

## DECOUPLING OF ENERGY TRANSMISSION BETWEEN SUBSYSTEMS OF A COMPLEX STRUCTURE

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**Abstract.** Experimental vibroacoustic measurements are very common for the study of emitted noise reduction and vibration energy isolation of structures. The most important case is when structures are subjected to an aerodynamic excitation as Turbulent Boundary Layer (TBL). In this paper, a preliminary study is performed on the energy transmission between subsystems of a structure subjected to TBL. A numerical test is developed on a three-plates-in-row system at high frequencies, through the application of Statistical Energy Analysis (SEA). Parameters such as surface dimensions, thickness and damping loss factor are evaluated in different configurations for a first design of a testbench used for vibroacoustic measurements in a wind tunnel.

### 1 INTRODUCTION

In the aerospace engineering field, the study of vibroacoustic behaviour of complex structures is very popular as academic research, as well as for industrial purposes.

In fact, main topics are the emitted noise and the structural vibration of systems which are subjected to an aerodynamic excitation (i.e. Turbulent Boundary Layer, TBL), with the aim of guarantee a good comfort in terms of reduced emitted noise and isolation of structural vibration. Focusing the attention on the effect of a TBL at high flow speeds, it is important to have a relevant testbench, mounted in the wind tunnel that, for this specific excitation, can guarantee the quality and robustness of the measured experimental data. In other words, it is required that ideally no contamination effects, related to the impedance breaks between testbench and sample panel, affect the measurements.

For this reason, the decoupling of energy transmission between subsystems of a structure is the main topic of this paper. A similar work is done by Finnveden in [1,2]. The wind tunnel, at KTH laboratories, consists in a suspended flow duct in which an air flow is blown; the test panel is mounted on the wall of the above-mentioned duct. The validation of the wind tunnel,

which Finnveden [1,2] proposed through the application of the Statistical Energy Analysis (SEA), assumed that it is possible to consider the vibration of the test panel decoupled by the vibration of the flow duct.

In the present paper, a verification of the prior assumption is made, followed by a numerical procedure based only on SEA. The procedure uses the description of an aerodynamic excitation, such as TBL, as an equivalent “rain on the roof” excitation in the mid-high frequencies [3,4].

The following sections are organized in this order: first, the presentation of the adopted SEA method, second, the presentation of the Equivalent TBL model, and finally the presentation of results per categories: influence of surface, thickness and damping, respectively

## 2 STATISTICAL ENERGY ANALYSIS

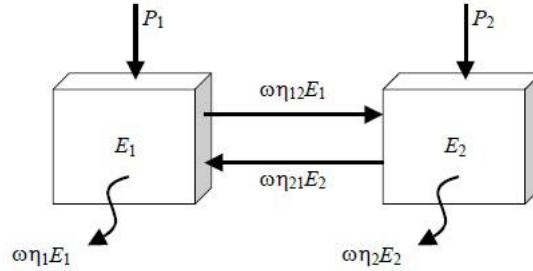
Statistical Energy Analysis is based on the principle of energy balance between subsystems which are assumed to be linearly coupled. This energetic method has been studied and theorized by Lyon [5]. Among the assumptions thus theorized, it is considered the one for which the excitation spectrum is broadband, and the excitation forces are statistically independent: there are no pure tones in the input spectra.

The energy balance between two subsystems  $i$  and  $j$  can be expressed as

$$P_{in,j,i} = P_{diss,i} + (P_{ij} - P_{ji}) \quad (1)$$

$$P_{in,j,i} = \omega \eta_i E_i + \sum_{j=1}^2 \omega \eta_{ij} n_i \left( \frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \quad (2)$$

where  $\eta_{ij}$  is the coupling loss factor (CLF),  $n_i$  is the modal density,  $E_i$  and  $E_j$  are the uncoupled total subsystem energies (Fig. 1).



**Figure 1:** Simple SEA system.

SEA affirms that power always flows from the subsystem which has a higher energy to the one having lower energy

$$P_{ij} = \omega (\eta_{ij} E_i - \eta_{ji} E_j) = \omega \eta_{ij} n_i \left( \frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \quad (3)$$

$$\eta_{ij} n_i = \eta_{ji} n_j \quad (4)$$

For a complex structure, the relation between the injected powers and the modal energies can be written as follows

$$\begin{Bmatrix} P_{inj,1} \\ \vdots \\ P_{inj,s} \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 n_1 + \sum_{j=1}^s \omega \eta_{1j} n_j & \cdots & -\eta_{1s} n_s \\ \vdots & \ddots & \vdots \\ -\eta_{s1} n_s & \cdots & \eta_s n_s + \sum_{j=1}^{s-1} \omega \eta_{sj} n_s \end{bmatrix} \begin{Bmatrix} E_{m,1} \\ \vdots \\ E_{m,s} \end{Bmatrix} \quad (5)$$

$$\{P_{inj}\} = \omega[L]\{E_m\} \quad (6)$$

The energy distribution inside the subsystems of a complex structure is the result of a simple algebraic matrixial equation in which it is needed to know only three parameters, which are the injected powers in terms of Power Spectral Densities (PSD), the modal densities and the CLFs.

However, SEA presents some limitations of applicability, defined by Mace in [6,7]. It is here mentioned the modal overlap factor

$$m(\omega) = \eta \omega n(\omega) \quad (7)$$

as instrument for SEA validity. If  $m \gg 1$  for all subsystems of the structure, in fact, it can be said that it is possible to use SEA. The frequency for which the modal overlap factor is equal to unity, is the limit frequency for SEA validity.

### 3 EQUIVALENT TBL EXCITATION

With the aim of designing a testbench for vibroacoustic measurements in wind tunnel, it is necessary to describe the effect of an aerodynamic load as a TBL excitation. But, as mentioned in the previous section, the type of excitation applied to a structure in SEA method should be a broadband statistically independent excitation, such as ‘rain on the roof’ excitation (ROF). For this reason, the Equivalent TBL presented by Ichchou in [3] is considered.

Referring to the Corcos model described as

$$C(x - x', y - y'; \omega) = e^{-\delta_x |x-x'|} \cos(\gamma_x (x - x')) e^{-\delta_y |y-y'|} \quad (8.a)$$

$$\begin{cases} \gamma_x = \omega/U_c \\ \delta_{x,y} = a_{x,y} \gamma_x \end{cases} \quad (8.b)$$

where  $U_c$  is the convective speed and  $a_{x,y}$  are empirical coefficients, it is possible to define the Equivalent TBL auto-spectrum (Eq. 10) in function of an equivalent correlation function  $C^{eq}(\omega)$  (Eq. 9) and ROF auto-spectrum

$$C^{eq}(\omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\delta_x |\zeta|} \cos(\gamma_x \zeta) e^{-\delta_y |\chi|} d\zeta d\chi = \frac{4\delta_x}{\delta_y (\delta_x^2 + \gamma_x^2)} \quad (9)$$

$$S_{pp}^{ETBL}(\omega) = C^{eq}(\omega) S_{pp}^{ROF}(\omega) = \frac{4\delta_x}{\delta_y (\delta_x^2 + \gamma_x^2)} S_{pp}^{ROF}(\omega) \quad (10)$$

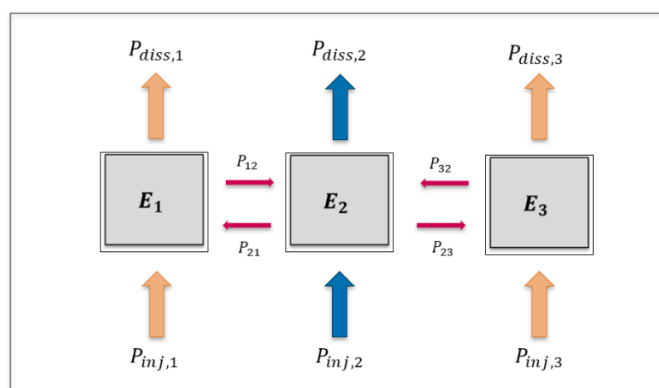
In this way, it is possible to describe at high frequencies, the effect of an aerodynamic load on a structure as vibration levels, thanks to the application of Equivalent TBL (ETBL) excitation in a SEA approach.

## 4 NUMERICAL APPLICATIONS OF EQUIVALENT TBL (ETBL)

### 4.1 Generalities

Ref. [2] shows how to evaluate the modal density of a flow duct. Particularly at high frequencies, it has been observed that the dispersion curves of the flow duct follow the analytical thin-plate theory. At high frequencies, this means that it is possible to approximate a complex structure as an ensemble of thin plates.

According with this assumption, the energy transmission between a flow duct and a test panel can be described as an energy transmission among simple plates: the energy transmission between a testbench and a test panel can be also simplified as said before. Therefore, the preliminary study of energy transmission between three plate in a row (Fig. 3) is carried out.



**Figure 2:** Equivalent SEA model.

Three is the lowest number of plates that can be chosen for the first study of energy flow direction among subsystems: the energy transmission referred to a test panel would be analyzed between more than one interface.

The plates of equivalent SEA system (Fig. 2) have the characteristics described in Table 1.

**Table 1:** Geometry and physical properties of the plate.

<b>Geometry:</b> Plate	<b>Material:</b> Aluminium
$L_x = 1.5m$	$E = 7.1e10 Pa$
$L_y = 0.9m$	$\nu = 0.33$
$h = 5mm$	$\rho = 2500 kg/m^3$
	$\eta = 0.005 (0.5\%)$

The energy transmission analysis of the equivalent SEA system is organized as follows.

A first solution has been evaluated keeping all three plates with the same identical properties; the other solutions calculated for different configurations would be compared with this as reference case ('case 0'). The SEA frequency limit for this configuration is 2500Hz. The first comparison is in function of difference of surface: the middle plate – which represents the test

panel – would have a reduced length  $L_x = 0.5m$ , having a consequential reduction of surface of 33% respect to the side plates surfaces.

The second comparison is in function of difference of thickness: the middle plate would have a reduced thickness  $h = 1mm$ , 1/5 of the side plates thickness.

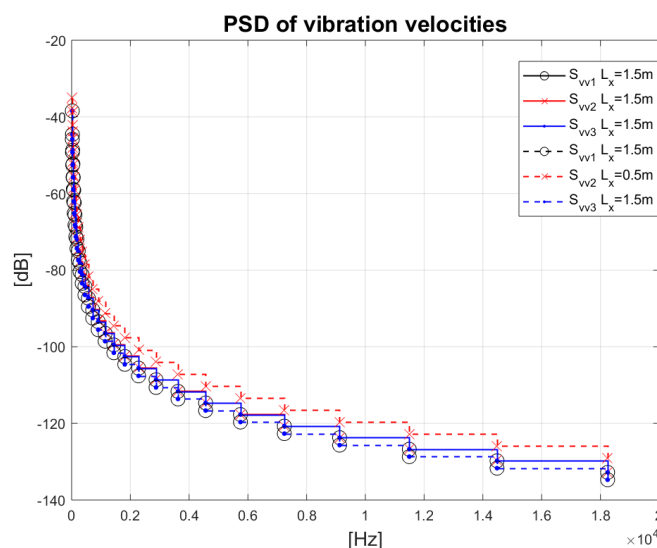
The third and last comparison is in function of the damping loss factor: the side plates would present an increase of damping from 0.5% to 7%.

All the cases described above will be compared with ‘case 0’ in terms of: (i) Vibration velocity levels  $S_{VV}$ , in order to obtain a velocity gap between middle plate and side plates of at least 20 dB; (ii) Powers levels which act on subsystem ‘plate 2’, in order to understand how the energy spreads inside the system. The power levels are not presented as absolute values, but as ratio with the injected power on ‘plate 2’.

## 4.2 SEA Test-Cases

### 4.2.1 Surfaces

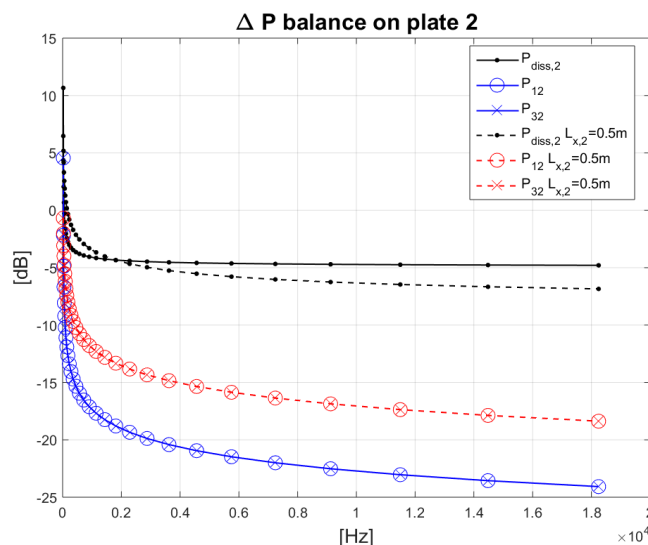
As first case, the length of middle plate is reduced from the initial value  $L_x = 1.5m$  to  $L_x = 0.5m$ , as mentioned above. The reduction of surface applied at the middle plate changes its modal overlap factor and, consequently, increases the value of SEA frequency to  $\approx 7000$  Hz. The velocity gap obtained for a surface reduction of 33% is only of 6 dB (Fig. 3), which is too low for an energy isolation of the middle plate in terms of vibration velocity.



**Figure 3:** Vibration velocities of three plate in row subjected to an Equivalent TBL: same properties (-), subsystem ‘plate 2’ with  $L_x = 0.5m$  (--).

The consequences of surface reduction of the middle plate can be seen in Fig. 4: the dissipated power,  $P_{diss,2} = \omega \eta_2 E_2 = f[\langle v_2 \rangle^2]$ , of ‘plate 2’ has reduced, while the transmitted energy of side plates has increased. This is not an acceptable configuration, because the aim is the reduction of the transmitted power from the other subsystems to the test panel, not the

opposite.



**Figure 4:** Difference of powers calculated in function of injected power acting on the subsystem 'plate 2': same properties (-), subsystem 'plate 2' with  $L_x = 0.5m$  (--).

#### 4.2.2 Thicknesses

In this second configuration, the middle panel had a reduction of thickness from  $h = 5mm$  to  $h = 1mm$ . With this change of thickness, the modal overlap factor of the middle plate leads to a wider frequency range in which the SEA method is valid. But, because it must be considered the modal overlap factors of all the subsystems, the SEA frequency limit still remains over 2500Hz. The reduction of thickness leads also to a velocity gap between the middle plate and the side plates of 20 dB and more (Fig. 5), which can ensure an energy isolation of the middle plate from the side plates.

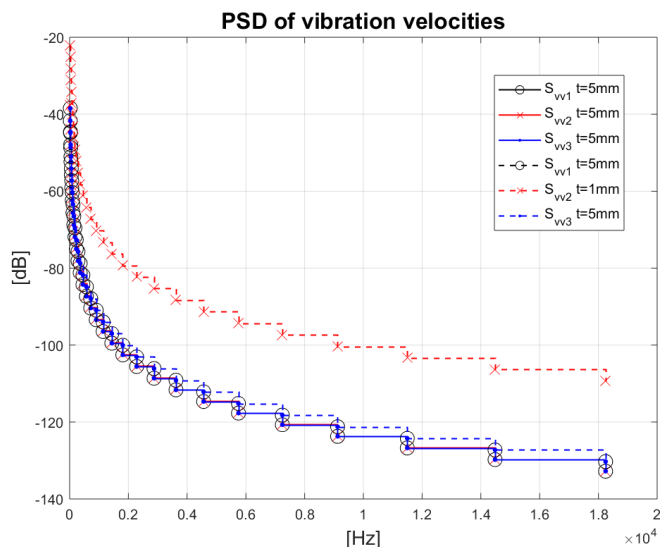
Observing Fig. 6, it is possible to notice that the dissipated power of 'plate 2' reaches almost the same value of the injected power; this could mean that the energy flows inside the middle plate and then it is directly dissipated without almost any transmission to the near subsystems. Moreover, the values of the transmitted energy by side plates is consistently reduced, which could mean that the velocity response of the middle plate it is a direct effect of the TBL.

Hence, it can be said that a change of thickness can ensure the energy isolation of a test panel from its testbench.

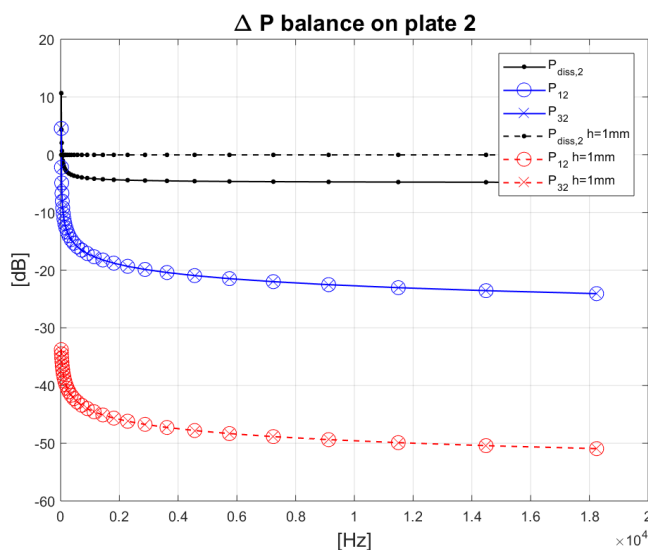
#### 4.2.3 Damping loss factors

The last comparison refers to a change of damping loss factor; this has been increased in side plates, to see the effect of energy transmission between a high damped subsystem with a low damped subsystem. The increase of damping loss factor in a subsystem has a direct effect on the modal overlap factor, which even this time ensures a wider frequency range of SEA validity. But, because the modal overlap factor of the middle plate must be considered too, the SEA frequency limit still remains 2500 Hz.

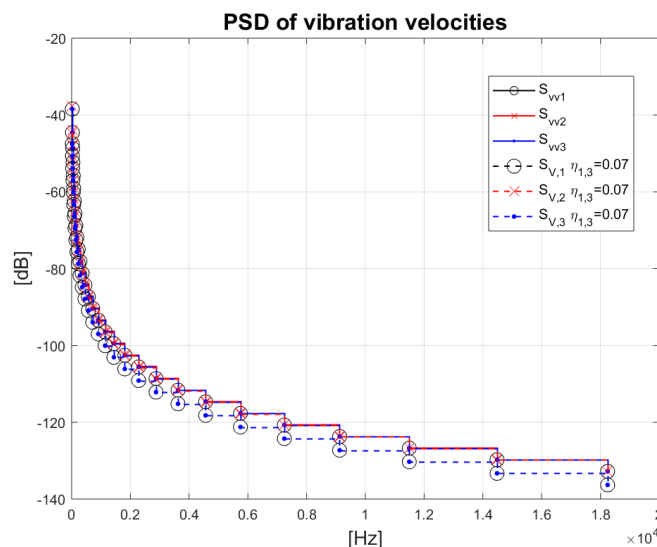
In Fig. 7, a velocity gap of nearly 4 dB is shown. While the vibration velocity of the middle plate remains the same, the side plates present a reduction of vibration velocity. Considering the case of the testbench, it means that a damped frame system does not ensure a huge difference of vibration velocities between itself and the test panel. On the other hand, it is shown in Fig. 8 that an increase of damping loss factor in the side plates ensures a reduction of transmitted energy from them to the middle plate. As expected, the dissipated power of ‘plate 2’ seems not changed.



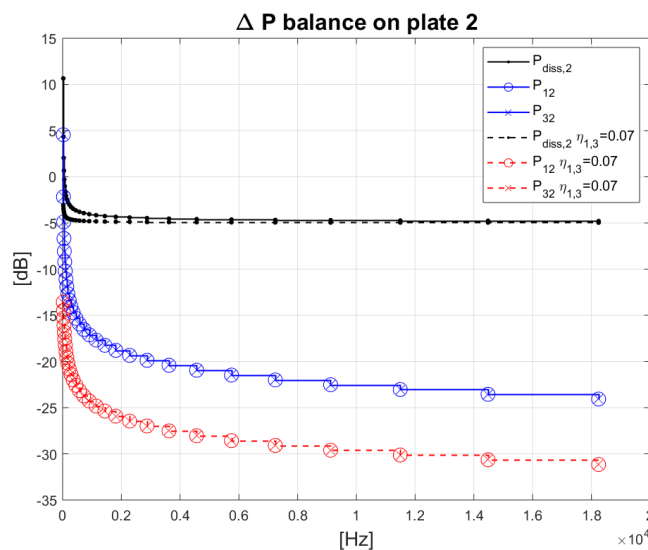
**Figure 5:** Vibration velocities of three plate in row subjected to an Equivalent TBL: same properties (-), subsystem ‘plate 2’ with  $h = 1mm$  (--).



**Figure 6:** Difference of powers calculated in function of injected power acting on the subsystem ‘plate 2’: same properties (-), subsystem ‘plate 2’ with  $t = 2.5mm$  (--).



**Figure 7:** Vibration velocities of three plate in row subjected to an Equivalent TBL: same properties (-), subsystems 'plate 1' and 'plate 3' with  $\eta = 0.07$  (7%) (--).



**Figure 8:** Difference of powers calculated in function of injected power acting on the subsystem 'plate 2': same properties (-), subsystems 'plate 1' and 'plate 3' with  $\eta = 0.07$  (7%) (--).

## 5 CONCLUSIONS

From the above numerical tests based on SEA, it can be seen that there are different ways of changing the energy transmission between subsystems of a structure. Reducing the surface of the middle plate does not ensure its energetic isolation, but it implies only an increase of the energy transmission from the side plates to the middle one. On the other hand, the reduction of thickness is the best solution for a large velocity gap between the subsystems; in fact, the change



of thickness influences directly the impedance values of the subsystems, which consequently alters the way the energy is transmitted. Finally, modifying the damping loss factor only leads to a reduction of the transmitted energy; it could be taken in consideration as final option for the optimization of the structure in terms of energy isolation of the test panel.

This is only a preliminary study which can be conducted for a first design of a testbench. As following step, it is necessary to consider the indirect energy transmission between subsystems which are not physically connected each other. This second test can be carried on through the utilization of SEA-like method, which not only is able to estimate the indirect CLF, but it can also extend the study of energy transmission in the middle frequency range.

## ACKNOWLEDGEMENTS

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