.53	43.80	11.70	48.90	-86.86	101.41	140.47	243.42	241.91	-207.52
4	0.12298	13	23	-33.44	22.72	-20.58	-83.71	-28.20	26.93	-49.89	-23.37	5.08	30.67	41.29	-12.06	29.96	18.78	-18.25	-25.39	-11.15	24.25	-30.27	-32.46	-69.54	-47.73	69.14
5	0.08397	19	15	103.18	-20.58	151.71	309.52	154.64	-72.39	211.25	143.09	117.95	-13.18	-4.98	175.78	123.65	38.42	64.75	30.01	96.02	-23.30	86.28	82.98	215.75	121.71	-84.95
6	0.12355	12	1	357.15	-83.71	309.52	994.94	319.49	-256.26	506.20	330.71	180.68	-80.33	-150.14	196.06	242.33	104.77	141.95	30.33	265.53	-191.27	289.89	255.03	580.89	582.97	-439.25
7	0.10918	15	10	130.40	-28.20	154.64	319.49	214.93	-52.42	242.89	132.09	116.21	2.72	-27.60	174.76	110.31	40.92	74.11	17.36	40.56	-64.13	103.26	191.30	265.01	198.00	-85.40
8	0.12139	14	9	-2.63	26.93	-72.39	-256.26	-52.42	286.04	-150.36	-159.48	-59.29	87.30	65.48	-54.05	186.09	-107.09	-53.42	-5.66	-145.07	45.49	-93.48	1.98	-83.91	-135.31	32.72
910	0.10513	17	7	172.30	-49.89	211.25	506.20	242.89	-150.36	334.24	214.16	140.54	-45.78	-61.98	219.37	100.41	54.79	103.90	49.68	132.56	-80.51	155.35	168.22	356.69	263.29	-183.23
11	0.15546	5	13	78.72	-23.37	143.09	330.71	132.09	-159.48	214.16	170.68	105.55	-41.72	-20.75	159.08	39.43	66.68	67.64	34.08	140.23	-28.58	96.53	35.27	195.17	136.22	-92.66
12	0.14625	7	14	42.37	5.08	117.95	180.68	116.21	-59.29	140.54	105.55	157.15	20.46	30.10	138.48	93.49	80.57	42.97	-44.53	41.32	0.13	53.93	119.43	94.17	52.56	97.13
13	0.13009	11	21	-5.57	30.67	-13.18	-80.33	2.72	87.30	-45.78	-41.72	20.46	67.37	55.81	-10.23	109.40	21.11	-21.77	-49.30	-47.66	12.41	-30.65	28.62	-50.44	-15.27	82.69
14	0.13998	8	18	-60.58	41.29	-4.98	-150.14	-27.60	65.48	-61.98	-20.75	30.10	55.81	99.54	50.27	97.15	17.46	-21.75	-12.41	-0.67	72.58	-61.43	-89.00	-96.29	-136.07	117.99
15	0.16382	4	8	48.30	-12.06	175.78	196.06	174.76	-54.05	219.37	159.08	138.48	-10.23	50.27	291.43	120.56	16.84	75.57	85.04	103.56	42.18	48.00	10.53	202.62	-19.12	-27.85
16	0.13354	9	4	163.80	29.96	123.65	242.33	110.31	186.09	100.41	39.43	93.49	109.40	97.15	120.56	494.53	0.97	7.32	-10.42	55.49	6.12	24.28	30.91	171.52	103.76	-129.04
17	0.14863	6	17	-0.53	18.78	38.42	104.77	40.92	-107.09	54.79	66.68	80.57	21.11	17.46	16.84	0.97	114.49	11.79	-73.19	44.60	-23.42	36.47	54.04	-4.52	92.57	93.55
18	0.06675	21	22	43.80	-18.25	64.75	141.95	74.11	-53.42	103.90	67.64	42.97	-21.77	-21.75	75.57	7.32	11.79	34.58	22.86	39.03	-19.76	45.74	48.39	107.20	64.01	-48.92
19	0.19481	2	16	11.70	-25.39	30.01	30.33	17.36	-5.66	49.68	34.08	-44.53	-49.30	-12.41	85.04	-10.42	-73.19	22.86	116.36	57.93	25.12	0.64	-106.76	87.41	-50.66	-149.75
20	0.22410	1	11	48.90	-11.15	96.02	265.53	40.56	-145.07	132.56	140.23	41.32	-47.66	-0.67	103.56	55.49	44.60	39.03	57.93	182.07	1.54	60.67	-116.10	121.32	76.03	-147.89
21	0.06585	22	20	-86.86	24.25	-23.30	-191.27	-64.13	45.49	-80.51	-28.58	0.13	12.41	72.58	42.18	6.12	-23.42	-19.76	25.12	1.54	88.17	-69.83	-131.31	-112.53	-199.20	97.88
22	0.08072	20	19	101.41	-30.27	86.28	289.89	103.26	-93.48	155.35	96.53	53.93	-30.65	-61.43	48.00	24.28	36.47	45.74	0.64	60.67	-69.83	92.93	111.77	172.59	188.67	-111.95
24	0.13264	10	6	140.47	-32.46	82.98	255.03	191.30	1.98	168.22	35.27	119.43	28.62	-89.00	10.53	30.91	54.04	48.39	-106.76	-116.10	-131.31	111.77	419.10	191.31	276.40	47.83
26	0.10629	16	5	243.42	-69.54	215.75	580.89	265.01	-83.91	356.69	195.17	94.17	-50.44	-96.29	202.62	171.52	-4.52	107.20	87.41	121.32	-112.53	172.59	191.31	446.67	331.85	-330.52
27	0.18598	3	3	241.91	-47.73	121.71	582.97	198.00	-135.31	263.29	136.22	52.56	-15.27	-136.07	-19.12	103.76	92.57	64.01	-50.66	76.03	-199.20	188.67	276.40	331.85	509.55	-270.30
28	0.03112	23	2	-207.52	69.14	-84.95	-439.25	-85.40	32.72	-183.23	-92.66	97.13	82.69	117.99	-27.85	-129.04	93.55	-48.92	-149.75	-147.89	97.88	-111.95	47.83	-330.52	-270.30	511.30
Cor. with			(Var.)																							
$\hat{r}_i$	1.0000		-0.0042	0.2151	-0.1842	0.1846	0.2376	0.1248	-0.1874	0.1951	0.2300	-0.0963	-0.2262	-0.1539	0.1192	0.1626	0.0047	0.1698	0.2939	0.3617	-0.1539	0.1933	-0.1215	0.2234	0.2057	-0.3498
$\hat{Var}(\hat{r}_i)$	-0.0042		1.0000	0.5766	-0.3975	0.5158	0.5491	0.5510	-0.2513	0.5110	0.4318	0.5146	-0.1323	-0.4190	0.2334	0.4161	0.3347	0.4555	-0.1009	0.2772	-0.5371	0.5454	0.5583	0.5132	0.5670	-0.3692
$\bar{s}_{Ki}$	-0.2633		-0.3592	-0.0704	-0.0795	-0.0886	-0.0394	-0.1046	-0.0824	-0.0493	-0.0295	-0.1623	-0.1929	-0.0994	-0.1021	-0.2396	-0.0974	-0.0217	0.0823	0.0046	0.0133	-0.0211	-0.0800	-0.0475	-0.0458	-0.0128
$\bar{s}_{Ri}$	0.1460		-0.3527	-0.0803	-0.0679	-0.0761	-0.0264	-0.1466	-0.1204	-0.0498	0.0086	-0.2130	-0.2481	-0.0643	-0.0666	-0.2085	-0.1158	-0.0245	0.1860	0.1220	0.0629	-0.0316	-0.2205	-0.0475	-0.0688	-0.0684
$\bar{s}_{ni}$	0.4227		-0.1990	-0.0910	-0.0504	-0.0372	-0.0145	-0.1500	-0.1549	-0.0316	0.0597	-0.1984	-0.2770	-0.0119	0.0206	-0.1704	-0.1080	-0.0040	0.2737	0.2194	0.1170	-0.0379	-0.3211	-0.0385	-0.0948	-0.0952
$\frac{\overline{K_i}}{n_i}$	-0.3263		-0.2256	-0.0492	-0.0541	-0.0434	-0.0343	-0.0399	-0.0375	-0.0213	-0.0173	-0.0843	-0.1112	-0.0511	-0.0207	-0.1536	-0.0798	0.0023	0.0729	-0.0176	0.0186	-0.0189	-0.0341	-0.0219	-0.0401	0.0032
$\frac{\overline{R_i}}{n_i}$	-0.2963		-0.2279	-0.0306	-0.0609	-0.0212	-0.0203	-0.0219	-0.0285	-0.0055	-0.0039	-0.0729	-0.1077	-0.0460	0.0086	-0.1148	-0.0936	0.0162	0.0966	-0.0012	0.0184	-0.0104	-0.0364	-0.0027	-0.0334	-0.0164
Kgr	0.1280		0.2297	-0.0760	-0.0350	-0.1740	-0.0521	-0.0239	-0.1527	-0.0631	-0.1006	-0.0094	-0.0185	-0.2259	-0.3584	-0.4474	0.2910	-0.0511	-0.4166	-0.2672	-0.2165	0.0492	0.3260	-0.1019	0.1087	0.1942
Rgr	-0.1211		0.0573	-0.5791	0.5010	-0.6137	-0.5611	-0.5240	0.2239	-0.5638	-0.5374	-0.2376	0.3826	0.3220	-0.5740	-0.5482	0.1231	-0.5393	-0.5834	-0.5895	0.3219	-0.4820	-0.0945	-0.6154	-0.4254	0.6543
ngr	0.0310		0.0927	-0.0272	-0.0851	-0.2233	-0.0488	-0.0703	-0.0617	-0.0952	-0.1645	-0.1906	-0.0424	-0.2811	-0.4559	-0.3885	0.1021	-0.0952	-0.3327	-0.2836	-0.2453	0.0393	0.2871	-0.0834	0.1244	0.0895
(K/n)gr	0.1880		0.2829	-0.0987	0.0614	0.0110	-0.0233	0.0598	-0.1881	0.0248	0.0585	0.2636	0.0287	0.0014	0.0169	-0.2451	0.3811	0.0469	-0.2709	-0.0702	-0.0342	0.0321	0.1726	-0.0633	0.0153	0.2228
(R/n)gr	-0.1678		0.0213	-0.7077	0.6710	-0.6446	-0.6735	-0.6156	0.3127	-0.6518	-0.5814	-0.1933	0.5002	0.5538	-0.4693	-0.4736	0.0983	-0.6212	-0.5477	-0.5818	0.5343	-0.6225	-0.2733	-0.7225	-0.5980	0.7677
rgr	-0.2893		-0.1009	-0.8196	0.7916	-0.7918	-0.8116	-0.7783	0.4661	-0.8068	-0.7359	-0.3542	0.5977	0.6754	-0.5804	-0.4682	-0.0510	-0.7794	-0.5471	-0.6787	0.6675	-0.7743	-0.4110	-0.8536	-0.7368	0.8372

Table 1.A. *Industry Economic Return Rates: Means, Variances and Covariances (iii)*

$\hat{Cov}(r_{itl}, r_{jtl})$ *1000000

Sector	$\hat{r}_i$	Or of R Magnit.	Or of V Magnit.	3	4	5	6	7	8	910	11	12	13	14	15	16	17	18	19	20	21	22	24	26	25	28
Total	0.14486																									
3	0.09904	18	21	25013.8	-28.62	83.05	298.96	110.71	-1.54	146.84	63.14	33.54	-6.28	-50.27	41.03	131.33	-5.07	31.85	11.50	35.92	-60.77	72.52	87.37	188.19	134.15	-150.18
4	0.12297	13	22	-28.62	19815.2	-18.48	-72.42	-26.79	24.12	-45.70	-20.35	4.41	26.16	36.28	-10.56	26.32	16.63	-14.96	-21.26	-9.78	18.15	-24.09	-15.30	-57.66	-30.76	56.79
5	0.08401	19	3	83.05	-18.48	348403.5	267.76	132.26	-63.26	182.86	124.01	99.45	-12.05	-5.43	150.98	103.66	31.73	54.59	27.50	82.12	-11.43	69.97	59.40	183.66	67.39	-61.81
6	0.12301	12	1	298.96	-72.42	267.76	702500.0	292.39	-217.33	454.30	286.46	152.28	-65.24	-129.49	170.91	230.41	88.89	115.11	30.70	225.69	-146.29	233.39	158.67	493.98	389.73	-354.61
7	0.10898	15	10	110.71	-26.79	132.26	292.39	86264.0	-48.15	216.78	114.93	97.85	1.63	-28.50	146.84	94.06	34.03	62.09	11.69	34.35	-49.71	84.65	154.74	227.64	132.38	-56.93
8	0.12145	14	6	-1.54	24.12	-63.26	-217.33	-48.15	118977.1	-130.16	-136.73	-50.84	75.28	58.51	-46.03	160.64	-94.10	-45.53	-5.77	-125.31	33.97	-78.42	19.23	-69.27	-111.15	25.76
910	0.10467	17	13	146.84	-45.70	182.86	454.30	216.78	-130.16	63204.2	185.55	121.29	-38.85	-58.39	186.98	89.94	45.74	87.57	40.28	111.04	-59.74	128.26	126.54	306.56	174.33	-138.09
11	0.15542	5	20	63.14	-20.35	124.01	286.46	114.93	-136.73	185.55	36917.4	92.55	-35.73	-17.12	139.91	37.65	56.66	57.57	30.51	121.18	-14.18	78.77	16.27	164.90	83.03	-67.74
12	0.14613	7	8	33.54	4.41	99.45	152.28	97.85	-50.84	121.29	92.55	93210.6	19.60	26.86	120.18	82.25	71.93	35.58	-35.70	36.03	6.32	43.96	97.25	83.82	21.87	88.96
13	0.13013	11	18	-6.28	26.16	-12.05	-65.24	1.63	75.28	-38.85	-35.73	19.60	48459.1	47.64	-7.92	90.63	19.31	-17.74	-43.60	-45.16	6.53	-24.90	46.91	-40.46	-10.08	77.33
14	0.13996	8	19	-50.27	36.28	-5.43	-129.49	-28.50	58.51	-58.39	-17.12	26.86	47.64	44823.3	42.89	83.66	15.79	-15.73	-10.88	-1.17	56.96	-46.56	-44.70	-74.32	-93.53	95.76
15	0.16382	4	16	41.03	-10.56	150.98	170.91	146.84	-46.03	186.98	139.91	120.18	-7.92	42.89	57116.0	104.99	14.33	67.40	71.40	91.05	42.89	43.04	20.52	180.12	-29.33	-14.84
16	0.13339	9	9	131.33	26.32	103.66	230.41	94.06	160.64	89.94	37.65	82.25	90.63	83.66	104.99	91248.8	-0.54	6.00	-6.84	44.95	7.70	19.85	41.33	149.22	38.74	-96.39
17	0.14863	6	15	-5.07	16.63	31.73	88.89	34.03	-94.10	45.74	56.66	71.93	19.31	15.79	14.33	-0.54	62505.9	9.07	-64.46	34.97	-18.97	29.34	53.07	-7.29	71.47	92.58
18	0.06651	21	23	31.85	-14.96	54.59	115.11	62.09	-45.53	87.57	57.57	35.58	-17.74	-15.73	67.40	6.00	9.07	2269.49	20.97	34.54	-8.93	33.38	25.88	85.24	31.54	-32.42
19	0.19564	2	5	11.50	-21.26	27.50	30.70	11.69	-5.77	40.28	30.51	-35.70	-43.60	-10.88	71.40	-6.84	-64.46	20.97	206784.7	53.88	23.65	1.58	-100.08	68.74	-38.44	-122.85
20	0.22409	1	2	35.92	-9.78	82.12	225.69	34.35	-125.31	111.04	121.18	36.03	-45.16	-1.17	91.05	44.95	34.97	34.54	53.88	605270.0	8.67	51.11	-109.44	101.32	43.94	-124.95
21	0.06597	22	12	-60.77	18.15	-11.43	-146.29	-49.71	33.97	-59.74	-14.18	6.32	6.53	56.96	42.89	7.70	-18.97	-8.93	23.65	8.67	65554.9	-44.22	-82.94	-70.02	-127.27	58.91
22	0.08051	20	17	72.52	-24.09	69.97	233.39	84.65	-78.42	128.26	78.77	43.96	-24.90	-46.56	43.04	19.85	29.34	33.38	1.58	51.11	-44.22	54304.4	56.67	126.24	102.62	-72.68
24	0.13199	10	4	87.37	-15.30	59.40	158.67	154.74	19.23	126.54	16.27	97.25	46.91	-44.70	20.52	41.33	53.07	25.88	-100.08	-109.44	-82.94	56.67	336452.0	112.28	129.13	98.16
26	0.10605	16	7	188.19	-57.66	183.66	493.98	227.64	-69.27	306.56	164.90	83.82	-40.46	-74.32	180.12	149.22	-7.29	85.24	68.74	101.32	-70.02	126.24	112.28	109412.2	177.83	-234.70
27	0.18417	3	14	134.15	-30.76	67.39	389.73	132.38	-111.15	174.33	83.03	21.87	-10.08	-93.53	-29.33	38.74	71.47	31.54	-38.44	43.94	-127.27	102.62	129.13	177.83	62992.5	-132.74
28	0.03152	23	11	-150.18	56.79	-61.81	-354.61	-56.93	25.76	-138.09	-67.74	88.96	77.33	95.76	-14.84	-96.39	92.58	-32.42	-122.85	-124.95	58.91	-72.68	98.16	-234.70	-132.74	81500.5
Cor. w			(Var.)																							
$\hat{r}_i$	1.0000		0.304	-0.1230	-0.0096	-0.2001	-0.0079	-0.0769	-0.0167	-0.0967	0.1532	0.1051	0.0257	0.0742	0.1926	0.0433	0.1175	-0.2742	0.3494	0.4894	-0.2891	-0.2165	0.0355	-0.0904	0.2932	-0.4590
$V\hat{a}r(r_i)$	0.3047		1.000	-0.1336	-0.1506	0.2423	0.6619	-0.0671	-0.0317	-0.0914	-0.1219	-0.0600	-0.1154	-0.1204	-0.1017	-0.0618	-0.0964	-0.1340	0.0741	0.5466	-0.0949	-0.1040	0.2276	-0.0386	-0.0923	-0.0779
$V\hat{a}r(\hat{r}_i)$	-0.0438		0.538	-0.0751	-0.2252	-0.0945	0.7599	-0.0300	0.0419	0.0917	-0.0704	-0.0877	-0.1790	-0.1479	0.0486	0.2512	-0.1277	-0.1737	-0.1472	-0.0651	-0.1672	-0.1626	0.0689	0.2019	0.1212	0.2403
$\bar{s}_{Ki}$	-0.2625		-0.039	-0.1281	0.0395	0.0342	-0.1352	-0.1220	-0.0197	0.0187	-0.1295	-0.0183	-0.0870	-0.1205	-0.0998	-0.0725	-0.1384	0.5920	-0.0367	0.1639	-0.0467	0.6883	0.0210	-0.1525	-0.1544	-0.1071
$\bar{s}_{Ri}$	0.1339		0.236	-0.1563	0.0817	-0.0073	-0.1530	-0.1436	-0.0070	0.0262	-0.1356	0.0406	-0.0852	-0.1283	-0.0773	-0.0582	-0.1543	0.3634	0.0690	0.5873	-0.1151	0.5425	0.0469	-0.1826	-0.1772	-0.1850
$\bar{s}_{ni}$	0.4265		0.657	-0.1530	0.0388	0.4290	-0.0057	-0.0851	-0.0834	-0.1425	-0.1344	-0.0343	-0.0006	-0.0731	-0.1408	-0.1447	-0.0504	-0.1734	0.3675	0.7063	0.0250	-0.0115	0.0904	-0.1279	-0.1481	-0.1500
$\overline{K_i}/n_i$	-0.3273		-0.204	-0.0467	-0.0482	-0.0627	-0.0666	-0.0600	-0.0362	0.0241	-0.0540	-0.0468	-0.0606	-0.0610	-0.0370	-0.0215	-0.0659	0.9881	-0.0649	-0.0614	-0.0566	0.0502	-0.0540	-0.0650	-0.0638	-0.0359
$\overline{R_i}/n_i$	-0.2973		-0.215	-0.0512	-0.0453	-0.0775	-0.0798	-0.0700	-0.0236	0.0601	-0.0482	-0.0348	-0.0671	-0.0663	-0.0046	0.0124	-0.0763	0.9807	-0.0701	-0.0557	-0.0735	0.0577	-0.0545	-0.0785	-0.0677	-0.0704
Kgr	0.1251		-0.005	-0.0481	-0.2528	-0.2323	-0.1975	-0.1061	-0.1036	-0.1978	0.0698	-0.0519	-0.1140	-0.0874	-0.0566	-0.2403	0.0795	-0.2634	0.0999	0.0807	-0.0359	0.1405	0.3727	0.1598	0.5009	0.4605
Rgr	-0.1200		-0.169	-0.0897	-0.0691	-0.2387	-0.2995	-0.1452	0.0499	-0.2482	-0.0038	-0.0621	-0.0038	0.0212	-0.0680	-0.1446	0.0567	-0.2282	0.0937	0.0566	0.0974	0.0063	0.1521	-0.0508	0.2347	0.8002
ngr	0.0275		-0.064	-0.0020	-0.2419	-0.1684	-0.2240	-0.2921	-0.1668	-0.2322	-0.0352	-0.0983	-0.0274	-0.1491	-0.1387	-0.1192	-0.0535	0.0974	0.2120	-0.0133	0.0994	0.2058	0.4316	0.0697	0.5459	0.2674
(K/n)gr	0.1880		0.086	-0.0849	-0.1053	-0.1762	-0.0309	0.2364	0.0564	-0.0192	0.1793	0.0500	-0.1679	0.0600	0.1009	-0.2633	0.2239	-0.6244	-0.1296	0.1670	-0.2118	-0.0464	0.0450	0.1891	0.1109	0.4472
(R/n)gr	-0.1646		-0.176	-0.1108	0.0447	-0.2067	-0.2524	-0.0230	0.1525	-0.1840	0.0143	-0.0242	0.0101	0.1071	-0.0098	-0.1159	0.0997	-0.3373	0.0022	0.0777	0.0677	-0.1035	-0.0438	-0.1011	-0.0026	0.8535
rgr	-0.2853		-0.254	-0.0972	0.1018	-0.1732	-0.2943	-0.1341	0.1608	-0.2160	-0.0630	-0.0520	0.0877	0.1038	-0.0572	-0.0233	0.0212	-0.1317	0.0608	0.0200	0.1777	-0.1055	-0.0737	-0.2083	-0.0530	0.8412

Table 1.B. *Industry Economic Return Rates: Means, Variances and Covariances (iii)*

$\hat{Cov}(\hat{r}_i, \hat{r}_j)$ *1000000

Sector	$\hat{r}_i$	Or of Rate Magnit.	Or of V Magnit.	3	4	5	6	7	8	910	11	12	13	14	15	16	17	18	19	20	21	22	24	26	25	28
Total	0.14486																									
3	0.09904	18	13	22.763	-4.081	11.862	42.612	15.767	-0.220	20.937	9.038	4.795	-0.900	-7.179	5.861	18.767	-0.726	4.575	1.654	5.146	-8.721	10.433	12.626	27.000	19.404	-21.619
4	0.12297	13	23	-4.081	3.268	-2.641	-10.361	-3.836	3.447	-6.538	-2.903	0.629	3.732	5.185	-1.509	3.760	2.373	-2.129	-3.023	-1.395	2.586	-3.425	-2.169	-8.214	-4.353	8.069
5	0.08401	19	15	11.862	-2.641	21.481	38.255	18.905	-9.039	26.130	17.709	14.205	-1.721	-0.776	21.569	14.809	4.532	7.796	3.924	11.729	-1.632	9.984	8.471	26.217	9.604	-8.815
6	0.12301	12	1	42.612	-10.361	38.255	143.510	41.862	-31.046	64.998	40.875	21.746	-9.307	-18.506	24.417	32.913	12.684	16.378	4.369	32.192	-20.849	33.197	22.509	70.407	55.224	-50.452
7	0.10898	15	10	15.767	-3.836	18.905	41.862	30.332	-6.881	31.048	16.384	13.964	0.232	-4.073	20.979	13.433	4.851	8.818	1.658	4.895	-7.067	12.005	21.842	32.383	18.643	-8.067
8	0.12145	14	9	-0.220	3.447	-9.039	-31.046	-6.881	41.189	-18.601	-19.523	-7.261	10.750	8.358	-6.575	22.943	-13.438	-6.501	-0.823	-17.894	4.846	-11.189	2.741	-9.886	-15.836	3.672
910	0.10467	17	6	20.937	-6.538	26.130	64.998	31.048	-18.601	47.359	26.484	17.303	-5.543	-8.348	26.719	12.847	6.519	12.448	5.734	15.838	-8.504	18.244	17.942	43.701	24.677	-19.649
11	0.15542	5	12	9.038	-2.903	17.709	40.875	16.384	-19.523	26.484	24.210	13.224	-5.114	-2.445	19.991	5.382	8.101	8.248	4.386	17.349	-2.033	11.312	2.344	23.642	11.975	-9.745
12	0.14613	7	14	4.795	0.629	14.205	21.746	13.964	-7.261	17.303	13.224	22.334	2.801	3.835	17.165	11.751	10.289	5.095	-5.106	5.151	0.905	6.289	13.934	11.983	3.135	12.726
13	0.13013	11	21	-0.900	3.732	-1.721	-9.307	0.232	10.750	-5.543	-5.114	2.801	9.539	6.804	-1.132	12.952	2.762	-2.543	-6.267	-6.466	0.936	-3.577	6.762	-5.800	-1.454	11.123
14	0.13996	8	17	-7.179	5.185	-0.776	-18.506	-4.073	8.358	-8.348	-2.445	3.835	6.804	14.249	6.127	11.951	2.254	-2.244	-1.554	-0.167	8.131	-6.647	-6.379	-10.615	-13.342	13.677
15	0.16382	4	8	5.861	-1.509	21.569	24.417	20.979	-6.575	26.719	19.991	17.165	-1.132	6.127	41.725	15.000	2.047	9.627	10.205	13.009	6.128	6.149	2.932	25.737	-4.191	-2.121
16	0.13339	9	2	18.767	3.760	14.809	32.913	13.433	22.943	12.847	5.382	11.751	12.952	11.951	15.000	70.689	-0.077	0.858	-0.979	6.425	1.102	2.839	5.915	21.335	5.545	-13.794
17	0.14863	6	16	-0.726	2.373	4.532	12.684	4.851	-13.438	6.519	8.101	10.289	2.762	2.254	2.047	-0.077	16.653	1.302	-9.239	5.003	-2.719	4.207	7.631	-1.044	10.289	13.275
18	0.06651	21	22	4.575	-2.129	7.796	16.378	8.818	-6.501	12.448	8.248	5.095	-2.543	-2.244	9.627	0.858	1.302	4.653	3.029	4.957	-1.287	4.825	3.769	12.256	4.607	-4.687
19	0.19564	2	18	1.654	-3.023	3.924	4.369	1.658	-0.823	5.734	4.386	-5.106	-6.267	-1.554	10.205	-0.979	-9.239	3.029	14.039	7.753	3.423	0.229	-14.708	9.936	-5.670	-17.958
20	0.22409	1	11	5.146	-1.395	11.729	32.192	4.895	-17.894	15.838	17.349	5.151	-6.466	-0.167	13.009	6.425	5.003	4.957	7.753	25.745	1.245	7.348	-15.803	14.536	6.351	-17.992
21	0.06597	22	19	-8.721	2.586	-1.632	-20.849	-7.067	4.846	-8.504	-2.033	0.905	0.936	8.131	6.128	1.102	-2.719	-1.287	3.423	1.245	11.463	-6.391	-12.082	-10.079	-18.582	8.540
22	0.08051	20	20	10.433	-3.425	9.984	33.197	12.005	-11.189	18.244	11.312	6.289	-3.577	-6.647	6.149	2.839	4.207	4.825	0.229	7.348	-6.391	11.344	8.315	18.224	15.114	-10.594
24	0.13199	10	7	12.626	-2.169	8.471	22.509	21.842	2.741	17.942	2.344	13.934	6.762	-6.379	2.932	5.915	7.631	3.769	-14.708	-15.803	-12.082	8.315	44.819	16.313	19.378	14.482
26	0.10605	16	4	27.000	-8.214	26.217	70.407	32.383	-9.886	43.701	23.642	11.983	-5.800	-10.615	25.737	21.335	-1.044	12.256	9.936	14.536	-10.079	18.224	16.313	63.412	25.889	-33.985
27	0.18417	3	5	19.404	-4.353	9.604	55.224	18.643	-15.836	24.677	11.975	3.135	-1.454	-13.342	-4.191	5.545	10.289	4.607	-5.670	6.351	-18.582	15.114	19.378	25.889	51.645	-19.670
28	0.03152	23	3	-21.619	8.069	-8.815	-50.452	-8.067	3.672	-19.649	-9.745	12.726	11.123	13.677	-2.121	-13.794	13.275	-4.687	-17.958	-17.992	8.540	-10.594	14.482	-33.985	-19.670	70.018
Cor. wit			(Var.)																							
$\hat{r}_i$	1.0000		-0.0438	0.1510	-0.1465	0.1287	0.1872	0.0640	-0.1685	0.1386	0.2003	-0.1180	-0.2314	-0.1173	0.1095	0.1130	-0.0250	0.1266	0.2953	0.3645	-0.1101	0.1416	-0.2586	0.1539	0.1899	-0.3519
$V\hat{a}r(r_i)$	0.3047		0.5381	0.4034	-0.3906	0.4258	0.4995	0.3409	-0.3633	0.4114	0.4306	0.2537	-0.3726	-0.3820	0.2154	0.1665	0.1830	0.3895	0.1760	0.4682	-0.3479	0.4494	0.0926	0.3913	0.3748	-0.3839
$V\hat{a}r(\hat{r}_i)$	-0.0438		1.0000	0.5779	-0.3766	0.5282	0.5961	0.5589	-0.2172	0.5262	0.4418	0.4952	-0.1058	-0.3707	0.2976	0.4725	0.2844	0.4403	-0.0308	0.2947	-0.5060	0.5313	0.4708	0.5301	0.5338	-0.3071
$\bar{s}_{Ki}$	-0.2625		-0.3554	-0.0834	-0.0761	-0.0838	-0.0568	-0.1137	-0.0787	-0.0523	-0.0355	-0.1612	-0.1934	-0.1018	-0.1012	-0.2205	-0.1074	-0.0117	0.0881	0.0251	0.0265	0.0005	-0.1311	-0.0697	-0.0716	-0.0348
$\bar{s}_{Ri}$	0.1339		-0.3520	-0.0900	-0.0671	-0.0640	-0.0393	-0.1520	-0.1163	-0.0484	0.0112	-0.1966	-0.2604	-0.0679	-0.0554	-0.1837	-0.1321	-0.0056	0.2030	0.1740	0.0820	-0.0056	-0.2996	-0.0650	-0.0965	-0.1114
$\bar{s}_{ni}$	0.4265		-0.1953	-0.0954	-0.0526	-0.0035	-0.0117	-0.1510	-0.1553	-0.0391	0.0605	-0.1876	-0.2857	-0.0228	0.0080	-0.1641	-0.1135	0.0116	0.2799	0.2678	0.1349	-0.0230	-0.3953	-0.0458	-0.1057	-0.1375
$\overline{K_i}/n_i$	-0.3273		-0.2175	-0.0517	-0.0586	-0.0431	-0.0405	-0.0424	-0.0399	-0.0199	-0.0179	-0.0880	-0.1129	-0.0565	-0.0201	-0.1402	-0.0838	0.0279	0.0788	-0.0127	0.0223	-0.0167	-0.0533	-0.0276	-0.0471	-0.0083
$\overline{R_i}/n_i$	-0.2973		-0.2185	-0.0332	-0.0653	-0.0226	-0.0280	-0.0264	-0.0300	-0.0033	-0.0045	-0.0762	-0.1113	-0.0515	0.0099	-0.1015	-0.0996	0.0417	0.1018	0.0035	0.0240	-0.0079	-0.0570	-0.0102	-0.0447	-0.0304
Kgr	0.1251		0.1143	-0.1618	0.0257	-0.2317	-0.1332	-0.0735	-0.1378	-0.1184	-0.1408	-0.0498	0.0399	-0.1591	-0.3392	-0.4651	0.3026	-0.1246	-0.4256	-0.2857	-0.1930	-0.0289	0.2962	-0.1597	0.1444	0.3097
Rgr	-0.1200		-0.0091	-0.6233	0.5318	-0.6305	-0.5907	-0.5354	0.2189	-0.5818	-0.5418	-0.2341	0.4094	0.3626	-0.5360	-0.5204	0.1678	-0.5598	-0.5781	-0.5789	0.3121	-0.5208	-0.0045	-0.6303	-0.3651	0.7253
ngr	0.0275		-0.0397	-0.1262	-0.0108	-0.2881	-0.1440	-0.1439	-0.0532	-0.1640	-0.2181	-0.2392	0.0163	-0.2036	-0.4405	-0.4067	0.1032	-0.1779	-0.3388	-0.3076	-0.1993	-0.0583	0.2134	-0.1693	0.1393	0.1928
(K/n)gr	0.1880		0.2671	-0.1096	0.0628	0.0012	-0.0311	0.0777	-0.1733	0.0253	0.0640	0.2614	0.0487	0.0093	0.0292	-0.2505	0.4007	0.0343	-0.2783	-0.0687	-0.0590	0.0332	0.2268	-0.0423	0.0586	0.2817
(R/n)gr	-0.1646		0.0101	-0.7092	0.6692	-0.6305	-0.6589	-0.5899	0.3019	-0.6370	-0.5578	-0.1625	0.5019	0.5625	-0.4303	-0.4291	0.1535	-0.6021	-0.5378	-0.5557	0.4972	-0.6181	-0.1210	-0.6946	-0.5307	0.8004
rgr	-0.2853		-0.1075	-0.8166	0.7887	-0.7702	-0.7904	-0.7550	0.4463	-0.7889	-0.7097	-0.3156	0.5908	0.6824	-0.5383	-0.4114	0.0076	-0.7504	-0.5317	-0.6475	0.6334	-0.7694	-0.2495	-0.8290	-0.6741	0.8507

A first attempt to isolate some pattern in the covariance structure involved the extraction of principal components [11] of the covariance matrix itself. The factor loadings – simple correlations between the components (linear combinations of the variables – here, the columns of the covariance matrix – designed to be linearly independent and explain decreasing importance of the total variance) and the variables are depicted in Table 2 for case ii), and 2.A. for the covariance of the estimates of case iii) (the factor loadings for the decomposition of $Cov(r_{itl}, r_{jtl})$ revealed themselves quite sparse – resulting from the very small correlations between the original elements – and all the eigenvalues are close to 1. So we only report the results of the decomposition of $Cov(\hat{r}_i, \hat{r}_j)$). We shade darkest the highest loading in each row, mildest the second highest – the most important components contributing for the variable's explanation. Correlations with the other series are in the last rows.

[11] Computed directly by TSP 4.4 PRIN routine, which standardizes the input variables – see Hall & Cummins (1997) and (1998).

Table 2. *Principal Components: (Unweighted) Covariance Matrix, ii)*

Sector	PC1	PC2	PC3	PC4	PC5
Eigenv.	15.846333	2.7318894	2.1066607	1.6309184	0.64726145
% Cum. Exp Var.	0.68897100	0.80774880	0.89934274	0.97025224	0.99839404
Factor Loadings:					
3	0.92105	0.025139	0.37846	0.050718	0.070096
4	-0.93595	0.089876	-0.10038	0.27766	0.16563
5	0.95929	-0.097942	-0.069040	0.25239	-0.038378
6	0.98778	0.023115	0.090049	0.017406	0.12085
7	0.95583	0.10389	0.084197	0.17070	-0.18479
8	-0.76450	-0.11303	0.57164	0.24056	-0.13403
910	0.99507	-0.015713	-0.031166	0.076666	-0.050166
11	0.95086	-0.089440	-0.27484	0.088203	0.067467
12	0.63165	0.36936	-0.37403	0.54338	-0.14509
13	-0.76423	0.29174	0.34946	0.44815	0.047639
14	-0.86456	-0.18877	-0.15791	0.43293	0.060115
15	0.65875	-0.42719	-0.33871	0.44753	-0.25814
16	0.33356	-0.23152	0.61982	0.62099	0.25294
17	0.38922	0.67639	-0.52567	0.14468	0.29989
18	0.97822	-0.053728	-0.13658	0.040812	-0.14043
19	0.42147	-0.89219	-0.038287	-0.093359	-0.11587
20	0.80066	-0.39023	-0.28761	0.049184	0.34794
21	-0.85677	-0.35684	-0.29232	0.22132	-0.052460
22	0.98657	0.13776	0.048966	-0.057903	0.042433
24	0.62691	0.66432	0.28011	0.013829	-0.29496
26	0.97838	-0.089132	0.17732	0.038940	-0.040249
27	0.92156	0.25315	0.24191	-0.098975	0.13016
28	-0.85085	0.34588	-0.32986	0.13991	-0.16687
Cor. with					
$\hat{r}_i$	0.21261	-0.24588	0.03256	-0.09414	0.33360
$Var(\hat{r}_i)$	0.51124 *	0.29503	0.20053	0.26420	0.06375
$\bar{s}_{Ki}$	-0.02685	-0.07695	-0.07604	-0.32090	-0.04399
$\bar{s}_{Ri}$	-0.02333	-0.20708	-0.12739	-0.32789	0.09768
$\bar{s}_{ni}$	-0.01079	-0.30895	-0.19869	-0.26630	0.17046
$\overline{\frac{K_i}{n_i}}$	-0.01487	-0.05911	-0.05066	-0.17245	-0.10448
$\overline{\frac{R_i}{n_i}}$	-0.00162	-0.08271	-0.03582	-0.13959	-0.10616
Kgr	-0.03499	0.51462 *	-0.07256	-0.38653 **	-0.10871
Rgr	-0.55093 *	0.43269 *	-0.17721	-0.22557	-0.05490
ngr	-0.04977	0.40939 **	0.10646	-0.51997 *	-0.08025
(K/n)gr	0.00938	0.33692	-0.28926	0.05981	-0.08035
(R/n)gr	-0.66029 *	0.31820	-0.27866	0.00000	-0.02506
rgr	-0.81022 *	0.23728	-0.21040	-0.02683	0.00546

In both cases, four components have eigenvalues larger than 1 – a possible criteria to choose the number of relevant components -, explaining more than 95% of the total variance. The first (most important) component seems to be in line with the sectors' variance of the rate of return estimate and with a negative trend of the rate of return. The covariances of sectors 3, 5, 6, 7, 910, 11, 12, 15, 18, 20, 22, 26 and 27 are strongly and positively related to it; those of sectors 4, 8, 13, 14, 21, and 28, negatively.

The second component moves oppositely to the number of firms in the sector. Sectors 17 and 24's covariances are positively related to it, sector 19's negatively.

The fourth – associated to sector 16's covariances - moves oppositely to the sectors' capital and profits importance and to capital and number of firms' growth in the economy. The fifth appears slightly positively correlated with the estimated rates of return.

Table 2.A. *Principal Components: Covariance Matrix, ii),* $Cov(\hat{r}_i, \hat{r}_j)$

Sector	PC1	PC2	PC3	PC4	PC5
Eigenv.	15.157883	3.0995653	2.0962431	1.7937795	0.72412471
% Cum. Exp Var.	0.65903840	0.79380211	0.88494311	0.96293352	0.99441720
Factor Loadings:					
3	0.90841	0.025576	0.40609	0.013973	0.063052
4	-0.93483	0.11315	-0.079591	0.25107	0.20471
5	0.95752	-0.052869	-0.039204	0.27384	-0.023915
6	0.97844	0.036525	0.11463	-0.00056511	0.14536
7	0.94354	0.17324	0.11423	0.16816	-0.17271
8	-0.75831	-0.10939	0.58797	0.22355	-0.11579
910	0.99315	0.030872	-0.0039299	0.085126	-0.055300
11	0.95187	-0.066665	-0.26251	0.11469	0.072013
12	0.59887	0.43473	-0.32240	0.57306	-0.061627
13	-0.76193	0.37564	0.35615	0.37296	0.074364
14	-0.85614	-0.13392	-0.11170	0.46947	0.10643
15	0.67488	-0.31874	-0.26857	0.54900	-0.24188
16	0.29435	-0.15064	0.66235	0.58390	0.32848
17	0.31368	0.71697	-0.52345	0.067356	0.32334
18	0.96870	-0.060413	-0.15872	0.097423	-0.14998
19	0.46611	-0.87394	-0.032730	0.0046347	-0.11931
20	0.80294	-0.40108	-0.27865	0.044643	0.33556
21	-0.79548	-0.40912	-0.29779	0.32368	-0.052354
22	0.98750	0.12341	0.012724	-0.067480	0.048352
24	0.41222	0.80259	0.27793	0.075926	-0.31914
26	0.96777	-0.089952	0.18925	0.069453	-0.072180
27	0.86549	0.31599	0.19950	-0.25513	0.17159
28	-0.80643	0.43040	-0.33003	0.16005	-0.15014
Cor. with					
$\hat{r}_i$	0.17636	-0.27321	-0.00654	-0.11439	0.32669
$Var(r_i)$	0.43971 *	-0.02306	-0.02696	-0.03727	0.21942
$Var(\hat{r}_i)$	0.50914 *	0.27675	0.24045	0.26254	0.13798
$\bar{s}_{Ki}$	-0.02829	-0.12680	-0.10365	-0.27066	-0.07989
$\bar{s}_{Ri}$	-0.01501	-0.27150	-0.16121	-0.25849	0.06411
$\bar{s}_{ni}$	0.00179	-0.34988	-0.22450	-0.20319	0.14651
$\frac{K_i}{n_i}$	-0.01205	-0.08085	-0.06149	-0.13885	-0.12880
$\frac{R_i}{n_i}$	0.00112	-0.10075	-0.04389	-0.10613	-0.12588
Kgr	-0.09995	0.45957 *	-0.14511	-0.40008 **	-0.13801
Rgr	-0.57967 *	0.37416	-0.23727	-0.21191	-0.07181
ngr	-0.12801	0.32873	0.01443	-0.53006 *	-0.12244
(K/n)gr	0.00597	0.35518 **	-0.28623	0.04992	-0.07172
(R/n)gr	-0.65380 *	0.28884	-0.30378	0.02250	-0.02332
rgr	-0.80077 *	0.19325	-0.24242	0.00507	0.00371

Principal components on the average returns themselves (using only the 7 yearly observations) yielded the results in Tables 3 to 3.B, where the loadings of the first five components are depicted. We complemented the analysis computing the explanation [12] of each PC (principal component) in the subsequent five columns – and dashed the highest contributors accounting at least 90% of each PC's variance (Usually, shaded cells are also dashed cells, even if not conversely.) We completed the analysis inspecting the correlations with yearly outside series – a Trend, total capital (K_t), profits (R_t), number of firms in the sample (n_t), average firm size in terms of assets ($\frac{K_t}{n_t}$) and of profits ($\frac{R_t}{n_t}$), reported in the last seven rows of the Tables - , and with the yearly capital shares of each specific sector and of the average firm asset size - reported in the last 10 columns of the Tables.

(For 7 observations, correlations are significant at the 5% level for ǀ r ǀ larger than 0.7545450; at the 10% level, for ǀ r ǀ larger than 0.6694306.)

Table 3 presents the results for standard principal components (with only 7 observations to generate the correlation matrix, a total of 6 components are extracted…). With high loadings in the:

- first component (all positive) are sectors 3, 5, 6, 7, 910, 11, 18, 22, 26 and 27. The component is negatively and very strongly related to the trend in the series (that also drives the outside series), negatively (but weakly) to the average rate of return.

- second component (all positive), 4, 12, 13, 14 and 28 - including metallurgy in general. It appears to be negatively (even if weakly) affected by the number of firms in the industry, positively (weakly) by the rate of return level.

- third component, 15, 19, 20, 21 and 24 (the only one positive);

- fourth component 8, 16 and 17 (the only one positive).

Table 3.A is derived from the application of principal components to the residuals of equation (42) – those presumed homoscedastic -, a procedure that subscribes to Weighted Principal Components [13] applied to those same average returns – decomposing the cross-moment matrix of the (hence, transformed) residuals of regression (42) divided by the estimated standard error. Results are in Table 3.A and implied the same clustering of the sectors and general results as the standard algorithm.

Finally, using the correlation matrix inferred from the covariance matrix of the weighted mean returns in the internal decomposition generated the results of Table 3.B, closer to those of Table 3: the first and second components remain mostly unaltered relative to Table 3, but not the others: with high loadings in the:

- first component are sectors 3, 4 (the only negative), 5, 6, 7, 910, 11, 18, 22, 26 and 27; negatively related to the trend.

[12] Computations were programmed with TSP 4.4, relying on matrix and database facilities – see Hall & Cummins (1997) and (1998). See Martins (2004) for further explanation and examples.

[13] Computations were programmed with TSP 4.4, relying on matrix and database facilities – see Hall & Cummins (1997) and (1998). See Martins (2004) for a more detailed explanation of the method and some examples.

- second component, 12, 13, 14, 19 (the only positive), 24 and 28 - including metallurgy in general; it is positively related to the number of firms but also to total capital and profits level – even if weakly.
- third component, 8 and 16 (all positive); positively (weakly) related to firm size.
- fourth component (all positive), 15, 20 and 21.
- fifth component, 17.

The correlations of the components with the sectors capital shares and average firm size revealed a consistent importance of the first component and insignificance of relations to the others. For capitals shares, the only ones not related to the first (trended…) component were sector's 3, 7, 12, 15 and 21. Industry firm size of sectors 18 and 24 were also the only not importantly related to that main component.

Table 3. Principal components: Rates of Return, ii)

Sector	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5
Eigenv.	10.9156	4.0100	3.4172	2.62908	1.61885	10.916	4.0100	3.4173	2.6290	1.6188
% Cum Exp Var	0.474593	0.64894	0.797520	0.91182	0.982213	0.47459	0.648943	0.797520	0.91182	0.98221
	Factor Loadings:					Explained Component Variance:				
3	0.80833	-0.08943	0.29967	-0.45060	0.18126	~~0.059858~~	0.001994	~~0.026279~~	~~0.077228~~	0.020296
4	-0.65842	0.66393	0.08383	0.02951	0.31876	~~0.039716~~	~~0.109926~~	0.002056	0.000331	~~0.062767~~
5	0.86690	0.41758	-0.22291	-0.10727	-0.07652	~~0.068847~~	~~0.043483~~	0.01454	0.004376	0.003617
6	0.94873	-0.00444	0.08377	-0.01892	0.26306	~~0.082459~~	4.92E-06	0.002053	0.000136	~~0.042746~~
7	0.84007	0.31407	0.10577	-0.17817	-0.29745	~~0.064652~~	0.024598	0.003274	0.012074	~~0.054655~~
8	-0.51203	0.08923	0.20373	-0.81370	-0.14422	~~0.024018~~	0.001985	0.012146	~~0.251837~~	0.012847
910	0.97426	0.15886	-0.10135	-0.00914	-0.10958	~~0.086956~~	0.006294	0.003006	3.18E-05	0.007417
11	0.85572	0.26470	-0.33438	0.27576	0.09891	~~0.067083~~	0.017473	~~0.03272~~	~~0.028925~~	0.006044
12	0.50219	0.76798	0.05083	0.18788	-0.21954	0.023104	~~0.14708~~	0.000756	0.013426	~~0.029774~~
13	-0.37607	0.67161	0.45307	-0.38852	0.12545	0.012957	~~0.112481~~	~~0.060068~~	~~0.057414~~	0.009722
14	-0.49842	0.77619	-0.29598	-0.18484	0.13412	0.022758	~~0.15024~~	~~0.025636~~	0.012996	0.011111
15	0.53636	0.51450	-0.56418	-0.14722	-0.30969	~~0.026355~~	~~0.066011~~	~~0.093146~~	0.008244	~~0.059244~~
16	0.22010	0.51806	0.07787	-0.72200	0.36672	0.004438	~~0.066927~~	0.001774	~~0.198275~~	~~0.083075~~
17	0.25646	0.57288	0.30028	0.63830	0.29727	0.006025	~~0.081843~~	~~0.026386~~	~~0.15497~~	~~0.054588~~
18	0.91838	0.08732	-0.23231	0.08111	-0.29321	~~0.077267~~	0.001901	0.015792	0.002502	~~0.053105~~
19	0.21494	-0.39948	-0.79630	-0.33831	-0.10619	0.004232	~~0.039795~~	~~0.185554~~	~~0.043534~~	0.006966
20	0.54670	0.08621	-0.57403	0.20901	0.56258	~~0.027381~~	0.001854	~~0.096426~~	0.016617	~~0.195507~~
21	-0.61560	0.37518	-0.66362	-0.05575	-0.11162	~~0.034718~~	~~0.035102~~	~~0.128873~~	0.001182	0.007696
22	0.95654	-0.09747	0.21345	0.13929	0.07224	~~0.083821~~	0.002369	0.013332	0.00738	0.003224
24	0.47179	0.10510	0.74147	-0.02054	-0.46267	0.020391	0.002755	~~0.160881~~	0.00016	~~0.132232~~
26	0.94175	-0.06954	-0.03602	-0.31549	-0.06098	~~0.08125~~	0.001206	0.00038	~~0.037858~~	0.002297
27	0.75954	-0.15376	0.53752	0.00917	0.29206	~~0.052851~~	0.005896	~~0.084549~~	3.2E-05	~~0.052692~~
28	-0.56130	0.56207	0.18825	0.43044	-0.37825	~~0.028863~~	~~0.078782~~	0.01037	~~0.070471~~	~~0.088378~~
Cor. w										
Trend	-0.91838	-0.30072	-0.02322	-0.02484	0.17323					
K_t	-0.92645	-0.30691	-0.09455	-0.01912	0.14665					
R_t	-0.91532	-0.25251	-0.18193	-0.05786	0.20643					
n_t	-0.77505	-0.50907	-0.23886	0.16735	0.05151					
$\frac{K_i}{n_i}$	-0.96118	-0.16286	-0.04779	-0.10910	0.17579					
$\frac{R_i}{n_i}$	-0.91984	-0.01464	-0.14492	-0.21070	0.29008					
r_t	0.91694	0.36373	-0.10051	-0.10209	0.07401					

Sector	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5
	Correlations with Capital Shares					Correlations with Firm Capital Size				
3	0.31341	-0.51010	-0.28349	0.23316	-0.64914	-0.87437	-0.24559	-0.08757	-0.03824	0.19307
4	0.97409	0.11894	0.12455	0.06561	-0.06697	-0.91548	-0.31281	-0.04802	0.03590	0.21163
5	0.91389	0.34967	-0.06216	0.07568	-0.04326	-0.88452	-0.19690	-0.22318	-0.06008	0.35304
6	0.86330	0.46453	0.08040	-0.15962	-0.07392	-0.98004	-0.07257	-0.08698	-0.11544	0.02165
7	0.51880	0.73741	-0.03025	0.33033	0.08804	-0.96613	-0.15636	-0.10722	0.03591	0.15444
8	0.70134	0.51651	-0.41633	0.07033	-0.17374	-0.89336	-0.22465	-0.20222	0.00056	0.16216
910	0.90565	0.40226	0.05320	-0.07163	0.09403	-0.92082	-0.02085	-0.27155	-0.10062	0.22105
11	-0.92727	0.15185	-0.03225	0.01165	0.16575	-0.96901	-0.08920	-0.09081	-0.03579	0.17089
12	0.28137	0.45665	-0.06386	-0.21976	0.67738	-0.89470	-0.29248	0.02949	-0.00613	0.23147
13	0.95208	-0.07523	0.25396	0.05788	-0.04805	-0.92014	-0.27479	-0.07200	-0.07710	0.22976
14	0.70939	0.23548	-0.15770	-0.41708	0.39886	-0.93199	-0.21051	-0.20473	-0.05432	0.19231
15	0.27342	0.59087	-0.30853	-0.13133	0.60873	-0.93519	-0.20928	-0.15199	-0.03774	0.23273
16	0.88413	0.08712	0.41596	-0.05502	-0.13823	-0.86536	-0.32060	0.28603	-0.16356	0.19292
17	-0.77453	0.11353	-0.55716	-0.24540	0.03799	-0.92777	-0.28621	-0.14584	-0.16254	0.08993
18	0.91584	0.15769	0.25745	0.05208	-0.20595	0.52776	-0.39435	0.65488	-0.29378	0.18471
19	-0.85165	-0.20992	-0.11900	0.32873	0.27901	-0.92379	0.10010	-0.12173	-0.02192	0.23832
20	-0.95180	-0.12444	-0.11659	0.05999	0.10145	-0.96206	-0.15337	-0.01071	-0.03804	0.16709
21	-0.04766	-0.04661	-0.75472	0.60855	-0.20666	-0.97503	-0.12698	0.01886	0.00886	0.09455
22	-0.95328	-0.15442	-0.08676	-0.02762	-0.06954	-0.95784	-0.08762	-0.09898	-0.17118	0.06234
24	-0.74884	-0.40024	-0.17671	-0.25727	0.36070	-0.68327	-0.27310	-0.08589	-0.40385	0.46682
26	-0.97770	0.06198	-0.11885	0.15377	0.04850	-0.99088	0.02531	-0.10205	-0.00824	0.08382
27	-0.92755	-0.22076	-0.21899	-0.05253	0.12388	-0.91862	-0.20114	-0.25371	-0.12718	0.17417
28	-0.82634	-0.30219	-0.17666	0.13904	0.06527	-0.92920	-0.13569	-0.21717	0.01677	0.02306

Table 3.A. *Weighted principal components: Rates of Return, iii)*

Sector	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5
Eigenv.	10.76024	4.05767	3.50367	2.63361	1.62232	10.76024	4.05767	3.50367	2.63361	1.62232										
% Cum. Exp Var.	0.46784	0.64426	0.79659	0.91109	0.981626	0.46784	0.64426	0.79659	0.91109	0.981626										
	Factor Loadings:					Explained Component Variance:					Correlations with Capital Shares					Correlations with Firm Capital Size				
3	0.80229	-0.07445	0.29929	-0.46243	0.17771	~~0.05982~~	0.00137	~~0.02557~~	~~0.08120~~	0.01947	0.31482	-0.51744	-0.20892	0.22728	-0.67559	-0.87611	-0.24824	-0.06227	-0.02248	0.17616
4	-0.65558	0.65940	0.00566	0.03719	0.34126	~~0.03994~~	~~0.10716~~	0.00001	0.00053	~~0.07179~~	0.97352	0.12443	0.12465	0.05503	-0.06071	-0.91766	-0.31398	-0.02475	0.04668	0.19755
5	0.87422	0.38458	-0.25927	-0.10246	-0.06540	~~0.07103~~	~~0.03645~~	0.01919	0.00399	0.00264	0.91839	0.33463	-0.08699	0.07122	-0.02975	-0.88329	-0.22083	-0.21622	-0.04214	0.34653
6	0.94490	-0.00607	0.09974	-0.03326	0.27412	~~0.08298~~	0.00001	0.00284	0.00042	~~0.04632~~	0.86521	0.46670	0.04488	-0.16421	-0.05753	-0.97979	-0.07328	-0.09348	-0.10671	0.01787
7	0.84727	0.31379	0.10027	-0.18499	-0.28407	~~0.06671~~	0.02427	0.00287	0.01299	~~0.04974~~	0.52704	0.72139	-0.09610	0.32971	0.10785	-0.96584	-0.16288	-0.10214	0.04708	0.14584
8	-0.51767	0.11762	0.15565	-0.82136	-0.11976	~~0.02490~~	0.00341	0.00691	~~0.25616~~	0.00884	0.71233	0.47439	-0.44417	0.08085	-0.16611	-0.89276	-0.23853	-0.18045	0.01783	0.14659
910	0.97862	0.14056	-0.08434	-0.01494	-0.10907	~~0.08900~~	0.00487	0.00203	0.00008	0.00733	0.90754	0.39715	0.02740	-0.07209	0.10668	-0.91752	-0.04551	-0.27909	-0.08148	0.21721
11	0.86173	0.22719	-0.34191	0.28220	0.09529	~~0.06901~~	0.01272	~~0.03337~~	~~0.03024~~	0.00560	-0.92618	0.15292	-0.04867	0.02614	0.16006	-0.96880	-0.09430	-0.09257	-0.02352	0.16451
12	0.50412	0.77489	-0.02722	0.17892	-0.19330	0.02362	~~0.14798~~	0.00021	0.01216	~~0.02303~~	0.28334	0.43304	-0.08955	-0.19300	0.68207	-0.89838	-0.28547	0.05450	0.00501	0.21582
13	-0.37752	0.71693	0.37970	-0.38671	0.13231	0.01324	~~0.12667~~	~~0.04115~~	~~0.05678~~	0.01079	0.94781	-0.05651	0.27102	0.04286	-0.04609	-0.92204	-0.27890	-0.05586	-0.06504	0.21945
14	-0.48906	0.74108	-0.37945	-0.16958	0.15803	0.02223	~~0.13535~~	~~0.04110~~	0.01092	0.01539	0.71224	0.20076	-0.19154	-0.40964	0.42290	-0.93099	-0.22749	-0.19567	-0.03979	0.18423
15	0.54958	0.46236	-0.60303	-0.13604	-0.29494	~~0.02807~~	~~0.05269~~	~~0.10379~~	0.00703	~~0.05362~~	0.28250	0.53621	-0.37564	-0.11173	0.63511	-0.93498	-0.22233	-0.14364	-0.02409	0.22484
16	0.22247	0.51127	0.00309	-0.71450	0.40054	0.00460	~~0.06442~~	0.00000	~~0.19385~~	~~0.09889~~	0.87837	0.12406	0.41180	-0.07554	-0.12925	-0.87325	-0.29024	0.29563	-0.16595	0.18870
17	0.24976	0.59432	0.27009	0.64329	0.28924	0.00580	~~0.08705~~	0.02082	~~0.15713~~	~~0.05157~~	-0.76525	0.06132	-0.58940	-0.22677	0.04806	-0.92851	-0.29449	-0.13432	-0.15296	0.08343
18	0.91025	0.07716	-0.25696	0.08200	-0.29596	~~0.07700~~	0.00147	0.01885	0.00255	~~0.05399~~	0.91376	0.17971	0.24929	0.03405	-0.19681	0.51227	-0.33444	0.69582	-0.30971	0.17904
19	0.22756	-0.47605	-0.75652	-0.31536	-0.10762	0.00481	~~0.05585~~	~~0.16335~~	~~0.03776~~	0.00714	-0.85021	-0.22445	-0.10865	0.33787	0.26897	-0.92074	0.08488	-0.15653	-0.01301	0.24679
20	0.55346	0.00406	-0.57703	0.23872	0.54911	~~0.02847~~	0.00000	~~0.09503~~	0.02164	~~0.18586~~	-0.95126	-0.12916	-0.10739	0.07291	0.08907	-0.96373	-0.14988	-0.00547	-0.02801	0.15847
21	-0.56892	0.31471	-0.73460	-0.04040	-0.11218	~~0.03008~~	0.02441	~~0.15402~~	0.00062	0.00776	-0.03371	-0.11458	-0.72460	0.62679	-0.22621	-0.97601	-0.12195	0.00950	0.01122	0.09384
22	0.95607	-0.07474	0.21365	0.14248	0.07254	~~0.08495~~	0.00138	0.01303	0.00771	0.00324	-0.95353	-0.15128	-0.07743	-0.01951	-0.08067	-0.95794	-0.08990	-0.10228	-0.16011	0.05764
24	0.42093	0.25581	0.73344	-0.06838	-0.46100	0.01647	0.01613	~~0.15354~~	0.00178	~~0.13100~~	-0.75026	-0.42207	-0.16413	-0.24627	0.36048	-0.68570	-0.29012	-0.09087	-0.39317	0.47456
26	0.94076	-0.06056	-0.03671	-0.32017	-0.06429	~~0.08225~~	0.00090	0.00038	~~0.03892~~	0.00255	-0.97461	0.05499	-0.13601	0.16226	0.04542	-0.98903	0.01970	-0.11978	0.00088	0.08308
27	0.72119	-0.12197	0.57698	0.04235	0.30198	~~0.04834~~	0.00367	~~0.09502~~	0.00068	~~0.05621~~	-0.92648	-0.23639	-0.20526	-0.03779	0.11321	-0.91699	-0.22253	-0.24729	-0.11168	0.16805
28	-0.53581	0.61029	0.15590	0.41332	-0.37623	~~0.02668~~	~~0.09179~~	0.00694	~~0.06487~~	~~0.08725~~	-0.82642	-0.30991	-0.13742	0.15470	0.04129	-0.92727	-0.14786	-0.20564	0.03131	0.01025
Cor. witl																				
Trend	-0.92114	-0.29774	-0.00053	-0.01448	0.15928															
K_t	-0.92794	-0.31079	-0.07329	-0.00834	0.13386															
R_t	-0.91521	-0.26679	-0.16583	-0.04276	0.19553															
n_t	-0.77535	-0.52380	-0.18501	0.18006	0.02633															
$\frac{K_t}{n_t}$	-0.96244	-0.16396	-0.04670	-0.09917	0.17064															
$\frac{R_t}{n_t}$	-0.91885	-0.02931	-0.15991	-0.19474	0.29110															
r_t	0.92102	0.34348	-0.12499	-0.09995	0.08786															

Table 3.B. *Principal Components: Rates of Return, iii) (From* $Cov(\hat{r}_i, \hat{r}_j)$*)*

Sector	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5
Eigenv.	10.75944	3.07108	2.83469	2.49330	1.44148	10.75944	3.07108	2.83469	2.49330	1.44148										
% Cum. Exp Var.	0.46780	0.60133	0.72458	0.83298	0.94138	0.46780	0.60133	0.72458	0.83298	0.94138										
	Factor Loadings:					Explained Component Variance:					Correlations with Capital Shares					Correlations with Firm Capital Size				
3	0.78379	-0.01410	0.21270	-0.37543	0.07821	~~0.07509~~	0.000711	~~0.022112~~	~~0.042458~~	0.006275	0.21304	0.37630	-0.36830	0.20523	0.79326	-0.86537	0.34329	0.31108	-0.12718	-0.28283
4	-0.53368	-0.42584	0.16812	0.08254	-0.51379	0.010002	0.022445	0.002956	0.001242	0.026021	0.96597	-0.19870	-0.38803	0.05436	0.29931	-0.91196	0.38776	0.25267	-0.17212	-0.33446
5	0.80372	-0.26868	0.09050	0.34836	0.06287	~~0.061491~~	0.026182	0.00388	~~0.035094~~	0.003687	0.91488	-0.31716	-0.26252	0.29954	0.16038	-0.86102	0.38641	0.44063	-0.01380	-0.44226
6	0.94838	0.06986	-0.04254	-0.08725	-0.18343	~~0.232206~~	0.003932	-0.00096	0.006986	0.059409	0.88727	-0.49021	-0.11241	0.15001	0.20374	-0.96872	0.12598	0.41411	-0.08039	-0.27576
7	0.78282	-0.37384	-0.05836	0.02364	0.31409	~~0.039891~~	~~0.0814~~	0.000942	0.001884	~~0.152329~~	0.53386	-0.59398	-0.28813	0.44625	-0.35277	-0.96010	0.25777	0.30642	-0.06093	-0.38314
8	-0.39951	-0.29047	0.66509	-0.40806	0.22319	0.015401	0.020127	~~0.188186~~	~~0.079286~~	0.004343	0.69409	-0.33925	-0.06797	0.66167	0.17312	-0.89054	0.36142	0.33312	-0.00792	-0.28787
910	0.90637	-0.12366	-0.09938	0.17646	0.18126	~~0.08294~~	-0.00041	0.003731	0.011155	0.001286	0.93226	-0.36806	-0.16615	0.16502	0.09717	-0.89700	0.21602	0.50011	0.09223	-0.40997
11	0.77100	-0.02799	-0.15665	0.49468	-0.11524	~~0.053759~~	0.000472	0.007954	~~0.074839~~	0.002515	-0.90301	-0.02599	0.31530	-0.01091	-0.49168	-0.95301	0.19459	0.36943	-0.07187	-0.39876
12	0.44840	-0.67080	-0.14994	0.34752	-0.12708	~~0.022232~~	~~0.085772~~	0.005675	0.020648	0.031544	0.36702	-0.16530	0.27794	0.13658	-0.47035	-0.88451	0.34736	0.24896	-0.24294	-0.32989
13	-0.25025	-0.69575	0.26720	-0.30438	-0.22819	0.003141	~~0.147458~~	0.018586	0.002585	-0.04395	0.93939	-0.07763	-0.45019	-0.13831	0.35586	-0.90559	0.35998	0.36639	-0.17195	-0.32698
14	-0.38856	-0.47971	0.40996	0.37614	-0.31660	0.014188	0.029522	~~0.038287~~	~~0.026159~~	0.055194	0.77444	-0.08803	0.31435	0.13595	-0.01875	-0.92133	0.34541	0.41049	-0.02375	-0.33501
15	0.48218	-0.33025	0.22234	0.66290	0.25298	0.011946	0.019704	0.016194	~~0.153684~~	0.018267	0.35111	-0.23389	0.35578	0.42739	-0.57651	-0.92142	0.33673	0.38412	-0.06796	-0.37811
16	0.35294	-0.41521	0.73704	-0.16381	-0.32197	~~0.033809~~	~~0.069226~~	~~0.321061~~	0.022012	~~0.109538~~	0.88436	-0.31621	-0.39684	-0.24809	0.34869	-0.84220	0.23039	0.27097	-0.51750	-0.27853
17	0.22449	-0.39316	-0.54022	0.12952	-0.55256	0.005062	~~0.084272~~	~~0.072206~~	0.014118	-0.07789	-0.75379	0.12712	0.69491	0.37703	-0.24103	-0.91952	0.34485	0.44867	-0.12650	-0.19623
18	0.76366	-0.06485	-0.13431	0.39098	0.29654	0.0138	-0.00369	0.004005	0.005797	-0.02592	0.90316	-0.32867	-0.43026	-0.04363	0.35023	0.55339	0.10488	-0.21342	-0.77010	0.29479
19	0.19119	0.56177	0.44110	0.45533	0.37630	0.004247	-0.03996	~~0.048985~~	~~0.092385~~	~~0.253911~~	-0.86029	0.35719	0.06187	0.01349	-0.53394	-0.89431	0.04665	0.41502	0.02824	-0.56083
20	0.53105	0.36214	0.09338	0.54051	-0.42481	~~0.034083~~	~~0.053164~~	0.003398	~~0.068237~~	~~0.146273~~	-0.95076	0.22663	0.27275	-0.02683	-0.34489	-0.94780	0.21808	0.32539	-0.16541	-0.36723
21	-0.49002	-0.10940	0.37458	0.64413	-0.03628	~~0.019865~~	-0.0101	~~0.026626~~	~~0.044664~~	0.011631	-0.12538	0.33268	-0.22804	0.82823	0.02259	-0.96693	0.15156	0.28590	-0.16361	-0.38250
22	0.85188	0.04969	-0.27184	-0.04329	-0.00884	~~0.054233~~	0.000195	0.014896	0.001088	6.93E-05	-0.95985	0.18090	0.29965	-0.07525	-0.17942	-0.94066	0.15378	0.46000	-0.09321	-0.26341
24	0.33585	-0.63783	-0.34730	-0.46507	0.31076	0.014514	~~0.142265~~	~~0.058285~~	~~0.100496~~	~~0.068405~~	-0.71957	0.51239	0.53222	-0.17901	-0.27088	-0.62752	0.38352	0.63434	-0.25501	-0.34427
26	0.86497	0.00115	0.15374	0.00713	0.32542	~~0.106015~~	-3.8E-05	0.013283	-4.4E-06	~~0.104602~~	-0.97768	0.04836	0.23290	0.07511	-0.45847	-0.97775	0.07114	0.36471	-0.00526	-0.41677
27	0.64233	0.08294	-0.33943	-0.48933	-0.18049	~~0.045236~~	0.019786	~~0.044136~~	~~0.168259~~	~~0.10293~~	-0.92334	0.34289	0.39562	-0.00807	-0.26912	-0.90468	0.34413	0.48542	0.00372	-0.29273
28	-0.52177	-0.68167	-0.40007	0.17417	-0.08879	~~0.046847~~	~~0.247565~~	~~0.085574~~	~~0.02693~~	-0.01046	-0.84510	0.40441	0.15346	-0.00389	-0.19383	-0.93389	0.24918	0.32604	0.05239	-0.24732
Cor. wi																				
Trend	-0.91244	0.35356	0.28531	-0.20345	-0.28775															
K_t	-0.92450	0.37670	0.31015	-0.14111	-0.27082															
R_t	-0.90520	0.37936	0.39208	-0.05996	-0.31351															
n_t	-0.80807	0.59488	0.11999	-0.01597	-0.09558															
$\frac{K_t}{n_t}$	-0.94208	0.23349	0.40397	-0.15885	-0.35025															
$\frac{R_t}{n_t}$	-0.87963	0.16906	0.55582	-0.05358	-0.44370															
r_t	0.93934	-0.28674	-0.08605	0.27691	0.14952															

4. The unrestricted optimal portfolios

With mean returns, vector μ containing $\hat{r}_i$'s, and corresponding covariance matrix, $\Omega = [Cov(r_i, r_j)]$, we could proceed to the inference of the unrestricted optimal portfolio W, that is satisfying:

$$\underset{W}{Min} \quad W' \Omega W \qquad \text{s.t.} \quad \mu' W \geq \alpha \,, \qquad W_i \geq 0 \,, \; i = 1, 2, \ldots, n \qquad (45)$$

(of course, $W_i < \frac{\sum_{t=1}^{T} K_{it}}{T}$, or on average, it will be infeasible.) The general solution for the included assets/titles will be of the type [14]:

$$W^* = \Omega^{-1} \mu \; (\mu' \Omega^{-1} \mu)^{-1} \alpha \qquad (46)$$

provided that, being g the budget to be allocated and L a column vector of 1's [15] $\frac{\alpha}{g} < (\mu' \Omega^{-1} \mu) / (L' \Omega^{-1} \mu)$ – in other words, with unrestrictive funds.

We will concentrate on solution (46) and report the optimal shares:

$$\frac{W_\alpha^*}{L' W_\alpha^*} = \frac{\Omega^{-1} \mu}{L' \Omega^{-1} \mu} \qquad (47)$$

If the problem has a unique solution, there will be a typical "optimal portfolio", the shares of which are mean (α) invariant, that every investor buys a portion of and – under free market conditions - we would expect that, for any i, W_i^* to be close to observed K_{it} [16] (or $\frac{W_i^*}{\sum_{i=1}^{n} W_i^*}$ to $\frac{K_{it}}{\sum_{i=1}^{n} K_{it}}$). Also:

$$\frac{\alpha}{\sigma *} = (\mu' \Omega^{-1} \mu)^{1/2} \qquad (48)$$

where $\sigma *$ denotes the square root of the optimized minimand; then $\frac{\alpha}{\sigma *}$ (or its inverse) is independent of α. The unit return of the optimal portfolio, r*, will also be α invariant:

$$r^* = \frac{\mu' W_\alpha^*}{L' W_\alpha^*} = \frac{\alpha}{L' W_\alpha^*} = \frac{\mu' \Omega^{-1} \mu}{L' \Omega^{-1} \mu} \qquad (49)$$

[14] See Tobin (1958), or Martins (2005).
[15] See Martins (2005).
[16] Again, one can argue that such assertion is only compatible with assumption ii) of section 2.

Implicitly, investors will combine the optimal bundle, yielding this unit rate, with borrowing or lending at a zero rate… Of course, its standard deviation will obey (48).

The solution of (45) is that of a classical quadratic programme [17]. We relied on EXCEL's SOLVER for optimization to generate the efficient portfolio. Corner solutions – i.e., $W_i^* = 0$ – were found for several sectors and cases [18]. We therefore proceeded to a stepwise elimination of the highest optimal share sector and recalculated the optimal portfolio shares; we report in the Tables 4 and 4.A below (now, second moments were not multiplied by 1000000) - 4.A also contains the shares for the overall optimal portfolio when using the covariances of the returns themselves under case iii, once this resulted in no corner solutions - the results of such procedure applied to the covariance of average returns, for unweighted and weighted according hypothesis iii) respectively – where the iteration/order in which the sector was eliminated is registered, as well as the implicit number of "iterations" in which the sector was either included in the optimal portfolio or had already been eliminated, with the associated rankings. In two subsequent rows in the Tables, the inverse and the optimal standard deviation of the unitary mean yield is registered - α/◎* (that coincides with Sharpe's performance index of the optimal portfolio when the risk-free rate is zero [19]) and ◎*/α [20] - the slopes of the implicit opportunity locus or market line- below the effective shares we report the effective ratio (for case ii, the ratio of the arithmetic mean over the standard deviation of the yearly aggegate rates; for case iii, obtained dividing by the square root of T=7 the t-ratio of the coefficient of the regression, weighted by number of firms, of the yearly aggregate rates of return on a constant term). The next two rows report r* and its standard deviation (the latter inferred by the product of r* by the reported ◎*/α). The last seven rows register the usual correlations with the outside series.

The most important sectors in the sample period in terms of tangible and intangible assets – (actual) average industry capital shares are reported in the first column of the Tables, its descending order in the second - were 22, 18, 20, 4, 5, 910 and 24. The least important, 7, 3, 11, 6, 17, 26 and 27. (Correlation between the rankings of the actual sector shares weighted and unweighted estimates is very high: 0.99901.)

[17] See Intriligator (1971), Taha (1982).

[18] Notice that for case ii the estimated covariance matrix of all 23 sectors must be singular once only 7 observations are being used for its calculation – we would require at least 24 observations for its non-singularity. That implies that under such case we would not expect more than 6 (T – 1, for the corresponding estimated covariance matrix to be nonsingular - and hence invertible) sectors to be relevant to achieve an optimal portfolio…

[19] Rolling tests based on the comparison of adjacent Sharpe ratios – in the spirit of Gibbons, Ross, & Shanken (1989)'s statistic (see also Glen & Jorion (1993), Campbell *et al* (1997), p.196) - may not be appropriate – both because n is smaller than T – 1, as even included sectors are sometimes smaller T – 1… That is, because we invariably register corner solutions and effectively included sectors do not intersect.

[20] For aggregate profits, the ratios are 7.26089 and 0.137724.

For unweighted estimates, the correlations of the effective capital shares with: sector returns are –0.26331; estimated variances, -0.35924. For weighted estimates, the correlations of the effective capital shares with: sector average returns are –0.26250; estimated variances, -0.039259; variances of the mean return: -0.35543. That is, most signs are negative, but only significant (at 10%) for variances. Industry capital shares are close to profits shares, but not very much to the sector's number of firms; they are also positively related to firms' size but negatively related to size growth.

The first sectors to be found most representative in the optimal portfolio of unweighted estimates – Table 4 - were 4, 19, 21, 20, 17, 14 and 11; the last, 16, 5, 7, 910, 24, 28 and 6. Therefore, it appeared to be no correspondence between these and the actual relative importance of the sectors in the aggregate economy's assets (of course, in an open economy that would not be expected in any case). In fact, the Spearman's rank correlation test with the effective capital share order - on the rows labeled "Rank Corr" of the tables, along with the two-tailed critical value – inferred from Newbold (1995), Table 9, p. 843 - generated a positive but non-significant relation between the two series. The importance of a sector in the portfolio (negatively represented by the rank "Order") would appear – as expected – to be negatively related to the variances and positively to the average sector rate; no outside variables influence is significant.

For weighted estimates, we report two types of results in Table 4.A: those for $Cov(r_{itl}, r_{jtl})$, which generated a portfolio *per se*, and the stepwise results on $Cov(\hat{r}_i, \hat{r}_j)$ – the correlations between the rankings of sector importance implied by the two procedures is positive, but low: 0.38735.

The highest representation in the optimal portfolio from $Cov(r_{itl}, r_{jtl})$ of weighted estimates were found for 18, 4, 11, 3, 14, 27 and 15; the smallest for 19, 26, 28, 24, 20, 5 and 6. Spearman's rank correlation with the effective shares is negative and non-significant; yet, the (standard) correlation between the effective and optimal shares themselves – reported in the Table - is positive and significant: 0.54748. Moreover, firm size appears to be strongly and positively related to representation in the optimal bundle – with the correlations with the sectors ranking suggesting: a strong and negative influence of the variance of the returns, and of the number of firms in the industry in the determination of the sectors' portfolio representation.

The first sectors to be found most representative in the optimal portfolio from $Cov(\hat{r}_i, \hat{r}_j)$ of weighted estimates - were 4, 19, 21, 20, 18, 14 and 8; the last, 24, 7, 28, 910, 16, 26 and 6. Spearman's rank correlation test with the actual capital shares is again positive and almost significant at the 10% significance level.

The correlation between the sector portfolio rankings of the unweighted with those of the weighted estimates for $Cov(\hat{r}_i, \hat{r}_j)$ is, in any case, – and as expected - very high: 0.92292. We recover most traits of the pattern of association with outside variables: it is the variance – sector risk –

(negatively), and rate of return growth (positively) that are the main determinants of the sectors importance. Otherwise, only a slight positive relation between the profits share and the industry's optimal importance is registered.

The "efficient" portfolio would appear – for the iterated cases, comparable to a general unique portfolio – to be four to five times as efficient as – would exhibit a 4 to 5 times α/◎* than - the actual market line.

One could argue that the effective ratio α/◎* under iii when $Cov(r_{itl}, r_{jtl})$ is used should compare with the optimal one multiplied by the square root of the number of firms in each sector (which averages 29.05884), the square root of the number of times it would be independently replicated - i.e., with 2.70826 x 29.05884 = 78.69889402. Again a 4 to 1 relation is found to the effective Sharpe ratio.

A final set of results, adequate under assumption iii), are reported in Table 4.B. – closely reproducing those of Table 4.A. On the one hand, it presents the optimal portfolio considering $Cov(r_{itl}, r_{jtl})$ the first-step estimator $\hat{\sigma}_{ij}$ - no corner solutions were generated. Additionaly, optimal portfolios for a covariance matrix using such estimators off the main diagonal and $\frac{\hat{\sigma}_{ii}}{n_i}$ in the diagonal – the covariance matrix of a vector of unit applications that, in each sector i, would be equally partitioned by the existing n_i firms.

Table 4. *Stepwise Optimal Portfolios (ii)*

Sector	Effective Share	Ord	Ord. Port	Numb. App	Ord.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Obliterated						4	19	21	20	17	14	11	18	22	13	8	27	3	12	15	26	16	5	7	910	24	28
3	0.01054	18	13	11	16.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.53453	-	-	-	-	-	-	-	-	-
4	0.05414	4	1	23	1	0.52277	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0.05261	5	18	6	21	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.53317	-	-	-	-
6	0.00845	20	23	4	23	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.02822	0.00000	0.27481	0.47411
7	0.01201	17	19	7	20	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.10222	0.10009	0.66383	-	-	-
8	0.03850	9	11	22	3	0.00000	0.14121	0.03210	0.26535	0.28825	0.11893	0.33636	0.05252	0.10546	0.12066	0.29277	-	-	-	-	-	-	-	-	-	-	-
910	0.04869	6	20	5	22	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.22934	0.00000	0.00000	0.48764		
11	0.00992	19	7	18	10.5	0.00000	0.00000	0.00000	0.00000	0.14460	0.00000	0.45057	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	0.03887	8	14	10	18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.49617	-	-	-	-	-	-	-	-
13	0.02117	13	10	17	13	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01961	0.37576	0.38256	0.41845	-	-	-	-	-	-	-	-	-	-	-	-
14	0.01238	16	6	18	10.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.32606	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0.01771	14	15	15	14	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.07769	0.21900	0.25832	0.34192	0.20778	0.26119	0.47882	-	-	-	-	-	-	-
16	0.02508	12	17	11	16.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.06526	0.00000	0.09296	0.15239	0.21906	0.27878	-	-	-	-	-
17	0.00763	21	5	19	8	0.00000	0.00000	0.00000	0.00000	0.29782	-	-	-	-	-	-	-	-	-	-	-	-	-	-			
18	0.19905	2	8	18	10.5	0.00000	0.00000	0.00000	0.00000	0.26932	0.23221	0.00000	0.53887	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	0.03383	10	2	23	2	0.26593	0.32835	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.08572	3	4	21	5	0.00000	0.13771	0.00000	0.45760	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	0.03132	11	3	21	5	0.00000	0.00000	0.65775	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	0.22104	1	9	18	10.5	0.06492	0.00000	0.00000	0.00000	0.00000	0.23215	0.00000	0.03057	0.39958	-	-	-	-	-	-	-	-	-	-	-	-	-
24	0.04853	7	21	13	15	0.06153	0.00000	0.04856	0.10743	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.14967	0.27086	0.08797	0.16796	0.20829	0.09703	0.26811	0.40957	-
26	0.00394	22	16	8	19	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.35665	-	-	-	-	-	-
27	0.00343	23	12	20	7	0.04468	0.17309	0.25627	0.03921	0.00000	0.06114	0.08326	0.00000	0.00000	0.18811	0.28470	0.34200	-	-	-	-	-	-	-	-	-	-
28	0.01543	15	22	21	5	0.04017	0.21964	0.00533	0.13041	0.00000	0.02952	0.11020	0.00228	0.03471	0.05379	0.16421	0.25082	0.25769	0.00000	0.09792	0.33632	0.22171	0.15844	0.21092	0.24425	0.31563	0.52589
Corr.	Rank Cc		0.20059 (10%:0.351)		0.17456																						
α/σ*	20.59691					97.2201	88.8562	51.2035	47.3343	33.2963	30.4514	29.5414	26.4611	23.8673	22.0015	21.2218	15.4042	14.0245	12.3869	11.9501	9.6209	9.5538	8.6003	8.2716	7.7958	7.1177	6.2019
σ*/α	0.048551					0.01029	0.01125	0.01953	0.02113	0.03003	0.03284	0.03385	0.03779	0.04190	0.04545	0.04712	0.06492	0.07130	0.08073	0.08368	0.10394	0.10467	0.11627	0.12090	0.12827	0.14050	0.16124
r*	0.11023					0.13906	0.15100	0.10147	0.16036	0.11971	0.10661	0.13234	0.09376	0.10863	0.14162	0.13592	0.13614	0.09508	0.14762	0.13777	0.089297	0.10167	0.08826	0.09540	0.09443	0.09810	0.07494
σ_r*	0.005352					0.00143	0.00170	0.00198	0.00339	0.00360	0.00350	0.00448	0.00354	0.00455	0.00644	0.00640	0.00884	0.00678	0.01192	0.01153	0.00928	0.01064	0.01026	0.01153	0.01211	0.01378	0.01208
$\hat{r}_i$	-0.2633	0.1409	-0.3454	0.2024	-0.1920																						
$Var(\hat{r}_i)$	-0.3592	0.4423	0.7117	-0.449	0.3990																						
$\bar{z}_{K_i}$	1.0000	-0.752	-0.2211	0.1822	-0.1365																						
$\bar{z}_{R_i}$	0.8522	-0.805	-0.3380	0.2691	-0.2510																						
$\bar{z}_{n_i}$	0.1449	-0.445	-0.2910	0.1485	-0.2048																						
$\overline{K_i/n_i}$	0.6705	-0.381	-0.1070	0.0874	-0.0337																						
$\overline{R_i/n_i}$	0.6782	-0.403	-0.1045	0.0753	-0.0197																						
Kgr	-0.1353	0.3185	0.0984	0.3340	-0.3120																						
Rgr	-0.1492	0.1663	-0.0280	0.5864	-0.5801																						
ngr	0.1749	0.0646	-0.0429	0.3986	-0.3521																						
(K/n)gr	-0.5045	0.4865	0.2428	0.0228	-0.0511																						
(R/n)gr	-0.2807	0.1726	-0.0118	0.5158	-0.5331																						
rgr	-0.1163	-0.007	-0.1233	0.6194	-0.6278																						

Table 4.A. *Stepwise Optimal Portfolios, iii)* $\hat{Cov}(\hat{r}_i, \hat{r}_j)$ * T

Sector	Effective Share	Ord	Opt Port Ord $\hat{Cov}(\hat{r}_i, r_j)$	$\hat{Cov}(\hat{r}_i, \cdot)$	Ord Portf	Numb. App	Ord.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Obliterate								4	19	21	20	18	14	8	13	17	27	11	22	3	12	15	5	24	7	28	910	16	26
3	0.01055	18	0.05555	4	13	15	16.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.15335	0.05899	0.18615	0.21751	0.36642	-	-	-	-	-	-	-	-	-
4	0.05387	4	0.08951	2	1	23	3.5	0.46266	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0.05237	5	0.00330	22	16	8	19.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.31879	-	-	-	-	-	-
6	0.00842	20	0.00234	23	23	2	23	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.04844
7	0.01198	17	0.01741	14	18	8	19.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.07780	0.20720	0.42836	-	-	-	-
8	0.03845	9	0.01493	15	7	23	3.5	0.00902	0.09515	0.15948	0.21544	0.16975	0.21222	0.28354	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
910	0.04850	7	0.02252	10	20	7	21	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.09904	0.06748	0.28240	0.53343	-	-
11	0.00994	19	0.05913	3	11	18	11.5	0.00000	0.00000	0.00000	0.00000	0.03093	0.15299	0.26597	0.25502	0.00000	0.21125	0.43452	-	-	-	-	-	-	-	-	-	-	-
12	0.03884	8	0.02205	11	14	13	18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.04844	0.00000	0.16895	0.30923	0.44561	-	-	-	-	-	-	-	-
13	0.02113	13	0.03861	8	8	20	9.5	0.00000	0.07131	0.00966	0.04555	0.00000	0.00000	0.09793	0.60053	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	0.01236	16	0.04495	5	6	20	9.5	0.00000	0.00000	0.00000	0.03242	0.19064	0.23082	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0.01770	14	0.04018	7	15	16	15	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00434	0.20823	0.17309	0.05217	0.15910	0.18983	0.26261	0.47595	-	-	-	-	-	-	-
16	0.02495	12	0.02055	13	21	15	16.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03516	0.07739	0.07856	0.04141	0.00429	0.09001	0.13985	0.20538	0.25971	0.24828	0.28279	0.46657	0.57125	-
17	0.00765	21	0.03386	9	9	23	3.5	0.04805	0.21898	0.06208	0.06217	0.15081	0.15231	0.24882	0.00214	0.47029	-	-	-	-	-	-	-	-	-	-	-	-	-
18	0.19803	2	0.41491	1	5	21	8	0.00000	0.00000	0.00099	0.13292	0.30495	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	0.03394	10	0.01354	17	2	23	3.5	0.29608	0.37448	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.08596	3	0.00526	21	4	23	3.5	0.00844	0.03595	0.24915	0.31842	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	0.03134	11	0.01461	16	3	22	7	0.00000	0.04158	0.27152	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	0.22197	1	0.02070	12	12	17	13.5	0.03661	0.00000	0.00000	0.00000	0.08355	0.17099	0.05967	0.09937	0.00000	0.00000	0.00000	0.27773	-	-	-	-	-	-	-	-	-	-
24	0.04898	6	0.00553	20	17	17	13.5	0.09956	0.07488	0.12036	0.07030	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.09781	0.01054	0.03101	0.20178	0.35206	0.30074	0.27348	-	-	-		
26	0.00396	22	0.01319	18	22	6	22	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.02535	0.04302	0.14987	0.00000	0.42875	0.95156
27	0.00347	23	0.04149	6	10	23	3.5	0.02974	0.04563	0.10709	0.06004	0.06936	0.08067	0.04408	0.03861	0.10341	0.26362	-	-	-	-	-	-	-	-	-	-	-	-
28	0.01563	15	0.00588	19	18	18	11.5	0.00983	0.04205	0.01967	0.06273	0.00000	0.00000	0.00000	0.00000	0.02955	0.16723	0.15079	0.12475	0.09923	0.00000	0.03215	0.09730	0.13521	0.21286	0.28495	-	-	-
Corr.		Rank C		-0.0839	0.33103		0.15570																						
α/σ*	20.55538		2.70826		(5%:0.418)			84.7629	65.7614	41.6973	38.8062	31.9816	30.7276	29.1327	24.9053	17.8377	16.2030	14.8030	13.8301	13.5596	12.9293	12.1298	9.8462	9.6357	9.1879	8.7600	7.5156	6.8419	5.0395
σ*/α	0.048649		0.36924					0.01180	0.01521	0.02398	0.02577	0.03127	0.03254	0.03433	0.04015	0.05606	0.06172	0.06755	0.07231	0.07375	0.07734	0.08244	0.10156	0.10378	0.10884	0.11416	0.13306	0.14616	0.19843
r*	0.109979		0.10523					0.14682	0.15706	0.13989	0.14838	0.11431	0.13312	0.13842	0.13392	0.14387	0.13825	0.12266	0.10551	0.12037	0.14678	0.14410	0.10542	0.11064	0.09814	0.09216	0.11807	0.12167	0.10687
σ_r*	.0053504		0.03886					0.00173	0.00239	0.00335	0.00382	0.00357	0.00433	0.00475	0.00538	0.00807	0.00853	0.00829	0.00763	0.00888	0.01135	0.01188	0.01071	0.01148	0.01068	0.01052	0.01571	0.01778	0.02121
$\hat{r}_i$	-0.26251	0.1341	-0.2385	-0.0450	-0.28658	0.2900	-0.3519																						
$\hat{Var}(r_i)$	-0.0393	-0.1025	-0.3071	0.7350	0.2025	-0.320	0.1735																						
$\hat{Var}(\hat{r}_i)$	-0.3554	0.4265	-0.3133	0.4992	0.7274	-0.592	0.5030																						
$\bar{s}_{Ki}$	1.00000	-0.7521	0.54748	-0.1133	-0.25589	0.1720	-0.1246																						
$\bar{s}_{Ri}$	0.86250	-0.8117	0.31271	0.0523	-0.36239	0.2619	-0.2501																						
$\bar{s}_{ni}$	0.14605	-0.4522	-0.2587	0.5353	-0.34205	0.1494	-0.2500																						
$\overline{K_i}/n_i$	0.66674	-0.3787	0.95758	-0.3667	-0.19529	0.1338	-0.1001																						
$\overline{R_i}/n_i$	0.67447	-0.3999	0.95323	-0.3879	-0.18676	0.1307	-0.0904																						
Kgr	-0.13185	0.3009	-0.2919	0.2025	0.0718	0.2665	-0.2443																						
Rgr	-0.14691	0.1542	-0.2010	0.1362	-0.1230	0.4788	-0.4312																						
ngr	0.17692	0.0442	0.0656	0.0762	-0.1161	0.3950	-0.3382																						
(K/n)gr	-0.50130	0.4845	-0.6296	0.2575	0.3021	-0.095	0.0519																						
(R/n)gr	-0.27899	0.1684	-0.2863	0.1287	-0.0907	0.3834	-0.3549																						
rgr	-0.11568	-0.011	-0.0670	0.0416	-0.2462	0.5107	-0.4565																						

Table 4.B. *Stepwise Optimal Portfolios, iii)* σ_{ij}

Sector	Effective Share	Ord	Opt Port Ord $\hat{\sigma}_i$	$\hat{\sigma}_i$	Ord Portf	Numb. App	Ord.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Obliterate								4	19	20	21	18	8	14	13	17	11	27	22	3	12	15	5	16	7	24	26	910	28
3	0.01055	18	0.05646	4	13	16	14.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01636	0.00000	0.16681	0.09616	0.10317	0.22665	0.35775	-	-	-	-	-	-	-	-	-
4	0.05387	4	0.09110	2	1	23	3	0.47430	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0.05237	5	0.00340	22	16	8	19.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.29674	-	-	-	-	-	-
6	0.00842	20	0.00240	23	23	4	23	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01266	0.06589	0.48494
7	0.01198	17	0.01764	14	18	8	19.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.15232	0.24734	0.42914	-	-	-	-
8	0.03845	9	0.01525	15	6	23	3	0.03571	0.12207	0.17281	0.14207	0.17928	0.22148	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
910	0.04850	7	0.02270	10	21	7	21.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.10060	0.03395	0.19542	0.26926	0.60060	-
11	0.00994	19	0.06006	3	10	19	12	0.00000	0.00000	0.00000	0.00886	0.04789	0.16564	0.10659	0.24800	0.00000	0.23539	-	-	-	-	-	-	-	-	-	-	-	-
12	0.03884	8	0.02265	11	14	14	18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.05638	0.19444	0.18801	0.32008	0.48686	-	-	-	-	-	-	-	-
13	0.02113	13	0.03932	8	8	20	10	0.00000	0.08845	0.05929	0.03552	0.00000	0.00000	0.26537	0.59573	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	0.01236	16	0.04604	5	7	20	10	0.00000	0.00000	0.00000	0.02592	0.17986	0.21406	0.32148	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	0.01770	14	0.04142	7	15	16	14.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00963	0.20526	0.15338	0.24985	0.16291	0.19013	0.25724	0.49651	-	-	-	-	-	-	-
16	0.02495	12	0.02114	13	17	15	16.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03325	0.07392	0.04811	0.03906	0.00626	0.09175	0.15266	0.20932	0.25878	-	-	-	-	-
17	0.00765	21	0.03453	9	9	22	6.5	0.04008	0.20968	0.09893	0.15047	0.16351	0.16970	0.00000	0.01231	0.48485	-	-	-	-	-	-	-	-	-	-	-	-	-
18	0.19803	2	0.41145	1	5	21	8	0.00000	0.00000	0.11961	0.20099	0.28402	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	0.03394	10	0.01198	17	2	23	3	0.22775	0.30584	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.08596	3	0.00534	21	3	23	3	0.01987	0.07138	0.20148	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	0.03134	11	0.01447	16	4	22	6.5	0.00000	0.03385	0.16525	0.23662	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	0.22197	1	0.01996	12	12	20	10	0.10070	0.03534	0.04109	0.11607	0.08658	0.15936	0.21112	0.09656	0.00000	0.00000	0.00000	0.24945	-	-	-	-	-	-	-	-	-	-
24	0.04898	6	0.00475	20	19	15	16.5	0.05159	0.03678	0.04468	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01407	0.03054	0.16415	0.29977	0.21815	0.19760	0.22971	0.33282	-		
26	0.00396	22	0.01405	18	20	7	21.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03410	0.13572	0.26015	0.36149	-	-
27	0.00347	23	0.03790	6	11	23	3	0.03402	0.04888	0.07205	0.08348	0.05887	0.06976	0.07908	0.03777	0.08471	0.21797	0.27599	-	-	-	-	-	-	-	-	-	-	-
28	0.01563	15	0.00598	19	22	17	13	0.01598	0.04772	0.02482	0.00000	0.00000	0.00000	0.00000	0.00000	0.02512	0.16680	0.12845	0.11985	0.09523	0.00000	0.05106	0.12348	0.16158	0.17147	0.21162	0.35659	0.33351	0.51506
Corr.		Rank C		-0.0839	0.32213		0.19856																						
α/σ*	20.55538		2.47446		(5%:0.418)			65.1396	53.8679	39.3477	34.7507	31.5444	30.3656	26.9585	24.7116	17.7522	15.8936	15.5299	13.7749	13.5261	12.8240	11.8604	9.6614	9.4883	8.3362	8.0478	7.2239	6.7272	5.4847
σ*/α	0.048649		0.40413					0.01535	0.01856	0.02541	0.02878	0.03170	0.03293	0.03709	0.04047	0.05633	0.06292	0.06439	0.07260	0.07393	0.07798	0.08431	0.10351	0.10539	0.11996	0.12426	0.13843	0.14865	0.18233
r*	0.109979		0.10522					0.13931	0.15377	0.13067	0.10294	0.11539	0.13350	0.12928	0.13420	0.14304	0.13474	0.14086	0.10754	0.12122	0.14719	0.14288	0.10214	0.10679	0.10044	0.09864	0.07932	0.08148	0.07589
σ_r*	.0053504		0.04252					0.00566	0.00755	0.00879	0.00784	0.00968	0.01163	0.01269	0.01437	0.02132	0.02243	0.02400	0.02066	0.02371	0.03037	0.03187	0.02797	0.02978	0.03188	0.03243	0.02905	0.03205	0.03661
$\hat{r}_i$	-0.26251	0.13418	-0.2405	-0.0450	-0.34610	0.29147	-0.3385																						
$\hat{Var}(r_i)$	-0.0362	-0.1152	-0.3131	0.7402	0.1941	-0.3098	0.1852																						
$\hat{Var}(\hat{r}_i)$	-0.3526	0.4198	-0.3192	0.5019	0.7101	-0.5816	0.5157																						
$\bar{s}_{Ki}$	1.00000	-0.7522	0.54470	-0.1134	-0.25608	0.23363	-0.2071																						
$\bar{s}_{Ri}$	0.86250	-0.8118	0.31038	0.05232	-0.38406	0.31294	-0.3250																						
$\bar{s}_{ni}$	0.14605	-0.4523	-0.2612	0.53533	-0.35005	0.14023	-0.2572																						
$\overline{K_i}/n_i$	0.66674	-0.3788	0.95585	-0.3668	-0.19555	0.14073	-0.1109																						
$\overline{R_i}/n_i$	0.67447	-0.4000	0.95172	-0.388	-0.19399	0.13881	-0.1041																						
Kgr	-0.13185	0.30094	-0.2994	0.20246	0.16073	0.24093	-0.2040																						
Rgr	-0.14691	0.15420	-0.2048	0.13623	-0.01960	0.43629	-0.3787																						
ngr	0.17692	0.04417	0.05548	0.07618	-0.04009	0.38083	-0.3247																						
(K/n)gr	-0.50130	0.48452	-0.6285	0.25753	0.35255	-0.1208	0.1056																						
(R/n)gr	-0.14691	0.15420	-0.2048	0.13623	-0.01960	0.43629	-0.3787																						
rgr	0.17692	0.04417	0.05548	0.07618	-0.04009	0.38083	-0.3247																						

5. The effective market "Betas"

A complementary question, entailing a quite different perspective, would unravel how far away is each sector from the economy's "market line" – assuming the current distribution of asset values across sectors to be optimal (of course, this can only be assumed in the long-run) – and if far enough to reject the possibility of aggregate "optimality". To some extent, the positive correlations found between the optimal portfolio's shares and actual market shares and on the corresponding sectors' rankings previously advanced can be seen as a sign that on the aggregate the argument could hold – and the inspection of the significance of such correlations as tests on the market financial efficiency. Another perspective is derived from theoretical implications of optimization – implying regression analysis, sometimes pursued in empirical work:

With a riskless asset of zero yield, with u_i denoting the aggregate return of sector i – u the corresponding column vector for the n assets -, and R^M = u'W* that of the optimal portfolio W*, we expect that [21]:

$$E[u_i] = E[R^M]\,\frac{Cov(u_i, u'W^*)}{Cov(u'L, u'W^*)}$$

If we are assuming the current portfolio to be the optimal one, W* = L and Cov(u'L, u'W*) = Var(u'W*). This suggests:

$$\overline{R_i} = \overline{R}\,\frac{Cov(R_i, R)}{Var(R)} \quad (50)$$

where $R_t = \sum_{i=1}^{n} R_{it}$, approximating total trading profits in year t. This suggests calculating $\frac{\overline{R_i}}{\overline{R}}$ and $\frac{Cov(R_i, R)}{Var(R)}$ by sector, then, regressing, over i (the whole n sectors), one on the other – excluding the intercept – and testing the equality to 1 of the unique parameter.

Of course, the expression also applies to unit returns [22], r_{it}'s – interchangeability being appropriate under assumption ii) of section 2. Then:

$$\overline{r_i} = \overline{r}\,\frac{Cov(r_i, r)}{Var(r)} \quad (51)$$

[21] See Martins (2005). Also, Campbell, Lo & MacKinlay (1997).
[22] See Brealey & Myers (2003).

with $\frac{Cov(r_i, r)}{Var(r)}$ = β_i corresponding to the well-known beta of asset i.

We considered two approaches to the estimation of $\frac{Cov(R_i, R/n)}{Var(R/n)}$ - for which we used for R = $\sum_{i=1}^{n} R_{it}$ and n = 23 [23] -, and of $\frac{Cov(r_i, r)}{Var(r)}$:

a) directly, calculating the numerator and denominator of such "betas" (generating estimator B1 below).

b) from the (n) coefficient(s) of the regression(s) $r_{it} = a_i + b_i r_t + v_{it}$. As the estimate of b_i would approach the former, a_i was fixed to 0 (estimator B2).

Optimality testing then relies on the fact – arising from (50) and (51) - that the ratio between mean returns - $\frac{\overline{r_i}}{\overline{r}}$, $n\frac{\overline{R_i}}{\overline{R}}$ - should equal such estimates [24].

The several indicators are depicted in Table 5 below – where we also register in two middle columns the relative firm size of the sector in terms of both assets and profits (positively related to sector's capital and profit shares).

A first general test on the significance of a trend in a regression on the yearly rates by sector – in the last columns of the table - resulted in the non rejection of the null for most (but not all) cases – negative and significant at 5% only for sectors 5, 6, 7, 910, 11, 12, 18, 22 and 26. The same is true for its squared deviation from the general mean (only five sectors show a significant trend at 5%). Hence, stationarity of mean unit returns, and possibly of their variances – is suggested. The shares (s_{it}'s) on total assets did exhibit a significant trend for the majority of the 23 sectors, though (correlation between the trend coefficients of rates and share regressions were non-significant: -0.038410). Interestingly, the size of the trend coefficient on the rates of return regressions is significantly negatively related to the sector's rate of return variance. Of the capital share regressions, negatively and strongly related to average firm size.

The correlation of B1 and B2 is high for aggregate profits (0.81556), but low and even negative for rates (-0.32617). Correlation between the two B1's is again positive, 0.39925, and between the two B2's 0.38581 (both significant at 10%) – yet, as noted, we would not expect the latter coincidence. The correlations of the relative rates and betas with outside variables only exhibited a consistent (positive) sign for number of firms in the sector.

[23] That provides some standardization; even so, $\frac{Cov(r_i, r)}{Var(r)}$ and $\frac{Cov(R_i, R/n)}{Var(R/n)}$ are, of course, expected to differ.

[24] One can argue that such test would be more reliable for direct betas, once indirect betas already use (50) and (51). Nevertheless, it would stand in this case as an overall performance test.

Otherwise, positive influence of other sector shares and of firm size (for B2 only) was found for relative profits, but none for relative rates.

In the last two rows, the coefficient estimates of the regression of the ratio of mean returns in the column betas (with standard errors in parenthesis) are reported along with the F-test and its significance probability for the equality to 1. The regression (case b) usually did not lead to a rejection of the equality to 1, even if the direct betas estimates did.

Table 5. *Betas by Industry, (ii)*

Sector	$\bar{R}_i / \bar{\bar{R}}$	23 $\bar{R}_i / \bar{\bar{R}}$	B1 (a)	B2 (b)	$\bar{r}_i / \bar{\bar{r}}$	B1 (a)	B2 (b)	$\frac{\bar{K}_i}{n_i} / \frac{\bar{K}}{n}$	$\frac{\bar{R}_i}{n_i} / \frac{\bar{R}}{n}$	Trend on s_{Kit} (Coef.;	p-val)	Trend on r_{it} (Coef.;	p-val)	Tr. on $(r_{it}-r_i)^2$ (Coef.;	p-val)
3	0.00947	0.21779	0.14573	0.21527	0.89992	1.81152	0.90176	1.11807	1.00094	-0.00002	0.68096	-0.00424	0.07926	0.00002	0.73014
4	0.06017	1.38400	0.96478	1.37037	1.11572	-0.31527	1.11283	1.03327	1.16074	-0.00269	0.00036	0.00106	0.27412	0.00008	0.02115
5	0.04023	0.92535	-0.24100	0.88789	0.76180	2.24872	0.76480	0.37502	0.28398	-0.00230	0.00005	-0.00538	0.00145	0.00017	0.00418
6	0.00951	0.21874	-0.15981	0.20627	1.12086	5.22173	1.12913	0.20086	0.21953	-0.00030	0.00193	-0.01248	0.01434	-0.00038	0.07436
7	0.01187	0.27310	0.06877	0.26651	0.99055	2.36250	0.99332	0.49787	0.48488	-0.00018	0.05738	-0.00589	0.01129	0.00001	0.70655
8	0.04243	0.97593	1.45164	0.99173	1.10126	-1.21002	1.09660	1.55750	1.73552	-0.00056	0.03545	0.00329	0.34807	0.00014	0.44578
910	0.04660	1.07173	-0.38935	1.02416	0.95374	3.25367	0.95838	4.24990	3.99424	-0.00176	0.00244	-0.00803	0.00114	0.00004	0.02595
11	0.01396	0.32106	0.38295	0.32322	1.41038	2.18129	1.41193	0.76407	1.07070	0.00025	0.00714	-0.00512	0.01635	-0.00025	0.07911
12	0.05153	1.18512	0.91329	1.17704	1.32683	1.65706	1.32750	1.08774	1.43674	-0.00012	0.68715	-0.00456	0.03605	-0.00014	0.33420
13	0.02498	0.57463	0.63615	0.57665	1.18018	-0.15776	1.17748	0.48353	0.57415	-0.00037	0.00689	0.00076	0.66793	0.00012	0.14197
14	0.01574	0.36192	0.46447	0.36557	1.26994	-0.22201	1.26693	0.45987	0.58927	-0.00013	0.08418	0.00128	0.54742	0.00017	0.17401
15	0.02630	0.60493	0.48223	0.60147	1.48624	2.31107	1.48790	1.51492	2.24214	-0.00006	0.41397	-0.00525	0.10387	-0.00030	0.37527
16	0.03036	0.69829	0.18023	0.68173	1.21149	2.02660	1.21314	2.23896	2.72129	-0.00118	0.00667	-0.00315	0.50509	-0.00007	0.79435
17	0.01030	0.23682	0.35543	0.24085	1.34844	0.73158	1.34720	0.23906	0.32180	0.00020	0.09294	-0.00175	0.43786	0.00006	0.76482
18	0.12078	2.77800	-0.00077	2.68600	0.60554	0.95134	0.60624	47.28765	28.81184	-0.00995	0.00056	-0.00247	0.00471	0.00001	0.85813
19	0.06002	1.38035	2.51417	1.41715	1.76733	0.30816	1.76439	0.26924	0.47755	0.00107	0.01408	-0.00017	0.94219	0.00036	0.36288
20	0.17454	4.01451	6.25341	4.08834	2.03314	1.54446	2.03215	0.42801	0.87059	0.00233	0.00023	-0.00271	0.33032	-0.00008	0.88233
21	0.01883	0.43301	0.75887	0.44424	0.59739	-0.63017	0.59491	0.63827	0.38573	0.00002	0.92258	0.00183	0.34713	-0.00042	0.05622
22	0.16072	3.69661	4.59820	3.72395	0.73236	1.46545	0.73383	5.40412	3.92307	0.00900	0.00076	-0.00383	0.01333	0.00008	0.27439
24	0.05757	1.32402	2.83069	1.37206	1.20331	1.39838	1.20370	0.76041	0.90402	0.00449	0.01839	-0.00540	0.18123	-0.00023	0.29003
26	0.00373	0.08579	0.07866	0.08555	0.96434	3.42676	0.96930	0.26862	0.25189	0.00017	0.00818	-0.00816	0.01955	-0.00001	0.86384
27	0.00573	0.13179	0.35394	0.13897	1.68728	2.54085	1.68900	0.32972	0.55138	0.00039	0.00018	-0.00610	0.16882	-0.00067	0.29513
28	0.00463	0.10650	0.35733	0.11500	0.28234	-1.68551	0.27837	1.55800	0.46967	0.00171	0.00170	0.00264	0.58513	-0.00085	0.29886
Corr.															
$\hat{r}_i$	0.14599	0.14598	0.40141	0.15981	1.00000	0.21210	0.99997	-0.32634	-0.29628	0.16383	-0.14374	-0.13337	0.23871	0.17398	0.14806
$V\hat{a}r(\hat{r}_i)$	-0.35271	-0.35270	-0.21833	-0.34957	-0.00421	0.54288	0.00019	-0.22557	-0.22789	0.12492	-0.22794	-0.56618	-0.16604	-0.55070	-0.03587
$\bar{s}_{Ki}$	0.85219	0.85220	0.47567	0.84180	-0.26332	-0.08714	-0.26356	0.67048	0.67822	0.01794	-0.16884	0.06657	-0.24832	0.24434	0.12344
$\bar{s}_{Ri}$	1.00000	1.00000	0.79843	0.99958	0.14598	-0.07447	0.14512	0.40685	0.42439	0.16950	-0.19635	0.08495	-0.09041	0.29505	0.20705
$\bar{s}_{ni}$	0.52445	0.52445	0.65174	0.53568	0.42271	-0.04342	0.42162	-0.20742	-0.21687	0.20277	-0.13858	0.07776	0.23982	0.29569	-0.01222
$\frac{\bar{K}_i}{n_i}$	0.40685	0.40685	-0.10574	0.38581	-0.32634	-0.04960	-0.32617	1.00000	0.99694	-0.62534	-0.11676	0.03394	-0.22714	0.08966	0.31472
$\frac{\bar{R}_i}{n_i}$	0.42438	0.42439	-0.09749	0.40307	-0.29628	-0.02929	-0.29600	0.99694	1.00000	-0.62434	-0.11587	0.01908	-0.24001	0.10858	0.31615
Kgr	-0.07418	-0.07419	0.30534	-0.05678	0.12801	-0.13540	0.12668	-0.26153	-0.28822	0.57369	-0.09617	0.03647	0.16111	-0.56783	0.05907
Rgr	-0.11442	-0.11443	0.20614	-0.10010	-0.12110	-0.60385	-0.12578	-0.18820	-0.22204	0.37008	-0.02370	0.52162	0.47920	-0.57454	-0.01913
ngr	0.12179	0.12177	0.30518	0.13147	0.03104	-0.17001	0.02961	0.10994	0.08530	0.34828	-0.04735	0.08591	0.18106	-0.44120	0.06387
(K/n)gr	-0.31479	-0.31479	0.10821	-0.29727	0.18804	0.00309	0.18774	-0.63948	-0.65193	0.53479	-0.10590	-0.05990	0.02760	-0.38726	0.01382
(R/n)gr	-0.20863	-0.20863	0.09199	-0.19600	-0.16785	-0.66123	-0.17292	-0.29425	-0.32313	0.27316	-0.00394	0.60417	0.49977	-0.47792	-0.05842
rgr	-0.11346	-0.11346	0.06375	-0.10590	-0.28925	-0.80855	-0.29530	-0.07233	-0.10199	0.09354	0.04270	0.76439	0.59769	-0.40967	-0.07752
Coef.			.675878	.998249		.384629	.999668								
(s.e.)			(.084195)	.00219072		(.091282)	.00055528								
F			14.81974	0.6386925!		45.44659(	0.356656								
[p-v.]			[0.00087]	[0.43273]		[0.00000]	[0.55647]								

6. (Weighted) Method of order statistics under truncated normal assumptions

. If one advances a particular cdf for the distribution of the unit rates of return, weighted principles can again be applied to produce adequate inference. Of general importance, truncated distribution assumptions, specially lower truncation, have meaning in finance, where a lower bound – even if it does not have to…-may correspond to cut-off, acceptance, points of investment profitability. Moreover, range of the random variable – implicit in double truncation - is also important as a dispersion (risk) measure. We consider below the three cases of a truncated normal – that is, of a truncated standard normal of the standardized residuals of adequate single-parameter regression models. We use the method of order statistics (MOS) [25], providing the Minimum Distance estimators, corresponding standard errors, and the identifying restrictions statistic $e' W^{-1} e$ where W denotes the appropriate covariance matrix.

Table 6. provides the inference for form (29), i.e. compatible with case ii) of section 2:

$$\frac{Rank_i(r_{it})}{T+1} \approx E\{\frac{\Phi(\frac{r_{it}-r_i}{\sigma_i})-\alpha_i}{\beta_i-\alpha_i}\} \text{ , t = 1,2,…,T [26] ,} \quad \alpha_i < \Phi(\frac{r_{it}-r_i}{\sigma_i}) < \beta_i \text{ for all i} \qquad (52)$$

For each sector i, the lower and upper truncation probabilities were translated into truncation points on the rates of return range through:

$$a_i = \hat{r}_i + \hat{\sigma}_i\, \Phi^{-1}(\hat{\alpha}_i) < r_{it} < \hat{r}_i + \hat{\sigma}_i\, \Phi^{-1}(\hat{\beta}_i) = b_i, \text{ for all i} \qquad (53)$$

Estimates are obtained from nonlinear least squares applied to (51). The covariance matrix of the estimates for sector i from $\hat{Cov}(\hat{\theta}_{MDMOS}) = [G(\hat{\theta})' G(\hat{\theta})]^{-1} G(\hat{\theta})' W G(\hat{\theta}) [G(\hat{\theta})' G(\hat{\theta})]^{-1}$, where $W = [\ w_{tl}\ ] = [\ \frac{\frac{Min[Rank_i(r_{it}), Rank_i(r_{il})]}{T+1}\{1-\frac{Max[Rank_i(r_{it}), Rank_i(r_{il})]}{T+1}\}}{T+2}\ ]$ and $G(\hat{\theta})$ the matrix of derivatives of $\frac{\Phi(\frac{r_{it}-r_i}{\sigma_i})-\alpha_i}{\beta_i-\alpha_i}$ with respect to the four parameters. In $e' W^{-1} e$, the identifying restrictions test, e refers to the vector estimated differences (residuals…) between the left and right hand-side of

[25] Martins (2005b). The range restriction was not embedded in estimation.

[26] Ranks are measured ascendingly here: Rank of r_{it} goes, for each sector i, from 1 (smallest r_{it}) to T (largest).

(52), expected to have under the correct distribution $\chi^2_{(T-k)}$, where k denotes the number of estimated parameters.

In Table 6.A stages an analogous setting to form (42) – case iii): denoting by e^*_{it} the (first-step) OLS residual for the t-th observation of the i-th sector of such equation, MOS fits [27]:

$$\frac{Rank_i(e^*_{it})}{T+1} \approx E\{\frac{\Phi[\frac{\sqrt{n_{it}}(r_{it}-r_i)}{\sigma_i}]-\alpha_i}{\beta_i-\alpha_i}\}, \ t = 1,2,\ldots,T, \ \alpha_i < \Phi[\frac{\sqrt{n_{it}}(r_{it}-r_i)}{\sigma_i}] < \beta_i \text{ for all i} \quad (54)$$

The truncation points were evaluated at [28]:

$$a_i = \hat{r}_i + \frac{\hat{\sigma}_i}{\sqrt{\underset{t}{Max_i}\, n_{it}}}\Phi^{-1}(\hat{\alpha}_i) < r_{it} < \hat{r}_i + \frac{\hat{\sigma}_i}{\sqrt{\underset{t}{Min_i}\, n_{it}}}\Phi^{-1}(\hat{\beta}_i) = b_i, \text{ for all i} \quad (55)$$

For each Table, we produce the untruncated distribution estimates (for fixed $\alpha = 0$, $\beta = 1$); the lower single - (for fixed $\beta = 1$) truncation case, and the doubly-truncated one.

The estimates, including the range ones, exhibit general consonance regardless of the heteroscedasticity assumption (ii or iii) – of course, they differ for each according to the truncation context. Also, identifying restrictions would not seem to reject normality (at – upper tail - 5%, $\chi^2_{(5)} = 11.07$; $\chi^2_{(4)} = 9.49$; $\chi^2_{(3)} = 7.81$; therefore, the order restrictions would not be rejected in most cases at that significance level – moreover, the tests are only asymptotically valid…)

Not always truncation seems required – evident from the number of cases with empty cells inferring truncation points, corresponding to estimates of α smaller than 0 or β larger than 1.

(Notice that, say for case ii), standard errors of the plain mean estimates would be the reported sd divided by $\sqrt{7}$ – which are, for our sample, in general, smaller than the reported standard errors of $\hat{r}_i$ from the untruncated MDMOS estimates… We are not implying, therefore, that the latter should be used instead – even if GMOS would eventually perform better. For truncated distributions, they are an alternative available to infer the true distribution parameters.)

[27] For case i, the same procedure could – and was – applied, with n_{it} replaced by K_{it}.

[28] Notice that, effectively, the distribution and distribution range changes with $\sqrt{n_{it}}$. We evaluate the extremes at such points...

Table 6. *Minimum Distance Method of Order Statistics, ii)*

Sect	$\bar{r}_i$ (sd)[1]	$\hat{r}_i$	$\hat{\sigma}_i$	e'W-1	$\hat{r}_i$	$\hat{\sigma}_i$	$\hat{\alpha}_i$	a_i	e'W-1	$\hat{r}_i$	$\hat{\sigma}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	a_i	b_i	$b_i - a_i$	e'W-1
3	0.099195 (0.013076)	.097905 (0.0058708)	.016629 (0.0055970)	5.45435	-.053673 (1.41547)	.057793 (0.24042)	.991133 (0.35241)	0.083362	2.89408	-.015105 (5.40702	.066949 (1.54810)	.925012 (6.73811)	.984800 (1.16501)	0.081276	0.12983	0.048554	3.66681
4	0.12298 (0.0047668)	.123129 (0.0018831)	.00538172 (0.0019104)	3.53233	.123821 (0.0017576)	.00449690 (0.0018706)	-.101857 (0.15345)		2.42665	.123216 (0.0014990)	.00335331 (0.0015085)	-.155622 (0.16845)	1.14013 (0.17942)				1.65845
5	0.083970 (0.012317)	.084350 (0.0056765)	.016338 (0.0049260)	2.95711	.085652 (0.0086854)	.015125 (0.0076236)	-.054213 (0.25660)		3.00626	.159986 (5.52831)	.075456 (2.63761)	.088262 (4.15803)	.227940 (14.31672)	0.058005	0.10372	0.045715	2.45561
6	0.12355 (0.031543)	.121469 (0.014300)	.040866 (0.012889)	4.54727	.00600200 (0.55415)	.089449 (0.16430)	.788736 (1.39613)	0.077744	3.46271	-.150408 (18.22669)	.169666 (5.61613)	.910002 (10.25686)	.980678 (1.80136)	0.077074	0.20045	0.123376	3.58702
7	0.10918 (0.014660)	.109526 (0.0066217)	.019068 (0.0058478)	2.82670	.110696 (0.0094460)	.018012 (0.0083078)	-.042091 (0.22950)		2.66571	.118333 (0.090906)	.028409 (0.098408)	.045832 (0.52105)	.729487 (2.01368)	0.070416	0.13570	0.065284	2.66579
8	0.12139 (0.016913)	.117447 (0.0038852)	.011053 (0.0036989)	7.36954	.000670237 (1.00856)	.046940 (0.18652)	.986950 (0.42614)	0.10510	6.61777	.106752 (0.047801)	.014841 (0.022588)	.413424 (1.51748)	1.08069 (0.22773)	0.10351			3.53110
910	0.10513 (0.018282)	.104601 (0.0090416)	.026780 (0.0076794)	7.31086	-.102207 (3.00190)	.087098 (0.55376)	.981661 (0.95735)	0.079770	5.75890	.00675649 (20.64300)	.164603 (17.23500)	.657265 (30.64066)	.792895 (11.56742)	0.073422	0.14116	0.067738	6.54219
11	0.15546 (0.013065)	.155725 (0.0062165)	.018008 (0.0052690)	2.81589	.158301 (0.011685)	.015627 (0.010442)	-.099322 (0.36930)		2.80955	.215897 (10.83509)	.116262 (10.33668)	.231982 (8.65862)	.371303 (24.27164)	0.13075	0.17772	0.04697	2.09158
12	0.14625 (0.012536)	.147244 (0.0057468)	.016471 (0.0052187)	5.46759	.151805 (0.0032060)	.00839159 (0.0033807)	-.222512 (0.17363)		3.25175	.150619 (0.0033132)	.00659396 (0.0037986)	-.258169 (0.20068)	1.12167 (0.23429)				3.15498
13	0.13009 (0.0082076)	.130209 (0.0039762)	.011683 (0.0033674)	6.01664	.134394 (0.0030960)	.00669713 (0.0036735)	-.287705 (0.20902)		4.91778	.109522 (7.65570)	.076251 (12.70337)	.531224 (34.89370)	.682265 (7.81134)	0.11550	0.14567	0.03017	5.89244
14	0.13998 (0.0099769)	.140607 (0.0041522)	.011831 (0.0039125)	2.76878	.142276 (0.0041340)	.010065 (0.0037940)	-.104386 (0.15659)		1.87169	.141083 (0.0063155)	.00852311 (0.0067992)	-.133690 (0.18842)	1.10313 (0.42182)				1.75362
15	0.16382 (0.017071)	.163264 (0.0070741)	.020411 (0.0067989)	4.68377	.167340 (0.0034100)	.00869094 (0.0042658)	-.222527 (0.17228)		3.99400	.165649 (0.0029002)	.00682496 (0.0030542)	-.271265 (0.20853)	1.17909 (0.18598)				1.69404
16	0.13354 (0.022238)	.131610 (0.010025)	.028519 (0.0093900)	4.14657	.047207 (0.33629)	.062787 (0.099402)	.809046 (1.09931)	0.10211	3.60677	.125773 (0.0042666)	.00964138 (0.0048109)	-.144505 (0.19460)	1.26675 (0.20344)				2.25482
17	0.14863 (0.010700)	.148196 (0.0045699)	.013132 (0.0042200)	3.82304	.146068 (0.014396)	.015021 (0.011002)	.100277 (0.53361)	0.12684	3.66699	.148556 (0.0035324)	.00737925 (0.0048371)	-.196074 (0.23238)	1.16833 (0.18397)				3.83690
18	0.066747 (0.0058808)	.066557 (0.0028966)	.00856514 (0.0024785)	6.70314	.00355340 (0.98888)	.027037 (0.18434)	.979314 (1.13582)	0.058702	5.54932	.031649 (4.71087)	.042096 (2.71658	.726230 (24.46188)	.869387 (8.39566)	0.056967	0.078944	0.021977	5.79678
19	0.19481 (0.010787)	.194742 (0.0041084)	.011864 (0.0044513)	5.71779	.195904 (0.0037302)	.00992148 (0.0046069)	-.084677 (0.15480)		5.07379	.193988 (0.0022255)	.00530596 (0.0024712)	-.159967 (0.17441)	1.20900 (0.18076)				4.41182
20	0.22410 (0.013493)	.224484 (0.0062857)	.018054 (0.0054023)	1.85824	.227056 (0.0094756)	.015525 (0.0087859)	-.099182 (0.27664)		1.87764	.296041 (7.58387)	.095011 (4.92963)	.149433 (6.03754)	.300969 (18.43615)	0.19734	0.24648	0.04914	1.28233
21	0.065848 (0.0093898)	.066222 (0.0044653)	.012927 (0.0038456)	3.08274	.068987 (0.0058344)	.00999493 (0.0059348)	-.151962 (0.23870)		3.30763	.127538 (5.18860)	.061816 (2.51493)	.094663 (5.15257)	.224631 (15.99321)	0.046400	0.080766	0.034366	1.98992
22	0.080725 (0.0096401)	.079884 (0.0044032)	.012545 (0.0040729)	4.65418	-.027125 (0.82119)	.042457 (0.14710)	.987250 (0.38352)	0.067713	3.21231	.077167 (0.0023205)	.00467340 (0.0026624)	-.119640 (0.23096)	1.25835 (0.19979)				2.87178
24	0.13264 (0.020472)	.131281 (0.0079840)	.023256 (0.0082347)	7.90074	.125560 (0.027592)	.029260 (0.022518)	.143147 (0.52457)	0.094361	6.92270	.134073 (0.0021528)	.00526648 (0.0022791)	-.271585 (0.21015)	1.18249 (0.18566)				4.63101
26	0.10629 (0.021135)	.105505 (0.0099694)	.028724 (0.0085242)	2.62160	-.028184 (0.93542)	.075716 (0.22100)	.914844 (1.31503)	0.075639	1.69874	-.018418 (12.28811)	.150965 (7.39094)	.725325 (17.42580)	.865244 (5.93958)	0.071970	0.14828	0.07631	1.78983
27	0.18598 (0.022573)	.183959 (0.010361)	.029565 (0.0096337)	4.59456	-.084246 (2.52184)	.101810 (0.42736)	.991439 (0.34476)	0.15848	2.68511	-.00887117 (10.76710)	.121830 (3.30757)	.909749 (8.48680)	.978517 (1.70518)	0.15428	0.23772	0.08344	3.14416
28	0.031121 (0.022612)	.030002 (0.0092574)	.026400 (0.0088799)	2.88780	.026403 (0.024357)	.029864 (0.020606)	.085460 (0.44293)		2.64466	.029995 (0.0090665)	.017201 (0.010337)	-.141565 (0.23833)	1.14312 (0.17321)				2.05361

Table 6.A. *Minimum Distance Method of Order Statistics, iii)*

Sect	$\bar{r}_i$ (sd)[1]	$\hat{r}_i$	$\hat{\sigma}_i$	e'W-1	$\hat{r}_i$	$\hat{\sigma}_i$	$\hat{\alpha}_i$	a_i	e'W-1	$\hat{r}_i$	$\hat{\sigma}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	a_i	b_i	$b_i - a_i$	e'W-1
3	0.098665 (0.17190)	.097706 (0.0056945)	.219805 (0.0055534)	5.41730	.053322 (0.057168)	.444510 (0.22991)	.812570 (0.37564)	0.081215	3.48005	.071307 (0.060202)	.290868 (0.37945)	.706999 (0.75268)	1.03891 (0.15077)	0.082509			3.07154
4	0.12286 (0.15269)	.123114 (0.0018780)	.170231 (0.0019234)	3.72599	.123844 (0.0017195)	.140724 (0.057435)	-.108949 (0.15044)		2.37118	.123223 (0.0014803)	.105525 (0.047068)	-.158584 (0.16780)	1.13875 (0.17961)				1.64449
5	0.084089 (0.63824)	.084408 (0.0056657)	.848281 (0.0049297)	3.08552	.085711 (0.0087049)	.784984 (0.39751)	-.054435 (0.25811)		3.13154	.173841 (0.15430)	3.57653 (5.89889)	.044940 (0.19676)	.152304 (0.36403)	0.058981	0.10244	0.043459	2.27246
6	0.12470 (0.91095)	.121815 (0.014446)	1.17655 (0.012923)	4.57894	-.129240 (0.60207)	3.78890 (4.62211)	.935945 (0.33555)	0.064664	3.13660	-.131452 (0.63239)	3.82138 (5.69199)	.936286 (0.32959)	.999805 (0.020552)	0.064465	0.35509	0.290625	3.13415
7	0.10988 (0.32026)	.109775 (0.0067077)	.418549 (0.0058403)	2.88505	.110469 (0.010613)	.404755 (0.20315)	-.024274 (0.25829)		2.80493	.114097 (0.051940)	.527404 (1.37305)	.032017 (0.50810)	.842305 (1.57115)	0.071265	0.13928	0.068015	2.82315
8	0.12137 (0.37288)	.117410 (0.0039060)	.242931 (0.0037092)	7.46984	.092595 (0.081172)	.531091 (0.66666)	.680462 (1.01689)	0.10376	7.27860	.109178 (0.029247)	.296444 (0.32839)	.326833 (1.10023)	1.09328 (0.18089)	0.10322			3.63500
910	0.10574 (0.27522)	.104886 (0.0091062)	.401583 (0.0076408)	6.84381	.039829 (0.31325)	.813337 (1.60217)	.744724 (1.46552)	0.073950	5.29440	.041302 (0.21797)	.409061 (0.68077)	.935136 (0.70752)	1.01917 (0.20978)	0.080819			4.17947
11	0.15506 (0.20893)	.155524 (0.0062465)	.288278 (0.0052669)	3.00657	.158518 (0.010955)	.244338 (0.15617)	-.115116 (0.34394)		2.99415	.167757 (0.11672)	.383670 (1.63728)	-.018953 (0.77688)	.668192 (2.68810)		0.17851	0.17851	2.68235
12	0.14618 (0.33027)	.147243 (0.0057322)	.433948 (0.0052119)	5.64684	.151789 (0.0031904)	.220052 (0.088945)	-.223776 (0.17405)		3.25565	.150603 (0.0032311)	.171966 (0.097740)	-.259311 (0.20083)	1.12191 (0.22941)				3.14365
13	0.13004 (0.23876)	.130211 (0.0039603)	.338734 (0.0033526)	5.69719	.134059 (0.0036906)	.212382 (0.12506)	-.262176 (0.22085)		4.79991	.188828 (0.041373)	8.33223 (446.99587)	.397253 (5.38562)	.440704 (3.15359)	0.11657	0.14471	0.02814	2.93885
14	0.13985 (0.22887)	.140575 (0.0041401)	.270191 (0.0039185)	2.86586	.142271 (0.0040956)	.229321 (0.085693)	-.106650 (0.15542)		1.90939	.141026 (0.0061933)	.192726 (0.15211)	-.136568 (0.18651)	1.10742 (0.41563)				1.78483
15	0.16378 (0.25819)	.163253 (0.0070927)	.308908 (0.0067974)	4.66518	.167341 (0.0033942)	.130538 (0.063867)	-.222803 (0.17238)		3.98708	.165655 (0.0028888)	.102634 (0.045915)	-.271346 (0.20858)	1.17906 (0.18601)				1.69311
16	0.13332 (0.32646)	.131563 (0.010050)	.418799 (0.0093801)	4.11147	-.00383192 (0.27943)	1.07194 (0.84696)	.930644 (0.37394)	0.10269	3.41453	.125771 (0.0042176)	.140449 (0.069313)	-.146943 (0.19276)	1.26757 (0.20386)				2.28254
17	0.14861 (0.27110)	.148191 (0.0046302)	.330671 (0.0042489)	3.97635	.147813 (0.010558)	.339716 (0.23456)	.019303 (0.43435)	0.12034	3.95819	.148725 (0.0032637)	.176262 (0.10755)	-.214286 (0.21767)	1.17135 (0.18204)				3.89208
18	0.066326 (0.052416)	.066280 (0.0028518)	.075862 (0.0024901)	6.74340	.028151 (0.012087)	.145135 (0.037526)	.979122 (0.042137)	0.059650	0.82512	.026456 (0.012569)	.120548 (0.040285)	.995530 (0.017098)	1.00057 (0.0019069)	0.060051			0.71317
19	0.19494 (0.52912)	.194902 (0.0041428)	.596161 (0.0042917)	4.70345	.196196 (0.0038393)	.503792 (0.22909)	-.091853 (0.15854)		3.89731	.194083 (0.0023515)	.268459 (0.13185)	-.167724 (0.17535)	1.20819 (0.18188)				4.06089
20	0.22395 (0.84549)	.224460 (0.0063187)	1.13490 (0.0054215)	2.33352	.227814 (0.0083789)	.920752 (0.49971)	-.131310 (0.24499)		2.29709	.259896 (0.055881)	2.42465 (2.91480)	.023099 (0.20092)	.356914 (0.44240)	0.18528	0.24508	0.05980	1.71578
21	0.066305 (0.28111)	.066365 (0.0043394)	.390863 (0.0037864)	3.27957	.067909 (0.0075492)	.343999 (0.22243)	-.083784 (0.32702)		3.44515	.072788 (0.035831)	.561153 (2.23538)	.043237 (1.02344)	.698286 (1.53728)	0.043663	0.083075	0.039412	2.95422
22	0.079782 (0.26020)	.079521 (0.0041997)	.336204 (0.0041021)	3.97326	.072612 (0.013852)	.429778 (0.22632)	.329247 (0.46232)	0.066465	3.34597	.077151 (0.0023419)	.137483 (0.081165)	-.119822 (0.23899)	1.25380 (0.19792)				2.85290
24	0.13013 (0.68565)	.131022 (0.0079578)	.830225 (0.0079587)	6.17248	.132901 (0.0088827)	.734715 (0.42286)	-.061554 (0.22593)		6.81972	.133778 (0.0020236)	.179187 (0.078568)	-.267644 (0.21133)	1.19209 (0.18645)				4.18820
26	0.10512 (0.35224)	.104976 (0.0098288)	.480215 (0.0085629)	2.78303	.073284 (0.052703)	.734794 (0.39285)	.506905 (0.51137)	0.073987	1.99393	.070224 (0.066418)	.827554 (1.44180)	.528918 (0.50596)	.973472 (0.35317)	0.073539	0.16951	0.095971	1.96148
27	0.18270 (0.28765)	.182486 (0.0089724)	.373069 (0.0091882)	3.07542	.165532 (0.020859)	.489950 (0.18841)	.362091 (0.30647)	0.15514	3.03466	.168650 (0.022874)	.373518 (0.42544)	.284340 (0.55814)	1.07603 (0.25650)	0.15586			3.07382
28	0.031375 (0.30793)	.030217 (0.0092602)	.370990 (0.0086385)	3.89231	.017954 (0.026953)	.508257 (0.26026)	.241008 (0.36752)	-0.0045112	3.08744	.030507 (0.010625)	.243661 (0.18012)	-.146930 (0.29748)	1.14102 (0.18482)				3.21957

Looking at the first Table, and the single truncated cases produced lower bounds (a_i's) for rates of return [29] ranging from 5.8% to 15.8% [30]; from highest to lowest, the estimates denote lower bounds for sectors 27, 17, 8, 16, 24, 3, 910, 6, 26, 22 and 18. Interestingly, such rating does not seem to be related to the mean rates of return (reported in the first column, close to the MOS estimate of the second column), highest for sectors 20, 19, 27, 17, 11, 17,… Of course, when truncation was found significant, the mean estimates changed (of course the "truncated mean" would still be close to the first column number…)

For some cases – but not completely -, when lower truncation was justified, upper truncation estimates also were. Under ii), double truncation estimates imply (significant) ranges ($b_i – a_i$) between 2.2 to 12.3% in sectors (from longer to shorter range) 6, 27, 26, 910, 7, 20, 11, 5, 21, 13, 18. Under iii),

[29] We always work with gross rates… To have some correspondence to net rates, we would have to deduct the appropriate depreciation rate.

[30] Estimates of a_i's and b_i's appear more meaningful than those of α_i and β_i … Nevertheless, - recall that the number of observations is very small - convergence was not met in some truncated – specially, doubly truncated - cases.

they range (even if the range has a lesser meaning under iii [31]) between 2.8 to 29.1%, in sectors (from longer to shorter range) 6, 11, 26, 7, 20, 5, 21, 13. Some sectors of short rate of return range are also sectors of low (untruncated variance estimates) volatility.

[31] See footnote 30.

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