Eur. Phys. J. Plus (2022) 137:521
https://doi.org/10.1140/epjp/s13360-022-02705-8

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Nonlinear convective motion of the asthenosphere and the lithosphere melting: a model for the birth of a volcano

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Received: 14 March 2021 / Accepted: 7 April 2022

Abstract The processes of heat transfer occurring between the Earth's asthenosphere and lithosphere are responsible for partial melting of rocks, leading to the magma generation and its migration and segregation in the crust and, possibly, to volcanoes generation at the surface. Convection is the dominant mechanism regulating the heat transfer from the asthenosphere to the lithosphere, although many aspects of the whole process are not yet clear. Therefore, the knowledge of the physical processes leading to the melting of the lithospheric rocks has important consequences in understanding the interior Earth dynamics, the surface volcanic dynamics, and its related hazards. Rock melting occurs when the temperature gradient meets the rock solidus. Here, we propose a nonlinear convective 1D analytical model (representing an approximation of more 3D complex models). The steady-state solution of our equation is in good agreement with the estimated geotherms of the asthenosphere. A perturbative approach leads to a heat swelling at the boundary between asthenosphere and lithosphere able to determine its melting and the birth of a volcano.

1 Introduction

Volcanic activity represents the most evident manifestation of the heat contained in the Earth interior that is constantly transferred from depth of hundreds of kilometers to the surface [1]. Temperature within the Earth increases with depth generating a different degree of rock melting. At large scale, a partial melting of rocks occurs in the asthenosphere where temperatures are up to 1600-1900 K and heat is mainly transferred by convection. The latter is the most efficient mechanism of heat transfer in the Earth. The asthenosphere represents the zone of Earth's upper mantle lying beneath the lithosphere and extending from about 100 km to about 700 km [2]. The boundary between the asthenosphere and the lithosphere represents the crucial zone of the upper mantle where most primitive magma initiates its journey towards the surface. The amount of melting of the upper mantle controls the ability to produce eruptible magmas [2]. There are different factors controlling the mantle melting: temperature, pressure, rock composition and the presence of fluids (mainly water). As a consequence, there are three main ways leading to melt the mantle: the first one is to lower the pressure, the second one is to change the chemical composition of the mantle rocks, and the third one is to raise the temperature [3–8]. These mechanisms are generally connected with different tectonic regimes associated with the tensile or the compressive ones [9]. A further mechanism governing the heating of the asthenosphere is generated by the viscous dissipation (frictional heating) which is generally small enough to be neglected [8].

Melting the mantle by lowering its pressure (decompression melting) is the most common and, probably, best understood mechanism. It occurs when the adiabat of the convecting mantle crosses the rock solidus. The difference between the temperature that would be achieved if the mantle rose along the adiabat without melting and the temperature that is achieved if the mantle stayed on the solidus provides the thermal energy necessary to generating a phase transition from solid to solid+liquid [10–14]. This mechanism is generally considered responsible for the generation of magmas in a tensile tectonic regime at mid-ocean ridges, in the backarc of subduction zones, at ocean islands, and in the interior of many continents [15].

Melting by changing composition occurs when the chemical nature of rocks is changed and the solidus temperature for the new composition is lower than the current temperature. The addition of a solute to a solvent expands the liquid stability field and lowers the freezing point of the solvent [16–18]. The most common cause of this type of melting in the Earth's mantle is the addition of water to mantle rocks at subduction zones.

A third rock melting mechanism is an anomalously high temperature. In this case, a steady-state mantle upwelling, called plume, is observed and a *hot spot* is generated at the Earth surface [6,8,19]. Mantle plumes are large convective systems, driven by buoyancy forces, that cannot be directly associated with the plate tectonic process. In fact, their relative fixity with respect

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to the oceanic plates highlights that they are poorly affected by the large-scale mantle circulation associated with the subduction zones and oceanic ridges [6,15]. The occurrence of mantle plumes or vertically extended zones of anomalous high temperature have been seismically recognized in the asthenosphere of different areas of our Planet (for instance, see [20–23]), whereas their thermodynamical/mechanical behavior has been investigated by laboratory analogue experiments and numerical modeling (see [2], and references therein). At the surface, hot spots, related to mantle plumes, were capable of generating a huge amount of basaltic magma, along the so-called *Large Igneous Provinces* (LIPs) which also result in a local crustal thickening [24,25]. These spots have been recognized in the Red Sea and Gulf of Aden in eastern Africa, Greenland in the North Atlantic, Deccan in northwestern Indian Ocean, Parana in South Atlantic, Karoo in southwestern Indian Ocean, Central Atlantic Margin, Siberian in northern Asia and Emeishan in China [26]. In particular, the presence of LIPs (whose geological age spans from few ten millions to few hundred millions of years) is recognized along the ancient rims of continents before they break up and subsequent ocean sea floor spread. Thus, many authors agree in recognizing the component of a mantle plume (and resulting flood basalt) as a prerequisite for the break up of a major oceanic basin and onset of spreading and rifting of the plates [19,24,26]. At the present geological time, isolated mantle plumes are responsible for the generation of volcanic hot spots such as the Hawaiian chain of islands in the Pacific Ocean, the Iceland along the Atlantic Ocean ridge and the Reunion Island in the Indian Ocean. Although the volcanism related to hot plumes represents only a part of the whole Earth's volcanism, its understanding is fundamental to reconstruct the processes governing the dynamics of the mantle and the plate tectonic.

In what follows, we focus on the third of the above mentioned mechanisms of mantle melting, assessing the importance of a thermal disturbance during the evolution of a thermal plume. This aspect has been widely investigated including thermo-mechanic effects [27–31], although many analytical aspects remain still unsolved [22,32]. Here, we introduce a very simplified analytical model for increasing the temperature at the boundary between asthenosphere and lithosphere, which is a crucial zone for the heat transfer and magmatic migration towards the surface. Of course, an increase of temperature at this boundary could be a melting mechanism in any tectonic regime and can be invoked as a good model when convection is involved in the process of heat transfer.

2 The model

The convection is the main process from which the heat of the Earth is transported by means of mass transfer [6]. Hot, partial melted and buoyant material is thus vertically moved by convection forming magma feeding the volcanic activity at the Earth surface. Thus, our starting point is represented by a standard Reynolds approach which states that the temperature T in a convective cell obeys to the equation

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T = D \nabla^2 T, \tag{1}$$

where $\mathbf{v}$ is the flux velocity and D is the thermal diffusivity.

In the sequel, we assume that the flux velocity $\mathbf{v}$ depends on the temperature T. Without loss of generality, we assume that v varies slowly as a function of T. Therefore, second- and higher order terms in the Taylor expansion of v can be neglected, so providing

$$\mathbf{v} = \mathbf{v}_0 + \boldsymbol{\alpha}(T - T_0), \tag{2}$$

where $\boldsymbol{\alpha}$ is the the derivative of velocity with respect to T evaluated in the reference temperature T_0.

Let us now assume that the temperature vertical gradient is much larger than the radial one. This could seem as an oversimplification compared to other studies that includes thermomechanical effects [27–31]. Nevertheless, we would like to remark that:

– rock melting is mainly due to the vertical temperature gradient and not to the radial one [6];
– the horizontal temperature gradient is negligible as compared to the vertical one; indeed, in an ideal convective cell, it should be zero at the middle plane cutting the cell;
– the presented model represents a first step providing a very simplified scheme as a prelude to more complex descriptions of the phenomenon.

In such a framework, Eq. (1) reads

$$\frac{\partial \Theta}{\partial t} + v_0 \frac{\partial \Theta}{\partial z} + \alpha \Theta \frac{\partial \Theta}{\partial z} = D \frac{\partial^2 \Theta}{\partial z^2}, \tag{3}$$

where $\Theta = T - T_0$, i.e., the difference between the actual temperature and the reference temperature.

2.1 The steady-state solution

Mantle convection is a very stable mechanism at the time scale of some million of years justifying plate tectonics and many of the geodynamic observed processes. As a consequence, we investigate the steady-state solutions of Eq. (3):

$$v_0 \frac{\partial \Theta}{\partial z} + \alpha \Theta \frac{\partial \Theta}{\partial z} = D \frac{\partial^2 \Theta}{\partial z^2}. \tag{4}$$

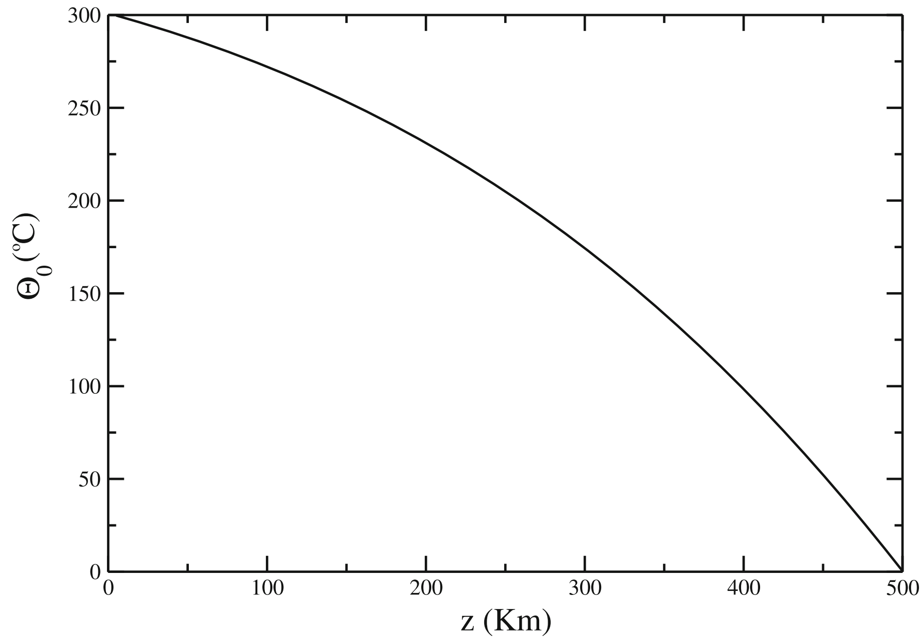

Fig. 1 Θ_0 versus z, where we used the parameters $v_0 = 5$cm/y$\simeq 5.8 \cdot 10^{-7}$m/s, $\Delta T = 300°$C, $D = 5 \cdot 10^{-4}$m^{2}s^{-1}, $\alpha = 3 \cdot 10^{-8}$m s^{-1} ^{o}C^{-1} and $\ell = 5 \cdot 10^5$m

Equation (4) admits the analytical solution

$$\Theta_0 = -\frac{v_0}{\alpha} - \frac{v_0 + \alpha \Delta T}{\alpha} \tanh\left[\frac{v_0 + \alpha\Delta T}{2D}(z - \ell)\right], \tag{5}$$

where we have used the boundary conditions $\Theta_0(0) = \Delta T$ and $\Theta_0(\ell) = 0$. Here, the origin is the bottom of the convective asthenosphere with thickness is ℓ, whereas ΔT is the temperature difference between bottom and top of the convective asthenosphere. Figure 1 shows the profile of Θ_0 versus z.

Almost all these values can be found in literature: v_0 has been fixed at the plate velocity at the earth surface, ΔT has been evaluated on the basis of the estimated temperature gradient with z, and ℓ has been fixed on the basis of global tomography (see, as an example, [8]). The diffusivity D has been extensively investigated and we use a value reported in literature (see, among the others, [33]). The value of α has been estimated through a linear interpolation assuming that v at the bottom of the convective asthenosphere is $\approx 10 v_0$.

2.2 A perturbative approach

Let us perturb the steady-state solution, say

$$\Theta = \Theta_0 + \Theta_1, \tag{6}$$

where Θ_1 is the perturbation.

Inserting the ansatz (6) into (3), and neglecting second order terms, we obtain

$$\frac{\partial \Theta_1}{\partial t} + (v_0 + \alpha\Theta_1)\frac{\partial \Theta_0}{\partial z} + (v_0 + \alpha\Theta_0)\frac{\partial \Theta_1}{\partial z} = D\frac{\partial^2 \Theta_1}{\partial z^2}. \tag{7}$$

Finally, using the steady solution (5), we get

$$\begin{aligned}\frac{\partial \Theta_1}{\partial t} - (v_0 + \alpha\Theta_1)\frac{(v_0 + \alpha\Delta T)^2}{\Delta T \alpha}\mathrm{sech}^2\left[\frac{v_0 + \alpha\Delta T}{2D}(z - \ell)\right] \\ - \alpha\Delta T \tanh\left[\frac{v_0 + \alpha\Delta T}{2D}(z - \ell)\right]\frac{\partial \Theta_1}{\partial z} = D\frac{\partial^2 \Theta_1}{\partial z^2}.\end{aligned} \tag{8}$$

Introducing the new independent variables

$$\xi = \frac{v_0 + \alpha\Delta T}{2D}(z - \ell), \qquad \tau = \frac{(v_0 + \alpha\Delta T)^2}{4D}t, \tag{9}$$

we simplify Eq. (8) that now reads

$$2\mathrm{sech}^2(\xi)\Theta_1 + 2\tanh(\xi)\frac{\partial \Theta_1}{\partial \xi} + \frac{\partial^2 \Theta_1}{\partial \xi^2} - \frac{\partial \Theta_1}{\partial \tau} = 0. \tag{10}$$

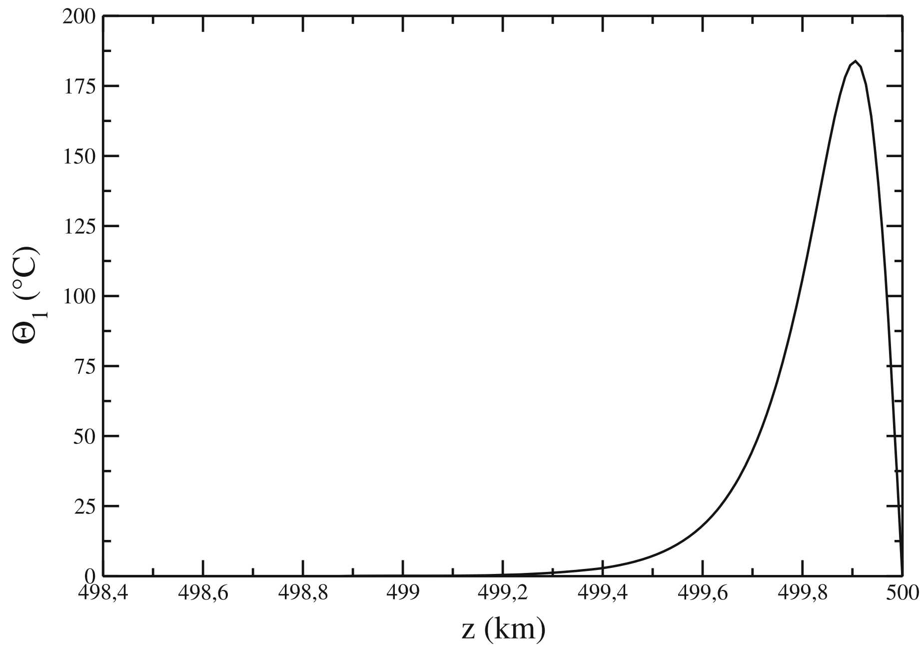

Fig. 2 The section at $t = 0$ of the solution (15) along with (9)

Let us look for a solution of Eq. (9) with the method of separation of variables, namely

$$\Theta_1(\tau, \xi) = f(\tau)g(\xi); \tag{11}$$

in this case, we obtain two ordinary differential equations that can be easily solved, providing

$$\begin{aligned} f(\tau) &= k_2 \exp(k_1 \tau), \\ g(\xi) &= k_3 P_1^m \tanh(\xi)\sqrt{1 - \tanh^2(\xi)} + k_4 Q_1^m \tanh(\xi)\sqrt{1 - \tanh^2(\xi)} \end{aligned} \tag{12}$$

($m = \sqrt{1 + k_1}$), where $P_n^m(x)$ is the associated Legendre polynomial,

$$P_n^m(x) = (-1)^m (1 + x^2)^{m/2} \frac{d^m P_n(x)}{dx^m}, \tag{13}$$

$P_n(x)$ being the standard Legendre polynomial, and $Q_n^m(x)$ the associated Legendre polynomial of second kind,

$$Q_n^m(x) = (-1)^m (1 + x^2)^{m/2} \frac{d^m Q_n(x)}{dx^m}, \tag{14}$$

$Q_n(x)$ being the standard Legendre polynomial of second kind; moreover, k_i $(i = 1, \ldots, 4)$ are arbitrary constants.

Notice that, in order to have a solution not diverging with time, k_1 has to be less than 0. In particular, the choice $k_1 = -1$ considerably simplifies the expression of Legendre polynomials and, consequently, of the solution. Moreover, the constant k_2 can be adsorbed in k_3 and k_4, so that there is no loss of generality in fixing $k_2 = 1$.

Thus, the solution assumes the form

$$\Theta_1(\tau, \xi) = \exp(-\tau)\mathrm{sech}(\xi)\tanh(\xi)(k_4(1 - \xi) - k_3). \tag{15}$$

We would like to remark that, due to (9), this solution has to be considered for negative values of ξ (corresponding to the values of $z \in [0, \ell]$) and that $\Theta_1 = 0$ for $z = 0$ and assumes very small values at $z = \ell$. Hereafter, we adopt $k_3 = 400$ and $k_4 = 20$. Of course, this choice is arbitrary and can be modified without loss of generality. For $\tau = 0$ (corresponding to $t = 0$), the solution attains a maximum for $\xi \approx -0.8546$ (corresponding to $z \approx 499.9$ Km, very close to $z = \ell$), where $\Theta_1 \approx 181.321\,°\mathrm{C}$ (see Fig. 2). The location and sharpness of the peak are controlled by v_0, α, ΔT and D, which are experimental parameters not susceptible of large variations. The peak of Θ_1, greater than 100 °C, added to the steady-state value $\Theta_0(\ell) = 0$, and assuming $T_0 \approx 1200\,°\mathrm{C}$, is sufficient to cause the melting of the rocks [8] at the top of the asthenosphere.

Figure 3 shows the 3D plot of $\Theta_1(z, t)$ for the above selected set of constants. Thus, the melting of rocks at the border between asthenosphere and lithosphere can occur. At this point, the melted rocks can start their buoyant journey through the lithosphere and towards the upper crust.

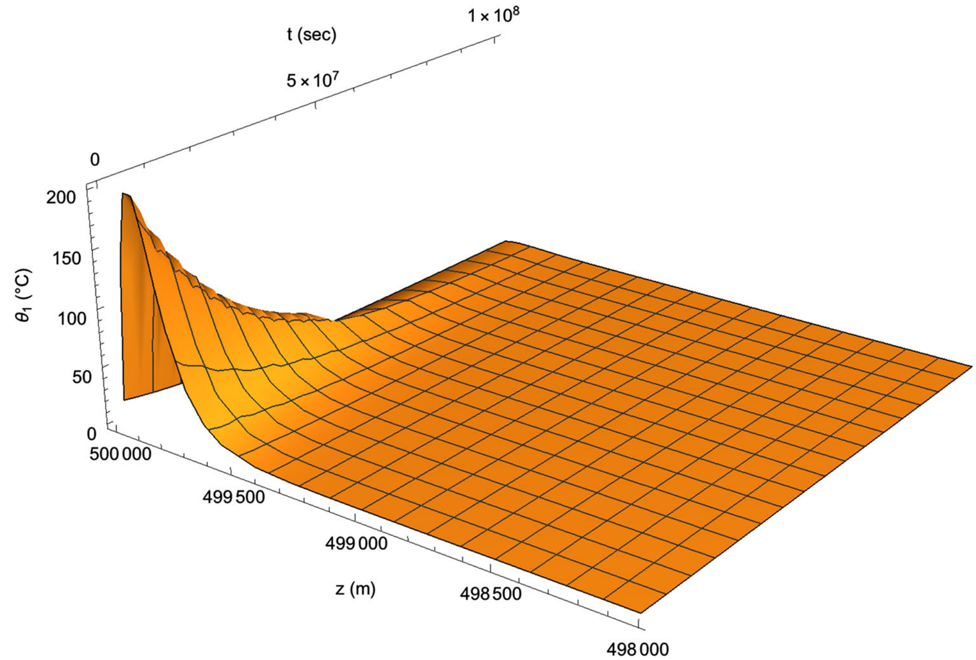

Fig. 3 Plot of $\Theta_1(t, z)$ given by solution (15) along with (9). Here, we used the same parameters as in Fig. 1 (see text for the choice of the parameters k_3 and k_4)

3 Conclusions

We have presented a nonlinear convective 1D model representing a very simple approximation of more complex 3D models. The steady state solution of our model is in good agreement with the estimated temperature gradients in the earth mantle. Then this steady-state solution has been perturbed, and a new linearized equation for such a perturbation has been obtained. The solution of this equation represents a possible model for the birth of a volcano. Indeed, at the boundary between the asthenosphere and the lithosphere, the thermal escalating can lead to further rocks melting enhancing their ability to move upwards. At this point, a high-temperature primitive magma batch can rise, up to the crust, where it can accumulate at different depths and finally erupts. This picture agrees with the evidence that most crustal magmatism is driven, ultimately, by the influx of mantle-derived basaltic melt [34].

Notice that at $z \approx \ell$ both asthenosphere and lithosphere can melt and the ratio between the mass of melted asthenosphere and the mass of melted lithosphere could determine the initial composition of high-temperature magmas. Our model predicts that high-temperature magmas are expected to dominate the first eruptive products of a new plume, a result which is confirmed by many observations (see, among the others, [24], and references therein). Furthermore, the temperature swelling at the top of the plume of our solution represents a transient stage which can be associated with the onset of large magmatic volcanism such that observed in the LIPs. Thus, a transient stage allows huge magma generation in the lithosphere and large volcanic activity, whereas steady-state condition of the plume corresponds to subsequent volcanism. This can continue, as an intermittent phenomenon, for millions of years, like what is observed in the Hawaiian island, up to the complete depletion of the plume tail [35].

As a final observation, we would like to remark that the solution (15) is independent of the tectonic regime. Therefore, it can explain not only hot spots, but also any type of volcanism in which the remelting of subducted crust (such as backarc volcanism) or a branch of convective cell (such as mid-ocean ridges) generate a super-heated plume of melted and low-density material.

Acknowledgements We would like to thank the MIUR project PRIN 2017 WZFT2p for partial financial support.

Funding Our work was partially supported by the MIUR project PRIN 2017 WZFT2p.

Data Availability No data or materials has been used in the present manuscript.

Code availability No specific code has been used in the present manuscript.

Declarations

Conflict of interest The authors declare that they have no conflict of interest. This research would be used only for scientific research and would not passed on to third parties.

References

1. F. Press, R. Siever, J. Grotzinger, T.H. Jordan, *Understanding Earth* (Macmillan, London, 2004), p. 784
2. G. Schubert, D.L. Turcotte, P. Olson, *Mantle Convection in the Earth and Planets* (Cambridge University Press, Cambridge, 2001)
3. P.D. Asimow, C.H. Langmuir, The importance of water to oceanic mantle melting regimes. Nature **421**, 815–820 (2003)
4. D.P. Asimow, A measure of mantle melting. Science **355**(6328), 908–909 (2017)
5. G.C. Brown, A.E. Mussett, *The inaccesible earth.* (An integrated view of its structure and composition, p. 1993 (1993)
6. C. Jaupart, J.-C. Marechal, *Heat Generation and Transport in the Earth* (Cambridge University Press, Cambridge, 2011)
7. K.A. Kelley, T. Plank, T.L. Grove, E.M. Stolper, S. Newman, E. Hauri, Mantle melting as a function of water content beneath back-arc basins. J. Geophys. Res. Solid Earth 111(B9), (2006)
8. D.L. Turcotte, G. Schubert, *Geodynamics* (Cambridge University Press, Cambridge, 2017)
9. F. Press, R. Siever, *Earth* (WH Freeman, New York, 1986)
10. R.V. Conceição, D.H. Green, Derivation of potassic (shoshonitic) magmas by decompression melting of phlogopite+pargasite lherzolite. Lithos **72**(3), 209–229. (2004) http://www.sciencedirect.com/science/article/pii/S0024493703001774. – ISSN 0024-4937
11. J.A. Conder, A.D. Wiens, J. Morris, On the decompression melting structure at volcanic arcs and back-arc spreading centers. Geophys. Res. Lett. **29**(15), 17-1-17–4 (2002)
12. J. Malcolm, The generation of continental flood basalts by decompression melting of internally heated mantle. Geology **43**(04), 311–314 (2015)
13. P.J. Adrian, G.D. Price, J.P. Neville, S.D. Paul, A.C. Richard, Impact induced melting and the development of large igneous provinces. Earth Planet. Sci. Lett. **202**(3), 551–561 (2002)
14. M.J. Raddick, E.M. Parmentier, D.S. Scheirer, Buoyant decompression melting: a possible mechanism for intraplate volcanism. J. Geophys. Res. Solid Earth **107**(B10), ECV7-1-ECV7-14 (2002)
15. P. Kearey, K.A. Klepeis, F.J. Vine, *Global Tectonics* (Wiley, Hoboken, 2009)
16. R.M. Palin, R.W. White, E.C.R. Green, J.F.A. Diener, R. Powell, T.J.B. Holland, High-grade metamorphism and partial melting of basic and intermediate rocks. J. Metamorphic Geol. **34**(9), 871–892 (2016)
17. R.W. White, N.E. Pomroy, R. Powell, An in situ metatexite-diatexite transition in upper amphibolite facies rocks from Broken Hill. Australia. J. Metamorphic Geol. **23**(7), 579–602 (2005)
18. H.M. Williams, A.H. Peslier, C. McCammon, A.N. Halliday, S. Levasseur, N. Teutsch, J.-P. Burg, Systematic iron isotope variations in mantle rocks and minerals: the effects of partial melting and oxygen fugacity. Earth Planet. Sci. Lett. **235**(1), 435–452 (2005)
19. R.I. Hill, I.H. Campbell, G.F. Davies, R.W. Griffiths, Mantle plumes and continental tectonics. Science **256**(5054), 186–193 (1992)
20. H. Bijwaard, W. Spakman, Tomographic evidence for a narrow whole mantle plume below Iceland. Earth Planet. Sci. Lett. **166**(3), 121–126 (1999)
21. F. Gerlings, H.R. Joanna, J. Thomas, E.L. Keith, F. Klingelh, Seismic evidence for plume-derived volcanism during formation of the continental margin in southern Davis Strait and northern Labrador Sea. Geophys. J. Int. **176**(03), 980–994 (2009)
22. N. Henri-Claude, Seismic imaging of mantle plumes. Ann. Rev. Earth Planet. Sci. **28**(1), 391–417 (2000). https://doi.org/10.1146/annurev.earth.28.1.391
23. Yang Shen, Sean C. Solomon, Ingi T. Bjarnason, Cecily J. Wolfe, Seismic evidence for a lower-mantle origin of the Iceland plume. Nature **395**, 62–65 (1998)
24. I.H. Campbell, Large igneous provinces and the mantle plume hypothesis. Elements **1**(5), 265–269 (2005)
25. P. Francis, C. Oppenheimer, *Volcanoes* (Oxford University Press, Oxford, 2004)
26. V. Courtillot, C. Jaupart, I. Manighetti, P. Tapponnier, J. Besse, On causal links between flood basalts and continental breakup. Earth Planet. Sci. Lett. **166**(3), 177–195 (1999)
27. A.Y. Babeyko, S.V. Sobolev, High-resolution numerical modeling of stress distribution in visco-elasto-plastic subducting slabs. Lithos **103**(1), 205–216 (2008)
28. T.V. Gerya, *Introduction to Numerial Geodynamic Modelling* (Cambridge University Press, New York, 2019), p. 2019
29. A.A. Popov, S.V. Sobolev, SLIM3D: a tool for three-dimensional thermomechanical modeling of lithospheric deformation with elasto-visco-plastic rheology. Phys. Earth Planet. Interiors **171**(1), 55–75 (2008)
30. T. Roubíček, Thermodynamically consistent model for poroelastic rocks towards tectonic and volcanic processes and earthquakes. Geophys. J. Int. **227**(0)8(3), 1893–1904. (2021)
31. V.M. Yarushina, Y.Y. Podladchikov, (De)compaction of porous viscoelastoplastic media: model formulation. J. Geophys. Res. Solid Earth **120**(6), 4146–4170 (2015)
32. K.C. Condie, *Mantle Plumes and Their Record in Earth History* (Cambridge University Press, Cambridge, 2001), p. 2001
33. G. Benoit, R.S. Frank, T. Gratz, A. Kristin, Thermal diffusivity of olivine single crystals and a dunite at high temperature: evidence for heat transfer by radiation in the upper mantle. Phys. Earth Planet. Interiors **151**(1), 129–141 (2005)
34. K.V. Cashman, R.S. Sparks, J.D. Blundy, Vertically extensive and unstable magmatic systems: a unified view of igneous processes. Science **eaag355**(6331), 3055 (2017)
35. W. Jason Morgan, Hotspot tracks and the early rifting of the Atlantic. Tectonophysics **94**(1), 123–139 (1983)