This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2022.3205268

User Pairing and Power Allocation for IRS-Assisted NOMA Systems with Imperfect Phase Compensation

Pavan Reddy M., Abhinav Kumar

***Abstract*—In this letter, we analyze the performance of the intelligent reflecting surface (IRS) assisted downlink non-orthogonal multiple access (NOMA) systems in the presence of imperfect phase compensation. We derive an upper bound on the imperfect phase compensation to achieve minimum required data rate for each user. Using this bound, we propose an adaptive user pairing algorithm to maximize the network throughput. We then derive bounds on the power allocation factors and propose power allocation algorithms for the paired users to achieve the maximum sum rate or ensure fairness. Through extensive simulations, we show that the proposed algorithms significantly outperform the state-of-the-art algorithms in the presence of phase imperfections.**

***Index Terms*—Intelligent-reflecting surfaces (IRS), non-orthogonal multiple access (NOMA), power allocation, spectral efficiency, user pairing.**

I. Introduction

Non-orthogonal multiple access (NOMA) is considered as a key radio access technique for fifth-generation (5G) and beyond 5G networks [1]. In NOMA, the users are allocated the same time and frequency resources but are multiplexed across the power domain to achieve multi-fold improvement in the network capacity. At the receiver side of NOMA systems, the successive interference cancellation is employed to decode the transmitted data. Similar to NOMA, intelligent reflecting surface (IRS) is another key technology to improve coverage for the beyond-5G networks [2]. An IRS consists of a large number of passive antenna elements where the reflection from each antenna is controlled to direct the signal towards a particular user. Note that, unlike the spatial multiplexing scenario, NOMA is preferred in situations where the channel vectors of users are in the same direction [3]. However, this is not always possible in conventional wireless systems, whereas, in the case of IRS-assisted systems, the network operator can control direction of the user channel vectors by tuning the IRS [3]. Motivated by this, IRS has been analyzed along with NOMA to achieve better network capacity and enhanced coverage [2]–[5].

The practical IRS systems have imperfections in the phase control because of hardware limitations and channel estimations errors [6], [7]. These imperfections in the phase compensation have a significant impact on the data rates observed by the users. However, limited works in the literature consider these imperfections while analyzing the network performance [3], [6]. In [3], the authors have proposed a novel design for IRS-assisted NOMA transmissions and have analyzed the impact of hardware impairments. In [7], the authors have presented a joint optimal training sequence and reflection pattern for IRS systems to minimize the mean squared error of the channel estimation. In [6], the authors have evaluated the performance of orthogonal multiple access (OMA) systems in the presence of imperfect phase compensation. In [8]–[12], the authors have shown that network performance of the NOMA systems is heavily dependent on the user pairing. Hence, IRS-assisted NOMA systems have to consider the imperfections in the phase compensation while pairing the users. Otherwise, the enhanced network throughputs will not be realized in practice. To the best of our knowledge, none of the existing works in the literature have proposed user pairing and power allocation for IRS-assisted NOMA systems with imperfect phase compensation.

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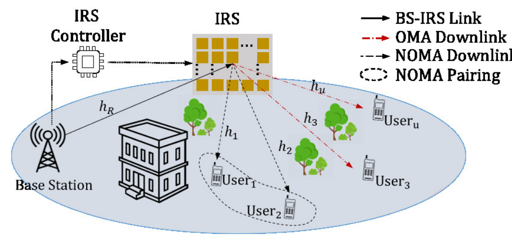


Fig. 1: System Model.

In view of the aforementioned details, this letter presents the first work that discusses the following contributions.

- We derive bounds on the power allocation factors and the imperfect phase compensation to achieve minimum required data rates in IRS-assisted NOMA systems.
- Using the derived bounds, we propose adaptive user pairing algorithm for IRS-assisted downlink NOMA systems.
- We then derive optimal power allocation factors that maximize the achievable sum rate (ASR) or ensure fairness.
- Through numerical evaluation, we validate the derived bounds and show that the proposed adaptive user pairing algorithm and power allocation factors significantly outperform the state-of-the-art algorithms.

The organization of the paper is as follows. We present system model in Section II. In Section III, we derive various bounds on the imperfect phase compensation and power allocation factors. We propose adaptive user pairing and power allocation algorithms in Section IV. In Section V, we present numerical results for various scenarios. We then provide concluding remarks and directions for future work in Section VI.

II. System Model

We consider a base station (BS) with M antennae and an IRS with N antennae, where IRS is activated by a controller connected to the BS as shown in Fig. 1. The channel coefficients between the BS to IRS and i^{th} user to IRS are denoted by $\mathbf{h}_R$ and $\mathbf{h}_i$, respectively, and are defined as follows [6]:

$$\begin{aligned}\mathbf{h}_R &= \beta_I \mathbf{a}_N(\phi_I^a, \phi_I^e)\mathbf{a}_M^H(\psi_B^a, \psi_B^e), \\ \mathbf{h}_i &= \beta_i \mathbf{a}_N(\psi_I^a, \psi_I^e),\end{aligned} \tag{1}$$

where $\{\cdot\}^H$ is the Hermitian of the matrix, β_I and β_i are the distance dependent losses of BS to IRS link and IRS to i^{th} user link, respectively, ϕ_I^a and ϕ_I^e are the angle of arrival (AoA) in azimuth and elevation at the IRS, respectively, ψ_B^a and ψ_B^e are the angle of departure (AoD) in azimuth and elevation at the BS, respectively, ψ_I^a and ψ_I^e are the AoD in the azimuth and elevation at the IRS, respectively, and $\mathbf{a}_X(\upsilon^a, \upsilon^e)$ is the array factor that captures the beamforming gain. For a planar array with X antenna elements, we assume $\sqrt{X}$ elements in the horizontal and vertical direction of the planar array, and thus, define the array factor as follows [6]:

$$\mathbf{a}_X(\upsilon^a,\upsilon^e) = [1,\ldots,\ e^{j\frac{2\pi d}{\lambda}(x\sin\upsilon^a\sin\upsilon^e + y\cos\upsilon^e)}, \ldots, e^{j\frac{2\pi d}{\lambda}\left((\sqrt{X}-1)\sin\upsilon^a\sin\upsilon^e+(\sqrt{X}-1)\cos\upsilon^e\right)}]^T \quad (2)$$

where $0 \le x, y \le (\sqrt{X}-1)$ are the indices of antenna elements in the planar array, d is the spacing between antenna elements, λ is the wavelength, υ^a and υ^e are the desired directions in azimuth and elevation, respectively.

We denote the diagonal matrix that captures the reflection of the IRS as $\mathbf{\Theta}$ and define each diagonal element of $\mathbf{\Theta}$ as $e^{j\theta_k}$ [3], where $k \in [1, N]$ is the antenna index and $\theta_k \in [0, 2\pi)$ is the phase reflection coefficient. Ideal phase control is difficult to achieve in real-time because of various factors like imperfect channel estimation [7], finite resolution while applying phase shifters, etc. All these factors result in imperfect phase compensation, and hence, we consider the actual reflection matrix to be $\widetilde{\mathbf{\Theta}}$ with each diagonal element defined as $e^{j\tilde{\theta}_k}$, where $\tilde{\theta}_k = \theta_k + \hat{\theta}_k$, $\hat{\theta}_k$ being the phase noise. We consider $\hat{\theta}_k$ to be uniformly distributed over $[-\delta, \delta]$ with $\delta \in [0, \pi)$. With all this information, the signal received by the i^{th} user in an OMA system is formulated as [3]

$$y_i^{\text{OMA}} = \mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R \sqrt{P_t} s_i + n,$$

where s_i is the data transmitted to the i^{th} user. Without loss of generality, we assume s_i's to be independent and identically distributed with zero mean and unit variance, n denotes the additive white Gaussian noise with zero mean and variance of σ^2, and P_t is the available transmit power at the BS. The signal-to-interference-plus-noise ratio (SINR) of the i^{th} user in the OMA system is formulated as

$$\gamma_i^{\text{OMA}} = \frac{P_t ||\mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R||^2}{I + \sigma^2}, \quad (3)$$

where I is the interference power received at the user. In case of an IRS-assisted downlink NOMA system, we consider that the BS transmits $\sqrt{P_t}(\sum_{i=1}^{U}\sqrt{\alpha_i}s_i)$, where U represents the number of users paired together, and s_i and α_i denote transmitted data and the fraction of power allocated to the i^{th} user, respectively. We assume the channel gains of the users satisfy $||h_i \mathbf{\Theta} \mathbf{h}_R||^2 > ||h_j \mathbf{\Theta} \mathbf{h}_R||^2, \ \forall j > i$. We consider that user i correctly decodes and removes the interference from the user j, where $j > i$. Further, $0 < \alpha_i < 1$ and $\sum_{i=1}^{U} \alpha_i = 1$. The signal received by the i^{th} user in NOMA is given by [3]

$$y_i^{\text{NOMA}} = \mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R \sqrt{P_t}\left(\textstyle\sum_{i=1}^{U}\sqrt{\alpha_i}s_i\right) + n.$$

Thus, we define the SINR of i^{th} user in a NOMA as follows:

$$\gamma_i^{\text{NOMA}} = \frac{\alpha_i P_t ||\mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R||^2}{\sum_{j=1}^{i-1}\alpha_j P_t ||h_j \widetilde{\mathbf{\Theta}} \mathbf{h}_R||^2 + I + \sigma^2}. \quad (4)$$

III. Computation of Bounds

Typically, the ideal phase compensation $\mathbf{\Theta}$ is calculated such that it maximizes the received signal power as follows [6].

$$\begin{aligned}\mathbf{\Theta} &= \arg\max_{\mathbf{\Theta}^*} |\mathbf{h}_i^H \mathbf{\Theta}^* \mathbf{h}_R|^2, \\ &= \arg\max_{\mathbf{\Theta}^*} |\mathbf{a}_N^H(\psi_I^a, \psi_I^e)\mathbf{\Theta}^* \mathbf{a}_N^H(\phi_I^a, \phi_I^e)|^2, \\ &= \arg\max_{\mathbf{\Theta}^*} \Bigg| \sum_{\substack{0 \le x,y \le \sqrt{N} \\ n = \sqrt{N}x + y + 1}} e^{j2\pi\frac{d}{\lambda}(xp+yq)+j\theta_n} \Bigg|^2. \end{aligned} \quad (5)$$

where, (x, y) represents the index of antenna element in the two-dimensional planar array, p and q are the deterministic values corresponding to $\mathbf{h}_i^H$ and $\mathbf{h}_R$ values. The ideal phase compensation on the n^{th} antenna element is given by $\theta_n = -2\pi\frac{d}{\lambda}(xp + yq)$. With imperfections in phase compensations, we consider $\tilde{\theta}_n = \theta_n + \hat{\theta}_n$, where $\hat{\theta}_n$ is the error. From (1)-(2) and (5), we get

$$\mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R = \beta_i \beta_I \textstyle\sum_{n=1}^{N} e^{j\hat{\theta}_n} \mathbf{a}_M^H(\psi_B^a, \psi_B^e), \quad (6)$$

$$||\mathbf{a}_M^H(\psi_B^a, \psi_B^e)||^2 = M, \quad (7)$$

$$||\mathbf{h}_i^H \widetilde{\mathbf{\Theta}} \mathbf{h}_R||^2 = |\beta_i\beta_I|^2 \Big|\textstyle\sum_{n=1}^{N} e^{j\hat{\theta}_n}\Big|^2 M. \quad (8)$$

We define channel state information (CSI) of i^{th} user (γ_i^{CSI}) as

$$\gamma_i^{\text{CSI}} = \frac{P_t ||\mathbf{h}_i^H \mathbf{\Theta} \mathbf{h}_R||^2}{I + \sigma^2} = \frac{P_t |\beta_i \beta_I|^2 |N^2 M}{I + \sigma^2}. \quad (9)$$

Lemma 1. *As $N \longrightarrow \infty$, the normalized achievable data rates in an IRS-assisted OMA and NOMA systems are as follows:*

$$R_i^{\text{OMA}} = \frac{1}{U}\log_2\left(1 + \gamma_i^{\text{CSI}} \operatorname{sinc}^2(\delta)\right), \quad (10)$$

$$R_i^{\text{NOMA}} = \log_2\left(1 + \frac{\alpha_i \gamma_i^{\text{CSI}} \operatorname{sinc}^2(\delta)}{\sum_{j=1}^{i-1}\alpha_j \gamma_j^{\text{CSI}} \operatorname{sinc}^2(\delta) + 1}\right). \quad (11)$$

Proof. We adopt the SINR approximation formulated in [6] and define the following:

$$\left|\frac{1}{N}\sum_{n=1}^{N} e^{j\hat{\theta}_n}\right|^2 \xrightarrow{(a)} \left|\mathbb{E}[e^{j\hat{\theta}_n}]\right|^2 \overset{(b)}{=} \left|\mathbb{E}[\cos(\hat{\theta}_n)]\right|^2 \overset{(c)}{=} \operatorname{sinc}^2(\delta), \quad (12)$$

where (a) is based on the law of large numbers [6], (b) is obtained by integrating the odd symmetrical function $\sin(\hat{\theta}_n)$ for $\hat{\theta}_n \in [-\delta, \delta]$, and (c) uses the probability density function of $\hat{\theta}_n$ which is defined as $f(\hat{\theta}_n) = 1/2\delta, \forall \hat{\theta}_n \in [-\delta, \delta]$, and $\operatorname{sinc}(\delta) = \sin(\delta)/\delta$. Substituting (6)-(9), (12) in (3)-(4), we get

$$\gamma_i^{\text{OMA}} = \frac{P_t |\beta_i\beta_I|^2 \left|\sum_{n=1}^{N} e^{j\hat{\theta}_n}\right|^2 M}{I + \sigma^2} = \gamma_i^{\text{CSI}} \operatorname{sinc}^2(\delta), \quad (13)$$

$$\begin{aligned}\gamma_i^{\text{NOMA}} &= \frac{\alpha_i P_t |\beta_i\beta_I|^2 \left|\sum_{n=1}^{N} e^{j\hat{\theta}_n}\right|^2 M}{\sum_{j=1}^{i-1}\alpha_j P_t |\beta_j\beta_I|^2 \left|\sum_{n=1}^{N} e^{j\hat{\theta}_n}\right|^2 M + I + \sigma^2}, \\ &= \frac{\alpha_i \gamma_i^{\text{CSI}} \operatorname{sinc}^2(\delta)}{\sum_{j=1}^{i-1}\alpha_j \gamma_j^{\text{CSI}} \operatorname{sinc}^2(\delta) + 1}. \end{aligned} \quad (14)$$

Assuming the full bandwidth allocation for all the users in case of NOMA and equal bandwidth allocation for each user in OMA, and substituting (13)-(14) while calculating the normalized data rates completes the proof of Lemma 1. ■

A. Bounds on α_1 and α_2

We define $\overline{R}_1$ and $\overline{R}_2$ as the minimum rates required by strong and weak user, respectively. For the lower bound on α_1, we assume that rate of strong user in NOMA (R_1^{NOMA}) should be greater than or equal to the minimum rate required by strong user ($\overline{R}_1$). Thus, by considering $R_1^{\text{NOMA}} \ge \overline{R}_1$, we get

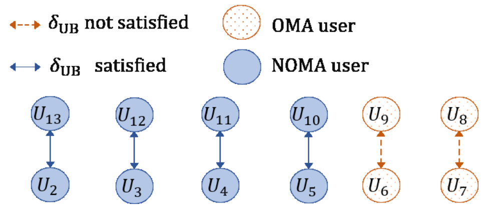


Fig. 2: Adaptive user pairing.

$$\log_2\left(1+\alpha_1\gamma_1^{\text{CSI}}\operatorname{sinc}^2(\delta)\right) \geq \overline{R}_1,$$

$$\alpha_1 \geq \frac{2^{\overline{R}_1}-1}{\gamma_1^{\text{CSI}}\operatorname{sinc}^2(\delta)} \triangleq \alpha_{1_{\text{LB}}}. \quad (15)$$

Similarly, for the upper bound, by using $R_2^{\text{NOMA}} > \overline{R}_2$, we get

$$\log_2\left(1+\frac{\alpha_2\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta)}{\alpha_1\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta)+1}\right) \geq \overline{R}_2. \quad (16)$$

Substituting $\alpha_2 = 1-\alpha_1$ in (16), we obtain

$$\alpha_1 \leq \frac{\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta)-(2^{\overline{R}_2}-1)}{2^{\overline{R}_2}\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta)} \triangleq \alpha_{1_{\text{UB}}}. \quad (17)$$

Note that $\alpha_{1_{\text{LB}}}$ in (15) specifies the minimum power required for the strong user to achieve the minimum required data rates, whereas, $\alpha_{1_{\text{UB}}}$ in (17) specifies the maximum power that could be allocated for the strong user to ensure minimum required data rates for the weak user. Similar bounds can be achieved for the power allocation factor of the weak user by substituting $\alpha_2 = 1-\alpha_1$ in (15)-(17).

B. Upper Bound on the Imperfect Phase Compensation (δ_{UB})

For the upper bound on δ, we consider that the upper bound of α_1 in (17) should be greater than or equal to the lower bound of α_1 in (15). Using (15)-(17) and solving $\alpha_{1_{\text{UB}}} \geq \alpha_{1_{\text{LB}}}$, we get

$$\operatorname{sinc}^2(\delta) \geq \frac{(2^{\overline{R}_1}-1)2^{\overline{R}_2}}{\gamma_1^{\text{CSI}}} + \frac{(2^{\overline{R}_2}-1)}{\gamma_2^{\text{CSI}}} \triangleq \operatorname{sinc}^2(\delta_{\text{UB}}). (18)$$

The constraint in (18) specifies that, there exists a practically feasible α_1 (i.e., $0 < \alpha_1 < 1$ and $\alpha_2 = 1-\alpha_1$) which achieves minimum data rate requirements for both the users for any $\delta \leq \delta_{\text{UB}}$. Note that when we consider $\overline{R}_i = R_i^{\text{OMA}}$, δ_{UB} is computable at the base station as it is only dependent on γ_i^{CSI}. From (18), we conclude that it is beneficial to pair the users in IRS-assisted NOMA systems only when δ_{UB} with that user pair is greater than or equal to δ. Otherwise, the data rates achieved by the users in NOMA will not be higher than their OMA counterparts.

IV. Proposed Algorithms

In this section, we initially present an adaptive user pairing (AUP) algorithm for the IRS-assisted NOMA system based on the δ_{UB} derived in (18). We then propose maximum ASR achieving power allocation (MPA) and fair power allocation (FPA) algorithms for the paired users.

A. AUP

In [9], [12], [13], the authors have shown that pairing of near users with far users achieves better data rates. Motivated by this, we initially sort the users based on their SINRs and group the near users with far users. For each user, we then define the

Algorithm 1: Proposed AUP, MPA, and FPA

Input : Set of users U_i and corresponding SINRs γ_i^{CSI}, imperfection in phase compensation δ at the IRS.
Variables: i is a variable representing user pair index.

Sort the users based on their SINRs ($\gamma_1^{\text{CSI}} \geq \ldots \geq \gamma_G^{\text{CSI}}$);
for $i = 1 \rightarrow \frac{G}{2}$ **do**
  $\overline{R}_1 = \frac{1}{2}\log_2(1+\gamma_i^{\text{CSI}}\operatorname{sinc}^2(\delta))$;
  $\overline{R}_2 = \frac{1}{2}\log_2(1+\gamma_{G-i+1}^{\text{CSI}}\operatorname{sinc}^2(\delta))$;
  $\operatorname{sinc}^2(\delta_{UB}) = \frac{(2^{\overline{R}_1}-1)2^{\overline{R}_2}}{\gamma_1^{\text{CSI}}} + \frac{(2^{\overline{R}_2}-1)}{\gamma_{G-i+1}^{\text{CSI}}}$;
  if $\operatorname{sinc}^2(\delta_{UB}) > \operatorname{sinc}^2(\delta)$ **then**
    Consider the users for OMA;
  else
    Consider the users for NOMA;
    if *MPA* **then**
      $\alpha_1 = \frac{\gamma_{G-i+1}^{\text{CSI}}\operatorname{sinc}^2(\delta)-(2^{\overline{R}_2}-1)}{2^{\overline{R}_2}\gamma_{G-i+1}^{\text{CSI}}\operatorname{sinc}^2(\delta)}$;
    else
      FPA;
      $\alpha_1 = \frac{(2^{\overline{R}_1}-1)}{(2^{\overline{R}_2}-1+2^{\overline{R}_1}(2^{\overline{R}_1}-1))}$;
    end
    $\alpha_2 = 1-\alpha_1$;
  end
end

rate achievable with OMA as the minimum required rate (i.e., $\overline{R}_i = R_i^{\text{OMA}}$). From (18), it is evident that pairing two users in IRS-assisted NOMA with imperfect phase compensation will not always ensure that achievable data rates are better than OMA rates. Hence, to exploit the benefits from NOMA, we pair only those users whose achievable date rates outperform the OMA counterparts. Thus, for users in each group, we check if the imperfect phase compensation (δ) is less than or equal to δ_{UB} formulated in (18). If this criterion is satisfied, we consider the users in that group to be a NOMA pair. Otherwise, we consider them to be OMA users. This procedure will ensure that each user achieves at least OMA rates. All this procedure is pictorially presented in Fig. 2 for a set of 14 users with $\gamma_1^{\text{CSI}} \geq \ldots \geq \gamma_{14}^{\text{CSI}}$. Next, we present MPA procedure in detail.

B. MPA

In this section, we present maximum ASR achieving power allocation procedure for the IRS-assisted NOMA systems.

Lemma 2. *The power allocation factors for NOMA pair that maximize the ASR and also ensure each user achieves at least OMA rates are as follows, $\alpha_1 = \alpha_{1_{UB}}$ and $\alpha_2 = 1-\alpha_1$.*

Proof. We formulate ASR for a NOMA pair as $R_1^{\text{NOMA}} + R_2^{\text{NOMA}}$.

$$\frac{d(ASR)}{d\alpha_1} = \frac{\gamma_1^{\text{CSI}}\operatorname{sinc}^2(\delta)-\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta)}{(1+\alpha_1\gamma_1^{\text{CSI}}\operatorname{sinc}^2(\delta))(1+\alpha_2\gamma_2^{\text{CSI}}\operatorname{sinc}^2(\delta))}. \quad (19)$$

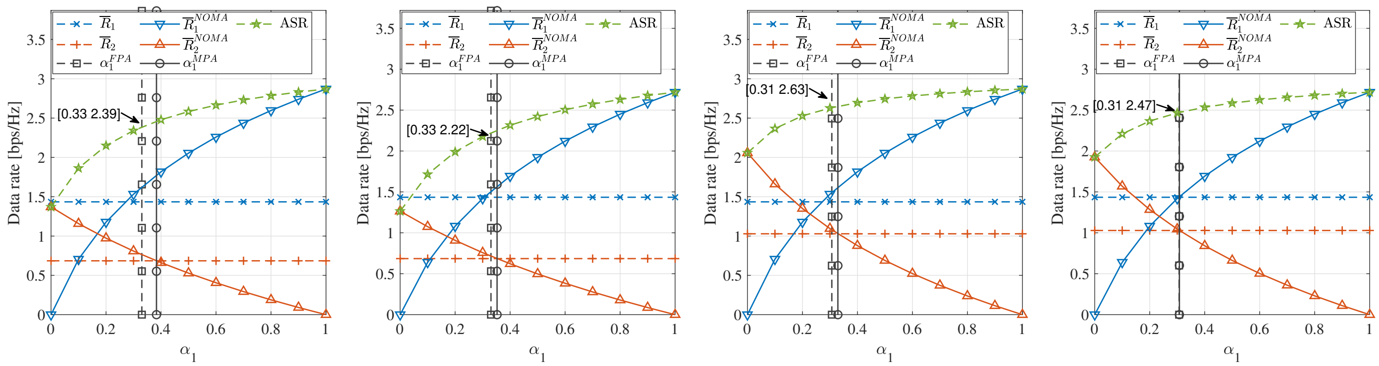
Note that as per our formulation in (4), $\gamma_1^{\text{CSI}} \geq \gamma_2^{\text{CSI}}$, and thus, $\frac{d(ASR)}{d\alpha_1} \geq 0$. Hence, ASR is a non-decreasing function and $\alpha_1 = \alpha_{1_{\text{UB}}}$ will result in maximum ASR. This completes the proof of Lemma 2. ∎

Thus, in MPA, we allocate $\alpha_1 = \alpha_{1_{\text{UB}}}$ and $\alpha_2 = 1-\alpha_1$ to strong and weak users, respectively, to maximise ASR.

C. FPA

We define the $\mathcal{O}_i$ as an event of outage for i^{th} user, where $Pr(\mathcal{O}_i) = Pr(R_i^{\text{NOMA}} < \overline{R}_i), \forall i = 1, 2$. Using (11), we get

This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2022.3205268



(a) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 2]$ dB, $\delta = 0°$. (b) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 2]$ dB, $\delta = 11°$. (c) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 5]$ dB, $\delta = 0°$. (d) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 5]$ dB, $\delta = 11°$.

Fig. 3: Comparison of achievable data rates with varying α_1.

$$Pr(\mathcal{O}_1) = Pr(R_1^{\text{NOMA}} < \overline{R}_1) = Pr\Big(\gamma_1^{\text{CSI}} < \frac{(2^{\overline{R}_1} - 1)}{\alpha_1 \,\text{sinc}^2(\delta)}\Big). \quad (20)$$

Similarly, for the weak user in NOMA, we get

$$Pr(\mathcal{O}_2) = Pr(R_2^{\text{NOMA}} < \overline{R}_2),$$
$$= Pr\Big(\gamma_2^{\text{CSI}} < \frac{(2^{\overline{R}_2} - 1)}{(\alpha_2 - \alpha_1(2^{\overline{R}_2} - 1))\,\text{sinc}^2(\delta)}\Big). \quad (21)$$

In NOMA, with an increase in power allocation for a user, the probability of outage increases for the other paired user. Hence, to ensure fairness in power allocation, we consider allocating power levels such that the probability of outage is same for both the users. Thus, for a given probability distribution function for γ_i^{CSI}, using (20)-(21), when the probability of outage is same for both the paired users, the following holds:

$$\frac{(2^{\overline{R}_1} - 1)}{\alpha_1 \,\text{sinc}^2(\delta)} = \frac{(2^{\overline{R}_2} - 1)}{(\alpha_2 - \alpha_1(2^{\overline{R}_2} - 1))\,\text{sinc}^2(\delta)}. \quad (22)$$

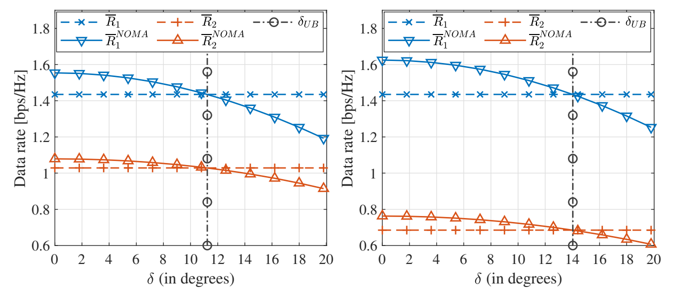
Substituting $\alpha_2 = 1 - \alpha_1$ and solving it further, we obtain

$$\alpha_1 = \frac{(2^{\overline{R}_1} - 1)}{(2^{\overline{R}_2} - 1 + 2^{\overline{R}_1}(2^{\overline{R}_1} - 1))}. \quad (23)$$

Note that we also obtain (23) by considering the upper bound in (17) equal to the lower bound in (23). Further, α_1 obtained in (15) satisfies $0 < \alpha_1 < 1$. The power allocation in both FPA and MPA can be visualized in two steps. In the first step, we allocate minimum required power to both the users to achieve desired data rates. In the next step, MPA allocates the remaining transmit power to the strong user to maximise ASR, whereas, FPA allocates the remaining power proportionally to both the users to ensure fairness. A pseud-code to implement the proposed AUP, MPA, and FPA is presented in Algorithm 1. The AUP algorithm with MPA or FPA requires sorting of the users, and hence, the complexity is of the order $\mathcal{O}(G \log_2 G)$. Next, we present the simulation results.

V. Numerical Results

For the evaluation, we have considered Poisson point distributed BSs and users with densities 25 BS/km^2 and 2000 users/km^2, respectively. Further, we have assumed $M = 8,\ N = 32,\ \delta = 11°, \overline{R}_i = R_i^{\text{OMA}}$, and the urban cellular path loss model as presented in [14]. In Fig. 3, we present the comparison of data rates with varying α_1. We consider $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 2]$ dB with $\delta = 0°$ and $11°$ in Fig. 3a and 3b, respectively. Since the SINRs of the users in NOMA pair are same, the minimum required rates by the users ($\overline{R}_1$ and $\overline{R}_2$) are same in both the cases. However, the individual NOMA rates vary with δ and are better in case of $\delta = 0°$. As shown in Fig. 3a and 3b, the power allocation α_1 chosen by MPA is the upper bound, beyond which the data rates of weak user will be less than the minimum required rate. Further, observe that ASR is a non-decreasing function as presented in (19). Hence, the ASR obtained at α_1^{MPA} is always higher than α_1^{FPA}. Additionally, ASR is better when $\delta = 0°$ in Fig. 3a as compared to $\delta = 11°$ in Fig. 3b. In Fig. 3c and 3d, we consider $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 5]$ dB with $\delta = 0°$ and $11°$, respectively. Compared to Fig. 3a and 3b, the ASR at α_1^{FPA} is higher in Fig. 3c and 3d. This is because, α_1^{FPA} is comparatively less in the latter case, and hence, the interference observed by the weak user is less which results in better ASR. Note that with increasing δ, the gap between the lower bound in (15) and the upper bound in (17) decreases, and thus, the gap between α_1^{FPA} and α_1^{MPA} also decreases.



(a) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 5]$ dB. (b) $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 2]$ dB.

Fig. 4: Comparison of achievable data rates with varying δ.

In Fig. 4, we present the comparison of data rates of strong and weak users for varying δ. For analyzing the impact of δ, we have considered a set of NOMA user pairs with $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 5]$ dB and $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}}] = [8, 2]$ dB in Fig. 4a and Fig. 4b, respectively. Note that a similar behaviour holds for any NOMA user pair. In Fig. 4a and 4b, the minimum required rate for strong user ($\overline{R}_1$) is same, however, its NOMA rates (R_1^{NOMA}) vary as they depend on the SINR of the other paired user. As per (18), δ_{UB} is different for both the pairs as it is a function of individual SINRs of the users in the NOMA pair. Further, only when $\delta < \delta_{\text{UB}}$, the NOMA rates for both strong and weak users are better than the minimum required rates. Hence, for NOMA rates to be better than OMA rates, the base station should consider δ_{UB} while pairing the users.

This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2022.3205268

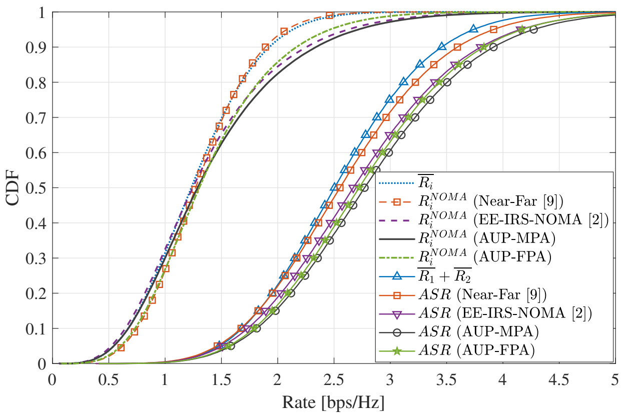


Fig. 5: CDF of achievable data rates with various algorithms.

In Fig. 5, we present the cumulative distribution function (CDF) of achievable data rates with various algorithms. With the Near-Far algorithm [9] and energy efficient (EE) algorithm EE-IRS-NOMA [2], the individual NOMA rates for some users are worse than the minimum required rates. However, when we consider the proposed AUP, the individual rates are never worse than the minimum required rates. Further, when AUP is used along with MPA, the strong users observe higher data rates and the weak users observe the minimum required data rates, as the algorithm allocates more power to the strong user. Even though Near-Far [9] has poor individual rates for some users, it has higher ASR as compared to the minimum required ASR. The EE-IRS-NOMA algorithm tries to maximize the data rates of the paired NOMA users, and thus, has higher ASR than the Near-Far algorithm. The proposed AUP with FPA and MPA algorithms have significant improvements in terms of ASR as compared to the minimum required ASR. Further, MPA has comparatively higher ASR than FPA, as it allocates more power to the strong user.

In Fig. 6, we present the comparison of the mean ASR with all the algorithms for varying δ. The performance of Near-Far [9] and EE-IRS-NOMA [2] decreases with increase in δ. Since these algorithms do not consider the imperfection in phase compensation while pairing the users, the ASR achieved is less than the required ASR for higher δ. However, the proposed algorithms consider the imperfection in phase compensation while pairing. Thus, for higher δ, the proposed AUP uses OMA and achieves minimum required ASR. Further, the performance of MPA is better than the FPA because of more power allocation to the strong user.

We have evaluated the probability of outage and presented the results for the same in Table. I. It can be observed that AUP with FPA has similar levels of outage for both strong and weak users. In AUP with MPA, the outage for weak users is higher as compared to the strong user. This is because MPA allocates more power to the strong user to achieve higher ASR, and hence, results in this unfairness. Further, in the evaluation of Near-Far [9], we have assumed $\alpha_1 = \alpha_{1_{\text{LB}}}$. Thus, it results in a higher outage for strong user. Both Near-Far and EE-IRS-NOMA have higher overall outage than the AUP algorithms. As shown in Fig. 5, 6, and Table I the proposed AUP with MPA and FPA outperform the state-of-the-art algorithms and provide trade-offs between maximum ASR and fairness.

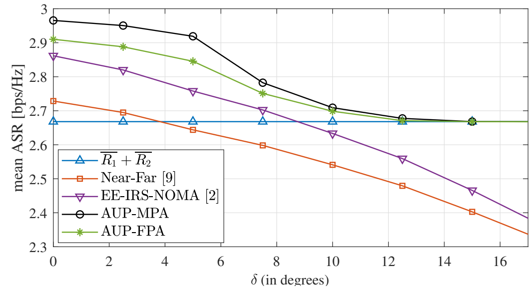


Fig. 6: Comparison of mean ASR with varying δ.

Algorithm \ Metric	Strong User Outage	Weak User Outage	Overall Outage
Near-Far [9]	0.1479	0.0895	0.1105
EE-IRS-NOMA [2]	0.0874	0.1331	0.1087
MPA	0.0844	0.1301	0.1072
FPA	0.0989	0.0980	0.0984

TABLE I: Probability of outage with various algorithms

VI. Conclusion

In this letter, we have derived bounds on the imperfection in the phase compensation and the power allocation factors for IRS-assisted NOMA systems. Using these bounds, we have proposed an adaptive user pairing algorithm to improve the achievable data rates of each user. We have then proposed power allocation algorithms to achieve maximum sum rate or ensure fairness. Through extensive simulations, we have shown that the proposed power allocation algorithms offer trade-offs between the achievable data rates and the fairness. In future, we plan to jointly solve the phase compensation and user pairing for the practical IRS-assisted NOMA systems.

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