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Influence of Hydrodynamic Regimes on Mixing of Waters of Confluent Rivers

T. P. Lyubimova[a,b],*, A. P. Lepikhin[b,c],, Ya. N. Parshakova[a],***, C. Gualtieri[d],****, S. Lane[e],*****, and B. Roux[f],********

[a] *Institute of Continuous Media Mechanics, Ural Branch, Russian Academy of Sciences, Perm, 614013, Russia*
[b] *Perm State University, Perm, 614990 Russia*
[c] *Mining Institute, Ural Branch, Russian Academy of Sciences, Perm, 614007 Russia*
[d] *University of Naples Federico II, Naples, Italy*
[e] *Institute of Earth Surface Dynamics, University of Lausanne, Lausanne, CH-1015 Switzerland*
[f] *Aix-Marseille Université and Ecole Centrale Marseille, Marseille, 13007 France*
**e-mail: lyubimovat@mail.ru*
***e-mail: lepihin49@mail.ru*
****e-mail: parshakova@icmm.ru*
*****e-mail: carlo.gualtieri@unina.it*
******e-mail: stuart.lane@unil.ch*
*******e-mail: broux@L3M.univ-mrs.fr*

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Abstract—At present, a significant weakening of the intensity of transverse mixing at the confluence of large rivers, which is observed in a number of cases, is widely discussed. Since the observed features of the confluence of large watercourses are not only of research interest but also of significant economic importance associated with the characteristics of water management at these water bodies, a large number of works are devoted to their study. Water resources management requires measures for the organization of water use which can be rational only under the understanding of processes occurring in water basins. To explain the phenomenon of suppression of the transverse mixing, which is interesting and important from the point of view of ecology, a wide range of hypotheses is proposed, up to the negation of turbulence in rivers. One of the possible mechanisms for explaining the suppression of transversal mixing can be the presence of transverse circulation manifesting itself as Prandtl's secondary flows of the second kind. The characteristic velocity of these circulation flows is very small and difficult to measure directly by instruments; however, in our opinion, they can significantly complicate the transverse mixing at the confluence. The proposed hypothesis is tested in computational experiments in the framework of the three-dimensional formulation for dimensions of a real water object at the mouth of the Vishera River where it meets the Kama. Calculations demonstrate that, at sufficiently large flow rates, the two waters practically do not mix in the horizontal direction throughout the depth over long distances from the confluence. It has been found that a two-vortex flow is formed downstream the confluence, which just attenuates the mixing; the fluid motion in the vortices is such that, near the free surface, the fluid moves from the banks to the middle of the riverbed.

Keywords: confluence of two rivers, secondary flows, three-dimensional numerical modeling, weakening of transverse mixing.

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1. INTRODUCTION

It was noticed long ago that dimensions of zones at confluences of two rivers are different even when their relative dimensions are close to each other [1]. The interest in this problem significantly increased with the availability of space images in which these effects are very demonstrative and available for primary analysis [2].

Confluence of rivers is a quite natural phenomenon. However, in some cases, it significantly deviates from traditional ideas, which can have far reaching consequences for the economic use of the water objects under consideration. One can observe situations in which waters are mixed very intensely; at the same time, in other cases, confluent water streams retain their properties over ten kilometers. Sometimes, water masses in a confluent flow retain high homogeneity over the flow width; however, they are also characterized by inhomogeneity of physical and chemical characteristics over the depth.

In this work, the Kama and Vishera rivers — two large rivers with basins situated in Western Ural — are examples used for the process of confluence. Although the Kama is officially the largest tributary of the Volga, the Europe's longest river, it comes short of water flow rate and river valley age compared to the Vishera at the confluence. The average annual river flow of the Kama upstream its confluence with the Vishera is about 385 m^3/s with a characteristic flow rate of about 144 m^3/s in summer; the average annual river flow of the Vishera is about 508 m^3/s with the summer flow rate of about 267 m^3/s. The confluence zone is about 300 km upstream the Kama hydroelectric plant, i.e., beyond the backwater zone of this hydrotechnical construction. The characteristic hydraulic slope of the bed downstream the confluence of the Kama and Vishera is 8 cm per 1 km of the length of the zone under consideration. Both streams have rather close water densities, which is related to the closeness of their temperature regimes and water mineralization. For the Kama, the mineralization is 80–100 mg/L; for the Vishera, 140–180 mg/L. In connection with the fact that the Kama river passes through a very large wetland before the confluence with the Vishera, it is characterized by a higher content of organic substances and iron; as a consequence, the water color in the Kama is darker than in the Vishera, which is well seen in space and aerial images.

Transverse mixing is caused to a considerable extent by the character and structure of secondary flows. In 1933, Makkaveyev [3], based on works of 1917 and 1926 by Schmidt [4], assumed that mixing of waters in rivers occurred according to the Fick diffusion scheme. Later, that hypothesis was considerably developed and became dominating in Russian hydrology [5]. However, it does not take into account the complex structure of flows in bed streams, which results in anisotropic diffusion. As early as in 1953, Taylor [6] discovered that the nonuniformity of the field of averaged flow velocities could have a very strong effect on the character of the distribution of both longitudinal and transverse diffusion. Prandtl [7] demonstrated that secondary flows called the secondary flows of the second kind could appear even on rectilinear areas, due to the nonuniform distribution of tangential stresses. Although the velocities of these flows are low and amount to ~1% of the velocity of the averaged longitudinal flow, they can have an effect of the displacement of flows.

As follows from materials of field observations, a significant inhomogeneity of physical and chemical characteristics over the flow width is retained through at least 10 km downstream the confluence of the Kama and Vishera rivers, which is corroborated by space images.

According to the simplest K model of diffusion, the characteristic linear scale cane be estimated based on the relationship $L = kVB^2/D_{xx}$, where k is a certain empirical coefficient less than unity (according to [8], $k \approx 0.18$); x is the horizontal coordinate directed transversely to the flow; and V, B, and D_{xx} are the velocity, width, and lateral dispersion typical for the stream interval under consideration. It is well known that the lateral dispersion depends on the collection of the turbulent and Taylor diffusions [8, 9]. These two types of diffusion are of principally different nature: the turbulent diffusion is determined by pulsations of the velocity field; the Taylor diffusion, by nonuniformity of the averaged velocity field over the cross section of the flow [6]. As applied to river flows, the Taylor diffusion was analyzed for the first time in [8].

In the general case, the effective coefficient of lateral dispersion with allowance for the turbulent and Taylor diffusion can be written by the relationship $D_{xx} \approx p_1 V_* B^{\alpha} H^{1-\alpha}$, where V_* is the dynamic velocity of the flow; H and B are the characteristic values of the flow depth and width, respectively; p_1 is an empirical constant; and α is the governing parameter. As a rule, $B/H \gg 1$ and $\alpha > 1$. Substituting the last expression into the formula for the characteristic linear scale L and taking into account that, by definition, $V_*/V = \sqrt{g}/C$, where C is the Chézy coefficient and g is the gravitational acceleration, we obtain $L = k\left(\sqrt{g}/C\right)B^{2-\alpha}H^{\alpha-1}/p_1$.

According to [8], the value taken for α is 0.75. At the same time, according to [9], $\alpha \approx 1.378$. These values of the governing parameter do not make it possible to explain the causes of the observed difference in dimensions of the mixing zones at rather close morphometric indicators of confluent water streams within the framework of the K diffuse approximation.

The problem is much more complicated. It consists of the incorrect usage of the K model for describing the processes under consideration. It is well known that, according to this model, the turbulent flow is parameterized. In a sufficiently rigorous formulation, the relationship $L \sim k\left(\sqrt{g}/C\right)B^{2-\alpha}H^{\alpha-1}/p_1$ is

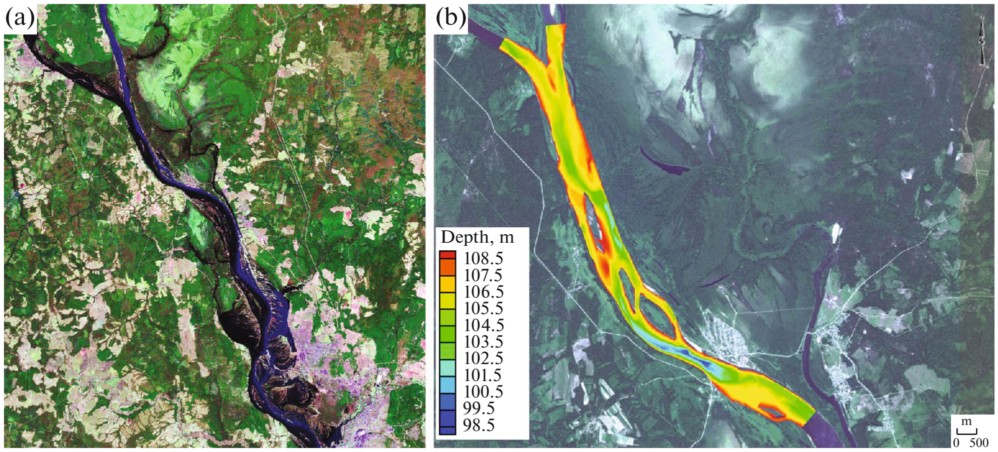

Fig. 1. (a) Cartographical representations of the confluence of the Kama and Vishera and (b) morphometry.

applicable in the case where the averaging scales $L_0 \gg L_p$, where L_p is the characteristic scale of turbulent pulsations. In the problem under consideration, the inequality $L_0 \gg L_p$ is not satisfied and, therefore, there are no grounds for the a priori suitability of the K model of diffusion. In connection with this, it becomes topical to consider alternative models of the interaction between water streams. One of such models is discussed in this work.

In the present-day literature, there are works on studying peculiarities of confluences of rivers with different characteristics. For example, in the case of confluence presented in [10, 11], a tributary with a low depth, about a half meter deep, penetrates downward into the main channel. The tributary is characterized by the presence of a sediment whereas the main flow is almost homogeneous. It was shown that, as a result of confluence with a tributary containing a large amount of rough sediments, the area of the main flow downstream the confluence decreases. This causes an acceleration of the main flow, which favors a faster transport of the sediment.

The confluence of shallow water rivers (with a depth varying from 0 to 3 m) with flows of the same velocity and density was studied in [12]. Results of modeling a turbulent flow and visualization of vortices forming at the interface of waters from two rivers were presented.

In our works [13–15], three-dimensional modeling of hydrodynamic regimes of large water objects was carried out with allowance for the density stratification. In this work, the developed and approbated mathematical apparatus is applied to studying the mechanism of confluence of the two largest rivers of Perm Krai, the Kama and Vishera, to verify the adequateness of the proposed alternative model of the interaction between water streams.

2. FORMULATION OF THE PROBLEM

The zone of confluence of the Vishera and Kama rivers, studied as a water object by the mechanisms of the formation of transverse circulation, was considered (Fig. 1). The salient feature of these rivers is that their flow rates are rather close to each other. At the same time, their waters are significantly different in optical density, which considerably simplifies the observation their mixing processes.

To clarify the mechanism of stream mixing in the area of the confluence, calculations were carried out for a model configuration in the case of two rivers characterized by the same spatial dimensions as the Kama and Vishera but with rectilinear intervals of the riverbed and constant depth (Fig. 2).

The calculations were carried out for an area with an extension of 11 km. The computational domain included river stretches with an extension of 1 km to the confluence and a stretch with an extension of 10 km downstream from the confluence. The width of the riverbeds to the confluence was considered similar and equal to 250 m; the width of the riverbed downstream the confluence was considered equal to 500 m. The depth of the rivers was assumed to be constant through the whole interval under consideration and equal to 8 m. The dependence of the transverse mixing rate on the water flow rates typical for mixing rivers at different seasons was studied. The computational grid in the horizontal direction consisted of tetragonal cells uniformly distributed over the whole length and having a characteristic linear size of 10 m

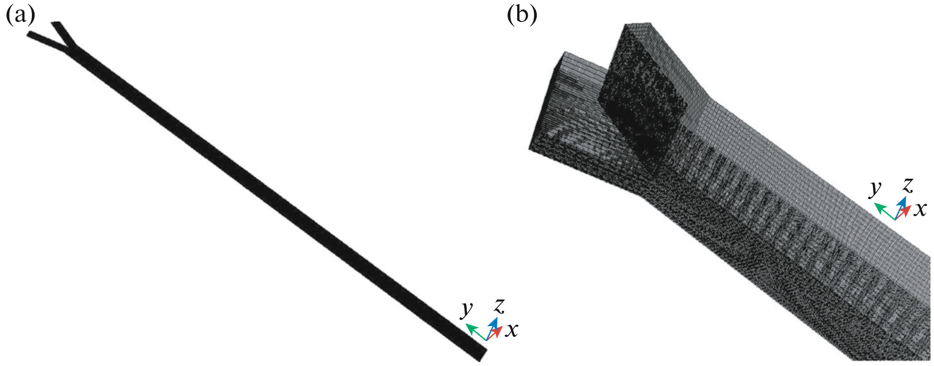

Fig. 2. (a) Computational domain and (b) fragment of the computational grid; for better visualization, the vertical size is magnified by 100 times.

(Fig. 2). Along the vertical, the grid contained 20 nodes positioned at equal distances from each other; the total number of the grid nodes was 1400000.

3. COMPUTATIONAL EXPERIMENT

The numerical simulation was carried out within the framework of the three-dimensional approach. The computations were carried out using the ANSYS Fluent computational hydrodynamics package with the use of the k–ε model describing turbulent pulsations. The problem was solved within the framework of the nonstationary isothermal approach.

The problem formulation included the following equations:

—equations of motion in the tensor form

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho \mathrm{v}_i) = 0, \tag{1}$$

$$\begin{gathered} \frac{\partial}{\partial t}(\rho \mathrm{v}_i) + \frac{\partial}{\partial x_j}(\rho \mathrm{v}_i \mathrm{v}_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \mathrm{v}_i}{\partial x_j} + \frac{\partial \mathrm{v}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial \mathrm{v}_l}{\partial x_l}\right)\right] \\ + \frac{\partial}{\partial x_j}\left[\mu_t\left(\frac{\partial \mathrm{v}_i}{\partial x_j} + \frac{\partial \mathrm{v}_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial \mathrm{v}_l}{\partial x_l}\right)\delta_{ij}\right] + \rho g_i; \end{gathered} \tag{2}$$

—equations for the turbulent kinetic energy and its dissipation rate

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k \mathrm{v}_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k - \rho\varepsilon, \tag{3}$$

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon \mathrm{v}_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}G_k - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}. \tag{4}$$

In Eqs. (1)–(4), the following notation is taken: ρ is the fluid density in the flow; v_i are components of the velocity vector ($i = 1, 2, 3$); μ is the kinematic viscosity; δ_{ij} is the Kronecker delta; the turbulent viscosity μ_t is a function of the turbulent kinetic energy k and its dissipation rate ε: $\mu_t = \rho C_\mu k^2/\varepsilon$; C_μ is a constant; g_i is the gravitational acceleration; $G_k = \mu_t S^2$ is the generation of the turbulent kinetic energy due to the average velocity gradient; $S = \sqrt{2S_{ij}S_{ij}}$ is the norm of the average deformation rate tensor; $S_{ij} = \dfrac{1}{2}\left(\dfrac{\partial \mathrm{v}_j}{\partial x_i} + \dfrac{\partial \mathrm{v}_i}{\partial x_j}\right)$; Pr_t is the turbulent Prandtl number; and $C_{1\varepsilon}$, $C_{2\varepsilon}$, σ_k, and σ_ε are constants.

To estimate the effectiveness of using the k–ε turbulence model, test calculations were carried out using a higher order model, namely, the Reynolds stress model in which seven additional equations for

Reynolds stresses were solved. It has been found that the difference in data obtained by these models does not exceed 5%; in connection with this, further investigations involved the k–ε model.

The admixture was assumed to be passive and its concentration c served as an indicator showing the boundary of the mixing of rivers; the density assumed to be constant, not depending on the mineralization of water: $\rho = \rho_{\text{const}} = 1000.196$ kg/m^3. To take into account its transport, the corresponding equation was solved:

$$\frac{\partial}{\partial t}(\rho c) + \nabla \cdot (\rho \mathbf{v} c) = -\nabla \cdot \mathbf{J}. \tag{5}$$

Here, ∇ is the nabla operator; $\mathbf{J}$ is the admixture diffusion current vector determined by the expression

$$\mathbf{J} = -\rho (D_m + D_t) \nabla c, \tag{6}$$

where D_m is the molecular diffusion coefficient, D_t is the effective turbulence diffusivity coefficient connected with the turbulent viscosity μ_t by the relationship $D_t = (\mu_t/\rho)/\mathrm{Sc}_t$; here, Sc_t is the Schmidt turbulent number.

Equations (1)–(6) were complemented by the following boundary conditions:

—the lower and lateral boundaries of the computational domain imitating the river bottom and river banks, respectively, were assumed to be hard; the no-slip and impermeability conditions for the substance were imposed on them:

$$\mathrm{v}_1 = \mathrm{v}_2 = \mathrm{v}_3 = 0, \quad \frac{\partial c}{\partial n} = 0; \tag{7}$$

—the upper boundary of the domain corresponding to the free surface of the fluid was assumed to be nondeformable; on this boundary, the conditions of the absence of the normal velocity component, tangential stresses, and admixture flow were assumed to be satisfied:

$$(\mathbf{vn}) = 0, \quad \frac{\partial \mathrm{v}_x}{\partial x_z} + \frac{\partial \mathrm{v}_z}{\partial x_x} = 0, \quad \frac{\partial \mathrm{v}_y}{\partial x_z} + \frac{\partial \mathrm{v}_z}{\partial x_y} = 0, \quad \frac{\partial c}{\partial \mathbf{n}} = 0; \tag{8}$$

—at the exit from the computational domain, conditions of the mass balance and zero diffusion flow for all variables of the flow were imposed;

—at the entrances into the computational domain, we took similar, constant over the whole cross section, main flow velocities $\mathrm{V}_{j=1,2}$ having a single nonzero component and background concentrations of the admixture in the rivers

$$\begin{aligned} &\text{riverbed}_1: \quad \mathrm{v}_1^2 + \mathrm{v}_2^2 = \mathrm{V}_1^2, \quad \mathrm{v}_3 = 0, \quad c = C_1, \\ &\text{riverbed}_2: \quad \mathrm{v}_1^2 + \mathrm{v}_2^2 = \mathrm{V}_2^2, \quad \mathrm{v}_3 = 0, \quad c = C_2. \end{aligned} \tag{9}$$

For the initial conditions of problem (1)–(9), the uniformly distributed background concentration of the admixture $C_{\text{eff}} = 0$ and constant velocity of the main flow $\mathbf{v} = 0$ were specified at internal nodes of the computational domain.

Values of the parameters Pr_t, Sc_t, $G_{1\varepsilon}$, $C_{2\varepsilon}$, C_μ, σ_k, and σ_ε were taken as follows [16]: $\mathrm{Pr}_t = 0.85$; $\mathrm{Sc}_t = 0.7$; $C_{1\varepsilon} = 1.44$; $C_{2\varepsilon} = 1.92$; $C_\mu = 0.09$; $\sigma_k = 1.0$; and $\sigma_\varepsilon = 1.3$. The kinematic viscosity was taken to be $\mu = 9.34 \times 10^{-7}$ m^2/s and the molecular diffusion coefficient $D = 1.0 \times 10^{-9}$ m^2/s.

For the spatial discretization of the equations, the second order accuracy scheme was used. The time evolution was modeled by the explicit scheme of the second order.

4. RESULTS OF THE COMPUTATIONAL EXPERIMENT

Vector fields of the velocity and fields of the contaminant concentration have been obtained in several transverse cross sections (at different distances from the confluence of rivers). Calculations demonstrate that waters of the rivers after the confluence almost do not intermix in the horizontal direction over the whole depth at sufficiently high rates through large distances (Fig. 3). Such behavior below the confluence is explained by the formation of a two-vortex flow in the transverse cross section (Fig. 4a). The fluid motion in the vortices is such that the fluid near the free surface moves from shores to the middle of the riverbed. With an increase in the distance from the confluence zone, the intensity of secondary vortices decreases (Fig. 4b) and degradation of the interface is observed.

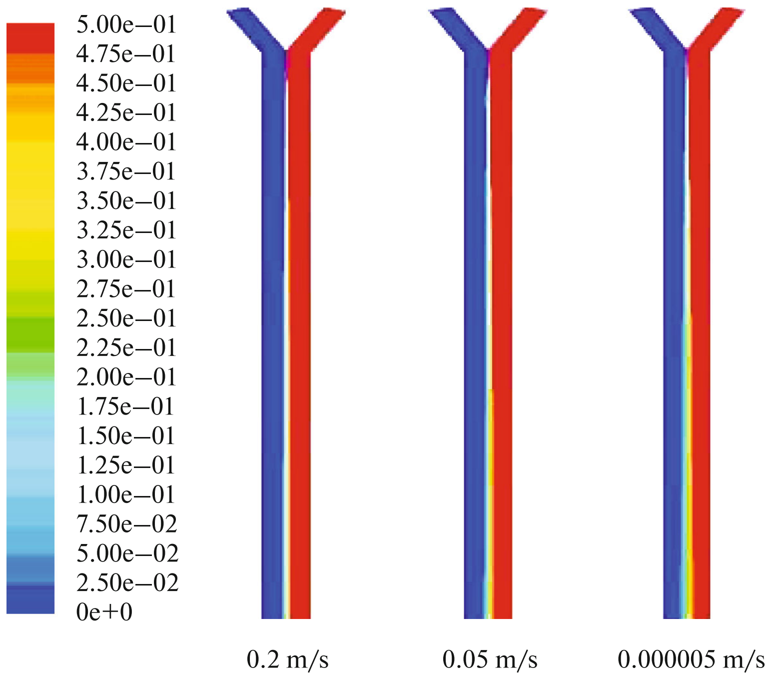

Fig. 3. Fields of the contaminant concentration at the lower boundary of the computational domain at different velocities of the confluent rivers. The extension of the river bed after the confluence is 10 km. The spatial scale along the river bed is decreased by two times.

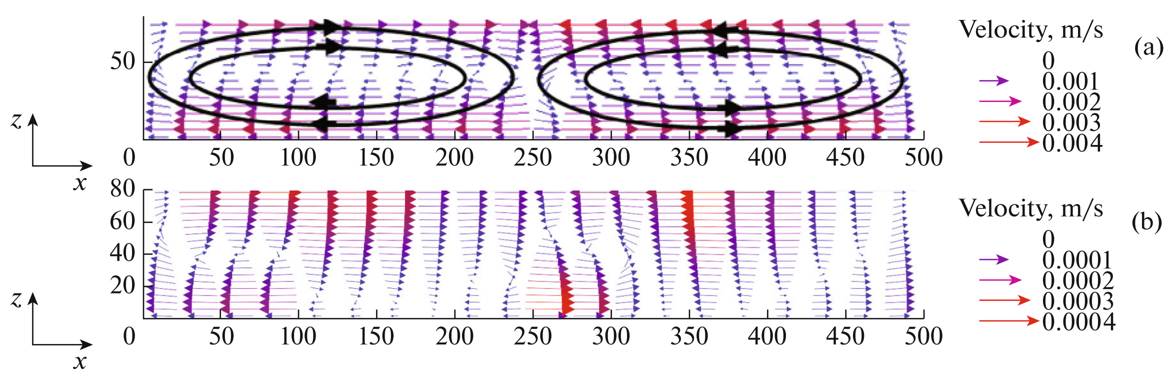

Fig. 4. Structure of the secondary flow in the transverse cross section for the flow velocity of waters of the confluent rivers of 0.2 m/s at different distances from the confluence, m: (a) 700 and (b) 10 000; for better visualization, the vertical size of the grid is magnified by 10 times.

In particular, for the flow velocity of waters of confluent rivers of 0.2 m/s, the maximum velocity of the transverse flow is 0.04 m/s at a distance of 200 m from the confluence and 0.004 m/s at a distance of 700 m; for the flow velocity of waters of confluent rivers of 0.05 m/s, the maximum velocity of the transverse flow is 0.001 m/s at a distance of 200 m from the confluence and 0.0004 m/s at a distance of 700 m. As a result, at low intensities of water flows in the rivers, the degradation of the interface between flows is observed at lesser distances from the confluence (Fig. 5).

5. CONCLUSIONS

This work is devoted to studying the influence of hydrodynamic regimes on processes of transverse mixing of waters at the confluence of rivers. The zone where the Vishera meets the Kama served as a model area on which mechanisms of mixing were studied. The salient feature of these water objects are water flow rates rather close to each other. At the same time, their waters are significantly different in opti-

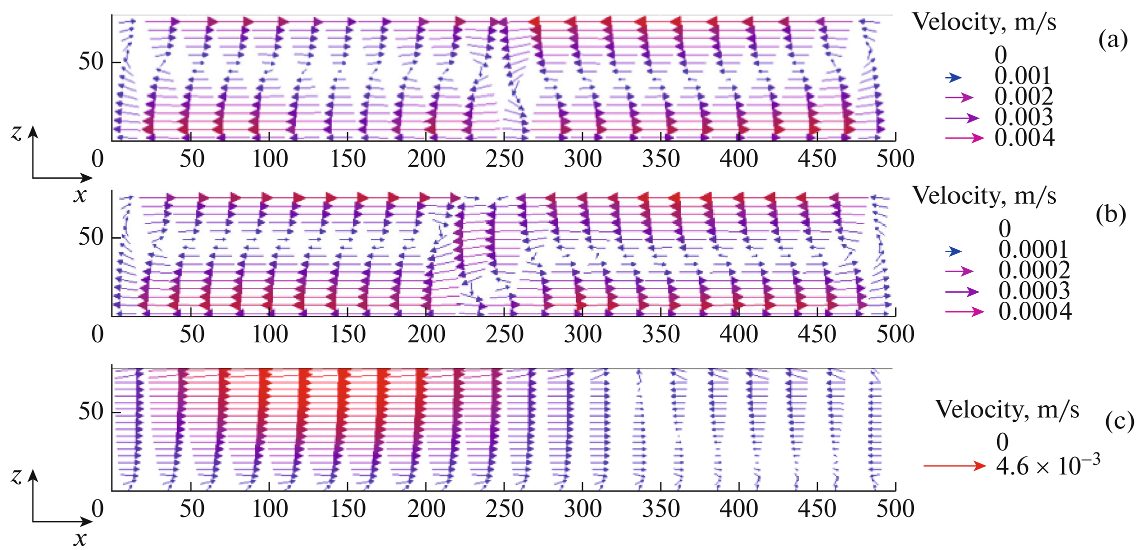

Fig. 5. Vector fields of the velocity in the vertical cross section of waters of confluent rivers at a distance of 700 m from the confluence at different velocities, m/s: (a) 0.2; (b) 0.05; and (c) 0.000005; for better visualization, the vertical size of the grid is magnified by 10 times.

cal density, which considerably simplifies visualization of mixing processes. The confluence of the rivers is upstream the largest water consumer in the basin of the Volga River—the mining industry complex of the Solikamsk–Berezniki industrial hub mining the Verkhnekamskoe field of potassium and magnesium ores, one of largest in the world.

Three-dimensional numerical simulation demonstrates that stable transverse secondary vortex structures can be formed in the zone of confluence of two rivers with close densities and chemical composition of waters. In spite of the fact that the intensity of this secondary flow is much lower than the intensity of the longitudinal flow, it can lead to a significant attenuation of the transverse mixing. At the same time, if secondary structures do not arise or if their formation is blocked, waters of confluent flows are mixed more intensely.

Thus, the formation of the so-called secondary flows of the second kind which are stable transverse vortex structures have been successfully reproduced and visualized for the first time within the framework of a computational experiment in the three-dimensional formulation on a model with dimensions of a real large water object. The presence of such structures allows one to explain the considerable attenuation of the intensity of the transverse mixing observed at confluences of some rivers. The main factors of the formation of secondary structures are peculiarities of the morphometry of the area under consideration and its hydrological regime.

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