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DOI:

https://doi.org/10.1109/ICHQP46026.2020.9177873

Document Version

Accepted author manuscript

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Rodrguez-Pajaron, P., Hernandez-Bayo, A., & Milanovic, J. V. (2020). Probabilistic Assessment of Harmonics in a Residential Network. In 2020 19th International Conference on Harmonic and Quality of Power (ICHQP) (pp. 23-28). IEEE. https://doi.org/10.1109/ICHQP46026.2020.9177873

Published in:

2020 19th International Conference on Harmonic and Quality of Power (ICHQP)

Citing this paper

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Probabilistic Assessment of Harmonics in a Residential Network

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Abstract—This paper presents a framework to assess voltage harmonic levels of future distribution networks with increasing participation of nonlinear loads and photovoltaic distributed generation. The methodology proposed is used to assess harmonic impact up to 7^{th} order and THD degradation in future scenarios up till year 2030 in low voltage residential networks. Stochastic behaviour of loads and photovoltaic generation is considered in the model by applying a probabilistic approach that accounts for uncertainties in electrical devices allocation and performance. A Monte Carlo simulation is performed for every scenario studied in order to obtain a probabilistic assessment of harmonic impact.

The methodology proposed is applied to the IEEE European Low Voltage network and results are compared to standard EN50160 limits as the reference framework for low voltage power quality indices.

Index Terms—distribution network, EN50160, harmonic analysis, power quality, probabilistic analysis.

I. INTRODUCTION

In recent years, Power Quality (PQ) issues have attained considerable attention in distribution networks [1]. This interest has aroused due to the widespread use of nonlinear, power electronics based loads [2], and increasing integration of power electronic interfaced generation such as photovoltaic (PV) in distribution networks [3]. Among all PQ events, harmonics are one of the most concerning disturbances in residential networks since power electronic based devices connected in domestic environments produce non sinusoidal currents [2]. The loads, linear and nonlinear, switch randomly and involve high level of uncertainties in both, performance and harmonic emission. PV generation is also characterized by a stochastic behaviour. To assess the impact of these technologies, deterministic formulations for harmonic analysis are not representative of the true performance of the distribution system if the uncertainty associated with input data are accounted for [4]. This paper aims to assess voltage harmonic levels of future distribution networks with increasing participation of nonlinear loads and photovoltaic distributed generation unsing a probabilistic approach in order to take into account the

uncertainties related to the random performance and allocation of harmonic sources.

In past studies, [5], [6] harmonic assessment in distribution networks has been conducted mostly in a deterministic way. In [7] a probabilistic approach for the analysis of low voltage networks is proposed but not focusing on PQ disturbances. In [8], [9] harmonic distortion is studied from a probabilistic point of view but PV generation and its associated uncertainty are not considered. Reference [10] presents a very detailed method for stochastic harmonic source modeling but, in this case, the compatibility of emission levels with applied PQ standard limits is not studied. The method proposed here focuses on the probabilistic assessment of harmonic voltage distortion for a Low Voltage (LV) distribution network and the analysis of compliance with present and future emission limits. This study focuses on PV distributed generation and generic nonlinear domestic loads. The methodology proposed can be extended for assessing the impact of other technologies that may degrade power quality and even for different power quality disturbances.

II. OVERVIEW OF THE PROBABILISTIC METHODOLOGY

As previously explained, in order to take into account the stochastic behaviour of devices connected to distribution network, a probabilistic approach based on Monte Carlo method is used for the assessment of the harmonic impact. Due to all the uncertainties existing in the input parameters of the simulation, one single simulation would not be representative of the real state of the whole network. Monte Carlo methodology allows repeating simulations with different, random generated, inputs which yield the output results that can be assessed and compared to the limits specified in international standards using probabilistic measures and indices.

In Fig. 1, the general flowchart of the proposed framework is presented. The proposed process follows these steps:

- 1) Definition of the scenario: the whole process shown in this flowchart is run for each scenario analysed.
- 2) Generation of stochastic power demand profiles for residential loads.

978-1-7281-3697-4/20/\$31.00 ©2020 IEEE

This work was supported by the Ministerio de Ciencia, Innovación y Universidades, Spain, under Project RTI2018-097424-B-I00, and by the ETSII-UPM under project Primeras Ayudas de Investigación ETSII-UPM19-PM02.

- 3) Random allocation of load profiles to network buses.
- 4) Random allocation of PV generation to network buses and calculation of PV generated power profile.
- Simulation of harmonic injection caused by nonlinear loads and PV generation.
- 6) Assessment of harmonic levels.
- Steps 3) 6) are repeated numerous times in the Monte Carlo simulation in order to get probabilistic assessment of PQ harmonic indices.

The steps indicated in the flowchart are described in more detail in the following sections.



Fig. 1. Flowchart of the proposed probabilistic harmonics assessment method.

III. SCENARIOS AND STOCHASTIC INPUTS

A. Studied Scenarios

In this study, the evolution from the present situation to a expected future perspective (year 2030) is analysed. Therefore, scenarios for years 2018, 2021, 2024, 2027 and 2030 are considered. In addition, for each year, winter and summer studies are carried out separately so that conclusions can be extracted for different seasons. Therefore, the total number of simulated scenarios is 10 (five years and two seasons). Each year scenario is characterized by different penetration level of nonlinear domestic loads, different penetration of PV generation and different average domestic occupancy according to the following estimations:

- Nonlinear percentage of domestic loads: According to estimations proposed in [2], up to 60% of residential loads in 2030 will be power-electronic-based. In 2018 the estimated proportion is 40% and linear interpolation is assumed for the years in-between.
- PV penetration: Penetration level of a PV technology can also be considered as probability of finding it in a specific dwelling. In this study, a penetration varying from 10% in 2018 to 40% in 2030 is assumed, based on [10].
- Domestic occupancy: Number of occupants per residence is updated before generating domestic load curves.

Estimations are extracted from Spanish statistics and forecasts [11] as it can be seen in Table I.

 TABLE I

 FORECAST DISTRIBUTION OF OCCUPANTS PER DWELLING [11]

| Number of occupants | Year | | | | |
|---------------------|-----------------------------|------|------|------|------|
| | 2018 | 2021 | 2024 | 2027 | 2030 |
| | Percentage of dwellings (%) | | | | |
| 1 | 25.5 | 26.2 | 26.8 | 27.5 | 28.4 |
| 2 | 30.4 | 30.5 | 30.7 | 31.0 | 31.0 |
| 3 | 20.8 | 20.5 | 20.0 | 19.6 | 19.0 |
| 4 | 17.5 | 17.2 | 16.7 | 16.2 | 16.0 |
| >4 | 5.8 | 5.6 | 5.8 | 5.7 | 5.6 |

B. Residential Loads Profiles

The residential load loading profile is characterized by the stochastic behaviour of domestic consumers. In order to simulate realistically this behaviour, the model provided by [12] is used in this study. This tool is based on Markov chains [13] and provides 1-minute resolution profiles for demand of residential loads based on the behavior of domestic customers. In this study, the profiles provided by [12] have been modified to a resolution of 10 minutes as required by standard EN50160 [14].

In order to randomize the process, a pool of 100 weekly load profiles have been generated for three occupancy levels (few, many and average) and for both seasons, summer and winter. In total, 600 weekly load profiles have been generated. An example of one residential load curve for both summer and winter is shown in Fig. 2. According to the scenario considered (winter or summer and year characterized by a different distribution of occupancy level) weekly load profiles are randomly selected from the appropriate pool.



Fig. 2. Weekly loading profile for a random dwelling in winter and summer.

C. PV generation

In this study, PV generation is installed at some of the dwellings. In the proposed methodology there are several variables randomly selected:

- PV size: Considering that the rating of PV generation can vary appreciably, in this study, according to [7], the rated powers of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 or 4.0 kW are assumed with probabilities of 1%, 8%, 13%, 14%, 14%, 12%, 37% respectively. All PV devices are single-phase and low voltage.
- Weekly temperature and solar irradiation are considered as stochastic variables. In this case, as in [12], the values

considered are the same as those that have been used in a certain week for the generation of the load profile.

With these variables randomly selected, a weekly power generation profile is determined with resolution of 10 minutes for each PV device. This profile is calculated by taking into account efficiency variation of panels with temperature and irradiation according to the model provided by the applied simulation tool (OpenDSS). For all PV devices simulated power factor is assumed to be 1 for the PV and inverter device. In Fig. 3, the weekly generation profile of a random PV device is showed for winter and summer.



Fig. 3. Weekly PV generation profile for a random PV device.

D. Allocation of Load Profiles and PV Generation

According to the penetration level of PV in each study, buses with installed PV devices are randomly selected and allocated. Load profile curves according to the scenario considered (winter or summer and level of occupancy of the studied year) are also randomly allocated to network buses.

IV. HARMONIC ANALYSIS

According to the scenario (year) considered, the appropriate percentage of non linear loads in the total load level of each dwelling is considered. This nonlinear load level varies from 40% in 2018 up to 60% in 2030 [2]. In order to estimate the harmonic emission of non linear loads, the estimations of [4], [15] are adopted. According to this approach, harmonic spectrum of aggregated residential profiles follows a distribution with random harmonic magnitudes and phases with different composition of load types for different time of the day. Two types of loads are considered (type a and type b). The magnitude and phase of 3^{rd} , 5^{th} and 7^{th} follow a uniform distribution between the limit interval values shown in Table II for both types of loads. The contribution of loads of type a and type b to the total non linear load is obtained from a normal distribution with mean and typical standard deviation that vary depending on the time of the day as shown in Table III.

For assigning harmonic spectrum for each PV device, the injected harmonic currents are considered according to [16].

For the study of harmonic propagation, residential loads and PV devices are modelled as constant P and Q for the fundamental frequency, and as Norton equivalent for higher frequencies, as proposed in [17].

 TABLE II

 HARMONIC SPECTRUM FOR RESIDENTIAL LOADS [4]

| Harmonic current spectrum for residential loads | | | | | |
|---|-----------|------------|-----------|--|--|
| Harmonic order | | Type a | Type b | | |
| 3^{rd} | Mag [%] | 10 - 20 | 50 - 70 | | |
| | Phase [°] | 230 - 290 | 120 - 180 | | |
| 5^{th} | Mag [%] | 5 - 10 | 40 - 60 | | |
| | Phase [°] | 90 - 150 | 200 - 260 | | |
| 7^{th} | Mag [%] | 2 - 4 | 30 - 40 | | |
| | Phase [°] | 40 - (-10) | 200 - 260 | | |

 TABLE III

 WEIGHTED CONTRIBUTION OF EACH OF THE HARMONIC TYPE [4]

| Period (hrs) | f_a | | f_b | |
|---------------|-------|------|-------|------|
| r choù (ilis) | μ | σ | μ | σ |
| 8:00 - 18:00 | 0.1 | 0.03 | 0.9 | 0.3 |
| 18:00 - 8:00 | 0.6 | 0.2 | 0.4 | 0.12 |

V. POWER QUALITY PROBABILISTIC INDICES

Load profiles and PV generation show a great variability along the time and from one week to another. In order to take into account this stochastic behaviour in the assessment of harmonic levels, it is necessary to approach the problem by means of a probabilistic method based on repeating many iterations, each corresponding to a different week simulated with random input values according to the probability distributions previously described. Therefore, following a Monte Carlo approach, harmonic propagation in all the 10-minute intervals of a week is repeated over a certain number of weeks until a reliable probability distribution of output values is obtained. In this case, the outputs of the method are voltage harmonics of order 3, 5 and 7 and THD at all system buses. To assess power quality in distribution networks a common technique is comparing the simulated values to a threshold or standard value. In this study, the commonly used in Europe EN50160 [14] is adopted. According to this standard, during 95% of the week, harmonic disturbances calculated in 10 minutes intervals should not exceed the limit values for each harmonic voltage magnitude or voltage THD, shown in Table IV. In each iteration or simulation of the Monte Carlo method a value for the 95 percentile of voltage harmonics is calculated. This value is subjected to uncertainty since it has been obtained from uncertain inputs. In order to assess statistically its probability, the simulation is rerun over a large number of iterations so that the results obtained in the set of simulations can be representative of all possible PQ output values.

VI. CASE STUDY

A. Network studied

For this study, the IEEE European low voltage (LV) test feeder has been adopted [18]. This is a low voltage radial test grid based on a distribution system with fundamental frequency of 50 Hz, that represents the typical configuration of European low voltage networks. The simplified one-line diagram of the feeder is shown in Fig. 4. This test feeder consists

TABLE IV EN50160 compliance limits (95% of time during one week, 10 minutes intervals)

| Voltage harmonics | | |
|-------------------------------|------------------------|--|
| Harmonic order h | Relative amplitude (%) | |
| 3 | 5 | |
| 5 | 6 | |
| 7 | 5 | |
| Total Harmonic Distortion (%) | | |
| THD | 8 | |

on 906 buses and 55 residential loads each representing an individual domestic consumer. The feeder is connected to the medium voltage by a Dy 800 kVA transformer substation that steps the voltage down from 11 kV to 416 V. All the residential loads are single-phase and are allocated to the phase of the system indicated by the specification of the network model.



Fig. 4. One-line diagram of the LV European test feeder.

B. Simulation Characteristics

All simulations carried out in this study have been performed according to the methodology proposed and by means of the power system simulator OpenDSS [19]. OpenDSS is a tool for distribution system analysis, including power flow and harmonic propagation analysis among others. For each scenario defined, a Monte Carlo approach consisting on 100 simulations has been applied. The generation of random input data and the control of the iteration loop has been performed through interaction of OpenDSS with Matlab. Simulations are run with Intel Core i7-7700 CPU @ 3.6 GHz with 32 GB RAM. When performing one study considering odd harmonic orders up to 7th, the computation time is around 180 minutes.

VII. RESULTS

As previously mentioned, simulations have been run for years 2018, 2021, 2024, 2027 and 2030 and for winter and summer scenarios. Furthermore, for each scenario, a base case with no PV generation has been established in order to analysed the effect of PV generation. Therefore, the performed study allows to draw conclusions, among others, about the following aspects:

- 1) Comparison of summer and winter scenarios.
- 2) Influence on harmonic levels of PV generation.

- 3) Analysis of future evolution of harmonic distortion trends in the network.
- 4) Identification of the worst locations and maps of problems detection.

The main results of this study are shown in the following sub-sections.

A. Aggregated Power Curves for Summer and Winter Scenarios

In Fig. 5, the mean value of aggregated residential load profiles and PV generation profiles among all 100 simulations are shown for year 2018 scenario. Considering a different year does not imply any significant changes in demand except those caused by the small variation in the expected average dwelling occupancy. As years move forward, the share of PV generation will increase, because PV penetration is increasing. The curves shown in Fig.5, show a great difference between summer and winter average power demand in dwellings, mainly motivated by high electric heating penetration. The other interesting fact that can be observed is the reduced PV generation during the winter. Both of these effects are deeply influenced by the model selected for generating load profiles [12]. This study is based on climate data from Sweden, and so winters are extremely cold and with low sun irradiation. These considerations influence all the results presented next.



Fig. 5. Mean weekly aggregated load profile and PV generation profile for all 55 residences in 2018.

B. Evolution of Harmonics System Performance

THD and voltage harmonic magnitude determined through the simulation can be compared to EN50160 limits, already described in Section V.

Results for the 5^{th} and 7^{th} harmonic show that the injection of both harmonics cause small voltage harmonic magnitudes far from the limit threshold. Therefore, voltage limits for the 5^{th} and 7^{th} are never exceeded for more than 5% of the time (i.e., the 95 percentile compliant with the standard) and figures showing values of these two harmonic orders are omitted in this section.

For the 3^{rd} harmonic and for THD in the summer scenario, there is a similar situation (no limits exceedance). However, in winter scenario, a very different situation is obtained. Fig.6 and Fig.7 show the percentage of clients where the harmonic limits are violated, both, the THD and 3^{rd} harmonic limits, and its evolution along the years.

These figures represent the average percentage \pm three times the standard deviation of consumers with non-compliance of

limits. This interval corresponds to 99.75% of the samples. In other words, the percentage of clients with THD or 3^{rd} harmonic limit exceedance is inside the marked interval with 99.75% probability. In Fig. 6, it can be seen that although the number of clients with distortion problems is reduced in 2018, this number is expected to grow significantly along the years to reach an important value in 2030. This effect is even sharper for the 3^{rd} harmonic as shown in Fig. 7, where in 2030, around that 70% of the costumers will experience voltage levels of 3^{rd} harmonic over the established threshold.

On the other hand, Fig.6 and Fig.7 also show a very low influence of the PV generation on harmonic levels. This is mainly caused by small magnitude of harmonic currents injected by PV generation in winter scenario (with very low generation) compared to domestic loads.



Fig. 6. Percentage of clients with THD exceeding limits in winter scenarios.



Fig. 7. Percentage of clients with 3^{rd} harmonic exceeding limits in winter scenarios.

C. Harmonic Performance at Specific Sites

Previous analyses have been focused on the system performance as a whole, but they can disguise differences between buses inside the grid. To allow identification of individual buses performance, Fig. 8 shows the box plot corresponding to the THD value exceeded during 5% of the week (considering the 100 simulations performed). In the upper part, the results for 2018 are shown and it can be seen that the most buses have a 95th THD percentile below the 8% EN50160 limit and that this limit is violated only at some atypical weeks and at some buses. However, in 2030 the situation is drastically different. Different buses have different behaviours but many of them exceed the 8% limit. Similar conclusions but even more severe are drawn for the 3^{rd} harmonic voltages according to the box plot shown in Fig. 9.

As a summary, Fig. 10 presents a colour coded graphical interpretation of the results in the network studied. In the figure, each load bus is coloured according to the probability of occurrence of a harmonic problem (non-compliance of THD or 3^{rd} harmonic limits). A dark red colour corresponds to 100% probability of exceeding any of the EN50160 harmonic limits. Scenarios corresponding to winter 2018 and 2030 are shown in Fig. 10. It can be seen that 2030 scenario is much more



Fig. 8. Box plot of THD values exceeded 5% of time at buses for network load buses in winter 2018 and 2030 (100 simulations).



Fig. 9. Box plot of 3^{rd} harmonic levels exceeded 5% of time at network load buses in winter 2018 and 2030 (100 simulations).

severe with many load buses with high probability of noncompliance occurrence. In addition, it can also be observed that nodes that are closest to the substation appear to have less problems than nodes that are further away.

D. Detailed Analysis at a Specific Site

According to previous analysis, the load bus 52 is identified as the worst load bus regarding harmonic levels. Therefore, more detailed analysis of its harmonic performance is carried out. In Fig. 11, statistical distribution of 3^{rd} harmonic voltage levels is shown for year 2018 and 2030. In both scenarios, it can be seen very clearly that the frequency of occurrence follows a typical bi-modal distribution. This fact is caused by the existence of two different groups of loads (type *a* and type *b*) with different harmonic emission, as shown in III. It can be seen that the EN50160 limit value (also plotted) is exceeded mainly because of the distribution of voltage harmonics frequencies caused by the loads of Type *b*. This effect is even more pronounced in the 2030 scenario.



Fig. 10. Probability of harmonic problems for load nodes in winter 2018 and 2030 PV scenarios.



Fig. 11. Frequency of occurrence of 3^{rd} harmonic relative magnitude (U_3/U_1) at node 52 for winter 2018 and 2030.

VIII. CONCLUSION

This paper presented a probabilistic method to assess harmonic impact of nonlinear load and PV generation on distribution residential networks and illustrated it on a range of different scenarios considering future increase of penetration of nonlinear load and PV generation.

The results have shown that, while 5^{th} and 7^{th} harmonic do not jeopardise the harmonic limit compliance of low voltage network with the standard EN50160, increasing amount of nonlinear loads may lead to voltage harmonic problems (both at individual harmonic level and THD) caused by the increased injection of the 3^{rd} harmonic.

It has been found that the nonlinear load penetration is critical for harmonic distortion levels, and that PV generation influences very little those levels.

The procedure proposed here can be easily extended to investigate the effect of other technologies on harmonic levels, as well as on other types of PQ phenomena and on other types of costumers (commercial, industrial).

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