Italian Association of Aeronautics and Astronautics
XXVI International Congress
31st August – 3rd September 2021
On-line event hosted by the Tuscany AIDAA Section in Pisa

A LINEAR TRANSFORMATION FOR THE RECONSTRUCTION OF THE RESPONSES BETWEEN SYSTEMS IN SIMILITUDE

F. Tavasso[1], A. Casaburo[1*], G. Petrone[1], F. Franco[1], S. De Rosa[1]
[1] PASTA-Lab (Laboratory for Promoting experiences in Aeronautical Structures and Acoustics), Department of Industrial Engineering – Aerospace Section, University of Naples "Federico II",Via Claudio 21, 80125, Naples, Italy

*f.tavasso@studenti.unina.it

ABSTRACT
The increasing attention towards the possibility of scaling structures and, therefore, systems in similitude in engineering field has led to plenty of methods which allow to reconstruct the response of a system, starting from that of a reference one. In fact, this approach would help to overcome the obstacles associated with full-scale testing, such as cost and setup. However, the associated predictions may not be fully reliable, due to some intrinsic limitations characterizing the traditional similitude methods (based on the definition of similitude conditions and scaling laws), such as: size effects, rate sensitivity phenomena, distorted similitudes. For this reason, a new method, called VOODOO (Versatile Offset Operator for the Discrete Observation of Objects), has been proposed; such a method is based on the definition of a transformation matrix which links the outputs between two sets of points belonging to a linear systems.
The applications of VOODOO to plates and systems of plates demonstrate that an exact estimation of the frequency response is obtained when the degrees of freedom involved in the definition of the transformation are considered. Therefore, this work aims at investigating, by means of a sensitivity analysis, the method's strengths and limitations when other degrees of freedom are considered, in order to identify the direction for further developments.

Keywords: similitude, linear transformation, plate

1 INTRODUCTION

The use of similitudes has expanded into many engineering fields since the analytical, numerical, and experimental analyses can be executed more easily in a transformed solution domain. In fact, by defining a set of similitude conditions and scaling laws, which allow to pass from the response of a model to that of the full-scale prototype (and vice versa), it is possible to test directly the scaled-up (or down) model in order to overcome many of the disadvantages characterizing experimental tests and numerical simulations, such as financial costs, set-up management, computational power, etc. Similitudes proved to be useful in the investigation of many types of structures, as well as perform several types of analyses, such as static and dynamic behaviour, impact response and, therefore, failure analysis, etc. [1]

Despite the number of similitude methods, there are inevitably limitations in the applicability of the scaling laws they provide: size effects, structural complexity, alteration of the modes succession [2], etc. Thus, similitude theory provides good reconstructions only when models in complete similitude (namely, models satisfying completely the set of similitude conditions) are taken into account. Nonetheless, the problem of partial similitudes (models which do not satisfy at least one similitude condition) must not be underestimated, since it may arise due to manufacturing errors and/or limits. Therefore, there is a need to research and formulate alternative methods to overcome this limit, which has led to the definition of VOODOO method, already proposed in a work by De Rosa et al. [3]. The purpose of this work is to investigate the potentialities of VOODOO in off-design condition, with specific attention to single plates and assemblies of two plates.

2 SUMMARY OF THE VOODOO METHOD

The main purpose of VOODOO method is to construct a linear transformation matrix T, called VOODOO matrix, between the frequency responses of a prototype, $\pi(\omega)$, and a model, $\mu(\omega)$, both subjected to the same load. For a given number N of degrees of freedom, equal for both systems, if $T(\omega)$ is the transformation matrix between the two systems (dependent on the excitation frequency, ω) and if the output vector of the prototype is known, it is possible to derive the response of the model and vice versa, using the inverse matrix of T:

$$\mu(\omega) = \mathbf{T}(\omega)\,\pi(\omega) \tag{1}$$

$$\mathbf{\Theta}(\omega) = \mathbf{T}(\omega)^{-\mathbf{1}} \tag{2}$$

$$\pi(\omega) = \mathbf{\Theta}(\omega)\,\mu(\omega) \tag{3}$$

The vectors $\mu(\omega)$ and $\pi(\omega)$ have size [N × 1] and the matrix $T(\omega)$ and its inverse have [N × N].
The transformation matrix is constructed by considering the responses of the prototype to a unitary excitation applied to one point (of the N points chosen) at time. These are called VOODOO points. As a consequence, N linear systems like

$$\begin{cases} \mu_1^{(F_{\mu,1}=1)} = T_{1,1}\pi_1^{(F_{\pi,1}=1)} + T_{1,2}\pi_2^{(F_{\pi,1}=1)} + \ldots + T_{1,N}\pi_N^{(F_{\pi,1}=1)} \\ \mu_2^{(F_{\mu,1}=1)} = T_{2,1}\pi_1^{(F_{\pi,1}=1)} + T_{2,2}\pi_2^{(F_{\pi,1}=1)} + \ldots + T_{2,N}\pi_N^{(F_{\pi,1}=1)} \\ \vdots \\ \mu_N^{(F_{\mu,1}=1)} = T_{N,1}\pi_1^{(F_{\pi,1}=1)} + T_{N,2}\pi_2^{(F_{\pi,1}=1)} + \ldots + T_{N,N}\pi_N^{(F_{\pi,1}=1)} \end{cases}$$

$$\vdots \tag{4}$$

$$\begin{cases} \mu_1^{(F_{\mu,N}=1)} = T_{1,1}\pi_1^{(F_{\pi,N}=1)} + T_{1,2}\pi_2^{(F_{\pi,N}=1)} + \ldots + T_{1,N}\pi_N^{(F_{\pi,N}=1)} \\ \mu_2^{(F_{\mu,N}=1)} = T_{2,1}\pi_1^{(F_{\pi,N}=1)} + T_{2,2}\pi_2^{(F_{\pi,N}=1)} + \ldots + T_{2,N}\pi_N^{(F_{\pi,N}=1)} \\ \vdots \\ \mu_N^{(F_{\mu,N}=1)} = T_{N,1}\pi_1^{(F_{\pi,N}=1)} + T_{N,2}\pi_2^{(F_{\pi,N}=1)} + \ldots + T_{N,N}\pi_N^{(F_{\pi,N}=1)} \end{cases}$$

can be written. Thus, the outputs of the points of the model are seen as a linear combination of the outputs of the points of the prototype, through the coefficients of the VOODOO matrix.
Using Betti's reciprocity theorem, by rearranging the unknown terms, i.e. the elements of the transformation matrix, a $[N^2 \times N^2]$ block diagonal matrix is obtained. The r-th block on the diagonal, [N × N], can be written in this way:

$$\begin{bmatrix} \pi_1^{(F_{\pi,1}=1)} & \pi_2^{(F_{\pi,1}=1)} & \ldots & \pi_N^{(F_{\pi,1}=1)} \\ \text{symm.} & \pi_2^{(F_{\pi,2}=1)} & \ldots & \pi_N^{(F_{\pi,2}=1)} \\ \ldots & \ldots & \ldots & \ldots \\ \text{symm.} & \text{symm.} & \ldots & \pi_N^{(F_{\pi,N}=1)} \end{bmatrix} \begin{bmatrix} T_{r,1} \\ T_{r,2} \\ \vdots \\ T_{r,N} \end{bmatrix} = \begin{bmatrix} \mu_r^{(F_{\mu,1}=1)} \\ \mu_r^{(F_{\mu,2}=1)} \\ \vdots \\ \mu_r^{(F_{\mu,N}=1)} \end{bmatrix} \tag{5}$$

$$[\mathbf{\Gamma}]\, T^{(r)} = \mu_r \tag{6}$$

The i-th column of the matrix Γ represents the response of the i-th point (of the prototype) to the unit load applied on each of the N points. The right-hand column vector represents the response of the r-th point (of the model) to the unit load applied on each point. The column vector $T^{(r)}$ represents the r-th row of the unknown transformation matrix. It represents, therefore, the unknown terms of the problem:

$$\begin{bmatrix} T_{r,1} \\ T_{r,2} \\ \vdots \\ T_{r,N} \end{bmatrix} = [\boldsymbol{\Gamma}]^{-1} \begin{bmatrix} \mu_1{}^{(F_{\mu,r}=1)} \\ \mu_2{}^{(F_{\mu,r}=1)} \\ \vdots \\ \mu_N{}^{(F_{\mu,r}=1)} \end{bmatrix} \rightarrow \quad T^{(r)} = [\boldsymbol{\Gamma}]^{-1}\mu_r \tag{7}$$

It may be noted that the matrix Γ depends only on one system, the prototype. It includes all the responses of each point to the unit load applied on each point at a fixed frequency.
Moreover, the design of the VOODOO matrix is based on the outputs of the points to a unit excitation applied to each point. Since they are outputs, material properties, damping, and boundary conditions are indirectly taken into account.

3 VALIDATION OF VOODOO METHOD

Once the properties and characteristics of the two systems are defined, the VOODOO matrix is constructed. To validate the method, the same systems are subjected to a different load and the VOODOO matrix is used to predict the response curve of the model from that of the prototype and vice versa.

3.1 Test case 1: two simple plates

The first test concerns two panels; the prototype has sizes equal to 0.656x0.400x0.001 mm, while the model has dimensions equal to 0.460x0.364x0.003 mm. Three DOFs are considered for each panel (Figure 1). Both the test articles are simply supported on all four sides. The material is a typical aircraft material, an aluminium alloy, with Young's modulus, density, and Poisson's ratio equal to 71 GPa, 2723 kg/m^3 and 0.33, respectively.

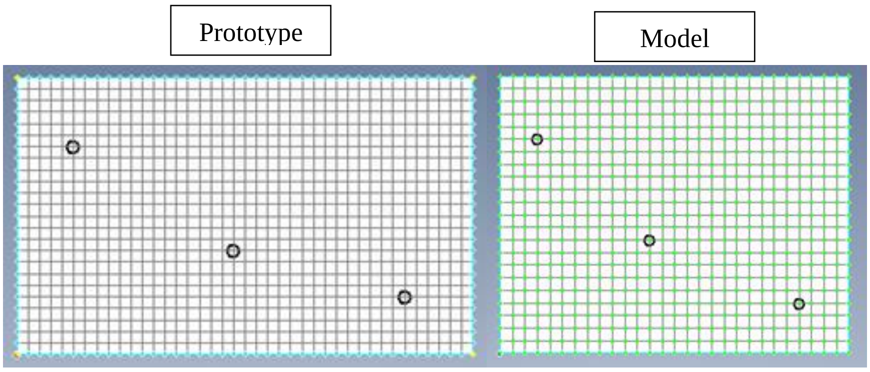
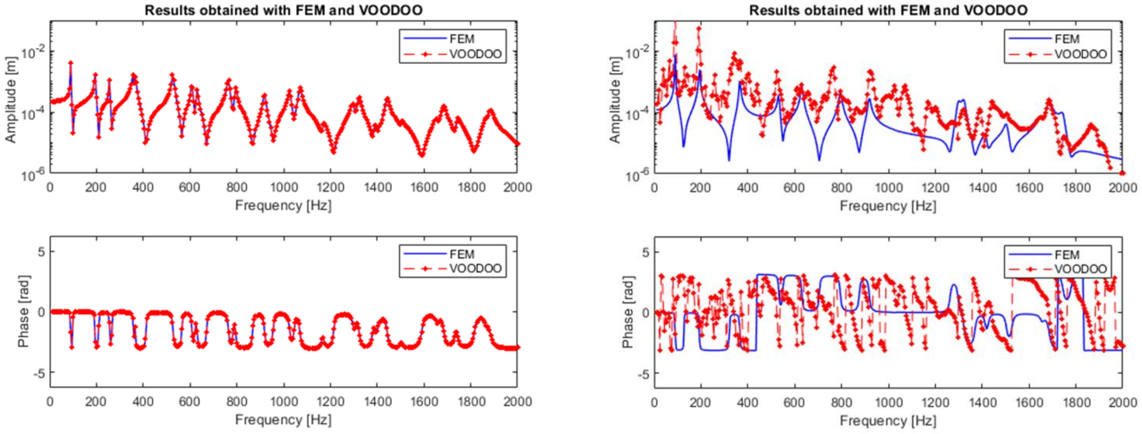


Figure 1 – Prototype and model of test case 1.

The two systems are subjected to the same concentrated deterministic load (F=100 N) and, at first, the excitation and acquisition points coincide with the VOODOO points. The results, in the case of same excitation and acquisition points, are shown in Figure 2(a); each graph shows the comparison between the numerically calculated model response and the prediction provided by VOODOO. The figure consists of two sub-figures, showing the amplitude and phase of the displacement response as a function of frequency. The plots exhibit a perfect overlap between the reference and predicted curves. This is due to the fact that the excitation and acquisition points coincide with the points used to construct VOODOO. The same happens if the behaviour of the prototype is derived from the model.
Figure 2(b) shows the results obtained when the excitation point does not coincide with a VOODOO point. Since there is no spatial correspondence between the VOODOO points and the excitation/acquisition points, the prediction is not accurate as in Figure 2(a).

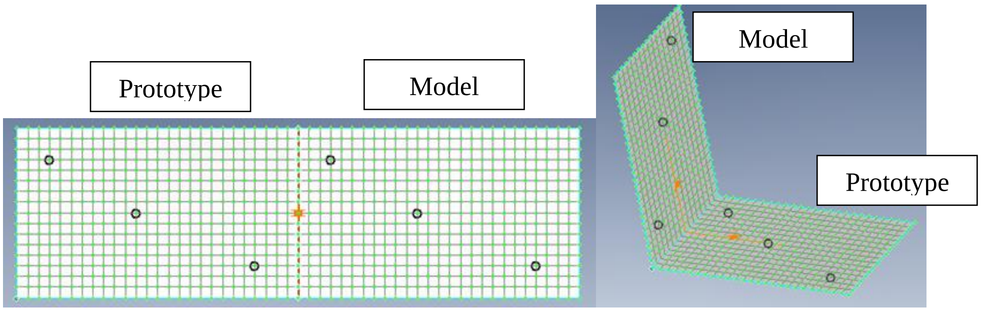


(a) (b)

Figure 2 – Displacement response (amplitude and phase) of the model (a) the excitation and acquisition points coincide with the VOODOO point, (b) the excitation point does not coincide with any of the VOODOO points (b).

3.2 Test case 2: assembly of 2 plates

Test case 2 concerns two joined panels in two different configurations: coplanar and orthogonal. The dimensions of each plate composing the assembly, as well as the sizes, are the same of the previous case (Figure 3).

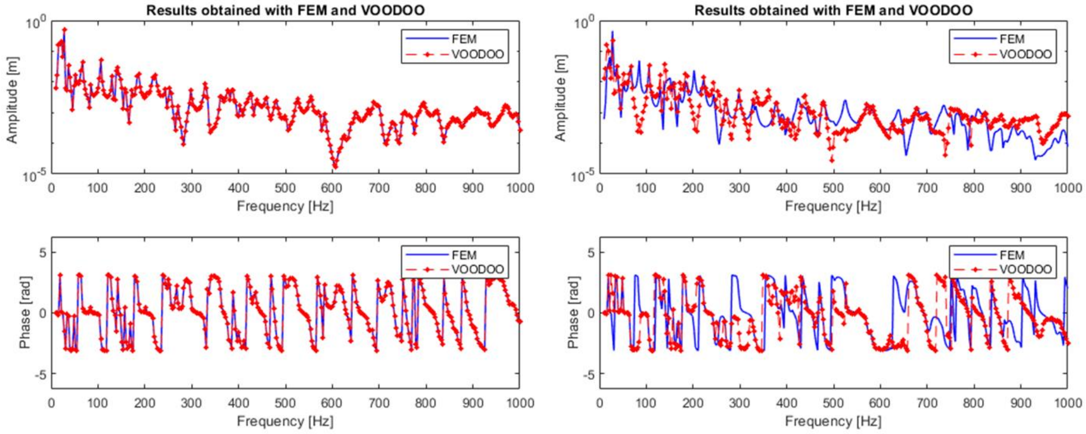


(a) (b)

Figure 3 – Prototype and model of test case 2: (a) coplanar plates, (b) orthogonal plates.

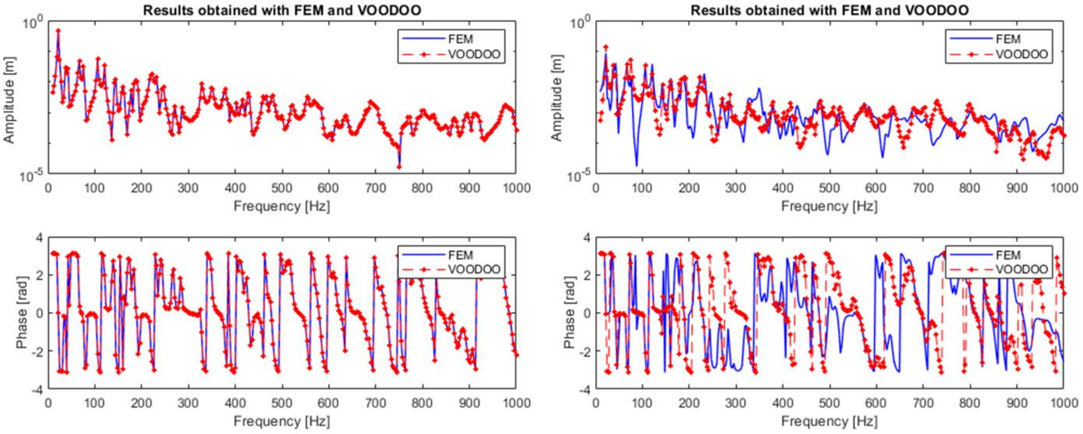
Figure 4(a)-(b) illustrates the results when the excitation and acquisition points coincide with VOODOO points and when the acquisition points does not, respectively. The predictions are accurate in the first case, while they are not in the second case, like in the previous case. The reasons behind the discrepancies are the same.

Moreover, the same results can be obtained when the panels are arranged orthogonally (Figure 3), with one side in common, so that the out-of-plane displacements act along two different directions.



(a) *(b)*

Figure 4 - Displacement response (amplitude and phase) of the model, joined and coplanar to the prototype, (a) the excitation and acquisition points coincide with the VOODOO points, (b) the acquisition point does not coincide with any of the VOODOO point.



(a) *(b)*

Figure 5 - Displacement response (amplitude and phase) of the model, joined and orthogonal to the prototype, (a) the excitation and acquisition points coincide with the VOODOO point, (b) the acquisition point does not coincide with any of the VOODOO points.

4 CONCLUSIONS

The results show that VOODOO method works perfectly when there is a correspondence between the VOODOO points, used for the construction of the transformation matrix, and the excitation and acquisition points, chosen during validation. The promising outcomes open the way to further investigations concerning the limits of VOODOO and the extension of the analysis more complex structures. In addition, there is the possibility to reproduce responses for stochastic excitation. This is a starting point for analysing the vibro-acoustic behaviour of structural components excited by turbulent boundary layers, for example.

REFERENCES

[1] A. Casaburo, G. Petrone, F. Franco, S. De Rosa. A review of similitude methods for structural engineering. *Appl. Mech. Rev.*, **71**, pp. xx-yy (year).

[2] V. Meruane, S. De Rosa e F. Franco. Numerical and experimental results for the frequency response of plates in similitude. *Journal of Mechanical Engineering Science* (2015).

[3] S. De Rosa, F. Franco, G. Petrone, A. Casaburo e F. Marulo. A versatile offset operator for the discrete observation of objects. *Journal of Sound and Vibration*, **500** (2021).