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Utilizing Post-Newtonian Expansion to Determine Parameters of Compact Binary Black Hole Mergers

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UTILIZING POST-NEWTONIAN EXPANSION TO DETERMINE PARAMETERS OF COMPACT BINARY BLACK HOLE MERGERS

by

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ABSTRACT

The process of determining parameters of black hole mergers requires complicated formulae like the Einstein Field Equations (EFEs) that can only be solved numerically with the help of supercomputers. This paper sought to explore an alternative method to prediction of parameters through the use of 1st order Post-Newtonian Expansion (PNE), which is a way of approximating solutions to the EFEs. Two binary-black hole mergers, GW170814 and GW170809 were analyzed with the use of 1st order PNE to obtain the chirp mass and radiated energy parameters. These parameters were then compared with the parameters obtained using numerical solutions to the EFEs and it was found that 1st order PNE is insufficient in the case of these two mergers. This does not entirely discount the use of 1st order PNE for prediction, but higher order approximations may yield better predictive results.

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1. INTRODUCTION

Uncovering the secrets of black holes and the early universe is a major endeavor for astrophysicists, and within the past decade, major strides have been made in detector technology. These strides are in the area of gravitational wave detection, which is itself a major subject of research today with several projects both already running and planned to further study them. The current detection facilities include LIGO, Virgo, and KAGRA though the main focus of this particular project is on data from the LIGO-Hanford and Virgo facilities. LIGO was the first to receive a gravitational wave signal, on September 14, 2015, and they denoted the event GW150914 [7].

Gravitational waves are produced by massive objects undergoing extreme accelerations, such as orbiting black holes/neutron stars. Even the Earth-Sun system produces gravitational waves, though these are beyond the sensitivity of current detectors which barely reach into the kHz range. For more details on high frequency detection, see [3]. We will define the specifics of a gravitational wave in a later section. This paper will focus on orbiting black holes, and even more specifically on black hole mergers, which occur when these orbiting objects are inspiral over long periods of time. At the point of merger, they produce a large burst of gravitational waves that "chirps" in frequency when detected. Readers are assumed to have a basic understanding of Newtonian dynamics and calculus, the dynamics behind binary black holes will be covered in the background section.

These gravitational waves can be modeled and described in depth by the Einstein field equations (EFEs), which are sets of dense differential equations describing how the local geometry of space-time changes due to the local energy in the area. Due to the nature of these equations, solving them exactly requires simplified symmetry conditions, such as a single stationary black hole, or other simple object. More complex systems, such as the orbiting black holes that this paper will deal with, require computational solutions to the EFEs that can create precise models. With these models, one can match experimental and analytical data to determine parameters of the system being analyzed, such as mass, energy released, and any other variables of interest.

1.1 Computational Alternatives

Computational solutions of the EFEs have some drawbacks, such as the length of time necessary to ensure precision and the hardware required to reduce said time to months instead of years. Consequently, the EFEs are not the main focus of this paper, and they will not be described in depth. Instead, see [14] for more details on numerical relativity.

Until computational times decrease significantly, when seeking basic analysis and confirmation of detected events, the downsides of numerical relativity outweigh the benefits, and an alternative method is preferred. This paper will instead focus on one such alternative method, Post-Newtonian Expansion (PNE), and how well it can be used to predict parameters of black hole mergers. This method is used to analyze processed experimental data and obtain parameters based on arguments using classical physics combined with some relativistic arguments. PNE are approximate solutions to the EFEs that consist of power-series that can be expanded to as many orders as we might want for different parameters. This paper focuses entirely on the first-order approximation and how well we can utilize the singular parameter obtained to gather more information on the observed event through the use of some reasonable assumptions.

1.2 Literature Examples

The literature surrounding black hole mergers has some examples of PNE being used to do analysis of a system of two orbiting black holes, usually going to higher-order approximations of at least third order.

L. Blanchet [6] discusses using high order PNE to model compact binary systems. He describes the reasons why PNE can be utilized to describe compact binary systems and goes into the assumptions necessary for accuracy in both higher and lower order approximations.

Campanelli, Manuela, et al. [8] compare models created using PNE to numerical relativity models and conclude that higher order PNE waveforms have advantages over lower order ones. Higher order waveforms yield better waveforms in analysis.

1.3 Outline

This paper focuses on two gravitational events, GW170814 [1] and GW170809 [2] which have already been confirmed to be black hole mergers, and their specific parameters. A direct analysis of LIGO data was done using Python, specifically PyCBC [4] which is designed to analyze gravitational wave data. This analysis involved band-passing and frequency filtering to clean the signal, and then the frequencies associated with this data were determined. This frequency data was then plotted versus time with linear regression to determine the slope. The slope was then used to determine chirp mass and, through some analysis, energy emitted by gravitational waves. These parameters were then compared to the parameters already found to determine effectiveness of 1st order PNE analysis.

1.4 Outline

In this paper we will start in Section 2 by describing the basics of gravitational waves and compact binary systems. We will then move on to a brief overview of first order PNE in Section 3, deriving the expressions we will use later to determine our parameters. Next, we will describe our process of analyzing gravitational wave data in Section 4, and then move on to a discussion of results and our conclusions in Section 5.

2. BACKGROUND

The majority of the background of this paper revolves around general relativity and Einstein's predictions surrounding the theory. The theoretical framework of general relativity is far beyond the scope of this paper, but the beauty of our first order PNE methods is that a rigorous understanding of general relativity is not required to still be able to make parameter predictions of compact two body systems.

2.1 Gravitational Waves

Gravitational waves are a perturbation of space created by massive objects undergoing acceleration, and like all waves, they have a frequency, and an amplitude, sometimes called strain, a dimensionless quantity. These waves transmit energy as they perturb spacetime, and this energy transmission is the reason for inspiral and merger, as the gravitational waves take energy from the orbit and cause it to decay. The specifics of the energy decay will be discussed later. Any orbiting system has this energy decay, though for systems like the Earth orbiting the Sun, the energy decay is on a long time scale.

Einstein first predicted the existence of gravitational waves in 1916 [10], later providing a rigorous solution of the EFE for cylindrical gravitational waves [9]. We note here that this phenomenon is not predicted by standard Newtonian gravitational analysis, which assumes that physical interactions such as gravity propagate instantaneously, which is one reason PNE is called “Post-Newtonian” as it stands in-between the assumptions of general relativity and Newtonian mechanics.

As mentioned in section one, even the Earth-Sun system generates gravitational waves, but in this paper, we will be looking at specific compact two body systems so that we can properly apply PNE.

2.2 Two-Body Systems

We will wait until our section on black holes to properly define compactness in our case, but we can define a two-body system. A two-body system in our case will be a system of two black holes, though the more generic case is any system of two orbiting objects. For example, the Earth-Sun system is a two-body system.

We assume that neither object is spinning, which only means that our energy radiated value will be a lower bound prediction. We also assume that their orbits are Keplerian, so that we can use some arguments using Kepler's laws and remain generally circular until merger. There are very few examples of elliptical mergers as the energy radiated will tend to circularize the orbit, but there have been very recent estimates of orbital eccentricity in numerical relativity simulations due to the GW190521 event [11]. As these events are rare and under large mass conditions, our circular orbit assumption can still hold. For more information on PNE assumptions, see [13] which describes the mass range in which non-chaotic behavior occurs that allows for the use of PNE.

2.3 Black Holes

Black holes come in four flavors, which come from a combination of charge and rotation. For the purposes of this paper, we will focus on Schwarzschild black holes, which are uncharged and non-rotating which comes from an earlier assumption. At a most basic level, black holes are massive objects which have been compressed to below a metric known as the Schwarzschild radius, at which irreversible gravitational collapse occurs. The Schwarzschild radius is defined as follows,

$$r_s = \frac{2GM}{c^2}.$$

Equation 1

With G being the gravitational constant, c being the speed of light, and M being the mass of the object. With this metric we can now complete our definition of a compact two body system, in which compactness in our case will be distances of within an order of magnitude to the Schwarzschild radius of each black hole. The orbit of these objects will be constrained to a single plane. Now with black holes defined, we have one thing left to define before moving on to our section on PNE.

2.4 Gravitational Quadrupole Moment and Chirp Mass

Before going into the details of Post-Newtonian expansion, we will first talk about the gravitational quadrupole moment, and chirp-mass of a compact binary system. The gravitational quadrupole is a measure of how stretched out a mass along a particular axis, and we will give the mathematical definition for our system in our section on PNE.

The chirp mass of our system is related to the two masses of our black hole system. It effectively is a reduced mass that measures the gravitational wave emission from a binary system. The chirp mass will be defined as,

$$\mathcal{M} = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}}.$$

Equation 2

With m_1 and m_2 being the masses of each object in our two-body system. The reason why we are using chirp mass instead of attempting to determine the separate masses of each individual black hole, is because we only need a 1st order Post-Newtonian expansion to determine chirp-mass, and it still provides detail about the system that can be used in conjunction with some assumptions to calculate a few parameters. We do not need to derive this form because it will appear in our derivation of 1st order PNE and we will rewrite the expression as the chirp mass. We now have a solid framework in place of our system, so we can move on to the theoretical framework behind parameter estimation.

3. POST-NEWTONIAN EXPANSION

Post-Newtonian expansions are designed as approximate solutions of Einstein's field equations for the metric tensor. They are power series solutions to the EFE that can be expanded or contracted depending on which parameters are desired to be extracted. For a thorough derivation, see [17] which provides a rigorous description of the different mathematical steps involved for general PNE of any order. Since this paper is focused on the first order expansion, there will be a derivation of this expansion in simpler terms, utilizing the quadrupole moment of the two-body system in question, which is related to the stress-energy tensor of the system. We are using this because it is the lowest order term in the Newtonian limit, in which $\frac{1}{c} \longrightarrow 0$, with c being the speed of light. Time variation in the quadrupole moment can also produce gravitational radiation, unlike time variation in the lower multipole or monopole moments, which will give us our gravitational wave source. This limit is possible since c is very large ($\sim 10^8$).

Also note that there are a couple of assumptions made under PNE, namely that the system is slowly moving and weakly stressed. These assumptions are quite relative with regards to the systems in question, since this paper is dealing with compact-binary systems which are moving at speeds in excess of 50% the speed of light, with large changing gravitational fields, but PNE still holds under these conditions [5].

3.1 Derivation of 1st Order PNE

The majority of this derivation comes from [15]. Start with the general two body system we have described, orbiting in a Cartesian coordinate system (x, y, z), with masses m_1 and m_2. The Quadrupole moment of this system is,

$$Q_{ij} = \int d^3x\rho(x,y,z)(x_i x_j - \tfrac{1}{3}r^2\delta_{ij}),$$

Equation 3

with r being the radial distance from the origin (the system center of mass), δ_{ij} being the Kronecker delta, and $\rho(x,y,z)$ being the mass density. For a two-body system A $\in (1,2)$ rotating in the x-y plane, this equivalent to,

$$Q_{ij} = \sum_{A\in(1,2)} m_A \begin{bmatrix} \frac{2}{3}x_A^2 - \frac{1}{3}y_A^2 & x_A y_A & 0 \\ x_A y_A & \frac{1}{3}y_A^2 - \frac{2}{3}x_A^2 & 0 \\ 0 & 0 & -\frac{1}{3}r_A^2 \end{bmatrix}$$

Equation 4

Einstein found [9] that the gravitational wave strain h at a distance $d_L$is defined as,

$$h_{ij} = \frac{2G}{c^4 d_L}\frac{d^2Q_{ij}}{dt^2},$$

Equation 5

with the rate at which the energy of the gravitational wave is changing due to these waves over time given by

$$\frac{dE_{GW}}{dt} = \frac{c^3}{16\pi G}\int\int \left|\frac{dh}{dt}\right|^2 dS = \frac{G}{5c^5}\sum_{i,j=1}^{3}\frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3},$$

Equation 6

with $\left|\frac{dh}{dt}\right|^2 = \sum_{i,j=1}^{3}\frac{dh_{ij}}{dt}\frac{dh_{ij}}{dt}$, the integral being over a sphere at radius d_L, G being the gravitational constant, and c being the speed of light. Utilizing some trigonometry, we can simplify Equation 4. The orbit is assumed to be in the x-y plane, and circular, which means $r = r_1 + r_2$ and frequency $f = \frac{\omega}{2\pi}$, giving us

$$Q_{ij}^{A}(t) = \frac{m_A r_A^2}{2} I_{ij},$$

Equation 7

with $I_{xx} = \cos(2\omega t) + \frac{1}{3}$, $I_{yy} = \frac{1}{3} - \cos(2\omega t)$, $I_{yx} = I_{xy} = \sin(2\omega t)$ and $I_{yx} = \frac{1}{3}$. We can combine our expressions in Equation 7 to find $Q_{ij}(t) = \frac{\mu r^2}{2} I_{ij}$ with μ being the reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ and apply that to Equation 6 to find,

$$\frac{dE_{GW}}{dt} = \frac{32G}{5c^5} \mu^2 r^4 \omega^6.$$

Equation 8

The energy loss from this system mainly takes energy from the orbital energy,

$$E_{orb} = -\frac{GM\mu}{2r},$$

Equation 9

and therefore

$$\frac{dE_{orb}}{dt} = \frac{GM\mu}{2r^2}\frac{dr}{dt} = -\frac{dE_{GW}}{dt}.$$

Equation 10

Assuming the energy radiated away over each orbit is small in comparison to E_{orb}, each orbit can be described as approximately Keplerian. We can use Kepler's third law, $r^3 = \frac{GM}{\omega^2}$ [16], and its derivative, $\frac{dr}{dt} = -\frac{2}{3} r \frac{d\omega}{dt}\frac{1}{\omega}$, in combination with Equations 8 and 10 to obtain our final equation,

$$(\frac{d\omega}{dt})^3 = (\frac{96}{5})^3 \frac{G^5 \mu^3 M^2}{c^{15}} \omega^{11} = (\frac{96}{5})^3 \frac{(G\mathcal{M})^5}{c^{15}} \omega^{11},$$

Equation 11

where $M = m_1 + m_2$ and $\mathcal{M}$ is the chirp mass from Equation 2. This can be converted into a frequency equation, to find

$$\frac{df}{dt} = \frac{96}{5}\pi^{\frac{8}{3}}\left(\frac{G\mathcal{M}}{c^3}\right)^{\frac{5}{3}} f^{\frac{11}{3}}.$$

Equation 12

This finishes our derivation of our equation that we will use to perform parameter estimation, which we can now move on to.

4. PARAMETER ESTIMATION

Now that we have an expression that contains chirp mass, we can do some rearranging and use it alongside experimental data to obtain some parameters.

4.1 Chirp Mass Estimation

From Equation 12, we can rearrange terms to get an equation of the form,

$$f^{-\frac{11}{3}}df = \frac{96}{5}\pi^{\frac{8}{3}}(\frac{G\mathcal{M}}{c^3})^{\frac{5}{3}}dt,$$

Equation 13

which can then be integrated to find

$$-\frac{3}{8}f^{-\frac{8}{3}} = \frac{96}{5}\pi^{\frac{8}{3}}(\frac{G\mathcal{M}}{c^3})^{\frac{5}{3}}t + c.$$

Equation 14

We can rearrange this again and isolate frequency to give our final equation

$$f^{-\frac{8}{3}} = -\frac{256}{5}\pi^{\frac{8}{3}}(\frac{G\mathcal{M}}{c^3})^{\frac{5}{3}}t + c.$$

Equation 15

This equation relates the frequency of the gravitational waveform, which increases as the orbiting black holes inspiral towards a merger. The constant of integration is known as the time of coalescence [15] which is beyond the scope of this paper. By measuring the $f^{-\frac{8}{3}}$, and plotting against time, we can use simple linear regression to find a best fit line. From that line we can obtain a slope and solve for chirp mass. We can do this without any assumptions of a particular waveform model by measuring the t between successive zero crossings and estimating the frequency as $f = \frac{1}{2\Delta t}$ since $f = \frac{1}{T}$ and we will be finding half the time to complete one cycle.

We can do all of this using Python and PyCBC [4], the specific code used can be found in Appendix A. The data from LIGO-Hanford and VIRGO was whitened, band-passed and frequency filtered at the frequencies recommended by PyCBC. After analysis, the VIRGO data was found to be too noisy for the analysis without assumption of waveform and so only LIGO-Hanford (L1 and H1 respectively) was used. The data was first high pass filtered, to remove frequencies lower than what is reasonable for the data sets we are working with. The data was then whitened to bring the waveform into better focus, and then the data was high and low passed to reduce noise. The result of this filtering can be seen below, in Figure 1.

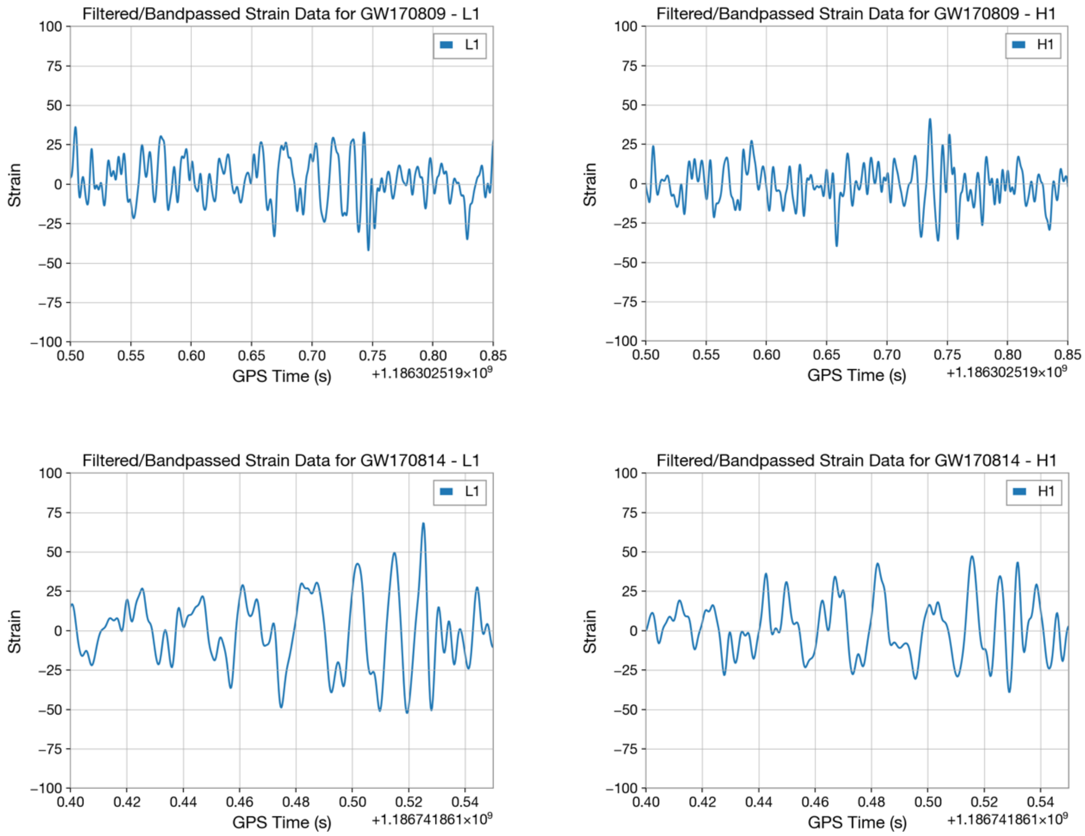

Figure 1. The strain data for both GW170814 and GW170809 plotted against GPS time, which is shown in seconds form, note that for GW170814, the data was whitened slightly more than GW170809 though there still is some noise present.

This data was then transferred into R for regression analysis, the results of which can be seen below, in Figure 2.

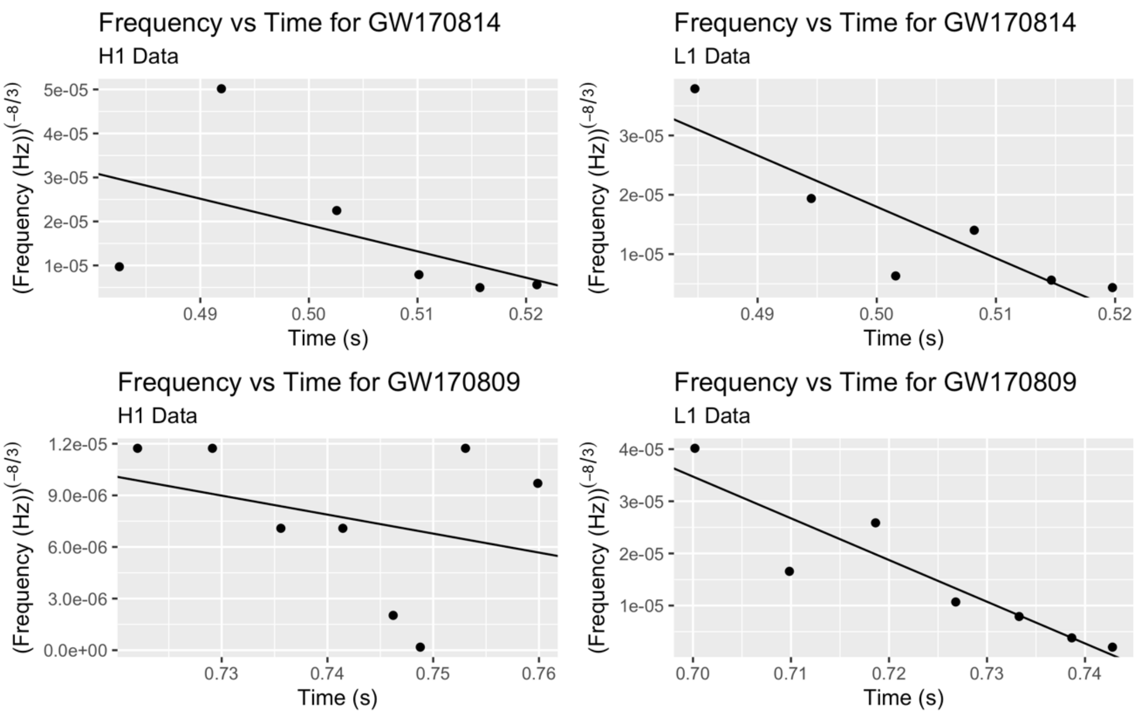

Figure 2. The regression models for each merger, fitting $f^{-\frac{8}{3}}$ to time and creating a best fit line for the purpose of finding slope.

The main reason R was chosen for regression over Python is that R is better suited for this kind of data analysis. Python was still required since R does not easily analyze the main data format that the gravitational wave strain data comes in. The calculated chirp-masses can be seen below, in Table 1, with R^2 values and uncertainties.

	Chirp Mass ($M_{\odot}$)	R^2	Uncertainty ($\pm M_{\odot}$)
GW170814 (H1)	172.6	0.2494	158.6
GW170814 (L1)	215.5	0.7756	98.06
GW170809 (H1)	62.6	0.09525	71.95
GW170809 (L1)	205.5	0.8362	77.81

Table 1. Chirp masses with uncertainties and R^2 calculated from the slopes of the linear regression models.

See Appendix B for how these values were calculated using R. We will discuss these further in the results section. From these chirp masses, we can move on to estimating radiated energy.

4.2 Radiated Energy Estimation

To estimate the maximum radiated energy, we need the frequency of gravitational waves at maximum amplitude, the distance between the two black holes, and an estimation of the mass of each black hole. The frequency of gravitational waves at maximum amplitude was found for each data set using the zero-crossing data already collected. These were then averaged for each object.

We can then use these to estimate the orbital angular velocity as $\frac{2\pi f_{max}}{2} = \omega$. From here we turn to Kepler's third law, where

$$r^3 = \frac{GMT^2}{4\pi^2} \text{ [16]},$$

Equation 16

with M as the orbited mass, T as the orbit period, and r as the distance between the binary objects. Equation 16 can be rearranged using $T = \frac{2\pi}{\omega}$ to obtain $r = (\frac{GM}{\omega^2})^{\frac{1}{3}}$, where r is the distance between the black holes and M is the combined black hole masses. We can finally find the mass of each black hole by assuming the black holes have equal masses, which is reasonable as they do to an order of magnitude. From this assumption we can use the chirp mass to find the mass as

$$m = 2^{\frac{1}{5}}\mathcal{M},$$

Equation 17

where m is the mass of either black holes and $\mathcal{M}$ is the chirp mass, this follows directly from the chirp mass formula.

Finally, we can use our orbital energy formula from Section 3, and all of our values obtained above to calculate the energy radiated away assuming all of it is $E_{orb} = -\frac{GM\mu}{2r}$ from Equation 9. This value will be positive as we are looking at it from the perspective of the energy of the gravitational wave and not the energy lost. The tabulated results of all of these calculations can be seen in Table 2, below.

	f_{max} (Hz)	m ($M_{\odot}$)	r (km)	E_{orb} ($M_{\odot}c^2$)
GW170814 (L1)	136.5	247.5	709.7	63
GW170809 (L1)	102.4	236.1	846.1	49

Table 2. Parameters found using the previous analysis, uncertainties not included as these values are entirely approximations.

Note that for these parameters, only the chirp-masses with an $R^2 > 0.5$ were used to estimate parameters as these are the only values of statistical significance. With this, we have our estimated parameters of both mergers, and we can move on to a discussion of our results.

5. CONCLUSION

Our final estimated parameters compared to the parameters found from formal sources can be seen in Table 3, below.

	Chirp Mass ($M_{\odot}$)	Radiated Energy ($M_{\odot}c^2$)
GW170814 (predicted)	215.5 ± 98.06	63
GW170809 (predicted)	205.5 ± 77.81	49
GW170814 (actual)	$24.1^{+1.4}_{-1.1}$	$2.7^{+0.4}_{-0.3}$
GW170809 (actual)	$24.9^{+2.1}_{-1.7}$	$2.7^{+0.6}_{-0.6}$

Table 3. Actual and predicted values of parameters for GW170814 and GW170809 [1], [2].

For GW170814 and GW170809, only the chirp masses found for the data from the L1 detector were of statistical significance, and they were both an order of magnitude off of the actual chirp masses determined by numerical relativity methods. The same was true of the radiated energy parameters obtained. The main reason for this comes down to noise filtering in the data, and since we are dealing with an $f^{-\frac{8}{3}}$, we require better data resolution than the methods employed were able to give us. This would lead us to the conclusion that a 1st order PNE approximation is insufficient for parameter predication the two mergers we explored in this paper. This does not entirely discount the use of 1st order PNE approximation, but the use cases must be carefully analyzed, and it is perhaps best left in the classroom to introduce students to gravitational wave analysis, instead of being used in any official capacity.

5.1 Further Avenues

There are some avenues to take to improve this analysis, instead of using zero crossings to estimate frequency, we could combine a tangent line analysis of the strain data to find derivative values and combine that with Equation 8 to find a better calculated value for chirp mass that does not rely on linear regression [15]. This would require more sophisticated analysis of strain data, and if the data is still noisy then it might prove just as unreliable as our regression model, but it is an alternative that could be explored.

We also could do matched waveform filtering, where we build up our own gravitational waveform in Python using PyCBC [4], compare it to our model until we match the waveform as best we can, and then perform our frequency analysis on our model. This is more work/time as the analysis requires some amount of trial and error and relies on an underlying assumption of a specific waveform, but can be effective if more detailed understanding is required. This is also how numerical relativity results are compared to experimental data, except instead of building a waveform from some assumptions, you instead numerically solve the EFEs.

There are also other parameters we could estimate using some basic physics methods and assumptions. These parameters are the distance to the source of gravitational waves and luminosity, the latter of which would enable us to take redshift into account in our calculations. In the case of this paper, redshifts for these sources were already found to be >0.2 which does not greatly affect our conclusions, as our values in the detector frame would not change significantly enough to affect our results in the source frame. The physics and assumptions involved with these parameters were also beyond the scope of this paper. For more information on how to estimate luminosity or distance from the information we have obtained in this paper, [15] describes the entire process in detail.

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APPENDIX A: Python Code

```
from pycbc.filter import highpass_fir, lowpass_fir
from pycbc.psd import welch, interpolate
from pycbc import catalog
import matplotlib.pyplot as pylab
import numpy
from gwpy.time import tconvert
import datetime
from math import floor

def gravitational_function_GW170814_H1(z):
    m = catalog.Merger("GW170814")
    mchirp = m.median1d('mchirp')
    print(mchirp)

    ifo = 'H1'

    #Read data, remove low freq, content
    data = catalog.Merger("GW170814").strain(ifo)
    data = highpass_fir(data, 15, 10)

    #Calculate the noise spectrum
    psd = interpolate(welch(data), 1.0 / data.duration)

    #Whiten
    white_strain = (data.to_frequencyseries() / psd ** 0.5).to_timeseries()

    #remove some of the high and low
    smooth = highpass_fir(white_strain, 35, 10)
    smooth = lowpass_fir(white_strain, 150, 10)

    #Time shift and flip
    if ifo == 'L1':
        smooth *= -1
        smooth.roll(int(.007 / smooth.delta_t))

    zoom = smooth.time_slice(m.time - 0.026, m.time + 0.025)

    zero_crossings = numpy.where(numpy.diff(numpy.sign(zoom)))[0]

    zero_times = []
    for i in zero_crossings:
        zero_times.append(zoom.sample_times[i])

    delta_zeroes = []
```

```
    for i in range(len(zero_times) - 1):
        delta_zeroes.append(zero_times[i + 1] - zero_times[i])

    frequencies = []
    for i in delta_zeroes:
        frequencies.append(float((1 / (i * 2))**(-8/3)))

    decimal_times = []
    for i in zero_crossings:
        decimal_times.append(zoom.sample_times[i] -
floor(zoom.sample_times[i]))

    decimal_avg_times = []
    for i in range(len(zero_times) - 1):
        decimal_avg_times.append(float((decimal_times[i + 1] +
decimal_times[i])/2))

    max_frequency = 1/(2*min(delta_zeroes))
    print(max_frequency)
    #datetime.datetime.now().second
    #print(delta_zeroes)
    #print(frequencies)
    #print(decimal_avg_times)
    #print(whitened)

    if z == 1:
        pylab.plot(smooth.sample_times, smooth, label=ifo)
        pylab.xlim(m.time - 0.1, m.time + 0.05)
        pylab.ylim(-100, 100)
        pylab.ylabel('Strain')
        pylab.xlabel('GPS Time (s)')
        pylab.legend()
        pylab.title("Filtered/Bandpassed Strain Data for GW170814 - H1")
        pylab.show()

    return frequencies, decimal_avg_times

def gravitational_function_GW170814_L1(z):
    m = catalog.Merger("GW170814")
    mchirp = m.median1d('mchirp')
    print(mchirp)

    ifo = 'L1'

    #Read data, remove low freq, content
    data = catalog.Merger("GW170814").strain(ifo)
    data = highpass_fir(data, 15, 10)
```

```
    #Calculate the noise spectrum
    psd = interpolate(welch(data), 1.0 / data.duration)

    #Whiten
    white_strain = (data.to_frequencyseries() / psd ** 0.5).to_timeseries()

    #remove some of the high and low
    smooth = highpass_fir(white_strain, 35, 10)
    smooth = lowpass_fir(white_strain, 150, 10)

    #Time shift and flip
    if ifo == 'L1':
        smooth *= -1
        smooth.roll(int(.007 / smooth.delta_t))

    zoom = smooth.time_slice(m.time - 0.026, m.time + 0.025)
    zero_crossings = numpy.where(numpy.diff(numpy.sign(zoom)))[0]

    zero_times = []
    for i in zero_crossings:
        zero_times.append(zoom.sample_times[i])

    delta_zeroes = []
    for i in range(len(zero_times) - 1):
        delta_zeroes.append(zero_times[i + 1] - zero_times[i])

    frequencies = []
    for i in delta_zeroes:
        frequencies.append(float((1 / (i * 2))**(-8/3)))

    decimal_times = []
    for i in zero_crossings:
        decimal_times.append(zoom.sample_times[i] -
floor(zoom.sample_times[i]))

    decimal_avg_times = []
    for i in range(len(zero_times) - 1):
        decimal_avg_times.append(float((decimal_times[i + 1] +
decimal_times[i])/2))

    max_frequency = 1/(2*min(delta_zeroes))
    print(max_frequency)
    #datetime.datetime.now().second
    #print(delta_zeroes)
    #print(frequencies)
    #print(decimal_avg_times)
    #print(whitened)
```

```
    if z == 1:
        pylab.plot(smooth.sample_times, smooth, label=ifo)
        pylab.xlim(m.time - 0.1, m.time + 0.05)
        pylab.ylim(-100, 100)
        pylab.ylabel('Strain')
        pylab.xlabel('GPS Time (s)')
        pylab.legend()
        pylab.title("Filtered/Bandpassed Strain Data for GW170814 - L1")
        pylab.show()

    return frequencies, decimal_avg_times

def gravitational_function_GW170809_H1(z):
    m = catalog.Merger("GW170809")
    mchirp = m.median1d('mchirp')
    print(mchirp)

    ifo = 'H1'

    #Read data, remove low freq, content
    data = catalog.Merger("GW170809").strain(ifo)
    data = highpass_fir(data, 15, 15)

    #Calculate the noise spectrum
    psd = interpolate(welch(data), 1.0 / data.duration)

    #Whiten
    white_strain = (data.to_frequencyseries() / psd ** 0.5).to_timeseries()

    #remove some of the high and low
    smooth = highpass_fir(white_strain, 35, 15)
    smooth = lowpass_fir(white_strain, 150, 15)

    #Time shift and flip
    if ifo == 'L1':
        smooth *= -1
        smooth.roll(int(.007 / smooth.delta_t))

    zoom = smooth.time_slice(m.time - 0.085, m.time - 0.035)
    zero_crossings = numpy.where(numpy.diff(numpy.sign(zoom)))[0]

    zero_times = []
    for i in zero_crossings:
        zero_times.append(zoom.sample_times[i])

    delta_zeroes = []
    for i in range(len(zero_times) - 1):
        delta_zeroes.append(zero_times[i + 1] - zero_times[i])
```

```
    frequencies = []
    for i in delta_zeroes:
        frequencies.append(float((1 / (i * 2))**(-8/3)))

    decimal_times = []
    for i in zero_crossings:
        decimal_times.append(zoom.sample_times[i] -
floor(zoom.sample_times[i]))

    decimal_avg_times = []
    for i in range(len(zero_times) - 1):
        decimal_avg_times.append(float((decimal_times[i + 1] +
decimal_times[i])/2))

    max_frequency = 1/(2*min(delta_zeroes))
    print(max_frequency)
    #datetime.datetime.now().second
    #print(delta_zeroes)
    #print(frequencies)
    #print(decimal_avg_times)
    #print(whitened)

    if z == 1:
        pylab.plot(smooth.sample_times, smooth, label=ifo)
        pylab.xlim(m.time - 0.3, m.time + 0.05)
        pylab.ylim(-100, 100)
        pylab.ylabel('Strain')
        pylab.xlabel('GPS Time (s)')
        pylab.legend()
        pylab.title("Filtered/Bandpassed Strain Data for GW170814 - H1")
        pylab.show()

    return frequencies, decimal_avg_times

def gravitational_function_GW170809_L1(z):
    m = catalog.Merger("GW170809")
    mchirp = m.median1d('mchirp')
    print(mchirp)

    ifo = 'L1'

    #Read data, remove low freq, content
    data = catalog.Merger("GW170809").strain(ifo)
    data = highpass_fir(data, 15, 15)

    #Calculate the noise spectrum
    psd = interpolate(welch(data), 1.0 / data.duration)
```

```
    #Whiten
    white_strain = (data.to_frequencyseries() / psd ** 0.5).to_timeseries()

    #remove some of the high and low
    smooth = highpass_fir(white_strain, 35, 15)
    smooth = lowpass_fir(white_strain, 150, 15)

    #Time shift and flip
    if ifo == 'L1':
        smooth *= -1
        smooth.roll(int(.007 / smooth.delta_t))

    zoom = smooth.time_slice(m.time - 0.11, m.time - 0.053)
    zero_crossings = numpy.where(numpy.diff(numpy.sign(zoom)))[0]

    zero_times = []
    for i in zero_crossings:
        zero_times.append(zoom.sample_times[i])

    delta_zeroes = []
    for i in range(len(zero_times) - 1):
        delta_zeroes.append(zero_times[i + 1] - zero_times[i])

    frequencies = []
    for i in delta_zeroes:
        frequencies.append(float((1 / (i * 2))**(-8/3)))

    decimal_times = []
    for i in zero_crossings:
        decimal_times.append(zoom.sample_times[i] -
floor(zoom.sample_times[i]))

    decimal_avg_times = []
    for i in range(len(zero_times) - 1):
        decimal_avg_times.append(float((decimal_times[i + 1] +
decimal_times[i])/2))

    max_frequency = 1/(2*min(delta_zeroes))
    print(max_frequency)
    #datetime.datetime.now().second
    #print(delta_zeroes)
    #print(frequencies)
    #print(decimal_avg_times)
    #print(whitened)

    if z == 1:
        pylab.plot(smooth.sample_times, smooth, label=ifo)
```

```
        pylab.xlim(m.time - 0.3, m.time + 0.05)
        pylab.ylim(-100, 100)
        pylab.ylabel('Strain')
        pylab.xlabel('GPS Time (s)')
        pylab.legend()
        pylab.title("Filtered/Bandpassed Strain Data for GW170814 - L1")
        pylab.show()

    return frequencies, decimal_avg_times

gravitational_function_GW170809_L1(1)
#gravitational_function_GW170809_H1(1)
#gravitational_function_GW170814_L1(1)
#gravitational_function_GW170814_H1(1)
```

APPENDIX B: R Code

```
library(reticulate)
library(tidyverse)
library(ggplot2)
library(glue)
library(gridExtra)

c = 3*10^8 #Speed of light [m][s^-1]
G = 6.6743*10^(-11) #Gravitational constant [m^3][kg^-1][s^-2]
M_sun = 1.9891*10^30 #Mass of sun [kg]

setwd("~/Desktop/Honors Thesis Project")
gobj <- import('gravitational_object_function_old')

#Importing data
frequency_data_GW170814_H1 <-
gobj$gravitational_function_GW170814_H1(integer(1))
frequency_data_GW170814_L1 <-
gobj$gravitational_function_GW170814_L1(integer(1))
frequency_data_GW170809_H1 <-
gobj$gravitational_function_GW170809_H1(integer(1))
frequency_data_GW170809_L1 <-
gobj$gravitational_function_GW170809_L1(integer(1))

data.frame(
  time = frequency_data_GW170814_H1[[2]],
  frequency = frequency_data_GW170814_H1[[1]]
) -> merger_GW170814_H1

data.frame(
  time = frequency_data_GW170814_L1[[2]],
  frequency = frequency_data_GW170814_L1[[1]]
) -> merger_GW170814_L1

data.frame(
  time = frequency_data_GW170809_H1[[2]],
  frequency = frequency_data_GW170809_H1[[1]]
) -> merger_GW170809_H1

data.frame(
  time = frequency_data_GW170809_L1[[2]],
  frequency = frequency_data_GW170809_L1[[1]]
) -> merger_GW170809_L1
```

```
#Trimming data

#merger_GW170814_H1 = merger_GW170814_H1[-c(), ]
#merger_GW170814_L1 = merger_GW170814_L1[-c(), ]
#merger_GW170809_H1 = merger_GW170809_H1[-c(), ]
#merger_GW170809_L1 = merger_GW170809_L1[-c(), ]

gravfit1.m <- lm(frequency ~ time, data = merger_GW170814_H1)
gravfit2.m <- lm(frequency ~ time, data = merger_GW170814_L1)
gravfit3.m <- lm(frequency ~ time, data = merger_GW170809_H1)
gravfit4.m <- lm(frequency ~ time, data = merger_GW170809_L1)

slope_coefficients <- c(coef(gravfit1.m)[2], coef(gravfit2.m)[2], coef(gravfit3.m)[2],
coef(gravfit4.m)[2])


chirp_mass <- vector("integer", 4)
for(i in seq_len(4)) {
  chirp_mass[i] <- ((slope_coefficients[i]*(-5/256)*(pi)^(-
3/8))^(3/5)*(1/G)*(c^3))/M_sun
}
chirp_mass

ggplot(
  data = merger_GW170814_H1,
  aes(x = time, y = frequency)
) +
  geom_point() +
  labs(
    x = "Time (s)", y = expression("(Frequency (Hz))"^(-8/3)),
    title = glue("Frequency vs Time for GW170814"),
    subtitle = glue("H1 Data")
  ) +
  geom_abline(aes(intercept = coef(gravfit1.m)[1],
                slope = coef(gravfit1.m)[2])) -> x1

ggplot(
  data = merger_GW170814_L1,
  aes(x = time, y = frequency)
) +
  geom_point() +
  labs(
    x = "Time (s)", y = expression("(Frequency (Hz))"^(-8/3)),
    title = glue("Frequency vs Time for GW170814"),
    subtitle = glue("L1 Data")
  ) +
```

```
  geom_abline(aes(intercept = coef(gravfit2.m)[1],
                slope = coef(gravfit2.m)[2])) -> x2

ggplot(
  data = merger_GW170809_H1,
  aes(x = time, y = frequency)
) +
  geom_point() +
  labs(
    x = "Time (s)", y = expression("(Frequency (Hz))"^(-8/3)),
    title = glue("Frequency vs Time for GW170809"),
    subtitle = glue("H1 Data")
  ) +
  geom_abline(aes(intercept = coef(gravfit3.m)[1],
                slope = coef(gravfit3.m)[2])) -> x3

ggplot(
  data = merger_GW170809_L1,
  aes(x = time, y = frequency)
) +
  geom_point() +
  labs(
    x = "Time (s)", y = expression("(Frequency (Hz))"^(-8/3)),
    title = glue("Frequency vs Time for GW170809"),
    subtitle = glue("L1 Data")
  ) +
  geom_abline(aes(intercept = coef(gravfit4.m)[1],
                slope = coef(gravfit4.m)[2])) -> x4

big1 <- grid.arrange(x1, x2, x3, x4, ncol = 2)

summary(gravfit1.m)
summary(gravfit2.m)
summary(gravfit3.m)
summary(gravfit4.m)

ggsave("figure5.png", plot = big1, width = 8, height = 5)
```

AUTHOR'S BIOGRAPHY

Jarrod E. Rudis was born in Bethlehem, PA and has lived both there and Washington state in his elementary school years before ending up in Berwick, ME for the rest of grade school. He graduated from Noble High School in 2018. His love of Star Trek and desire for understanding of the universe led him to study Physics and Mathematics at the University of Maine Orono, where he is a dual degree in Physics and Mathematics.

After graduation, Jarrod intends to take a break from being a student before pursuing a graduate degree in physics.