The Pricing of Skewness Over Different Return Horizons

The Pricing of Skewness Over Different Return Horizons*

Kevin Aretz[†]
Alliance Manchester Business School
kevin.aretz@manchester.ac.uk

Y. Eser Arisoy[‡]
NEOMA Business School
eser.arisoy@neoma-bs.fr

This Draft: October 21, 2022

Abstract

While recent theoretical and empirical work suggests that the physical skewness of a stock's future discrete return distribution prices stocks, it does not tell us over which return horizon(s) that physical skewness is priced. Developing a novel block bootstrap estimator that allows us to calculate realized return skewness over arbitrary horizons, we aim to identify those return horizons. In doing so, we first show that our block bootstrap estimator produces more accurate realized skewness estimates than other recent estimators do. Next, we report that the existing skewness proxies used in the empirical asset pricing literature differ in how well they predict skewness over short or long return horizons. Finally, we reveal that the skewness pricing evidence documented in the empirical asset pricing literature is mostly driven by skewness over short (and not long) return horizons.

Keywords: asset pricing, physical skewness, realized skewness, quantile regression models

JEL classification: G11, G12, G15

[†]Accounting and Finance Division, Room 5.013 AMBS, Booth Street West, Manchester M15 6PB, UK, tel.: +44(0)161 275 0122, fax: +44(0)161 275 4023, URL address: <www.kevin-aretz.com>.

[‡]Corresponding author, Department of Finance, 59 rue Pierre Taittinger, 51100 Reims, France, tel: +33(0)326 350 951, URL address: <https://sites.google.com/site/eserarisoy/>.

*We are indebted to Geert Bekaert (our editor) and two anonymous referees for valuable advice and feedback on our work. We are further indebted to Sofiane Aboura, David Allan, Diego Amaya, Turan Bali, Hendrik Bessembinder, Tolga Cenesizoglu, Dudley Gilder, Nishad Kapadia, Paul Karehnke, Sonia Konstantinidi, Alex Kostakis, Scott Murray, Anthony Neuberger, Andrew Patton, Richard Payne, Julien Penasse, Alberto Plazzi, Peter Pope, Aurelio Vasquez, Yuhang Xing, seminar participants at Dauphine University and Norwich Business School, and conference participants at the 2014 Amundi Asset Management Workshop (Paris), the 2016 Financial Risks Forum of the Louis Bachelier Institute (Paris), the 2016 European Financial Management Association Conference (Basel), the 2016 Financial Econometrics and Empirical Asset Pricing Conference (Lancaster), the 2016 Financial Management Association Conference (Las Vegas), the 8th French Econometrics Conference (Paris), the 2017 Global Finance Conference (New York), the 2017 Financial Management Association European Conference (Lisbon), and the 2017 World Finance Conference (Cagliari).

The Pricing of Skewness Over Different Return Horizons

This Draft: October 21, 2022

Abstract

While recent theoretical and empirical work suggests that the physical skewness of a stock's future discrete return distribution prices stocks, it does not tell us over which return horizon(s) that physical skewness is priced. Developing a novel block bootstrap estimator that allows us to calculate realized return skewness over arbitrary horizons, we aim to identify those return horizons. In doing so, we first show that our block bootstrap estimator produces more accurate realized skewness estimates than other recent estimators do. Next, we report that the existing skewness proxies used in the empirical asset pricing literature differ in how well they predict skewness over short or long return horizons. Finally, we reveal that the skewness pricing evidence documented in the empirical asset pricing literature is mostly driven by skewness over short (and not long) return horizons.

Keywords: asset pricing, physical skewness, realized skewness, quantile regression models

JEL classification: G11, G12, G15

1 Introduction

Recent theoretical studies suggest that the physical skewness of an asset's future discrete ("dollar")[1] return distribution can be negatively priced in equilibrium (see Brunnermeier et al. (2007), Mitton and Vorkink (2007), and Barberis and Huang (2008)). Boyer et al. (2010), Bali et al. (2011), Kozhan et al. (2013), Boyer and Vorkink (2014), Conrad et al. (2014), and Amaya et al. (2015) provide empirical evidence that historical or forward-looking skewness proxies negatively price stocks, stock indexes, and single-stock options.[2]

Notwithstanding the empirical evidence, the literature is notably silent on the return horizon(s) over which skewness is priced. For example, the theoretical literature only studies two-period models with an arbitrary return horizon. In turn, the empirical literature almost always relies on proxies for the skewness of shorter-horizon (often intraday or daily) returns. However, as skewness does not, in contrast to, for example, expected returns and variance, generally scale with the return horizon (see Neuberger (2012) and Fama and French (2018)), it is unclear whether the proxies used in the empirical literature also reflect skewness over other return horizons. Given this, it is then further unclear whether the pricing ability of those proxies comes from skewness being priced over the short return horizons used to estimate them, from it being priced over other return horizons, and/or from other factors.

In this study, we take a closer look at skewness proxies used in the empirical stock pricing literature, evaluating how well they capture the skewness of dollar returns over alternative horizons and how much of their pricing ability comes from them capturing skewness over a particular horizon. In doing so, we begin by developing a novel estimator of the realized skewness of dollar returns over arbitrary horizons. Relying on the estimates derived from that estimator as forecasting target, we then explore the ability of the skewness proxies to predict

[1] We define the gross dollar return as the ratio of the sum of an asset's value at the end of the investment period plus the dividends paid out over that period to the asset's value at the start of the period. Conversely, we define the log return as the natural log of the gross dollar return.

[2] Defining (as we do) the skewness premium as the mean spread return between high and low skewness assets, prior studies find a negative premium. If we instead defined that premium as the spread in skewness across the real-world and the equivalent martingale measure, they would find a positive premium.

the realized future skewness of the daily, monthly, quarterly, and annual dollar return. Finally, we decompose each proxy into the sum of a component capturing the expected future realized skewness of the dollar return over one of the four horizons and a residual component, separately studying each component's pricing power. As skewness proxy contenders, we use Boyer et al.'s (2010) cross-sectional least-squares prediction of daily dollar return skewness (*OLSSkew*); a cross-sectional counterpart of Ghysels et al.'s (2016) quantile regression prediction of annual dollar return skewness (*QuantileSkew*); daily dollar return skewness calculated over some prior period (*HistoricalSkew*); Conrad et al.'s (2014) logit-model prediction of the probability that a stock's log twelve-month-ahead return exceeds 100% (*LogitSkew*); and Bali et al.'s (2011) maximum of the daily dollar return over the prior month (*MaxRet*).

Building on Fama and French (2018), our novel estimator of the realized skewness of dollar returns starts by repeatedly using Politis and Romano's (1994) block bootstrap to form a large number of bootstrapped samples of a stock's short-horizon dollar returns. Using each bootstrapped sample, we calculate one artificial long-horizon dollar return. Finally, we obtain our realized skewness estimate by applying the sample skewness to the set of artificial long-horizon dollar returns. In contrast to Fama and French's (2018) and Farago and Hjalmarsson's (2022) estimators,[3] the advantage of our estimator is that it does not assume that returns are independent and identically distributed (i.i.d.), allowing it to capture the effects of return dependencies (as, e.g., Black's (1976) leverage effect). In contrast to Neuberger and Payne's (2021) estimator, our estimator has the further advantage of capturing the effects of compounding (see Bessembinder (2018)). Using a Monte Carlo simulation exercise based on geometric Brownian motion (GBM), stochastic volatility (SV), and stochastic volatility-jump (SVJ) stochastic processes, we confirm that our estimator yields less biased estimates with competitive standard errors relative to the other estimators.

[3]Whereas Fama and French's (2018) estimator is identical to ours except that they rely on a simple (and not a block) bootstrap, Farago and Hjalmarsson's (2022) estimator can essentially be interpreted as the closed-form equivalent of Fama and French's (2018) estimator.

Next, we assess how accurately the skewness proxies predict the realized skewness estimates obtained from our block-bootstrap estimator. Using portfolio sorts and Mincer-Zarnowitz (1969) regressions, we find that all skewness proxies are significantly positively related to the future realized skewness of short- and long-horizon dollar returns. However, in comparison, *OLSSkew*, *QuantileSkew*, and *LogitSkew* perform better than *HistoricalSkew* and *MaxRet* over short horizons, with them (the other proxies), for example, capturing 5% to 8% (2% to 3%) of the cross-sectional variation over the daily horizon. Moving to longer horizons, the predictive ability of *QuantileSkew*, *LogitSkew*, and *MaxRet* dramatically improves relative to that of *OLSSkew* and *HistoricalSkew*, which is consistent with the skewness proxies designed to directly capture the skewness of long-horizon returns performing better over such horizons. Whereas *QuantileSkew* and *LogitSkew*, for example, capture 18% of the cross-sectional variation over the annual horizon, the other proxies capture no more than 10%. In addition, once we omit the compounding effect from our realized skewness estimates, the predictive ability of all skewness proxies drops significantly, especially over long horizons, in line with the idea that the proxies mostly capture the compounding effect in the skewness of dollar returns.

Finally, we examine the extent to which the ability of a skewness proxy to reflect the expected future realized skewness of the dollar return over some horizon drives its stock pricing power. Using Hou and Loh's (2016) methodology, we first decompose the skewness proxy into the sum of a component reflecting expected skewness and an orthogonal component. To mitigate attenuation error, we conduct the decomposition at the portfolio level, sorting stocks into 50 portfolios based on the skewness proxy and running a cross-sectional regression of a portfolio's average skewness proxy value onto its average future realized skewness. We then assign the fitted regression value, which we interpret as the expected skewness component, to all the stocks in a portfolio. Finally, we gauge each component's ability to price stocks and calculate the proportions of the skewness proxy premium attributable to the components.

In line with the literature, our portfolio sorts and Fama-MacBeth (FM; 1973) regressions suggest that *OLSSkew*, *LogitSkew*, and *MaxRet*, but not *QuantileSkew* and *HistoricalSkew*, yield significantly negative simple or risk-adjusted premia. Decomposing the skewness proxies into components reflecting expected skewness over some horizon and orthogonal components, we find that the pricing of the skewness proxies generally comes from them predicting skewness over short horizons. For example, while the component in *OLSSkew* that predicts daily skewness is significantly negatively priced (t-statistic: –2.90) and explains 70% of *OLSSkew*'s pricing, the component that predicts annual skewness is insignificantly priced (t-statistic: –1.34) and explains only 51%. We obtain similar results for *QuantileSkew* and *LogitSkew*. Notably, the component in *MaxRet* predicting skewness is significantly negatively priced over all horizons, suggesting that *MaxRet* mostly captures the compounding effect in skewness, and that the compounding effect is generally significantly negatively priced. Taken together, we conclude that the skewness pricing evidence in the literature appears to come mostly from the skewness proxies used in that literature picking up skewness over short (and not long) horizons.

Our work extends the emerging literature on accurately estimating the skewness of long-horizon returns from short time-series of data. Neuberger (2012) proposes an estimator of the skewness of long-horizon *log* returns based on short-horizon return data on the primary asset and options written on it. Assuming i.i.d. returns, Fama and French (2018) propose a simple bootstrap method, and Farago and Hjalmarsson (2022) propose an equivalent closed-form estimator for the skewness of long-horizon *dollar* returns from short-horizon return data. Applying their estimators to single stocks, they find that dollar return skewness rises rapidly with the return horizon, consistent with Bessembinder (2018). Neuberger and Payne (2021) caution against those conclusions, showing that return dependencies (i.e., violations of the i.i.d. assumption) greatly affect skewness. Unfortunately, however, their own estimator also yields a biased estimate of the skewness of *dollar* returns because of approximation error. Motivated by these studies, we develop a block bootstrap estimator that rectifies the shortcomings of the existing estimators. Using a Monte Carlo simulation exercise, we confirm that our estimator has several desirable properties and noticeably improves on other existing estimators.

Our study also contributes to the literature on whether the physical skewness of an asset's future return distribution is priced. Scott and Horvath (1980) show that expected utility investors with wealth-independent preferences like skewness. As adding assets to a portfolio can lower skewness, Simkowitz and Beedles (1978) and Conine and Tamarkin (1981) establish that skewness preferences can explain portfolio underdiversification. Despite that, Rubinstein (1973) shows that monetary separation implies that investors who care about skewness but no other higher moments choose portfolios such that an asset's expected return is a function of its return covariance and coskewness with the excess market return, but not skewness.[4] To create a separate pricing role for skewness, recent theoretical studies rely on assumptions that violate monetary separation. Mitton and Vorkink (2007), for example, use heterogeneity in skewness preferences, whereas Brunnermeier et al. (2007) and Barberis and Huang (2008) use non-expected-utility preferences. While the aforementioned empirical studies often support these theoretical studies, neither they nor the theoretical studies precisely identify the return horizon(s) over which skewness is priced. We add to this literature by showing that the empirical evidence comes mostly from short-horizon return skewness being priced in stocks.[5]

We finally add to an emerging literature that uses quantile regressions to forecast an asset's future return distribution. Cenesizoglu and Timmermann (2008) show that quantile regressions efficiently estimate the conditional S&P 500 return distribution, improving market timing and option investment strategies. Ghysels et al. (2016) combine quantile regressions with a mixed-data sampling (MIDAS) model to predict stock index skewness. We transform their time-series estimator into a cross-sectional estimator; however, we use a simpler regression specification and alternative means to convert estimates into skewness predictions.

We proceed as follows. Section 2 introduces our block-bootstrap estimator for realized skewness. In Section 3, we review the skewness proxies used in previous empirical studies. Section 4 outlines our data sources. In Section 5, we evaluate the ability of the skewness proxies

[4]Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Dittmar (2002) show that physical coskewness negatively prices stocks. More recently, Chang et al. (2013) report that exposure to shocks in risk-neutral market skewness calculated from Bakshi et al.'s (2003) methodology does so too.

[5]A related strand of literature studies whether the risk-neutral skewness of an asset's future return distribution is priced (see, e.g., Rehman and Vilkov (2012), Conrad et al. (2013), and Stilger et al. (2017)).

to predict realized skewness. In Section 6, we examine whether the skewness proxies are priced because they predict skewness over a particular return horizon. Section 7 concludes.

2 Calculating Realized Skewness

In this section, we introduce our block bootstrap estimator for the realized skewness of dollar returns over arbitrary horizons. We begin by reviewing the issues inherent in estimating realized skewness over long horizons and why other recent estimators do not satisfactorily address these issues. Next, we describe our estimator. Finally, we conduct a Monte Carlo simulation exercise to verify that our estimator improves on other recent estimators.

2.1 Skewness Estimators Proposed in the Recent Literature

It is well-known that it is generally infeasible to accurately estimate the skewness of long-horizon dollar returns from standard moments-based estimators (see Lau et al. (1989)).[6] To rectify the deficiencies of standard estimators, Neuberger (2012), Fama and French (2018), Neuberger and Payne (2021), and Farago and Hjalmarsson (2022) develop alternative estimators intended to more accurately measure long-horizon-return skewness from limited amounts of short-horizon return data. However, as Neuberger's (2012) estimator focuses on the skewness of log (and not dollar) returns, it cannot be used in cross-sectional asset pricing studies because these studies generally consider investable dollar (and not uninvestable log) returns.

Assuming i.i.d. returns, Fama and French (2018) advocate a simple bootstrap to estimate the skewness of long-horizon dollar returns. Specifically, they draw short-horizon dollar returns from an estimation window, ensuring that each return has an equal probability of being drawn and replacing the drawn returns. They then compound the drawn returns to construct an artificial long-horizon dollar return. Repeating those steps multiple times, they finally apply

[6]Neuberger and Payne (2021) calculate that, if monthly (annual) returns were i.i.d. standard normal, a moments-based estimator would require 50 (600) years of data to estimate skewness with a standard error of approximately 0.10. As the median U.S. stock is listed for only seven-and-a-half years (see Bessembinder (2018)), moments-based estimators cannot yield accurate estimates of long-horizon-return skewness.

the sample skewness estimator to the sample of artificial long-horizon returns to obtain their estimate. Using the same assumptions, Farago and Hjalmarsson (2022) derive the closed-form equivalent of Fama and French's (2018) simple bootstrap estimator.

Neuberger and Payne (2021) criticize Fama and French's (2018) and Farago and Hjalmarsson's (2022) estimators, arguing that the i.i.d. assumption destroys the effect of return dependencies (e.g., Black's (1976) leverage effect) on skewness. Assuming that asset prices are stationary martingales and approximating an asset's return moments, they analytically show that the skewness of long-horizon dollar returns equals the sum of the skewness of short-horizon dollar returns and the scaled covariance between the current short-horizon return volatility and past price changes divided by the square root of the length of the long horizon. Accordingly, they propose an estimator of the skewness of long-horizon dollar returns based on estimates of the skewness of short-horizon dollar returns and the scaled covariance term.

As Table 1 in Neuberger and Payne (2021) reveals, their estimator also yields a biased estimate of the skewness of long-horizon dollar returns because it approximates away the compounding effect in skewness. To see this, assume i.i.d. short-horizon *log* returns. In this case, the central limit theorem implies that the long-horizon log return (the sum of the short-horizon log returns) converges to a normal variable as the length of the return horizon increases to infinity. Moreover, the variance of the normal variable, σ^2, increases with the length of the return horizon. As the long-horizon *dollar* return is the exponential of the long-horizon log return, it converges to a lognormal variable whose skewness is known to be equal to $(e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$. It immediately follows that the long-horizon dollar return becomes more right-skewed with the length of the return horizon because of compounding. Despite that, Neuberger and Payne's (2021) estimator erroneously predicts that i.i.d. short-horizon returns imply that dollar return skewness must decline with the length of the return horizon, simply because the covariance term in the estimator's numerator is zero under such returns. See Appendix A for details on Neuberger and Payne's (2021) estimator.

Taken together, it is obvious that the estimators proposed in the recent literature suffer from either not capturing the effects of return dependencies or from compounding on the skewness of long-horizon dollar returns.

2.2 Our New Block Bootstrap Estimator

While it is correct that Fama and French's (2018) estimator does not incorporate the effects of return dependencies on skewness, a straightforward remedy is to use a modified version of that estimator relying on a block (and not a simple) bootstrap. Sampling blocks of consecutive observations, these bootstraps aim to ensure that bootstrap samples drawn from a population mimic the dependence structure in the population. Given its popularity and desirable statistical properties, we choose the stationary block bootstrap of Politis and Romano (1994) in our estimator. Then, we take the following steps to implement our block bootstrap estimator to obtain the skewness of the dollar return over horizon h, using the calculation of the skewness of the annual IBM dollar return from (hypothetical) daily IBM dollar return data over the start-2001 to end-2005 period (a total of 1,260 daily observations) as an example:

1. We collect data on an asset's short-horizon dollar returns over a sample window. In the IBM example, those data contain the 1,260 daily IBM dollar returns over the sample window ranging from start-2001 to end-2005.

2. We use Politis and Romano's (1994) block bootstrap to draw a bootstrap sample of short-horizon returns containing a sufficient number of short-horizon returns to be able to compound those up to the h horizon. In the IBM example, assuming that a year features 252 trading days, the bootstrap sample would thus contain 252 daily IBM dollar returns taken from the 1,260 daily dollar returns. To form the bootstrap sample:

   (a) We draw a short-horizon return from the sample window, ensuring that each short-horizon return is equally likely to be drawn and replacing the drawn return. We add that return to the bootstrap sample. In the IBM example, the drawn return could, for instance, be the 587th out of the 1,260 daily dollar returns.

(b) We next draw a number from the univariate distribution with support [0,1]. If the number is below a threshold p, we add the short-horizon return immediately after the last drawn return to the bootstrap sample and repeat step (ii).[7] In the IBM example, that return would be the 588th out of the 1,260 daily dollar returns. If it is not, we return to step (i). We continue until we have a sufficient number of short-horizon returns in the bootstrap sample (252 in our example).

3. We compound up the bootstrap sample returns to the dollar return over horizon h. In the IBM example, we ensure that the 252 drawn daily IBM dollar returns are gross returns and use their product to create an artificial annual IBM dollar return.

4. We repeat steps (2) and (3) until we have a large sample of horizon h dollar returns. In the example, we may, for instance, create 1,000 artificial annual IBM dollar returns.

5. We apply the sample skewness estimator to the sample of dollar returns over horizon h to obtain an estimate of the skewness of the dollar return over that horizon. In the IBM example, we apply the estimator to the 1,000 artificial annual IBM dollar returns.

To find the threshold p fixing the expected length of a consecutive block of observations in step 2(b), we follow Politis and White (2004) and set p equal to the value minimizing the theoretical mean-squared error of a bootstrap based estimator of the long-run variance of the underlying variable (in our case: the daily dollar return). Specifically, the bootstrap-based estimator estimates the variance using the square root of the number of observations over the sample window multiplied by the variance of the sample mean of the variable calculated from the bootstrap samples. Notwithstanding, we obtain similar skewness estimates by setting $p = 0.95$, in line with Politis and Romano's (1994) result that their block bootstrap's performance is

[7] The return after the final short-horizon return in the sample window is the first return in the window.

largely insensitive to the choice of the threshold p. We provide further details in Appendix B.[8]

In our Monte Carlo simulation exercise and empirical tests, we consistently use daily dollar returns as short-horizon dollar returns, the prior 60 months of data as the sample window, and 1,000 artificial long-horizon dollar returns to implement our block bootstrap estimator.[9] We use the estimator to estimate the skewness of monthly (22 daily returns), quarterly (66 daily returns), and annual (252 daily returns) dollar return. As our estimator (obviously) does not improve upon the sample skewness in estimating daily dollar return skewness, we stick to the sample skewness in estimating skewness over that horizon in our empirical tests.

2.3 A Monte Carlo Simulation Exercise

We conduct a Monte Carlo simulation exercise to study the unbiasedness and efficiency of our block bootstrap estimator and two recent competitors, Fama and French's (2018) and Neuberger and Payne's (2021) estimators. To achieve this goal, we simulate daily asset value paths from a geometric Brownian motion (GBM), stochastic volatility (SV), and stochastic volatility-jump (SVJ) process (see Bates (1996); Andersen et al. (2002); Broadie et al. (2009); and others). We can compactly write these asset value processes as follows:

$$\begin{aligned} dS(t) &= \alpha S(t)dt + S(t)\sqrt{V(t)}dW^S(t) + d\left(\sum_{j=1}^{N(t)} S(\tau_{j-})[e^{Z_j^s} - 1]\right) - \lambda\bar{\mu}S(t)dt, \quad (1) \\ dV(t) &= \kappa_v(\theta_v - V(t))dt + \sigma_v\sqrt{V(t)}dW^v(t), \quad (2) \end{aligned}$$

where $S(t)$ and $V(t)$ are, respectively, the asset value and asset variance at time t; α is the asset value drift rate; κ_v is the variance mean-reversion parameter; θ_v is the long-run variance; and σ_v is the volatility of variance. Conversely, $W^S(t)$ and $W^v(t)$ are two Brownian motions with

[8] We are grateful to Andrew Patton for distributing a Matlab code calculating Politis and White's (2004) optimal p value. Importantly, that code incorporates Patton et al.'s (2009) correction of the errors in the variance formulas originally used in Politis and White's (2004) derivations.

[9] We checked that the usage of 1,000 artificial long-horizon dollar returns is sufficient to obtain reasonably accurate bootstrap estimates. In particular, using 2,000 artificial returns to recalculate the bootstrap estimates for five random cross-sections of real-world U.S. stock returns, we find that the original and recalculated monthly (quarterly) [annual] estimates share a cross-sectional correlation above 0.95 (0.93) [0.90].

correlation coefficient equal to ρ (i.e., $dW^S(t)dW^v(t) = \rho dt$). Finally, $N(t)$ is an independent Poisson process with intensity λ, $Z_j^s \sim N(\mu_z, \sigma_z^2)$, and $\bar{\mu} = e^{\mu_z + \sigma_z^2/2} - 1$. While the SV process imposes $\lambda = 0$ (no jumps), the GBM process further imposes $\kappa_v = \sigma_v = 0$ (constant variance).[10]

Using standard values, we set the initial asset value, $S(0)$, to 30, the initial volatility, $\sqrt{V(0)}$, to 0.30, and $\alpha = 0.12$ in our base case specification. As we are ultimately interested in the skewness of *single* stocks, we employ the SV and SVJ process parameter estimates of Pollastri et al. (2022), who rely on a Monte Carlo Markov chain (MCMC) approach to obtain the estimates for all single stocks included in the S&P 100 index.[11] Specifically, we take the average estimate for each parameter taken over all single stocks as the basecase value, which yields $\kappa_v = 7.50$, $\theta_v = 0.30$, $\sigma_v = 0.40$, $\rho = -0.25$, $\mu_z = 0.01$, $\sigma_z = 0.05$, and $\lambda = 9.00$.[12] We then separately vary either one or two parameters from $\sqrt{V(0)}$, σ_v, ρ, μ_z, and σ_z, choosing low and high values of 0.15 and 0.45 for $\sqrt{V(0)}$ and the 2.5th and 97.5th percentiles of Pollastri et al.'s (2022) estimates for the others.[13] Given a stochastic process and parameter value set, we further apply the moment-generating functions of the process to calculate the true skewness of the dollar return from time 0 to one month (monthly return), three months (quarterly return), and a year (annual return) later, $S(t)/S(0)$. See Appendix C for further details.

For each stochastic process and parameter value set, we calculate the mean estimate and standard error of the skewness of the monthly, quarterly, and annual dollar returns produced by our block bootstrap, Fama and French's (2018) simple bootstrap, and Neuberger and Payne's (2021) closed-form estimator. To do so, we first simulate 10,000 daily asset-value paths per

[10] We acknowledge that our Monte Carlo simulation evidence explicitly depends on stochastic processes unable to generate time-varying conditional non-Gaussianity. Models able to produce such non-Gaussianity (and thus greater time-series variation in skewness) include the bad environment–good environment (BEGE) model of Bekaert et al. (2015). Given that our focus is on cross-sectional variation in skewness, we believe that we are not overly restrictive by relying on conditional Gaussian stochastic processes.

[11] We are indebted to Professor Michael Johannes for pointing us to Pollastri et al. (2022).

[12] In comparison to stock indexes, single stocks thus tend to have a slightly higher variance mean reversion, a higher volatility of variance, and a less negative asset value-volatility correlation. Moreover, they also tend to jump more, with, however, a positive (rather than negative) mean jump size and a higher jump volatility. See, for example, Eraker (2004), Broadie et al. (2009), Hurn et al. (2015), and Jacobs and Liu (2018).

[13] We highlight that whereas a higher volatility of variance and a lower (i.e., more negative) correlation between asset value and volatility produce stronger deviations from the i.i.d. assumption, variations in the mean jump size and jump volatility do not do so because the asset value jumps occur independently.

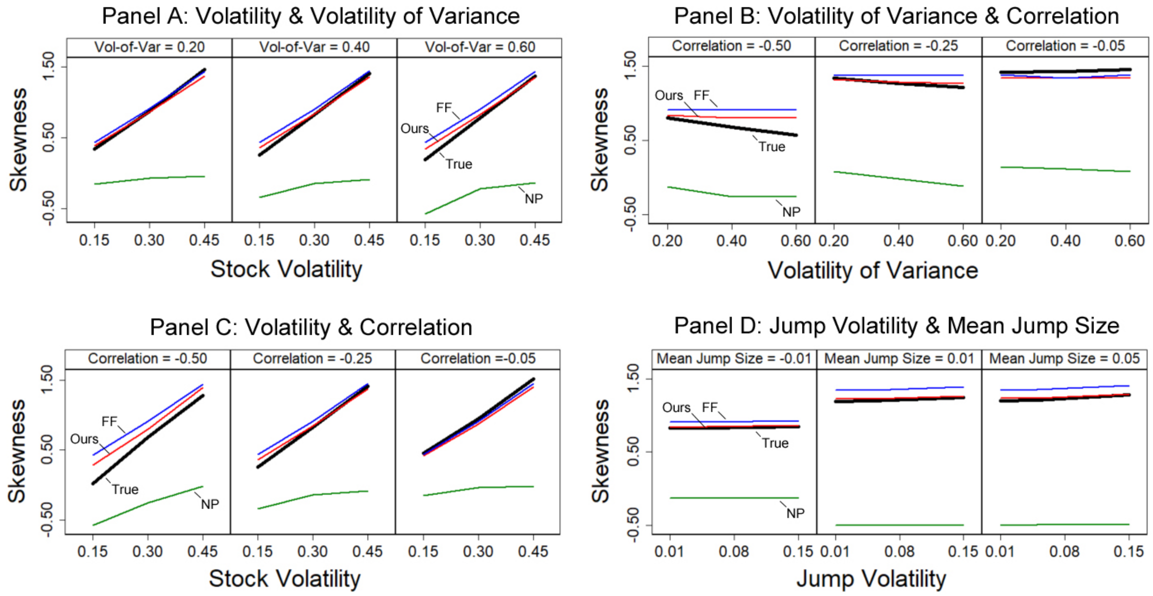

Figure 1. Monte Carlo Simulation Exercise Outcomes. The figure plots the true value (True) and the mean estimates of our block bootstrap (Ours), Fama and French's (2018) simple bootstrap (FF), and Neuberger and Payne's (2021) closed-form (NP) estimator of the skewness of the annual dollar return in the SV (Panels A to C) and SVJ (Panel D) worlds. We describe the basecase parameters in Section 2.3. In Panels A to D, we allow for simultaneous variations in two parameters out of asset volatility, the volatility of variance, the asset value-volatility correlation, the mean jump size, and jump volatility.

process and parameter value set. We then separately apply each of the three skewness estimators to the data from every single path, yielding nine estimates (three estimators times three return horizons) per path and 90,000 estimates in total. We follow the description in Section 2.2 in implementing our block bootstrap estimator, whereas we implement Fama and French's (2018) estimator exactly like ours, except for setting $p = 0$. We follow the description in Appendix A in implementing Neuberger and Payne's (2021) estimator. Finally, we compute the mean, the absolute bias (i.e., the absolute value of the mean minus the true value), and the standard deviation (standard error) of the estimate for each estimator and return horizon.

In Table 1 and Figure 1, we present the simulation exercise outcomes for annual dollar returns. In line with Bessembinder (2018), Panel A of the table suggests that in a GBM world without return dependencies, the skewness of the annual dollar return strongly rises with volatility $\sqrt{V(0)}$ because of the compounding effect, from 0.46 to 1.53. As our estimator and Fama and French's (2018) estimator, however, consider the compounding effect, they both yield almost unbiased and efficient estimates. Notwithstanding, Fama and French's (2018) estimator is slightly less biased and slightly more efficient than ours. More specifically, while the average absolute bias (standard error) of our estimator is about 0.07 (0.26), the corresponding number for Fama and French's (2018) is 0.04 (0.23). In contrast, because Neuberger and Payne's (2021) estimator does not consider the compounding effect, it yields a significantly biased estimate, with the average absolute bias (standard error) equal to 0.98 (0.14). On balance, our evidence in Panel A of Table 1 thus favors Fama and French's (2018) estimator over the other two estimators in a GBM world in which there are no return dependencies.

However, as there are strong return dependencies in the real world, Panels B and C of the table and the figure next turn to contrasting the skewness estimators in the SV and SVJ worlds. Starting with the base case values in either world, both table and figure suggest that while our estimator yields an essentially unbiased estimate, Fama and French's (2018) yields a markedly more biased estimate. Looking into the SV world, Panel B, for example, reveals that the absolute biases of our and Fama and French's (2018) estimator are now 0.01 and 0.08, respectively. Notwithstanding the reversal in unbiasedness, our estimator continues to attract a standard error only 0.03 higher than that of Fama and French (2018). Amplifying the return dependencies (through, e.g., raising the volatility of variance σ_v and/or lowering the asset value-volatility correlation ρ), the absolute bias spread can widen to 0.15, without accompanying changes in the spread in standard errors (see, e.g., the $\sigma_v = 0.60$ and $\rho = -0.50$ case in Panel B). Despite these results, we acknowledge that once return dependencies become extreme, our estimator also struggles to yield a close-to-unbiased estimate. In the $\sigma_v = 0.60$ and $\rho = -0.50$ case, the absolute bias of our estimator is, for example, 0.19. Finally, turning to Neuberger and Payne's (2021) estimator, the table and figure suggest that it yields as biased estimates in the SV and SVJ worlds as in the GBM world. Setting volatility $\sqrt{V(0)}$ to 0.30, its absolute bias is, for example, again close to 0.95 across all parameter value sets.

Overall, our evidence in Panels B and C of Table 1 and Figure 1 favors our estimator over the other two in worlds with notable return dependencies (as, e.g., ours), especially when unbiasedness is more important than efficiency. While the average absolute bias of our estimator is only about one-third of Fama and French (2018), its average standard error is approximately 19% higher. In comparison, Neuberger and Payne's (2021) estimator yields significantly more biased estimates than ours and Fama and French's (2018).[14] Looking into monthly and quarterly returns, Internet Appendix Tables IA.1 and IA.2 further support the conclusions drawn from the annual returns in Table 1 and Figure 1.

3 Calculating the Skewness Proxy Contenders

In this section, we introduce the skewness proxies used in empirical asset pricing studies and evaluated by us. The skewness proxies consist of Boyer et al.'s (2010) least-squares prediction of daily dollar return skewness; a modified version of Ghysels et al.'s (2016) quantile regression prediction of annual dollar return skewness; historical daily dollar return skewness; Conrad et al.'s (2014) logit model probability of a stock's one-year ahead log return exceeding 100%; and Bali et al.'s (2011) historical maximum daily dollar return. In short, we detail the construction of the variables underlying the skewness proxies in Appendix D.

3.1 Boyer et al.'s (2010) Least-Squares Forecast

Boyer et al. (2010) use a cross-sectional least-squares prediction of the skewness of the daily dollar return as skewness proxy ($OLSSkew$). To do so, they first compute the skewness coefficient of a stock from daily dollar return data over the 60 months prior to month t. Next, they conduct a cross-sectional least-squares regression of that coefficient on predictor variables measured until the start of the 60-month period. Among these predictor variables are historical volatility and the historical skewness coefficient, the intermediate-term past return ("momentum"),

[14] We highlight that our absolute bias estimates for Neuberger and Payne's (2021) estimator align with those reported in their own Table 1. Specifically, assuming that the CRSP market return's sample moments are equal to its population moments, they show that while the "true" skewness of the annual CRSP market dollar return is –0.07, the "true" value of their estimator is –0.93, yielding an absolute bias of 0.86. Despite examining single stocks (and not an index), we find a similar average absolute bias.

and average share turnover. They also use the 17 Fama-French (FF) industry dummies plus two market size dummies. Finally, they combine the regression estimates with the predictor variable values at the end of month t, yielding a forward-looking direct estimate of the skewness of the daily dollar return. See Appendix D for more details on the predictor variables.

3.2 Ghysels et al.'s (2016) Quantile-Regression Forecast

Motivated by Ghysels et al.'s (2016) time-series quantile-regression estimator of the skewness of the dollar return, we also investigate an analogous estimator adapted to our cross-sectional setting (*QuantileSkew*).[15] To form this estimator, we run panel data quantile regressions of the annual dollar return on predictor variables measured until the start of the return horizon over the past 20 years of monthly data, fitting the first, fifth, tenth, 25th, 50th, 75th, 90th, 95th, and 99th quantiles of the return distribution. If a stock is delisted over an annual horizon, we replace its final return with its delisting return and compound up until the delisting date to create the annual return. We choose Boyer et al.'s (2010) variables as predictor variables, excluding the market size dummies but adding (continuous) market size, company age, asset tangibility, sales growth, the book-to-market ratio, share issuances, asset growth, and total profitability. See Appendix D for more details on the predictor variables. Finally, we combine the regression estimates with the values of the predictor variables at the end of the 20-year estimation window to yield forward-looking estimates of a stock's one-year-ahead dollar return quantiles.

To convert the quantile estimates into the skewness proxy, we assume that a stock's dollar return distribution is uniform between two consecutive quantiles and ignore density outside the extreme quantiles. We then calculate the first three conditional moments of a stock's dollar

[15] In a quantile regression, we model the conditional quantiles of a random variable as a linear function of exogenous variables. See Koenker (2005) for more technical details about quantile regressions.

return from time t to $t+T$, denoted by $R_{t,t+T}$, as follows:

$$\hat{E}[R_{t,t+T}|\hat{q}_{\tau_{(j-1)}} \leq R_{t,t+T} < \hat{q}_{\tau_{(j)}}] = \frac{\hat{q}_{\tau_{(j-1)}} + \hat{q}_{\tau_{(j)}}}{2}, \tag{3}$$

$$\hat{E}[R^2_{t,t+T}|\hat{q}_{\tau_{(j-1)}} \leq R_{t,t+T} < \hat{q}_{\tau_{(j)}}] = \frac{\hat{q}^2_{\tau_{(j-1)}} + \hat{q}_{\tau_{(j-1)}} \times \hat{q}_{\tau_{(j)}} + \hat{q}^2_{\tau_{(j)}}}{3}, \tag{4}$$

$$\hat{E}[R^3_{t,t+T}|\hat{q}_{\tau_{(j-1)}} \leq R_{t,t+T} < \hat{q}_{\tau_{(j)}}] = \frac{\hat{q}^3_{\tau_{(j-1)}} + \hat{q}^2_{\tau_{(j-1)}} \times \hat{q}_{\tau_{(j)}} + \hat{q}_{\tau_{(j-1)}} \times \hat{q}^2_{\tau_{(j)}} + \hat{q}^3_{\tau_{(j)}}}{4}, \tag{5}$$

where $\hat{q}_{\tau_{(j)}}$ is an estimate of the τth quantile of $R_{t,t+T}$, $j = \{1, 2, \ldots, J\}$ indexes the estimated quantiles in ascending order (so $j = 1$ refers to the lowest estimated quantile), J is the number of estimated quantiles, and $\hat{E}[.]$ is the estimated conditional expectation. In turn, the stock's first three unconditional return moments can be estimated as follows:

$$\hat{E}[R^n_{t,t+T}] = \sum_{j=2}^{J} \frac{F^{-1}(\hat{q}_{\tau_{(j)}}) - F^{-1}(\hat{q}_{\tau_{(j-1)}})}{F^{-1}(\hat{q}_{\tau_{(J)}}) - F^{-1}(\hat{q}_{\tau_{(1)}})} \hat{E}[R^n_{t,t+T}|\hat{q}_{\tau_{(j-1)}} \leq R_{t,t+T} < \hat{q}_{\tau_{(j)}}], \tag{6}$$

where n is equal to one (two) [three] for the first (second) [third] moment. Essentially, Equation (6) estimates the unconditional moments by approximating the integrals taken over the conditional moments. However, because there is density mass outside the extreme estimated quantiles, we scale the approximated integral by $F^{-1}(\hat{q}_{\tau_{(J)}}) - F^{-1}(\hat{q}_{\tau_{(1)}})$ to ensure that density mass sums up to unity. Finally, we plug the approximated unconditional moments from Equation (6) into the formula for the skewness coefficient to obtain the skewness proxy:

$$QuantileSkew_t = \frac{\hat{E}[R^3_{t,t+T}] - 3\hat{E}[R_{t,t+T}](\hat{E}[R^2_{t,t+T}] - \hat{E}[R_{t,t+T}]^2) - \hat{E}[R_{t,t+T}]^3}{(\hat{E}[R^2_{t,t+T}] - \hat{E}[R_{t,t+T}]^2)^{\frac{3}{2}}}. \tag{7}$$

Similar to *OLSSkew*, *QuantileSkew* is a forward-looking direct estimate of the skewness of dollar returns, focusing, however, on the skewness of annual (not daily) returns.[16]

[16] We also used Kelly's measure of skewness, defined as the 90th plus the tenth quantile minus two times the median, to convert our quantile estimates into a skewness proxy. However, as the skewness proxy derived from Equation (7) strongly dominates the proxy derived from Kelly's measure in terms of predictability, we decided to report only the results from the skewness proxy derived from Equation (7).

As *QuantileSkew* has not been used in prior studies, at least not in its cross-sectional version, we offer more details about its construction in the Internet Appendix. More specifically, Internet Appendix Table IA.3 offers the average estimates and fractions of significant estimates obtained from the underlying quantile regressions. Conversely, Internet Appendix Table IA.4 suggests that the forward-looking annual dollar return quantiles generated through these regressions are well calibrated in the full sample and subsamples.[17]

3.3 Historical Skewness

Amaya et al. (2015) and Bali et al. (2016) use the skewness coefficient calculated from a stock's historical short (i.e., intraday or daily) dollar returns as skewness proxy. Spurred by them, we also consider such a proxy calculated over the previous 60 months of data (*HistoricalSkew*).[18] *HistoricalSkew* is a backward-looking direct estimate of the daily dollar return skewness.

3.4 Conrad et al.'s (2014) Logit Model Forecast

Conrad et al. (2014) use an estimate of the probability that a stock's twelve-month-ahead log return exceeds 100% as skewness proxy (*LogitSkew*). To form that proxy, they estimate a logit model of a dummy variable equal to one if a stock's log return from start-July of year t to end-June of year $t+1$ is above 100% (translating into a dollar return above 170%) and else zero, on predictor variables measured until the end of June of year t. They use market size, momentum, company age, asset tangibility, and sales growth as predictor variables. They further use historical volatility, historical skewness, and share turnover, but calculate those differently from Boyer et al. (2010). See Appendix D for more details on the predictor

[17] In the Internet Appendix, we further study quantile-regression proxies capturing the skewness of the daily, monthly, and quarterly dollar return. More specifically, Internet Appendix Table IA.5 shows that adapting the return horizon of the skewness proxy to the return horizon of the forecasting target improves predictability, whereas Internet Appendix Table IA.6 reveals that the short-horizon-return quantile-regression proxies are more significantly negatively priced than the long-horizon-return quantile-regression proxies.

[18] Using the skewness of daily returns over the prior month or monthly returns over the prior 60 months, we obtain asset pricing conclusions in line with those reported later. We thank Scott Murray for prompting us to look into alternative data frequencies and historical windows in calculating *HistoricalSkew*.

variables. Recursively estimating the logit model starting from June 1951 and using only June observations, they finally combine the model estimates with the values of the predictor variables over the twelve months directly after the estimation period. *LogitSkew* is thus a forward-looking indirect estimate of the skewness of the annual (either dollar or log) return.

3.5 Bali et al.'s (2011) Maximum Return

Bali et al. (2011) use the maximum of a stock's daily dollar return over the prior month to proxy for the stock's propensity to yield a "lottery-like return" (*MaxRet*). While *MaxRet* is not designed to capture skewness, we can nonetheless interpret it as an indirect backward-looking estimate of the skewness of the daily (either dollar or log) return.

4 Data Sources

We obtain stock data from CRSP, accounting data from Compustat, and data on the FF benchmark factors, the 17 FF industry portfolios, and the risk-free rate of return from Kenneth French's website.[19] We study the common stocks (share codes: 10 and 11) traded on the NYSE, AMEX, and NASDAQ. We exclude financial (SIC codes: 6000-6999) and utility (4900-4949) stocks. To avoid microstructure biases, we further exclude stocks with prices below $5 at the end of the forecast production/portfolio formation period. To ensure our data were available to real investors, we use the accounting variable values from the fiscal year ending in calendar year $t-1$ from start-July of year t to end-May of year $t+1$. We winsorize all variables involving accounting data at the 0.5th and 99.5th percentiles per month. We run our forecasting tests over the period for which we have data on all the skewness proxies (January 1988 to December 2016). To be fair to the studies advocating the skewness proxies, we run our asset pricing and decomposition tests over sample periods whose starting dates align with those in the studies, but whose ending date is always December 2016. Specifically, we start the sample data for the

[19] The URL address is <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html>.

later tests in January 1988 for *OLSSkew* and *QuantileSkew*, July 1963 for *HistoricalSkew* and *MaxRet*, and January 1972 for *LogitSkew*.

5 Predicting Realized Skewness

In this section, we study the ability of the skewness proxies introduced in Section 3 to predict the realized skewness of future short- and long-horizon dollar returns estimated using our block bootstrap estimator in Section 2. First, we present descriptive statistics and correlations for the realized skewness estimates and skewness proxies. Next, we evaluate how well the skewness proxies calculated using data until time t predict the realized skewness of the dollar return from time t to one day, one month, three months, and one year later.

5.1 Descriptive Statistics and Rank Correlations

Table 2 presents descriptive statistics on our estimates of the realized skewness of the daily, monthly, quarterly, and annual dollar return (Panel A) and the skewness proxies from prior studies (Panel B). Consistent with our later forecasting tests, we calculate the realized skewness estimates using data from the start of month $t+1$ to the end of month $t+60$ and the skewness proxies using data until the end of month t.[20] We exclude observations for which we are unable to calculate values for all realized skewness estimates and skewness proxies.

Panel A of Table 2 suggests that the realized dollar return skewness of the average single stock is positive and becomes more positive with the return horizon, consistent with Bessembinder's (2018) argument that compounding induces right skewness in stock returns. The skewness of the daily dollar return of the average (median) stock is, for example, 0.41 (0.30), whereas the skewness of that stock's annual dollar return is 1.47 (1.14). The panel further reveals that the cross-sectional volatility in the realized skewness estimates also rises with the return horizon,

[20] Our timing conventions ensure that we can interpret each skewness proxy as a (direct or indirect) forecast of the skewness of the dollar return from time t to some later time formed from only information available at time t. They further ensure that we can interpret each realized skewness estimate as an estimate of the realized dollar return skewness obtained using data over the window from time t to 60 months later.

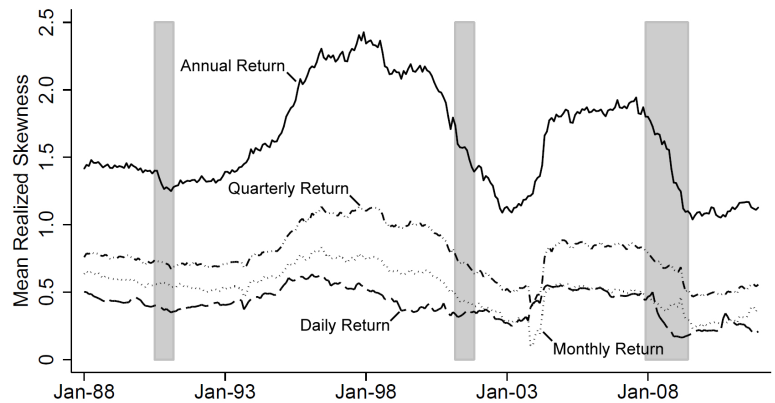

Figure 2. Evolution of Mean Realized Stock Skewness Over Time. The figure plots the cross-sectional average of our estimates of the realized skewness of the daily, monthly, quarterly, and annual dollar return over our sample period. The realized skewness estimates are calculated from the 60 months of daily data starting from the month shown in the figure. The gray areas are NBER-defined recession periods.

from 0.60 for the skewness of the daily dollar return to 1.22 for the skewness of the annual dollar return. Plotting the simple cross-sectional mean of each realized skewness estimate over our sample period, Figure 2 shows that, except for several months in 2004, the average realized skewness consistently rises with the return horizon. Finally, the figure reveals that all realized skewness estimates tend to drop over NBER recessions (gray bars).

Turning to the skewness proxies, Panel B shows that those designed to directly fit skewness, *OLSSkew*, *QuantileSkew*, and *HistoricalSkew*, attract positive mean values, in agreement with the positive mean values for the realized skewness estimates in Panel A. Interestingly, however, *OLSSkew* and *QuantileSkew* yield notably lower cross-sectional volatilities than *HistoricalSkew*, in line with these being statistical predictions of future skewness and the other being a historical realization. The mean values of the indirect skewness proxies, *LogitSkew* and *MaxRet*, suggest that the average stock has a close to one percent probability of yielding a lottery return, and that its maximum daily return over the prior month is approximately six percent.

Table 3 shows the Spearman rank correlations across the realized skewness estimates and skewness proxies. The first lesson to take away from the table is that the realized skewnesses of dollar returns computed over alternative horizons are only imperfectly correlated, with the correlation coefficient dropping with the difference in the return horizon. While the skewnesses of the daily and monthly dollar returns, for example, still share a correlation coefficient of 0.51, the coefficient between the skewnesses of the daily and annual returns is only 0.34. Another lesson to draw is that while *QuantileSkew*, *LogitSkew*, and *MaxRet* share far more positive correlation coefficients with the realized skewnesses of longer-horizon dollar returns, in line with at least the first two being fitted to longer-horizon returns, the coefficients of *OLSSkew* and *HistoricalSkew* form no discernable patterns with the return horizon used to calculate the realized skewness estimates. The final lesson is that the skewness proxies are often highly but never close to perfectly correlated, implying that they contain different information about the future realized skewness of dollar returns over alternative horizons.

5.2 Forecasting Tests

We next gauge the ability of the skewness proxies to predict cross-sectional variation in the future realized skewness of short- and long-horizon dollar returns. At the end of each sample month t, we first sort all sample stocks into portfolios according to one of the skewness proxies calculated from data until that date, using the tenth, 20th, 40th, 60th, 80th, and 90th percentiles as breakpoints. Separately for each portfolio and sample month t, we next form equally weighted averages of the future realized skewness estimates of the daily, monthly, quarterly, and annual dollar returns, where we calculate the future realized skewness estimates using data from start-month $t+1$ to end-month $t+60$. Separately for each portfolio, we take the simple average of the equally weighted averages over our sample period.

Table 4 presents the portfolio sort forecasting exercise results, with Panels A to D focusing on the skewness of daily, monthly, quarterly, and annual returns, respectively. The table further reports the spreads in the simple realized skewness estimate averages across the extreme portfolios, plus the accompanying t-statistics (in parentheses). To account for the overlapping nature of our data, we calculate the t-statistics from Newey and West (1987) standard errors with a 60-month lag length. The table shows that all skewness proxies are statistically significant predictors of future realized skewness over all return horizons. While we are unable to judge the absolute performance of the skewness proxies (as we do not know how much of the cross-sectional variation in future realized skewness is predictable), it is obvious that their relative performance varies across return horizons. For example, while Panel A shows that *OLSSkew* and *LogitSkew* perform best over the daily horizon, with spreads over the portfolios equal to 0.49 and 0.54, Panel B reveals that *QuantileSkew* and *LogitSkew* perform best over the annual horizon, with spreads equal to 1.47 and 1.56, respectively.

To supplement the portfolio sort forecasting exercises, we next perform stock-level Mincer-Zarnowitz (1969) regressions of the future realized skewness of the daily, monthly, quarterly, and annual dollar returns on each skewness proxy and a constant, where we again calculate future realized skewness using data from start-month $t+1$ to end-month $t+60$ and the skewness proxies using data until the end of month t. For each realized skewness estimate-skewness proxy combination, we first conduct cross-sectional regressions separately by sample month and then average the constant estimates, slope coefficient estimates, and model diagnostics over our sample period. While the Mincer-Zarnowitz (1969) regressions allow us to assess how much of the cross-sectional variation in a realized skewness estimate is captured by a skewness proxy through their R-squared values, they also allow us to formally test for unbiasedness, with an unbiased skewness proxy yielding a mean constant estimate insignificantly different from zero and a mean slope coefficient estimate insignificantly different from one.

Table 5 presents the regression results, with Panels A to D again focusing on the realized skewness of daily, monthly, quarterly, and annual dollar returns, respectively. While plain numbers are average estimates, those in parentheses are Newey-West (1987) t-statistics with a 60-month lag length. The table demonstrates that the Mincer-Zarnowitz (1969) regressions yield conclusions in complete agreement with the portfolio sort forecasting exercises. Although all skewness proxies relate significantly positively to future realized skewness over all return horizons, their relative ability to capture cross-sectional variation in future realized skewness varies across horizons. For example, Panel A reveals that *OLSSkew* and *LogitSkew* yield R-squared values of approximately 7% and 8% over the daily horizon, respectively, and no other skewness proxy yields an R-squared above 5% over that horizon. Conversely, while Panel D establishes that *QuantileSkew* and *LogitSkew* both yield R-squareds of approximately 18% over the annual horizon, no other skewness proxy yields an R-squared above 10% over that horizon.[21]

The table further establishes that the skewness proxies designed to directly capture the skewness of the daily dollar return, *OLSSkew* and *HistoricalSkew*, are significantly biased over all return horizons, including, surprisingly, the daily horizon. When forecasting future realized skewness, an unbiased predictor of skewness is expected to have a mean constant estimate that is not significantly different from zero and a mean slope coefficient estimate close to one. Notwithstanding, Panel A reveals that both *OLSSkew* and *HistoricalSkew* yield a mean constant estimate significantly above zero and a mean slope coefficient estimate significantly below one. In comparison, the skewness proxy designed to directly predict the skewness of the annual dollar return, *QuantileSkew*, is more weakly biased, at least over the annual return horizon. While Panel D shows that this proxy also produces a mean constant estimate significantly above zero, its mean slope coefficient estimate is only slightly below one, with the deviation of the mean estimate from one being only mildly significant (unreported).

In the Internet Appendix, we further examine the ability of the skewness proxies to predict the realized skewness of returns that is not attributable to the compounding effect over the same horizons. In particular, Internet Appendix Table IA.8 offers strong evidence that the lion's share of all proxies' predictive power comes from them capturing that effect, especially over the longer return horizons over which the effect is relatively more important.

Overall, this section offers strong evidence that the skewness of dollar returns only imperfectly scales with the return horizon, as clarified by the far-below-one correlations between the realized skewness estimates in Table 3. Notwithstanding, the skewness proxies advocated in the prior empirical asset pricing literature are all significant predictors of the realized skewness

[21] In the Internet Appendix, we also report the results from Mincer-Zarnowitz (1969) regressions on the joint set of skewness proxies. To be specific, Internet Appendix Table IA.7 suggests that the skewness proxies often embed independent information about future realized skewness over different horizons. While *QuantileSkew*, *LogitSkew*, and *MaxRet* are, for example, jointly significant in predicting the realized skewness of the annual dollar return, *OLSSkew* and *HistoricalSkew* are both insignificant over that horizon.

of the dollar return over short and long horizons. In line with intuition, the proxies designed to capture the skewness of short-horizon dollar returns (e.g., *OLSSkew*) are generally more successful in predicting skewness over shorter horizons, whereas those designed to capture the skewness of long-horizon returns (e.g., *QuantileSkew*) are generally more successful in predicting skewness over longer horizons. The two exceptions are *LogitSkew* and *MaxRet*. While *LogitSkew* performs best over all horizons, *MaxRet* performs better over longer horizons despite being derived from short-horizon (i.e., daily) returns. *MaxRet*'s higher predictive ability over longer horizons can be explained by it being a much stronger proxy for stock volatility than skewness, as shown by, for example, Hou and Loh (2016). As stock volatility critically conditions the compounding effect in skewness (see, e.g., our simulation evidence in Section 2.3), it is a strong determinant of dollar return skewness, especially over longer horizons, over which the compounding effect is more important.

6 The Pricing of Skewness

In this section, we investigate whether the skewness proxies advocated in prior studies and evaluated by us price stock returns because they contain information about the skewness of future dollar returns over alternative horizons. To do so, we first verify that the skewness proxies continue to be priced over our updated sample periods. Next, we decompose each proxy into the sum of a component reflecting the expectation of the skewness of the future dollar return over the daily, monthly, quarterly, or annual horizon and an orthogonal component and then separately evaluate the stock pricing power of those two components.

6.1 The Skewness Proxy Premiums Over Our Sample Periods

We first use portfolio sorts to establish whether the skewness proxies continue to price stocks over our updated sample periods. At the end of each sample month t, we sort the sample stocks into decile portfolios according to one of the skewness proxies calculated using data until then. We value-weight the portfolios and hold them over month $t + 1$. Next, we form a spread portfolio long the top and short the bottom portfolio. To adjust for risk, we regress the spread portfolio return on the excess market return (MKT; CAPM alpha) or MKT, SMB, and HML (FF3 alpha), and report the intercept. Finally, we compute Newey-West (1987) t-statistics with a twelve-month lag length for the spread portfolio mean returns and alphas.

Table 6 presents the portfolio sort results. While the plain numbers are mean returns and alphas, the numbers in parentheses are t-statistics. The table suggests that, in line with the literature, mean returns decline significantly over all decile portfolios, except for the *QuantileSkew* and *HistoricalSkew* portfolios. Interestingly, however, the decline is only close to monotonic over the *OLSSkew* portfolios, whereas it is close to zero over the earlier *LogitSkew* or *MaxRet* portfolios and only more pronounced over the latter. In agreement with this, the *OLSSkew*, *LogitSkew*, and *MaxRet* spread portfolios produce significantly negative mean returns and alphas. For example, the *OLSSkew* spread portfolio produces a mean monthly return of –1.03% (t-statistic: –2.50), a lower CAPM alpha of –1.10% (t-statistic: –2.88), and an even lower FF3 alpha of –1.28% (t-statistic: –3.23). Conversely, while the *QuantileSkew* spread portfolio produces an insignificant mean return but significantly negative CAPM and FF3 alphas, the *HistoricalSkew* spread portfolio does not produce significant mean returns or alphas.

In Table 7, we supplement our portfolio sort evidence with the results from FM regressions of single-stock returns over month $t + 1$ on each skewness proxy calculated using data until the end of month t. Whereas Panel A relies on a stock's dollar return as regressant, Panel B relies on the same return minus the dollar return of the value-weighted size and book-to-market portfolio to which the stock belongs (the so-called Daniel, Grinblatt, Titman, and Wermers (DGTW; 1997) return).[22] An advantage of using the DGTW return is that it parsimoniously

[22] We follow Fama and French (1993) in forming the value-weighted size and book-to-market portfolios. At the end of June in each calendar year t, we thus measure a stock's market size and the ratio of its book equity value from the fiscal year ending in calendar year $t - 1$ to its market size at the end of calendar year $t - 1$. We calculate the book equity value as total assets minus total liabilities plus deferred taxes (zero if missing) minus preferred stock (zero if missing). Using only NYSE stocks, we next separately derive quintile breakpoints for both market size and the book-to-market ratio. Using the intersection of the two sets of breakpoints, we sort all stocks (including financial and utility stocks) into 25 portfolios. We finally value-weight the 25 portfolios and hold them from start-July of calendar year t to end-June of calendar year $t + 1$.

controls for market size and book-to-market effects in stock returns. The table reveals that the FM regressions yield conclusions that are in close agreement with the portfolio sorts. In particular, the regressions also suggest that *OLSSkew*, *LogitSkew*, and *MaxRet* are significantly negatively priced in dollar and DGTW returns. However, deviating from our portfolio sort evidence, *QuantileSkew* is never significantly negatively priced, even in DGTW returns.

Taken together, our evidence that *OLSSkew*, *LogitSkew*, and *MaxRet* robustly negatively price stocks aligns with Boyer et al. (2010), Bali et al. (2011), and Conrad et al. (2014). By contrast, our evidence that *HistoricalSkew* only prices stocks more weakly aligns with Bali et al. (2016), who also find that most of their historical skewness estimates are not priced.

6.2 Why Do the Skewness Proxies Price Stocks?

We next examine whether some of the skewness proxies price stocks because they predict the skewness of the dollar return over one or more horizons. To do so, we rely on a slightly modified version of Hou and Loh's (2016) methodology and decompose each skewness proxy into the sum of a component reflecting the expectation of the realized skewness of the dollar return over some horizon, and an orthogonal component. As explained below, if a skewness proxy predicts skewness over some horizon and the expectation of skewness over that horizon is priced, the expectation component of the skewness proxy would be priced with the same sign as the expectation of skewness in our decomposition methodology.

To decompose the skewness proxies, we first conduct the cross-sectional regression:

$$SkewnessProxy_{i,t-1} = a_{t-1} + \delta_{t-1} ExpectedFutureSkewness_{i,t,t+T} + \mu_{i,t-1}, \tag{8}$$

where $SkewnessProxy_{i,t-1}$ is one of the five skewness proxies for stock i calculated using data until end-month t, $ExpectedFutureSkewness_{i,t,t+T}$ is the expected skewness of stock i's dollar return from start-month t to end-month $t+T$ (so that T is the return horizon), a_{t-1} and δ_{t-1} are parameters, and $\mu_{i,t-1}$ is the residual. Next, we treat $\delta_{t-1} ExpectedFutureSkewness_{i,t,t+T}$ as the expected skewness component, and $a_{t-1}+\mu_{i,t-1}$ as the residual component. While we separately use the components in FM regressions to contrast their pricing power, we further rely on them to decompose the $SkewnessProxy_{i,t-1}$ premium obtained from an FM regression of stock i's return over month t, $R_{i,t}$, on the same skewness proxy, γ_t, as follows:

$$
\begin{aligned}
\gamma_t &= \frac{\text{cov}[R_{i,t}, SkewnessProxy_{i,t-1}]}{\text{var}[SkewnessProxy_{i,t-1}]} \\
&= \frac{\text{cov}[R_{i,t}, a_{t-1} + \delta_{t-1} ExpectedFutureSkewness_{i,t,t+T} + \mu_{i,t-1}]}{\text{var}[SkewnessProxy_{i,t-1}]} \\
&= \frac{\delta_{t-1}\text{cov}[R_{i,t}, ExpectedFutureSkewness_{i,t,t+T}]}{\text{var}[SkewnessProxy_{i,t-1}]} + \frac{\text{cov}[R_{i,t}, \mu_{i,t-1}]}{\text{var}[SkewnessProxy_{i,t-1}]} \qquad (9) \\
&= \gamma_t^C + \gamma_t^R, \qquad (10)
\end{aligned}
$$

where γ_t^C is the first and γ_t^R is the second summand of Equation (9). Finally, we interpret the sample mean of γ_t^C/γ_t (γ_t^R/γ_t) as the fraction of the $SkewnessProxy_{i,t-1}$ premium due to the proxy capturing expected future skewness over the T return horizon (other factors).

An immediate problem with estimating regression (8) is that we observe only the ex-post realization of skewness, but not its expectation. While we could rely on the ex-post realization to proxy for the expectation, that strategy creates an attenuation bias. To understand this better, let $RealizedFutureSkewness_{i,t,t+T}$ be the realization of future skewness over the period from t to $t+T$, which we can write as the sum of $ExpectedFutureSkewness_{i,t,t+T}$ and an orthogonal error independent of $SkewnessProxy_{i,t-1}$, $Error_{i,t,t+T}$. We then have the following:

$$
\begin{aligned}
& \frac{\text{cov}(SkewnessProxy_{i,t-1}, RealizedFutureSkewness_{i,t,t+T})}{\text{var}(RealizedFutureSkewness_{i,t,t+T})} \\
= & \frac{\text{cov}(SkewnessProxy_{i,t-1}, ExpectedFutureSkewness_{i,t,t+T})}{\text{var}(ExpectedFutureSkewness_{i,t,t+T}) + \text{var}(Error_{i,t,t+T})} \\
< & \frac{\text{cov}(SkewnessProxy_{i,t-1}, ExpectedFutureSkewness_{i,t,t+T})}{\text{var}(ExpectedFutureSkewness_{i,t,t+T})}, \qquad (11)
\end{aligned}
$$

so that the slope coefficient obtained from the regression of $SkewnessProxy_{i,t-1}$ onto *Realized-FutureSkewness*$_{i,t,t+T}$ is downward-biased compared to that obtained from the regression of

$SkewnessProxy_{i,t-1}$ onto $ExpectedFutureSkewness_{i,t,t+T}$.

To mitigate the attenuation bias, we follow a well-known instrumental variable methodology also used in Black et al. (1972) and Fama and French (1992). To implement this methodology, we sort all stocks at the end of each sample month t into 50 portfolios according to one of the skewness proxies. Separately for each portfolio, we calculate the average of the skewness proxy and the future realized skewness estimate, where we again compute the skewness proxy using data until end-month t and the future realized skewness estimate using data from start-month $t+1$ to end-month $t+60$. Finally, we run a cross-sectional portfolio-level regression of the average skewness proxy on the average future realized skewness estimate. As averaging the future realized skewness estimates over observations with a similar expected future skewness diversifies away the orthogonal error without greatly reducing the expected future skewness variation, the estimates from the portfolio-level regression are likely to be close to those from the (infeasible) stock-level regression of the skewness proxy on expected future skewness. See Black et at. (1972) for the mathematical proof of this claim.

While the instrumental variable methodology allows us to obtain consistent estimates for a_{t-1} and δ_{t-1}, it does not allow us to calculate the expected skewness and residual component because we do not observe stock-level expected future skewness. To obtain an estimate of stock-level expected future skewness, we assume that a monotonic transformation of the skewness proxy is an unbiased predictor of skewness over the chosen horizon. Recognizing that all stocks within each of the 50 portfolios have a nearly identical skewness proxy value, we then take a simple average of the future realized skewness estimates of all stocks in a portfolio to obtain an estimate of their joint expected future skewness. By combining the a_{t-1} and δ_{t-1} estimates with the expected future skewness estimates, we can decompose each skewness proxy into expected skewness and residual components.[23]

[23] While our methodology requires a skewness proxy to be a decent predictor of future realized skewness over some horizon to yield reliable estimates, we stress that proxies with a low predictive ability do not lead us to draw wrong inferences. To see that, assume that a skewness proxy has almost no ability to predict future realized skewness over some horizon. In that case, the slope coefficient from the cross-sectional portfolio-level regression of average skewness proxy value on average future realized skewness, δ_{t-1}, will be strongly biased toward zero. In addition, there is almost no cross-sectional variation in the expected future skewness estimate

Table 8 reports the premium estimates obtained from univariate FM regressions of single-stock returns over month t on either of the two components extracted from each skewness proxy for each return horizon and calculated using data until month-end $t-1$ and the average fractions of the overall skewness proxy premium due to the components. Panels A-E focus on *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, and *MaxRet*, respectively. Conversely, columns (1) and (2) ((3) and (4)) offer the expected skewness (residual) component premium estimates and their t-statistics, while columns (5) and (6) ((7) and (8)) offer the average fractions attributable to the expected skewness (residual) component, γ_t^C/γ_t (γ_t^R/γ_t), and their t-statistics. We consistently calculate t-statistics using Newey-West (1987) standard errors with a twelve-month lag length. To aid interpretation, the panel headings repeat the overall skewness proxy premiums and t-statistics from Panel A of Table 7.

The table suggests that the stock pricing power of the skewness proxies usually originates from them predicting skewness over short horizons but not from them predicting skewness over long horizons. Panel A, for example, reveals that while the *OLSSkew* components predicting the skewness of the daily and monthly dollar returns yield significantly negative monthly premiums of –0.57% and –0.50% (t-statistics: –2.90 and –2.37), all respectively, the corresponding components predicting the skewness of the quarterly and annual dollar returns yield insignificant premiums. In complete agreement, the fraction of the *OLSSkew* premium owing to the expected skewness component markedly declines with the return horizon, from approximately 70% to 51%. We reach similar conclusions for *QuantileSkew* and *LogitSkew* (see Panels B and D, respectively). By contrast, Panel E indicates that the expected skewness component in *MaxRet* is significantly priced over all return horizons, with its premium capturing a larger fraction of the overall premium over longer horizons. Finally, Panel C shows that the expected skewness components in *HistoricalSkew* are never priced.

over our sample stocks. Both effects further reduce the low (absolute and relative) pricing ability of the expected skewness component derived from the skewness proxy (see Equation (9)). In other words, only skewness proxies with high predictive abilities are able to perform well in our decomposition tests.

The table further reveals that the orthogonal components in *OLSSkew*, *LogitSkew*, and *MaxRet* are significantly negatively priced over all return horizons, implying that these skewness proxies contain priced information that does not reflect skewness expectations. In contrast, the orthogonal components in *QuantileSkew* and *HistoricalSkew* are not priced, except for the *HistoricalSkew* component orthogonal to expected skewness over the daily horizon.

In Table 9, we re-evaluate the stock pricing ability of the two components controlling for size and book-to-market effects by repeating the tests in Table 8 using DGTW (and not dollar) returns. The table suggests that this strategy does not materially change our conclusions with two exceptions. First, the components reflecting the skewness of the daily return are always significantly negatively priced, even for *HistoricalSkew*. Second, the controls greatly boost the significance of the *LogitSkew* premium, inducing its expected skewness components to be consistently negatively priced over both short and long horizons.

In the Internet Appendix, we repeat the tests in Tables 8 and 9 controlling for realized skewness estimates over different horizons. We do so because the high correlations between the realized skewness estimates over alternative horizons may create concern that the relation between a skewness proxy and future realized skewness over some horizon may be spuriously driven by both variables also relating to future realized skewness over another horizon. To address this concern, we control for the non-systematic parts of the future realized skewness estimates over other horizons in the portfolio regressions used to calculate the expected skewness and orthogonal component. Internet Appendix Tables IA.9 and IA.10 reveal that this methodological variation does not materially alter our conclusions.

Taken together, this section suggests that the skewness proxies advocated in the empirical asset pricing literature often price stocks because they embed forward-looking information about the skewness of short-horizon dollar returns, but not because they embed the same information about the skewness of long-horizon dollar returns.

7 Conclusion

While recent theoretical and empirical asset pricing research suggests that the skewness of a stock's future dollar return distribution prices stocks, it is notably silent on the identity of the return horizon(s) over which skewness is priced. Whereas theoretical studies examine two-period models with an unspecified horizon, empirical studies generally rely on proxies for the skewness of shorter-horizon returns. However, as skewness does not scale with the time horizon, it is unclear whether the stock pricing ability of those skewness proxies comes from capturing the expected skewness of future short-horizon returns, the same skewness of future long-horizon returns, and/or information unrelated to expected skewness.

In this study, we comprehensively examine why the skewness proxies advocated in the empirical asset pricing literature price stocks. To do so, we first propose a new bootstrap estimator for the skewness of the dollar return over an arbitrary horizon. Using a Monte Carlo simulation exercise, we confirm that the estimator is less biased than the recent estimators of Fama and French (2018), Neuberger and Payne (2021), and Farago and Hjalmarsson (2022), with reasonable standard errors. Relying on the estimates obtained from our estimator, we next show that some of the skewness proxies from the literature are relatively better in predicting the skewness of short-horizon dollar returns, whereas others are relatively better in predicting the skewness of long-horizon dollar returns. Most importantly, we follow Hou and Loh (2016) in decomposing each skewness proxy into components that reflect the expectation of skewness over alternative horizons and orthogonal components. Using those components, we find that most skewness proxies are priced because they embed information about skewness over short horizons but not because they embed information about skewness over longer horizons.

References

Amaya, D., Christoffersen, P., Jacobs, K., Vasquez, A., 2015. "Does realized skewness predict the cross-section of equity returns?" *Journal of Financial Economics* 118, 135-167.

Andersen, T., Benzoni, L., Lund J., 2002. "An empirical investigation of continuous-time equity return models." *The Journal of Finance* 57, 1239-1284.

Bakshi, G., Kapadia, N., Madan, D., 2003. "Stock return characteristics, skew laws, and the differential pricing of individual equity options." *Review of Financial Studies* 16, 101-143.

Bali, T. G., Cakici, N., Whitelaw, R. F., 2011. "Maxing out: Stocks as lotteries and the cross-section of expected returns." *Journal of Financial Economics* 99, 427-446.

Bali, T. G., Engle, R. F., Murray, S., 2016. "Empirical asset pricing: The cross-section of stock returns." *Wiley, New Jersey.*

Barberis, N., Huang, M., 2008. "Stocks as lotteries: The implications of probability weighting for security prices." *American Economic Review* 98, 2066-2100.

Bates, D., 1996. "Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options." *Review of Financial Studies* 9, 69-107.

Bates, D., 2006. "Maximum likelihood estimation of latent affine processes." *Review of Financial Studies* 19, 909-965

Bekaert, G., Engstrom, E., Ermolov, A., 2015. "Bad environments, good environments: A non-Gaussian asymmetric volatility model." *Journal of Econometrics* 186, 258-275.

Bessembinder, H., 2018. "Do stocks outperform Treasury bills?" *Journal of Financial Economics* 129, 440-457.

Black, F., 1976. "Studies of stock price volatility changes." *Proceedings of the 1976 Meeting of the American Statistical Association, Business and Economic Statistics Section*, 177-181.

Black, F., Jensen, M. C., Scholes, M., 1972. "The capital asset pricing model: Some empirical tests." In M. C. Jensen (Ed.), *Studies in the Theory of Capital Markets. Praeger, New York.*

Boyer, B., Mitton, T., Vorkink, K., 2010. "Expected idiosyncratic skewness." *Review of Financial Studies* 23, 169-202.

Boyer, B., Vorkink, K., 2014. "Stock options as lotteries." *The Journal of Finance* 69, 1485-1528.

Broadie, M., Chernov, M., Johannes, M., 2009. "Understanding index option returns" *Review of Financial Studies* 22, 4493-4529.

Brunnermeier, M. K., Gollier, C., Parker, J. A., 2007. “Optimal beliefs, asset prices, and the preference for skewed returns.” *American Economic Review* 97, 159-165.

Cenesizoglu, T., Timmermann, A., 2008. “Is the distribution of stock returns predictable?” *Working Paper, University of California San Diego.*

Chang, B. Y., Christoffersen, P., Jacobs, K., 2013. “Market skewness risk and the cross section of stock returns.” *Journal of Financial Economics* 107, 46-68.

Conine, T. E., Tamarkin, M. J., 1981. “On diversification given asymmetry in returns.” *The Journal of Finance* 36, 1143-1155.

Conrad, J., Dittmar, R. F., Ghysels, E., 2013. “Ex ante skewness and expected stock returns.” *The Journal of Finance* 68, 85-124.

Conrad, J., Kapadia, N., Xing, Y., 2014. “Death and jackpot: Why do individual investors hold overpriced stocks?” *Journal of Financial Economics* 113, 455-475.

Cooper, M. J., Gulen, H., Schill, M. J., 2008. “Asset growth and the cross-section of stock returns.” *The Journal of Finance* 63, 1609-1651.

Cox, J., Ross, S. A., 1976. “The valuation of options for alternative stochastic processes.” *Journal of Financial Economics* 3, 145-166.

Daniel, K., Grinblatt, M., Titman, S., Wermers, R., 1997. “Measuring mutual fund performance with characteristic-based benchmarks.” *The Journal of Finance* 52, 1035-1058.

Dittmar, R. F., 2002. “Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns.” *The Journal of Finance* 57, 369-403.

Eraker, B., 2004. “Do stock prices and volatility jump? Reconciling evidence from spot and option prices.” *The Journal of Finance* 59, 1367-1403.

Fama, E. F., French, K. R., 1992. “The cross-section of expected stock returns.” *The Journal of Finance* 47, 427-465.

Fama, E. F., French, K. R., 1993. “Common risk factors in the returns on stocks and bonds.” *Journal of Financial Economics* 33, 3-56.

Fama, E. F., French, K. R., 2008. “Dissecting anomalies.” *The Journal of Finance* 63, 1653-1678.

Fama, E. F., French, K. R., 2018. “Long-horizon returns.” *The Review of Asset Pricing Studies* 8, 232-252.

Fama, E. F., MacBeth, J. D., 1973. “Risk, return, and equilibrium: Empirical tests.” *Journal of Political Economy* 71, 607-636.

Farago, A., Hjalmarsson, E., 2022. “Long-horizon stock returns are positively skewed.” *Review of Finance, forthcoming.*

Ghysels, E., Plazzi, A., Valkanov, R. I., 2016. "Why invest in emerging markets? The role of conditional return asymmetry." *The Journal of Finance* 71, 2145-2192.

Harvey, C. R., Siddique, A., 2000. "Conditional skewness in asset pricing tests." *The Journal of Finance* 55,1263-1295.

Heston, S. L., 1993. "A closed-form solution for options with stochastic volatility with applications to bond and currency options." *Review of Financial Studies* 6, 327-343.

Hou, K., Loh, R. K., 2016. "Have we solved the idiosyncratic volatility puzzle?" *Journal of Financial Economics* 121, 167-194.

Hurn, A. S., Lindsay, K. A., McClelland, A. J., 2015. "Estimating the parameters of stochastic volatility models using option price data." *Journal of Business and Economic Statistics* 33, 579-594.

Jacobs, K., Liu, Y., 2018. "Estimation and filtering with big option data." *Working Paper.*

Jegadeesh, N., Titman, S., 1993. "Returns to buying winners and selling losers: Implications for stock market efficiency." *The Journal of Finance* 48, 65-91.

Koenker, R., 2005. "Quantile regressions." *Cambridge University Press, New York.*

Kozhan, R., Neuberger, A., Schneider, P., 2013. "The skew risk premium in the equity index market." *Review of Financial Studies* 26, 2174-2203.

Kraus, A., Litzenberger, R. H., 1976. "Skewness preference and the valuation of risk assets." *The Journal of Finance* 31, 1085-1100.

Lau, H.-S., Wingender, J. R., Lau, A. H.-L., 1989. "On estimating skewness in stock returns." *Management Science* 35, 1029-1150.

Mincer, J., Zarnowitz, V., 1969. "The evaluation of economic forecasts."In J. Mincer (Ed.), *Economic Forecasts and Expectations. Columbia University Press, New York.*

Mitton, T., Vorkink, K., 2007. "Equilibrium underdiversification and the preference for skewness." *Review of Financial Studies* 20, 1255-1288.

Newey, W. K., West, K. D., 1987. "A simple positive-definite heteroscedasticity and autocorrelation consistent covariance matrix." *Econometrica* 55, 703-708.

Neuberger, A., 2012. "Realized skewness." *Review of Financial Studies* 25, 3423-3455.

Neuberger, A., Payne, R., 2021. "The skewness of the stock market at long horizons." *Review of Financial Studies* 34, 1572-1616.

Patton, A., Politis, D. N., White, H., 2009. "Correction to "Automatic block-length selection for the dependent bootstrap"." *Econometric Reviews* 28, 372-375.

Politis, D. N., Romano, J. P., 1994. "The stationary bootstrap." *Journal of the American Statistical Association* 89, 1303-1313.

Politis, D. N., White, H., 2004. “Automatic block-length selection for the dependent bootstrap.” *Econometric Reviews* 23, 53-70.

Pollastri, A., Rodrigues, P., Seeger, N. J., Schlag, C., 2022. “A jumping index of jumping stocks? An MCMC analysis of continuous-time models for individual stocks.” *Working Paper*.

Rehman, Z., Vilkov, G., 2012. “Risk-neutral skewness: Return predictability and its sources.” *Working Paper*.

Rubinstein, M., 1973.“The fundamental theorem of parameter-preference security valuation.” *Journal of Financial and Quantitative Analysis* 8, 61-69.

Scott, R. C., Horvath, P. A., 1980. “On the direction of preference for moments of higher order than the variance.” *The Journal of Finance* 35, 915-919.

Simkowitz, M. A., Beedles, W. L., 1978. “Diversification in a three-moment world.” *Journal of Financial and Quantitative Analysis* 13, 927-941.

Stilger, P. S., Kostakis, A., Poon, S.-h., 2017. “What does risk-neutral skewness tell us about future stock returns?” *Management Science* 63, 1814-1834.

Table 1.
Monte Carlo Simulation Exercise

The table presents the results from a Monte Carlo simulation exercise contrasting the unbiasedness and efficiency of our block bootstrap (Our), Fama and French's (2018) simple bootstrap (FF), and Neuberger and Payne's (2021) closed-form (NP) estimator for the skewness of the annual dollar return in the GBM (Panel A), SV (Panel B), and SVJ (Panel C) worlds. The table entries are the true skewness of the annual dollar return computed from the appropriate moment-generating functions (True Skew), the mean estimate of each estimator (Mean Est.), its absolute bias (the absolute difference between true skewness and the mean estimate; Abs. Bias), and its standard error (St.E.). To compute the mean estimate, absolute bias, and standard error, we simulate 10,000 daily asset value paths per world and parameter value set and then separately apply each estimator to each sample path. The basecase parameter values are: $S(0) = 30$, $\sqrt{V(0)} = 0.30$, $\alpha = 0.12$, $\kappa_v = 7.50$, $\theta_v = 0.30$, $\sigma_v = 0.40$, $\rho = -0.25$, $\lambda = 0.09$, $\mu_z = 0.01$, and $\sigma_z = 0.05$. In the subpanels, we vary either one or two parameters out of $\sqrt{V(0)}$, σ_v, ρ, μ_z, and σ_z.

	True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.
	Panel A. Geometric Brownian Motion (GBM) Model											
	Panel A.1 Volatility											
	0.15				0.30				0.45			
Our	0.456	0.428	(0.028)	[0.156]	0.950	0.887	(0.063)	[0.231]	1.528	1.398	(0.129)	[0.384]
FF	0.456	0.444	(0.012)	[0.130]	0.950	0.918	(0.031)	[0.195]	1.528	1.458	(0.070)	[0.352]
NP	0.456	−0.005	(0.461)	[0.175]	0.950	0.000	(0.950)	[0.137]	1.528	0.003	(1.525)	[0.130]

(*continued on next page*)

Table 1.
Monte Carlo Simulation Exercise (cont.)

		True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.
						Panel B. Stochastic Volatility (SV) Model							
						Panel B.1 Volatility of Variance							
Volatility		0.20				0.40				0.60			
	Our	0.345	0.392	(0.048)	[0.162]	0.258	0.364	(0.106)	[0.196]	0.194	0.341	(0.148)	[0.237]
0.15	FF	0.345	0.437	(0.092)	[0.131]	0.258	0.435	(0.177)	[0.133]	0.194	0.438	(0.245)	[0.143]
	NP	0.345	−0.154	(0.499)	[0.385]	0.258	−0.341	(0.599)	[0.775]	0.194	−0.566	(0.760)	[1.309]
	Our	0.881	0.869	(0.012)	[0.225]	0.826	0.841	(0.015)	[0.233]	0.786	0.830	(0.044)	[0.250]
0.30	FF	0.881	0.917	(0.036)	[0.195]	0.826	0.909	(0.083)	[0.200]	0.786	0.907	(0.121)	[0.204]
	NP	0.881	−0.067	(0.948)	[0.191]	0.826	−0.138	(0.964)	[0.298]	0.786	−0.212	(0.998)	[0.430]
	Our	1.463	1.377	(0.086)	[0.367]	1.412	1.361	(0.051)	[0.375]	1.372	1.353	(0.020)	[0.382]
0.45	FF	1.463	1.439	(0.024)	[0.331]	1.412	1.445	(0.033)	[0.339]	1.372	1.440	(0.068)	[0.349]
	NP	1.463	−0.042	(1.505)	[0.157]	1.412	−0.086	(1.498)	[0.210]	1.372	−0.131	(1.503)	[0.280]
						Panel B.2 Volatility of Variance							
Correlation		0.20				0.40				0.60			
	Our	0.807	0.842	(0.036)	[0.218]	0.682	0.804	(0.122)	[0.214]	0.574	0.761	(0.187)	[0.220]
−0.50	FF	0.807	0.910	(0.103)	[0.199]	0.682	0.913	(0.231)	[0.198]	0.574	0.906	(0.332)	[0.205]
	NP	0.807	−0.129	(0.936)	[0.193]	0.682	−0.259	(0.941)	[0.310]	0.574	−0.390	(0.964)	[0.455]
	Our	0.881	0.869	(0.012)	[0.225]	0.826	0.841	(0.015)	[0.233]	0.786	0.826	(0.040)	[0.250]
−0.25	FF	0.881	0.917	(0.036)	[0.195]	0.826	0.909	(0.083)	[0.200]	0.786	0.907	(0.121)	[0.205]
	NP	0.881	−0.067	(0.959)	[0.175]	0.826	−0.138	(0.964)	[0.298]	0.786	−0.212	(0.998)	[0.430]
	Our	0.941	0.880	(0.061)	[0.230]	0.947	0.877	(0.070)	[0.245]	0.967	0.881	(0.086)	[0.274]
−0.05	FF	0.941	0.912	(0.029)	[0.198]	0.947	0.877	(0.070)	[0.249]	0.967	0.912	(0.055)	[0.207]
	NP	0.941	−0.018	(0.959)	[0.190]	0.947	−0.040	(0.987)	[0.293]	0.967	−0.067	(1.035)	[0.420]

(*continued on next page*)

Table 1.
Monte Carlo Simulation Exercise (cont.)

		True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.	True Skew	Mean Est.	Abs. Bias	St.E.
		Panel B.3 Correlation											
Volatility		–0.50				–0.25				–0.05			
0.15	Our	0.022	0.289	(0.267)	[0.175]	0.258	0.361	(0.103)	[0.191]	0.454	0.417	(0.037)	[0.204]
	FF	0.022	0.432	(0.410)	[0.134]	0.258	0.439	(0.181)	[0.135]	0.454	0.437	(0.017)	[0.135]
	NP	0.022	−0.577	(0.600)	[0.828]	0.258	−0.341	(0.599)	[0.775]	0.454	−0.149	(0.603)	[0.743]
0.30	Our	0.682	0.801	(0.119)	[0.216]	0.826	0.841	(0.015)	[0.233]	0.947	0.877	(0.070)	[0.249]
	FF	0.682	0.912	(0.230)	[0.203]	0.826	0.909	(0.083)	[0.200]	0.947	0.909	(0.038)	[0.200]
	NP	0.682	−0.259	(0.941)	[0.310]	0.826	−0.138	(0.964)	[0.298]	0.947	−0.040	(0.987)	[0.293]
0.45	Our	1.280	1.398	(0.118)	[0.391]	1.412	1.374	(0.037)	[0.371]	1.524	1.403	(0.121)	[0.409]
	FF	1.280	1.445	(0.165)	[0.344]	1.412	1.450	(0.039)	[0.344]	1.524	1.453	(0.071)	[0.340]
	NP	1.280	−0.020	(1.300)	[0.175]	1.412	−0.086	(1.498)	[0.210]	1.524	−0.020	(1.544)	[0.209]
		Panel C. Stochastic Volatility-Jump (SVJ) Model											
		Panel C.1 Mean Jump Size											
Jump Vol.		–0.01				0.01				0.05			
0.01	Our	0.826	0.840	(0.014)	[0.233]	0.826	0.844	(0.017)	[0.236]	0.829	0.848	(0.019)	[0.234]
	FF	0.826	0.910	(0.083)	[0.206]	0.826	0.912	(0.085)	[0.202]	0.829	0.914	(0.085)	[0.202]
	NP	0.826	−0.128	(0.955)	[0.175]	0.826	−0.129	(0.955)	[0.296]	0.829	−0.128	(0.957)	[0.296]
0.05	Our	0.828	0.841	(0.013)	[0.233]	0.829	0.847	(0.018)	[0.234]	0.833	0.852	(0.019)	[0.231]
	FF	0.828	0.911	(0.083)	[0.206]	0.829	0.910	(0.082)	[0.202]	0.833	0.915	(0.082)	[0.201]
	NP	0.828	−0.128	(0.956)	[0.295]	0.829	−0.128	(0.957)	[0.296]	0.833	−0.127	(0.959)	[0.298]
0.15	Our	0.848	0.861	(0.012)	[0.260]	0.856	0.866	(0.011)	[0.264]	0.875	0.880	(0.006)	[0.275]
	FF	0.848	0.930	(0.082)	[0.236]	0.856	0.935	(0.079)	[0.235]	0.875	0.945	(0.070)	[0.258]
	NP	0.848	−0.126	(0.974)	[0.175]	0.856	−0.124	(0.979)	[0.355]	0.875	−0.118	(0.992)	[0.384]

Table 2.
Descriptive Statistics

The table offers descriptive statistics including the number of observations (Obs), the mean, the standard deviation (StD), the first percentile (Pct1), the first quartile (Q1), the median, the third quartile (Q3), and the 99th percentile (Pct99). While Panel A focuses on sample skewness or block bootstrap based estimates of the realized skewness of the daily (*DailySkew*), monthly (*MonthlyBSSkew*), quarterly (*QuarterlyBSSkew*), and annual (*AnnualBSSkew*) dollar return, Panel B focuses on the skewness proxies *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, and *MaxRet* (see Table D.1 in Appendix D for more details about those proxies). We calculate the realized skewness estimates from daily data over the period from month $t+1$ to $t+60$ and the skewness proxies from data until the end of month t. The sample excludes stocks with non-complete data on the realized skewness estimates and skewness proxies.

	Descriptive Statistics							
	Obs	Mean	StD	Pct1	Q1	Median	Q3	Pct99
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Realized Skewness								
DailySkew	641,528	0.41	0.60	−0.61	0.00	0.30	0.68	2.06
MonthlyBSSkew	641,528	0.47	0.64	−0.56	0.11	0.35	0.68	3.07
QuarterlyBSSkew	641,528	0.71	0.66	−0.18	0.32	0.56	0.90	3.42
AnnualBSSkew	641,528	1.47	1.22	0.30	0.78	1.14	1.72	6.80
Panel B: Skewness Proxies								
OLSSkew	641,528	0.83	0.50	−0.22	0.49	0.79	1.12	2.17
QuantileSkew	641,528	0.96	0.56	−0.50	0.60	0.97	1.34	2.21
HistoricalSkew	641,528	0.39	0.96	−1.79	0.01	0.30	0.64	4.20
LogitSkew	641,528	1.06	0.85	0.17	0.49	0.87	1.39	4.08
MaxRet	641,528	5.97	5.28	1.02	3.02	4.68	7.36	23.64

Table 3.
Spearman Rank Correlations

The table offers Spearman rank correlations between sample skewness and block bootstrap based realized skewness estimates and the skewness proxies. The realized skewness estimates are the realized skewness of the daily (*DailySkew*), monthly (*MonthlyBSSkew*), quarterly (*QuarterlyBSSkew*), and annual (*AnnualBSSkew*) dollar return. The skewness proxies are *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, and *MaxRet* (see Table D.1 in Appendix D for more details about those proxies). We calculate the realized skewness estimates from daily data over the period from months $t+1$ to $t+60$ and the skewness proxies from data until the end of month t. The sample excludes stocks with non-complete data on the variables.

	Realized Skewness				Skewness Proxies			
	Daily Skew	*Monthly BSSkew*	*Quar. BSSkew*	*Annual BSSkew*	*OLS Skew*	*Quantile Skew*	*Hist. Skew*	*Logit Skew*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
MonthlyBSSkew	0.51							
QuarterlyBSSkew	0.44	0.81						
AnnualBSSkew	0.34	0.63	0.79					
OLSSkew	0.18	0.24	0.23	0.20				
QuantileSkew	0.17	0.27	0.36	0.44	0.40			
HistoricalSkew	0.15	0.17	0.19	0.19	0.35	0.33		
LogitSkew	0.25	0.38	0.45	0.50	0.59	0.65	0.31	
MaxRet	0.08	0.22	0.30	0.37	0.22	0.51	0.18	0.52

Table 4.
The Forecasting Power of the Skewness Proxies: Portfolio Sorts

The table offers the average future realized skewness of portfolios univariately sorted on the skewness proxy *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, or *MaxRet* (see Table D.1 in Appendix D for more details about those proxies). At the end of each sample month t, we sort stocks into portfolios according to the tenth, 20th, 40th, 60th, 80th, and 90th percentiles of a skewness proxy's distribution on that date. We then compute a portfolio's future realized skewness by taking an average over the sample skewness or block bootstrap based realized skewness estimates of all stocks in the portfolio, where we separately consider the realized skewness of the daily (*DailySkew*; Panel A), monthly (*MonthlyBSSkew*; Panel B), quarterly (*QuarterlyBSSkew*; Panel C), and annual (*AnnualBSSkew*; Panel D) return. We calculate the realized skewness estimates from daily data over months $t+1$ to $t+60$. We also calculate the spread in average future realized skewness across the extreme portfolios (LS9010). Plain numbers are the time-series means of average portfolio realized skewness and the spreads, while the numbers in parentheses are Newey-West (1987) t-statistics with a 60 month lag length.

	Skewness Proxy Percentiles								
	00–10	10–20	20–40	40–60	60–80	80–90	90–100	LS9010	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(7)–(1)	
Panel A: Daily Skewness									
OLSSkew	0.23	0.27	0.29	0.35	0.47	0.60	0.72	0.49	[15.53]
QuantileSkew	0.22	0.29	0.34	0.38	0.45	0.54	0.65	0.43	[8.33]
HistoricalSkew	0.31	0.30	0.33	0.37	0.44	0.51	0.62	0.30	[12.02]
LogitSkew	0.16	0.24	0.29	0.39	0.50	0.58	0.70	0.54	[11.54]
MaxRet	0.35	0.33	0.34	0.38	0.43	0.48	0.56	0.21	[5.71]
Panel B: Monthly Skewness									
OLSSkew	0.26	0.29	0.31	0.41	0.56	0.69	0.83	0.57	[20.71]
QuantileSkew	0.21	0.27	0.33	0.42	0.54	0.70	0.87	0.66	[10.55]
HistoricalSkew	0.35	0.35	0.38	0.43	0.50	0.59	0.72	0.37	[14.78]
LogitSkew	0.14	0.23	0.30	0.44	0.59	0.72	0.91	0.77	[22.90]
MaxRet	0.33	0.32	0.35	0.43	0.52	0.63	0.75	0.42	[9.12]

(*continued on next page*)

Table 4.
The Forecasting Power of the Skewness Proxies: Portfolio Sorts (cont.)

	Skewness Proxy Percentiles								
	00–10	10–20	20–40	40–60	60–80	80–90	90–100	LS9010	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(7)–(1)	
Panel C: Quarterly Skewness									
OLSSkew	0.50	0.53	0.55	0.64	0.80	0.91	1.04	0.55	[10.83]
QuantileSkew	0.38	0.46	0.54	0.65	0.81	0.97	1.17	0.79	[11.35]
HistoricalSkew	0.58	0.58	0.61	0.66	0.74	0.84	0.96	0.37	[10.59]
LogitSkew	0.32	0.43	0.52	0.67	0.84	0.98	1.20	0.88	[15.72]
MaxRet	0.49	0.50	0.56	0.66	0.78	0.91	1.06	0.57	[8.19]
Panel D: Annual Skewness									
OLSSkew	1.15	1.20	1.24	1.35	1.63	1.74	1.96	0.81	[10.82]
QuantileSkew	0.84	1.00	1.15	1.37	1.67	1.98	2.31	1.47	[11.53]
HistoricalSkew	1.29	1.25	1.28	1.38	1.52	1.69	1.88	0.58	[7.68]
LogitSkew	0.77	0.97	1.15	1.41	1.69	1.94	2.33	1.56	[14.11]
MaxRet	0.99	1.06	1.19	1.39	1.63	1.87	2.15	1.16	[7.20]

Table 5.
The Forecasting Power of the Skewness Proxies: Regressions

This table shows the results from the cross-sectional regression:

$$RealizedSkew_{t+1,t+60} = \alpha + \beta\ SkewnessProxy_t + \epsilon_{t+1,t+60},$$

where *RealizedSkew* is a sample skewness or block bootstrap based estimate of the realized skewness of the daily (*DailySkew*; Panel A), monthly (*MonthlyBSSkew*; Panel B), quarterly (*QuarterlyBSSkew*; Panel C), or annual (*AnnualBSSkew*; Panel D) return, and *SkewnessProxy* is *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, or *MaxRet* (see Table D.1 in Appendix D for more details about those proxies). We calculate the realized skewness estimates using daily data over months $t+1$ to $t+60$ and the skewness proxies using data until the end of month t. We estimate the regressions separately by sample month t. The table entries are the time-series mean of each estimate (the plain numbers), the t-statistic of the mean (computed using Newey-West (1987) standard errors with a 60 month lag length; in square parentheses), and the time-series means of the adjusted R-squared (bottom row of each panel) and the number of sample observations (final row of table).

	Skewness Proxy				
	OLS Skew	Quantile Skew	Historical Skew	Logit Skew	Max Ret
	(1)	(2)	(3)	(4)	(5)
Panel A: Daily Skewness					
Coefficient	0.34	0.23	0.10	0.23	0.02
	[9.32]	[6.05]	[7.54]	[8.65]	[7.79]
Constant	0.12	0.19	0.37	0.18	0.31
	[2.45]	[3.87]	[13.57]	[5.04]	[10.34]
Mean Adj. R^2	0.07	0.05	0.03	0.08	0.02
Panel B: Monthly Skewness					
Coefficient	0.40	0.36	0.11	0.32	0.03
	[13.38]	[5.96]	[8.51]	[16.24]	[9.24]
Constant	0.13	0.13	0.42	0.14	0.29
	[2.92]	[3.22]	[13.56]	[4.58]	[12.52]
Mean Adj. R^2	0.09	0.11	0.03	0.14	0.05

(*continued on next page*)

Table 5.
The Forecasting Power of the Skewness Proxies: Regressions (cont.)

	Skewness Proxy				
	OLS Skew	Quantile Skew	Historical Skew	Logit Skew	Max Ret
	(1)	(2)	(3)	(4)	(5)
Panel C: Quarterly Skewness					
Coefficient	0.38	0.44	0.11	0.36	0.04
	[13.62]	[6.06]	[8.22]	[18.15]	[8.37]
Constant	0.38	0.29	0.65	0.34	0.48
	[7.51]	[4.74]	[17.43]	[11.93]	[13.12]
Mean Adj. R^2	0.09	0.15	0.03	0.18	0.08
Panel D: Annual Skewness					
Coefficient	0.56	0.82	0.18	0.62	0.07
	[13.01]	[6.21]	[7.70]	[18.83]	[7.91]
Constant	0.99	0.68	1.38	0.82	1.02
	[9.08]	[5.04]	[15.54]	[13.82]	[11.03]
Mean Adj. R^2	0.07	0.18	0.03	0.18	0.10
Mean Observations	2,210	2,210	2,210	2,210	2,210

Table 6.
The Skewness Proxy Premiums: Portfolio Sorts

The table presents mean excess returns and alphas for portfolios univariately sorted on the skewness proxy *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, or *MaxRet* (see Table D.1 in Appendix D for more details about those proxies). At the end of each sample month t, we sort stocks into portfolios according to the decile breakpoints of a skewness proxy's distribution on that date. We value-weight the portfolios and hold them over month $t+1$. We also form a spread portfolio long the highest skewness proxy portfolio and short the lowest. To adjust such a portfolio for risk, we regress its return on the excess market return (MKT; CAPM alpha) or MKT, SMB, and HML (FF3 alpha) and report the intercept. The plain numbers are the mean value-weighted monthly portfolio returns or spread portfolio alphas (in %), while the numbers in square parentheses are Newey-West (1987) t-statistics with a twelve month lag length.

	Sorting Variable				
Decile	OLS Skew	Quantile Skew	Historical Skew	Logit Skew	Max Ret
	(1)	(2)	(3)	(4)	(5)
1 (Low)	1.43	0.95	0.91	0.93	0.93
2	0.99	1.01	1.03	1.04	0.91
3	0.95	0.92	0.94	1.04	0.91
4	0.79	1.08	0.97	1.00	0.99
5	0.80	1.04	1.00	1.19	1.03
6	0.75	1.02	0.88	0.85	1.17
7	0.63	1.09	0.98	1.08	1.00
8	0.72	0.80	1.14	0.76	0.89
9	0.47	0.61	1.05	0.77	0.84
10 (High)	0.40	0.13	0.91	0.09	0.41
Spread Return	−1.03	−0.82	0.00	−0.83	−0.51
t-statistic	[−2.50]	[−1.50]	[0.00]	[−2.28]	[−1.90]
CAPM Alpha	−1.10	−1.35	−0.02	−1.17	−0.86
t-statistic	[−2.88]	[−2.67]	[−0.09]	[−3.35]	[−2.86]
FF3 Alpha	−1.28	−1.16	−0.13	−1.07	−0.76
t-statistic	[−3.23]	[−3.81]	[−0.91]	[−4.09]	[−3.38]

Table 7.
The Skewness Proxy Premiums: Fama-MacBeth Regressions

The table offers the results from Fama-MacBeth (1973) regressions of single-stock returns over month $t+1$ on each skewness proxy *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, or *MaxRet* calculated using data until the end of month t (see Table D.1 in Appendix D for more details about those proxies). While the regressant in Panel A is a stock's dollar return, the regressant in Panel B is that same dollar return minus the dollar return of the double-sorted value-weighted size and book-to-market portfolio to which the stock belongs ("DGTW Return"). We form the double-sorted portfolios as in Fama and French (1993). Plain numbers are monthly premium estimates (in %), while the numbers in square parentheses are Newey-West (1987) t-statistics with a twelve-month lag length.

	Skewness Proxy				
	OLS Skew	Quantile Skew	Historical Skew	Logit Skew	Max Ret
	(1)	(2)	(3)	(4)	(5)
Panel A: Dollar Return					
Premium	−0.53	−0.18	0.01	−0.20	−0.08
	[−2.63]	[−0.64]	[0.28]	[−2.35]	[−5.84]
Constant	1.45	1.30	1.11	1.38	1.57
	[4.29]	[3.97]	[4.71]	[6.06]	[7.42]
Panel B: DGTW Return					
Premium	−0.54	−0.07	−0.02	−0.22	−0.07
	[−2.62]	[−0.41]	[−0.79]	[−6.24]	[−7.99]
Constant	0.52	0.16	0.02	0.29	0.44
	[2.83]	[0.59]	[0.64]	[3.97]	[5.95]

Table 8.
Decomposing the Skewness Proxy Premiums in Dollar Returns

The table presents the results from Fama-MacBeth (1973) regressions of single-stock returns over month $t+1$ on the component of each skewness proxy capturing the expectation of future realized skewness (columns (1) to (2)) or the residual (columns (3) to (4)) calculated using data until the end of month t. The table further shows the results from splitting the total skewness proxy premium from Table 7 (repeated in the panel headings) into parts related to the two components (columns (5) to (8)). Panels A to E focus on *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, and *MaxRet*, respectively (see Table D.1 in Appendix D for more details about those proxies). We construct the components by sorting stocks into 50 portfolios according to each skewness proxy at the end of month t. We then run a cross-sectional regression of a portfolio's average skewness proxy value on its average future realized skewness. To calculate average future realized skewness, we average over the estimates of the realized skewness of the daily (*DailySkew*), monthly (*MonthlyBSSkew*), quarterly (*QuarterlyBSSkew*), or annual (*AnnualBSSkew*) return of all stocks in the portfolio, calculating the estimates using daily data from month $t+1$ to $t+60$. We assign the fitted value from the regression to each stock in a portfolio, interpreting it as the component of the skewness proxy predicting future realized skewness. The plain numbers in columns (1) and (3) are monthly premium estimates (in %), while those in columns (5) and (7) are the proportions of the skewness proxy premium attributable to each component. The numbers in square parentheses in columns (2), (4), (6), and (8) are Newey-West (1987) t-statistics with a twelve-month lag length.

	Premium Skewness Proxy Component Predicting Skewness		Premium Skewness Proxy Component Orthogonal to Skewness		Decomposition of Skewness Proxy Premium			
					Fitted Value		Residual	
	est.	t-stat.	est.	t-stat.	est.	t-stat.	est.	t-stat.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: *OLSSkew (Premium: –0.528; t-statistic: –2.63)*								
DailySkew	−0.57	[−2.90]	−0.38	[−1.72]	0.70	[7.03]	0.30	[2.97]
MonthlyBSSkew	−0.50	[−2.37]	−0.37	[−1.78]	0.65	[5.80]	0.35	[3.15]
QuarterlyBSSkew	−0.40	[−1.49]	−0.35	[−1.68]	0.61	[4.92]	0.39	[3.20]
AnnualBSSkew	−3.19	[−1.34]	−0.35	[−1.75]	0.51	[4.20]	0.49	[3.97]

(*continued on next page)*

Table 8.
Decomposing the Skewness Proxy Premiums in Dollar Returns (cont.)

	Premium Skewness Proxy Component Predicting Skewness		Premium Skewness Proxy Component Orthogonal to Skewness		Decomposition of Skewness Proxy Premium: Fitted Value		Residual	
	est.	*t*-stat.	est.	*t*-stat.	est.	*t*-stat.	est.	*t*-stat.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel B: *QuantileSkew (Premium: –0.182; t-statistic: –0.64)*								
DailySkew	−0.77	[−1.78]	0.18	[0.79]	1.49	[1.18]	−0.49	[−0.39]
MonthlyBSSkew	−0.28	[−0.86]	0.19	[1.20]	1.21	[2.31]	−0.21	[−0.41]
QuarterlyBSSkew	−0.27	[−0.81]	0.25	[1.53]	1.14	[3.15]	−0.14	[−0.39]
AnnualBSSkew	−0.26	[−0.77]	0.15	[0.93]	1.08	[4.06]	−0.08	[−0.30]
Panel C: *HistoricalSkew (Premium: 0.013; t-statistic: 0.28)*								
DailySkew	−0.06	[−0.88]	0.07	[2.23]	−1.46	[−0.16]	2.46	[0.26]
MonthlyBSSkew	−0.03	[−0.43]	0.04	[1.28]	−0.12	[−0.03]	1.12	[0.28]
QuarterlyBSSkew	−0.04	[−0.49]	0.02	[0.73]	0.21	[0.08]	0.79	[0.29]
AnnualBSSkew	−0.06	[−0.66]	−0.02	[−0.64]	1.58	[0.39]	−0.58	[−0.14]
Panel D: *LogitSkew (Premium: –0.196; t-statistic: –2.35)*								
DailySkew	−0.18	[−1.70]	−0.18	[−2.67]	0.67	[5.36]	0.33	[2.68]
MonthlyBSSkew	−0.16	[−1.56]	−0.22	[−3.64]	0.69	[5.94]	0.31	[2.64]
QuarterlyBSSkew	−0.15	[−1.47]	−0.30	[−5.29]	0.61	[3.75]	0.39	[2.40]
AnnualBSSkew	−0.15	[−1.38]	−0.32	[−4.98]	0.57	[3.02]	0.43	[2.32]
Panel E: *MaxRet (Premium: –0.080; t-statistic: –5.84)*								
DailySkew	−0.23	[−3.10]	−0.06	[−5.40]	0.47	[7.56]	0.53	[8.44]
MonthlyBSSkew	−0.12	[−6.43]	−0.05	[−5.04]	0.68	[11.54]	0.32	[5.49]
QuarterlyBSSkew	−0.11	[−5.68]	−0.06	[−6.43]	0.74	[15.70]	0.26	[5.61]
AnnualBSSkew	−0.03	[−2.76]	−0.06	[−7.22]	0.73	[15.83]	0.27	[5.79]

Table 9.
Decomposing the Skewness Proxy Premiums in DGTW Returns

The table presents the results from Fama-MacBeth (1973) regressions of single-stock DGTW returns over month $t+1$ on the component of each skewness proxy capturing the expectation of future realized skewness (columns (1) to (2)) or the residual (columns (3) to (4)) calculated using data until the end of month t. The table further shows the results from splitting the total skewness proxy premium from Table 7 (repeated in the panel headings) into parts related to the two components (columns (5) to (8)). A stock's DGTW return is its dollar return minus the value-weighted dollar return of the size and book-to-market portfolio to which the stock belongs. Panels A to E focus on *OLSSkew*, *QuantileSkew*, *HistoricalSkew*, *LogitSkew*, and *MaxRet*, respectively (see Table D.1 in Appendix D for more details about those proxies). We construct the components by sorting stocks into 50 portfolios according to each skewness proxy at the end of month t. We then run a cross-sectional regression of a portfolio's average skewness proxy value on its average future realized skewness. To calculate average future realized skewness, we average over the estimates of the realized skewness of the daily (*DailySkew*), monthly (*MonthlyBSSkew*), quarterly (*QuarterlyBSSkew*), or annual (*AnnualBSSkew*) return of all stocks in the portfolio, calculating the estimates using daily data from month $t+1$ to $t+60$. We assign the fitted value from the regression to each stock in a portfolio, interpreting it as the component of the skewness proxy predicting future realized skewness. The plain numbers in columns (1) and (3) are monthly premium estimates (in %), while those in columns (5) and (7) are the proportions of the skewness proxy premium attributable to each component. The numbers in square parentheses in columns (2), (4), (6), and (8) are Newey-West (1987) t-statistics with a twelve-month lag length.

	Premium Skewness Proxy Component Predicting Skewness		Premium Skewness Proxy Component Orthogonal to Skewness		Decomposition of Skewness Proxy Premium: Fitted Value		Decomposition of Skewness Proxy Premium: Residual	
	est.	t-stat.	est.	t-stat.	est.	t-stat.	est.	t-stat.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: *OLSSkew (Premium: –0.542; t-statistic: –2.62)*								
DailySkew	−0.64	[−3.15]	−0.43	[−2.03]	0.62	[8.39]	0.38	[5.04]
MonthlyBSSkew	−0.54	[−2.83]	−0.43	[−2.06]	0.62	[7.29]	0.38	[4.38]
QuarterlyBSSkew	−0.47	[−2.50]	−0.42	[−1.99]	0.57	[5.77]	0.43	[4.42]
AnnualBSSkew	−3.94	[−1.27]	−0.41	[−2.08]	0.46	[4.87]	0.54	[5.71]

(*continued on next page)*

Table 9.
Decomposing the Skewness Proxy Premiums in DGTW Returns (cont.)

	Premium Skewness Proxy Component Predicting Skewness		Premium Skewness Proxy Component Orthogonal to Skewness		Decomposition of Skewness Proxy Premium: Fitted Value		Decomposition of Skewness Proxy Premium: Residual	
	est.	*t*-stat.	est.	*t*-stat.	est.	*t*-stat.	est.	*t*-stat.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel B: *QuantileSkew (Premium: –0.072; t-statistic: –0.41)*								
DailySkew	−0.54	[−1.78]	0.30	[2.11]	2.71	[0.61]	−1.71	[−0.39]
MonthlyBSSkew	−0.15	[−0.77]	0.25	[1.74]	1.72	[0.98]	−0.72	[−0.41]
QuarterlyBSSkew	−0.13	[−0.67]	0.23	[1.41]	1.48	[1.28]	−0.48	[−0.42]
AnnualBSSkew	−0.12	[−0.60]	0.11	[0.68]	1.34	[1.60]	−0.34	[−0.41]
Panel C: *HistoricalSkew (Premium: –0.024; t-statistic: –0.79)*								
DailySkew	−0.10	[−2.22]	0.03	[1.23]	1.53	[0.98]	−0.53	[−0.34]
MonthlyBSSkew	−0.06	[−1.25]	0.01	[0.34]	1.20	[1.03]	−0.20	[−0.17]
QuarterlyBSSkew	−0.06	[−1.36]	0.00	[−0.01]	1.16	[0.98]	−0.16	[−0.13]
AnnualBSSkew	−0.07	[−1.45]	−0.02	[−0.75]	0.85	[0.92]	0.15	[0.17]
Panel D: *LogitSkew (Premium: –0.219; t-statistic: –6.24)*								
DailySkew	−0.24	[−5.97]	−0.20	[−4.00]	0.60	[8.16]	0.40	[5.53]
MonthlyBSSkew	−0.20	[−5.34]	−0.26	[−4.88]	0.65	[10.21]	0.35	[5.41]
QuarterlyBSSkew	−0.19	[−5.17]	−0.30	[−5.59]	0.64	[9.35]	0.36	[5.17]
AnnualBSSkew	−0.19	[−5.16]	−0.30	[−4.75]	0.63	[8.63]	0.37	[4.97]
Panel E: *MaxRet (Premium: –0.073; t-statistic: –7.99)*								
DailySkew	−0.16	[−2.52]	−0.06	[−6.94]	0.45	[8.39]	0.55	[10.13]
MonthlyBSSkew	−0.10	[−7.49]	−0.04	[−6.00]	0.67	[12.28]	0.33	[5.93]
QuarterlyBSSkew	−0.09	[−7.59]	−0.04	[−6.61]	0.76	[16.61]	0.24	[5.27]
AnnualBSSkew	−0.04	[−2.23]	−0.05	[−7.71]	0.76	[16.93]	0.24	[5.35]

A Neuberger and Payne's (2021) Estimator

In this appendix, we give more technical details about Neuberger and Payne's (2021) skewness estimator. Assuming an asset's value obeys a strictly stationary martingale, and approximating the moments of the asset's return, these authors show that the skewness of the dollar return from time $t-T$ to t is related to the skewness of the dollar return over each non-overlapping subperiod $q \in \{t-T, t-T+1, \ldots, t\}$ within the $T-t$ to t period as follows:

$$\text{skew}(R_{t-T,t}) = \left(\text{skew}(R_q) + 3\frac{\text{cov}(y_{q-1}, x^{(2,E)}(R_q))}{\text{var}^L(R_q)^{3/2}}\right)/\sqrt{T}, \tag{A1}$$

where R_m is the gross return, with $m \in \{t-T, t; q\}$; $\text{skew}(R_m)$ is the approximated skewness coefficient $\mathbb{E}[x^{(3)}(R_m)]/\text{var}^L(R_m)^{3/2}$, with $x^{(3)}(R_m) = 6((R_m+1)\ln R_m - 2(R_m-1))$; and $\text{var}^L(R_m)$ is the approximated variance of R_m, $\mathbb{E}[2(R_m - 1 - \ln R_m)]$. Conversely, $\text{cov}(.,.)$ is the covariance operator; $y_q = (1/T)\sum_{u=0}^{T-1}(R_{q-u,q} - 1)$, where $R_{q-u,q}$ is the gross return from subperiod $q-u$ to subperiod q; and $x^{(2,E)}(R_q) = 2(R_q \ln R_q + 1 - R_q)$.[24]

Interpreting y_{q-1} as the geometric average return over the prior T subperiods and $x^{(2,E)}(R_q)$ as an approximation of the squared net return over subperiod q, Equation (A1) neatly suggests why return dependence affects the skewness of long-horizon returns. More specifically, when higher past returns predict a higher (lower) volatility, the covariance in the equation is positive (negative), raising (lowering) the skewness of the long-horizon return. The higher (lower) skewness is caused by the fact that a positive (negative) past return-volatility relation can create a hump in the right (left) tail of the long-horizon-return distribution, as can also happen in other non-constant volatility models (e.g., Cox and Ross (1976) and Heston (1993)).

[24]To avoid bias, Neuberger and Payne (2021) advise that the covariance term should be calculated as the time-series average of the product between y_{q-1} and $x^{(2,E)}(R_q)$ minus the time-series average of $x^{(2,E)}(R_q)$, where the time-series average of $x^{(2,E)}(R_q)$ should, however, be calculated over some sample period ending before the start of the sample period used to calculate the time-series average of the product.

B Politis and White's (2004) Optimal p Choice

In this appendix, we offer more technical details about Politis and White's (2004) optimal choice for the p parameter in Politis and Romano's (1994) block bootstrap. Recalling that the expected block length in that bootstrap is $1/(1-p)$, Politis and White (2004) prove that, to minimize the theoretical mean-squared error of a block bootstrap based estimator of the long-run variance of the underlying variable (in our case: the short-horizon dollar return),[25] it is optimal to choose p such that this length becomes equal to:

$$\hat{b}_{opt} = N^{1/3} \left(\frac{2\hat{G}^2}{\hat{D}} \right)^{1/3}, \tag{B1}$$

where N is the number of (original) short-horizon returns, and $\hat{G}$ and $\hat{D}$ are, respectively:

$$\hat{G} = \sum_{k=-M}^{M} \lambda(k/M)|k|\hat{R}(k) \quad \text{and} \quad \hat{D} = \left(4\hat{g}^2(0) + \frac{2}{\pi} \int_{-\pi}^{\pi} (1+\cos w)\hat{g}^2(w)dw \right), \tag{B2}$$

where the flat-top lag-window function, $\lambda(t)$, is 1 if $|t| \in [0, 1/2]$, $2(1-|t|)$ if $|t| \in [1/2, 1]$, and else zero, $\hat{g}(w) = \sum_{k=-M}^{M} \lambda(k/M)\hat{R}(k)\cos(wk)$, and $\hat{R}(k)$, the kth-order autocorrelation, is $N^{-1}\sum_{i=1}^{N-|k|}(R_q - \bar{R}_q)(R_{q+|k|} - \bar{R}_q)$, with $\bar{R}_q$ the mean of the short-horizon return.

C Calculating True Skewness

In this appendix, we show how to analytically calculate the true skewness of dollar returns under the GBM, SV, or SVJ stochastic processes in Section 2.3 from the moment generating functions of those same processes. To do so, we first express the true skewness of the dollar

[25] To be specific, the estimator is the square root of the number of observations over the original sample window times the variance of the sample mean of the variable calculated from block bootstrap samples.

return from time t to $t+\tau$ as a function of the moment generating functions:

$$TrueSkewness_{t,t+\tau} = \frac{M^{(3;x)}_{t,t+\tau} - 3M^{(1;x)}_{t,t+\tau}M^{(2;x)}_{t,t+\tau} + 2(M^{(1;x)}_{t,t+\tau})^3}{(M^{(2;x)}_{t,t+\tau} - (M^{(1;x)}_{t,t+\tau})^2)^{\frac{3}{2}}}, \tag{C1}$$

where $M^{(n;x)}_{t,t+\tau}$ is the moment generating function for the nth moment of the dollar return from time t to $t+\tau$ of stochastic process x. As shown in, for example, Bates (2006), the moment generating functions $M^{(n;x)}_{t,t+\tau}$ associated with the three processes are given by:

$$M^{(n;GBM)}_{t,t+\tau} = e^{(\alpha\tau - \frac{1}{2}\sigma^2\tau)n + \frac{1}{2}\sigma^2\tau n^2}, \tag{C2}$$

$$M^{(n;SV)}_{t,t+\tau} = e^{\alpha\tau n + \phi^{SV}(n,\tau) + \psi(n,\tau)V(t)}, \tag{C3}$$

$$M^{(n;SVJ)}_{t,t+\tau} = e^{\alpha\tau n + \phi^{SVJ}(n,\tau) + \psi(n,\tau)V(t)}, \tag{C4}$$

where $\psi(n,\tau)$, $\phi^{SV}(n,\tau)$, and $\phi^{SVJ}(n,\tau)$ are defined as:

$$\psi(n,\tau) = \frac{Q(n)\alpha_+(n) - \alpha_-(n)e^{P(n)\tau}}{Q(n) - e^{P(n)\tau}}, \tag{C5}$$

$$\phi^{SV}(n,\tau) = \kappa_v\theta_v\left(\alpha_+(n)\tau + \frac{\alpha_-(n) - \alpha_+(n)}{P(n)} ln\frac{Q(n) - e^{P(n)\tau}}{Q(u) - 1}\right), \tag{C6}$$

$$\phi^{SVJ}(n,\tau) = \kappa_v\theta_v\left(\alpha_+(n)\tau + \frac{\alpha_-(n) - \alpha_+(n)}{P(n)} ln\frac{Q(n) - e^{P(n)\tau}}{Q(n) - 1}\right) + \lambda E(n), \tag{C7}$$

and:

$$Q(n) = \frac{\alpha_-(n)}{\alpha_+(n)}, \tag{C8}$$

$$\alpha_\pm(n) = \frac{\kappa_v - n\rho\sigma_v \pm P(n)}{\sigma_v^2}, \tag{C9}$$

$$P(n) = \sqrt{(\kappa_v - \sigma_v\rho n)^2 + \sigma_v^2(n - n^2)}, \tag{C10}$$

$$E(n) = e^{\mu_z n + \frac{1}{2}\sigma_z^2 n^2} - (1 + \bar{\mu}n). \tag{C11}$$

D The Variables Underlying the Skewness Proxies

In this appendix, we offer more details on the calculations of the variables underlying *OLSSkew*, *QuantileSkew*, and *LogitSkew*. Starting with *OLSSkew*, Boyer et al. (2010) use historical volatility ($HistoricalVolatility$) and the historical skewness coefficient ($HistoricalSkew$) calculated from the 60 months of daily data directly before month $t-59$, the compounded stock return over months $t-71$ to $t-61$ (*Momentum*), and the average of daily share volume to daily shares outstanding over month $t-60$ (*ShareTurnover*) as their continuous underlying variables. They further add several dummy variables, including a NASDAQ dummy equal to one if a stock is traded on the NASDAQ at the end of month $t-60$ and else zero ($NASDAQ$), 16 industry dummies indicating whether a stock operates in one of the 17 Fama-French industries at that time, and two market size dummies, the first (second) dummy equal to one if a stock's market size is in the bottom (middle) tercile of the market size distribution at that time and else zero.

In our calculations of *QuantileSkew*, we choose the same underlying variables as Boyer et al. (2010), with the minor difference that we use the log of market size (*MarketSize*), and not the market size dummies, to capture size effects. In line with Conrad et al. (2014), we further add the number of years since a stock became publicly traded (*CompanyAge*), the log ratio of gross property, plant, and equipment to total assets (*AssetTangibility*), and log sales growth over the prior year (*SalesGrowth*). We finally also use a stock's log book-to-market value ratio (*BookToMarket*), log share issuances (*ShareIssuances*), log total assets growth (*AssetGrowth*), and profitability (*Profitability*). *ShareIssuances* is the annual change in split-adjusted shares outstanding, *AssetGrowth* the annual change in total assets, and *Profitability* is the ratio of sales net of costs of goods sold, selling, general, and administrative expenses, and interest expenses to the book value of equity. Heeding Fama and French (1992), we compute the book equity value as total assets less total liabilities plus deferred taxes less preferred stock.

To compute *LogitSkew*, Conrad et al. (2014) also choose *MarketSize*, *Momentum*, *CompanyAge*, *AssetTangibility*, and *SalesGrowth* as underlying variables. They further also consider historical volatility, historical skewness, and share turnover, but calculate those differently from *HistoricalVolatility*, *HistoricalSkew*, and *ShareTurnover*. To be specific, Conrad et al. (2014) calculate historical volatility from daily dollar returns over the previous three months and historical skewness from daily log returns over that same three-month period, in each case assuming that the expected daily return is equal to zero. They calculate share turnover as the ratio of daily trading volume to daily shares outstanding averaged over the prior six months of data minus that same ratio averaged over the prior 18 months of data.

See Table D.1 in this appendix for a concise summary of how we construct the underlying variables, including the mnemonics used by the data providers.

Table D.1.
Variable Construction

The table details the construction of our skewness proxies. We calculate those variables indexed by an "M" on a monthly basis. Conversely, we calculate those indexed by an "A" on an annual basis, using the calculated values from June of calendar year t to May of calendar year $t+1$. We show the CRSP and Compustat database mnemonics in parentheses.

Name	Description
	Panel A: Skewness Proxies
$OLSSkew$ (M)	Prediction from a cross-sectional regression of the skewness coefficient of a stock's daily dollar return (ret) on predictor variables, where the coefficient is estimated over the prior 60 months and the predictors are measured at the start of the 60-month period (see Boyer et al. (2010)).
$QuantileSkew$ (M)	Skewness coefficient calculated from the predictions of quantile regressions of a stock's annual dollar return (ret) on predictor variables, where the predictors are measured at the start of the twelve-month return period and the regressions are run over the prior 20 years of monthly data.
$HistoricalSkew$ (M)	Skewness coefficient calculated from a stock's daily dollar returns (ret) over the prior 60 months.
$LogitSkew$ (M)	Prediction from a logit model of a dummy variable equal to one if a stock's twelve-month log return (ret) exceeds 100% and else zero, on predictor variables, where the predictors are measured at the start of the twelve-month period and the model is estimated recursively (starting in June 1951) and only on June values (see Conrad et al. (2014)).
$MaxRet$ (M)	A stock's maximum daily dollar return (ret) over the month (see Bali et al. (2011)).
	Panel B: Skewness Proxy Predictor Variables
$MarketSize$ (M)	Log of stock market capitalization (abs(prc) $\times$ shrout; see Fama and French (1992)).
$BookToMarket$ (A)	Log of the ratio of the book value to the market value of equity (abs(prc) $\times$ shrout), where the book value of equity is total assets (at) less total liabilities (lt) plus deferred taxes (txditc, zero if missing) less preferred stock (pstkl, pstkrv, prfstck, or zero, in that order of availability) and the variables are from the fiscal year end in calendar year $t-1$ (see Fama and French (1992)).
$Momentum$ (M)	A stock's dollar return (ret) compounded over the prior twelve months of monthly data, but excluding the most recent month (see Jegadeesh and Titman (1993)).
$ShareIssuances$ (A)	Log of the gross percent change in split-adjusted shares outstanding from the fiscal year end in calendar year $t-2$ to that in calendar year $t-1$, where split-adjusted shares outstanding is shares outstanding (csho) times the adjustment factor (ajex; see Fama and French (2008)).
$AssetGrowth$ (A)	Log of the gross percent change in total assets (at) from the fiscal year end in calendar year $t-2$ to that in calendar year $t-1$ (see Cooper et al. (2008)).
$Profitability$ (A)	Ratio of sales (sale) net of costs of goods sold (cogs), selling, general, and administrative expenses (xsge), and interest expenses (xint) to the book value of equity, where the book value of equity is total assets (at) less total liabilities (lt) plus deferred taxes (txditc, zero if missing) less preferred stock (pstkl, pstkrv, prfstck, or zero, in that order of availability) and the variables are from the fiscal year end in calendar year $t-1$ (see Fama and French (2008)).
$ShareTurnover$ (M)	Average of the ratio of share volume (vol) to shares outstanding (shrout), where the average is taken over all trading days over the month (see Boyer et al. (2010)).
$CompanyAge$ (M)	Number of years since a stock first appeared in CRSP (see Conrad et al. (2014)).
$AssetTangibility$ (A)	Log of the ratio of gross property, plant, and equipment (ppegt) to total assets (at), where both variables are taken from the fiscal year end in calendar year $t-1$.
$SalesGrowth$ (A)	Log of the gross percent change in sales (sale) from the fiscal year end in calendar year $t-2$ to that in calendar year $t-1$ (see Conrad et al. (2014)).
$HistoricalVolatility$ (M)	A stock's annualized volatility derived from daily dollar returns (ret) over the prior 60 months.
$HistoricalSkew$ (M)	A stock's skewness coefficient derived from daily dollar returns (ret) over the prior 60 months.