



The q_T subtraction method: electroweak corrections and power suppressed contributions

Luca Buonocore^{1,2,a} , Massimiliano Grazzini^{2,b}, Francesco Tramontano^{1,c}

¹ Dipartimento di Fisica, Università di Napoli Federico II and INFN, Sezione di Napoli, 80126 Naples, Italy

² Physik Institut, Universität Zürich, 8057 Zürich, Switzerland

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Abstract Building upon the formulation of transverse-momentum resummation for heavy-quark hadroproduction, we present the first application of the q_T subtraction formalism to the computation of electroweak corrections to massive lepton pairs through the Drell–Yan mechanism. We then study the power suppressed contributions to the q_T subtraction formula in the parameter r_{cut} , defined as the minimum transverse momentum of the lepton pair normalised to its invariant mass. We analytically compute the leading power correction from initial and final-state radiation to the inclusive cross section. In the case of initial-state radiation the power correction is *quadratic* in r_{cut} and our analytic result is consistent with results previously obtained in the literature. Final-state radiation produces *linear* contributions in r_{cut} that may challenge the efficiency of the q_T subtraction procedure. We explicitly compute the linear power correction in the case of the inclusive cross section and we discuss the extension of our calculation to differential distributions.

1 Introduction

The q_T subtraction formalism [1] is a method to handle and cancel the IR divergences appearing in QCD computations at next-to-next-to-leading order (NNLO) and beyond. In its original formulation it has been applied to carry out a variety of NNLO QCD computations for the production of colourless final states in hadronic collisions [2–16]. Most of the above computations are now publicly available in MATRIX [17]. A first application of q_T subtraction to the computation of the approximate next-to-next-to-next-to-leading order (N^3 LO) QCD corrections to Higgs boson production through gluon fusion has been presented recently [18].

In the last few years, thanks to the formulation of transverse-momentum resummation for heavy-quark production [19–23] the method has been extended and applied to the production of top-quark pairs [24–26]. The q_T subtraction counterterm is constructed by exploiting the universal behavior of the associated transverse-momentum (q_T) distribution. Therefore, the subtraction is intrinsically *non local* and in practice the computation is carried out by introducing a cut, r_{cut} on the transverse momentum of the colourless system normalised to its invariant mass. When evaluated at finite r_{cut} both the contribution of the real emission and the one of the counterterm exhibit logarithmically divergent terms plus additional power suppressed contributions that vanish as $r_{\text{cut}} \rightarrow 0$. The efficiency of the subtraction procedure crucially depends on the size of such power suppressed contributions.

In the inclusive production of a colourless final state the power suppressed contributions are known to be *quadratic* in r_{cut} (modulo logarithmic enhancements) [12]. This allows us to obtain precise predictions by either evaluating the cross section at sufficiently small r_{cut} , or carrying out the $r_{\text{cut}} \rightarrow 0$ extrapolation¹ [17]. The power suppressed contributions to the next-to-leading order (NLO) total cross section have been explicitly evaluated in Refs. [28,29]. In the case of heavy-quark production the r_{cut} dependence is found to be *linear* [25,26,30], and it is an interesting question to investigate the origin of this peculiar behavior.

Up to now the q_T subtraction formalism has been applied only to higher-order QCD computations. The formulation of the method for heavy-quark production can be straightforwardly extended to the computation of NLO electroweak (EW) corrections to the Drell–Yan process. The purpose of

^a e-mail: luca.buonocore@na.infn.it (corresponding author)

^b e-mail: grazzini@physik.uzh.ch

^c e-mail: francesco.tramontano@na.infn.it

¹ The only exception is the production of direct photons ($\gamma\gamma$ [3,4], $Z\gamma$ [6], $W\gamma$ [8]...), for which a fully inclusive cross section cannot be defined, and an isolation prescription is required. The interplay of the isolation prescription with the subtraction procedure makes the r_{cut} dependence stronger [17,27].

the present paper is twofold. We first present and discuss the first application of the q_T subtraction formalism to the computation of the NLO EW corrections to the production of massive lepton pairs. Our computation consistently includes initial-state radiation, final-state radiation from the massive leptons and their interference, and our results are compared with the ones obtained with the well established public generator SANC [31] and to an independent computation that we carry out with the dipole subtraction formalism [32]. Then, we present the analytic computation of the power suppressed contributions, and we confirm the linear r_{cut} behaviour by computing its NLO coefficient. We also extend our results to the case in which cuts are applied.

The paper is organised as follows. In Sect. 2 we review the q_T -subtraction formalism, by detailing its implementation up to NLO in the case of heavy-quark production. In Sect. 3 we apply the formalism to the computation of NLO EW corrections to the Drell–Yan process. In Sect. 4 we study the power suppressed contributions and we explicitly compute the leading power corrections in the case of final-state and initial-state radiation. In Sect. 5 we summarise our results.

2 The q_T subtraction formalism

The q_T subtraction formalism [1] is a method to handle and cancel the IR divergences appearing in higher-order QCD computations. The method uses IR subtraction counterterms that are constructed by considering and explicitly computing the transverse-momentum q_T distribution of the produced final-state system. At Born level such distribution is proportional to $\delta(q_T^2)$. At higher perturbative orders multiple radiation of soft and collinear partons makes the distribution divergent in the $q_T \rightarrow 0$ limit. If the produced final-state system is composed of non-QCD (colourless) partons (e.g., leptons, vector bosons or Higgs bosons), the small- q_T behaviour has a universal (process-independent) structure that is explicitly known up to the NNLO level (and, in part, at N³LO [18,33]) through the formalism of transverse-momentum resummation [34]. These results on transverse-momentum resummation are sufficient to fully specify the q_T subtraction formalism for this entire class of processes. By using the formulation of transverse-momentum resummation for heavy-quark production [19–23], the q_T subtraction formalism has been recently extended to this class of processes [24–26].

According to the q_T subtraction method [1], the parton level differential cross section $d\sigma_{(N)NLO}^{Q\bar{Q}}$ for the inclusive production process $pp \rightarrow Q\bar{Q} + X$ can be written as

$$d\hat{\sigma}_{(N)NLO}^{Q\bar{Q}} = \mathcal{H}_{(N)NLO}^{Q\bar{Q}} \otimes d\hat{\sigma}_{LO}^{Q\bar{Q}} + \left[d\hat{\sigma}_{(N)LO}^{Q\bar{Q}+\text{jet}} - d\hat{\sigma}_{(N)NLO}^{Q\bar{Q},CT} \right], \quad (1)$$

where $d\hat{\sigma}_{(N)LO}^{Q\bar{Q}+\text{jet}}$ is the $Q\bar{Q}$ +jet cross section at (N)LO accuracy. The square bracket term of Eq. (1) is IR finite in the limit $q_T \rightarrow 0$, but its individual contributions, $d\hat{\sigma}_{(N)LO}^{Q\bar{Q}+\text{jet}}$ and $d\hat{\sigma}_{(N)NLO}^{Q\bar{Q},CT}$, are separately divergent. The IR subtraction counterterm $d\hat{\sigma}_{(N)NLO}^{Q\bar{Q},CT}$ is obtained from the (N)NLO perturbative expansion (see, e.g., Refs. [24,35]) of the resummation formula of the logarithmically-enhanced contributions to the q_T distribution of the $Q\bar{Q}$ pair [19–21]: the explicit form of $d\hat{\sigma}_{(N)NLO}^{Q\bar{Q},CT}$ can be completely worked out up to NNLO accuracy.

In the following we will limit ourselves to consider Eq. (1) up to NLO accuracy. The explicit expression of $d\hat{\sigma}_{NLO}^{Q\bar{Q},CT}$ in the partonic channel $ab \rightarrow Q\bar{Q} + X$ reads [24]

$$d\hat{\sigma}_{NLO}^{Q\bar{Q},CT} = \sum_{c=q,\bar{q},g} \frac{\alpha_S}{\pi} \Sigma_{c\bar{c} \leftarrow ab}^{(1)} \otimes d\hat{\sigma}_{LO}^{Q\bar{Q}} \frac{dq_T^2}{M^2}, \quad (2)$$

where M is the invariant mass of the $Q\bar{Q}$ pair and the symbol \otimes denotes convolutions with respect to the longitudinal-momentum fractions z_1 and z_2 of the colliding partons. The functions $\Sigma_{c\bar{c} \leftarrow ab}^{(1)}$ in Eq. (2) can be written as

$$\Sigma_{c\bar{c} \leftarrow ab}^{(1)}(z_1, z_2; r) = \Sigma_{c\bar{c} \leftarrow ab}^{(1,2)}(z_1, z_2) \tilde{I}_2(r) + \Sigma_{c\bar{c} \leftarrow ab}^{(1,1)}(z_1, z_2) \tilde{I}_1(r) \quad (3)$$

where $r = q_T/M$, and the coefficients $\Sigma_{c\bar{c} \leftarrow ab}^{(1,k)}(z_1, z_2)$ ($k = 1, 2$) read

$$\begin{aligned} \Sigma_{c\bar{c} \leftarrow ab}^{(1,2)}(z_1, z_2) &= -\frac{1}{2} A_c^{(1)} \delta_{ca} \delta_{\bar{c}b} \delta(1-z_1) \delta(1-z_2) \\ \Sigma_{c\bar{c} \leftarrow ab}^{(1,1)}(z_1, z_2) &= -\left[\delta_{ca} \delta_{\bar{c}b} \delta(1-z_1) \delta(1-z_2) B_c^{(1)} \right. \\ &\quad + \delta_{ca} \delta(1-z_1) P_{\bar{c}b}^{(1)}(z_2) \\ &\quad + \delta_{\bar{c}b} \delta(1-z_2) P_{ca}^{(1)}(z_1) \left. \right] \\ &\quad - \delta_{ca} \delta_{\bar{c}b} \delta(1-z_1) \delta(1-z_2) \\ &\quad \times \frac{\langle \mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}} | (\mathbf{r}_t^{(1)} + \mathbf{r}_t^{(1)\dagger}) | \mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}} \rangle}{|\mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}}|^2}. \end{aligned} \quad (4)$$

The coefficients $A_c^{(1)}$ and $B_c^{(1)}$ are the first-order coefficients for transverse-momentum resummation ($A_q^{(1)} = C_F$, $A_g^{(1)} = C_A$, $B_q^{(1)} = -3/2 C_F$, $B_g^{(1)} = -(11/6 C_A - n_F/3)$).

The functions $P_{ab}^{(1)}(z)$ are the lowest-order DGLAP kernels (the overall normalisation is specified according to the notation in Eq. (41) of Ref. [35]). The functions $\tilde{I}_k(r)$ ($k = 1, 2$), which appear in Eq. (3), encapsulate the singular behavior at small q_T , and they read (see Appendix B of Ref. [35])

$$\begin{aligned}\tilde{I}_1(r) &= -\frac{b_0}{r} K_1(b_0 r), \\ \tilde{I}_2(r) &= \frac{2b_0}{r} \left[K_1(b_0 r) \ln r - \frac{\partial K_\nu(b_0 r)}{\partial \nu} \Big|_{\nu=1} \right],\end{aligned}\quad (6)$$

where $b_0 = 2e^{-\gamma_E}$ and we have introduced the modified Bessel function of imaginary argument

$$K_\nu(r) = \int_0^\infty dt e^{-r \cosh t} \cosh \nu t. \quad (7)$$

The coefficient $\Sigma_{c\bar{c} \leftarrow ab}^{(1,2)}(z_1, z_2)$ in Eq. (4) controls the leading logarithmic contribution at small q_T , while the coefficient $\Sigma_{c\bar{c} \leftarrow ab}^{(1,1)}(z_1, z_2)$ in Eq. (5) controls the next-to-leading logarithmic term. The latter has a first term (first line in Eq. 5) which is identical to what we have in the case of the production of a colour singlet. The second term (second line in Eq. 5) is due to soft radiation and it is an additional term that is specific of the q_T subtraction method for the case of heavy-quark pair production [24]. Here $\Gamma_i^{(1)}$ is the first-order contribution to the soft anomalous dimension for transverse-momentum resummation in heavy-quark production and its explicit expression is given in Eq. (33) of Ref. [21]. The soft anomalous dimension is a matrix acting on the colour indices of the four hard partons in the Born level scattering amplitude $|\mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}}\rangle$. At this perturbative order the soft anomalous dimension is expressed in terms of colour correlators $\mathbf{T}_i \cdot \mathbf{T}_j$ with definite kinematic dependence, where the indices i and j refer to the hard-scattering partons.

The first-order hard-collinear coefficients $\mathcal{H}_{NLO}^{Q\bar{Q}}$ in Eq. (1) are also completely known [19–21]. In the next Section we apply the method to the computation of EW corrections to the Drell–Yan process.

3 NLO EW corrections to the Drell–Yan process

We consider the hadroproduction of a dilepton pair through the Drell–Yan mechanism. NLO EW corrections to this process have been considered in Refs. [36–39]. A tuned comparison of various Monte Carlo codes is presented in Ref. [40].

The NLO q_T subtraction formalism for heavy-quark production reviewed in Sect. 2 can be straightforwardly extended to the computation of the NLO EW corrections to the Drell–Yan process. In this case the heavy-quark pair is replaced by a massive lepton pair and the abelian limit is carried out along the lines of Ref. [41]. The partonic cross section up to NLO EW can be evaluated by using

$$d\hat{\sigma}_{NLO} = \mathcal{H}_{NLO} \otimes d\hat{\sigma}_{LO} + \left[d\hat{\sigma}^R - d\hat{\sigma}^{CT} \right], \quad (8)$$

where $d\hat{\sigma}^R$ is the real emission cross section and the functions \mathcal{H}_{NLO} and $d\hat{\sigma}^{CT}$ are obtained from the corresponding functions appearing in Eq. (1) with the replacements

$$C_A \rightarrow 0 \quad C_F \rightarrow e_f^2 \quad \mathbf{T}_i^2 \rightarrow e_i^2 \quad \mathbf{T}_i \cdot \mathbf{T}_j \rightarrow e_i e_j. \quad (9)$$

As is well known, at LO (i.e. $\mathcal{O}(\alpha^2)$) both the $q\bar{q}$ and the $\gamma\gamma$ partonic channels contribute and we can write for the hadronic cross section

$$\sigma_{LO} = \sigma_{LO}^{q\bar{q}} + \sigma_{LO}^{\gamma\gamma}, \quad (10)$$

where $\sigma_{LO}^{q\bar{q}}$ and $\sigma_{LO}^{\gamma\gamma}$ are the Born level cross sections in the $q\bar{q}$ and $\gamma\gamma$ channels, respectively. At NLO EW we can write

$$\sigma_{NLO} = \sigma_{LO}^{q\bar{q}} + \sigma_{LO}^{\gamma\gamma} + \Delta\sigma_{q\bar{q}} + \Delta\sigma_{q\gamma} + \Delta\sigma_{\gamma\gamma} \quad (11)$$

where we have introduced the $\mathcal{O}(\alpha^3)$ correction in the $q\bar{q}$ channel, $\Delta\sigma_{q\bar{q}}$, the corresponding correction in the $q(\bar{q})\gamma$ channel, $\Delta\sigma_{q\gamma}$, and the correction in the $\gamma\gamma$ channel, $\Delta\sigma_{\gamma\gamma}$. Since the $\gamma\gamma$ channel provides only a very small contribution to the Drell–Yan cross section, $\Delta\sigma_{\gamma\gamma}$ will be neglected in the following discussion.

Our calculation is carried out by using an extension of the numerical program of Ref. [2]. All the required tree level matrix elements are computed analytically while the virtual EW corrections for $q\bar{q} \rightarrow l^+l^-$, which include vertex and box diagrams, are obtained by using GOSAM [42,43]. We use the setup of Ref. [44], and, in particular, we work in the \bar{G}_μ scheme with

$$\begin{aligned}G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2} & \alpha(0) &= 1/137.03599911 \\ m_W &= 80.403 \text{ GeV} & m_Z &= 91.1876 \text{ GeV} \\ \Gamma_W &= 2.141 \text{ GeV} & \Gamma_Z &= 2.4952 \text{ GeV}\end{aligned}\quad (12)$$

and use the complex-mass scheme [45] throughout. More precisely, real and virtual photons emissions are controlled by $\alpha(0)$, while the α^2 in the LO cross section is derived from G_F , m_Z and m_W . Following Ref. [44], the MRST2004qed [46] parton distribution functions (PDFs) are used setting the factorization scale to $\mu_F = M_Z$.

The following set of cuts are applied

$$m_{ll} > 50 \text{ GeV} \quad p_{T,l} > 25 \text{ GeV} \quad |y_l| < 2.5. \quad (13)$$

To validate our implementation, we have performed a tuned comparison with the public generator SANC setting the mass of the heavy lepton to the muon mass $m_l \equiv m_\mu = 105.658369 \text{ MeV}$. In Table 1 we report our result for the lowest order cross sections $\sigma_{LO}^{q\bar{q}}$ and $\sigma_{LO}^{\gamma\gamma}$, and the NLO EW corrections in the $q\bar{q}$ and $q\gamma$ channels, $\Delta\sigma_{q\bar{q}}$ and $\Delta\sigma_{q\gamma}$. The NLO correction $\Delta\sigma_{q\bar{q}}$ is obtained performing the calculation at different values of r_{cut} and extrapolating to $r_{\text{cut}} \rightarrow 0$

Table 1 Comparison of NLO EW corrections to Drell–Yan dimuon production ($m_l \equiv m_\mu = 105.658369 \text{ MeV}$) computed with q_T subtraction and with the SANC generator. In the $q\bar{q}$ channel the q_T result is obtained with a linear extrapolation in the $r_{\text{cut}} \rightarrow 0$ limit (see Fig. 2), while in the $q(\bar{q})\gamma$ channel it is obtained at $r_{\text{cut}} = 0.01\%$. The LO result in the $q\bar{q}$ and $\gamma\gamma$ channels is also reported for reference. The small discrepancy in the $\sigma_{LO}^{q\bar{q}}$ cross section, below 0.5 *per mille*, can be ascribed to the use of the complex mass scheme in our computation

	$q_T + \text{GoSam}$	SANC
$\sigma_{LO}^{q\bar{q}}$ (pb)	739.45 ± 0.02	739.17 ± 0.01
$\sigma_{LO}^{\gamma\gamma}$ (pb)	1.289 ± 0.005	1.29 ± 0.01
$\Delta\sigma_{q\bar{q}}$ (pb)	-29.18 ± 0.03	-29.23 ± 0.02
$\Delta\sigma_{q\gamma}$ (pb)	-0.777 ± 0.002	-0.78 ± 0.01

through a linear fit. This procedure will be fully motivated by an analytic computation presented in the next section. We find that the agreement is quite good, at few *per mille* on the NLO correction. The small discrepancy in the $\sigma_{LO}^{q\bar{q}}$ cross section, below 0.5 *per mille*, is consistent with the use of different schemes to handle unstable particles. In particular, at variance with what is done in SANC, complex couplings are used within our computations.

The comparison with SANC can be extended to differential distributions. In Fig. 1 we compare the invariant mass (m_{ll}) and rapidity (y_{ll}) distribution of the dilepton pair, the transverse momentum and rapidity of the positively charged lepton ($p_{T,l+}$ and y_{l+}) obtained with our implementation with the corresponding results obtained with SANC. For ease of reference, we plot also the LO result in black. We see that the agreement is good, the differences typically being well below the percent level. In addition, we have repeated our calculation by using the dipole subtraction method² [32] and the independent matrix-element generator RECOLA [48,49] for the virtual corrections, finding the same level of agreement as reported in Table 1.

4 Power corrections

In this section we are going to analytically study the power suppressed contributions to the q_T subtraction formula in Eq. (8). In particular, we will show that the leading power suppressed contributions have a *soft* origin. In order to avoid complications due to the small lepton mass, which are sensitive to *collinear* logarithmic contributions and could obscure the r_{cut} behavior, in the following we will consider a pair of heavy leptons of mass $m_l = 10 \text{ GeV}$. By using the setup of Sect. 3 (see Eqs. 12, 13), we have studied the dependence

² An early comparison of phase-space slicing and dipole subtraction methods in the case of final-state radiation off a heavy-quark pair was presented in Ref. [47].

of the NLO corrections for the fiducial cross section on r_{cut} . We have varied r_{cut} in the range $0.01\% \leq r_{\text{cut}} \leq 1\%$ and we have used the r_{cut} -independent cross section computed with our inhouse implementation of the dipole subtraction method as normalisation. The results for the r_{cut} dependent correction $\delta_{q_T} = \Delta\sigma/\sigma_{LO}^{q\bar{q}}$ in the $q\bar{q}$ and $q\gamma$ channels are shown in Fig. 2. A distinctive linear behavior in the dominant $q\bar{q}$ -annihilation channel emerges. Nonetheless, as reported in Ref. [17], it is known that symmetric cuts on the transverse momenta of the final state leptons challenge the convergence of q_T -subtraction leading to a stronger dependence on r_{cut} even in the case in which a charge-neutral final state is produced. In Fig. 3 we show the dependence of the NLO corrections for the inclusive cross section on r_{cut} when no cuts are applied. Again a distinct linear behavior in the dominant $q\bar{q}$ -annihilation channel emerges, in agreement with what has already been observed for the case of the $t\bar{t}$ cross section [25], which can be clearly interpreted as a genuine new effect due to the emission of radiation off the massive final-state leptons.

In the following we analytically study the behavior of NLO cross sections computed with q_T subtraction in the $r_{\text{cut}} \rightarrow 0$ limit. We are interested in determining the structure of the leading power correction to the inclusive cross section, and in identifying the origin of the linear behavior observed in Fig. 3. We recall that when applying the q_T subtraction formula the second term on the right hand side of Eq. (8) is computed by introducing a lower limit r_{cut} on the q_T/M ratio. With such a cutoff we can treat separately the real contribution $d\hat{\sigma}^R$ and the counterterm $d\hat{\sigma}^{CT}$. We start our discussion from the contribution of the counterterm. From Eq. (2) we have

$$d\hat{\sigma}_{ab}^{CT}(r_{\text{cut}}) = \sum_{c=q,\bar{q},\gamma} \int_{r_{\text{cut}}}^{\infty} 2r dr \frac{\alpha_S}{\pi} \Sigma_{c\bar{c} \leftarrow ab}^{(1)} \otimes d\hat{\sigma}_{LO}^{l+l-} \quad (14)$$

The NLO coefficient $\Sigma_{c\bar{c} \leftarrow ab}^{(1)}$ depends on $r = q_T/M$ only through the functions $\tilde{I}_i(r)$. Therefore we have

$$\frac{d\hat{\sigma}_{ab}^{CT}(r_{\text{cut}})}{dr_{\text{cut}}} = -2r_{\text{cut}} \frac{\alpha_S}{\pi} \left(\Sigma_{c\bar{c} \leftarrow ab}^{(1,2)} \tilde{I}_2(r_{\text{cut}}) + \Sigma_{c\bar{c} \leftarrow ab}^{(1,1)} \tilde{I}_1(r_{\text{cut}}) \right) \otimes d\hat{\sigma}_{LO}^{l+l-} \quad (15)$$

In the small r limit the integrals $\tilde{I}_1(r)$ and $\tilde{I}_2(r)$ read

$$\begin{aligned} \tilde{I}_1(r) &= -\frac{1}{r^2} + \frac{b_0^2}{4} (1 - 2 \ln r) + O(r^2), \\ \tilde{I}_2(r) &= \frac{4 \ln r}{r^2} + \frac{b_0^2}{2} (-1 + 2 \ln^2 r) + O(r^2), \end{aligned} \quad (16)$$

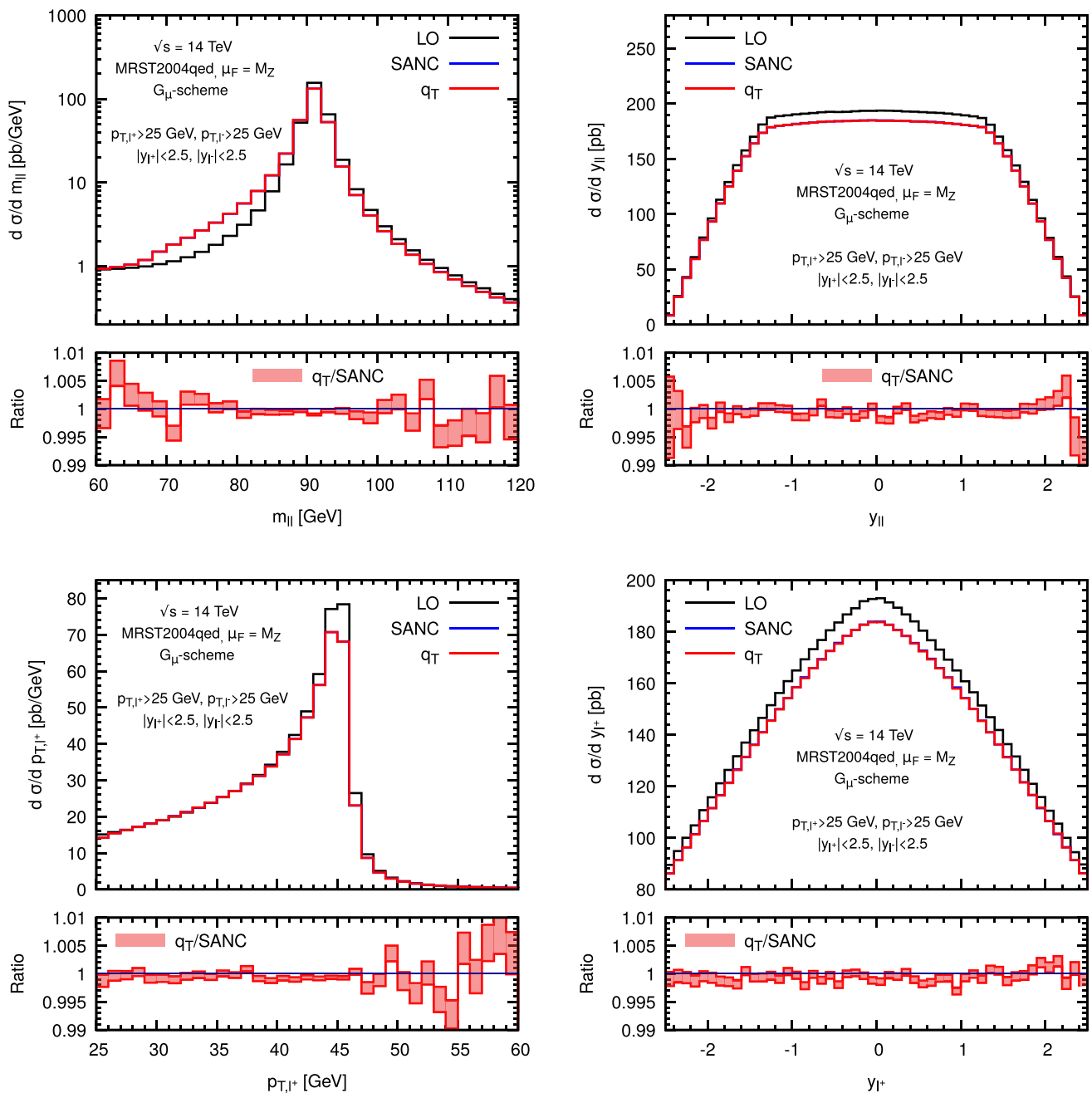


Fig. 1 Tuned comparison for the dilepton invariant mass distribution (top left), rapidity distribution (top right), p_T distribution of the positively charged lepton (bottom left) and rapidity distribution (bottom

right) with the SANC generator. The q_T result is obtained by fixing $r_{\text{cut}} = 0.01\%$ and with $m_l = m_\mu = 105.658369$ MeV. For reference, also the LO result is showed (black line)

i.e., they depend quadratically on r modulo logarithmic terms. This results holds also at NNLO and beyond. It follows that the leading power corrections from the counterterm are always quadratic in r_{cut} , independently on the perturbative order. As a consequence, the linear behavior with r_{cut} that we observe in heavy-quark production and in the EW corrections to dilepton production must be due to the real emission. In the following we analytically compute the real-emission contribution at small values of r_{cut} .

We consider the production of a massive lepton pair in pure QED in the diagonal channel

$$q(p_1) + \bar{q}(p_2) \rightarrow l^+(p_3) + l^-(p_4) + \gamma(k) \quad (17)$$

with $p_3^2 = p_4^2 = m^2$. We define the variables

$$\begin{aligned} s &= (p_1 + p_2)^2 & M^2 &= (p_3 + p_4)^2 & t &= (p_1 - k)^2 \\ u &= (p_2 - k)^2 & q_T^2 &= ut/s \end{aligned} \quad (18)$$

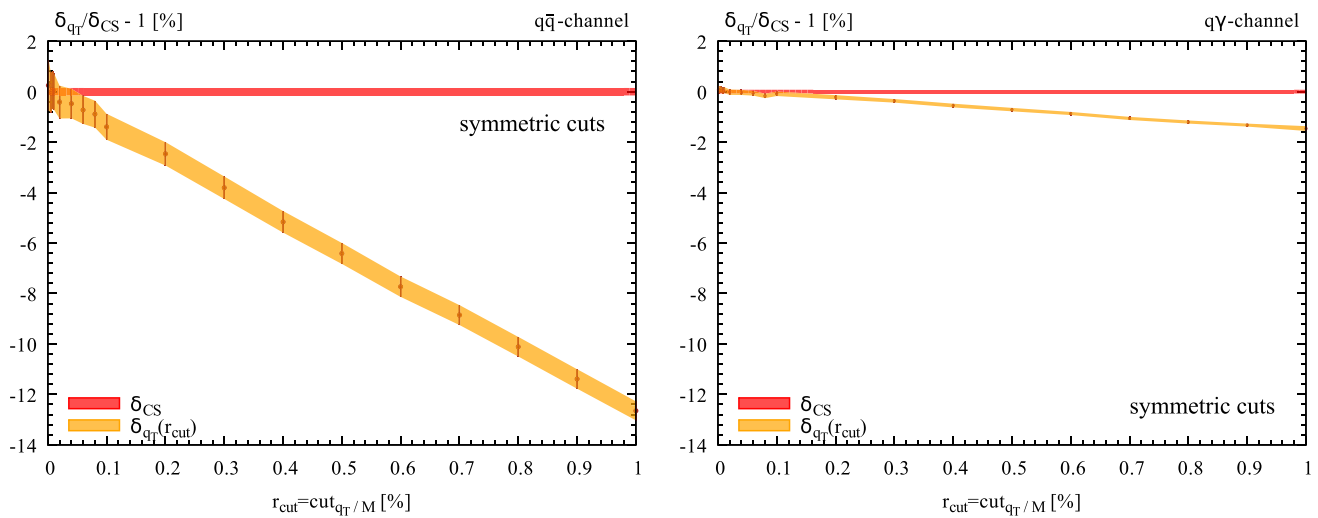


Fig. 2 NLO EW correction as a function of r_{cut} in the dominant $q\bar{q}$ diagonal channel (left panel) and in the off-diagonal $q(\bar{q})\gamma$ channel (right panel) at 14 TeV. The NLO result is normalised to the r_{cut} -

independent cross section computed with dipole subtraction. The lepton mass is fixed to $m_l = 10$ GeV. The fiducial cuts in Eq. (13) are applied

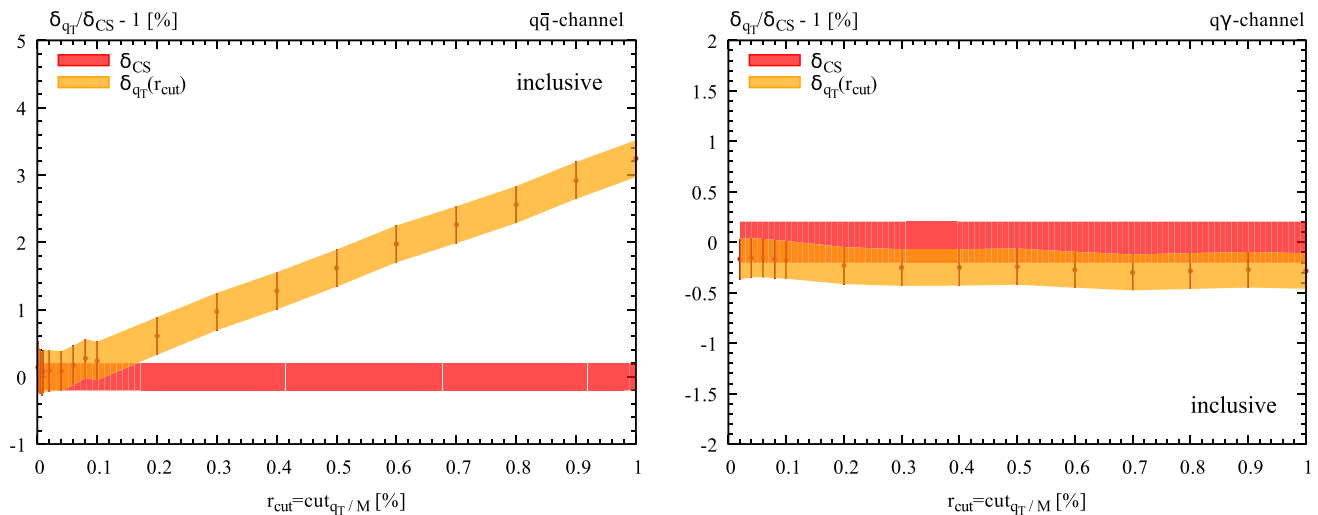


Fig. 3 NLO EW correction as a function of r_{cut} in the dominant $q\bar{q}$ diagonal channel (left panel) and in the off-diagonal $q(\bar{q})\gamma$ channel (right panel) at 14 TeV. The NLO result is normalised to the r_{cut} -

independent cross section computed with dipole subtraction. The lepton mass is fixed to $m_l = 10$ GeV. No cuts are applied

and

$$z = M^2/s. \quad (19)$$

Since there is a lower limit on the ratio $r = q_T/M$ we can safely work in $d = 4$ dimensions. The differential cross reads

$$\begin{aligned} \frac{d^2\hat{\sigma}_{q\bar{q}}}{dM^2 dq_T^2} &= \frac{1}{32s^2} \frac{1}{(2\pi)^4} \frac{1}{\sqrt{(1-z)^2 - 4zq_T^2/M^2}} \\ &\times \sqrt{1 - \frac{4m^2}{M^2}} \int d\Omega |\mathcal{M}|^2 \end{aligned} \quad (20)$$

and the angular integral is defined in the centre-of-mass frame of the final-state leptons. By integrating Eq. (20) over q_T^2 and M^2 and keeping into account the phase space constraints we obtain

$$\begin{aligned} \frac{d\hat{\sigma}_{q\bar{q}}}{dr_{\text{cut}}^2} &= -\frac{1}{32} \frac{1}{(2\pi)^4} \int_{z_{\min}}^{z_{\max}} \frac{z dz}{\sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} \\ &\times \sqrt{1 - \frac{z_{\min}}{z}} \int d\Omega |\mathcal{M}|^2. \end{aligned} \quad (21)$$

where

$$z_{\min} = \frac{4m^2}{s} \quad z_{\max} = 1 - 2r_{\text{cut}}\sqrt{1 + r_{\text{cut}}^2} + 2r_{\text{cut}}^2. \quad (22)$$

The matrix element squared $|\mathcal{M}|^2$ can be divided into three separate gauge invariant contributions: final state radiation, initial state radiation and interference. The interference contribution is odd under the exchange $p_3 \leftrightarrow p_4$ and therefore vanishes after angular integration. We now discuss the final- and initial-state contributions in turn.

4.1 Final-state radiation

The integration of the matrix element squared corresponding to final state radiation over the angular variables can be carried out along the lines of Ref. [50]. After partial fractioning, the required angular integrals have the form

$$I^{(k,l)} = \int_0^\pi \sin \vartheta_1 d\vartheta_1 \int_0^\pi d\vartheta_2 (a + b \cos \vartheta_1)^{-k} \times (A + B \cos \vartheta_1 + C \sin \vartheta_1 \cos \vartheta_2)^{-j} \quad (23)$$

where the coefficients a, b, A, B, C are functions of the invariants s, M^2, u, t . The ensuing contribution to $d\hat{\sigma}_{q\bar{q}}/dr_{\text{cut}}^2$ can be expressed in the following form

$$\frac{d\hat{\sigma}_{q\bar{q}}^{\text{FS}}}{dr_{\text{cut}}^2} = -\frac{4\alpha^3 e_q^2}{3s} \int_{z_{\min}}^{z_{\max}} dz \left[\frac{K_1(z; z_{\min})}{(1-z)^2 \sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} + \frac{K_2(z; z_{\min})r_{\text{cut}}^2}{(1-z)^4 \sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} \right] \quad (24)$$

in terms of two coefficient functions, K_1 and K_2 , which are regular at $z = 1$ (soft limit) and do not depend on the cut-off parameter r_{cut} :

$$K_1(z; z_{\min}) = -\left[z_{\min} z^2 + z(1+z)^2 \right] \sqrt{1 - \frac{z_{\min}}{z}} + z \left(1 + z^2 + z_{\min} z - \frac{z_{\min}^2}{2} \right) \ln \frac{1 + \sqrt{1 - \frac{z_{\min}}{z}}}{1 - \sqrt{1 - \frac{z_{\min}}{z}}}, \quad (25)$$

and

$$K_2(z; z_{\min}) = 2z^2 \left\{ [1 + z(6+z) + z_{\min} z] \sqrt{1 - \frac{z_{\min}}{z}} \right.$$

$$\left. - \left(1 + z^2 + z_{\min}(2+z) - \frac{z_{\min}^2}{2} \right) \times \ln \frac{1 + \sqrt{1 - \frac{z_{\min}}{z}}}{1 - \sqrt{1 - \frac{z_{\min}}{z}}} \right\}. \quad (26)$$

In the small- r_{cut} limit the integral in Eq. (24) can be computed by using the expansions

$$\begin{aligned} \frac{\Theta(z_{\max} - z)\Theta(z - z_{\min})}{(1-z)^2 \sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} &= \frac{1}{4} \delta(1-z) \frac{1}{r_{\text{cut}}^2} \\ &+ \frac{\pi}{8} [\delta(1-z) + 2\delta'(1-z)] \frac{1}{r_{\text{cut}}} + \mathcal{O}(1) \\ \frac{\Theta(z_{\max} - z)\Theta(z - z_{\min})r_{\text{cut}}^2}{(1-z)^4 \sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} &= \frac{1}{24} \delta(1-z) \frac{1}{r_{\text{cut}}^2} \\ &+ \frac{\pi}{64} [3\delta(1-z) + 2\delta'(1-z)] \frac{1}{r_{\text{cut}}} + \mathcal{O}(1) \end{aligned} \quad (27)$$

and we obtain for the r_{cut} dependence of the partonic cross section

$$\begin{aligned} \hat{\sigma}_{q\bar{q}}^{\text{FS}}(s; r_{\text{cut}}) &= \sigma_0(s) \frac{\alpha}{2\pi} \left\{ \left[2 - \frac{(1+\beta^2)}{\beta} \ln \frac{1+\beta}{1-\beta} \right] \ln(r_{\text{cut}}^2) \right. \\ &- \frac{3\pi}{8} \left[\frac{6(5-\beta^2)}{3-\beta^2} + \frac{-47+8\beta^2+3\beta^4}{\beta(3-\beta^2)} \right. \\ &\quad \left. \left. \times \ln \frac{1+\beta}{1-\beta} \right] r_{\text{cut}} \right\} + \mathcal{O}(r_{\text{cut}}^2) \\ &\equiv \hat{\sigma}_{\text{LP}}^{\text{FS}}(s; r_{\text{cut}}) + \hat{\sigma}_{\text{NLP}}^{\text{FS}}(s; r_{\text{cut}}) + \mathcal{O}(r_{\text{cut}}^2) \end{aligned} \quad (28)$$

where we have dropped terms which do not depend on r_{cut} and we have introduced the Born cross section

$$\sigma_0(s) = \frac{2\pi}{9s} \alpha^2 e_q^2 \beta(3-\beta^2) \quad (29)$$

with $\beta = \sqrt{1 - \frac{4m^2}{s}}$.

Equation (28) shows that the final-state contribution to the NLO cross section, integrated down to r_{cut} , contains the expected single logarithmic term in r_{cut} , which is due to soft emission and will be cancelled by the subtraction counterterm [more precisely, by the abelian limit of the term in the second line in Eq. (5)]. The next-to-leading power contribution $\hat{\sigma}_{\text{NLP}}^{\text{FS}}(s; r_{\text{cut}})$ is linear in r_{cut} and it is responsible for the behavior observed in Fig. 3.

4.2 Initial-state radiation

The integration of the matrix element squared corresponding to initial-state radiation over the angular variables is straightforward and we obtain

$$\frac{d\hat{\sigma}_{q\bar{q}}^{\text{IS}}}{dr_{\text{cut}}^2} = -\frac{4\alpha^3 e_q^4}{9s} \int_{z_{\min}}^{z_{\max}} dz \left[\frac{K_3(z; z_{\min})}{r_{\text{cut}}^2 \sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} + \frac{K_4(z; z_{\min})}{\sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} \right] \quad (30)$$

where the coefficient functions K_3 and K_4 now read

$$\begin{aligned} K_3(z; z_{\min}) &= \sqrt{1 - \frac{z_{\min}}{z}} \left(z + \frac{z_{\min}}{2} \right) \frac{1+z^2}{z^2} \\ K_4(z; z_{\min}) &= -2K_3(z; z_{\min}) \frac{z}{1+z^2}. \end{aligned} \quad (31)$$

The coefficient function $K_3(z; z_{\min})$ controls the most singular term, and is proportional to the Altarelli–Parisi splitting function. In order to evaluate the integral in Eq. (30) we have to expand the distribution

$$T(z, r_{\text{cut}}, z_{\min}) = \frac{\Theta(z - z_{\min})\Theta(z_{\max} - z)}{\sqrt{(1-z)^2 - 4zr_{\text{cut}}^2}} \quad (32)$$

in the small r_{cut} limit. Since we know that linear terms in r_{cut} are absent, we have to expand up to $\mathcal{O}(r_{\text{cut}}^2)$. Such expansion is tricky, because the functions $K_3(z; z_{\min})$ and $K_4(z; z_{\min})$ contain a square root which vanishes at $z = z_{\min}$. At variance with the case of final-state radiation, this leads to spurious singularities when the distributions appearing in the expansion involve derivatives at $z = z_{\min}$. We found it convenient to split the integration over z as follows

$$\int_{z_{\min}}^{z_{\max}} dz = \int_{z_{\min}}^a dz + \int_a^{z_{\max}} dz, \quad z_{\min} < a < z_{\max}. \quad (33)$$

The integral from z_{\min} to a can be safely computed by expanding in r_{cut} . The integral from a to z_{\max} can be computed by using the expansion

$$\begin{aligned} T(z, r_{\text{cut}}, a) &= -\frac{1}{2}\delta(1-z) \ln r_{\text{cut}}^2 + \left(\frac{1}{1-z} \right)_a \\ &\quad + \ln(1-a)\delta(1-z) \\ &\quad - \frac{1}{2} \left(\delta^{(2)}(1-z) - 2\delta^{(1)}(1-z) \right) r_{\text{cut}}^2 \ln r_{\text{cut}}^2 \\ &\quad + \left[(1 + \ln(1-a))\delta^{(2)}(1-z) \right. \\ &\quad \left. - [1 + 2\ln(1-a)]\delta^{(1)}(1-z) - \frac{1}{2}\delta(1-z) \right. \\ &\quad \left. + \frac{1-2a}{(1-a)^2} \delta(z-a) - \frac{a}{1-a} \delta^{(1)}(z-a) \right. \\ &\quad \left. + D^{(2)}(z, a) + 2D^{(1)}(z, a) \right] r_{\text{cut}}^2 + \mathcal{O}(r_{\text{cut}}^4). \end{aligned} \quad (34)$$

where we have defined the distributions $\delta^{(n)}(z-b)$, $\left(\frac{1}{1-z} \right)_a$, $D^{(1)}(z, a)$ and $D^{(2)}(z, a)$ through their action on a test function $f(z)$ as

$$\int_0^1 dz f(z) \delta^{(n)}(z-b) = (-1)^n f^{(n)}(b), \quad b \in [0, 1], \quad (35)$$

$$\int_0^1 dz f(z) \left(\frac{1}{1-z} \right)_a = \int_a^1 dz \frac{f(z) - f(1)}{1-z}, \quad (36)$$

$$\int_0^1 dz f(z) D^{(1)}(z, a) = \int_a^1 dz \frac{f^{(1)}(z) - f^{(1)}(1)}{1-z}, \quad (37)$$

$$\int_0^1 dz f(z) D^{(2)}(z, a) = \int_a^1 dz \frac{zf^{(2)}(z) - f^{(2)}(1)}{1-z}. \quad (38)$$

By combining the two contributions $z_{\min} < z < a$ and $a < z < z_{\max}$ the dependence on a cancels out and we obtain for the r_{cut} dependence of the partonic cross section

$$\begin{aligned} \hat{\sigma}_{q\bar{q}}^{\text{IS}}(s; r_{\text{cut}}) &= \sigma_0(s) \frac{\alpha}{2\pi} e_q^2 \left\{ \ln^2 r_{\text{cut}}^2 \right. \\ &\quad - 4 \left(2 \ln 2 - \frac{4}{3} - \ln \frac{1-\beta^2}{\beta^2} - \frac{1}{\beta(3-\beta^2)} \ln \frac{1+\beta}{1-\beta} \right) \ln r_{\text{cut}}^2 \\ &\quad - \frac{3}{2} \frac{(1+\beta^2)(1-\beta^2)^2}{\beta^4(3-\beta^2)} \\ &\quad \times \left(1 - 4 \ln 2 + 2 \ln \frac{(1-\beta^2)r_{\text{cut}}}{\beta^2} \right) r_{\text{cut}}^2 \left. \right\} + \dots \\ &\equiv \hat{\sigma}_{\text{LP}}^{\text{IS}}(s; r_{\text{cut}}) + \hat{\sigma}_{\text{NLP}}^{\text{IS}}(s; r_{\text{cut}}) + \dots \end{aligned} \quad (39)$$

where we have dropped terms which do not depend on r_{cut} and the dots stand for terms that vanish faster than r_{cut}^2 as $r_{\text{cut}} \rightarrow 0$. At variance with Eq. (28), Eq. (39) contains a double and a single logarithmic term in r_{cut} , which will be cancelled by the subtraction counterterm. As expected, the next-to-leading power contribution $\hat{\sigma}_{\text{NLP}}^{\text{IS}}(s; r_{\text{cut}})$ is quadratic in r_{cut} , modulo logarithmic enhancements.

As a byproduct of our calculation, we can reobtain the power suppressed terms in the case of the production of a vector boson of mass M . To get rid of the decay it is enough to carry out the $m \rightarrow 0$ limit, while the constraint on the mass of the vector boson M eliminates the integration over the z variable. By using the expansion in Eq. (34) with $a = 0$ the contributions from the functions $K_3(z; z_{\min})$ and $K_4(z; z_{\min})$ agree with the results in Eq. (4.7) and (4.8) in Ref. [29].

4.3 Numerical validation

In order to check the results presented in Sects. 4.1, 4.2 we have numerically implemented the exact real emission contribution to the cross section and the expansions in Eqs. (28) and (39).

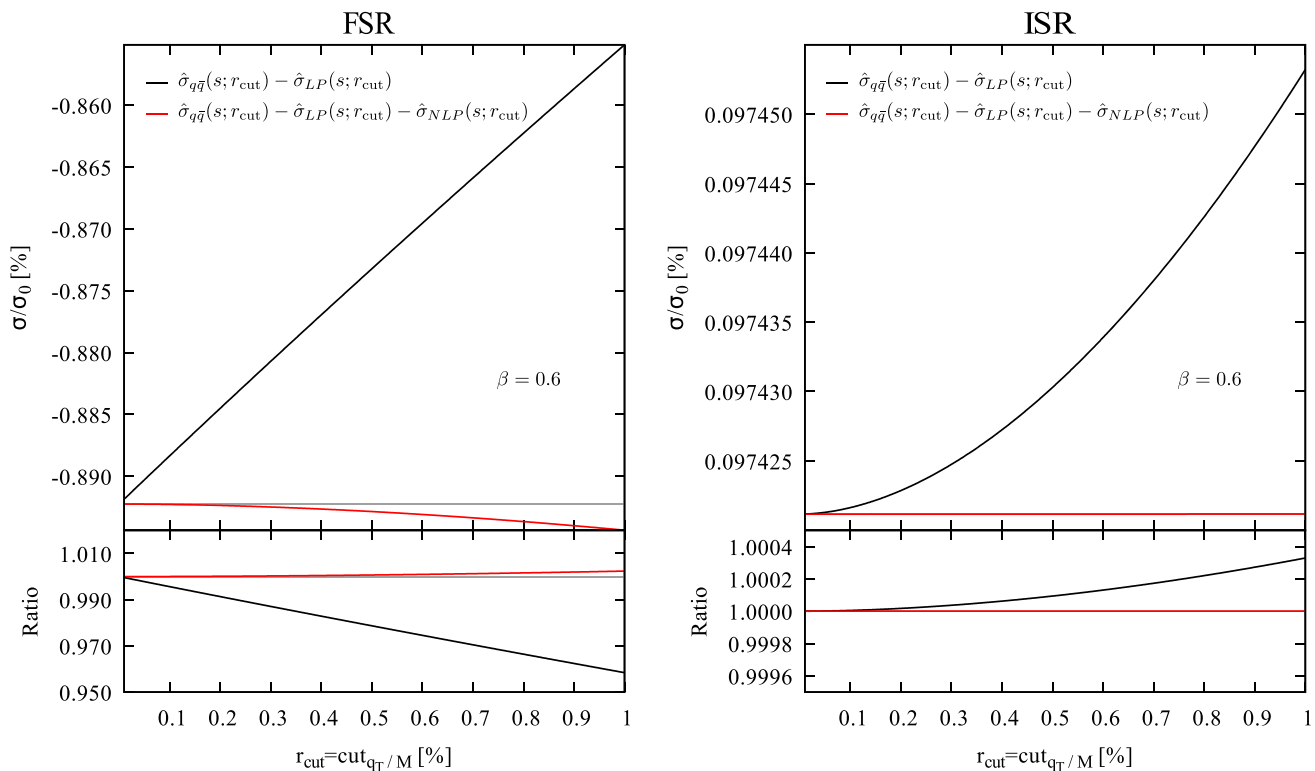


Fig. 4 Subtracted partonic cross section for final-state radiation (left panel) and initial-state radiation (right panel). The solid lines represent the subtraction of the leading-power term, while the red solid line is obtained by subtracting also the next-to-leading power terms in Eqs. (28)

In Fig. 4 we report the exact real emission partonic cross section in the $q\bar{q}$ channel for $\beta = 0.6$ as a function of r_{cut} from which we have subtracted the leading-power contribution (black curve) and both the leading and next-to-leading power contributions (red curve). The numerical computation is separately carried out for the final-state radiation (left panel) and initial-state radiation (right panel) contributions. Both for final-state radiation and initial-state radiation the leading-power contribution exactly matches the divergent behavior of the real emission cross section which is finite in the small- r_{cut} limit. The subtraction of the leading-power contribution exactly corresponds (up to quadratic terms in r_{cut} , see Eq. 16) to the second term on the right hand side of Eq. (8) and it is thus what is usually done in the standard q_T subtraction procedure. In the case of final-state radiation (left panel) the subtracted cross section exhibits the expected linear behavior, while for initial-state radiation (right panel) the subtracted cross section scales quadratically with r_{cut} . When besides the leading-power contribution, also the next-to-leading power (linear) term is subtracted the final-state subtracted cross section (red curve) behaves quadratically with r_{cut} , consistently with the result in Eq. (28). In the case of initial-state radiation, the additional subtraction of the next-

to-leading power (quadratic) term makes the subtracted cross section almost independent on r_{cut} .

4.4 Hadronic cross section

We now briefly comment upon the behavior of the hadronic cross section. In the following, we will show that when the fully inclusive cross section is considered, the convolution with the PDFs potentially introduces an additional linear term in r_{cut} . In the case of final-state radiation such contribution could modify the parton level result. In the case of initial-state radiation such contribution could potentially change the power counting, by making the power correction linear. However, we will argue that, both for final-state and initial-state radiation, such additional term vanishes.

The real contribution to the hadronic cross section reads

$$\sigma(S, r_{\text{cut}}) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \times \hat{\sigma}_{ab}(s, r_{\text{cut}}) \delta(x_1 x_2 S - s) \quad (40)$$

where S is the hadronic CM-energy. The presence of a finite r_{cut} implies that

$$s > \frac{4m^2}{z_{\max}}. \quad (41)$$

where z_{\max} , defined in Eq. (22), behaves linearly with r_{cut}

$$z_{\max} = 1 - 2r_{\text{cut}} + \mathcal{O}(r_{\text{cut}}^2). \quad (42)$$

The hadronic cross section in Eq. (40) can be rewritten as

$$\begin{aligned} \sigma(S, r_{\text{cut}}) &= \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \\ &\quad \times \Theta \left(x_1 x_2 S - \frac{4m^2}{z_{\max}} \right) \hat{\sigma}_{ab} \left(s = x_1 x_2 S, r_{\text{cut}} \right) \\ &= z_0 \sum_{a,b} \int_{z_0}^{z_{\max}} \frac{dz}{z^2} \int_{\ln \sqrt{z_0/z}}^{-\ln \sqrt{z_0/z}} dy \\ &\quad \times f_a \left(\sqrt{\frac{z_0}{z}} e^y, \mu_F \right) f_b \left(\sqrt{\frac{z_0}{z}} e^{-y}, \mu_F \right) \\ &\quad \times \hat{\sigma}_{ab} \left(s = \frac{4m^2}{z}, r_{\text{cut}} \right) \\ &\equiv \sum_{a,b} \int_{z_0}^{z_{\max}} dz \mathcal{L}_{ab}(z, z_0; \mu_F) \hat{\sigma}_{ab} \left(s = \frac{4m^2}{z}, r_{\text{cut}} \right) \end{aligned} \quad (43)$$

where in the last step, we have performed the change of variables

$$x_1 = \sqrt{\frac{z_0}{z}} e^y, \quad x_2 = \sqrt{\frac{z_0}{z}} e^{-y}, \quad z_0 \equiv \frac{4m^2}{S}. \quad (44)$$

The presence of z_{\max} as an upper integration limit in Eq. (43) could potentially induce an additional linear term in r_{cut} when the hadronic cross section is evaluated. However, the partonic cross section vanishes at the kinematical limit $z = z_{\max}$

$$\hat{\sigma}_{ab} \left(s = \frac{4m^2}{z_{\max}}, r_{\text{cut}} \right) = 0. \quad (45)$$

This is a sufficient mathematical condition to prevent the appearance of a further linear term through integration. We thus conclude that, as anticipated, in the case of final-state radiation the linear term in r_{cut} is completely driven by the parton level result, while for initial-state radiation the convolution with PDFs will not produce linear terms in r_{cut} .

4.5 Final-state radiation at next-to-leading power: beyond inclusive observables

In Sect. 4.1, we have established by means of an analytical computation that in the case of final-state radiation off massive emitters the leading power corrections are linear in r_{cut}

and we have explicitly evaluated the coefficient of the linear term. The feasibility of our computation and the methods involved crucially rely on the fact we consider a sufficiently inclusive observable as the total cross section. In the following we propose a way to promote the calculation of the final-state radiation contribution to differential level.

Our starting point is the expansion of the real contribution to the differential cross section in the soft limit. According to the discussion in Appendix A, the leading soft contribution allows us to obtain the leading-power term in r_{cut} , while the next-to-soft contribution will allow us to obtain the next-to-leading power. By inspection of Eq. (27) which involves the derivative of the δ -distribution, we indeed expect that higher order terms in the soft expansion contribute to the next-to-leading power.

In the following we propose a strategy to numerically prove the above result, which in turns provides a procedure to compute the next-to-leading power in a fully differential way. From the soft contributions we can construct a local counterterm which cancels the singularities of the real cross section but does not contribute to the next-to-leading power. Then the subtracted cross section is finite in four dimensions and can be integrated numerically in the unresolved region $r < r_{\text{cut}}$. By construction, since the standard q_T subtraction counterterm does not lead to linear contributions in r_{cut} (see Eq. 16), the combination of the standard q_T subtraction formula for $r > r_{\text{cut}}$ with such an additional subtracted term for $r < r_{\text{cut}}$ will be free of linear terms in r_{cut} .

To construct the additional counterterm we need a mapping which reabsorbs the radiation into a Born-like configuration. Among the available mappings at NLO, we choose the mapping proposed in Ref. [51]. It is a massive FKS [52] mapping dedicated to the case of the radiative emission off final state massive emitters and present some peculiar features:

- the radiation is reabsorbed in such a way not to modify the partonic CM energy;
- the energy of the radiation (in the CM frame), which controls the way the soft limit is approached, appears explicitly among the variables of the mapping.

In the above mapping, we identify an emitter and a radiated parton. The radiation variables are given by the radiation energy fraction $\xi = 2E_{\text{rad}}/\sqrt{s}$ (s is the partonic CM energy), the cosine of the angle between the emitter and the radiated partons y and an azimuthal angle ϕ (we refer to Ref. [51] for more details). The phase space reads

$$d\Phi_R = d\Phi_B \times J(\xi, y, \phi) d\xi dy d\phi \quad (46)$$

where $d\Phi_B$ is the Born phase space element and the jacobian J is given in Eq. (49) of Ref. [51], and reduces to

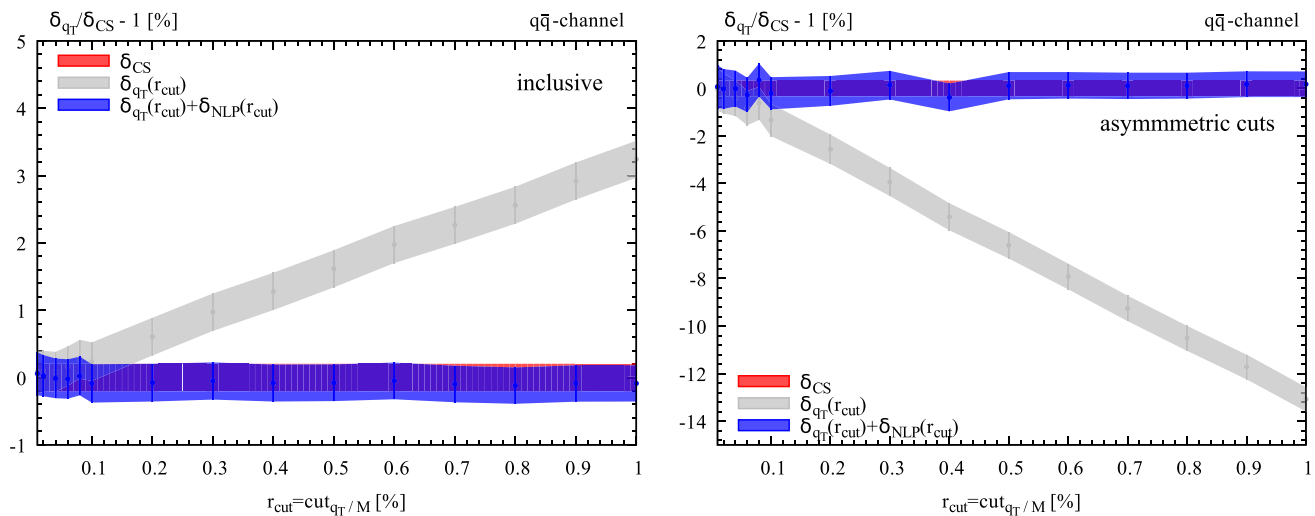


Fig. 5 NLO EW correction as a function of r_{cut} for the complete Drell–Yan process in the dominant $q\bar{q}$ diagonal channel without cuts (left panel) and with asymmetric cuts (right panel) at 7 TeV. The standard result obtained with q_T subtraction (grey band) is compared with

$J^{(0)} = s \xi / (4\pi)^3$ in the soft limit. The local soft counterterm is defined as

$$d\hat{\sigma}_S^{CT} = d\hat{\sigma}_{LO}(\Phi_B) \times \frac{e^2}{4\pi^3 s} \frac{d\xi}{\xi} dy d\phi \times \left[\frac{s - 2m^2}{(1 - \beta y_{\text{phy}})(1 + \beta y_{\text{phy}})} - \frac{m^2}{(1 - \beta y_{\text{phy}})^2} - \frac{m^2}{(1 + \beta y_{\text{phy}})^2} \right] \quad (47)$$

where $\beta = \sqrt{1 - 4m^2/s}$ and y_{phy} is the cosine of the physical angle between the emitted photon and the leptons in the Born configuration (in practice we have either $y_{\text{phy}} = y$ or $y_{\text{phy}} = -y$ [51]). The next-to-leading power correction as function of r_{cut} is obtained by subtracting the new soft counterterm from the real emission contribution in the unresolved region $r < r_{\text{cut}}$

$$d\hat{\sigma}_{\text{NLP}}^{\text{FS}}(r_{\text{cut}}) = d\hat{\sigma}^{\text{FS}} \Theta(r_{\text{cut}} M(\Phi_R) - q_T) - d\hat{\sigma}_S^{CT} \Theta(r_{\text{cut}} \sqrt{s} - q_T) \quad (48)$$

where in the argument of the second theta function, we take the soft limit $M(\Phi_R) \rightarrow M(\Phi_B) = \sqrt{s}$.

The expression in Eq. (48) is fully differential, so that it can be used also in the case in which cuts on the final state are applied. The contribution in Eq. (48) can be combined with the standard q_T subtraction formula in Eq. (8) to obtain an improved subtraction procedure. In Fig. 5 we study the r_{cut} dependence of the NLO EW correction to the complete Drell–Yan process when the q_T subtraction formula is supplemented with the next-to-leading power term in Eq. (48). We consider pp collisions at $\sqrt{s} = 7$ TeV and we compute the r_{cut} dependent correction $\delta_{q_T}(r_{\text{cut}})$ in the case in which no cuts are applied (Fig. 5 (left)) and when asymmet-

ric cuts on the transverse momenta and rapidities are applied: $p_{T,l^-} > 25$ GeV, $p_{T,l^+} > 20$ GeV and $|y_l| < 2.5$ (Fig. 5 (right)). We see that in both cases the linear dependence with r_{cut} is nicely cancelled.³ The crucial point for this additional subtraction to be effective is that the additional counterterm in Eq. (47) scales like $d\xi/\xi$, thereby leading to purely logarithmic contributions in r_{cut} . We have checked that alternative local subtractions which do not fulfill this property do not lead to a cancellation of the linear term.

We conclude this section with few comments on the above results. The subtraction of the linear r_{cut} behavior through Eq. (48) does not require any analytic integration. It just requires an appropriate phase space mapping. The reader may of course argue that there is no need to introduce the modification of Eq. (48) to achieve a smooth cancellation of the soft singularity. Indeed, at NLO one can simply use a local subtraction scheme like FKS or dipole subtraction to carry out the fully differential computation. Nonetheless, the strategy adopted here could prove itself useful when extending the computation to the mixed QCD-EW corrections with the q_T subtraction formalism. In this case, given that we aim at the computation of an effect of the order of few *per mille*, having a quadratic instead of linear r_{cut} behavior could dramatically improve the numerical control of the $\mathcal{O}(\alpha\alpha_s)$ contribution.

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5 Summary

In this paper we have considered an application of the q_T subtraction formalism to the production of massive lepton

³ As discussed in Sect. 3, when symmetric cuts are applied a linear dependence on r_{cut} appears in the contribution from initial-state radiation.

pairs in hadron collisions, and we have used this process as a case study to investigate the power suppressed contributions in the parameter r_{cut} . We have shown that q_T subtraction can be applied to evaluate NLO EW corrections to this process through a straightforward abelianisation procedure from heavy-quark production in QCD. The computation can be carried out for lepton masses as small as the one of the muon without substantial complications, and we have been able to successfully reproduce inclusive and differential results obtained with the numerical program SANC. Our calculation paves the way to possible applications to the computation of mixed QCD-EW corrections [41, 53–56] and to NNLO QED corrections [41] to the Drell–Yan process.

We have then studied the power suppressed contributions to the q_T subtraction formula in the parameter r_{cut} . As is known, in the case of the inclusive cross section, initial state radiation leads to power corrections quadratic in r_{cut} , and we have explicitly evaluated the corresponding NLO coefficient. Generally speaking, linear power suppressed terms arise when cuts on the lepton transverse momenta are applied. We have shown that, even in the case of the inclusive cross section, final state radiation leads to a linear power correction in r_{cut} . We have explicitly computed the coefficient of such linear term in the case of the inclusive cross section. By exploiting the purely soft nature of final state singularities, we have also discussed how the result can be extended to differential distributions. The method used to carry out such extension could be used in future applications of the q_T subtraction formalism at $\mathcal{O}(\alpha\alpha_S)$.

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Appendix A: Soft Power Counting

In this Appendix, we discuss the soft power counting for final-state radiation. In the strictly soft limit, the phase space of the emitted photon with momentum k exactly factorizes

$$d\Phi_3 = d\Phi_2 \times \frac{d^3k}{(2\pi)^3 2k^0}. \quad (\text{A.1})$$

The leading power contribution to final-state radiation is given by the soft-factorisation formula

$$|\mathcal{M}(p_1, p_2, p_3, p_4, k)|_{\text{FSR}}^2 \sim \left(e_3^2 \mathcal{S}_{33} + e_4^2 \mathcal{S}_{44} + 2e_3 e_4 \mathcal{S}_{34} \right) |\mathcal{M}(p_1, p_2, p_3, p_4)|^2 \quad (\text{A.2})$$

where

$$\mathcal{S}_{ij} = \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}. \quad (\text{A.3})$$

The power counting is more easily understood if we consider light-cone coordinates

$$k^\pm = \frac{k^0 \pm k^3}{\sqrt{2}}, \quad d^4k = dk^+ dk^- d^2\mathbf{k}_\perp \quad (\text{A.4})$$

Then, the 1-body phase space volume has the form

$$\begin{aligned} \int \frac{d^4k}{(2\pi)^3} \delta_+(k^2) &= \int \frac{dk^+ dk^- d^2\mathbf{k}_\perp}{(2\pi)^3} \delta_+(2k^+ k^- - \mathbf{k}_\perp^2) \\ &= \frac{1}{(2\pi)^3} \int_0^\infty \frac{dk^+}{2k^+} \int d^2\mathbf{k}_\perp \end{aligned} \quad (\text{A.5})$$

with $k^- = \mathbf{k}_\perp^2 / 2k^+$. Considering for example the contribution from \mathcal{S}_{34} , the leading power unconstrained soft integral is given by

$$\begin{aligned} I_{34}^{\text{soft}} &= \frac{1}{(2\pi)^3} \int_0^\infty \frac{dk^+}{2k^+} \int_0^\infty \frac{dk_\perp^2}{2} \int_0^{2\pi} d\theta \mathcal{S}_{34} \Theta(k_\perp^2 - sr_{\text{cut}}^2) \\ &= \frac{p_3 \cdot p_4}{(2\pi)^3} \int_{sr_{\text{cut}}^2}^\infty \frac{dk_\perp^2}{2} \int_0^\infty \frac{dk^+}{2k^+} \\ &\quad \times \int_0^{2\pi} d\theta \frac{1}{p_\perp^2 k_\perp^2} \frac{1}{(a_3 - \cos\theta)(a_4 + \cos\theta)} \end{aligned} \quad (\text{A.6})$$

$$a_i = \frac{1}{p_\perp k_\perp} \left(p_i^+ \frac{k_\perp^2}{2k^+} + p_i^- k^+ \right). \quad (\text{A.7})$$

In the above formula, we have enforced the soft kinematic with two back-to-back massive leptons. The azimuthal average is straightforward, after disentangling the product occurring in the denominator by means of the partial fractioning relation

$$\frac{1}{(a_3 - \cos \theta)(a_4 + \cos \theta)} = \frac{1}{a_3 + a_4} \times \left(\frac{1}{a_3 - \cos \theta} + \frac{1}{a_4 + \cos \theta} \right), \quad (\text{A.8})$$

and it gives

$$I_{34}^{\text{soft}} = \frac{p_3 \cdot p_4}{(2\pi)^2} \int_{sr_{\text{cut}}^2}^{\infty} \frac{dk_{\perp}^2}{2} \int_0^{\infty} \frac{dk^+}{2k^+} \frac{1}{p_{\perp}^2 k_{\perp}^2} \frac{1}{a_3 + a_4} \times \sum_{i=3,4} \frac{1}{\sqrt{a_i^2 - 1}}. \quad (\text{A.9})$$

To make the scaling with the transverse momentum manifest, we apply the following change of variables at fixed k_{\perp} :

$$x = \left(\frac{k^+}{k_{\perp}} \right)^2, \quad dk^+ = k_{\perp} \frac{dx}{2\sqrt{x}}. \quad (\text{A.10})$$

The soft integral becomes

$$I_{34}^{\text{soft}} = \frac{p_3 \cdot p_4}{(2\pi)^2} \frac{1}{\sqrt{2s}} \int_{sr_{\text{cut}}^2}^{\infty} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^{\infty} \frac{dx}{1+2x} \times \sum_{i=3,4} \frac{1}{\sqrt{4(p_i^-)^2 x^2 + 2(m^2 - p_{\perp}^2)x + (p_i^+)^2}} \quad (\text{A.11})$$

where s is the partonic CM energy.

We can complete the calculation of the leading power contribution by performing the integration over the x variable. The relevant integrals are of the form

$$T(a, b, c) = \int_0^{\infty} \frac{dx}{1+2x} \frac{1}{\sqrt{ax^2 + 2bx + c}} = \frac{1}{\sqrt{a-4b+4c}} \times \ln \left[\frac{-2b+4c+2\sqrt{c}\sqrt{a-4b+4c}}{-a+2b+\sqrt{a}\sqrt{a-4b+4c}} \right] \quad (\text{A.12})$$

under the conditions $b^2 - ac < 0$ and $a, c > 0$. Then, it is straightforward to compute

$$\int_0^{\infty} \frac{dx}{1+2x} \sum_{i=3,4} \frac{1}{\sqrt{4(p_i^-)^2 x^2 + 2(m^2 - p_{\perp}^2)x + (p_i^+)^2}} = \frac{1}{\sqrt{2p}} \ln \frac{1+\beta}{1-\beta}, \quad p = \sqrt{E^2 - m^2}. \quad (\text{A.13})$$

We get the final expression

$$I_{34}^{\text{soft}} = \frac{1}{4(2\pi)^2} \frac{1+\beta^2}{\beta} \ln \frac{1+\beta}{1-\beta} \int_{sr_{\text{cut}}^2}^{\infty} \frac{dk_{\perp}^2}{k_{\perp}^2} \quad (\text{A.14})$$

which exactly matches the coefficient of the leading logarithmic divergence proportional to the charge product $e_3 e_4 = -1$ in Eq. (28). The contributions from I_{33}^{soft} and I_{44}^{soft} can be obtained in a similar way and reproduce the remaining term in Eq. (28). The power counting for the linear power correction follows now easily observing that the energy of the radiation scales with the transverse momentum

$$k^0 = \frac{k^+ + k^-}{\sqrt{2}} = k_{\perp} \sqrt{\frac{x}{2}} \left(1 + \frac{1}{2x} \right). \quad (\text{A.15})$$

This implies that corrections to the soft approximation will produce linear terms in r_{cut} .

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