OPTIMAL DESIGN OF FILTERLESS OPTICAL NETWORKS

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A thesis
in
The Department
of
Computer Science and Software Engineering

Presented in Partial Fulfillment of the Requirements
For the Degree of Doctor of Philosophy
Concordia University
Montréal, Québec, Canada

March 2021

Concordia University
School of Graduate Studies

This is to certify that the thesis prepared

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Entitled: **Optimal Design of Filterless Optical Networks**

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Computer Science)

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Abstract

Optimal Design of Filterless Optical Networks

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Filterless optical network has been widely used in recent years. The incentive of this technology is only the passive equipment will be used, which requires no electricity. By using this technology, not only the cost reductions, but also the environment preservation will be achieved.

In literature, a lot of researchers studied the design of filterless optical network. But due to the complexity and scalability limits of this problem, most of the works are based on heuristic or meta-heuristic methods. We were seeking exact solutions to achieve the design of filterless optical networks.

First we proposed a one step solution scheme, which combines tree decomposition and network provisioning, i.e.,routing and wavelength assignment within a single mathematical model, called CG_FOP. We propose a decomposition with two different sub-problems, which are solved alternately, in order to design an exact solution scheme. The first sub-problem generates filterless sub-nets while the second deals with their wavelength allocation.

Due to the complexity of the problem, significant time will be consumed if applied our model on a large and more connected network. In order to improve the performance, we proposed Dantzig-Wolfe decomposition model, called DW_FOP in which the sub-problem consists in generating a potential filterless optical sub-network, with a directed tree topology. In this new model, single pricing problem was formed which compute the network provisioning along with wavelength assignment together. In this way, master problem would be simplified, no longer contains complicated logic to build conflicts among requests. With this approach, computation time significantly

reduced.

To further improve the design, we proposed a nested column generation model, called NCG_FOP, in order to speed up the solution process. We break down the solution into two level of pricing, the upper level pricing computes selected paths which assigned to granted requests, network provisioning and wavelength assignment for granted requests. The upper level pricing itself is a column generation process, which includes a lower level pricing generated improved path for each granted requests.

Acknowledgments

First and foremost, I would like to express my sincere gratitude to my supervisor Dr. Jaumard. Without her continuous guidance, support, and patience, this work would not have accomplished. Her wide knowledge and brilliant ideas have been exceptionally invaluable to me.

I would like to extend my gratitude to my examination committee for taking the time to read and evaluate this thesis. Special thanks to Dr. Shiri Nematollah taking time to comment and suggest modifications of my thesis.

I am grateful to all my student colleagues for their help and support during these years.

Last but not least, I want to express my heartfelt gratitude and appreciation to my family, I could not have completed my thesis without their support and encouragement.

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Glossary

CG Column Generation. 2

ILP integer linear programming. 4

LR Lagrangian Relaxation. 4

NCG Nested Column Generation. 4

PP pricing problem. 3

RMP restricted master problem. 3

ROADMs Re-configurable OpticalAdd-Drop Multiplexers. 1

WDM wavelength-division multiplexing. 1

Chapter 1

Introduction

1.1 General Background

Internet traffic has been growing rapidly for many years. Compared to copper cables, optical networks provide enormous capacities in the network. It is the preferred medium for transmission of data due to its high speed. Optical networks are widely deployed today in all kinds of telecommunications networks.

The components used in modern optical networks include couplers, lasers, photodetectors, optical amplifiers, optical switches, and filters and multiplexers [26].

Couplers are the simple components used to combine or split optical signals, which usually were called splitters/combiners. They are passive optical devices.

Filters and multiplexers are used to multiplex and de-multiplex signals at different wavelengths in WDM systems.

In traditional active switching networks, filters and multiplexers are used on the intermediate nodes, in order to perform Re-configurable Optical Add-Drop Multiplexers (ROADMs). This would result in increasing the cost.

Filterless optical networks are based on advanced transmission technologies and passive optical interconnections between nodes. Passive splitters and combiners are used at the intermediate nodes to interconnect the fiber links, which could eliminate or minimize the usage of active photonic re-configurable components. They exhibit several advantages. In particular, they reduce power consumption and consequently, allow both reduced costs and footprint. In addition, they improve the network robustness, while simplifying several aspects of impairment-aware design [36].

The major advantage of filterless optical network is using the passive equipment only, which requires no electricity. This could not only achieve cost reductions, but also enable the environment friendly as well. The disadvantage of this architecture is that more wavelength would be used, at the cost of wasting spectrum on the network. Pavon-Marino *et al.* [25] provided the analysis result of the wasted spectrum in the static planning case (filterless) vs dynamic planning case (filtered node). There is no significant extra penalty in spectrum efficiency caused by filterless nodes. It concluded that the filterless optical network is the most cost effective architecture.

Due to the nature of filterless optical networks, the signal of the lightpaths will continue downstream even after the intended destination has been reached. While this drawback can be overcome with blockers, in order to maximize the cost-effective, we still focus on the minimization of the number of required wavelengths, without assuming the usage of blockers.

1.2 Research Projects

In this research, we studied the optimal design of filterless optical networks, in order to minimize the wavelength usage across the whole network.

We proposed different exact solution schemes for the provisioning of filterless networks, which combined the network routing and wavelength assignment into the same mathematical model.

In our approach, column generation technique has been used as a basis of solution schemes, in order to efficiently solve large scale optimization problems.

Column Generation(CG) method is a well-known technique for solving large-scale optimization problems efficiently. The key step in the design of exact algorithms for a large class of integer programs is to embed column generation techniques within a linear-programming-based branch-and-bound framework[20]. The basic idea of column generation is, it converted the original problem into a restricted version, only contains very limited variables/columns, which defined as restricted master problem(RMP), and a pricing problem(PP) which generates improved column based on linear programming principle. V. Chvatal *et al.* [7] present how to create an improved column during the linear relaxation.

In order to improve the quality of the solution, a one step solution scheme was proposed, which combines network provisioning, i.e., routing and wavelength assignment within a single mathematical model. A more efficient model was proposed to improve the performance. At the last stage of our research, we enhanced our design, replaced the detailed link formulation by path formulation inside our solution schema, which improved performance significantly. We also introduced nested column generation into our solution schema, in order to efficiently generate qualified paths. Lagrangian Relaxation technique is also used to obtain valid bound for nested column generation model.

In order to verify the genericity of our design approach, we also applied our solution to the networks with non-single unit traffic cases.

1.3 Contributions of the Thesis

This thesis presents the design of filterless optical networks, proposed different models to solve the problem efficiently. The main contributions of this thesis are concluded into three journal papers:

1. Paper 1 (submitted in [17]): In the literature, a lot of researchers solved the problem by meta-heuristic. In this paper, we proposed a one step solution scheme, which combines network provisioning, i.e., routing and wavelength assignment within a single mathematical model, called CG_FOP. Decomposition into two different types sub-problems is then used in order to conceive an exact solution scheme. The first type of sub-problem relies on the generation of filterless subnetworks while the second one takes care of their wavelength assignment. These two pricing problem provided the configuration into the single

master problem. Compared to the literature, the results of our solution are improved.

2. Paper 2 (submitted in [15]): In this paper, we proposed a Dantzig-Wolfe decomposition model, called DW_FOP, in which the single sub-problem consists in generating a potential filterless optical sub-network, with a directed tree topology. We design an exact solution process of the resulting Integer Linear Program(ILP) model. We improved our design, with an simplified mathematical programming model, as well as an efficient exact solution process. This translated into significant reduced computational times and further improved accuracy for the output designs.

3. Paper 3 (submitted [14]): We continue to improve the design, proposed nested column generation(NCG) model, called NCG_FOP, to speed up solution process. We break down the solution into two level of pricing, the upper level pricing computing selected paths which assigned to granted requests, network provisioning and wavelength assignment for granted requests. The upper level pricing itself is a column generation process, which consist a lower level pricing generated improved path for each granted requests. In order to get valid bound for our problem, Lagrangian relaxation(LR) technique was used to improve LP bound. Further more, we also applied our solution to the networks which generated non-single unit traffic, in order to verify the generality of our solution scheme.

1.4 Organization of the Thesis

The thesis is organized as follows. This chapter provides a general introduction to filterless networks, the goal of the research, i.e., the optimal design of filterless optical networks using large scale optimization models and methods, and presents the key contributions.

Each of the next 3 chapters corresponds to a paper submitted for publication. Therefore, the thesis contains some repetitions, in particular for the literature review, together with the problem statement and the notation. Chapter 2 describes a first large scale optimization model, with two types of sub-problems, one for the design of filterless subnet, and the other for their "coloring", i.e., the wavelength assignment. This first model allows a first improvement over the state of the art, with the improvement of the solution for several open source data sets.Chapter 3 presents a new optimization model, contains only one single sub-problem for the design of filterless subnet with the coloring functionality included. The improvement of this model aligned on the performance and scalability. Chapter 4 proposes a nested column generation model, which contains two levels of pricing problems. Upper level pricing generated the network provisioning based on path formulations, and the lower level pricing provided upper level pricing improved paths. And also embedded Lagrangian relaxation technique into the solution to improve the solution bound. Chapter 5 concludes the thesis and provides possible directions of future work for further improvement of the optimal design of filterless optical networks.

Chapter 2

A Two Sub-problem Decomposition for the Optimal Design of Filterless Optical Networks

This chapter contains a paper submitted for publication into [17]. A short preliminary version of it was published in the conference ICTON [16], which proposed a mathematical mode to minimize the maximum link capacity of the network, and perform wavelength assignment as a post-processing step.

In this chapter, We propose a one step solution scheme, which combines tree decomposition and network provisioning, i.e., routing and wavelength assignment within a single mathematical model. We propose a decomposition with two different sub-problems, which are solved alternately, in order to design an exact solution scheme. The first sub-problem generates filterless sub-nets while the second deals with their wavelength allocation

Numerical experiments demonstrate the relevance of the model and the performance of the proposed optimization algorithm compared to the state of the art, with the improvement of the solution for several open source data sets.

2.1 Introduction

The idea of filterless optical networks goes back to the seminal articles of [37, 29]. These networks rely on broadcast-and-select nodes equipped with coherent transceivers, as opposed to active switching networks, which today use Reconfigurable Optical Add-Drop Multiplexers (ROADMs). Filterless optical networks exhibit several advantages and are currently considered for, e.g., metro regional networks [25] or submarine networks [24]. They allow a reduction in energy consumption and therefore lead to both a reduction in cost and in the carbon footprint. In addition, they improve the network robustness, while simplifying several aspects of impairment-aware design [36], although with a reduced spectral efficiency due to their inherent channel broadcasting which may be attenuated with the use of blockers, see, e.g., Dochhan *et al.* [8].

Tremblay *et al.* [34] proposed a two-step solution process for the design of filterless network: (1) A Genetic Algorithm to generate fiber trees, (2) A Tabu search algorithm for routing and allocation of wavelengths on selected trees generated in step (1). A Tabu Search algorithm for routing and assigning wavelengths on the selected trees generated by step (1). Both steps are solved with a heuristic, while efficient exact algorithms exist today, at least for the routing and wavelength assignment problem, see, e.g., [10, 13].

Ayoub *et al.* [5] also devised a two-step algorithm: (1) A first heuristic algorithm to generate edge-disjoint fiber trees, (2) An ILP (Integer Linear Program) model to solve the routing and spectrum assignment problem. Unfortunately, they did not compare their results with those of [34].

In this paper, we improve on the solution process we had previously proposed in [16], in which we had an exact algorithm to generate fiber trees, but we were still with a two-step process in which the second step, as in previous studies, solves the routing and wavelength assignment problem. We are now able to propose a first exact one-step solution process using a large scale optimization modelling and algorithm, which are described in Sections 2.3 and 2.4, respectively. Numerical results then show improved filterless network designs over all the previous studies.

The plan of the paper is as follows. We first state formally the design of filterless optical networks in Section 2.2. We next describe in Section 2.3, the one-step mathematical model. The proposed solution scheme is elaborated in Section 2.4: initial solution construction, column generation scheme for the solution of the continuous relaxation and deduction of an integer solution. We describe the numerical results in Section 2.5, including a comparison with the results of [34]. We conclude the paper in Section 2.6.

2.2 Filterless Optical Network Design

We represent the physical layer of an optical network by a graph $G = (V, L)$, where V is the set of nodes (indexed by v), and L is the set of fiber links (indexed by ℓ). We denote by $\omega^-(v)$ and $\omega^+(v)$ the set of incoming and outgoing links of v, respectively.

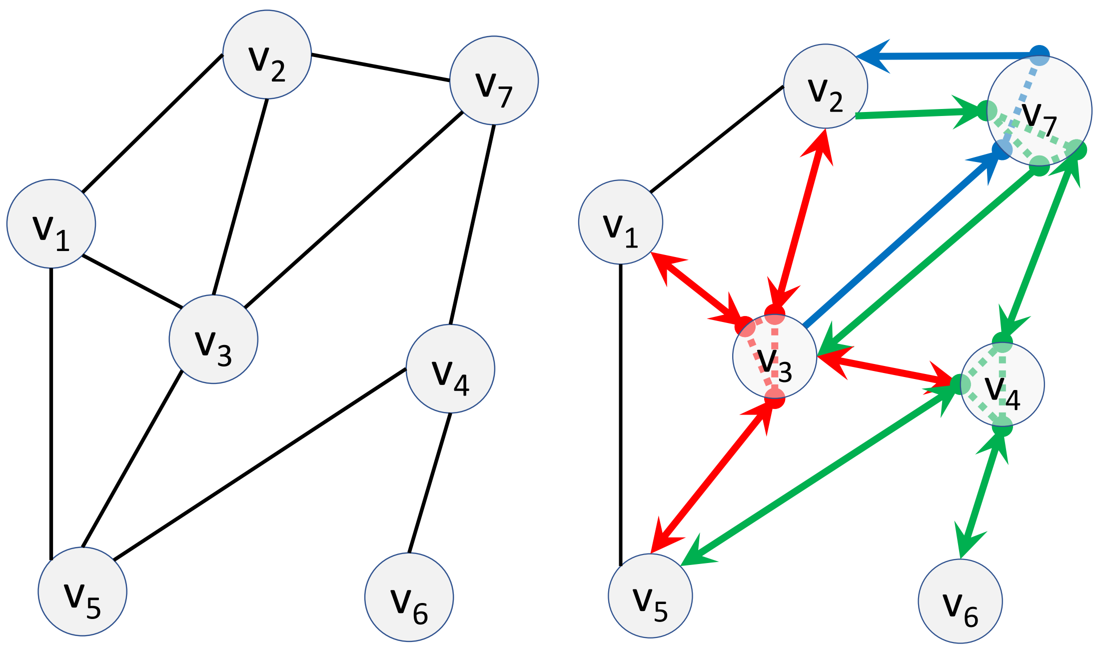
In the sequel, we will use undirected trees and will adopt the following terminology: we use the term "edge" for designating an undirected link, and the term link for designating a directed link. We denote by $\omega(v)$ the co-cycle of v, i.e., the set of adjacent edges of v in an undirected graph.

The traffic is described by a set K of unit requests where each request k is described by its source and destination nodes, v_s and v_d respectively.

The optimized design of a filterless network consists of a set of filterless subnets (f-subnets or FSNs for short), where each f-subnet is a digraph supported by an undirected tree (not necessarily a spanning tree), and f-subnets are pairwise fiber link disjoint. We provide an illustration in Figure 1. Figure 1(a) represents the node connectivity of the original optical network. We assume that each pair of connecting nodes has two directed fibers, one in each direction. Figure 1(b) illustrates a filterless optical network with three f-subnets, each supported by an undirected tree: FSN_1 (in red), with tree backbone $T_1 = \{\{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_3, v_5\}\}$, FSN_2 (in blue), with tree backbone $T_2 = \{\{v_3, v_7\}, \{v_2, v_7\}\}$ and FSN_3 (in green), with tree backbone $T_3 = \{\{v_3, v_7\}, \{v_2, v_7\}, \{v_4, v_7\}, \{v_4, v_5\}, \{v_4, v_6\}\}$. Fiber links with two arrows indicate that the filterless optical network uses fiber in both directions, while links with a single arrow only use fiber in one direction. Required passive splitters and combiners are added to interconnect the fiber links supporting the request provisioning, see, e.g., node v_4 requires a three splitters/combiners with two ports in order to provision the requests of the filterless subnet FSN_3, represented in green. As required by the filterless network design constraints, the subnets do not share any fiber link. Observe that two node connections are unused: edges $\{v_1, v_2\}$ and $\{v_1, v_5\}$ of the physical network, which may be an indication of a poor network design.

Each request is provisioned on one of the f-subnets. When the decomposition includes more than one f-subnet, there may be more than one option for routing a request. For instance, in Figure 1(b), request from v_2 to v_3 can be provisioned either on FSN_3 (green f-subnet), or on FSN_1 (red f-subnet).

The optimal design of optical filterless networks consists of the establishment of a set of pairwise fiber link disjoint f-subnets such that the request provisioning minimizes the number of wavelengths.



(a) Undirected graph supporting the physical network

(b) Three different FSNs

Figure 1: A filterless optical network solution by Using CG_FOP Model

The objective is to find f-subnets and a provisioning such that we minimize the overall number of wavelengths. Each request must be provisioning on a lightpath, i.e., a route and a wavelength such that the continuity constraint is satisfied, i.e., the same wavelength from source to destination, while considering the broadcast effect, i.e., same wavelength on all outgoing links of the source node and their descendants.

We next illustrate the impacts of the broadcast effect on the wavelength assignment. Consider the following example with 7 requests on FSN_1 in Figure 1(b): k_{13} from v_1 to v_3, and with the same notations, k_{53}, k_{35}, k_{21}, k_{12}, k_{42}, and k_{34}. Figure

2 depicts a provisioning of them, which requires 4 wavelengths. Plain lines correspond to the provisioning between the source and the destination nodes, while dotted lines indicate the broadcast effect beyond the destination nodes. Observe that the

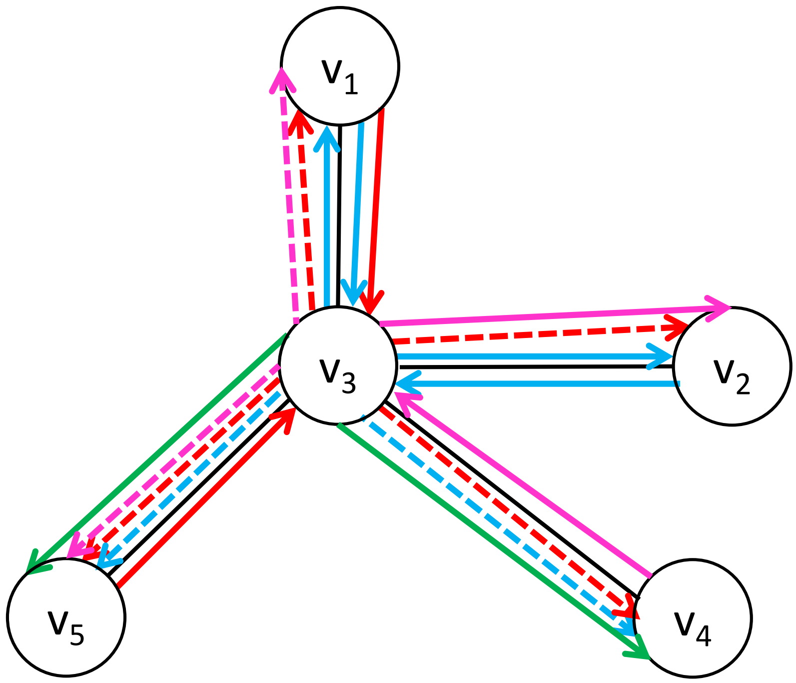


Figure 2: Wavelength request provisioning by Using CG_FOP Model

wavelength assignment takes into account that requests that conflict only in their broadcast effect beyond their destination nodes can share the same wavelength. For instance, request k_{13} can share the same wavelength as request k_{53}.

2.3 Mathematical Programming Decomposition Model

We propose a mathematical programming decomposition model, called CG_FOP, which uses two different sets of configurations: the FSN configurations and the wavelength configurations. Each FSN configuration consists of: *(i)* a FSN backbone made up of an undirected tree, and *(ii)* a subset of requests provisioned on FSN. Each wavelength configuration consists of a potential wavelength assignment for the overall set of requests.

The design of a filterless network then consists in selecting a set of FSN configurations, as many as the selected number of FSNs, and wavelength configurations, as many as the number of wavelengths required, which together minimize the number of wavelengths.

The proposed decomposition consists of a master problem coordinating a set of pricing problems which are solved alternately, see Section 2.4 for the details of the solution scheme. We next describe the master problem and the pricing problems.

2.3.1 Master Problem

Let FSN denote a f-subnet configuration. It is formally characterized by:

- $a_\ell^{\text{FSN}} = 1$ if link ℓ is in f-subnet FSN, 0 otherwise
- $b_k^{\text{FSN}} = 1$ if request k is routed on FSN, 0 otherwise
- $\theta_{kk'}^{\text{FSN}} = 1$ if request k and request k' are in conflict on FSN, 0 otherwise.

Let $\mathcal{FSN}$ be the set of f-subnets, and $n_{\mathcal{FSN}}$ be the desired number of f-subnets. Note that backbones of f-subnets are not necessarily spanning trees.

Let λ be a wavelength configuration. It is characterized by:

- $\beta_k^\lambda = 1$ if request k uses wavelength (color) configuration λ, 0 otherwise.

There are two sets of decision variables, one for each set of configurations:

- $z_{\text{FSN}} = 1$ if FSN, and its provisioned requests, is selected as a f-subnet, 0 otherwise.
- $x_\lambda = 1$ if wavelength (color) configuration λ is used, 0 otherwise. One wavelength configuration is defined by all the requests with the same wavelength assignment.

The objective is to minimize the total number of wavelengths:

$$\min \sum_{\lambda \in \Lambda} x_\lambda \tag{1}$$

subject to:

$$\sum_{\text{FSN} \in \mathcal{FSN}} b_k^{\text{FSN}} z_{\text{FSN}} \geq 1 \qquad k \in K \tag{2}$$

$$\sum_{\text{FSN} \in \mathcal{FSN}} a_\ell^{\text{FSN}} z_{\text{FSN}} \leq 1 \qquad \ell \in L \tag{3}$$

$$\sum_{\text{FSN} \in \mathcal{FSN}} z_{\text{FSN}} \leq n_{\mathcal{FSN}} \tag{4}$$

$$\sum_{\lambda \in \Lambda} \beta_k^\lambda \, \beta_{k'}^\lambda x_\lambda \leq 1 - \sum_{\text{FSN} \in \mathcal{FSN}} \theta_{kk'}^{\text{FSN}} z_{\text{FSN}} \qquad k, k' \in K \tag{5}$$

$$\sum_{\lambda \in \Lambda} \beta_k^\lambda x_\lambda \geq 1 \qquad k \in K \tag{6}$$

$$z_{\text{FSN}} \in \{0, 1\} \qquad \text{FSN} \in \mathcal{FSN} \tag{7}$$

$$x_\lambda \in \{0, 1\} \qquad \lambda \in \Lambda. \tag{8}$$

Constraints (2) enforce the provisioning of each request on at least one f-subnet. Note that the (2) constraints could be written as equality constraints. However, in practice, the inequalities make the model easier to solve for LP (Linear Program) / ILP (Integer Linear Program) solvers. Due to objective minimization, a request will not be routed more than once unless sufficient transport capacity is available to allow it. In such a case, we can easily eliminate unnecessary provisioning in a post-processing phase. Constraints (3) prohibit the sharing of links between f-subnets. Constraint (4) limits the number of FSN configurations. Constraints (5) guarantee the conflicted requests could not share the same wavelength (color). Constraints (6) express that each request will use at least one wavelength (color). Again, inequality is justified as for Constraints (2), and a post-processing can easily eliminate a double wavelength assignment. Post-processing is justified in view of the enhanced convergence of the solution scheme thanks to the inequalities.

The above master cannot be solved directly as it would requires the exhaustive list of f-subnet and wavelength configurations. As it will be explained in Section 2.4, we will only use an implicit enumeration of the configurations jointly with an optimality condition, to be checked using the sign of the objective functions of the configuration generator subproblems, which are next described.

2.3.2 $\mathbf{PP_{fsn}}$ FSN Pricing Problem

The role of the FSN pricing problem is to find a new FSN configuration that could further reduce the value of the objective function of the continuous relaxation of the restricted master problem.

Variables of the pricing problem directly or indirectly defines the coefficients of the z_{FSN} decision variables of the master problem, and are defined as follows:

- $a_\ell = 1$ if link ℓ is in the FSN under construction, 0 otherwise, for all $\ell \in L$
- $a_v = 1$ if node v belongs to FSN under construction, 0 otherwise, for all $v \in V$.
- $\alpha_e = 1$ if edge e belongs to the tree backbone of the FSN under construction, 0 otherwise, for all $e \in E$.
- $x_k = 1$ if request k is routed on the FSN under construction, 0 otherwise, for all $k \in K, t \in T$.
- $\varphi_{k\ell} = 1$ if the routing of request k goes through link ℓ, or if the wavelength channel used in the routing of request k propagates on link ℓ because it is not filtered, 0 otherwise, for all $\ell \in L, k \in K$.
- $\psi_{k\ell} = 1$ if the routing of request k goes through link ℓ between its source and destination, 0 otherwise, for all $\ell \in L, k \in K$. We need both variables $\varphi_{k\ell}$ and $\psi_{k\ell} = 1$ in order to identify the unfiltered wavelength links.
- $\theta_{kk'} = 1$ if request k and request k' are in conflict on the f-subnet under construction, 0 otherwise.
- $\omega_{kk'\ell} = \psi^{\text{FSN}}_{k\ell}\varphi^{\text{FSN}}_{k'\ell}$, for all $\ell \in L, k, k' \in K$. In other words, with $\omega_{kk'\ell} = 1$ identifying a link on which k and k' are conflicting either between their source and destination nodes, or with one of them being routed to the broadcast of the other request, and 0 otherwise (no conflict).

Objective: it corresponds to the reduced cost of variable z_{FSN}, i.e., the objective function of the pricing problems, i.e., the decomposition sub-problems. The reader who is not familiar with linear programming concepts, and in particular with the concept of reduced cost is referred to the book of Chvatal [7]. The coefficient 0 indicates that the variable z_{FSN} does not appear in the objective function 1 of the master problem, and therefore its coefficient contribution to the reduced cost is null.

$$\min\ 0 - \sum_{k\in K} u_k^{(2)} x^k + \sum_{\ell \in L} u_\ell^{(3)} \alpha_\ell + u^{(4)} + \sum_{k,k' \in K} u_{kk'}^{(5)} \theta_{kk'} \quad (9)$$

We now describe the set of constraints.

Building an undirected tree

$$\sum_{\substack{e=\{v,v'\}\in E:\\ v,v' \in V'}} \alpha_e \le |V'| - 1 \qquad V' \subset V, |V'| \ge 3 \quad (10)$$

$$\sum_{v\in V} a_v = \sum_{e\in E} \alpha_e + 1 \quad (11)$$

$$2\alpha_e \le a_v + a_{v'} \qquad v, v' \in V, e = \{v, v'\} \quad (12)$$

$$\sum_{e\in\omega(v)} \alpha_e \ge a_v \qquad v \in V \quad (13)$$

$$a_\ell \le \alpha_e,\ a_{\bar\ell} \le \alpha_e \qquad \ell = (v, v'), \bar\ell = (v', v),\ v, v' \in V, e = \{v, v'\} \quad (14)$$

Routing of the provisioned requests

$$\varphi_{k\ell} \le a_\ell \qquad k \in K, \ell \in L \quad (15)$$

$$a_\ell \le \sum_{k\in K} \varphi_{k\ell} \qquad \ell \in L \quad (16)$$

$$\varphi_{k\ell} + \varphi_{k\bar\ell} \le 1 \qquad \ell = (v, v'),\ \bar\ell = (v', v) : v \in V, v' \in V \quad (17)$$

$$\sum_{\ell\in\omega^-(d_k)} \varphi_{k\ell} = \sum_{\ell\in\omega^+(s_k)} \varphi_{k\ell} = x_k \qquad k \in K \quad (18)$$

$$\sum_{\ell\in\omega^-(s_k)} \varphi_{k\ell} = 0 \qquad k \in K \quad (19)$$

Propagation of unfiltered channels

$$\varphi_{k\ell'} \leq \sum_{\ell \in \omega^-(v)} \varphi_{k\ell} + 1 - a_{\ell'} \qquad k \in K, \; v \in V \setminus \{s_k\}, \ell' \in \omega^+(v) \tag{20}$$

$$\varphi_{k\ell} \leq \varphi_{k\ell'} + 2 - a_\ell - a_{\ell'} \qquad k \in K, \; v \in V, \ell \in \omega^-(v), \ell' \in \omega^+(v) \setminus \{\bar{\ell}\} \tag{21}$$

"Filtered" provisioning of requests

$$\sum_{\ell \in \omega^-(d_k)} \psi_{k\ell} = \sum_{\ell \in \omega^+(s_k)} \psi_{k\ell} = x_k \qquad k \in K \tag{22}$$

$$\sum_{\ell \in \omega^-(v)} \psi_{k\ell} = \sum_{\ell \in \omega^+(v)} \psi_{k\ell} \leq x_k \qquad k \in K, \; v \in V \setminus (s_k, d_k) \tag{23}$$

$$\sum_{\ell \in \omega^+(d_k)} \psi_{k\ell} = \sum_{\ell \in \omega^-(s_k)} \psi_{k\ell} = 0 \qquad k \in K \tag{24}$$

Reach distance

$$\sum_{\ell \in L} \text{DIST}_\ell \psi_{k\ell} \leq \text{REACH\_DIST} \qquad k \in K \tag{25}$$

Identifying wavelength conflicting lightpaths

$$\psi_{k\ell} \leq \varphi_{k\ell} \qquad k \in K, \ell \in L \tag{26}$$

$$\theta_{kk'} \geq \psi^{\text{FSN}}_{k\ell} + \psi^{\text{FSN}}_{k'\ell} - 1 \qquad \ell \in L, \; k, k' \in K \tag{27}$$

$$\theta_{kk'} \geq \psi^{\text{FSN}}_{k\ell} + \varphi^{\text{FSN}}_{k'\ell} - 1 \qquad \ell \in L, \; k, k' \in K \tag{28}$$

$$\theta_{kk'} \geq \varphi^{\text{FSN}}_{k\ell} + \psi^{\text{FSN}}_{k'\ell} - 1 \qquad \ell \in L, \; k, k' \in K \tag{29}$$

$$\theta_{kk'} \leq \sum_{\ell \in L} (\omega_{kk'\ell} + \omega_{k'k\ell}) \qquad k, k' \in K \tag{30}$$

$$\omega_{kk'\ell} \leq \psi^{\text{FSN}}_{k\ell} \qquad \ell \in L, k, k' \in K \tag{31}$$

$$\omega_{kk'\ell} \leq \varphi^{\text{FSN}}_{k'\ell} \qquad \ell \in L, k, k' \in K \tag{32}$$

$$\omega_{kk'\ell} \geq \psi^{\text{FSN}}_{k\ell} + \varphi^{\text{FSN}}_{k'\ell} - 1 \qquad \ell \in L, k, k' \in K \tag{33}$$

Domains of the variables

$$\alpha_e \in \{0,1\} \qquad e = \{v, v'\} \in E \tag{34}$$

$$a_v \in \{0,1\} \qquad v \in V \tag{35}$$

$$a_\ell \in \{0,1\} \qquad \ell \in L \tag{36}$$

$$\varphi_{k\ell} \in \{0,1\} \qquad \ell \in L, k \in K \tag{37}$$

$$\psi_{k\ell} \in \{0,1\} \qquad \ell \in L, k \in K \tag{38}$$

$$x_k \in \{0,1\} \qquad k \in K \tag{39}$$

$$\theta_{kk'} \in \{0,1\} \qquad k, k' \in K \tag{40}$$

$$\omega_{kk'\ell} \in \{0,1\} \qquad k, k' \in K, \ell \in L. \tag{41}$$

Constraints (10) are the classical subtour elimination in order to build a tree (i.e., a tree backbone for the FSN under construction) [23]. Constraints (11) make sure that a single FSN is generated. Constraints (12) and (13) make sure that the values of the node and edge variables are consistent: an edge (undirected link) is used in the tree backbone if and only if its two endpoints belong to it. Constraints (14) again make sure of the consistency between the values of the variables: check that if a link is used, its corresponding supporting edge belongs to the tree.

The next set of constraints (15)-(19) take care of the routing of the requests, in particular of the links with the unfiltered channels. Constraints (15) and (16) ensure that if a request is routed over link ℓ, then ℓ belongs to the FSN under construction and vice versa, if a link belongs to the FSN under construction, then it belong to the routing of at least one request. Assuming $\overline{\ell}$ denotes the link in the opposite direction of ℓ, Constraints (17) forbid the use of both ℓ and $\overline{\ell}$ in the routing of any request. Constraints (18)-(19) are flow constraints in order to take care of the useful routing (no unfiltered channel) of the requests.

The next two sets of constraints (20)-(21) deal with the propagation of the unfiltered channels. For every request k and node different from its source node (s_k), constraints (20) make sure that if none of the incoming link of a node is on the route or broadcast effect of request k, then none of its outgoing links can be used for either the routing or the broadcast effect of request k. For every request k and every node v, constraints (21) make sure that outgoing links of v will carry some traffic for k if there is incoming traffic belonging to k. This is to enforce that the broadcast effect will carry over to the traffic going forward. In other words, if $\varphi_{k\ell} = a_\ell = 1$, it will force $\varphi_{k\ell'} = 1$ if $a_{\ell'} = 1$, i.e., if link ℓ' belongs to the FSN under construction.

Constraints (22)-(23) are flow constraints that define the routing of request k between its source and destination. Constraints (24) impose no wavelength assignment on the incoming links of the source and on the outgoing links of the destination for the "filtered" routing of k.

Constraint (25) is the reach constraint, for each request the routing distance between source and destination must not exceed a maximum distance (1,500 km).

Constraints (26) define the relation between variables $\varphi_{k\ell}$ and $\psi_{k\ell}$. Constraints (27)-(29) are conflict wavelength constraints expressing that: either k and k' share a link between their source and destination nodes, see (27), or that one of the requests is routed to the broadcast part of the other request, see (28) and (29). Constraints (30)-(33) identify the no conflict case, i.e., when the routes between their source and destination are not overlapping, and then none of the requests is routed to the broadcast part of the other request.

2.3.3 $\mathbf{PP}^{\mathbf{H}}_{\mathbf{color}}$ Relaxed Wavelength Pricing Problem

The role of the wavelength pricing problem is to check whether there exists a new wavelength configuration that could further reduce the value of the objective function of the continuous relaxation of the restricted master problem.

The output of this pricing problem contains a set of requests which could share the same wavelength.

Variables

- $\beta_k = 1$ if request k uses the wavelength associated with the current wavelength pricing problem, 0 otherwise, for all $k \in K$
- $\alpha_{kk'} = \beta_k \beta_{k'}$. It is equal to 0 if k and k' cannot be assigned the same wavelength, 1 otherwise.

Objective:

$$\min\ 1 - \sum_{k \in K} u_k^{(6)} \beta_k + \sum_{k,k' \in K} u_{kk'}^{(5)} \alpha_{kk'} \tag{42}$$

subject to:

$$\beta_k + \beta_{k'} \leq 1 \quad \text{if} \sum_{\text{FSN} \in \mathcal{FSN}} \theta_{kk'}^{\text{FSN}} z^{\text{FSN}} > 0 \qquad k, k' \in K \tag{43}$$

$$\alpha_{kk'} \leq \beta_k \qquad k, k' \in K \tag{44}$$

$$\alpha_{kk'} \leq \beta_{k'} \qquad k, k' \in K \tag{45}$$

$$\beta_k + \beta_{k'} \leq \alpha_{kk'} + 1 \qquad k, k' \in K \tag{46}$$

$$\alpha_{kk'} \in \{0, 1\} \qquad k, k' \in K \tag{47}$$

$$\beta_k \in \{0, 1\} \qquad k \in K. \tag{48}$$

Constraints (43) identify wavelength conflicts between two requests, and consequently make sure that wavelength conflicting requests are not assigned the same wavelength. Constraints (44)-(46) are the linearization of $\alpha_{kk'} = \beta_k \, \beta_{k'}$.

Note that the above wavelength pricing problem is not exact due to constraints (43). It was however found very useful in practice in order to speed up the convergence of the solution process. We next describe the exact wavelength pricing problem.

2.3.4 $\mathrm{PP}^{\mathrm{E}}_{\mathrm{color}}$ Exact Coloring Pricing Problem

The exact wavelength pricing problem is the heuristic one with the omission of constraints (43).

2.4 Solution Scheme

We now describe the solution process of the Master problem described in Section 2.3.1, in coordination with the pricing problems PP_{FSN}, $\mathrm{PP}^{\text{H}}_{\text{COLOR}}$ and $\mathrm{PP}^{\text{E}}_{\text{COLOR}}$, described in Sections 2.3.2, 2.3.3 and 2.3.4.

2.4.1 Column Generation and ILP Solution

Column Generation technique allows the efficient solution of large-scale Linear programs [20]. In order to derive an ILP solution, we can then use either a branch-and-price method [6] or heuristic methods [28], the accuracy of which can be estimated.

We first discuss the column generation technique and the novelty we introduced with our mathematical formulation. It allows the solution of the continuous relaxation of the Master Problem, i.e., (1)-(8). In order to do so, we first define the so-called Restricted Master Problem (RMP), i.e., the Master Problem with a very limited number of variables or columns, and a so-called Pricing Problem, i.e., a configuration generator. Here, in our modelling, contrarily to the classical Dantzig-Wolfe decomposition, we consider two different types of pricing problems, the FSN one, and the wavelength one.

The objective function of the Pricing Problems is defined by the Reduced Cost of the decision variable associated with the newly generated configuration: if its minimum value is negative, the addition of the latter configuration will allow a decrease of the optimal LP value of the current RMP, otherwise, if its minimum value is positive for both pricing problems (i.e., the LP optimality condition), the optimal LP value ($z^{\star}_{\text{LP}}$) of problem (1)-(8) has been reached. In other words, the solution process consists in solving the RMP and the pricing problems alternatively, until the LP optimality condition is satisfied.

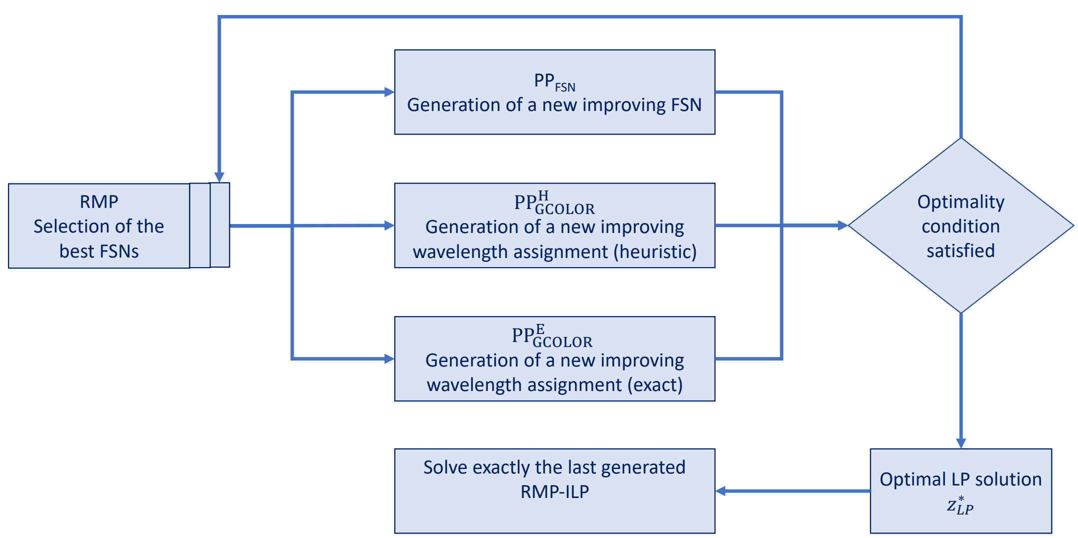


Figure 3: Flowchart: column generation with three different pricing problems

Once the optimal LP solution of the MP has been reached, we can then derive an ILP solution, i.e., a selection of pairwise link disjoint filterless sub-networks, which provision all the requests, jointly with a wavelength assignment for all the requests. It is done with the ILP solution of the last RMP in which domains of the variables has been changed from continuous to integer requirements. Denote by $\tilde{z}_{\text{ILP}}$ the value of that ILP solution: it is not necessarily an optimal ILP solution, but is guaranteed to have an accuracy not larger than

$$\varepsilon = (\tilde{z}_{\text{ILP}} - z^{\star}_{\text{LP}})/\ z^{\star}_{\text{LP}}.$$

2.4.2 Detailed Flowchart and Algorithm

After outlining the general process of a solution process with a column generation algorithm in the previous section, and on the derivation of an ε-optimal ILP solution, we now provide the detailed flowchart of our solution process in Figure 3. Solution alternance of the three pricing problems is sought until the LP optimality condition is satisfied, i.e., no configuration with a negative reduced cost can be found. Different strategies can be defined for this alternance, and we describe below the one which gave the best results, among the ones we tested.

1. Generate an initial solution with a single FSN supported by a spanning tree, and a first set of wavelength assignment. We use the algorithm of Welsh-Powell [39] to minimize the number of wavelengths, using a wavelength conflict graph on which we minimize the number of colors.

2. Apply the column generation algorithm in order to solve the linear relaxation of model CG_FOP:

   (a) Solve restricted master problem with current FSN and wavelength assignment configurations

   (b) Solve pricing problem PP_{FSN}. If it generates an improving FSN configuration, add it to the RMP and return to Step 2a

   (c) Solve pricing problem $\text{PP}^{\text{H}}_{\text{COLOR}}$. If it generates an improving wavelength configuration, add it to the RMP and return to Step 2a

   (d) Solve pricing problem $\text{PP}^{\text{E}}_{\text{COLOR}}$. If it generates an improving wavelength configuration, add it to the RMP and return to Step 2a

3. As the continuous relaxation of the Master Problem has been solved optimally, solve the last generated restricted master problem with integer requirements for the variables, derive an ILP solution.

2.5 Numerical Results

We report now the performance of our proposed mathematical model and algorithm, and compare the quality of its solutions with those of Tremblay *et al.* [34].

2.5.1 Data Sets

We use the Italy, California and Germany topologies as in Tremblay *et al.* [34] (see [2] for distances), as well as the Cost239 and USA topologies, using the link distances of [11] and [32], respectively. The characteristics of the network topologies are recalled in Table 1. We consider unit uniform demands, as in [34], unless otherwise indicated.

Table 1: Data set characteristics

Networks	# nodes	# edges
Italy	10	15
California	17	20
Germany17	17	26
Cost239	11	26
USA	12	15

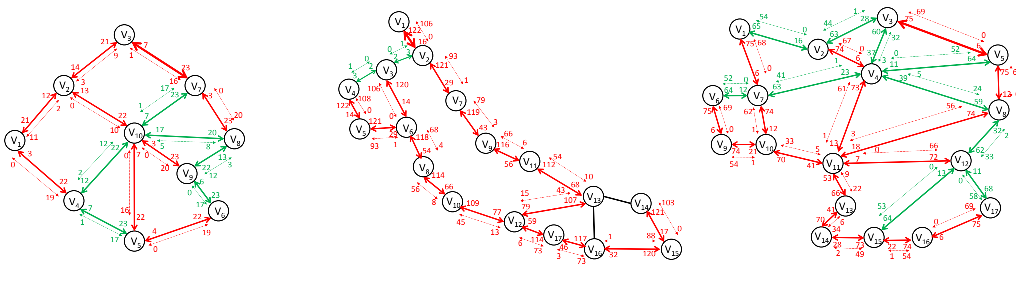
2.5.2 Performance of the Proposed CG_FOP Model

We tested our model/algorithm on the same data sets as Tremblay *et al.* [34]. Results are summarized in Table 2.

Available numerical results of [34] are reported in columns 2 and 3, while results or our CG_FOP model are reported in the remaining columns. We provide the results of the CG_FOP model in Columns 4 to 7 (2 trees for Italian and Germany17, 3 trees for California), for the first three data sets. Note that there is no solution with three trees for the Italy, Germany17 and USA networks.

Table 2: Network parameters on CG_FOP model for filterless solutions

	Tremblay *et al.* [34]		Model CG_FOP											
			a single tree				two trees				three trees			
	# Trees	W	$z^\star_{\text{LP}}$	W	Max Load R	Max Load N	$z^\star_{\text{LP}}$	W	Max Load R	Max Load N	$z^\star_{\text{LP}}$	W	Max Load R	Max Load N
Italy	2	25	41.0	41	9	32	20.0	23	7	16	-	-	-	-
California	3	120	125.6	126	16	110	117.4	122	16	106	113.4	120	16	104
Germany17	2	88	120.5	125	16	109	62.3	73	6	67	-	-	-	-
Cost239	-	-	49.3	51	10	41	22.7	28	6	22	15.7	25	7	18
USA	-	-	61.0	61	11	50	41.2	53	7	46	-	-	-	-



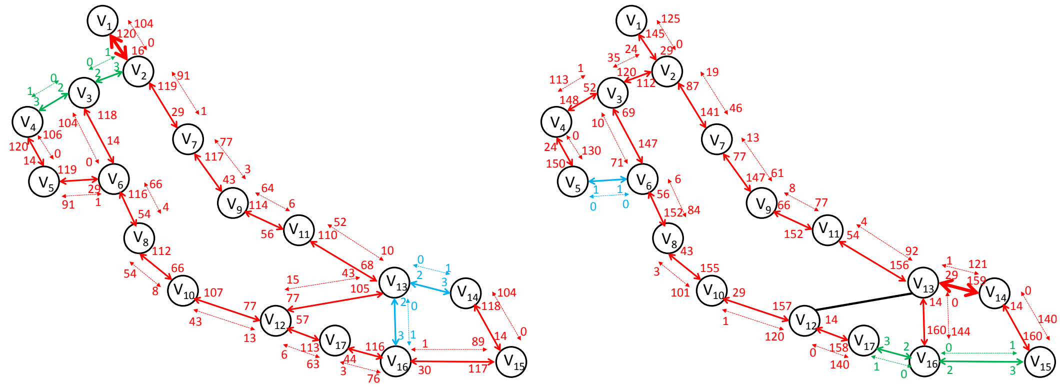
(a) Italy (b) California (c) Germany17

Figure 4: Two filterless sub-networks solutions with optimized trees compared to those of [34]

The last two columns report the spectrum use, R reports the number of fiber links used between the source and destination of the requests, while N corresponds to the number of wasted fiber links due to the broadcast effect.

We observe that we improve the values of Tremblay *et al.* [34] when using the same 2 supporting trees for 2 for the FSNs of two data sets, i.e., 23 wavelengths instead of 25 for the Italy network, and 73 wavelengths instead of 88 for Germany network In Figure 4, we provide the details of the solutions of Model CG_FOP with different trees than those of [34]. The numbers on the bidirectional arrows indicate the number of fiber links in each direction, differentiating between "filtered" and "unfiltered" fiber links, respectively thick and thin double arrows. The numbers on the bidirectional arrows indicate the number of fiber links, differentiating between "filtered" and "unfiltered" fiber links, respectively thick and thin double arrows.

We did not implement a branch-and-price algorithm and we limited the number of



(a) Uniform Traffic (272 unit requests, 1 request per node pair)

(b) Non Uniform Traffic (338 unit requests, several requests per node pair)

Figure 5: Three filterless sub-networks on the California network

iterations in the column generation algorithm. Consequently, the solutions of Model CG_FOP are usually not optimal, except when $z^{\star}_{\text{LP}}$ is equal to the ILP optimal value. We then observe that several solutions with one tree are optimal, i.e., those for the Italy, California and USA networks.

We also provided results with one tree (optimized selection): we observe that the number of wavelengths for one tree is increasing over the required number for 2 or 3 trees.

Lastly, in Figure 5, we provide results with three f-subnets for the California network, first with uniform traffic in Figure 5(a), and then with non uniform traffic in Figure 5(b) (number of requests randomly generated between 1 and 3 for each node pair). It is interesting to observe that the optimal solution for the non uniform traffic does not use one link, and that while one f-subnet is always a spanning tree, the two other f-subnets are very small, most likely due to the characteristics of the California topology. The bold red arrow indicates the link with the largest number of wavelengths.

2.6 Conclusions

This paper presents a one step decomposition model and algorithm, which can solve exactly the filterless network design problem. While computational times are sometimes quite high (i.e., sometimes a couple of hours), we hope to be soon able to improve further the modelling and algorithm in order to provide more scalable solutions. However, this is already a very significant first step towards the exact design of filterless optical networks.

Chapter 3

Dantzig-Wolfe Decomposition for the Design of Filterless Optical Networks

This chapter contains a paper submitted for publication into [15].

In this chapter, we propose a Dantzig-Wolfe decomposition model in which each subproblem aims to generate a potential filterless optical subnetwork, with a directed tree topology. The master problem then selects the best combination of subnetworks.

Numerical experiments illustrate significant performance improvement over previous work, reducing previous computational results by a factor of 2 to 10 depending on the size of the data instances.

3.1 Introduction

Optical networks are widely deployed today in all kinds of telecommunications networks, with a volume of user traffic on carrier networks growing at a rate between 20% and 40% per year during the last decade. As exponential traffic growth was set to slow, the rise of 5G introduced new use cases, with capacity growth set to continue. In this context, from a techno-economic point of view, filterless optical networks offer opportunities for cost and energy reduction, at least for certain types of networks, for example access networks [25].

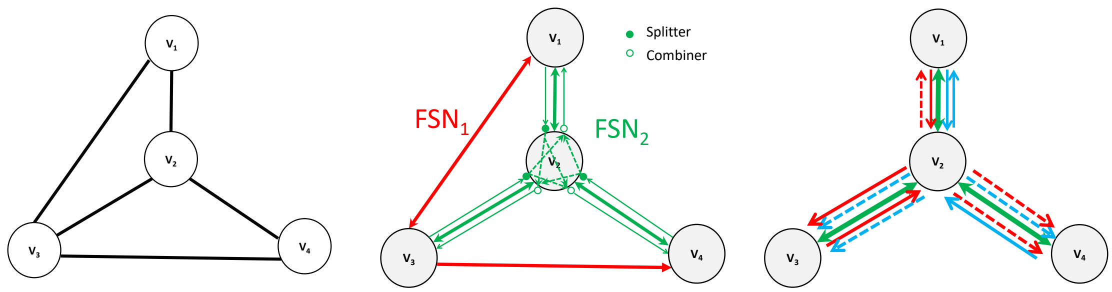
There is a compromise to be found between the usage of more spectrum efficient technologies thanks to, e.g., pure optical layer switching using expensive Reconfigurable Optical Add-Drop Multiplexers (ROADMs) (see, e.g., Simons [31]), and filterless optical networks which rely on cheap couplers/splitters at the expense of a reduction in the spectral efficiency performance [30].

Tzanakaki *et al.* [37] was one of the pioneers to investigate the filterless network components. The basic design principle of filterless optical networks was described by Savoie *et al.* [29] in the early work on filterless optical networks with a motivation for cost reduction and environment friendly networks. In spite of a decrease in the spectrum efficiency, more authors are studying filterless optical networks today, and we can see studies investigating the wasted spectrum on the static planning case (filterless) vs. dynamic planning case (filtered node), see, e.g., Pavon-Marino *et al.* [25]. The addition of blockers offers a mean to improve the efficiency of spectrum utilization, see, e.g., Dochhan *et al.* [8].

Several researchers study the design of optimized filterless optical networks. Tremblay *et al.* [34], Ayoub *et al.* [5] and Jaumard *et al.* [16] all proposed a two-step solution process with, for the first two references, a first step aiming at generating potential/promising fiber trees, and then a second step to provision the resulting trees. The quality of the resulting filterless optical networks is however difficult to assess, both for the two-step process, but also because one or both steps are solved using heuristics. In [17], there is a first attempt for designing an exact method for the optimal design of filterless optical networks. In this paper, we improve on that latter study, with an improved mathematical programming model, as well as an improved exact solution process. This translates into significant reduced computational times and further improved accuracy for the output designs.

The paper is organized as follows. In Section 3.2, we present the problem statement and the concept of filterless optical network design. In Section 3.3, we propose a new mathematical model DW_FOP for the optimal design of filterless optical networks. The solution process, which relies on Dantzig-Wolfe decomposition (or column generation techniques), is described In Section 3.4. Computational results are summarized in Section 3.5, including a comparison with the results of previous works. Conclusions are drawn in the last section.

3.2 Design of Filterless Networks



(a) Physical network

(b) Two FSNs

(c) Wavelength provisioning of FSN_2

Figure 6: Construction of a filterless network solution by Using DW_FOP Model

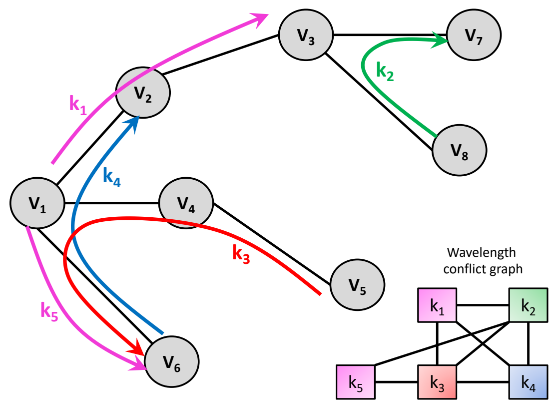


Figure 7: Requests with wavelength conflicts

Consider an optical network represented by its physical network $G = (V, L)$, where V is the set of nodes (indexed by v), and L is the set of fiber links (indexed by ℓ). We denote the set of incoming and outgoing links of v as $\text{IN}(v)$ and $\text{OUT}(v)$, respectively.

The traffic is described by a set of unit requests K, where each request k is characterized by its source and destination nodes, v_s and v_d respectively. Indeed, non unit requests are considered as a set of unit requests.

A filterless network solution consists of a set of filterless optical sub-network (FSN for short) solutions, where each FSN relies on an undirected tree, called the *backbone* of an FSN, deduced from a subgraph of the physical network G. FSNs need to be pairwise fiber link disjoint in order to avoid closed loops, i.e., indesirable laser effects in optically amplified links [34]. Wavelength assignment follows the usual rule of avoiding wavelength conflicts, see the discussion below. On the undirected tree, we use the term "edge" to represent an undirected link, which can host multiple directed links or links for short, possibly in both directions.

Figure 6 illustrates the construction of a filterless optical network made of two FSNs: the red one (FSN_1) is supported by the undirected tree $T_1 = \{\{v_1, v_3\}, \{v_3, v_4\}\}$, and the green one ($\text{FSN}_2$), by the undirected tree $T_2 = \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_2, v_4\}\}$.

In Figure 6(b), fiber links are represented by directed arrows, where a bidirected arrow indicates two fiber links, one in each direction. The first FSN, i.e., FSN_1, is made of one bi-directional arrow, and a single fiber link on the second tree edge supporting the FSN. The second FSN, i.e., FSN_2 has two opposite fiber links on each edge of its supporting undirected tree.

On the FSN intermediate nodes, passive splitters and combiners are required to interconnect the fiber links supporting the request provisioning, see, e.g., node v_2 requires three splitters/combiners with two ports in order to accommodate the provisioning of the requests using the green filterless network, see Figure 6(b).

On our example, each request can be provisioned on any one of the sub-networks. While this is not necessarily the case for all requests in general, this may happen in practice for some requests. In the example of Figure 6(b), the request from node v_1 to node v_4 can be provisioned either on FSN_1 or FSN_2. The decision is usually driven by the objective, e.g., number of wavelengths, number of fiber trees, number of passive optical divider. In this study, we are interested in establishing a set of FSNs such that the provisioning of all the requests minimizes the number of wavelengths.

Figure 6(c) depicts a possible wavelength assignment, with the provisioning of 4 requests on FSN_2 in Figure 6(b): k_{13} from v_1 to v_3, and with the same notations, k_{12}, k_{41}, and k_{32}.

We now consider the broadcast effect. When the destination is reached, the lightpath will continue downstream. For instance, when the optical signal supporting request k_{12} reaches v_2 through the fiber link (v_1, v_2), the signal is broadcast through fiber links (v_2, v_3) and (v_2, v_4).

Wavelength assignment should be done in order to avoid wavelength conflicts.

Such conflicts occur for two requests if there is a shared fiber link on one of their routing path, i.e., between source and destination nodes. For instance, in Figure 7, there are conflicts for the requests k_1 and k_4 because their routes between the source and destination nodes share a fiber link. Requests k_1 and k_2 are also wavelength conflicting as one of the fiber links of k_2 lies on the broadcast route of k_1 with unfiltered channels. Similarly, requests k_3 and k_4 are wavelength conflicting, as k_4 is on the broadcast of k_3. Non conflicting requests are for instance k_1 and k_5, or k_4 and k_5. Note that once a FSN is defined, wavelength assignment can be done with a graph coloring algorithm (see, e.g., [34]), using a wavelength graph conflict, as illustrated in Figure 7.

In Figure 6(c), requests $k_{1 \mapsto 3}$ and $k_{3 \mapsto 2}$ are not conflicting and so can be assigned the same wavelength. The same holds between requests $k_{1 \mapsto 2}$ and $k_{4 \mapsto 1}$. For this example, 2 wavelengths are sufficient to satisfy the 4 requests.

3.3 Mathematical Model

3.3.1 Definitions and Notations

We introduce the concept of FSN configuration as a provisioned directed tree characterized by:

- $a_\ell^{\text{FSN}} = 1$ if link ℓ is in configuration FSN, 0 otherwise.
- $x_k^{\text{FSN}} = 1$ if request k is routed on configuration FSN, 0 otherwise.
- $\beta_\lambda^{\text{FSN}} = 1$ if wavelength λ is used in configuration FSN, 0 otherwise.

Denote by $\mathcal{FSN}$ the overall set of FSN configurations, and let $n_{\mathcal{FSN}}$ be an upper bound on the number of FSNs in the network.

3.3.2 Mathematical Model

We propose an integer linear program that has two sets of decision variables:

- $z^{\text{FSN}} = 1$ if FSN configuration is selected as a filterless sub-network, 0 otherwise.
- $x_\lambda = 1$ if wavelength λ is used by any of the selected FSN configurations, 0 otherwise.

The objective corresponds to the minimization of the total number of wavelengths.

$$\min \sum_{\lambda \in \Lambda} x_\lambda \tag{49}$$

subject to:

$$\sum_{\text{FSN} \in \mathcal{FSN}} \beta_\lambda^{\text{FSN}} z^{\text{FSN}} \leq n_{\mathcal{FSN}} x_\lambda \qquad \lambda \in \Lambda \tag{50}$$

$$\sum_{\text{FSN} \in \mathcal{FSN}} x_k^{\text{FSN}} z^{\text{FSN}} \geq 1 \qquad k \in K \tag{51}$$

$$\sum_{\text{FSN} \in \mathcal{FSN}} a_\ell^{\text{FSN}} z^{\text{FSN}} \leq 1 \qquad \ell \in L \tag{52}$$

$$\sum_{\text{FSN} \in \mathcal{FSN}} z^{\text{FSN}} \leq n_{\mathcal{FSN}} \tag{53}$$

$$z^{\text{FSN}} \in \{0, 1\} \qquad \text{FSN} \in \mathcal{FSN} \tag{54}$$

$$x_\lambda \in \{0, 1\} \qquad \lambda \in \Lambda. \tag{55}$$

Constraints (50) indicate that a wavelength is used in the solution if it is used by any of the selected FSN. Since the FSNs are link disjoint, the same wavelength λ can be used by several FSNs. The constraint is presented as a "big-M" constraint, that is a constraint in which the right-hand side is a large enough constant (i.e., that is always larger than the value of the left-hand side) times a binary variable. Here, the value $n_{\mathcal{FSN}}$ is sufficient since Constraints (53) limits the the number of FSNs to be selected to $n_{\mathcal{FSN}}$. Constraints (51) ensure that each request is routed on at least one selected FSN. Observe that, although a request should be routed on "a unique" FSN, it is faster in practice for LP/ILP solvers to find a solution when the constraint is relax to "at least one" FSN, and then to use a simple post-processing procedure to extract a unique route per request from the solution. Constraints (52) ensure that selected FSNs are link disjoint (i.e., that a link can be used by at most one selected FSN). Constraint (53) limits the number of FSNs to be selected. Finally, Constraints (54) and (55) define the domains of the variables.

As can be observed, the above model (49)-(55) has an exponential number of variables, and corresponds to a Dantzig-Wolfe decomposition model, which can be solved by a column generation algorithm, and, e.g., another algorithm to derive an ILP solution, without the requirement of explicitly enumerating all the variables.

3.4 Solution Process

In our solution process, column generation technique is used to solve the linear relaxation of the master problem. Section 3.3 presented the mathematical model of our master problem. We proposed two pricing models to compute the new improved FSN configuration. In section 3.4.1, we propose a one step full pricing model, which includes routing and wavelength assignment inside the same mathematical formulation. In section 3.4.2, we present a simplified version of pricing model, which leads to a routing only configuration as a first step. Then, we apply graph coloring on the obtained configuration to assign wavelengths to requests, and finally we adjust the reduce cost calculation accordingly. In section 3.4.3, we provide a detailed description on how to coordinate these two models.

3.4.1 FSN Pricing Problem

This pricing problem aims to find new "improving" FSN configurations, i.e., a configuration such that if added to the current restricted master problem will improve the value of its linear relaxation, where a FSN configuration is defined by the FSN backbone (in-directed tree supporting the FSN), its set of links supporting the routing of the requests, and the wavelength assignment.

Variables. Note that several variables of the pricing problem corresponds to parameters of the Master Problem. In order to alleviate the notations, and with a slight abuse of notations, we denote them the same way.

- $a_\ell = 1$ if link ℓ is in the FSN under construction, 0 otherwise, for all $\ell \in L$
- $a_v = 1$ if node v belongs to the FSN under construction, 0 otherwise, for all $v \in V$
- $\alpha_e = 1$ if edge e belongs to the backbone of the selected tree under construction, 0 otherwise, for all $e \in E$.
- $x_k = 1$ if request k is routed on the FSN under construction, 0 otherwise, for all $k \in K$
- $\varphi_{k\ell} = 1$ if the routing of request k goes through link ℓ, or if the channel used in the routing of request k propagates on link ℓ because it is not filtered, 0 otherwise, for all $\ell \in L, k \in K$
- $\psi_{k\ell} = 1$ if the routing of request k goes through link ℓ between its source and destination, 0 otherwise, for all $\ell \in L, k \in K$

- $\theta_{kk'} = 1$ if request k and request k' are in conflict on the FSN under construction, 0 otherwise.
- $\omega_{kk'\ell} = \psi^{\text{FSN}}_{k\ell} \varphi^{\text{FSN}}_{k'\ell}$, for all $\ell \in L, k, k' \in K$. In other words, with $\omega_{kk'\ell} = 1$ identifying a link on which k and k' are conflicting either between their source and destination nodes, or with one of them being routed to the broadcast of the other request, and 0 otherwise (no conflict).
- $\beta_\lambda = 1$ if the wavelength λ is used on the FSN under construction, 0 otherwise, for all $\lambda \in \Lambda$
- $\beta_{\lambda k} = 1$ if request k is assigned the wavelength λ on the FSN under construction, 0 otherwise, for all $k \in K, \lambda \in \Lambda$

The objective of the pricing problem with respect to the z^{FSN} variables (or columns made up of their coefficients in the constraint matrix of the master problem) corresponds to the reduced cost of these variables, following the Dantzig-Wolfe decomposition algorithm which equals the simplex algorithm with an implicit enumeration of variables Therefore, variables of the pricing problem are weighted with the values of dual variables of the master problem, and we refer the reader who is not familiar with (delayed) linear programming (aka Dantzig-Wolfe algorithm) to the seminal book of Chvátal [7].
Objective:

$$\min \quad 0 \quad + \quad \sum_{\lambda \in \Lambda} u^{(50)}_\lambda \beta_\lambda \quad - \quad \sum_{k \in K} u^{(51)}_k x^k \quad + \quad \sum_{\ell \in L} u^{(52)}_\ell a_\ell \quad + \quad u^{(53)} \tag{56}$$

Subject to:

Construct an undirected tree

$$\sum_{\substack{e=\{v,v'\} \in E: \\ v,v' \in V'}} \alpha_e \leq |V'| - 1 \qquad V' \subset V, |V'| \geq 3 \tag{57}$$

$$\sum_{v \in V} a_v = \sum_{e \in E} \alpha_e + 1 \tag{58}$$

$$2\alpha_e \leq a_v + a_{v'} \qquad v, v' \in V, e = \{v, v'\} \tag{59}$$

$$\sum_{e \in \omega(v)} \alpha_e \geq a_v \qquad v \in V \tag{60}$$

$$a_\ell \leq \alpha_e,\ a_{\overline{\ell}} \leq \alpha_e \qquad \ell = (v, v'), \overline{\ell} = (v', v) \quad v, v' \in V, e = \{v, v'\} \tag{61}$$

Routing of the requests

$$\varphi_{k\ell} \le a_\ell \qquad k \in K, \ell \in L \tag{62}$$

$$a_\ell \le \sum_{k\in K} \varphi_{k\ell} \qquad \ell \in L \tag{63}$$

$$\varphi_{k\ell} + \varphi_{k\bar{\ell}} \le 1 \qquad \ell = (v, v'), \overline{\ell} = (v', v) : v \in V, v' \in V \tag{64}$$

Flow constraints for broadcast stream

$$\sum_{\ell\in\textsc{in}(d_k)} \varphi_{k\ell} = \sum_{\ell\in\textsc{out}(s_k)} \varphi_{k\ell} = x_k \qquad k \in K \tag{65}$$

$$\sum_{\ell\in\textsc{in}(s_k)} \varphi_{k\ell} = 0 \qquad k \in K \tag{66}$$

$$\varphi_{k\ell'} \le \sum_{\ell\in\textsc{in}(v)} \varphi_{k\ell} + 1 - a_{\ell'} \qquad k \in K, v \in V \setminus \{s_k\}, \ell' \in \textsc{out}(v) \tag{67}$$

$$\varphi_{k\ell} \le \varphi_{k\ell'} + 2 - a_\ell - a_{\ell'} \qquad k \in K, v \in V, \ell \in \textsc{in}(v), \ell' \in \textsc{out}(v) \setminus \{\overline{\ell}\} \tag{68}$$

Flow constraints between source and destination

$$\sum_{\ell\in\textsc{in}(d_k)} \psi_{k\ell} = \sum_{\ell\in\textsc{out}(s_k)} \psi_{k\ell} = x_k \qquad k \in K \tag{69}$$

$$\sum_{\ell\in\textsc{in}(v)} \psi_{k\ell} = \sum_{\ell\in\textsc{out}(v)} \psi_{k\ell} \le x_k \qquad k \in K, v \in V \setminus (s_k, d_k) \tag{70}$$

$$\sum_{\ell\in\textsc{out}(d_k)} \psi_{k\ell} = \sum_{\ell\in\textsc{in}(s_k)} \psi_{k\ell} = 0 \qquad k \in K \tag{71}$$

$$\psi_{k\ell} \le \varphi_{k\ell} \qquad k \in K, \ell \in L \tag{72}$$

$$\sum_{\ell\in L} \textsc{dist}_\ell \psi_{k\ell} \le \textsc{reach\_dist} \qquad k \in K \tag{73}$$

Identify wavelength conflicting paths

$$\theta_{kk'} \geq \psi_{k\ell}^{\text{FSN}} + \psi_{k'\ell}^{\text{FSN}} - 1 \qquad \ell \in L, k, k' \in K \tag{74}$$

$$\theta_{kk'} \geq \psi_{k\ell}^{\text{FSN}} + \varphi_{k'\ell}^{\text{FSN}} - 1 \qquad \ell \in L, k, k' \in K \tag{75}$$

$$\theta_{kk'} \geq \psi_{k'\ell}^{\text{FSN}} + \varphi_{k\ell}^{\text{FSN}} - 1 \qquad \ell \in L, k, k' \in K \tag{76}$$

$$\theta_{kk'} \leq \sum_{\ell \in L} (\omega_{kk'\ell} + \omega_{k'k\ell}) \qquad k, k' \in K \tag{77}$$

$$\omega_{kk'\ell} \leq \psi_{k\ell}^{\text{FSN}} \qquad \ell \in L, k, k' \in K \tag{78}$$

$$\omega_{kk'\ell} \leq \varphi_{k'\ell}^{\text{FSN}} \qquad \ell \in L, k, k' \in K \tag{79}$$

$$\omega_{kk'\ell} \geq \psi_{k\ell}^{\text{FSN}} + \varphi_{k'\ell}^{\text{FSN}} - 1 \qquad \ell \in L, k, k' \in K \tag{80}$$

Wavelength Assignment

$$\sum_{\lambda \in \Lambda} \beta_{\lambda k} = x_k \qquad k \in K \tag{81}$$

$$\beta_{\lambda k} \leq \beta_\lambda \qquad \lambda \in \Lambda, k \in K \tag{82}$$

$$\beta_{\lambda k} + \beta_{\lambda k'} \leq 2 - \theta_{kk'} \qquad \lambda \in \Lambda, k, k' \in K \tag{83}$$

Definition of variables

$$\alpha_e \in \{0, 1\} \qquad e = \{v, v'\} \in E \tag{84}$$

$$a_v \in \{0, 1\} \qquad v \in V \tag{85}$$

$$a_\ell \in \{0, 1\} \qquad \ell \in L \tag{86}$$

$$\varphi_{k\ell} \in \{0, 1\} \qquad \ell \in L, k \in K \tag{87}$$

$$\psi_{k\ell} \in \{0, 1\} \qquad \ell \in L, k \in K \tag{88}$$

$$x_k \in \{0, 1\} \qquad k \in K \tag{89}$$

$$\theta_{kk'} \in \{0, 1\} \qquad k, k' \in K \tag{90}$$

$$\beta_\lambda \in \{0, 1\} \qquad \lambda \in \Lambda \tag{91}$$

$$\beta_{\lambda k} \in \{0, 1\} \qquad \lambda \in \Lambda, k \in K. \tag{92}$$

$$\omega_{kk'\ell} \in \{0, 1\} \qquad k, k' \in K, \ell \in L. \tag{93}$$

Constraints (57) - (61) are aim to restrict the undirected tree condition. Constraints (57) are the classical subtour elimination constraints in order to guarantee an acyclic graph structure (i.e., a supporting tree for the f-subnet under construction) [23]. As the number of these constraints is exponential, we will enumerate them only implicitly throughout a so-called lazy constraint procedure [1]. Constraint (58) ensures that the selected edges correspond to the edges of a tree and so that a unique f-subnet will be generated in the entire network. Constraints (59) and (60) guarantee the consistency between node and edge variables: an edge is used in the undirected tree if and only if its two endpoints belong to it. Constraints (61) enforce the consistency between link and edge variables: if a link is used, its associated edge belongs to the un-directed tree. Here, the notation $\overline{\ell}$ denotes the link in the opposite direction of link ℓ.

Constraints (62)-(64) take care of the routing of the requests, including the links hosting the unfiltered channels. Constraints (62) and (63) ensure that a request can be routed over link ℓ if and only if that link ℓ belongs to the f-subnet under construction and vice versa, if a link belongs to the output f-subnet structure, then at least one routing uses it. Constraints (64) prevent the use of both ℓ and $\overline{\ell}$ in the routing of a given request, and so ensure that the signal can not be sent back to the sender.

Constraints (65)-(68) are the flow constraints for the requests, but include the broadcast effect. Constraints (65) states for each request, the total incoming traffic which associated to this request on the destination node should be 1 if this request is routed on this f-subnet, 0 otherwise, so as the total outgoing traffic for the source node of the request. Constraints (66) ensure that no flow supporting a request k can enter its source node s_k.

The next two sets of constraints (67)-(68) are for taking care of the propagation of the unfiltered channels. Constraints (67) enforce for every request k and node different from its source node (s_k), that if none of its incoming link is on the route or broadcast effect of request k, then none of the outgoing links can be used for either the routing or the broadcast effect of request k. Constraints (68) make sure on each node, for every request k, any used outgoing links will carry the traffic for this request if there is incoming traffic of this request. This is enforce the broadcast effect will carry over to the traffic going forward. In other words, if $\varphi_{k\ell} = a_\ell = 1$, it will force $\varphi_{k\ell'} = 1$ if $a_{\ell'} = 1$, i.e., if link ℓ' belongs to the FSN under construction.

Constraints (69)-(70) are flow constraints that define the routing of request k between its source and destination. Constraints (71) impose no wavelength assignment on the incoming links of the source and on the outgoing links of the destination for the "filtered" routing of k.

Constraint (73) is the reach constraint, for each request the routing distance between source and destination must not exceed a maximum distance (1,500 km). Constraints (72) define the relation between variables $\varphi_{k\ell}$ and $\psi_{k\ell}$, which indicate $\psi_{k\ell}$ is the subset of $\varphi_{k\ell}$.

Constraints (74)-(76) are wavelength conflicts constraints expressing that: either k and k' share a link between their source and destination nodes, see (74), or that one of the requests is routed to the broadcast part of the other request, see (75) and (76). Constraints (77)-(80) identify the no conflict case, i.e., when the routes between their source and destination are not overlapping, and then none of the requests is routed to the broadcast part of the other request.

Constraints (81) express that one and only one wavelength is used per granted request, and that no wavelength is used if the request is not granted. Constraints (82) indicate that a wavelength is used in the FSN configuration if any of the granted request uses it. Constraints (83) enforce conflicting requests to use different wavelengths.

3.4.2 Relaxed Pricing Problem

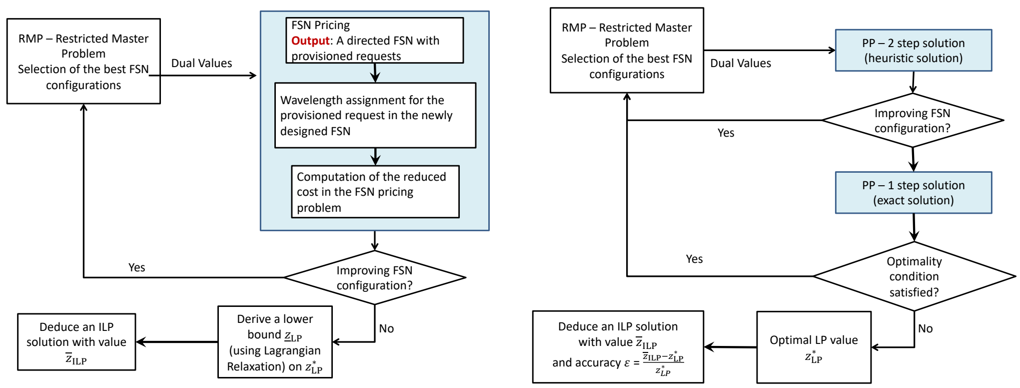
The pricing problem described in Section 3.4.1 is a quite complex optimization problem. In order to speed up the solution process, we proposed a two step process to solve the pricing problem.

The two step solution process consists in solving first a relaxation of the pricing problem defined as the original pricing problem in which we omit the constraints related to the wavelength assignment, i.e., constraints (74) to (83). We can then simplify the objective function of the pricing problem, and omit the term: $\sum\limits_{\lambda \in \Lambda} u_\lambda^{(50)} \beta_\lambda$.

This relaxation of the pricing problem outputs a FSN backbone, jointly with a routing for the granted requests. We next perform the wavelength assignment, which we reformulate as a graph coloring problem on the wavelength conflict graph. We then use the graph coloring algorithm of [22] and recompute the reduced cost in order to determine if we end up with an improving FSN configuration.

3.4.3 Detailed Solution Process

Figure 8(a) present the detailed process on a two steps pricing model. And Figure 8(b) demonstrate how we put these two model together. The advantage of two steps pricing model is lighting the computation load, especially when we apply our solution to large scale network. After the two steps pricing model reach the optimality, one step full pricing model will be applied on top of it, in order to guarantee the optimal solution.



(a) Flowchart1 (b) Flowchart2

Figure 8: Flowcharts of DW_FOP Model

Solution process steps:

1. Generate an initial solution:

   (a) Compute an FSN supported by a spanning tree, which granted all the requests

   (b) Use an ILP formulaltion which Anuj Mehrotra *et al.* presented in [22] to calculate the wavelength assignment on the FSN which step 1a generated.

2. Apply the column generation algorithm in order to solve the linear relaxation of the model of DW_FOP:

   (a) Solve restricted master problem with current FSNs

   (b) Solve two steps pricing problem:

      i. Compute an FSN with math model of 3.4.2

ii. Calculate the wavelength assignment on this FSN
iii. Re-compute the reduced cost on the generated results
iv. If it generates an improving FSN configuration, add it to the RMP and return to Step 2a

(c) Solve one step full pricing problem with the mathematical model of 3.4.1. If it generates an improving wavelength configuration, add it to the RMP and return to Step 2a

3. Continuous relaxation of the Master Problem has been solved optimally
4. Solve the last generated restricted master problem with integer requirements for the variables, derive an ILP solution.

3.5 Numerical Results

In this section, we present our computational results, which we compare both to those of the literature [34] and to our own previous results [17]

3.5.1 Data Sets

We used the same network topologies as Tremblay *et al.* [34], i.e., Italy, California, and Germany. We also consider two additional networks, also widely used in other papers on optical networking: Cost239 and USA, from [32]. We recall their characteristics in Table 3, i.e., the number of nodes and edges of the network topologies, as well as their number of requests. We consider a one unit wavelength requirement for each pair of nodes, as in [34].

Networks	# nodes	# edges	# requests
Italy	10	15	90
California	17	20	272
Germany	17	26	272
Cost239	11	26	110
USA	12	15	132

Table 3: Characteristics of network topologies and requests

3.5.2 Comparative computational results

Table 4: Comparative results when using only one FSN

	CG_FOPmodel (Jaumard *et al.* [17])				New DW_FOP model				CPU
	$\tilde{z}_{\text{LP}}$	# col.	W	CPU	$z^{\star}_{\text{LP}}$	# col.	W	CPU	ratio
Italy	41.0	180/516	41	23h10m	40	1	40	10m06s	99.27 %
California	125.6	13/890	126	1d6h46m47s	125	2	125	12m46s	99.31 %
Germany.	120.5	7/813	125	2d6h35m35s	123	2	123	50m30s	98.46%
Cost239	49.25	69/557	51	1d9h35m20s	51	1	51	20m20s	98.99 %
USA	61.0	118/620	61	6d11h45m43s	61	2	61	10m11s	99.89 %
								average	99.18 %

We summarize our computational results on different number of FSNs in Table 4, 5, and 6.

Each table contains the following results:

- z_{LP}, i.e., the linear programming value
  - Model CG_FOP: as we limited the number of solutions of the wavelength pricing problem to 500, the resulting LP value is the last computed one, and not necessarily an optimal one. It is denoted by $\tilde{z}_{\text{LP}}$ and is not a lower bound.
  - Model DW_FOP: it is the optimal one, and is denoted by $z^{\star}_{\text{LP}}$.
- the number of generated configurations for each model
  - Model CG_FOP: the number of generated FSN configurations and the number of generated wavelength configurations, i.e., 180 FSN configurations and 516 wavelength configurations for the Italy data instance in Table 4.
  - Model DW_FOP: the number of generated FSN configurations
- W: an upper bound on the minimum number of required wavelengths in order to provision all requests
- CPU: computational times for reaching the ε-optimal solution associated with the W value.

Single FSN case

For the single FSN case, we did not recourse to an initial solution, and solution of both models are done starting with an empty initial set of variables/columns.

The first striking result is the reduction of the computation time of 99% in average. We observe that all instances are solved exactly with the new DW_FOP model as the lower and upper bounds are equal. This was not the case with the CG_FOP model for which the accuracy of the solutions was ranging between 0% and 3.7%. While the number of selected configurations is one in the ILP solution, we observe that the number of generated columns is pretty large for the previous CG_FOP model, which is likely to be the key explanatory factor for the huge differences for the computational results.

Two FSN case

Table 5: Comparative results with two FSNs

	Provisioned filterless sub-networks on two trees									
	Tremblay *et al.* [34]	CG_FOP model (Jaumard *et al.* [17])				DW_FOP model Result				CPU
	W	$z^\star_{\text{LP}}$	# col.	W	CPU	$\underline{z}_{\text{LP}}$	# col.	W	CPU	ratio
Italy	25	19.98	115/653	23	58m57s	20	340	21	54m26s	7.66 %
California	-	117.4	23/952	122	1d15h37m	62.5	155	122	20h18m22s	48.74 %
Germany17	88	62.3	29/1610	73	2d22h41m31s	61.5	106	73	1d8h53m44s	53.47 %
Cost239	-	22.74	347/1183	28	4d22h17m37s	23	270	25	1d0h47m07s	79.05 %
USA	-	41.24	117/1011	53	11h51m43s	31.25	302	53	1h40m05s	85.94 %
									average	54.97 %

In the case of two FSNs, we use an initial solution made of a single FSN. We generate a single FSN to satisfy all the requests with the objective to minimize the link usage.

In addition to the comparison with the CG_FOP model [17], we report some additional computational results of the literature, i.e., the numerical results of Tremblay *et al.* [34]. As for the single FSN case, we observe that the computation time needed for solving the new DW_FOP model is significantly smaller than for the CG_FOP model (average speedup of 55%). Moreover, the solutions obtained with the DW_FOP model use less wavelengths than the solutions of both [17] and [34].

An other interesting observation is the reduction of the number of wavelengths when moving from a single FSN to two FSNs. More precisely, we reduce on average the number of wavelengths by 21, and the improvements range from 3 to 50 depending on the instance.

Three FSN case

Table 6: Network parameters for filterless solutions of three trees case

	Provisioned filterless sub-networks on three trees									
	Tremblay *et al.* [34]	CG_FOP model (Jaumard *et al.* [17])				DW_FOP model Result				CPU
	W	$z^{\star}_{\text{LP}}$	# col.	W	CPU	$\underline{z}_{\text{LP}}$	# col.	W	CPU	ratio
California	120	113.4	115/1040	120	5d10h27m29s	41.67	85	120	19h24m37s	85.12 %
Cost239	-	15.7	357/1386	25	3d19h 9m34s	15.33	201	17	39m44s	99.27 %
									average	92.195 %

In this experiment, we have changed Constraint (53) in order to select *exactly* $n_{\mathcal{FSN}} = 3$ FSNs instead of *at most* $n_{\mathcal{FSN}}$ FSNs. With this modification, we observe that some instances are no longer feasible, i.e., for some instances it is not possible to partition the network and the set of requests into three FSNs while a solution with two FSNs is possible. Consequently, we analyse only the two instances for which a solution using three FSNs exists, namely California and Cost239.

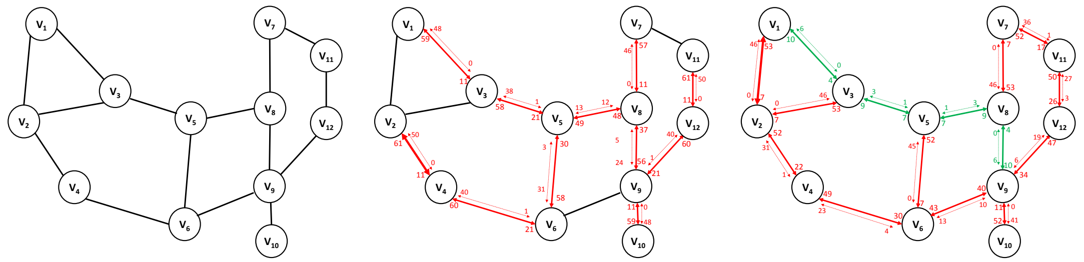
As for the other experiments, we observe a significant reduction of the resolution time compared to the CG_FOP model (92% faster in average). Furthermore, we obtain a solution for the Cost239 instance using only 17 wavelengths instead of 25, which is a significant improvement.

3.5.3 Impact of the number of FSNs

In this last experiment, we compare the solutions for the USA network, when moving from one to two FSNs.

Figure 9(a) represent the physical fiber connectivity of USA network. Figure 9(b) depicts the single spanning tree solution of USA network, and Figure 9(c) describes the solution with two FSNs.

In both Figure 9(b) and 9(c), on each link that is used in the network provisioning, we indicate different numbers. Bi-directed links indicate that the edge is used is both



(a) Physical network (b) Single spanning FSN provisioning (c) Two FSN fiber trees provisioning

Figure 9: Solution of US 12 nodes network by Using DW_FOP Model

directions, while uni-directed links mean the edge is used in only one direction. For each direction, the numbers define the number of used wavelengths on the plain links, while they indicate the number of wasted wavelengths (unfiltered channels) on the dash links.

We observe that even in the case of two FSNs, one of them has a spanning tree as a backbone tree.

3.6 Conclusions

In this paper, we proposed a new mathematical model, DW_FOP, for the design of filterless optical networks that significantly improves upon the mathematical model and algorithm proposed in [17], thanks to a decomposition into a single pricing problem combining the definition of a filterless optical sub-network and its full provisioning, including the wavelength assignment. The DW_FOP model is much faster to solve and provides better solutions than previous proposals.

In the future, we plan to enhance further the model in order to handle multi-unit requests, without breaking them into single unit requests.

Chapter 4

A Nested Path Decomposition Scheme for the Optimal Design of Filterless Optical Networks

This chapter contains a paper submitted for publication into [14].

In this chapter, we proposed a nested column generation model, called NCG_FOP, in order to speed up solution process. We break down the solution into two levels of pricing, the upper level pricing computing selected paths which assigned to granted requests, network provisioning and wavelength assignment for granted requests. The upper level pricing itself is a column generation process, which consist of a lower level pricing generated improved path for each granted requests.

4.1 Introduction

Internet traffic has been growing rapidly for many years. Due to Optical fiber offers much higher bandwidth than copper cables, optical networks provide enormous capacities in the network. It is the preferred medium for transmission of data, therefore it is widely deployed today in all kinds of telecommunications networks. Filterless optical networks is one of the deliver methods for optical networks.

The basic idea of filterless optical networks goes back to the seminal articles of [37, 29]. These networks rely on broadcast-and-select nodes equipped with coherent transceivers, as opposed to active switching networks, which today use Reconfigurable Optical Add-Drop Multiplexers (ROADMs).

The major advantage of filterless optical network is only the passive equipment will be used, i.e. the passive optical splitters/couples, which requires no electricity. Therefore it will lead to the cost reduction and environment friendly.

In this paper, we propose a nested column generation into our design, called NCG_FOP. The sub-problem was breaken down into two level of pricing problems. The upper level pricing compute selected paths for each granted requests, along with the network provisioning and wavelength assignment. The upper level pricing also rely on the lower level pricing to provided improved routing paths of the request.

The paper is organized as follows. In Section 4.2, we describe the problem statement and the concept of filterless optical network design. In Section 4.4, a mathematical model is proposed to solve filterless optical network design problem. In Section 4.5, we discuss our solution process, describe the process of our nested column generation schema, present the Lagrangian Relaxation formulation of our model. In Section 4.6, we summarize the computational results, including a comparison with the results of our previous work. Conclusions are drawn in the last section.

4.2 Design of Filterless Networks

Given an optical network, its underneath physical network is represented by a graph $G = (V, L)$, where V is the set of nodes (indexed by v), and L is the set of fiber links (indexed by ℓ). We use $\text{IN}(v)$ and $\text{OUT}(v)$ to represent the set of incoming and outgoing links of node v, respectively.

The traffic is presented by a set of unit requests, denoted as K. Each request k is characterized by its source and destination nodes, v_s and v_d respectively.

Each fiber link is limited to a transport capacity of W wavelengths (indexed by λ).

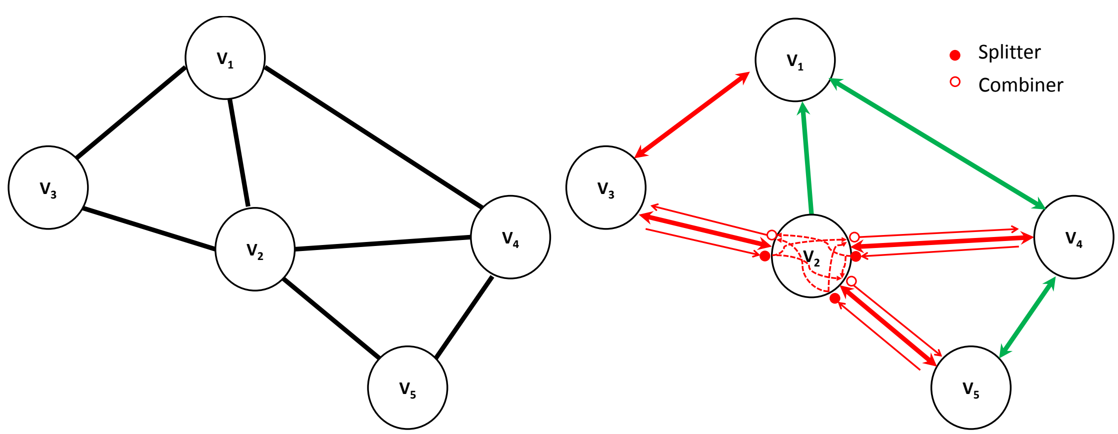
A filterless network solution consists of a set of filterless sub-network (FSN for short) solutions, where each FSN is based on an undirected tree which constructed by a set of nodes and a set of edges (represent undirectd links, which contains two directed links). The undirected tree is the subgraph of the physical network G.

Figure 10 illustrates the construction of a filterless optical network. Two FSNs are built, see Figure 10(b) the red one (FSN_1) is supported by the undirected tree $T_1 = \{\{v_1, v_3\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}\}$, and the green one ($\text{FSN}_2$), by the undirected tree $T_2 = \{\{v_1, v_2\}, \{v_1, v_4\}, \{v_4, v_5\}\}$.

In Figure 10(b), each FSN consists of a set of nodes and a set of directed fiber links. We used arrows to represent the direction of the links. Both Bi-directional fiber links, and single directional fiber link could be used to construct the FSNs. E.g., FSN_1 is constructed by only Bi-directional fiber links, while FSN_2 is built by both Bi-directional fiber links and single directional fiber link.

There might be passive splitters and combiners used on some intermediate nodes, in order to carry over the provisioning traffic. See, e.g., node v_2 requires two ports splitters/combiners to accommodate the provisioning of the requests. Figure 10(b) present the detailed internal connection on node v_2, demonstrate how the incoming traffic distributed to outgoing links through splitters/combiners.

Each request could be provisioned on one of the sub-networks. For instance, the request from node v_2 to node v_4 could be provisioned either on FSN_1 through the link of (v_2, v_4) or FSN_2 through the link of (v_2, v_1) and link of (v_1, v_4). The result is not necessarily unique, depends on the objective and the balance of the traffic. Our objective is establishing a set of FSNs such that the provisioning of all the requests which the total number of wavelength over the entire network minimized.



(a) physical network (b) Two FSNs

Figure 10: Construction of a filterless network solution by Using NCG_FOP Model

Figure 11 demonstrates the wavelength assignment for the example of 5 requests on the FSN_1 in Figure 10(b):k_{13} from v_1 to v_3, and with the same notations, k_{21}, k_{54},

k_{42}, and k_{35}.

Due to the broadcast effect, which is the nature of filterless network, when the destination is reached, the lightpath will continue downstream. See k_{13}, when the signal reached v_3 through link of (v_1, v_3), the light signal will continue going through link of (v_3, v_2), link of (v_2, v_4), and link of (v_2, v_5).

The conflict between requests defined as: if there is any shared link on their routing path among these requests, then we call them conflicted. The shared link only refer to the links which exist on the path from source to destination of one of the request. Which implies if both request going through the link due to the broadcast effect, won't be considered as conflicted. e.g. k_{54}, and k_{42}, on link of (v_2, v_3), and link of (v_3, v_1), both requests going through these links by broadcast effect, so we did not consider these links as shared link for these two requests.

In Figure 11, we conclude request k_{13} could be assigned the same wavelength with request k_{21}, as they are not conflicted. The same result between request k_{54} and k_{42}. For this example, we could use 3 wavelength to satisfy 5 requests.

From Figure 11, we used dashed line to represent the broadcast effect. On each link, there are two type of wavelength passing through it: regular signal which accommodates the traffic between source and destination, another type of signal is broadcast signal. The wavelength is used for broadcast effect, we considered it as wasted wavelength, as the same wavelength could not be re-used for the regular signal on this link. E.g. On link of (v_2, v_4), there are 3 wavelength going through this link, but only one wavelength carry the regular traffic for k_{54}; the rest 2 wavelength are the broadcast effect of request k_{35}, and k_{13}, should treated them as wasted wavelength.

4.3 Literature Review

There exist quite a number of studies on the design of filterless optical networks, and the approach we proposed is using nested column generation technique which is also widely studied in literature.

4.3.1 Literature Review on Filterless Networks

In literature, there are a lot of research works on the filterless optical networks.

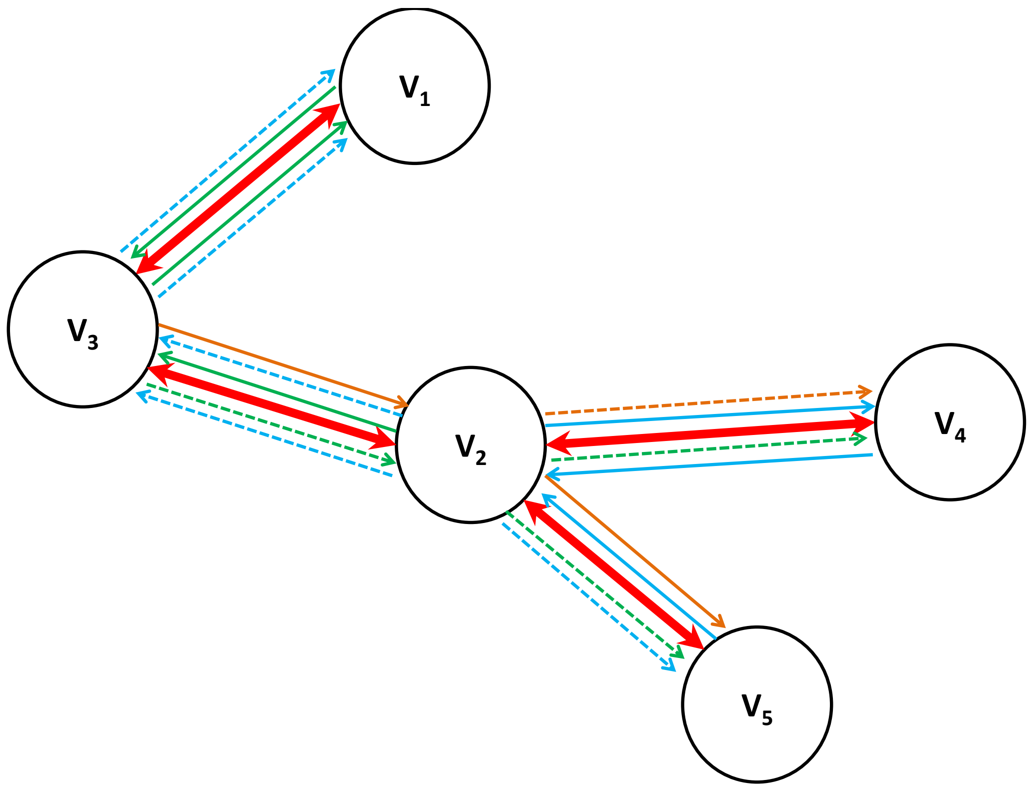


Figure 11: Wavelength request provisioning by Using NCG_FOP Model

In order to solve the filterless network design and planning problem, Tremblay *et al.* on their works:[34], [3], [35], divided the problem into two steps: (1) a genetic algorithm (GA) is first used to generate fiber trees which could connect all the nodes in the network. (2) Routing and wavelength assignment is performed on the selected trees (generated by step (1)) with a tabu search meta-heuristic approach. This strategy is based on complete knowledge of a traffic matrix

In order to achieve agility of filterless optical network design, the author of [21] propose a dynamic RWA algorithm that attempts to maximize network resources while establishing lightpaths for random connection requests. They were using load balancing strategy to perform the traffic shifting during maintenance procedures to prevent wavelength exhaustion.

In our earlier work, we proposed a column generation model in the work of [16] to minimize the maximum link network capacity on the entire network, and perform wavelength assignment on the provisioning result as a post processing step. In the work of [17], we propose a one step mechanism, which combine network provisioning and wavelength assignment into a single mathematical model by using Multi-Decomposition technique to obtain exact solution. In order to improve performance, in the paper of [15] we proposed a simplified model to ensure the scalability of our solution.

In order to improve spectrum utilization, decrease the wasted wavelength, the authors of [19], [5] also introduced Semi-Filterless Solutions, by adding wavelength filters at some selected nodes.

By recent emerging 5G technology, in order to satisfy high dynamic traffic demands, researcher started to study how to perform the network reconfiguration on the filterless network. They proposed different ways to mitigate spectrum waste. [4] proposes to deploy programmable optical switches which perform the re-provisioning in order to minimize overall spectrum consumption. On the other hand, researchers in [27], and [30] focus on monitoring the healthiness of the active lightpaths to tuning the signal power levels during the re-provisioning.

4.3.2 Column Generation and Nested Column Generation

Column Generation method is a well-known technique for solving efficiently large-scale optimization problems. The key step in the design of exact algorithms for a large class of integer programs is to embed column generation techniques within a linear-programming-based branch-and-bound framework[20]. The basic idea of column generaton is, it converted the original problem into a restricted version, only contains very limited variables/columns, which defined as restricted master problem (RMP). By adding more improved variables/columns which generated by a pricing problem into RMP based on the reduced cost, the solution process will stop until no more improvement.

Nested Column Generation implies the column-generation sub-problem itself is solved using a column-generation procedure. The underlying principle is when we take a difficult problem to solve, break it down into pieces that are tractable to solve, but the breaking up (decomposition) must be done in such a way that the solutions to the pieces can be recombined into solutions for the original problem (through the column-generation mechanism)[38].

Nested Column Generation is widely studied in the related literature. Researchers use this technique to solve different complex problems[38, 18, 33, 9, 12].

4.4 Mathematical Model: Master Problem

Let FSN be a provisioned f-subnet configuration. It is characterized by:
- $a_\ell^{\text{FSN}} = 1$ if link ℓ is in f-subnet FSN, 0 otherwise.
- $x_k^{\text{FSN}} = 1$ if request k is routed on FSN, 0 otherwise.
- $\beta_\lambda^{\text{FSN}} =$ if wavelength λ is used in f-subnet FSN, 0 otherwise.
- $n^{\text{FSN}} =$ Number of f-subnets allowed for the network
Variables:
- $z_{\text{FSN}} = 1$ if f-subnet FSN, together with its provisioned requests, is selected as a filterless sub-network, 0 otherwise.
- $x_\lambda = 1$ if wavelength λ is used, o otherwise.

The objective corresponds to the minimization of the total number of wavelengths.

$$\min \sum_{\lambda \in \Lambda} x_\lambda \tag{94}$$

subject to:

$$\sum_{\text{FSN}\in\mathcal{FSN}} \beta_\lambda^{\text{FSN}} z_{\text{FSN}} \le n_{\mathcal{FSN}} x_\lambda \qquad \lambda \in \Lambda \tag{95}$$

$$\sum_{\text{FSN}\in\mathcal{FSN}} x_k^{\text{FSN}} z_{\text{FSN}} \ge 1 \qquad k \in K \tag{96}$$

$$\sum_{\text{FSN}\in\mathcal{FSN}} a_\ell^{\text{FSN}} z_{\text{FSN}} \le 1 \qquad \ell \in L \tag{97}$$

$$\sum_{\text{FSN}\in\mathcal{FSN}} z_{\text{FSN}} \le n_{\mathcal{FSN}} \tag{98}$$

$$z_{\text{FSN}} \in \{0,1\} \qquad \text{FSN} \in \mathcal{FSN} \tag{99}$$

$$x_\lambda \in \{0,1\} \qquad \lambda \in \Lambda. \tag{100}$$

Constraints (95) represent each wavelength only will be used if any of the selected f-subnet assigned to this wavelength. Constraints (96) enforce every request will be granted. We wrote it as an inequality form, in order to ease the LP/ILP solvers. Our objective could eliminate the case of one request will be routed more than once. Constraints (97) guarantee the link disjoint between sub-networks. Constraints (98) indicate the number of FSNs could be allowed on the entire network.

4.5 Solution Process

In our solution process, nested column generation method is used to solve the linear relaxation of the so-called Master Problem. Section 4.5.1 proposed the nested column generation algorithm which we were using to solve our problem, and also illustrated the solution process through a flow chat. Section 4.5.2 presented the upper level pricing, which provides mathematical model to construct a FSN and perform network provisioning for granted requests on this FSN. Section 4.5.3 demonstrated the construction of lower level pricing. Section 4.5.4 applied the well-known lagrangian relaxation technique to our problem, in order to get valid lower bound.

4.5.1 Nested Column Generation

Column Generation method is a well-known technique to solve large-scale optimization problems efficiently [7]. We use it to solve the linear relaxation of the original problem, defined as the so-called Master Problem, i.e., (94)-(100). In order to solve the linear relaxation of master problem, we use a column generation algorithm. It consists of defining a Restricted Master Problem (RMP), i.e., the Master Problem with a very limited number of variables, and a so-called Pricing Problem, i.e., a configuration generator.

Our pricing problem is solved in turn by a decomposition algorithm (combination of column generation and ILP solution), and therefore called a nested pricing problem. It contains upper level of the pricing problem which serves as a configuration generator for master problem, lower level of pricing problem which serves as a configuration generator for the upper level of the pricing problem. Inside column generation algorithm, the pricing problem itself constructed from another column generation.

Due to the nature of nested column generation, the LP value is no longer a valid lower bound. We used Lagrangian relaxation technique to calculate the valid lower bound. Section 4.5.4 describes details on how to compute the Lagrangian relaxation bound: It corresponds to the Lagrangian bound described in chapter 12 of [23] to be checked, except that we do not need to write explicitly the compact formulation.

Fig. 12 presents the solution process of our nested column generation.

4.5.2 Upper Level FSN Pricing Problems

Pricing problem aims to find a new FSN configuration which could potentially improve the optimization objective of the restricted master problem. Each configuration will contain network provisioning and wavelength assignment.

Complete Pricing Problem

This complete Pricing problem aims to find a new FSN configuration with single mathematical model, which contains network routing and wavelength assignment for granted requests.

Variables (i.e., Parameters of the Master Problem)

$a_\ell = 1$ if link ℓ is in the f-subnet t under construction, 0 otherwise, for all $\ell \in L$

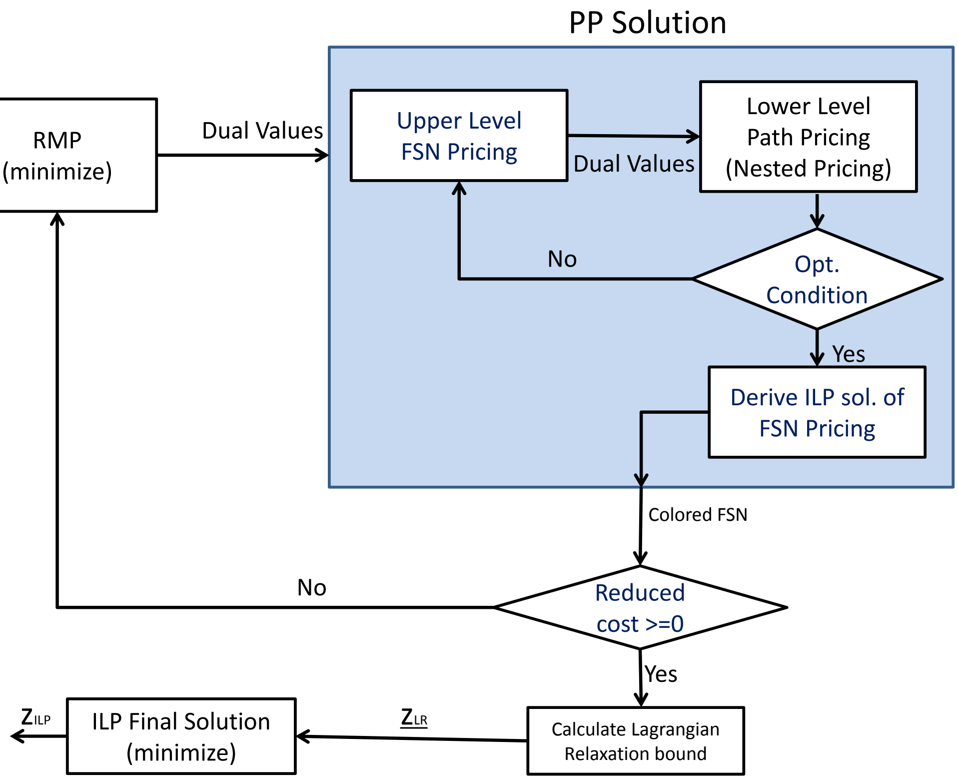


Figure 12: Flowchart of nested column generation

$a_v = 1$ if node v belongs to f-subnet under construction, 0 otherwise, for all $v \in V$

$\alpha_e = 1$ if edge e belongs to the backbone of the selected tree under construction, 0 otherwise, for all $e \in E$.

$x_k = 1$ if request k is routed on the f-subnet under construction, 0 otherwise, for all $k \in K, t \in T$

$y_p = 1$ if the path p is used for routing on the f-subnet under construction, 0 otherwise.

$a^{\text{BT}}_{\ell k} = 1$ if ℓ belongs to the broadcast tree of k, 0 otherwise.

$\theta_{kk'} = 1$ if request k and request k' are in conflict on the f-subnet under construction, 0 otherwise.

$\beta_\lambda = 1$ if the wavelength λ is used on the f-subnet under construction, 0 otherwise, for all $\lambda \in \Lambda$

$\beta_{\lambda k} = 1$ if request k is assigned the wavelength λ on the f-subnet under construction, 0 otherwise, for all $k \in K, \lambda \in \Lambda$

$\omega_{kk'\ell} = (\sum\limits_{p \in P_k} \delta^p_\ell y_p)\, a^{\text{BT}}_{\ell k'} \qquad k, k' \in K, \ell \in L.$

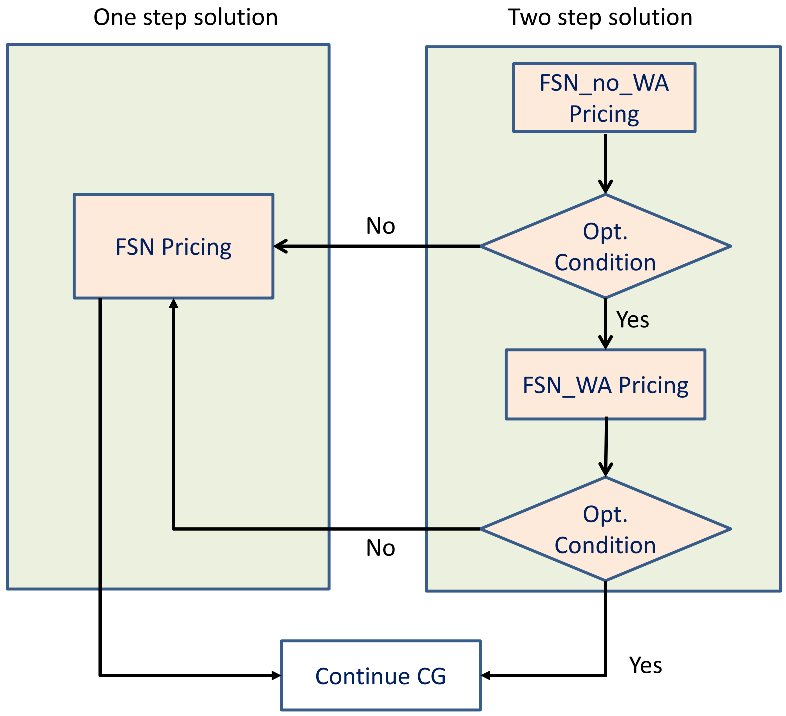


Figure 13: Column generation steps of NCG_FOP Model

objective:

$$\min \quad 0 \quad + \quad \sum_{\lambda \in \Lambda} u_{\lambda}^{(95)} \beta_{\lambda} \quad - \quad \sum_{k \in K} u_{k}^{(96)} x_{k} \quad + \quad \sum_{\ell \in L_T} u_{\ell}^{(97)} a_{\ell} \quad + \quad u^{(98)} \tag{101}$$

subject to:

$$\sum_{\substack{e=\{v,v'\}\in E:\\ v,v'\in V'}} \alpha_e \leq |V'| - 1 \qquad V' \subset V, |V'| \geq 3 \tag{102}$$

$$\sum_{v\in V} a_v = \sum_{e\in E} \alpha_e + 1 \tag{103}$$

$$2\alpha_e \leq a_v + a_{v'} \qquad v, v' \in V, e = \{v, v'\} \tag{104}$$

$$\sum_{e\in\omega(v)} \alpha_e \geq a_v \qquad v \in V \tag{105}$$

$$\sum_{p\in P_k} y_p = x_k \qquad k \in K \tag{106}$$

$$\sum_{\ell\in\text{IN}(s_k)} a^{\text{BT}}_{\ell k} = 0 \qquad k \in K \tag{107}$$

$$\sum_{\ell\in\text{IN}(d_k)} a^{\text{BT}}_{\ell k} = \sum_{\ell\in\text{OUT}(s_k)} a^{\text{BT}}_{\ell k} = x_k \qquad k \in K \tag{108}$$

$$a^{\text{BT}}_{\ell' k} \leq \sum_{\ell\in\text{IN}(v)} a^{\text{BT}}_{\ell k} + 1 - a_{\ell'} \qquad v \in V \setminus s_k, \ell' \in \text{OUT}(v) \tag{109}$$

$$a^{\text{BT}}_{\ell k} \leq a^{\text{BT}}_{\ell' k} + 2 - a_\ell - a_{\ell'} \qquad k \in K, v \in V, \ell \in \text{IN}(v), \ell' \in \text{OUT}(v) \tag{110}$$

$$a^{\text{BT}}_{\ell k} \leq a_\ell \qquad \ell \in L, k \in K \tag{111}$$

$$a_\ell \leq \alpha_e \qquad \ell \in L, e \equiv \{\ell, \overline{\ell}\} \in E, k \in K \tag{112}$$

$$a^{\text{BT}}_{\ell k} \geq \sum_{p\in P_k} \delta^p_\ell y_p \qquad k \in K, \ell \in L \tag{113}$$

$$a^{\text{BT}}_{\ell k} + a^{\text{BT}}_{\overline{\ell} k} \leq 1 \qquad k \in K, e \equiv \{\ell, \overline{\ell}\}, e \in E \tag{114}$$

$$\theta_{kk'} \geq \sum_{p\in P_k} \delta^p_\ell y_p + a^{\text{BT}}_{\ell k'} - 1 \qquad \ell \in L, k, k' \in K \tag{115}$$

$$\theta_{kk'} \leq \sum_{\ell\in L} (\omega_{kk'\ell} + \omega_{k'k\ell}) \qquad k, k' \in K_T \tag{116}$$

$$\omega_{kk'\ell} \leq \sum_{p\in P_k} \delta^p_\ell y_p \qquad \ell \in L, k, k' \in K \tag{117}$$

$$\omega_{kk'\ell} \leq a^{\text{BT}}_{\ell k'} \qquad \ell \in L, k, k' \in K \tag{118}$$

$$\omega_{kk'\ell} \geq \sum_{p\in P_k} \delta^p_\ell y_p + a^{\text{BT}}_{\ell k'} - 1 \qquad \ell \in L, k, k' \in K \tag{119}$$

$$\sum_{\lambda \in \Lambda} \beta_{\lambda k} = x_k \qquad k \in K \tag{120}$$

$$\beta_{\lambda k} \leq \beta_{\lambda} \qquad \lambda \in \Lambda, k \in K \tag{121}$$

$$\beta_{\lambda k} + \beta_{\lambda k'} \leq 2 - \theta_{kk'} \qquad \lambda \in \Lambda, k, k' \in K \tag{122}$$

$$\alpha_e \in \{0,1\} \qquad e = \{v, v'\} \in E \tag{123}$$

$$a_v \in \{0,1\} \qquad v \in V \tag{124}$$

$$a_\ell \in \{0,1\} \qquad \ell \in L \tag{125}$$

$$x_k \in \{0,1\} \qquad k \in K \tag{126}$$

$$y_p \in \{0,1\} \qquad p \in P_k, k \in K \tag{127}$$

$$a_{\ell k}^{\mathrm{BT}} \in \{0,1\} \qquad \ell \in L, k \in K \tag{128}$$

$$\theta_{kk'} \in \{0,1\} \qquad k, k' \in K \tag{129}$$

$$\omega_{kk'\ell} \in \{0,1\} \qquad \ell \in L, k, k' \in K \tag{130}$$

$$\beta_\lambda \in \{0,1\} \qquad \lambda \in \Lambda \tag{131}$$

$$\beta_{\lambda k} \in \{0,1\} \qquad \lambda \in \Lambda, k \in K. \tag{132}$$

Constraints (102) are the constraints to guarantee there is no cycles on any sub-graph of current network. Constraints (103) enforce to generate only one single f-subnet. Constraints (104) and (105) make sure two endpoints of the edge will be included into the tree if the edge constructed to the tree, and also enforce the node will be constructed into the tree if and only if any of the connected edges belong to the tree. Constraints (106) restrict each request only could use maximum one path if the request granted on the current FSN. Constraints (107) - (110) are flow constraints for broadcast tree of each request. Constraints (111) indicate the broadcast tree of each granted request only could use the links which belongs to the current FSN under construction. Constraints (112) represent each link only could be used if the associated edge belongs to the current FSN under construction. Constraints (113) represent the relation between the link usage for the path of granted request and the link usage for the broadcast tree of this request. Constraints (114) guarantee there is no loop back on the broadcast path of each request. Constraints (115) - (119) are the conflict constraints between requests. Constraints (120) - (122) are wavelength assignment constraints.

Two steps pricing

This is a simplified version of our Pricing Problem. By moving the wavelength assignment into separate step, it decreases the computing load of the mathematical model. It aims to speed up our pricing process.

In the two steps approach, we simplified our pricing model, by omitting the wavelength assignment related constraints: constraints (115) to (122), and removed $\sum_{\lambda \in \Lambda} u_{\lambda}^{(95)} \beta_{\lambda}$ term from the objective function. The simplified version of pricing model only compute the network routing for the requests as first step. Then on the second step, based on the computing result from mathematical model, conflict graph will be built accordingly, and therefore the wavelength assignment will be applied on the conflicted graph by using graph coloring algorithm [22]. The solution improvement decision will be made after the reduced cost been re-calculated.

4.5.3 Second Level Pricing Problem

The second level pricing serves as a configuration generator for the upper level pricing. It provides the path of the requests, such that the reduced cost of the upper level been minimized.

$$\text{Reduced cost} = \underbrace{\sum_{k \in K} u_k^{((106))}}_{\text{constant}} - \sum_{k \in K} \sum_{\ell \in L} \underbrace{u_{k\ell}^{(113)}}_{\leq 0} \delta_\ell^k$$

We could treated $-u_{k\ell}^{(113)}$ as the weight of the links for request k, which is all non-negative. The path will be shortest path which calculated using Dijkstra's algorithm with dual values as the weight of the links respectively.

4.5.4 Lagrangian Relaxation

Lagrangian relaxation is a relaxation method which converts a difficult problem of heavy constrained optimization to a simpler problem. In chapter 12 of [23], explained how to relax the linear constraints by bring them into the objective function.

We applied Lagrangian relaxation technique to our mathematical model, move all the constraints into objective, in order to retrieve valid bound. (133) is the objective to calculate the Lagrangian relaxation bound of our problem.

$$\begin{aligned}\min \sum_{\lambda\in\Lambda} x_\lambda + \sum_{\lambda\in\Lambda} u_\lambda^{(95)}(n_{\mathcal{FSN}} x_\lambda - \sum_{\text{FSN}\in\mathcal{FSN}} \beta_\lambda^{\text{FSN}} z^{\text{FSN}}) \\ + \sum_{k\in K} u_k^{(96)}(\sum_{\text{FSN}\in\mathcal{FSN}} x_k^{\text{FSN}} z^{\text{FSN}} - 1) \\ + \sum_{\ell\in L} u_\ell^{(97)}(1 - \sum_{\text{FSN}\in\mathcal{FSN}} a_\ell^{\text{FSN}} z^{\text{FSN}}) \\ + u^{(98)}(n_{\mathcal{FSN}} - \sum_{\text{FSN}\in\mathcal{FSN}} z^{\text{FSN}})\end{aligned} \tag{133}$$

subject to:

$$z^{\text{FSN}} \in \{0,1\} \qquad \text{FSN} \in \mathcal{FSN} \tag{99}$$

$$x_\lambda \in \{0,1\} \qquad \lambda \in \Lambda. \tag{100}$$

In order to simplify the formulation, in (134), we removed the terms which contains x_λ, they will be assigned to 0 anyway when this linear program calculated.

$$\begin{aligned}\min \cancel{\sum_{\lambda\in\Lambda} x_\lambda} + \sum_{\lambda\in\Lambda} u_\lambda^{(95)}(\cancel{n_{\mathcal{FSN}} x_\lambda} - \sum_{\text{FSN}\in\mathcal{FSN}} \beta_\lambda^{\text{FSN}} z^{\text{FSN}}) \\ + \sum_{k\in K} u_k^{(96)}(\sum_{\text{FSN}\in\mathcal{FSN}} x_k^{\text{FSN}} z^{\text{FSN}} - 1) \\ + \sum_{\ell\in L} u_\ell^{(97)}(1 - \sum_{\text{FSN}\in\mathcal{FSN}} a_\ell^{\text{FSN}} z^{\text{FSN}}) \\ + u^{(98)}(n_{\mathcal{FSN}} - \sum_{\text{FSN}\in\mathcal{FSN}} z^{\text{FSN}})\end{aligned} \tag{134}$$

We re-organize the terms by grouping them into constant terms, and variable terms as (135)

$$\begin{aligned}\min -\sum_{k\in K} u_k^{(96)} + \sum_{\ell\in L} u_\ell^{(97)} + u^{(98)} n_{\mathcal{FSN}} \\ - \sum_{\lambda\in\Lambda} u_\lambda^{(95)}(\sum_{\text{FSN}\in\mathcal{FSN}} \beta_\lambda^{\text{FSN}} z^{\text{FSN}}) + \sum_{k\in K} u_k^{(96)}(\sum_{\text{FSN}\in\mathcal{FSN}} x_k^{\text{FSN}} z^{\text{FSN}}) \\ - \sum_{\ell\in L} u_\ell^{(97)}(\sum_{\text{FSN}\in\mathcal{FSN}} a_\ell^{\text{FSN}} z^{\text{FSN}}) - u^{(98)}(\sum_{\text{FSN}\in\mathcal{FSN}} z^{\text{FSN}})\end{aligned} \tag{135}$$

We could use ub to represent the terms constructed by constant:

$$ub = -\sum_{k\in K} u_k^{(96)} + \sum_{\ell\in L} u_\ell^{(97)} + u^{(98)} n_{\mathcal{FSN}}$$

The terms which contains variables are formed reduced cost:

$$\begin{aligned}
\text{Reduced cost } = & \\
& -\sum_{\lambda\in\Lambda} u_\lambda^{(95)} \Big(\sum_{\text{FSN}\in\mathcal{FSN}} \beta_\lambda^{\text{FSN}} z^{\text{FSN}} \Big) \\
& + \sum_{k\in K} u_k^{(96)} \Big(\sum_{\text{FSN}\in\mathcal{FSN}} x_k^{\text{FSN}} z^{\text{FSN}} \Big) \\
& - \sum_{\ell\in L} u_\ell^{(97)} \Big(\sum_{\text{FSN}\in\mathcal{FSN}} a_\ell^{\text{FSN}} z^{\text{FSN}} \Big) \\
& - u^{(98)} \Big(\sum_{\text{FSN}\in\mathcal{FSN}} z^{\text{FSN}} \Big) \qquad (136)
\end{aligned}$$

So we could represent Lagrangian relaxation bound as: $\underline{z}_{LR}$ = ub + { Reduced cost }

4.5.5 Detailed Solution Process

Solution process steps:

1. Generate an initial solution:
   (a) Compute an FSN supported by a spanning tree, which granted all the requests
   (b) Use an ILP formulaltion which Anuj Mehrotra *et al.* presented in [22] to calculate the wavelength assignment on the FSN which step 1a generated.
2. Apply the nested column generation model NCG_FOP, in order to solve the linear relaxation of model:
   (a) Solve restricted master problem with current FSNs
   (b) Solve two steps pricing problem:
      i. Apply a lower level column generaton algorithm in order to get an improved FSN

A. Compute an FSN with the model of 4.5.2

B. Compute paths for all the eligible requests with 4.5.3

C. If it generates improved paths, add it to the FSN and return to Step 2(b)iA

ii. Calculate the wavelength assignment on this FSN

iii. Re-compute the reduced cost on the generated results

iv. If it generates an improving FSN configuration, add it to the RMP and return to Step 2a

(c) Solve one step full pricing problem with the mathematical model of 4.5.2. If it generates an improving wavelength configuration, add it to the RMP and return to Step 2a

3. Continuous relaxation of the Master Problem has been solved optimally

4. Compute Lagrangian Relaxation bound with the math formulation of (135)

5. Solve the last generated restricted master problem with integer requirements for the variables, derive an ILP solution.

4.6 Numerical Results

4.6.1 Data Sets

The network topologies which were used for our experiment are widely cited in literature: Italy(10 nodes with 15 bi-links), California(17 nodes with 20 bi-links), and Germany(17 nodes with 26 bi-links) as our data sets which Tremblay *et al.* [34] also used, plus two other networks: Cost239(11 nodes with 26 bi-links), and USA(12 nodes with 15 bi-links. We recall their characteristics in Table 7, which contains the number of nodes, the number of bi-links which connect the nodes to form a network, and also along with the number of requests which will be granted. We consider a uniform traffic matrix between nodes(e.g. one wavelength for every possible connection), as in [34]. So we generated request per each node pair for our uniform traffic case.

Table 7: Characteristics of 5 Cited Networks

Networks	# nodes	# edges	# requests
Italy	10	15	90
California	17	20	272
Germany	17	26	272
Cost239	11	26	110
USA	12	15	132

4.6.2 Computational Comparisons

We applied our solution against the data sets which listed on table 7 on a single spanning tree, two trees, and three trees cases. And also compare the results with our previous papers. Table 8, table 9, and table 10 demonstrate the result on single spanning tree, two trees, and three trees cases respectively.

From the results which presented inside the Table 8, table 9, and table 10, although the quality of new result solution is similar as our previous work, the performance (in terms of computation time) of the solution is significantly improved. Especially for the multiple tree cases.

Table 8: Network parameters for filterless solutions on a single spanning tree case

	Provisioned filterless sub-networks on single spanning tree											
	CG_FOP model [17]				DW_FOP model[15]				NCG_FOP model Result			
	$z^{\star}_{\text{LP}}$	#col.	W	CPU	z_{LP}	#col.	W	CPU	z_{LR}	#col.	W	CPU
Italy	41	180/516	41	23h10m	40	1	40	10m06s	40	2	40	10m3s
California	125.6	13/890	126	1d6h46m47s	125	2	125	12m46s	125	2	125	11m47s
Germany	120.5	7/813	125	2d6h35m35s	123	2	123	50m30s	123	2	123	12m51s
Cost239	49.25	69/557	51	1d9h35m20s	51	1	51	20m20s	51	3	51	10m06s
USA	61	118/620	61	6d11h45m43s	61	2	61	10m11s	61	2	61	10m09s

4.6.3 Detailed Provisioning Solution

In Figure 14, we illustrate the detailed provisioning solution for the cost239 network, for single spanning tree, two trees, and three trees cases respectively. On each routed

Table 9: Network parameters for filterless solutions on two tree case

	Provisioned filterless sub-networks on two trees											
	CG_FOP model [17]				DW_FOP model[15]				NCG_FOP model Result			
	$z^\star_{\text{LP}}$	#col.	W	CPU	$\underline{z}_{\text{LP}}$	#col.	W	CPU	$\underline{z}_{\text{LR}}$	#col.	W	CPU
Italy	19.98	115/653	23	58m57s	20	340	21	54m26s	20.06	324	21	43m04s
California	117.4	23/952	122	1d15h37m	62.5	155	122	20h18m22s	62.5	30	122	40m46s
Germany	62.3	29/1610	73	2d22h41m31s	61.5	106	73	1d8h53m44s	61.5	16	73	1h48m42s
Cost239	22.74	347/1183	28	4d22h17m37s	23	270	25	1d0h47m07s	23.91	323	25	22m26s
USA	41.24	117/1011	53	11h51m43s	31.25	302	53	1h40m05s	31.5	20	53	48m37s

Table 10: Network parameters for filterless solutions on three tree case

	Provisioned filterless sub-networks on three trees											
	CG_FOP model [17] Result				DW_FOP model[15] Result				NCG_FOP model Result			
	$z^\star_{\text{LP}}$	#col.	W	CPU	$\underline{z}_{\text{LP}}$	#col.	W	CPU	$\underline{z}_{\text{LR}}$	#col.	W	CPU
California	113.4	115/1040	120	5d10h27m29s	41.67	85	120	19h24m37s	42	9	120	1h23m11s
Cost239	15.68	357/1386	25	3d19h9m34s	15.33	201	17	39m44s	15.33	126	17	18m03s

link, we use arrows to indicate the direction of the link, the numbers beside the link represent the number of wavelength passing through this link on the directed direction. On each routed link, we used a dashed line, along with the numbers to represent the wasted wavelength, which caused by broadcast effect.

On the demonstrated solution of Figure 14, we could see the provisioning was very balanced. On the single spanning tree case, the max load is 51, which also the wavelength passing through 5 links. On the two trees case, the max load on both red FSN, and Green FSN are 25. On the three trees case, the max load on three different FSNs(Red, Green, Blue) are all 17. This observation indicate our solution is very balanced across the entire network.

4.6.4 Non-Uniform Traffic (not limited to one unit requests)

We also applied our solution into non-uniform traffic requests.

For a given node pair, we dynamically generate a number of units of request randomly in [0,3]. This role will apply on each node pair.

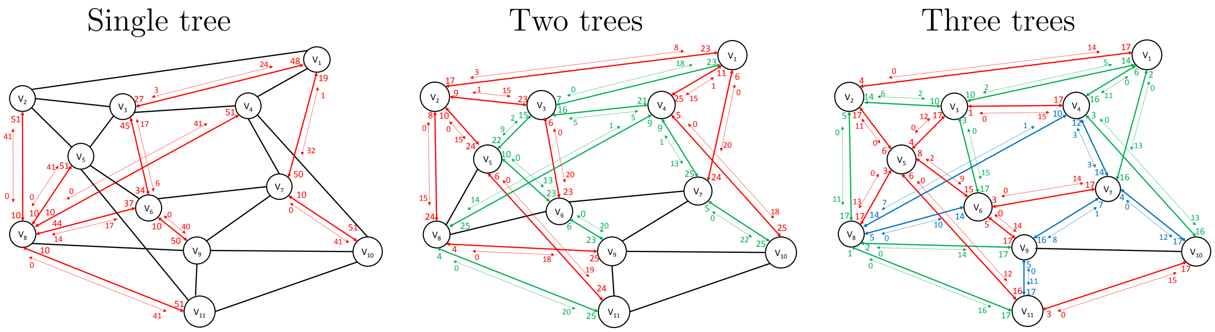


Figure 14: Filterless sub-networks Solutions on the cost239 11 nodes network

e.g. (v_1, v_7) 3 requests, k_1, k_2, k_3: all with same source & destination. Routing for these requests not necessarily in the same FSN although they have the same source & destination. If in the same FSN, it will be the same path, but then, it has to be with different wavelengths as we treated them as different requests.

Table 11 present the results for the non single unit traffic cases. On each network, first column is the total number of requests generated on that network. The following columns are the computation results to satisfy these requests on each network.

Table 11: Filterless solutions for non-single unit traffic

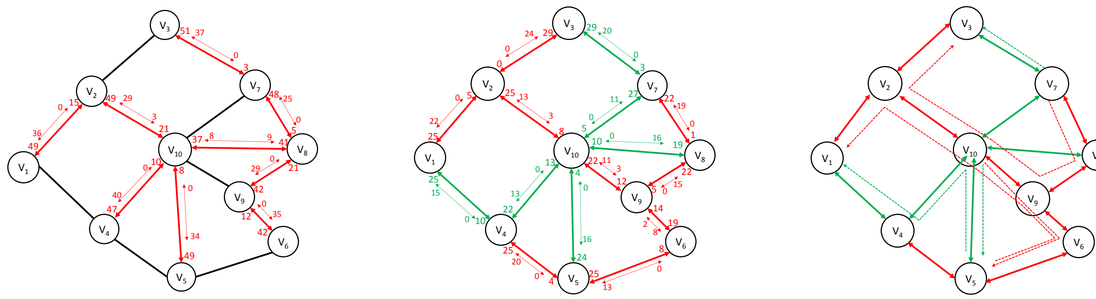
	# requests	Single spanning tree				Two trees				Three trees			
		$\underline{z}_{\mathrm{LR}}$	# col.	W	CPU	$\underline{z}_{\mathrm{LR}}$	# col.	W	CPU	$\underline{z}_{\mathrm{LR}}$	# col.	W	CPU
Italy	101	51	2	51	13m14s	27	16	29	17m9s	-	-	-	-
California	356	182	2	182	20m22s	94.5	13	176	4h36m11s	59.5	17	175	7h5m25s
Germany	374	177	2	177	39m28s	89.5	28	106	8h15m47s	-	-	-	-
Cost239	168	82	2	82	11m2s	41	19	57	11m59s	27	32	36	17m03s
USA	207	106	2	106	19m10s	63.5	16	91	19m29s	-	-	-	-

Table 12 demonstrate the detailed generated traffic and the provisioning on a two tree solution for Italian 10 nodes network. On each element, the number of the upper row present the unit of the traffic on this node pair; on the lower row illustrate the provisioning: A, B refer to tree A (red tree), tree B (green tree), which contains how many unit of traffic provisioned on this tree. For each node pair, if there is more than one unit of the traffic, they are not necessarily provisioned on the same tree. e.g. There are 2 unit of traffic on $v_5 \rightsquigarrow v_1$, one unit is provisioned on red tree (tree A), another unit is provisioned on green tree (tree B). See also $v_7 \rightsquigarrow v_3$, $v_{10} \rightsquigarrow v_5$.

Table 12: Non-Single unit traffic provision on two trees

		DESTINATION									
SOURCE		v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
v_1	Unit	0	3	1	0	3	1	3	3	1	0
	PROVISION		A(3)	B(1)		B(3)	A(1)	B(3)	B(3)	A(1)	
v_2	Unit	0	0	3	1	1	1	0	0	0	0
	PROVISION			A(3)	A(1)	A(1)	A(1)				
v_3	Unit	0	0	0	0	0	0	0	0	0	3
	PROVISION										B(3)
v_4	Unit	3	1	3	0	0	2	0	0	1	0
	PROVISION	B(3)	A(1)	B(3)			A(2)			A(1)	
v_5	Unit	2	0	0	0	0	0	2	0	3	1
	PROVISION	A(1) B(1)						A(1)		A(3)	B(1)
v_6	Unit	1	0	0	3	3	0	3	2	0	0
	PROVISION	A(1)			A(3)	A(3)		A(3)	A(2)		
v_7	Unit	1	0	3	0	0	0	0	0	0	1
	PROVISION	B(1)		A(1) B(2)							B(1)
v_8	Unit	2	3	1	0	3	0	1	0	1	3
	PROVISION	B(2)	A(3)	B(1)		B(3)		B(1)		A(1)	B(3)
v_9	Unit	1	0	1	1	2	0	0	2	0	2
	PROVISION	A(1)		A(1)	A(1)	A(2)			A(2)		A(2)
v_{10}	Unit	2	3	2	2	3	3	3	0	0	0
	PROVISION	B(3)	A(3)	B(2)	B(2)	A(1) B(2)	A(3)	B(3)			

Fig. 15 illustrates the result of provisioning and wavelength assignment for the non-single unit traffic case for Italian 10 nodes network. Fig. 15(a) demonstrates the single spanning tree, and Fig. Fig. 15(b) for the two trees case. The sub-figure Fig. 15(c) presents the node pairs which granted to the different trees on different unit of traffic, the detailed provisioning for the requests: $v_5 \rightsquigarrow v_1$, , $v_7 \rightsquigarrow v_3$, and $v_{10} \rightsquigarrow v_5$.



(a) Single tree (b) Two trees (c) Split traffic

Figure 15: Provision for non-single unit traffic on the Italian 10 nodes network

4.6.5 Wasted Bandwidth

Table 13 present the wasted bandwidth ratio, which contains the total number of wavelength used over all the links in order to satisfy the demands, total number of wasted wavelength over all the links due to the downstream, and also the wasted ratio (percentage). From the results, we could see the wasted ratio almost fall into the range 50%- 60%, regardless the complexity of the network, number of requests, number of trees to provision. The wasted wavelength is caused by the nature of filterless design.

Table 13: Wasted/Used wavelength of all links

	Single tree			Two trees			Three trees			Single tree(Multi)			Two trees(Multi)		
	Used	Wasted	%	Used	Wasted	%	Used	Wasted	%	Used	Wasted	%	Used	Wasted	%
Italy	236	203	46	224	203	47	-	-	-	265	285	52	218	244	53
California	1292	1349	51	1256	1220	49	1244	1205	49	1505	1860	55.3	1658	2007	54.7
Germany	1004	1449	59	856	1208	58	-	-	-	1364	2127	60.9	1187	1800	60.3
Cost239	320	349	52	301	314	51	236	295	55	539	521	49	414	554	57.2
USA	420	451	52	406	418	51	-	-	-	707	765	52	601	692	53.5

4.7 Conclusions

We developed a very efficient algorithm for solving the design of filterless network problem exactly. By using nested column generation technique, we could manage to solve the problem efficiently. Inside our nested column generation model, we tear down our solution process into two level of pricing problems, the upper level pricing computing selected paths which assigned to granted requests, network provisioning and wavelength assignment for granted requests. The upper level pricing itself is a column generation process, which consist a lower level pricing generated improved path for each granted requests. In order to obtain the valid bound for self-contained model, Lagrangian Relaxation technique was introduced to our model. Further more, we also validate our solution on the data instance which contains non-single unit traffic. Numerical results demonstrate the performance improvement compared to our previous works, in terms of CPU time.

Chapter 5

Conclusions and Future Work

In this thesis, we studied the design of filterless optical networks. We proposed several different solution algorithms which allow its exact solution to be reached efficiently. The results of the thesis have been submitted in [17, 15, 14].

5.1 Conclusions

In this thesis, we studied the design of filterless optical networks. In the literature, there are already some results published on this area. Most of them solve filterless optical network design problem by heuristic or meta-heuristic. We proposed mathematical models trying to solve it exactly.

In the earlier stage of the work, we proposed a mathematical model to solve network provisioning problem exactly, such that the total number of wavelength has been minimized. Wavelength assignment was applied on the resulting provisioning solution as a post-processing step. We applied our solution to the same data sets used by other researchers, some of our results already got improved. Since the wavelength assignment was a post-processing step, it could not be considered as exact optimal solution. We then improved our design in the following areas:

1. The quality of the solution

We propose a one step mathematic model, which combine network provisioning and wavelength assignment into the same mathematical model. Multi-Decompsition technique was used in our solution. We build three different sub-problems to generate different paramter set, and feed them into our master problem.

We applied our multi-decomposition solution to the same data set which Tremblay *et al.* [34] used, the results of the solution are improved.

2. The scalability of the solution

   Although we got optimal solution by using our multi-decomposition model, due to the complexity of the problem, the performance was not desired. When we applied our solution to the more connective networks, it would take several days to complete the calculation.

   We proposed a simplified mathematical model in our second paper. We extract the complicated wavelength assignment decision, move it to sub-problem. Sub-problem will generate an improved configuration which contains the routing, and the wavelength assignment on the current provisioning. Master problem only decide which configuration would be chosen, the problem got simplified significantly. The performance (in terms of CPU time) get improved. The improvement varied from network to networks. Some of them from days to hours, or from hours to minutes,..., etc.

3. The efficiency of the solution

   In order to improve the efficiency of the solution, we introduced nested column generation in the third part of our research work. The upper level pricing performed the network routing and wavelength assignment. The network provisioning will assign the paths to the granted requests. The paths were provided by lower level pricing. In this way, the performance get improved again.The improvement varied from network to networks.

5.2 Future Work

In our research, the designs we proposed, have still room to be improved.

Currently when we did the multi-unit traffic support on node pair, we added requests between source and destination nodes. By using this approach, the number of variables, and the number of constraints would be increased. We will improve our model to support multi-units traffic support on each request, by defining the unit of traffic as a parameter of the request.

By adding different techniques into our solution process, the performance of our optimization model get improved more and more. But on the other hand, when the network becomes more complex, or the number of requests increases dramatically, our model will hit some limitations. We will seek for more efficient heuristics to generate the higher quality of initial solutions, in order to speed up the column generation process.

In our design, we minimized the wavelength usage cross the whole network. Although we improved our mathematical models in order to try obtaining the exact optimal solutions, the ratio of wasted wavelength still on the range 50% to 60%. This is caused by nature of pure filterless network design. In order to exceed this limit, we have to introduce "white boxes/blockers" into our design. By using the "white boxes/blockers" on certain nodes of the network which will stop the wavelength from downstream, in order to make the stopped wavelength re-usable. Archambault *et al.* [3] also added blockers to allow wavelength reuse at strategic locations, in order to decrease the required number of wavelength. We need to modify the model in order to satisfy the following constraints:

1. limited number of blockers could be used on the network
2. where to put those limited number of blockers could make the overall wavelength usage across the entire network to be minimized.

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