EFFICIENT COMPUTATIONAL STRUCTURAL RELIABILITY ANALYSIS OF CONCRETE CONTAINMENTS

J. Sadeghi, M. de Angelis & **E. Patelli**, *University of Liverpool, UK*
N. K. Prinja, *Wood Plc, UK*

ABSTRACT

This paper presents probabilistic analysis of structural capacity of pre-stressed concrete containments subjected to internal pressure using the Advanced Line Sampling Method (an efficient advanced Monte Carlo simulation technique, which gives more accurate results than FORM). This task is an important part of level 2 probabilistic safety analysis in the nuclear industry, particularly when a reactor is undergoing design assessment. We compare our calculation with experimental results from two international round robin test exercises (Sandia National Laboratories and Bhabha Atomic Research Centre) and calculations using FORM, which are available in the literature. Since the ultimate structural collapse mode of the structures has already been established, we simply attempt to probabilistically determine the failure pressure of the containment. Our results show close agreement with previous calculations and demonstrate that structural engineers who are not specialists in risk engineering can obtain accurate probabilistic analyses of their models using freely available software.

NOMENCLATURE

FORM: First Order Reliability Method
PSA: Probabilistic Safety Analysis
SNL: Sandia National Laboratories
BARC: Bhabha Atomic Research Centre

1. INTRODUCTION

A pre-stressed concrete containment is a concrete structure designed to prevent the release of radiation from the core of a nuclear reactor to the environment. The structural reliability analysis of pre-stressed concrete containments is a key component of probabilistic safety analysis (specifically, level 2 PSA). The aim of this analysis is to determine the load conditions under which the structure will fail, for example we wish to determine if the structure will fail when a particular pressure is reached.

This analysis is usually conducted probabilistically since the contributing factors to the strength of the containment are subject to two forms of uncertainty. Firstly, the containment may be subject to aleatory uncertainty since there will be a natural variability of material properties throughout the containment, e.g. the concrete density may be subject to small variations. Secondly, even if there is no aleatory uncertainty, we may still suffer from a lack of knowledge regarding certain properties of the containment, and we refer to this as epistemic uncertainty [1]. Once these uncertainties are modelled we can calculate the failure probability of the containment due to a certain applied load (i.e. the fragility).

Various methods exist for calculating the probability of failure of a structural system. If the limit state function is linear and the random variables used to model the uncertainties are normally distributed, then the First Order Reliability Method (FORM) can be used. Even if these assumptions are not true, FORM is often a sufficiently accurate approximation for engineering purposes. However, if a more accurate calculation is required then the Monte Carlo Method can be used to obtain an unbiased estimate of the failure probability to arbitrary precision. Since the Monte Carlo Method relies upon random sampling there is an uncertainty in the failure probability proportional to the inverse of the square root of the number of samples. Assuming the samples are collected in serial this means that obtaining an accurate result can be very time consuming [2].

Fortunately, several 'Advanced Monte Carlo' methods exist to reduce the amount of time required by the Monte Carlo Method. These include Importance Sampling, Subset Simulation, Surrogate Model Methods (i.e. the Response Surface Methodology, Interval Predictor Models [3]) and Line Sampling. Each methodology has its advantages and disadvantages [4], for example Line Sampling is acknowledged as been the quickest but, in some situations, lacks accuracy. Subset Simulation is accurate and quick but is far more complex to implement than Monte Carlo Simulation. Surrogate Model Methods are subject to the weaknesses of the chosen surrogate model. Importance Sampling is simple to implement and quick, but often requires detailed knowledge of the engineering system being analysed (often this is the knowledge that we are trying to obtain by conducting structural reliability analysis!).

In [6] the structural reliability of a concrete containment was calculated using FORM, Importance Sampling and Subset Simulation. This was compared to two experimental test cases (Sandia National Laboratories and Bhabha Atomic Research Centre) which were conducted as a round robin international test exercise. In this paper we repeat the analysis using the Line Sampling Method available in the OpenCossan and COSSAN-X Software. OpenCossan is an open source and free generalised uncertainty quantification software package, which is jointly developed by the University of Liverpool and the University of Hannover, with modules for reliability analysis, sensitivity analysis, design optimisation and surrogate models. Crucially, the OpenCossan software can be used to determine the optimal design for the containment, to reduce the design cost whilst still providing a satisfactory probability of failure.
The software links to well-known structural engineering software automatically and can utilise high performance computing resources [5]. In this paper we aim to demonstrate that the Line Sampling Method available in OpenCossan is simple to use for novice users and obtains accurate results in a short amount of time. We also briefly describe the Response Surface method which is available in OpenCossan and may be useful when the performance function is not known explicitly.

2. STRUCTURAL MODEL

We consider a cylindrical concrete containment, and model the area and strength of the concrete, rebar, tendons and liner as normally distributed random variables. It would be more realistic to model these quantities as lognormally distributed random variables, however in this example it makes little practical difference as the Coefficient of Variation is reasonably low. In this study we will focus on the SNL containment. The properties of the Sandia National Laboratories containment are summarised in Table 1.

This is the same structural model considered in [6], where P was varied to produce a fragility curve for the containment. [6] also contains a sensitivity analysis of the coefficient of variation distribution parameter for the random variables used in the model and justifies why the choices made are conservative in an engineering sense. In [7] a similar analysis of a different containment model is conducted using imprecise probability techniques.

Table 1: SNL Containment Properties

Load and strength data	Mean Values	CoV
Concrete tensile strength, F_c	4.4	0.2
Liner yield, F_l	382	0.2
Rebar yield, F_s	465	0.2
Tendon yield, F_t	1740	0.2
Design pressure, P_d	0.39	0.2
Radius, R	5537.5	0.2
Concrete area, A_c	312.85	0.2
Liner area, A_l	1.6	0.2
Rebar area, A_s	6.85	0.2

Tendon area, A_t	3.7	0.2

The performance function of the containment can then be modelled using a simple load-strength relationship, i.e.

$$g = (A_s F_s + A_t F_t + A_l F_l + A_c F_c) - PR.$$

By convention, the structure is deemed to fail whenever *g*<0. The region in the space of the arguments of *g* where *g*<0 is known as the failure region. The surface where *g*=0 is known as the limit state surface. The first order reliability method (FORM) utilises an analytic relationship between the point of maximum probability density on a linear limit state surface (the design point) and the failure probability, when the random variables are normally distributed.

3. LINE SAMPLING METHOD

3.1 INTRODUCTION

The fundamental idea behind line sampling is to refine estimates obtained from the First-order reliability method (FORM), which may be incorrect due to the non-linearity of the limit state function. Conceptually, this is achieved by averaging the result of different FORM simulations [8]. In practice, this is made possible by identifying the importance direction in the input parameter space, which points towards the region which most strongly contributes to the overall failure probability. The importance direction can be closely related to the centre of mass of the failure region (or alternatively to the failure point with the highest probability density – the design point), which often falls at the closest point to the origin of the limit state function, when the random variables of the problem have been transformed into the standard normal space. Once the importance direction has been set to point towards the failure region, samples are randomly generated from the standard normal space and lines are drawn parallel to the importance direction in order to compute the distance to the limit state function, which enables the probability of failure to be estimated for each sample. This procedure is shown in Figure 1. These failure probabilities can then be averaged to obtain an improved estimate.

3.2 Algorithm

Firstly, the importance direction must be determined. This can be achieved by finding the design point, or the gradient of the limit state function.

A set of samples is generated using Monte Carlo simulation in the standard normal space. For each sample $\boldsymbol{x}$, the probability of failure in the line parallel to the important direction is defined as:

$$P_f(\boldsymbol{x}) = \int_{-\infty}^{\infty} I(\boldsymbol{x} + \beta \cdot \boldsymbol{\alpha}) \varphi(\beta) d\beta,$$

where $I(\cdot)$ is the indicator function returning 1 in the failure region and 0 otherwise, $\boldsymbol{\alpha}$ is the important direction, φ is the probability density function of a Gaussian distribution (and β is a real number). In practice the roots of a nonlinear function must be found to estimate the partial probabilities of failure along each line. This is either done by interpolation of a few samples along the line, or by using the Newton–Raphson method.

The global probability of failure is the mean of the probability of failure on the lines:

$$\tilde{P}_f = \frac{1}{N_L} \sum_{i=1}^{N_L} P_f^{(i)}$$

where N_L is the total number of lines used in the analysis and $P_f^{(i)}$ are the partial probabilities of failure estimated along all the lines.

For problems in which the dependence of the performance function is only moderately non-linear with respect to the parameters modelled as random variables, setting the importance direction as the gradient vector of the performance function in the underlying standard normal space leads to highly efficient Line Sampling. In general, it can be shown that the variance obtained by line sampling is always smaller than that obtained by conventional Monte Carlo simulation, and hence the line sampling algorithm converges more quickly.

It is unnecessary for engineers to have a detailed knowledge of the implementation of the algorithm, since all that is required to use the OpenCossan Algorithm is the choice of how to calculate the design point and the number of samples to be made.

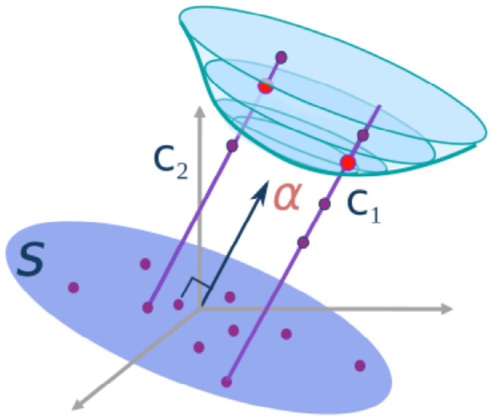

Figure 1: An illustration of the line sampling algorithm. Two line samples are shown approaching the limit state surface.

4. RESPONSE SURFACE METHOD

In some scenarios the exact performance function, and therefore limit state surface, may not be easily accessible. For example, this could be because a complex and time consuming finite element model is being used to predict structural response to a given load, or because only experimental data is available to describe the structural response of the system. In these cases it is desirable to construct an approximate performance function using mathematical methods from the field of machine learning. This process is known as surrogate modelling or metamodelling. Regression can be used to model the performance function or classification can be used to model the limit state surface. Once the model is constructed the appropriate analysis (e.g. Monte Carlo Simulation or Sensitivity Analysis) can be easily run on the metamodel at a negligible computational cost.

Specifically, the most well-known and widely used metamodelling technique is the response surface method where a polynomial (usually second degree) is regressed against the known structural response data [2]. The main advantages of the technique over more sophisticated strategies (e.g. neural networks or kriging models) are that the coefficients of the model have an easily interpretable physical meaning which allows engineers to qualitatively understand the behaviour of the structural system. The method is also simple and efficient to implement computationally. The accuracy of the approximation can be easily judged by 'holding out' a test set of data from the response surface training process, and later comparing the predictions of the model with this test data to obtain the so-called coefficient of determination and mean squared error statistics.

5. RESULTS

The failure probability was calculated by Line Sampling in OpenCossan using 100 lines, with 6 model evaluations on each line resulting in 600 samples of the performance function being made. This gave a failure probability of 2.6×10^{-8} when $P=P_d$ in a wall clock time of 2.2 seconds. The sampled lines are shown in Figure 2. The simulations concur with the values given in [6], which are summarised in Table 2.

Table 2: Summary of Failure Probabilities for SNL Containment, $P=P_d$

Method	P_f	Standard Deviation of P_f
Line Sampling	2.6×10^{-8}	7.0×10^{-9}
FORM	2.7×10^{-8}	Not Applicable
Importance Sampling	6.7×10^{-8}	1.8×10^{-9}
Subset Simulation	7.8×10^{-8}	2.4×10^{-9}

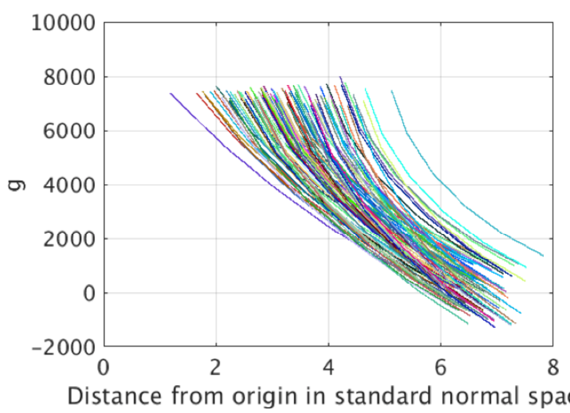

Figure 2: Plot of Lines used to calculate Failure Probability for SNL Containment, $P=P_d$

When $P=5.4P_d$ it is not appropriate to use the line sampling method, because the line sampling method is designed for problems with low failure probability (in other words when the limit state surface is far away from the mean of the random variables). When the failure probability is large it can be evaluated with the Monte Carlo Method. The results obtained in [6] are repeated in Table 3.

Table 3: Summary of Failure Probabilities for SNL Containment, $P=5.4P_d$

Method	P_f	Standard Deviation of P_f
Monte Carlo	0.489	0.005
FORM	0.507	Not Applicable

Our results show reasonable agreement with the results obtained from other methodologies.

6. CONCLUSIONS

The Line Sampling algorithm implemented in the OpenCossan software has been used to calculate the failure probability of a pre-stressed concrete containment. The failure probability is obtained accurately in a short amount of time, and the results obtained agree with experimental observation and previously calculated results. The software does not require specialist knowledge, outside of normal structural engineering domain knowledge.

We recommend that analysts with high performance computing resources attempt to connect their Finite Element Solvers to the OpenCossan software, as the sophisticated algorithms in OpenCossan allow PSA to be conducted using models which would previously only have been feasible for deterministic analysis, due to the high computational costs required.

REFERENCES

1. US NUCLEAR REGULATORY COMMISSION, 2007. ‘Guidance on the Treatment of Uncertainties Associated with PRAs in Risk-Informed Decision Making (Nureg-1855)’, Office of Nuclear Regulatory Research, Office of Nuclear Reactor Regulation.

2. SUNDARARAJAN, C.R., 2012. ‘Probabilistic Structural Mechanics Handbook: Theory and Industrial Applications’, Springer Science & Business Media.

3. SADEGHI, J., PATELLI, E., AND DE ANGELIS, M., 2018. ‘Robust Propagation of Probability Boxes by Interval Predictor Models’. Florianopolis, SC, Brazil. Proceedings of the Joint ICVRAM ISUMA Uncertainties Conference.

4. SHUËLLER, G.I., STIX, R., 1987 ‘A Critical Appraisal of Methods to Determine Failure’, Structural Safety, 4 (4), pp. 293-309

5. PATELLI, E., BROGGI, M., TOLO, S., SADEGHI, J., 2017 ‘Cossan Software: A Multidisciplinary and Collaborative Software for Uncertainty Quantification’, Proceedings of the 2nd ECCOMAS Thematic Conference on Uncertainty Quantification in Computational Sciences and Engineering, UNCECOMP 2017, Rhodes Island, Greece.

6. PRINJA, N.K., OGUNBADEJO, A., SADEGHI, J., PATELLI, E., 2016 ‘Structural Reliability of Pre-Stressed Concrete Containments’, Nuclear Engineering and Design, 323, Pp. 235-244.

7. SADEGHI, J., DE ANGELIS, M., PATELLI, E., 2018 ‘Analytic Imprecise-Probabilistic Structural Reliability Analysis’, ANS Best Estimate Plus Uncertainty International Conference (Bepu 2018). Real Collegio, Lucca, Italy, May 13-19, 2018.

8. DE ANGELIS, M., PATELLI, E., BEER, M., 2015 ‘Advanced Line Sampling for Efficient Robust Reliability Analysis’, Structural Safety, 52, Pp. 170-182.