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Implementing the reliability of data information in multi-criteria decision making process based on fuzzy TOPSIS and fuzzy entropy

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Abstract A multi-criteria decision-making process utilizes real-time data information, which is inherently uncertain and imprecise. To be relevant in the decision-making process, real-time data information must be reliable. Because fuzziness alone is insufficient to solve decision-making problems, measuring the information's reliability is critical. Z-number, which incorporates both restrictions and reliability in its definition is considered as a powerful tool to depict the imperfect information. In this paper, a new methodology is developed based on fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method and fuzzy entropy for solving the multi-criteria decision-making problems where the weight information for decision makers and criteria is incomplete. The evaluation of the information is represented in the form of linguistic terms and the following calculation is performed using Z-numbers. Fuzzy entropy is applied to determine the weights of the criteria and fuzzy TOPSIS is used to rank the alternatives. An empirical study of subjective well-being of working women is used to demonstrate the proposed methodology.

Keywords: Z-Numbers; Fuzzy Entropy; FTOPSIS; MCDM; Subjective Well-being

1. Introduction

Natural language (or linguistic terms) is the best way to explain decision makers' (DMs) opinions in the decision-making process. Since the natural language are usually vague and cannot be express with exact values, [1] introduced the concept of fuzzy set where it has the capability to measure uncertainties and vagueness expressed in natural language. The theory of fuzzy sets has since evolved into a variety of extensions based on its basic description, including type-2 fuzzy sets, fuzzy multisets, intuitionistic fuzzy sets, hesitant fuzzy sets, and various fuzzy numbers including triangular fuzzy numbers and trapezoidal fuzzy numbers [2].

Even though the fuzzy set and its generalizations are able to deal with vagueness in data information and human judgement, it does not consider the reliability of information of the DMs. The reliability of the evaluation is critical because it influences the final outcome of the decision-making process [3]. In order to overcome this limitation, [4] proposed the idea of Z-numbers that includes both the restriction



of the evaluation and reliability of the judgement in terms of fuzzy numbers. Z-numbers have more capability to explain human perception since it considers the level of confidence of the decision maker. Because of the reliability factor, Z-numbers have a higher level of uncertainty than fuzzy numbers. Furthermore, Z-numbers offer an extra degree of freedom for representing uncertainties and fuzziness in real-world situations. Therefore, using Z-numbers to model uncertainties instead of fuzzy numbers is more practical and relevant.

The contributions of reliability in Z-numbers have led the researchers to combine Z-numbers with various Multi Criteria Decision Making (MCDM) methodology. MCDM refers to find the best choice from all the possible criteria and alternatives in decision making problems. Various methods exist for MCDM such as technique for order preference by similarity to an ideal solution (TOPSIS) [5], Simple Additive Weighting (SAW) [6], Analytic Hierarchy Process (AHP) [7], and TODIM (an acronym in Portuguese for Interactive Multi Criteria Decision Making) [8].

TOPSIS is one of the most popular MCDM method introduced by [5] to determine the best choice of the alternatives that is chosen based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). This method has the capability of taking into consideration a variety of criteria with different alternatives at the same time [2]. However, the traditional MCDM method usually deals with crisp value in its rating and weights of the criteria. According to [9], it is impossible to achieve an accurate assessment if the data is immeasurable. In other words, crisp data are insufficient to model real-life problems because human evaluations are usually vague and cannot be expressed with exact values [10].

Hence, to overcome this issue, fuzzy sets can be used with the conventional TOPSIS method to allow experts to incorporate imperfect information into the model. Therefore, fuzzy TOPSIS has been introduced and implemented in various MCDM problems [11]. Since the introduction of fuzzy TOPSIS, many researchers proposed this method to solve MCDM related-problems such as [12] where the author proposed fuzzy TOPSIS methodology in selecting green suppliers based on GSCM practices and applied to a Brazilian electronics company. [13] did a comprehensive research on evaluation of sustainable acid rain control using fuzzy TOPSIS in multi-criteria decision analysis model framework. The latest research on fuzzy TOPSIS in MCDM problems can be found in the research done by [14], [15], [16], [17] and [18] where they used fuzzy TOPSIS in their respective areas including sustainable energy, credit risk evaluation, social networking and integrating with spherical fuzzy sets theory.

As mentioned earlier, the data information in real problem contains vagueness. When dealing with real-world information, it is insufficient to take into account the fuzziness in the data information only. The level of reliability in the data information is also a significant element to be considered [19]. Hence, the study on Z-numbers incorporating with fuzzy TOPSIS also has a great numbers of studies. For example, the integration of Z-numbers with TODIM and TOPSIS by [8] suggests that Z-numbers can tackle the MCDM problems. The authors claimed that information reliability is a very important aspect of decision analysis and that their approach seems to be a promising way of dealing with it. The other conclusion that highlight the importance of the reliability in data information comes from [20], where the authors conclude that using Z-TOPSIS method outperforms the existing TOPSIS method in terms of ranking performance. Another work carried out by [21] concludes that Z-numbers cooperating with TOPSIS and fuzzy similarity as an additional method, gives better selection in human decision making problems that can resolve human judgement.

In most MCDM methods, the relative weight of the criteria plays an important role in decision-making process for ranking suitable alternatives. The weights of criteria in decision making have different meanings and not all of them can be defined identical weight [22]. Thus, some methods were introduced to determine criteria weights, such as fuzzy entropy method, fuzzy AHP, etc. [22]. In this

paper, the concept of Z-numbers is applied in fuzzy TOPSIS (Z-FTOPSIS) and the weight of the criterion expressed in Z-numbers will be determine using fuzzy entropy method.

In other words, this study aims to propose a methodology on Z-numbers and fuzzy TOPSIS following the steps below:

- i) Define the criteria and alternatives of the decision maker and evaluate using Z-numbers data information.
- ii) Propose the criteria weights by applying fuzzy entropy method.
- iii) Apply the fuzzy TOPSIS method to rank the alternatives.

A case study of subjective well-being (SWB) of working women will be implemented to demonstrate the capability of the proposed methodology to handle knowledge of human being and uncertain information. The rest of this paper is structured as follows. In Section 2, an overview of the concept of the fuzzy set theory and Z-numbers is given. Section 3 presents the methodology of this research. A numerical example is provided in Section 4 and finally the discussion are drawn in Section 5.

2. Preliminaries

In this section, the definitions of the basic concept of fuzzy sets, followed by fuzzy numbers, z-numbers, fuzzy entropy and fuzzy FTOPSIS will be presented in the subsection below.

2.1 Basic Concepts of Fuzzy Sets

[1] Let X be a classical set of objects, called the universe, whose elements are denoted by x . The membership in a crisp subset of X is often viewed as characteristic function μ_A from X to $\{0,1\}$ such that:

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if } x \in A, \\ 0 & \text{otherwise,} \end{cases}$$

where $\{0,1\}$ is called a valuation set. If the valuation set is allowed to be the real interval $[0,1]$, A is called a fuzzy set and denoted by \tilde{A} and $\mu_{\tilde{A}}(x)$ is the degree of membership of x in \tilde{A} .

Definition 1 [1] If \tilde{A} be a fuzzy set, then \tilde{A} is completely characterized by the set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\} \tag{1}$$

2.2 Fuzzy Numbers

Definition 2 [31] A fuzzy numbers \tilde{A} is a convex normalized fuzzy set \tilde{A} of the real line \mathbb{R} with continuous membership function. Among the various kinds of fuzzy numbers shapes, triangular fuzzy numbers (TFN) and trapezoidal fuzzy numbers (TrFN) are the most popular [31].

Definition 2.1 [31] A TFN \tilde{A} can be defined by a triplet $\tilde{A} = (a, b, c)$, where $c \geq b \geq a$. The membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{x-c}{b-c} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Meanwhile, A TrFN \tilde{A} is defined by quadruplet $\tilde{A} = (a, b, c, d)$, where $d \geq c \geq b \geq a$. The TFN defined above is a subset of TrFN when $b = c$. The membership function of TrFN $\mu_{\tilde{A}}(x)$ given by [23]:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{x-d}{c-d} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Definition 3 [1] A linguistic variable is a variable whose values are words that are expressed in a natural languages. The concept of linguistic variable is useful in dealing with situations which are too complicated to be stated in quantitative values [24][11]. This linguistic variable (linguistic value) can be represented by FN as in Definition 2.

2.3 Z-Numbers

Definition 4 [4] The concept of a Z-numbers relates to the subject of reliability of information. A Z-numbers is a pair of FN, $Z = (\tilde{A}, \tilde{B})$ where \tilde{A} is a fuzzy restriction and \tilde{B} is a reliability of \tilde{A} . For simplicity, \tilde{A} and \tilde{B} are assumed to be TFN [31]. The membership function for \tilde{A} and \tilde{B} are as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{(x-b_1)}{(b_2-b_1)} & \text{if } b_1 \leq x \leq b_2 \\ 1 & b_2 \leq x \leq b_3 \\ \frac{(b_4-x)}{(b_4-b_3)} & \text{if } b_3 \leq x \leq b_4 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Because of lacking of basic properties in arithmetic operation in Z-numbers, several additional steps need to follow. The process of converting Z-numbers to fuzzy numbers based on intuitive vectorial centroid (IVC) proposed by [21], described as follows:

$$IVC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1+a_4)+7(a_2+a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right)$$

To convert a Z-numbers into a regular fuzzy number, Equations (6) - (8) could be used. Equation (6) converts the reliability part into a crisp number:

$$\alpha = \left(\frac{2(b_1+b_4)+7(b_2+b_3)}{18} \right) \tag{6}$$

The weighted Z-numbers can be denoted as \tilde{Z}^α by adding the weight of the reliability part to the restriction part:

$$\tilde{Z}^\alpha = \left\langle x, \mu_{\tilde{Z}^\alpha}(x) \right\rangle \mid \mu_{\tilde{Z}^\alpha}(x) = \alpha \mu_{\tilde{Z}}(x), x \in [0,1] \tag{7}$$

The weighted Z-numbers, can be converted to an ordinary fuzzy number \tilde{Z}^1 which is shown below:

$$\tilde{Z}^1 = \left\langle x, \mu_{\tilde{Z}^1}(x) \right\rangle \mid \mu_{\tilde{Z}^1}(x) = \mu_{\tilde{Z}}(\sqrt{\alpha} \times x), x \in [0,1] \tag{8}$$

2.4 Fuzzy Entropy Method

In this research, the concept of entropy is applied to determine the criteria weights. Entropy has been concerned as a measure of fuzziness [25]. The concept of Shannon’s entropy [26] has an important role in information theory and is used to refer to a general measure of uncertainty. The calculation steps are as follows: Suppose a decision matrix E with m alternatives and n criterion [27]:

Step 1: In matrix E , feature weight p_{ij} is of the i th alternatives to the j th factor:

$$p_{ij} = x_{ij} / \sum_{i=1}^m x_{ij} \quad (1 \leq i \leq m, 1 \leq j \leq n) \tag{9}$$

Step 2: The output entropy e_{ij} of the j th factor becomes

$$e_{ij} = -k \sum_{i=1}^m p_{ij} \ln p_{ij} \quad (k = 1 / \ln m; 1 \leq j \leq n) \tag{10}$$

Step 3: Calculate the variation coefficient

$$d_j = 1 - e_{ij} \quad (1 \leq j \leq n) \tag{11}$$

Step 4: Calculate the weight of entropy a_j :

$$w_j = d_j / \sum_{j=1}^n d_j \quad (1 \leq j \leq n) \tag{12}$$

2.5 Fuzzy TOPSIS Method

Consider a MCDM problem which is based on m alternatives $A = \{A_1, A_2, \dots, A_m\}$ and n criteria $C_j, (j = 1, 2, \dots, n)$. The weighting vector of criteria is denoted by $w = (w_1, w_2, \dots, w_n)^T$ where w_j is the weight of the criterion C_j , satisfying the normalization condition: $\sum_{j=1}^n w_j = 1$ and $w_j \geq 0$. Let x_{ij} be the criterion value of the alternative A_i with respect to the criterion C_j . Fuzzy TOPSIS method is a simple and useful method to address the classical MCDM problem. Its algorithm consists of the following steps [28]:

Step 1: Select the linguistic terms for alternatives with respect to criteria $(x_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n)$.

Step 2: Make the weighted normalized fuzzy decision matrix. The weighted normalised value is calculated as follow:

$$\tilde{v}_{ij} = w_j * \tilde{r}_{ij} \quad i = 1,2,\dots,m; j = 1,2,\dots,n \tag{13}$$

Step 3: Determine the positive ideal solution (PIS) A^+ and the negative ideal solution (NIS) A^- . The fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are shown as follows:

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+) \tag{14}$$

$$= ((\max_i v_{ij} / C_j \in J_I) \text{ or } (\min_i v_{ij} / C_j \in J_{II}))$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \tag{15}$$

$$= ((\min_i v_{ij} / C_j \in J_I) \text{ or } (\max_i v_{ij} / C_j \in J_{II}))$$

where J_I is a subset of benefit criteria and J_{II} is a subset of cost criteria, and $J_I \cup J_{II} = C$, $J_I \cap J_{II} = \emptyset$.

Step 4: Calculate the distances between the potential alternatives of the FPIS as well as the FNIS, respectively,

$$D_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^+) \quad i = 1,2,\dots,m \tag{16}$$

$$D_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-) \quad i = 1,2,\dots,m \tag{17}$$

Step 5: Compute the relative closeness index of each alternative to the FPIS:

$$CC_i^* = \frac{D_i^-}{D_i^- + D_i^+} \quad i = 1,2,\dots,m \tag{18}$$

Step 6: Rank the alternatives according to the closeness indices of alternatives: the bigger CC_i^* the better the alternative A_i .

3. Methodology

The procedure of this paper is summarised in Figure 1 below. The rest of the steps involves in this research elaborated below.

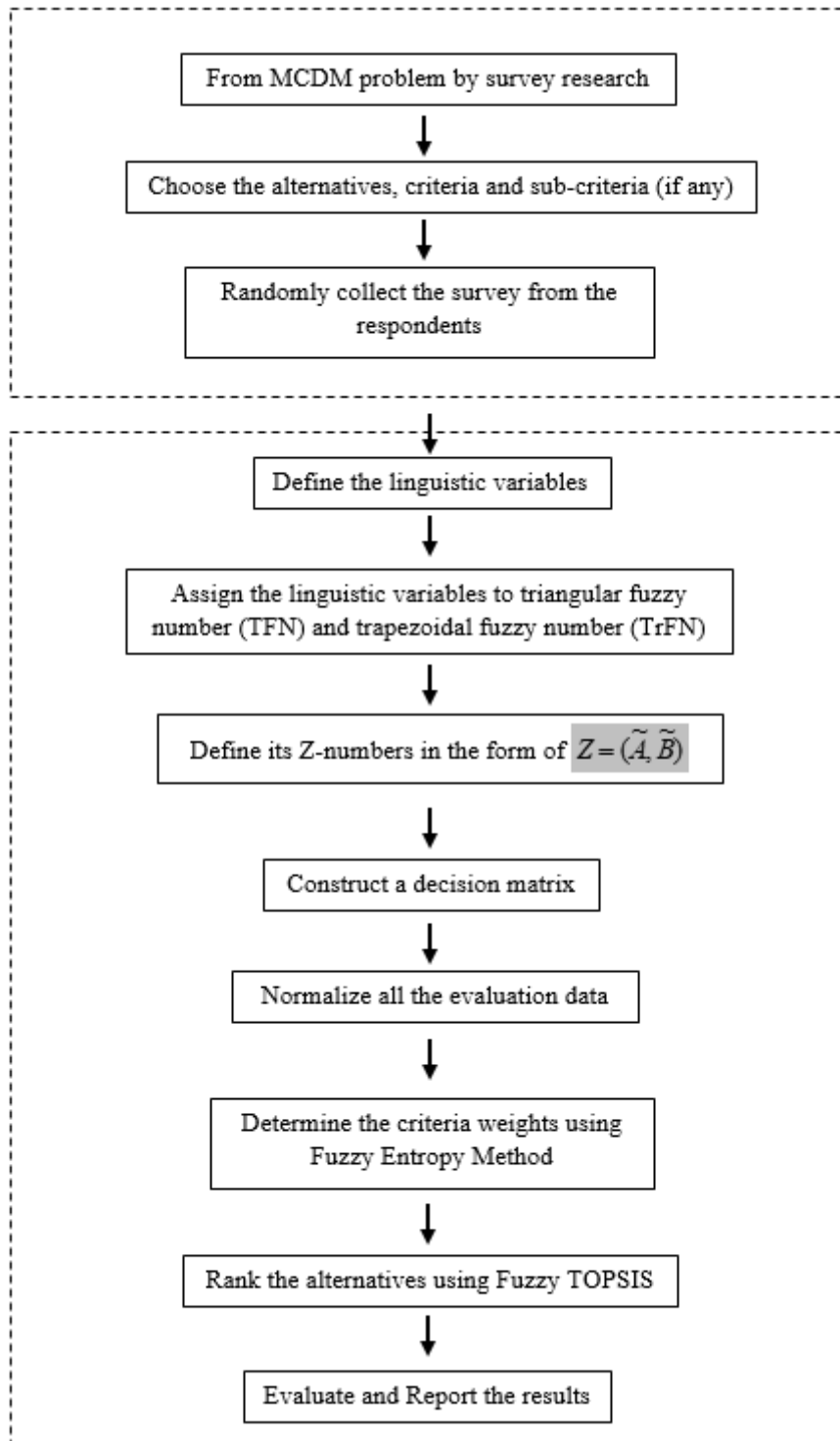


Figure 1: The framework of the methodology

The following are the steps involved:

Step 1: Choose the alternative, criteria and sub-criteria (if any). The decision maker(s) is then chosen among the expert in the area as well as the respondents that will be select randomly.

Step 2: Evaluate the data and define its Z-numbers.

The collected data which are in the form of linguistic variable needs to be converted into fuzzy number under the frame of fuzzy set. The first and second components of the Z-numbers, \tilde{A} and \tilde{B} are converted into a trapezoidal fuzzy number and triangular fuzzy number, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3)$, respectively. To convert the Z-numbers data information into generalized fuzzy numbers, the following steps are involved:

Step 2.1: Converting Z-numbers to generalized fuzzy numbers.

For this step, a method proposed by [21] is used to convert Z-numbers to generalized fuzzy numbers. From the paper, a reduction process to convert z-numbers to type-1 fuzzy numbers are proposed using intuitive vectorial centroid (IVC). For example, for the restriction (\tilde{A}) part, the formula for IVC can be computed as follows:

$$IVC (\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right) \tag{19}$$

where $\tilde{x}_{\tilde{A}}$: the centroid point on the horizontal x-axis

$\tilde{y}_{\tilde{A}}$: the centroid point on the vertical y-axis

$(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$: the centroid coordinate of restriction \tilde{A} .

To convert Z-numbers to generalised fuzzy numbers, the following steps are considered;

- i) Convert the reliability (\tilde{B}) part on x-coordinate into crisp number using the first part of Equation (19) which is:

$$IVC (\tilde{x}_{\tilde{B}}) = \left(\frac{2(b_1 + b_4) + 7(b_2 + b_3)}{18} \right) = \alpha \tag{20}$$

- ii) Add the weight of the reliability component to the restriction component using Equation (7).
- iii) The weighted restriction can be converted to an ordinary fuzzy number \tilde{Z}' using Equation (8).

Step 3: Construct a decision matrix.

Suppose there is a set of decision matrix, $\tilde{Z}_k = (x_{ij}^k)_{m \times n}$, consist of m alternatives, $A_i (i = 1, 2, \dots, m)$ evaluated against n selection criteria $C_j (j = 1, 2, \dots, n)$, and k^{th} decision maker $D_k (k = 1, 2, \dots, K)$ which are assessed in the form of triangular fuzzy numbers and their corresponding matrix is denoted by $\tilde{Z}_k = (a_{ij}^k, b_{ij}^k, c_{ij}^k) (i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, K)$ based on linguistic terms expressed in Z-numbers.

$$\tilde{Z}_k = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ A_1 & \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \cdots & \tilde{x}_{mn}^k \end{bmatrix} \end{matrix}, \quad x_{ij}^k = Z_{ij}^k(\tilde{A}, \tilde{B}) \tag{21}$$

where the weight vector $w = (w_1, w_2, \dots, w_n)^T$.

Step 4: Normalize the evaluation information.

In order to eliminate the influence of different in priority that different criteria may bring, then the normalization of all preference values is needed by dividing by maximum value of the right hand side. Normalize each attribute value $\tilde{x}_{ij} = (i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ in the matrix \tilde{Z}_{ik} into the corresponding element in the matrix called normalized fuzzy decision matrix $\tilde{z}_i = [\tilde{z}_{ij}]_{m \times n}$, where

$$\tilde{z}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right) \text{ and } c_j^+ = \max_i c_j \text{ (benefit criteria)} \tag{22}$$

Step 5: Determine the criteria weights using fuzzy entropy method. The steps as in Equation (9) – (12) are considered.

Step 6: Conducting fuzzy TOPSIS to do the ranking results. The steps for fuzzy TOPSIS are elaborated in sub-section 2.5 from Step 1 until Step 6 (equations (13) – (18)).

Step 7: Evaluate and report the results.

4. Numerical Example

The implementation of a case study of subjective well-being (SWB) of working women is considered to illustrate the numerical example of the proposed methodology. Generally, the SWB measurement were mainly focuses on statistical analysis [32] rather than measuring its criteria and alternatives using fuzzy numbers. By incorporating the suggested approach into SWB, it would be possible to learn a lot more about SWB and how to improve one's quality of life. Thus, the application of this case study into the proposed methodology aims to evaluate and identify which alternatives have the greatest effect on their subjective well-being (or also known as quality of life). For example, at the end of this research, among all 40 female respondents that takes part in the survey (randomly chosen), the ranking of the alternatives (see table 1) that based on the criteria chosen will be executed. The ranking will show us which alternatives that affect most to their subjective well-being and quality of life as a working woman. The survey will take part in the form of a questionnaire using Subjective Well-being (SWB) Questionnaire [29] and Positive Affect and Negative Affect (PANAS) Questionnaire [30].

Table 1: The relevance criteria, sub-criteria and alternatives

Criteria (C_n)	Sub-criteria (S_n)	Alternative (A_n)
Life Satisfaction (C_1)	In most ways, my life is close to my ideal. (S_{11}) The condition of my life is excellent. (S_{12}) I am satisfied with my life. (S_{13}) So far, I have gotten the important things I want in life. (S_{14}) If I could live my life over, I would change almost nothing. (S_{15})	
Positive Affect (C_2)	Interested (S_{21}) Enthusiastic (S_{22}) Strong (S_{23}) Inspired (S_{24}) Proud (S_{25}) Alert (S_{26}) Determined (S_{27}) Attentive (S_{28}) Active (S_{29})	Extremely Dissatisfied (A_1) Dissatisfied (A_2) Slightly Dissatisfied (A_3) Neutral (A_4) Slightly Satisfied (A_5) Satisfied (A_6) Extremely Satisfied (A_7)
Negative Affect (C_3)	Angry (S_{31}) Hostile (S_{32}) Nervous (S_{33}) Distressed (S_{34}) Irritable (S_{35}) Scared (S_{36})	

The proposed steps given in Section 3 were executed as follows:

Step 1: The criteria, sub-criteria and alternative are chosen (refer **table 1**). In this study, the respondents will be the decision maker to evaluate their own subjective well-being. The respondents are among working woman that are chosen randomly and voluntarily participate in the research questionnaire.

Step 2 and 3: The linguistic terms and its corresponding fuzzy numbers for each criteria are shown in **table 2**, **table 3** and **table 4**. Since the calculation is lengthy, only some of the calculation will be shown through figures and tables. For instance, the criteria and sub-criteria assessed by the respondents are shown in **table 5**, **table 6** and **table 7**, the criteria weights in **table 8** and the results in **table 9**.

Table 2: Linguistic terms and correspondent fuzzy numbers for Criteria 1 (Life Satisfaction)

Linguistic Terms	<i>For Restriction Part</i>	<i>For Reliability Part</i>
	Trapezoidal Fuzzy Number (TrFN)	Triangular Fuzzy Number (TFN)
Strongly disagree (SD)	(0,0.1,0.1,0.2;1)	(0,0.1,0.2;1)
Disagree (D)	(0.2,0.3,0.3,0.4;1)	(0.2,0.3,0.4;1)
Neither (N)	(0.4,0.5,0.5,0.6;1)	(0.4,0.5,0.6;1)
Agree (A)	(0.6,0.7,0.7,0.8;1)	(0.6,0.7,0.8;1)
Strongly agree (SA)	(0.8,0.9,0.9,1;1)	(0.8,0.9,1;1)

Table 3: Linguistic terms and correspondent fuzzy numbers for Criteria 2 (Positive Affect)

Linguistic Terms	<i>For Restriction Part</i>	<i>For Reliability Part</i>
	Trapezoidal Fuzzy Number (TrFN)	Triangular Fuzzy Number (TFN)
Very slightly or not at all (VS)	(0,0.1,0.1,0.2;1)	(0,0.1,0.2;1)
A little (AL)	(0.2,0.3,0.3,0.4;1)	(0.2,0.3,0.4;1)
Moderately (M)	(0.4,0.5,0.5,0.6;1)	(0.4,0.5,0.6;1)
Quite a bit (QAB)	(0.6,0.7,0.7,0.8;1)	(0.6,0.7,0.8;1)
Extremely (E)	(0.8,0.9,0.9,1;1)	(0.8,0.9,1;1)

Table 4: Linguistic terms and correspondent fuzzy numbers for Criteria 3 (Negative Affect)

Linguistic Terms	<i>For Restriction Part</i>	<i>For Reliability Part</i>
	Trapezoidal Fuzzy Number (TrFN)	Triangular Fuzzy Number (TFN)
Very slightly or not at all (VS)	(0,0.1,0.1,0.2;1)	(0,0.1,0.2;1)
A little (AL)	(0.2,0.3,0.3,0.4;1)	(0.2,0.3,0.4;1)
Moderately (M)	(0.4,0.5,0.5,0.6;1)	(0.4,0.5,0.6;1)
Quite a bit (QAB)	(0.6,0.7,0.7,0.8;1)	(0.6,0.7,0.8;1)
Extremely (E)	(0.8,0.9,0.9,1;1)	(0.8,0.9,1;1)

Table 5: Criteria assessed by ten respondents

Respondents	C ₁	C ₂	C ₃
1	((0.6,0.7,0.7,0.8),(0.4,0.5,0.6))	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))	((0.8,0.9,0.9,1),(0.6,0.7,0.8))
2	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))	((0.2,0.3,0.3,0.4),(0.4,0.5,0.6))	((0.6,0.7,0.7,0.8),(0.4,0.5,0.6))
3	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))	((0.4,0.5,0.5,0.6),(0.6,0.7,0.8))	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))
4	((0.2,0.3,0.3,0.4),(0.4,0.5,0.6))	((0.2,0.3,0.3,0.4),(0.4,0.5,0.6))	((0.4,0.5,0.5,0.6),(0.4,0.5,0.6))
5	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))	((0.8,0.9,0.9,1),(0.8,0.9,1))	((0.4,0.5,0.5,0.6),(0.8,0.9,1))
6	((0.8,0.9,0.9,1),(0.8,0.9,1))	((0.8,0.9,0.9,1),(0.8,0.9,1))	((0.6,0.7,0.7,0.8),(0.8,0.9,1))
7	((0.8,0.9,0.9,1),(0.8,0.9,1))	((0.6,0.7,0.7,0.8),(0.4,0.5,0.6))	((0.2,0.3,0.3,0.4),(0.4,0.5,0.6))
8	((0.4,0.5,0.5,0.6),(0.6,0.7,0.8))	((0.2,0.3,0.3,0.4),(0.4,0.5,0.6))	((0.4,0.5,0.5,0.6),(0.4,0.5,0.6))
9	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))	((0.2,0.3,0.3,0.4),(0.6,0.7,0.8))	((0.2,0.3,0.3,0.4),(0.6,0.7,0.8))
10	((0.4,0.5,0.5,0.6),(0.4,0.5,0.6))	((0.4,0.5,0.5,0.6),(0.6,0.7,0.8))	((0.6,0.7,0.7,0.8),(0.6,0.7,0.8))

Table 8: The criteria weights using Fuzzy Entropy method

	C ₁				C ₂				C ₃			
1	-0.08892	-0.08754	-0.08754	-0.08654	-0.11222	-0.10905	-0.10905	-0.10681	-0.16161	-0.14886	-0.14886	-0.14012
2	-0.10047	-0.09895	-0.09895	-0.09784	-0.04334	-0.05111	-0.05111	-0.05629	-0.11865	-0.11152	-0.11152	-0.10672
3	-0.10047	-0.09895	-0.09895	-0.09784	-0.08369	-0.08548	-0.08548	-0.08672	-0.13331	-0.12547	-0.12547	-0.12018
4	-0.0438	-0.05241	-0.05241	-0.05832	-0.04334	-0.05111	-0.05111	-0.05629	-0.08872	-0.08748	-0.08748	-0.08665
5	-0.10047	-0.09895	-0.09895	-0.09784	-0.14913	-0.14165	-0.14165	-0.13624	-0.10968	-0.1082	-0.1082	-0.10719
6	-0.13432	-0.12911	-0.12911	-0.12526	-0.14913	-0.14165	-0.14165	-0.13624	-0.14517	-0.13678	-0.13678	-0.13111
7	-0.13432	-0.12911	-0.12911	-0.12526	-0.09951	-0.09665	-0.09665	-0.09463	-0.05257	-0.05962	-0.05962	-0.06397
8	-0.10047	-0.09895	-0.09895	-0.09784	-0.04334	-0.05111	-0.05111	-0.05629	-0.08872	-0.08748	-0.08748	-0.08665
9	-0.10047	-0.09895	-0.09895	-0.09784	-0.04944	-0.0582	-0.0582	-0.06403	-0.05985	-0.06776	-0.06776	-0.07264
10	-0.06571	-0.06813	-0.06813	-0.06985	-0.08369	-0.08548	-0.08548	-0.08672	-0.13331	-0.12547	-0.12547	-0.12018
dj	0.036244	0.021854	0.021854	0.015424	0.039447	0.026291	0.026291	0.019478	0.051452	0.029404	0.029404	0.019984
	0.095375				0.111507				0.130245			
wj	0.380014	0.229134	0.229134	0.161718	0.353759	0.23578	0.23578	0.17468	0.395041	0.225761	0.225761	0.153437

Table 9: The FPIS, NPIS and relatives closeness index

	D ⁺	D ⁻	CC _i	Rank
A ₁	0.215	0.386	0.642	2
A ₂	0.461	0.141	0.234	6
A ₃	0.241	0.361	0.600	4
A ₄	0.523	0.078	0.130	7
A ₅	0.224	0.377	0.627	3
A ₆	0.181	0.421	0.700	1
A ₇	0.283	0.318	0.529	5

5. Discussion

In this paper, a methodology that integrates Z-numbers with fuzzy TOPSIS was suggested to solve the decision making problem by using Z-numbers. As mentioned earlier, Z-numbers, that has considered the reliability in data information has more capability to explain the uncertainty compared to classical fuzzy number. In the proposed method, the weights of the criteria were determined using fuzzy entropy method and the evaluated Z-numbers data information were then changed into classical fuzzy numbers to obtain the solution. Although, converting Z-number to traditional fuzzy number leads to loss of information [19], however, the significant of the suggested approach can lower the difficulty in computation. Then, the concept of fuzzy TOPSIS is applied to rank the alternatives. Based on the results shown in Section 4, A_7 (extremely satisfied) is in the first rank, A_1 (extremely dissatisfied) in the second rank, and the last rank belongs to A_4 (neutral). From these results, we can conclude that the working women is extremely satisfied with their quality of life and subjective well-being even though they are facing the hectic life of being an employee and at the same time have to manage their family at home. The proposed method provides an efficient approach which can be easily extended to deal with other decision-making problems. For the future work, the Z-numbers can be applied to other MCDM method which can concentrated on using different method of determining criteria weights and also, further research can be done in other area such as in data mining.

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