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ABSTRACT

We propose a novel primordial black hole (PBH) formation mechanism based on a first-order phase transition (FOPT). If a fermion species gains a huge mass in the true vacuum, the corresponding particles get trapped in the false vacuum as they do not have sufficient energy to penetrate the bubble wall. After the FOPT, the fermions are compressed into the false vacuum remnants to form non-topological solitons called Fermi-balls, and then collapse to PBHs due to the Yukawa attractive force. We derive the PBH mass and abundance, showing that for a $\mathcal{O}$(GeV) FOPT the PBHs could be $\sim 10^{17}$ g and explain all of dark matter. If the FOPT happens at higher scale, PBHs are typically overproduced and extra dilution mechanism is necessary to satisfy current constraints.

1. Introduction

Primordial black holes (PBHs) are hypothetical black holes which form prior to any galaxies and stars [1,2]. Although not yet confirmed by experiments, PBHs have been a source of interest, as they can serve as a good candidate for dark matter (DM) [2–8], can seed supermassive black holes [9–13], can generate the baryon asymmetry of the Universe [14–24], and can explain some gravitational wave (GW) signals at LIGO/Virgo [25–30], etc. While the most popular PBH formation mechanism is the collapse of the overdense region from primordial perturbations of inflation [31–33], there are other scenarios such as the collapse of cosmic topological defects [34–40], scalar field fragmentation [41–44], etc. PBHs can also form during a first-order phase transition (FOPT) in the early Universe via bubble collisions [45–53].

In this letter, we propose a novel PBH formation mechanism based on the collapse of non-topological solitons produced during a cosmic FOPT. The simplest realization of this mechanism consists of a real scalar ϕ and a Dirac fermion χ, with the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - U(\phi) + \bar{\chi} i \partial\!\!\!/ \chi - g_\chi \phi \bar{\chi} \chi , \tag{1}$$

which conserves the fermion number by a global $U(1)_Q$. The scalar potential $U(\phi)$ triggers a FOPT from $\langle\phi\rangle = 0$ to w_* at temper-

ature T_*. If $g_\chi w_* \gg T_*$, the fermions cannot penetrate into the new (true) vacuum bubbles, where they acquire a mass $M_\chi^* \equiv g_\chi w_*$ that significantly exceeds their thermal kinetic energy. Consequently, after the FOPT, fermions are trapped in the old (false) vacuum and compressed to form non-topological solitons called *Fermi-balls* [54] if there is an asymmetry between the number densities of χ and $\bar{\chi}$, such that only χ's survive the $\chi\bar{\chi} \to \phi\phi$ annihilation. The conditions for Fermi-ball formation can be easily satisfied in many new physics models [54]: $g_\chi w_* \gg T_*$ can be realized by a supercooled FOPT [55–61] or strong coupling [62,63] while the χ asymmetry can be generated by various asymmetric DM mechanisms [64–66].

Fermi-balls are macroscopic compact objects that collect huge Q-charge originating from the fermion asymmetry. Inside the Fermi-ball is the false vacuum $\langle\phi\rangle = 0$, in which χ's are massless fermions interacting with each other via the attractive Yukawa potential

$$V(r) = -\frac{g_\chi^2}{4\pi r} e^{-M_\phi r}, \tag{2}$$

whose range of force M_ϕ^{-1} is determined by the effective mass at the false vacuum

$$M_\phi^2 = \frac{d^2 U(\phi, T)}{d\phi^2}\bigg|_{\phi=0} = \mu^2 + c T^2, \tag{3}$$

where $U(\phi, T)$ is the thermal potential, c is the thermal coefficient contributed by the light degrees of freedom (DOF) in a model. At the beginning, the effect of Eq. (2) is negligible because its range
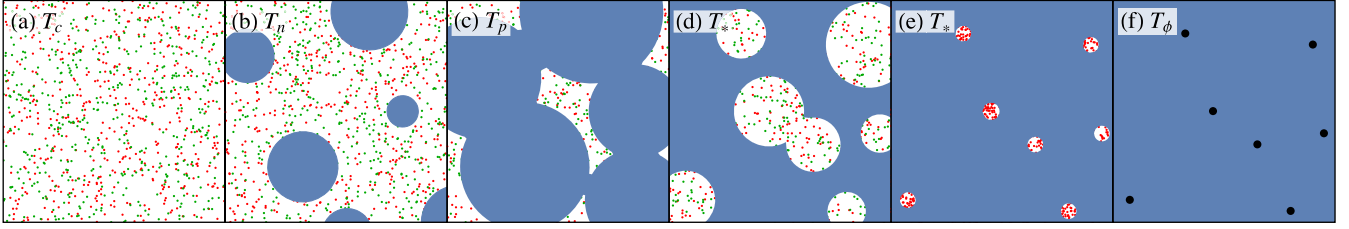
Fig. 1. Sketch of the mechanism. **(a)** T_c: χ (red points) and $\bar{\chi}$ (green points) live in the false vacuum (white). **(b)** T_n: true vacuum bubbles (blue) nucleate. Fermions cannot penetrate into the bubbles due to the large mass gap. **(c)** T_p: Fermions are trapped in the false vacuum remnants. **(d,e)** T_*: remnants shrink to form Fermi-balls. **(f)** T_ϕ: Fermi-balls collapse into PBHs.

of force M_ϕ^{-1} is very small, and we can treat χ's as independent particles obeying the Fermi-Dirac distribution. However, M_ϕ^{-1} increases as the Universe cools down. When M_ϕ^{-1} is comparable to the mean separation of χ's inside the Fermi-ball, the attractive Yukawa force dominates and Fermi-balls collapse into PBHs.

In short, the FOPT forms Fermi-balls, which in turn collapse into PBHs when the internal Yukawa force becomes dominant.[1] PBH formation via the attractive scalar force has been also studied in Refs. [68,69] where an ultralight scalar is introduced to provide a long-range force, causing the growth of density perturbation in the plasma. In our work, ϕ needs not to be light: the fermions inside a Fermi-ball are so dense that even a relatively short-range Yukawa force is able to cause instability, and collapse happens only in individual Fermi-balls. Since this situation can be easily realized in many particle physics models, we conclude that our scenario is rather generic. The mechanism is illustrated schematically in Fig. 1, and the details are discussed below.

2. Fermi-ball formation in FOPT

At critical temperature T_c, the two vacua are degenerate, and the Universe still stays in the false vacuum $\langle \phi \rangle = 0$, see Fig. 1a. Below T_c, the Universe acquires a probability of decaying to the true vacuum $\langle \phi \rangle = w(T)$ that has a lower energy, and the decay rate is dominated by the $O(3)$-symmetric bounce action $S_3(T)$ of the thermal potential $U(\phi, T)$ [70];

$$\Gamma(T) \sim T^4 e^{-S_3(T)/T} . \tag{4}$$

Bubbles start to nucleate at T_n when

$$\int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H^4(T)} \approx 1, \tag{5}$$

(see Fig. 1b), where the Hubble constant is $H(T)^2 = (8\pi/3M_{\rm Pl}^2) \times (\rho_R(T) + \rho_U(T))$. Here $\rho_R(T) = \pi^2 g_* T^4/30$ is the radiation energy density, g_* is the number of relativistic DOF, $\rho_U(T) = U(0, T) - U(w, 0)$ is the energy of the false vacuum with respect to the true vacuum at $T = 0$, and $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV.

Given $\Gamma(T)$ and $H(T)$, we can define the volume fraction of the false vacuum to the Universe as $p(T) = e^{-I(T)}$ where $I(T)$ is the amount of true vacuum volume per unit comoving volume [71–73]. The percolation temperature T_p is defined by $p(T_p) = 0.71$ [74] at which the bubbles form an infinite connected cluster, see Fig. 1c. The Fermi-balls form at T_* defined by $p(T_*) = 0.29$ [54] at which the false vacuum is separated into disconnected remnants, which first split and then shrink to Fermi-balls. The critical remnant is defined as a remnant that just ends

splitting and starts to shrink, and its radius R_* is determined by [54]

$$\Gamma(T_*) V_* \left(\frac{R_*}{v_b} \right) \sim 1 , \quad V_* = \frac{4\pi}{3} R_*^3 , \tag{6}$$

where v_b is the bubble wall velocity. Eq. (6) means that the critical remnant shrinks to a Fermi-ball before another true vacuum bubble is created inside it. The number density of the critical remnants is given by $n_{\rm rem.}^* = V_*^{-1} p(T_*)$ and it is also the number density of Fermi-balls right after the formation, $n_{\rm FB}^* = n_{\rm rem.}^*$.

Fermions are trapped in the false vacuum and the trapping fraction $F_\chi^{\rm trap.}$ is a function of v_b and $M_\chi^*/T_* = g_\chi w_*/T_*$ [54,75], where $w_* = w(T_*)$. For example, $M_\chi^* = 12 T_*$ and $v_b = 0.6$ yield $F_\chi^{\rm trap.} \approx 0.98$ in a benchmark model [54].[2] When the remnants shrink, the fermions are forced to annihilate via $\chi\bar{\chi} \to \phi\phi$. The ϕ bosons are free to pass through the wall to the true vacuum, and finally annihilate/decay to the Standard Model (SM) particles via the Higgs portal coupling $\phi^2 |H|^2$. Describing the χ asymmetry by $\eta_\chi \equiv (n_\chi - n_{\bar{\chi}})/s(T)$ with $s(T) = 2\pi^2 g_* T^3/45$ being the entropy density, the number of χ fermions surviving the annihilation in a critical remnant is

$$Q_{\rm FB} = F_\chi^{\rm trap.} \frac{n_\chi - n_{\bar{\chi}}}{n_{\rm FB}^*} = F_\chi^{\rm trap.} \frac{\eta_\chi s_* V_*}{p(T_*)} , \tag{7}$$

where $s_* = s(T_*)$. A remnant stops shrinking when the trapped fermions' degeneracy pressure is able to balance the vacuum pressure $U_0(T_*)$. Once such balance is built, the remnant together with its trapped fermions form a Fermi-ball, see Fig. 1d and e.

Let us now evaluate the profile of a Fermi-ball. Minimizing the Fermi-ball energy

$$E_{\rm FB} = \frac{3\pi}{4} \left(\frac{3}{2\pi} \right)^{2/3} \frac{Q_{\rm FB}^{4/3}}{R} + \frac{4\pi}{3} U_0(T_*) R^3, \tag{8}$$

yields the mass and radius as [54][3]

$$M_{\rm FB} = Q_{\rm FB} \left(12\pi^2 U_0(T_*) \right)^{1/4} , \quad R_{\rm FB}^3 = \frac{3}{16\pi} \frac{M_{\rm FB}}{U_0(T_*)} , \tag{9}$$

where $U_0(T) \equiv U(0, T) - U(w(T), T)$ is the vacuum energy difference between the interior and exterior of the Fermi-ball. Note that the first term in Eq. (8) only considers the degeneracy pressure of the Fermi gas and we neglect the subdominant contribution from

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[1] The collapse of Fermi-balls via gravity is recently studied in [67].

[2] More precisely, an FOPT by the generic singlet scalar potential $\mu^2\phi^2/2 + \mu_3\phi^3/3 + \lambda\phi^4/4$ was studied in Ref. [54]. In such a model, it is possible to realize $w_*/T_* \sim \mathcal{O}(10)$ if $|\mu_3|$ is fairly large.

[3] The FOPT-induced Fermi-ball is first proposed in Ref. [54]; the terminology "Fermi-ball" is also used in literatures with different physical meanings [76–78]. There are various DM mechanisms based on trapping particles into the false vacuum: quark nuggets [79–86], accidentally asymmetric DM [87], FOPT-induced Q-balls [88,89], etc.
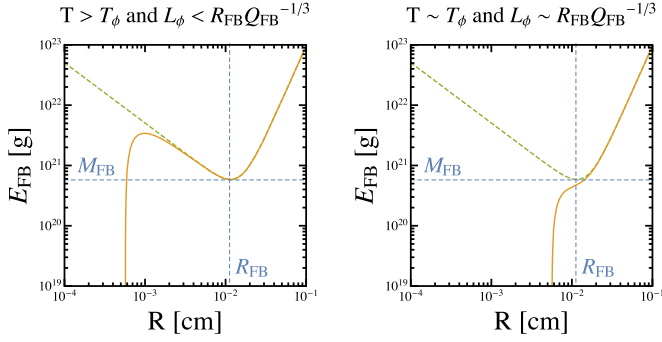
Fig. 2. The energy profile (orange lines) changes as T drops. Left: when $T > T_\phi$, the range of Yukawa force L_ϕ is small compared with the mean separation $R_{FB} Q_{FB}^{-1/3}$, and hence a stable Fermi-ball solution can exist. The green dashed line represents the energy profile without the Yukawa interaction. Right: when T cools down to T_ϕ, $L_\phi \sim R_{FB} Q_{FB}^{-1/3}$ and causes the instability. In that case there is no stable solution any more and the Fermi-ball collapses into a PBH.

thermal excitations. See the appendix for a more accurate Fermi-ball profile. As we will see, R_{FB} is typically much larger than the Schwarzschild radius of a Fermi-ball, thus gravity can be neglected in the calculation. Recently, it is proposed that during a FOPT the trapped fermions might be compressed to be a PBH without forming any stable soliton in Ref. [53]. Numerical simulations show that such a scenario requires a very large mass gap $M_\chi^*/T_* \gtrsim 200$ and small Yukawa $g_\chi \lesssim 10^{-3}$ [53]. Since we consider $M_\chi^*/T_* \sim \mathcal{O}(10)$ and $g_\chi \sim \mathcal{O}(1)$ in this letter, Fermi-balls can safely form during the FOPT in our scenario.

3. Collapse of Fermi-balls into PBHs

After formation, the Fermi-ball cools down by emitting light particles, e.g. $\chi \to \chi f \bar{f}$ where f could be electrons or neutrinos [79] (via the $\phi^2 |H|^2$ or other portal couplings). As shown in the appendix, the cooling time scale of the Fermi-ball is much shorter than the Universe expansion. Therefore, the Fermi-ball is able to track the temperature of the plasma, so that its profile changes slowly by replacing $T_* \to T$ in Eq. (9), and the number density is diluted as $n_{FB} = n_{FB}^* s(T)/s_*$. As the temperature drops, the Yukawa potential Eq. (2) starts to play a role because its range of force inside the Fermi-ball $L_\phi \equiv M_\phi^{-1} = (\mu^2 + c\,T^2)^{-1/2}$ correspondingly increases. Then, we should add the Yukawa potential energy of a Fermi-ball into Eq. (8):

$$\Delta E_{FB} \approx -\frac{3g_\chi^2}{20\pi} \frac{Q_{FB}^2}{R} \times \frac{5}{2}\left(\frac{L_\phi}{R}\right)^2 . \quad (10)$$

When $L_\phi/R_{FB} \sim Q_{FB}^{-1/3}$ (which means that the correlation length reaches the mean separation of χ's), the above energy dominates over the Fermi gas kinetic term in Eq. (8), and its negative sign causes the instability of Fermi-balls, resulting in the collapse into PBHs. See Fig. 2 and Fig. 1f for illustrations of the energy profile and PBH formation, respectively. A more accurate expression of the Yukawa energy is given in the appendix. The PBH mass and number density right after formation are given by $M_{PBH} = M_{FB}|_{T \to T_\phi}$ and $n_{FB}^* s(T_\phi)/s_*$, respectively, where T_ϕ is the collapse temperature.

Up to now, our PBH scenario has been completely described. Given the thermal potential $U(\phi, T)$, one can derive T_* by calculating $\Gamma(T)$ and $H(T)$. The Fermi-ball profile is determined by $\Gamma(T_*)$ and $U_0(T_*)$, while the PBH profile is derived by running the Fermi-ball profile to T_ϕ. Below we give some general estimates for the PBH profile based on a few simplified assumptions.

4. The PBH profile

We consider the radiation dominated era and approximate $T_* \approx T_p$ since the Fermi-ball formation is very close to the percolation. In this case, the bounce action can be obtained as [90]

$$\frac{S_3(T_*)}{T_*} \approx 131 - 4\ln\left(\frac{T_*}{100\,\text{GeV}}\right)$$
$$- 4\ln\left(\frac{\beta/H}{100}\right) + 3\ln v_b - 2\ln\left(\frac{g_*}{100}\right), \quad (11)$$

where β/H is the time scale ratio of the Universe expansion and the FOPT. Next, we assume $U_0(T_*) \approx \alpha \times \rho_R(T_*)$, where α is the ratio of the latent heat of FOPT to the radiation energy density. The parameters α and β/H are also crucial in the calculation of GWs [91–94] from a FOPT. For simplicity, we omit the temperature dependence of the Ferm-ball mass.

We are now able to express the Fermi-ball and PBH profiles as functions of $(v_b, \eta_\chi, T_*, \alpha, \beta/H)$. Let $F_\chi^{\text{trap.}} \approx 1$, the charge and radius of a Fermi-ball are given by

$$Q_{FB} \approx 1.0 \times 10^{42} \times v_b^3 \left(\frac{\eta_\chi}{10^{-3}}\right) \times$$
$$\left(\frac{100}{g_*}\right)^{1/2} \left(\frac{100\,\text{GeV}}{T_*}\right)^3 \left(\frac{100}{\beta/H}\right)^3, \quad (12)$$

$$R_{FB} \approx 4.8 \times 10^{-3}\,\text{cm} \times v_b \left(\frac{\eta_\chi}{10^{-3}}\right)^{1/3} \times$$
$$\left(\frac{100}{g_*}\right)^{5/12} \left(\frac{100\,\text{GeV}}{T_*}\right)^2 \left(\frac{100}{\beta/H}\right) \alpha^{-1/4}. \quad (13)$$

Note that by definition $\eta_\chi \lesssim n_\chi^{\text{eq}}(T)/s(T) \approx 4.2 \times 10^{-3} \times (100/g_*)$, where $n_\chi^{\text{eq}}(T) = 3\zeta(3)T^3/(2\pi^2)$ is the equilibrium distribution before the FOPT. The mass is

$$M_{FB} \approx M_{PBH} \approx 1.4 \times 10^{21}\,\text{g} \times v_b^3 \left(\frac{\eta_\chi}{10^{-3}}\right) \times$$
$$\left(\frac{100}{g_*}\right)^{1/4} \left(\frac{100\,\text{GeV}}{T_*}\right)^2 \left(\frac{100}{\beta/H}\right)^3 \alpha^{1/4}. \quad (14)$$

Therefore our scenario prefers PBHs with sublunar mass or below. Denoting $R_{\text{Sch}} \equiv 2M_{FB}/M_{\text{Pl}}^2$ as the Schwarzschild radius, one obtains

$$\frac{R_{FB}}{R_{\text{Sch}}} \approx 2.3 \times 10^4 \times v_b^{-2} \left(\frac{10^{-3}}{\eta_\chi}\right)^{2/3} \times$$
$$\left(\frac{100}{g_*}\right)^{1/6} \left(\frac{\beta/H}{100}\right)^2 \alpha^{-1/2}, \quad (15)$$

which confirms that gravity is negligible in the Fermi-ball formation. On the other hand, for the formation of Fermi-balls, the initial range of the Yukawa force has to be smaller than $R_{FB}/Q_{FB}^{1/3}$, i.e.

$$L_\phi^{-1}|_{T_*} = \sqrt{\mu^2 + c\,T_*^2} \gtrsim 3.3 \times g_\chi T_* \left(\frac{g_*}{100}\right)^{1/4} \alpha^{1/4}. \quad (16)$$

If the above condition is not satisfied, the old vacuum remnants directly collapse into PBHs instead of forming Fermi-balls as intermediate states. In that case, the PBH profile highly depends on the evolution trajectory of the remnants, and numerical simulation is necessary.

PBHs with mass between 10^9 g and 10^{17} g evaporate between the Big Bang Nucleosynthesis (BBN) and today, leaving impacts on the BBN, Cosmic Microwave Background and extragalactic and Galactic γ-ray backgrounds, which in turn put stringent constraints on the PBH abundance. Such constraints are usually shown

as upper limits for β'_{PBH} as a function of PBH mass, where β'_{PBH} is related to the energy fraction of PBH to the Universe at formation time [6–8]. For our scenario,

$$\beta'_{\text{PBH}} \approx 1.4 \times 10^{-15} \times v_b^{-3} \left(\frac{g_*}{100}\right)^{1/2} \times$$
$$\left(\frac{T_*}{100\,\text{GeV}}\right)^3 \left(\frac{\beta/H}{100}\right)^3 \left(\frac{M_{\text{PBH}}}{10^{15}\,\text{g}}\right)^{3/2} . \quad (17)$$

For $M_{\text{PBH}} \gtrsim 10^9$ g, the upper limit of β'_{PBH} varies from 10^{-29} to 10^{-17} [6–8]. For example, for $v_b = 0.6$, $g_* = 100$, $T_* = 100$ GeV, and $\beta/H = 100$, current bounds require $M_{\text{PBH}} < 10^{10}$ g, which corresponds to $\eta_\chi < 1.8 \times 10^{-14}$ if $\alpha = 1$.

PBHs with mass larger than 5.1×10^{14} g can survive until today. Such PBHs can be probed/constrained by Hawking radiation (if $M_{\text{PBH}} \lesssim 10^{17}$ g), gravitational lensing, dynamical processes, cosmic structure, etc [6–8]. The abundance is usually described by the fraction of PBH to DM, i.e. $f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{DM}} \leqslant 1$. While there are already stringent constraints for f_{PBH}, there is still a mass window 10^{17} g $\sim 10^{21}$ g that PBHs can account for all DM.[4] In our scenario, we have

$$f_{\text{PBH}} \approx 1.3 \times 10^3 \times v_b^{-3} \left(\frac{g_*}{100}\right)^{1/2} \left(\frac{T_*}{100\,\text{GeV}}\right)^3 \times$$
$$\left(\frac{\beta/H}{100}\right)^3 \left(\frac{M_{\text{PBH}}}{10^{15}\,\text{g}}\right) , \quad (18)$$

implying that non-evaporating PBHs can become a DM candidate when the formation temperature T_* is significantly lower than 100 GeV. For example, for $v_b = 0.6$, $g_* = 10$, $T_* = 1.1$ GeV, $\alpha = 1$, $\beta/H = 100$ and $\eta_\chi = 0.9 \times 10^{-10}$, one obtains $M_{\text{PBH}} = 4.2 \times 10^{17}$ g and $f_{\text{PBH}} = 1$, allowing a PBH DM candidate. Note that even in the mass region that PBHs cannot be the dominant DM component, they can still play an important role in the Universe evolution, such as seeding supermassive black holes or large scale structure formation.

We emphasize that above discussions on β'_{PBH} and f_{PBH} apply only to an adiabatically expanding Universe. If there is some process happening between the formation of PBHs and the BBN that enhances the entropy of the Universe by a factor of Δ, then the abundance is diluted as $\beta'_{\text{PBH}} \to \beta'_{\text{PBH}}/\Delta$ and $f_{\text{PBH}} \to f_{\text{PBH}}/\Delta$, and the constraints would be weakened. As there are various mechanisms generating significant entropy, such as thermal inflation [100–102], early matter era [103–106], late-time decay of domain walls [107–109], etc, we can say that our PBH DM scenario is possible in many particle physics models.

Finally, we comment on the wall velocity v_b. After nucleation, the bubble wall is accelerated for a short time, until the friction pressure P from the plasma balances the vacuum pressure $\Delta U = U(0, T) - U(w(T), T)$, and then it reaches the ultimate velocity v_b. In principle, v_b can be resolved by solving the Boltzmann equation provided that the particle content and interactions of the model are given. In our simple setup, only χ and $\bar{\chi}$ contribute to the pressure, and if we assume the thermal equilibrium of them and a 100% reflection rate (i.e. $F_\chi^{\text{trap.}} = 1$), P is given by [75]

$$P = \frac{7\pi^2}{180} \frac{(1 + v_b)^3}{1 - v_b^2} T^4 . \quad (19)$$

For example, when $\Delta U = (100\,\text{GeV})^4$ and $T = 80$ GeV, the balancing condition $P = \Delta U$ yields $v_b = 0.6$. For other values of ΔU

and T, v_b typically becomes $0.2 \sim 0.8$. If there are other species contributing to the friction pressure, v_b can be reduced, which corresponds to the lighter PBHs, as $M_{\text{PBH}} \propto v_b^3$.

5. Summary

We have discussed a novel PBH formation mechanism based on the collapse of Fermi-balls from a FOPT and illustrated that it can explain all of DM if $T_* \ll 100$ GeV. When $T_* \gtrsim 100$ GeV, our scenario typically predicts the overproduction of PBHs, and suitable dilution mechanism is needed to satisfy the experimental constraints. Since such a dilution is ubiquitous in new physics models, our scenario can be applicable in various models of particle physics.

Throughout the letter, we have assumed that a fermion χ has a vanishing bare mass and that the FOPT occurs by the transition of $\langle\phi\rangle$ from $\langle\phi\rangle = 0$ to w_* for simplicity. Thus, χ is massless and massive in the false and true vacua, respectively. However, even if the fermion has a bare mass M_0 and the FOPT is generally from $\langle\phi\rangle = w_1^*$ to w_2^*, our mechanism still applies as long as $M_\chi|_{\text{true}} = |M_0 + g_\chi w_2^*| \gg M_\chi|_{\text{false}} + T_* = |M_0 + g_\chi w_1^*| + T_*$. In this case, the Fermi-ball/PBH profile needs to be modified, but the qualitative picture remains the same.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

The Fermi-ball profile.– Given the charge Q_{FB}, the energy of a Fermi-ball with radius R at temperature T is

$$E_{\text{FB}} = \frac{3\pi}{4} \left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{\text{FB}}^{4/3}}{R} \left[1 + \frac{4\pi}{9} \left(\frac{2\pi}{3}\right)^{1/3} \frac{R^2 T^2}{Q_{\text{FB}}^{2/3}}\right]$$
$$+ 4\pi\sigma_0 R^2 + \frac{4\pi}{3} U_0(T) R^3, \quad (20)$$

where the first term is the Fermi gas kinetic energy derived from the low-temperature expansion of the Fermi integral, the second term is surface tension which is negligible since the Fermi-ball has a macroscopic size, and the third term is the bulk potential energy. By solving $dE_{\text{FB}}/dR|_{R_{\text{FB}}} = 0$ one obtains the Fermi-ball profile

$$M_{\text{FB}} = Q_{\text{FB}} \left(12\pi^2 U_0(T)\right)^{1/4} \left(1 + \frac{\pi T^2}{4\sqrt{3 U_0(T)}}\right), \quad (21)$$

$$R_{\text{FB}} = \left[\frac{3}{16} \left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{\text{FB}}^{4/3}}{U_0(T)}\right]^{1/4} \left(1 - \frac{\pi T^2}{12\sqrt{3 U_0(T)}}\right).$$

For a strong FOPT, $U_0^{1/4}(T) \gtrsim T$, the numbers in above brackets are very close to 1, leading to the expressions in Eq. (8) in the main text. The stable conditions for a Fermi-ball against decay and fission are respectively

$$\frac{dM_{\text{FB}}}{dQ_{\text{FB}}} < g_\chi w(T), \quad \frac{d^2 M_{\text{FB}}}{dQ_{\text{FB}}^2} < 0; \quad (22)$$

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[4] This window can be probed in the future [95–99].

While the second condition is automatically satisfied if the surface term ($\propto Q_{FB}^{2/3}$) is considered, the first condition needs to be checked for a specific model.

Cooling of Fermi-balls.– When Fermi-balls are shrinking, the old vacuum remnant is in equilibrium via the fast $\chi\bar{\chi} \to \phi\phi$ process. After the Fermi-ball formation, pair annihilation stops since only χ's are left inside Fermi-balls. Instead, the χ system cools down by emitting light fermions such as electrons or neutrinos [79]. Following the method in Ref. [86], according to Stefan-Boltzmann's law, the energy escaping from a Fermi-ball per unit time is

$$\mathcal{L}(T) = \frac{N_f}{4} \times \frac{7}{8} \times \left(\frac{\pi^2}{30} T^4\right) \left(4\pi R_{FB}^2\right), \tag{23}$$

where the N_f is the number of DOF of light fermions, e.g. 6 for 3 generations of neutrinos. We then can establish the cooling equation for a Fermi-ball as

$$\frac{dM_{FB}}{dt} = -\mathcal{L}(T), \tag{24}$$

where M_{FB} is the Fermi-ball mass given in Eq. (21). Eq. (24) can be solved analytically if we neglect the slow temperature dependence of $U_0(T)$. Defining the cooling time scale τ_{FB} as the time duration of a Fermi-ball cooling from T to $T/2$, we find

$$\frac{\tau_{FB}}{H^{-1}(T)} \approx 0.16 \times \left(\frac{3}{N_f}\right) v_b \left(\frac{g_*}{100}\right)^{7/12} \times$$
$$\left(\frac{\eta_\chi}{10^{-3}}\right)^{1/3} \left(\frac{100}{\beta/H}\right) \alpha^{1/4}, \tag{25}$$

which means τ_{FB} is typically shorter than the Universe expansion time scale and Fermi-balls can cool down to T_{PBH} and collapse.

The Yukawa potential energy.– Assume a uniform distribution for χ's in the Fermi-ball, the full expression for Yukawa energy is

$$\Delta E_{FB} = -\frac{3g_\chi^2}{20\pi} \frac{Q_{FB}^2}{R} f\left(\frac{L_\phi}{R}\right), \tag{26}$$

where

$$f(\xi) = \frac{5}{2}\xi^2 \left[1 + \frac{3}{2}\xi\left(\xi^2 - 1\right) - \frac{3}{2}\xi(\xi+1)^2 e^{-2/\xi}\right], \tag{27}$$

satisfying $f(0) = 0$ and $f(\infty) = 1$.

When $L_\phi/R \lesssim 0.01$, $f(\xi) \approx 5\xi^2/2$ is a very good approximation and the total Fermi-ball energy is

$$E_{FB} = \frac{3\pi}{4}\left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{FB}^{4/3}}{R} - \frac{3g_\chi^2}{8\pi} \frac{Q_{FB}^2 L_\phi^2}{R^3} + \frac{4\pi}{3} U_0 R^3. \tag{28}$$

To get the minimum of the energy, we take the first-order derivative, finding

$$R^4 \frac{dE_{FB}}{dR} \propto u^3 + pu + q, \tag{29}$$

where $u \equiv R^2$ and

$$p = -\frac{3}{16}\left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{FB}^{4/3}}{U_0}, \quad q = \frac{9g_\chi^2}{32\pi^2} \frac{L_\phi^2 Q_{FB}^2}{U_0}. \tag{30}$$

Define the discriminant

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 \propto \frac{36}{\pi^2} g_\chi^4 L_\phi^4 U_0 - 1, \tag{31}$$

when $\Delta < 0$, u has two positive roots, with the larger one being the Fermi-ball radius square R_{FB}^2, and the smaller one being

the local maximum of the energy. If $\Delta \geqslant 0$, then no Fermi-ball solution is found, and the old vacuum remnant just collapses to a PBH directly. This consideration already gives the condition for the Fermi-ball collapse,

$$L_\phi > \sqrt{\frac{\pi}{6}} \frac{U_0^{-1/4}}{g_\chi} = \frac{1}{g_\chi} \sqrt{\frac{2\pi}{3\sqrt{3}}} \left(\frac{2\pi}{3}\right)^{1/6} \frac{R_{FB}}{Q_{FB}^{1/3}}, \tag{32}$$

which is consistent with the naive estimate in the main text.

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