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HIDDEN ENTANGLEMENT AND UNITARITY AT THE PLANCK SCALE

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Attempts to go beyond the framework of local quantum field theory include scenarios in which the action of external symmetries on the quantum fields Hilbert space is deformed. We show how the Fock spaces of such theories exhibit a richer structure in their multi-particle sectors. When the deformation scale is proportional to the Planck energy, such new structure leads to the emergence of a "planckian" mode-entanglement, invisible to an observer that cannot probe the Planck scale. To the same observer, certain unitary processes would appear non-unitary. We show how entanglement transfer to the additional degrees of freedom can provide a potential way out of the black hole information paradox.

Keywords: Deformed symmetries, entanglement, black hole information paradox.

1. Introduction

The phenomenon of black hole quantum radiance discovered by Hawking[1] is a typical effect which characterizes the propagation of quantum fields on a classical curved background spacetime. As it is well known, the semiclassical understanding of black hole evaporation seems to be in contrast with the rules of ordinary quantum mechanics.[2] An evaporating black hole loses its mass, shrinks and eventually disappears. The emitted radiation is in a thermal state, i.e. a highly mixed state. Therefore starting with a black hole formed from a pure quantum state, at the end of the evaporation process, all we are left with is a mixed state. This conclusion

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violates the unitarity of evolution of closed quantum states, one of the basic pillars of quantum mechanics. Such a contradiction is known as the *black hole information paradox*. The problem can be clearly stated in the framework of quantum information theory. Vacuum fluctuations on a Schwarzschild background lead to the creation of entangled pairs, that can be approximatively seen as Bell pairs. One member of the pair is inside the horizon, while the other one is outside. All these qubits are entangled with respect to the bipartition (*inside the horizon*)–(*outside the horizon*). When the evaporation process is completed, the particles originally outside the horizon would be *entangled with nothing* because the bipartition has disappeared. It follows that either the fundamental theory is not unitary, or the entanglement has to decrease to zero during the evaporation.

Three decades of attempts to resolve the information paradox seem to indicate that the new ingredient needed beyond semiclassical gravity, i.e. local quantum field theory (QFT) on a classical curved background, is some form of *nonlocality* (see, e.g., Ref. 3). Indeed there seems to be at least two main indications that locality might be only an approximate concept in a quantum gravity framework. On the one hand, locality in QFT leads to an extensivity in the degrees of freedom which is in strident contrast with arguments based on the holographic principle,[4] in particular the *holographic entropy bound*.[5] On the other hand, there seems to be a fundamental difficulty, starting from a diffeomorphism invariant setting, in constructing observables that, in a certain limit, reproduce the familiar observables of local QFT.[6–8]

In this paper, we discuss how attempts to "stretch" the framework of local QFT, motivated by the introduction of nonlocal effects, can endow the Hilbert space of the theory with a richer structure and with degrees of freedom accessible only to observers which can resolve planckian scales. The new "planckian" degrees of freedom allow for unitary quantum processes which can appear to a macroscopic observer as non-unitary. Moreover, the entanglement created during black hole evaporation can then be transferred from the original *in–out* bipartition to a new bipartition outside the horizon. We argue that mode entanglement between "planckian" modes and ordinary "macroscopic" modes could help escaping the puzzling conclusions of the information paradox.

2. Nonlocality, Quantum Symmetries, and Momentum Dependent Statistics

If one is willing to adhere to some nonlocality paradigm, as discussed above, then the natural question is whether there exist some modifications of local QFT, whose features are qualitatively different in contexts like, for example, particle production in the presence of a horizon. Lower dimensional QFT models provide some useful insights in this regard. Nonlocal currents are well-studied objects in two-dimensional QFT and their associated charges exhibit a nontrivial algebraic structure which is described in terms of quantum groups.[10] The emergence of algebraic structures more complex than ordinary Lie algebras can be traced back to the possibility of relaxing one of the assumptions of the Coleman–Mandula theorem, namely the requirement that the charges (symmetry generators) act on multi-particle states following a generalized Leibnitz rule. As it will be discussed in what follows, such property is intimately related to the Lie algebra structure of the generators of symmetries, their (adjoint) action on observables of the theory, and ultimately, to locality. It has been argued[11] that relaxing the requirement of locality in a controlled way naturally leads to generalizing the description of the spacetime symmetries of quantum fields to the realm of quantum groups. Quantum fields with quantum group symmetries provide in turn a possible framework for noncommutative field theories (see, e.g. Refs. 12 and 13). Here we elaborate on the argument given in Ref. 11 and, focusing on a specific example, we discuss the nontrivial features that distinguish the new model from ordinary local QFT.

The Hilbert space of a linear scalar field is usually described in terms of "plane-wave" state vectors (the "wave-modes" of the field) labeled by their eigenvalues with respect to the space-translation generators. In the usual picture, such translation generators are identified with the charges associated to the translational symmetry of the classical theory. In particular such charges are suitable integrals of the quantized Noether currents. The fact that they generate the symmetry is most easily seen using the Ward identity involving the current and a product of n-fields.[14] Integrating such identity, one finds that the action of the translation generators P_μ on a basis "mode" vector $|\mathbf{k}\rangle = a^\dagger(\mathbf{k})|0\rangle$ is given in terms of the conserved charges Π_μ by

$$P_\mu \triangleright |\mathbf{k}\rangle = k_\mu |\mathbf{k}\rangle = [\Pi_\mu,\, a^\dagger(\mathbf{k})]\, |0\rangle\,. \tag{1}$$

From a mathematical point of view, the commmutator on the right-hand side represents the adjoint action of the (Lie) algebra of translation generators on the algebra of creation operators. Additivity of the translation generators on multiparticle states immediately follows from the well-known property of the commutator

$$[A, B_1 B_2 \cdots B_n] = \sum_{i=1}^{n} B_1 \cdots [A, B_i] \cdots B_n\,. \tag{2}$$

In 2D quantum integrable models, the complete nonperturbative knowledge of the S-matrix leads to the construction of nonlocal charges whose action on multi-particle states is non-additive.[10] Such behavior reveals an algebraic structure described by quantum algebras instead of ordinary Lie algebras. In Ref. 11 it was argued that an analogous situation might occur for the generators of spacetime symmetries in QFT when their associated charges experience an intrinsic nonlocality determined by dynamical gravity. To understand how quantum deformations of the algebra provide a generalization of the Leibnitz rule and of the adjoint action (1) we note that for a translation generator the action on a tensor product (two-particle) state is described by the operator

$$\Delta^{(0)}(P_\mu) = P_\mu \otimes 1 + 1 \otimes P_\mu\,. \tag{3}$$

In general, given a quantum deformed algebra, with deformation parameter h, such operator becomes

$$\Delta(P_\mu) = \Delta^{(0)}(P_\mu) + h\Delta^{(1)}(P_\mu) + O(h^2) \tag{4}$$

with the important property that given the "flip" operator σ: $\sigma(a \otimes b) = b \otimes a$ one has $\Delta(P_\mu) \circ \sigma \neq \Delta(P_\mu)$. The adjoint action is defined by

$$P_\mu \triangleright |\mathbf{k}\rangle = (\mathrm{id} \otimes S)\Delta(P_\mu) \diamond a^\dagger(\mathbf{k})|0\rangle\,, \tag{5}$$

where $(F\otimes G)\diamond a \equiv FaG$ and $S(P_\mu) = S^{(0)}(P_\mu) + h\, S^{(1)}(P_\mu) + O(h^2)$ with $S^{(0)}(P_\mu) = -P_\mu$. Note how $S(P_\mu)$ provides a deformed inversion map (involution). In the case $h = 0$, one recovers the usual adjoint action (1) of the Poincaré (Lie) algebra. The deformed quantum algebra can capture nonlocal effects that would manifest in Planck-scale suppressed corrections to the classical adjoint action (1)

$$\langle 0|P_\mu \triangleright |\mathbf{k}\rangle \equiv \langle 0|[P_\mu, a^\dagger(\mathbf{k})]|0\rangle + \alpha_1 E_p^{-1} F^{(1)}(\mathbf{k}) + O(E_p^{-2})$$

if one identifies the deformation parameter h with E_p^{-1}, the inverse Planck energy.[11]

In this paper we will focus on the particular example of the κ-Poincaré algebra, a well-known quantum Poincaré algebra with deformation parameter $h = 1/\kappa$.[15] For spatial momenta one has

$$\Delta(P_i) = P_i \otimes 1 + e^{-P_0/\kappa} \otimes P_i\,, \tag{6}$$

while the time-translation generator still acts according to the Leibnitz rule. Only recently a satisfactory understanding of the basic properties of free scalar quantum fields with such κ-symmetries has been reached.[16] The most peculiar feature of the Hilbert space of κ-bosons is that they obey a momentum dependent statistics[17] arising from the nontrivial structure of the multi-particle sector of the theory. Such structure is a direct consequence of the "non-symmetric" form of (6). Take for example the two states $|\mathbf{p}_1\rangle \otimes |\mathbf{p}_2\rangle$ and $|\mathbf{p}_2\rangle \otimes |\mathbf{p}_1\rangle$. Unlike the undeformed case they now have two *different* eigenvalues of the linear momentum, respectively $p_{i1} + e^{-p_{01}/\kappa} p_{i2}$ and $p_{i2} + e^{-p_{02}/\kappa} p_{i1}$. Clearly the usual "symmetrized" two-particle state

$$1/\sqrt{2}(|\mathbf{p}_1\rangle \otimes |\mathbf{p}_2\rangle + |\mathbf{p}_2\rangle \otimes |\mathbf{p}_1\rangle) \tag{7}$$

is no longer an eigenstate of the momentum operator. Rather, given two modes $\mathbf{p}_1$ and $\mathbf{p}_2$, we have two different κ-symmetrized two-particle states

$$|p_1 p_2\rangle_\kappa = \frac{1}{\sqrt{2}}[|\mathbf{p}_1\rangle \otimes |\mathbf{p}_2\rangle + |(1-\epsilon_1)\mathbf{p}_2\rangle \otimes |(1-\epsilon_1(1-\epsilon_2))^{-1}\mathbf{p}_1\rangle]\,,$$

$$|p_2 p_1\rangle_\kappa = \frac{1}{\sqrt{2}}[|\mathbf{p}_2\rangle \otimes |\mathbf{p}_1\rangle + |(1-\epsilon_2)\mathbf{p}_1\rangle \otimes |(1-\epsilon_2(1-\epsilon_1))^{-1}\mathbf{p}_2\rangle]\,,$$

with $\epsilon_i = \frac{|\mathbf{p}_i|}{\kappa}$.[16]

3. The Fine Structure of κ-Hilbert Space and Mode Entanglement

The construction of QFT using the deformed symmetries described above induces a radical modification in the number of degrees of freedom of the system. We show here how the new fine structure of the Hilbert space is determined by the splitting in the linear momenta between quantum states with different mode ordering. For the sake of simplicity, let us focus on the two-mode state case. We have already seen that the states $|p_1 p_2\rangle_\kappa$ and $|p_2 p_1\rangle_\kappa$ are not identical. In fact they are orthogonal for every finite κ, as it can be verified by taking the scalar product

$$\langle p_1 p_2 | p_2 p_1 \rangle_\kappa \simeq \frac{1}{2}\delta^{(3)}(\epsilon_2 \mathbf{p}_1)\delta^{(3)}\left(\frac{\epsilon_1(1-\epsilon_2)}{1-\epsilon_1(1-\epsilon_2)}\mathbf{p}_2\right) + 1 \leftrightarrow 2\,.$$

In the undeformed case ($\kappa \to \infty$) we recover the indistinguishability of the two states, i.e. $\langle p_2 p_1 | p_1 p_2 \rangle_\kappa = 1$. The two states can be distinguished by measuring the splitting in their linear momenta. The linear momentum for the state $|p_i p_j\rangle_\kappa$ is $\mathbf{P}_{ij} = \mathbf{p}_i + e^{-p_i^0/\kappa}\mathbf{p}_j$, where $p_i^0 = -\kappa \log(1 - |\mathbf{p}_i|/\kappa)$. The resulting splitting is thus

$$|\Delta \mathbf{P}_{12}| \equiv |\mathbf{P}_{12} - \mathbf{P}_{21}| = \frac{1}{\kappa}\big|\mathbf{p}_1|\mathbf{p}_2| - \mathbf{p}_2|\mathbf{p}_1|\big| \leq \frac{2}{\kappa}|\mathbf{p}_1||\mathbf{p}_2|\,. \tag{8}$$

Since such splitting is a quantity of the order $|\mathbf{p}_i|^2/\kappa$, we see that the states indeed become indistinguishable in the limit of $\kappa \to \infty$. In practice, this happens when the resolution of our measurements is much smaller than the splitting. We can see that only for very high momenta the splitting becomes relevant for an observer that has a reasonable resolution. Nevertheless, the Hilbert space has now a new degree of freedom: the mode of being of the type $|p_1 p_2\rangle_\kappa$ or $|p_2 p_1\rangle_\kappa$. Formally, the Hilbert space acquires the following tensor product structure: $\mathcal{H}^2_\kappa \cong \mathcal{S}_2\mathcal{H}^2 \otimes \mathbb{C}^2$, where $\mathcal{S}_2\mathcal{H}^2$ is the standard symmetrized two-particle Hilbert space. We can write the two states as

$$|E\rangle \otimes |0\rangle = |p_1 p_2\rangle_\kappa\,, \tag{9}$$

$$|E\rangle \otimes |1\rangle = |p_2 p_1\rangle_\kappa\,, \tag{10}$$

where $E = E(\mathbf{p}_1) + E(\mathbf{p}_2)$. The $\mathbb{C}^2$ degrees of freedom can only be seen under a planckian "magnifying glass". Far from the Planck scale, this variable is hidden. Many interesting phenomena can occur because of these additional degrees of freedom. First, one can have mode entanglement. The state of the superposition of two total "classical" energies $E_A = E(\mathbf{p}_A) + E(\mathbf{q}_A)$ and $E_B = E(\mathbf{p}_B) + E(\mathbf{q}_B)$ can be entangled with the additional hidden modes. We could have, for instance, a superposition of the form $|\Psi\rangle = 1/\sqrt{2}(|E_A\rangle \otimes |0\rangle + |E_B\rangle \otimes |1\rangle)$.

In the case of n particles with distinct momenta we will have $n!$ orthogonal plane wave states corresponding to $n!$ different orderings. In general, given n-particles, of which n_i have momentum p_i with $\sum_{i=1}^R n_i = n$, we denote their κ-symmetrized Hilbert space with $\mathcal{H}_\kappa^{(\mathbf{n}_R)}$ where $\mathbf{n}_R \equiv (n_1, \ldots, n_R)$. The number of orthogonal states in $\mathcal{H}_\kappa^{(\mathbf{n}_R)}$ is $N(\mathbf{n}_R) = n!/\prod_{i=1}^R n_i!$. For such space we can write the following decomposition

$$\mathcal{H}_\kappa^{(\mathbf{n}_R)} \cong \mathcal{S}_n \mathcal{H}^n \otimes \mathbb{C}^{N(\mathbf{n}_R)}\,, \tag{11}$$

where $\mathcal{H}^n = \underbrace{\mathcal{H}^1 \otimes \mathcal{H}^1 \cdots \otimes \mathcal{H}^1}_{n\,\text{times}}$ and $\mathcal{S}_n$ sums over all permutations of n objects. Here, $\mathcal{H}^1$ is the usual one-particle Hilbert space. The n-particle Hilbert space will be given by $\mathcal{H}_\kappa^n = \oplus_{n_1,\ldots,n_R} \mathcal{H}_\kappa^{(\mathbf{n}_R)}$ and we can thus have states containing up to $\log_2 N$ ebits with respect to the tensor product structure in (11).

4. Loss of Unitarity and the Black Hole Information Paradox

What are the implications of this construction for an observer that is unable to resolve the degrees of freedom on the right-hand side of the tensor product in Eq. (11)? To answer this question, consider a quantum system evolving unitarily, through a Hamiltonian H defined on the Hilbert space $\mathcal{H}_\kappa^n$. The quantum evolution of the system, described by the density matrix $\rho(t)$, would read

$$\rho(t) = U(t)\rho(0)U^\dagger(t)\,,$$

where $U(t) = \mathcal{T}e^{-iHt}$. Now assume we start with a pure state $\rho(0)$ factorized with respect to the bipartition in $\mathcal{H}_\kappa^{(\mathbf{n}_R)}$. If the unitary $U(t)$ acts as an entangling gate, the state $\rho(t)$ will be entangled. In order for this to happen, it is necessary that the Hamiltonian addresses simultaneously the degrees of freedom in the bipartition of $\mathcal{H}_\kappa^{(\mathbf{n}_R)}$. If the observer is not able to resolve the planckian degrees of freedom, what she will see is the reduced system obtained by tracing out the degrees of freedom in $\mathbb{C}^{N(\mathbf{n}_R)}$. Notice that if the pure state $\rho(0)$ is separable, also the reduced system $\rho_{\text{obs}}(0) = \mathrm{Tr}_{\text{Pl}}\,\rho(0)$ will be pure. As the system evolves, we have

$$\rho_{\text{obs}}(t) = \mathrm{Tr}_{\text{Pl}}\,\rho(t) = \mathrm{Tr}_{\text{Pl}}[U(t)\rho(0)U^\dagger(t)]\,. \tag{12}$$

For the observer, this evolution is not unitary. We have in fact started with a pure state $\rho_{\text{obs}}(0)$ and ended up with a mixed state $\rho_{\text{obs}}(t)$. This is analogous to evolution in open quantum systems. The reduced system evolves according to a completely positive map, taking density matrices into density matrices, but the process is not unitary. The entanglement with the environment makes the system dissipative.[a] Here, the environment is simply constituted by those internal degrees of freedom that the observer cannot measure. The environment is not "out" there, but "in" there. What we have obtained is an apparent loss of unitarity, related to the incapability of the observer to resolve the fine structure of the deformed Hilbert space of the theory. We could prepare a pure state, send it into a unitary channel, which is able to affect the planckian degrees of freedom, and measure the purity

[a] A somewhat similar framework has been proposed in Ref. 19, where it was argued that space-time foam effects, modeled as nonlocal interactions, would cause a loss of coherence in quantum evolution.

of the final state. If the final state is not pure, it is because it is mixed with the degrees of freedom which we cannot resolve. But what could be such a channel?

An example could be represented by a black hole. If we believe in quantum mechanics, a black hole that evaporates is a unitary channel, but nevertheless transforms a pure state into a mixed state. This is the black hole information paradox. The essence of the paradox can be seen in the nature of the emitted radiation consisting of entangled pairs (for a recent review see, e.g., Ref. 18). The mechanism for Hawking radiation has its roots in the properties of the vacuum of a quantum field. Quantum fluctuations of the vacuum can create a pair of particle–antiparticle straddling the black hole horizon. The particles inside the horizon are confined in the black hole, while outside particle can flow away to infinity. Particle creation will produce states of the type

$$|\psi\rangle_i = Ce^{\gamma_i \hat{b}_i^\dagger \hat{c}_i^\dagger}|0\rangle_{b_i}|0\rangle_{c_i}\,, \tag{13}$$

where C is a normalization constant and $\gamma_i = \exp(-8\pi\omega_i GM)$. Here, the b_i modes live inside the horizon, while the c_i modes are outside the horizon. The total state is the vacuum $|\psi\rangle_{\text{in}}$ for an observer at past null infinity that looks populated by a thermal spectrum of quanta to an observer at future null infinity. In the Heisenberg representation the total state is given by

$$|\psi\rangle_{\text{in}} = \exp\left(\sum_i \gamma_i b_i^\dagger c_i^\dagger\right)|\psi\rangle_{\text{fin}} = \bigotimes_{i=1}^{N}|\psi\rangle_i\,, \tag{14}$$

where the sum is ordered from lowest to higher energy. Low energy particles are emitted at earlier time slices than high energy ones. We can write the Hilbert space for the quantum field as

$$\mathcal{H} = \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}\,. \tag{15}$$

It is easy to see that the total state $|\psi\rangle_{\text{in}}$ is entangled in the bipartition (in–out) of this space. The evolution for the partial state $\rho_{\text{out}}(t)$, outside the black hole will not be unitary. When at time t_{f} the whole black hole has completely evaporated, the full system will be described by the density matrix $\rho_{\text{out}}(t_{\text{f}})$, representing a mixed state. Here we see that starting with a pure state we end up with a mixed one even if the evolution is unitary, thus leading to a contradiction.

Below we show how such contradiction could be evaded if the quantum fields Hilbert space exhibits a deformed n-particle structure as described in Eq. (11). Let us consider the creation of n pairs with distribution in momenta $\mathbf{n}_R$ at a certain stage of black hole evaporation. Their Hilbert space can be written as

$$\mathcal{H} = \mathcal{H}_{\text{in}} \otimes (\mathcal{S}_n\mathcal{H}^n \otimes \mathbb{C}^{N(\mathbf{n}_R)})_{\text{out}} \equiv \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C\,. \tag{16}$$

Let us consider terms in the expansion Eq. (14) that create pairs of modes, and for the sake of simplicity consider the situation in which first two pairs of momenta p_1, p_2 are created, namely by the term $T \equiv 1/2(\sum_{i=1,2}\gamma_i b_i^\dagger c_i^\dagger)^2$. And then, on a later time slice, pairs of momenta p_3, p_4. We have thus the following four states:

$|E_{12}, 0\rangle$, $|E_{12}, 1\rangle$, $|E_{34}, 0\rangle$, $|E_{34}, 1\rangle$ that are pairwise degenerate in energy, if we neglect the back reaction of gravity. In fact, the effects of gravity manifest through its coupling to the linear momentum and, consequently, to the spin degrees of freedom. We assume that such coupling can be effectively written as $V = \delta\sigma^z$, where σ^z is the Pauli matrix in of eigenvectors $|0\rangle$, $|1\rangle$. This induces a splitting in the energies: $E_{12} \pm \Delta_{12}, E_{34} \pm \Delta_{34}$. With this correction, the state $|\psi\rangle_{\text{in}}$ is a non-uniform superposition of the four states $|E_{12}, 0\rangle$, $|E_{12}, 1\rangle$, $|E_{34}, 0\rangle$, $|E_{34}, 1\rangle$ and therefore if we trace out the spin degrees of freedom we obtain a mixed state. The energy and spin degrees of freedom are entangled. Since the dimension of the spin Hilbert space grows at least exponentially in the number of the emitted particles, we expect that most of the entanglement is transferred from the partition (in)–(out) to the partition (energy)–(spin). In order for this to happen, the logarithm of the dimensions of the Hilbert spaces $\mathcal{H}_B$, $\mathcal{H}_C$ has to be greater than or equal to the amount of entanglement initially created in the state $|\psi\rangle_{AB}$. A reasonable estimate of this entanglement can be given by the following argument. Expanding (13), we can safely estimate the amount of entanglement for any emitted pair with a number of order unity, since higher terms in the expansion will be negligible. The total entanglement of the state will thus be given by an entropy $S \propto n$, where n is the number of emitted quanta.[18] On the other hand, the logarithm of the dimension of the Hilbert space H_C grows faster than n. For instance, in the sector $n_i = 1 \; \forall \; i$ it will be given by $\log_2 N \sim n \log_2 n$ and therefore all the entanglement can be in principle transferred. Now we can wrap up the result. We start with a completely separable pure state. The process of evaporation entangles the state in many ways, but the final state is a product with respect to the bipartition $A(B, C)$. The disappearance of the degrees of freedom in A does not change the purity of the state. Nevertheless the state is seen as a thermal state by an observer that is not able to resolve the Planck scale, because she is just observing a partial system. The whole process is unitary.

5. Conclusions and Outlook

We have explored the richer structure which emerges in the Hilbert space of certain deformed field theories which can be seen as attempts to go beyond the standard realm of local QFT. The new structure renders possible entanglement between the modes of the field and allows in principle to “hide” the unitarity of a quantum process in the fine structure of the new space. As an application, we have shown that in the new framework entanglement can be transferred away from the bipartition in–out for a radiating black hole. The presence of this entanglement lies at the basis for the information paradox. We also want to remark that the additional Hilbert space $\mathcal{H}_C$ outside the black hole could also be used to “lock” the information escaped from the black hole instead of hypothesizing a small remnant as proposed in Ref. 20. In this paper, we have not provided a complete physical mechanism for the entanglement transfer, but we have merely presented a sketch of how this could happen. We hope that this might open new roads for future efforts toward a resolution of the black hole information paradox.

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References

1. S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975) [Erratum-*ibid.* **46**, 206 (1976)].
2. S. W. Hawking, *Phys. Rev. D* **14**, 2460 (1976).
3. S. B. Giddings, *Phys. Rev. D* **74**, 106005 (2006).
4. R. Bousso, *Rev. Mod. Phys.* **74**, 825 (2002).
5. J. D. Bekenstein, *Phys. Rev. D* **23**, 287 (1981).
6. C. Rovelli, *Class. Quantum Grav.* **8**, 297 (1991).
7. B. Dittrich, *Class. Quantum Grav.* **23**, 6155 (2006).
8. S. B. Giddings, D. Marolf and J. B. Hartle, *Phys. Rev. D* **74**, 064018 (2006).
9. N. Arkani-Hamed, S. Dubovsky, A. Nicolis, E. Trincherini and G. Villadoro, *JHEP* **0705**, 055 (2007).
10. D. Bernard and A. Leclair, *Commun. Math. Phys.* **142**, 99 (1991).
11. M. Arzano, *Phys. Rev. D* **77**, 025013 (2008).
12. G. Amelino-Camelia and M. Arzano, *Phys. Rev. D* **65**, 084044 (2002).
13. M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu, *Phys. Lett. B* **604**, 98 (2004).
14. P. Di Francesco, P. Mathieu and D. Senechal, *Conformal Field Theory* (Springer, 1997).
15. S. Majid and H. Ruegg, *Phys. Lett. B* **334**, 348 (1994).
16. M. Arzano and A. Marciano, *Phys. Rev. D* **76**, 125005 (2007).
17. M. Arzano and D. Benedetti, *Int. J. Mod. Phys. A* **24**, 4623 (2009).
18. S. D. Mathur, *Lect. Notes Phys.* **769**, 3 (2009).
19. L. J. Garay, *Phys. Rev. Lett.* **80**, 2508 (1998).
20. J. A. Smolin and J. Oppenheim, *Phys. Rev. Lett.* **96**, 081302 (2006).