Predicted and empirical radii of RR Lyrae stars

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drafted November 9, 2018 / Received / Accepted

ABSTRACT

We present new theoretical Period-Radius-Metallicity (PRZ) relations for RR Lyrae stars. Current predictions are based on a large set of nonlinear, convective models that cover a broad range of chemical abundances and input parameters. We also provide new and homogeneous estimates of angular diameters for a sample of field RR Lyrae stars using a recent calibration of the Barnes-Evans surface brightness relation. Predicted and empirical radii are, within the errors, in reasonable agreement, but in the short-period range the latter present a larger scatter. As a working hypothesis we suggest that this discrepancy might be due to the occurrence either of nonlinear features such as bumps or a steep rising branch. New distance determination for RR Lyr itself is in very good agreement with HST trigonometric parallax and with pulsation parallax.

Subject headings: stars: evolution – stars: horizontal branch – stars: oscillations – stars: variables: others

1. Introduction

RR Lyrae stars are widely adopted not only as tracers of old, low-mass stellar populations but also as standard candles to estimate Galactic and extragalactic distances. They are ubiquitous across the Galaxy and they have been detected in all stellar systems that host a well-defined old population. This means that they can be adopted to constrain the intrinsic accuracy of current primary distance indicators, such as classical Cepheids, Tip of the Red

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Giant Branch, and Main Sequence fitting. These are the reasons why a countless number of theoretical and empirical investigations have been devoted to the RR Lyrae distance scale (Caputo et al. 1999; Bono et al. 2002,2003; Cacciari & Clementini 2003; Walker 2003; Catelan, Pritzl, & Smith 2004; Gratton et al. 2004).

Even though a paramount observational effort has been devoted to obtain Baade-Wesselink (BW) distances to a sizable sample of cluster and field RR Lyrae (Cacciari et al. 1989; Clementini et al. 1990; Carney et al. 1992, hereafter C92; Jones et al. 1992; Storm et al. 1994a,b) we still lack detailed empirical and theoretical Period-Radius (PR) relations for RR Lyrae stars. Note that radii are a by product of the BW method. There is only one exception, Burki & Meylan (1986) derived, using BW measurements, an empirical PR relation for Type II Cepheids that according to the authors could also be applied to RR Lyrae stars.

The same outcome applies to theoretical models. Pulsation properties of RR Lyrae stars have been investigated using both linear and nonlinear models, we still lack detailed theoretical predictions. To fill this gap we present new theoretical PR relations for RR Lyrae stars based on a detailed and homogeneous set of nonlinear pulsation models that cover a wide range of stellar masses and chemical compositions. We also investigate the dependence of the PR relation on metallicity and compare theoretical predictions with empirical radius estimates.

2. Predicted and Empirical radii of RR Lyrae

During the last few years we have been developing an homogeneous theoretical scenario for RR Lyrae stars by constructing an extensive grid of nonlinear, convective models (Bono et al. 1997, 2001, 2003). These models cover a wide range of stellar masses $(0.53 \leq M/M_{\odot} \leq 0.80)$, and chemical compositions $(0.24 \leq Y(\text{He abundance}) \leq 0.28; 0.0001 \leq Z \text{ (metal abundance}) \leq 0.02)$. The physical and numerical assumptions adopted in the model computations have already been discussed in previous papers (Bono & Stellingwerf 1994; Bono et al. 1997; Bono, Castellani, & Marconi 2000). Note that current nonlinear, convective models predict pulsation observables such as the variation along the pulsation cycle of luminosity, radius, velocity, and temperature to be compared with actual empirical data. We estimated for each model the mean radius as a time average over the predicted surface radius curve. Figure 1 shows predicted radii for fundamental (F) pulsators, the solid line displays the linear regression over the entire set of models:

$$\log R = 0.90(\pm 0.03) + 0.65(\pm 0.03) \log P \qquad \sigma = 0.03$$

where R is the mean RR Lyrae radius (solar units), P the pulsation period (days), and σ the intrinsic dispersion. For comparison Fig. 1 also shows the empirical PR relation (dashed line) for Type II Cepheids derived by Burki & Meylan (1986). The vertical error bar is the standard deviation of the predicted PR relation. The agreement between the empirical relation and current predictions is, within the intrinsic dispersion, quite good. This evidence supports the suggestion by Burki & Meylan concerning the similarity between the PR relation of Type II Cepheids and RR Lyrae. However theoretical radii present, at fixed period, a substantial spread along the best-fit line, thus strongly suggesting the dependence of the PR relation on a *second parameter*.

To further improve the intrinsic accuracy of the predicted PR relation we accounted for metal abundance. The dependence on this parameter is expected, since both theory and observations support the evidence that the mean magnitude of RR Lyrae stars depends on the metal content. We performed a linear regression over the entire set of F models and we found the following Period-Radius-Metallicity (PRZ) relation:

$$\log R = 0.774(\pm 0.009) + 0.580(\pm 0.007)\log P - 0.035(\pm 0.001)\log Z \qquad \sigma = 0.008$$

where the symbols have their usual meaning. Data plotted in the bottom panel of Fig. 2 show that the inclusion of the metallicity term causes a decrease in the intrinsic dispersion from 0.03 to 0.008 dex. To supply a homogeneous theoretical scenario for RR Lyrae radii we also estimated the PRZ relation for first overtone (FO) pulsators (see top panel of Fig. 2), and we found:

$$\log R = 0.883(\pm 0.004) + 0.621(\pm 0.004)\log P - 0.0302(\pm 0.001)\log Z \qquad \sigma = 0.004$$

The reason why we estimated an independent PR relation for FOs is twofold: i)) the width in temperature of the FO instability strip is narrower when compared with the F one. This means that the FO PR relation presents a smaller intrinsic dispersion when compared with the F one. ii) FO pulsators are systematically hotter than F ones. This means that the predicted FO PR relation is marginally affected by uncertainties in the treatment of convection.

To validate current predictions we collected a sample of RR Lyrae stars for which are available accurate photometric and spectroscopic data, namely the BW sample (Bono et al. 2003, hereafter B03). From these, we calculated angular diameters using the latest calibration (Nordgren et al. 2002) of the Barnes-Evans surface brightness relation (Barnes & Evans 1976). Together with radial velocities from spectroscopic data, we determined linear radii (and distances) for selected RR Lyrae (see Table 1). While all of these stars already had radii determined by various authors (Jones et al. 1992, C92, and see references in Table 1) this new sample has the benefit of uniformity and the most recent stellar interferometric angular diameter measurements.

We compiled published V and K photometry for the stars in Table 1 (see references in column 9). The mean values of these V and K magnitudes agree with those in B03 to within an average of 0.04 magnitudes (about 0.4% given an average V and K magnitude of 10). We performed dereddening of the photometric data using values for E(B-V) provided in Table 1, and extinction correction constants from Cardelli et al. (1989). We used both linear interpolation and polynomial-fitting in order to calculate K, and thus (V-K) values at those pulsation phases with V photometry. For RR Lyr itself, we used V and (V-R) photometry. We calculated angular diameters as a function of pulsation phase for stars in Table 1 using equations (1), (2), (5) and (6) of Nordgren et al. (2002) we obtain:

$$\log \theta = 0.5734 - 0.2V + 0.246(V - K)$$
$$\log \theta = 0.5914 - 0.2V + 0.730(V - R)$$

where θ is the angular diameter in milliarcseconds (mas). This version of the Barnes-Evans relation was calibrated using interferometric angular diameter observations of 57 nonvariable giant stars. Where available, angular diameters were calculated using V and (V-K) pairs, as opposed to V and (V-R), as the former relation has been shown to yield more precise results (Fouqué & Gieren 1997). Using polynomial fitting (with polynomial orders ranging from 9 to 11) we calculated radial velocities from spectroscopic measurements at the same pulsation phases as the calculated angular diameters (see references in Column 9 of Table 1). We calculated linear displacements of the stellar surface from:

$$\Delta R = -p \int (V_r - V^*) d\phi$$

where V^* is the radial velocity of the center of mass of the star and p is the pulsation projection factor. The value of V^* was found by integrating the V_r curve over the entire phase cycle (phase = 0 - 1), and demanding that $\Delta R_{0-1} = 0$. For all but RS Boo, our value of V^* agrees within the uncertainties with those published by Beers et al. (2000). For the pulsation factor, Fernley (1994) argues for p = 1.38 for field RR Lyrae stars and so we have used this value. The linear radius (R_o) and distance (d) for the star is found from $R_o + \Delta R = 1000d(\theta/2)$, where the angular diameter θ is in mas and the linear radius and radius displacement are in AU (yielding the distance, d, in parsecs). From this equation the radii (and distances) in Table 1 were calculated using a least-squares fit to our calculated angular diameters and linear displacements.

The above equation for ΔR shows that uncertainties in V^* will be propagated into the uncertainty in radius and distance. For each individual star this is a systematic error, but it is an error that is random from star to star within the sample. The uncertainties in radius and distance in Table 1 are the random errors, with the additional systematic error due to V^* given in parentheses. The total error for each star is the quadrature sum of the two errors. The comparison of random and systematic errors in Table 1 supports the importance of accurate radial velocity measurements in BW type analyses.

It should also be noted that there is an uncertainty in the value for the pulsation projection factor p. As the linear radius displacement scales linearly with p, so too will the final radius and distance. For instance, if one were to use p = 1.30 (Jones, Carney & Latham, 1988a,b) instead of our value of 1.38, all radii and distances in Table 1 will be smaller by a factor of 1.38/1.30 = 1.08. For each star in Table 1, our new empirical radius estimate is in agreement, within the errors, to that found by the authors of the original published photometry and radial velocities (see references in Table 1). This includes the star with the largest estimated radius, SS Leo. Fernley et al. (1990) estimated a radius of $6.63 R_{\odot}$, while Jones et al. (1992) estimate a radius of $7.32 R_{\odot}$, both of which, without better knowledge of their uncertainties, are in general agreement with our estimate of $7.2 \pm 0.4R_{\odot}$.

Figure 2 also shows the comparison between predicted and new empirical (open circles) radius estimates. Triangles display the radius estimates provided by C92. Homogeneous radius determinations for a larger sample of BW RR Lyrae with accurate radial velocity measurements will be provided in a forthcoming investigation. Observed radii have been plotted using the homogeneous compilation of metal abundances provided by Fernley et al. (1998) and listed in column 3) of Table 1. The reader interested in a detailed discussion concerning the metallicity measurements and the metallicity scale of RR Lyrae stars is referred to Dall'Ora et al. (2004) and to Gratton et al. (2004). Data plotted in Fig. 2 indicate that empirical radius estimates are affected by large scatter. However, theory and observations are, within current uncertainties, in reasonable agreement for periods longer than 0.42 days. The radius measurements by C92 do not include individual error estimates, and therefore, it is not clear whether the three objects with P > 0.63 days present a real discrepancy. However, observed radii show a larger scatter when moving toward shorter periods. The reason for this drift is not clear, however four shorter period RR Lyrae present

a well-defined bump along the decreasing branch and a steep rising branch in both light and velocity curves (RS Boo, TW Her, Jones et al. 1988b; V445 Oph, Fernley et al. 1990; W Crt, Skillen et al. 1993). Moreover, DH Peg is a FO RR Lyrae.

3. Discussion and final remarks

Recent improvements in optical and infrared interferometry have allowed for direct evaluation of the accuracy of radii and distances determined by BW analyses of pulsating stars, in particular Cepheid variables (Nordgren et al. 2002; Lane et al. 2002; Kervella et al. 2004). Since no RR Lyrae star currently has an angular diameter directly measured by interferometry, such a comparison of the accuracy of our RR Lyrae radii is not possible. However, one may compare the distances calculated using our method as a test of the accuracy of our surface brightness relation, provided there is a known distance to any of the stars in our sample. Alone of the stars in Table 1, RR Lyr has a distance known from high precision trigonometric parallax observations. Benedict et al. (2002, hereinafter B02) used the Hubble Space Telescope Fine Guidance Sensor to obtain a parallax of $\pi = 3.82 \pm 0.2$ mas yielding a distance of 262 ± 14 pc. As a check of our surface brightness analysis we compare this relatively model independent distance to the distance calculated in Table 1: 270 ± 35 pc (this uncertainty is the quadrature sum of the listed random and systematic uncertainties). Our distance is in excellent agreement with the HST distance and with the distance obtained using the K-band Period-Luminosity-Metallicity $(PLZ_{\rm K})$ relation $(260 \pm 5 \text{ pc})$ obtained by B03. This agreement gives us confidence in the accuracy of surface brightness relations and their results. It should be noted that no horizontal branch stars were included in the calibration of the surface brightness relation used here (Nordgren et al. 2002). That RR Lyrae radii determined from this calibration agree so well with the theoretically computed radii argues that the surface brightnesses of horizontal branch stars may be well computed from calibrations based on giant stars.

According to this evidence, we compared current distance determinations with distances estimated using the RR Lyrae visual magnitude metallicity relation ($M_V vs [Fe/H]$) provided by B02. Data plotted in the top panel of Fig. 3, show that the relative difference is within an average of 10%. As expected, the discrepancy is significantly larger in the shortperiod range. To constrain the intrinsic accuracy of current distances, the bottom panel of Fig. 3 shows the relative difference with the distances based on the PLZ_K relation. A glance at the data plotted in this panel confirms the quoted result.

Current findings suggest that predicted and observed radii of RR Lyrae stars are in reasonable agreement. The accuracy of empirical estimates do not allow us to constrain the plausibility of nonlinear, convective RR Lyrae models. Needless to say, that this analysis shall be extended to the entire sample of cluster and field RR Lyrae stars for which are available accurate spectroscopic and photometric (optical, NIR) measurements. In the future, more precise trigonometric and pulsation parallaxes together with new angular diameter measurements, will certainly improve the observational scenario not only for radii and distances but also for the pulsation factor p.

It is a pleasure to thank an anonymous referee for his/her positive comments and suggestions. This work was partially supported by PRIN 2003 and INAF 2003.

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Table 1.	Selected	Baade-	Wesselink	RR Lyrae
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Star	Period (days)	$[\mathrm{Fe}/\mathrm{H}]$	E(B-V)	V^{*a}	Phase	Distance (pc)	$\begin{array}{c} \text{Radius} \\ (\text{R}_{\odot}) \end{array}$	Ref.
$\rm DH~Peg^b$	0.2555	-1.24	0.08	-71 ± 1	0.0-0.8	$470 \pm 60(50)$	$3.8 \pm 0.5(0.5)$	е
RS Boo	0.3773	-0.36	0.02	-4 ± 2	0.35-0.8	$770 \pm 20(120)$	$4.1 \pm 0.1(0.7)$	g
TW Her	0.3996	-0.69	0.05	-5 ± 2	0.1 - 0.9	$1150 \pm 35(100)$	$4.2 \pm 0.1(0.5)$	g
V445 Oph	0.4000	-0.19	0.27	-18 ± 1	0.0 - 1.0	$700 \pm 15(30)$	$4.4 \pm 0.1(0.3)$	h
W Crt	0.4120	-0.54	0.05	59 ± 1	0.35-0.8	$1170 \pm 25(90)$	$3.7 \pm 0.1(0.4)$	c,d
UU Vir	0.4756	-0.87	0.03	-8 ± 1	0.1 - 0.9	$880 \pm 20(40)$	$4.6 \pm 0.2(0.4)$	g
BB Pup	0.4805	-0.64	0.10	130 ± 1	0.0 - 1.0	$1520 \pm 70(30)$	$4.3 \pm 0.2(0.1)$	c,d
RR Lyr	0.5668	-1.39	0.07	-72 ± 1	0.0 - 1.0	$270 \pm 25(25)$	$5.2 \pm 0.5(0.5)$	i,f
RV Oct	0.5711	-1.71	0.13	136 ± 1	0.0 - 0.85	$960 \pm 20(40)$	$5.3 \pm 0.2(0.2)$	c,d
WY Ant	0.5743	-1.48	0.05	204 ± 1	0.2 - 0.9	$1120 \pm 35(60)$	$5.7 \pm 0.2(0.4)$	c,d
SS Leo	0.6263	-1.50	0.01	161 ± 1	0.0-1.0	$1620 \pm 40(70)$	$7.2 \pm 0.2(0.4)$	h

^aMean radial velocity (kms⁻¹). ^b First overtone pulsator. References: (c) Skillen et al. (1993a); (d) Skillen et al. (1993b); (e) Jones, Carney & Latham (1988a); (f) Manduca & Bell (1981); (g) Jones, Carney & Latham (1988b); (h) Fernley et al. (1990); (i) Wilson (1953).



Fig. 1.— Predicted radii for fundamental RR Lyrae models as a function of the logarithmic period. Solid line shows the predicted linear regression over the entire set of models, while the dashed one the empirical PR relation for Type II Cepheids derived by Burki & Meylan (1986). The vertical error bar plotted in the left top corner shows the large intrinsic dispersion of predicted radii.



Fig. 2.— Period-Radius-Metallicity relation for first overtone (top) and fundamental (bottom) RR Lyrae models projected onto a two-dimensional plane. Filled and open circles display predicted and new radius estimates, while triangles the radius determinations by Carney et al. (1992). Empirical radii have been plotted assuming metal abundances by Fernley et al. (1998) and $Z_{\odot} = 0.02$. Individual error bars account for both random and systematic errors.



Fig. 3.— Top panel - Relative difference between current distances and distances estimated using the calibration of the $M_V vs[Fe/H]$ relation provided by B02. Individual error bars account for both random and systematic errors. The period of DH Peg (open circle) was fundamentalized. Bottom panel - Same as the top, but with distances estimated using the $PLZ_{\rm K}$ relation provided by B03.