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Bilevel aggregator-prosumers' optimization problem in real-time: A convex optimization approach

Koorosh Shomalzadeh [a,*], Jacquelien M.A. Scherpen [b], M. Kanat Camlibel [a]

[a] Jan C. Willems Center for Systems and Control, Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, Faculty of Science and Engineering, University of Groningen, Nijenborgh 9, 9747 AG, Groningen, the Netherlands

[b] Jan C. Willems Center for Systems and Control, Engineering and Technology Institute Groningen, Faculty of Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG, Groningen, the Netherlands

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ABSTRACT

This paper proposes a Real-Time Market (RTM) platform for an aggregator and its corresponding prosumers to participate in the electricity wholesale market. The proposed energy market platform is modeled as a bilevel optimization problem where the aggregator and the prosumers are considered as self-interested agents. We present a convex optimization problem which can capture a subset of the set of global optima of the bilevel problem as its optimal solution.

1. Introduction

Power systems are experiencing a fundamental transition. Previously, the energy was generated in the bulk power plants and it was flowing through transmission and distribution networks to the consumers. The massive installation of Renewable Energy Sources (RESs) at the household level has challenged this structure. Therefore, new schemes and models are needed to efficiently cope with this transition [3].

The emergence of the energy producing consumers, i.e., prosumers and recent Information and Communications Technology (ICT) developments in the paradigm of smart grid [11] have opened up new horizons for less grid-dependent households. Since output generation of RESs are volatile due to their intrinsic environmental dependency, researchers have proposed different approaches to address demand and supply matching for a group of prosumers. Utilization of storage devices [18], bilateral energy transactions between prosumers [1], and bilateral energy transaction between prosumers and the wholesale market [26] are among the most prominent of those approaches.

In this paper we focus on a real-time grid-prosumers energy transaction through an aggregator as the mechanism to address demand and supply matching. The aggregator's role is to gather and manage a group of prosumers in order to participate in the real-time wholesale market.

Many types of aggregator with dissimilar goals have been studied in different financial and market structures in the area of electricity markets [17]. In this work, the aggregator is a self-interested market participant who has the goal of participating in the real-time wholesale market in order to maximize its revenue. To do so, the aggregator considers each individual prosumer demand and supply situation and proposes a personalized price to buy its excess supply or provide the prosumer its energy deficiency at each time-step in a Real-Time Market (RTM).

On the other hand, each prosumer receives a price from the aggregator and responds optimally by considering its demand preferences and supply situations over a horizon. We assume that the aggregator can anticipate the reaction of the prosumers. This price oriented setup falls into the category of bilevel optimization problems [5] and Stackelberg games [23], where the lower level problem and the upper level problem are the problems related to the prosumers and the aggregator, respectively.

Bilevel optimization approach has been used in various applications with hierarchical nature [8,13,14]. Particularly, it has extensively used to model and solve energy systems problems [7]. The initial work [12] models strategic offering of a dominant generating firm as a bilevel optimization problem, where at the upper level a generator firm maximizes its profit and at the lower level a system operator maximizes social welfare or minimizes total system cost. This problem is rewritten as a Mathematical Program-

* Corresponding author.
E-mail addresses: k.shomalzadeh@rug.nl (K. Shomalzadeh), j.m.a.scherpen@rug.nl (J.M.A. Scherpen), m.k.camlibel@rug.nl (M.K. Camlibel).

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ming with Equilibrium Constraints (MPEC) and solved by a penalty interior point algorithm. More recent works (e.g., [26]) focus on the aggregator and prosumers problem. The state-of-the-art approach to solve these types of problems is to reformulate the bilevel optimization problem as a Mixed-Integer Programming (MIP).

Both the MPEC and MIP based methods are computationally expensive. One of the main challenges to implement an RTM is the computational efficiency. For an RTM the time intervals are in the order of a few minutes [22]. Therefore, new computational tools are needed for the aggregator's real-time control over the prosumers and its participation in the RTM. The paper [10] has addressed the computational efficiency of the dominant firm's strategic offering by introducing a convex relaxation for the bilevel optimization problem and has found a close to optimal solution. However, to the best of our knowledge, no study has been done on finding the global optimum of a bilevel optimization problem by solving a convex one in the field of prosumers integration in the wholesale energy markets.

In this paper, we define the problem of economic optimization of an aggregator and its corresponding prosumers for participation in an RTM over a time horizon as a bilevel optimization problem. The aggregator represents the prosumers to participate in the wholesale market in a real-time scenario. This problem, in general, is nonconvex [16]. We show that a subset of the set of global minimizers for the nonconvex problem can be obtained as the solution of a certain convex optimization problem. The convex problem has two main advantages. On the one hand, a convex formulation is attractive in real-time applications since the computation time is linear in the number of variables. On the other hand, off-the-shelf software packages can be used to solve the problem. In addition, replacing a bilevel optimization problem by a convex one is a key step towards devising decentralized or distributed algorithms [2]. This work is a continuation of the preliminary study by the authors [20] which dealt with a simple static model for balancing markets.

The paper is organized as follows. In Section 2, we define the aggregator and prosumers problems as a bilevel optimization problem. Section 3 presents a convex optimization problem that can capture a subset of the set of global minimizers for the bilevel one. Finally, the paper closes with the conclusions in Section 4.

Notation. We denote the set of real numbers by $\mathbb{R}$, n-vectors by $\mathbb{R}^n$ and $m \times n$ matrices by $\mathbb{R}^{m\times n}$. Throughout the paper, the inequalities for vectors are meant entrywise. The n-vectors of ones is denoted by $\mathbb{1}_n$. The $m \times m$ identity matrix is denoted by I_m. For a matrix $M \in \mathbb{R}^{m\times n}$ and index sets $\alpha \subseteq \{1, 2, \dots, m\}$, $\beta \subseteq \{1, 2, \dots, n\}$, the notation $M_{\alpha\beta}$ denotes the matrix $\left(M_{ij}\right)_{i\in\alpha,\, j\in\beta}$. If $\alpha = \{1, 2, \dots, m\}$, then we write $M_{\bullet\beta}$ and if $\beta = \{1, 2, \dots, n\}$, then we write $M_{\alpha\bullet}$. For any vector $x \in \mathbb{R}^n$ if $Mx = 0$, we write $x \in \ker M$. A symmetric matrix $M = M^T \in \mathbb{R}^{m\times m}$ is said to be positive semidefinite if $x^T Mx \geq 0$ for all $x \in \mathbb{R}^m$ and positive definite if $x^T Mx > 0$ for all $0 \neq x \in \mathbb{R}^m$. The symmetric square root of a positive definite matrix M is denoted by $M^{\frac{1}{2}}$. For a vector $v \in \mathbb{R}^n$, we write $\mathrm{diag}(v)$ for the diagonal matrix with diagonal entries $v_1, v_2, \dots, v_n$. Let $f : \mathbb{R}^n \to \mathbb{R}$ and $S \subseteq \mathbb{R}^n$. Consider the optimization problem

$$\mathrm{OP}: \quad \min_{x} \quad f(x) \qquad \text{subject to} \quad x \in S.$$

We say that $\bar{x}$ is feasible for OP if $\bar{x} \in S$. Also, we define the set of global optima for OP as

$$\mathrm{MIN(OP)} = \{x^* \in S \mid f(x^*) \leq f(x)\ \forall x \in S\}.$$

2. Problem formulation

In this section, we define a market model for an aggregator and the prosumers under its contract to participate in an RTM with the grid, i.e., the wholesale market. Here, the role of the aggregator is to act as an intermediary agent between the prosumers and the grid to facilitate the energy transactions. We consider the case where each prosumer can generate energy through some RESs with zero cost. Example of such energy sources are solar panels and wind turbines. Moreover, each prosumer's demand is elastic at each time-step. The aggregator goal is to propose the prosumers with a personalized price to deal their surplus or shortage energy with the grid in an optimal way. The advantages of a personalized price over a unique price have been addressed in many recent research (see e.g., [21,24]). Next, we explain the problem setting and market structure in detail.

2.1. Prosumer's problem

The main source of energy supply for a prosumer is its renewable energy units. Due to uncertain and uncontrollable nature of RESs, there might be a mismatch between supply and demand at each time-step. Each prosumer has two options to cancel this mismatch. One is to trade with the wholesale market through the aggregator. The other option is to use its demand elasticity. Therefore, each prosumer needs to find a trade-off between these two possible options for its optimal strategy. Before providing a mathematical formulation for the prosumer, we elaborate on demand elasticity.

We say that the demand of each prosumer is elastic if:

1. Each prosumer has a preference for its demand at each time-step.
2. Altering the demand from its preferred value causes dissatisfaction for the prosumer. Here, we model this dissatisfaction using a quadratic function.
3. Each prosumer has a lower bound and an upper bound for its demand at each time-step.
4. Total demand of each prosumer in a specific time period is constant.

These assumptions on the prosumer's demand are used extensively for the goal of load shifting in the literature under the name *demand-side management* [4,15,19].

As explained before, the prosumer's goal is to find a trade-off between two possible options to minimize its cost and maximize its comfort. We model this problem as an optimization problem. We define the set of prosumers by $\{1, 2, \dots, n\}$ and the set of time-steps by $\{1, 2, \dots, K\}$. Then, prosumer $i \in \{1, 2, \dots, n\}$ at time-step $k \in \{1, 2, \dots, K\}$ has three decision variables: its demand $h_i(k)$, the energy it sells to (buy form) the grid $y_i^+(k)$ ($y_i^-(k)$). For the ith prosumer, we consider the optimization problem (1) as

$$\mathrm{PP}_i: \min_{\substack{h_i(k),\, y_i^+(k),\, y_i^-(k)\\ \forall k\in\{1,2,\dots,K\}}} \sum_{k=1}^{K} \frac{1}{2} q_i(k)(h_i(k) - h_i^0(k))^2 + x_i^-(k) y_i^-(k) - x_i^+(k) y_i^+(k) \tag{1a}$$

$$\text{subject to} \quad y_i^+(k) - y_i^-(k) + h_i(k) = s_i(k) \quad \forall k \in \{1, \dots, K\} \tag{1b}$$

$$y_i^+(k),\, y_i^-(k) \geq 0 \quad \forall k \in \{1, \dots, K\} \tag{1c}$$

$$\underline{h}_i(k) \leq h_i(k) \leq \bar{h}_i(k) \quad \forall k \in \{1, \dots, K\} \tag{1d}$$

$$\sum_{k=1}^{K} h_i(k) = h_i^{\mathrm{tot}} \tag{1e}$$

where $x_i^+(k)$ ($x_i^-(k)$) is the proposed price by the aggregator to buy energy from (sell energy to) the prosumer at time-step k, $s_i(k) \geq 0$

is the generated energy by the prosumer at time-step k, which assumed to be known, and $q_i(k) > 0$ is the dissatisfaction parameter for the prosumer. Moreover, $h_i^0(k) \geq 0$, $\underline{h}_i(k) \geq 0$ and $\bar{h}_i(k) \geq 0$ are the preferred value, lower bound and upper bound for the demand $h_i(k)$, respectively. The parameter h_i^{tot} is the total demand for the prosumer over the period $k \in [1, K]$.

In (1a), the first term models the dissatisfaction the prosumer experiences by changing its demand from the preferred value. The second term is the cost of buying energy from the grid through the aggregator and the third term is the revenue the prosumer can obtain by selling energy through the aggregator. The constraint (1b) indicates that the total demand should be equal to the total supply for each prosumer at each time-step. The constraints (1c) and (1d) specify the lower bound and upper bound for the decision variables. Finally, (1e) captures the assumption that the total demand over a period is constant.

Assumption 1. The sum of preferred values $h_i^0(k)$s over the period from $k=1$ to $k=K$ is equal to h_i^{tot}, i.e., $\sum_{k=1}^{K} h_i^0(k) = h_i^{\text{tot}}$.

From (1b), we can write $h_i(k)$ as $h_i(k) = s_i(k) - (y_i^+(k) - y_i^-(k))$. Thus, the variable $h_i(k)$ can be eliminated from the problem PP_i and we can rewrite it as the optimization problem (2) where $c_i(k) = q_i(k)(h_i^0(k) - s_i(k))$.

$$
\begin{aligned}
\text{PP}_i' : \min_{\substack{y_i^+(k), y_i^-(k) \\ \forall k \in \{1,2,\ldots,K\}}} \quad & \sum_{k=1}^{K} \frac{1}{2} q_i(k)(y_i^+(k) - y_i^-(k))^2 + c_i(k)(y_i^+(k) - y_i^-(k)) \\
& + x_i^-(k) y_i^-(k) - x_i^+(k) y_i^+(k) + \frac{1}{2} q_i(k)(h_i^0(k))^2 && \text{(2a)} \\
\text{subject to} \quad & y_i^+(k), y_i^-(k) \geq 0 \quad \forall k \in \{1,\ldots,K\} && \text{(2b)} \\
& s_i(k) - \bar{h}_i(k) \leq y_i^+(k) - y_i^-(k) \leq s_i(k) - \underline{h}_i(k) \\
& \forall k \in \{1,\ldots,K\} && \text{(2c)} \\
& -\sum_{k=1}^{K} (y_i^+(k) - y_i^-(k)) = h_i^{\text{tot}} - \sum_{k=1}^{K} s_i(k) && \text{(2d)}
\end{aligned}
$$

Moreover, since these optimization problems are independent, we can add them and rewrite them in a vector form. To do so, for $\xi_i(k)$ with $i \in \{1, 2, \ldots, n\}$ and $k \in \{1, 2, \ldots, K\}$, we define the vector $\xi \in \mathbb{R}^{nK}$ as

$$
\xi = [\,\xi_1(1) \;\cdots\; \xi_1(K) \;\; \xi_2(1) \;\cdots\; \xi_2(K) \;\cdots\; \xi_n(1) \;\cdots\; \xi_n(K)\,]^T. \tag{3}
$$

Also, we drop the constant terms $\frac{1}{2} q_i(k)(h_i^0(k))^2$ for all $k \in \{1, 2, \ldots, K\}$ from (2a). Accordingly, the vector form of (2) is given by

$$
\begin{aligned}
\text{PP} : \min_{y^+, y^-} \quad & \frac{1}{2}(y^+ - y^-)^T Q (y^+ - y^-) + c^T (y^+ - y^-) \\
& + (x^-)^T y^- - (x^+)^T y^+ && \text{(4a)} \\
\text{subject to} \quad & y^+, y^- \geq 0 && \text{(4b)} \\
& \ell \leq y^+ - y^- \leq u && \text{(4c)} \\
& E(y^+ - y^-) = d && \text{(4d)}
\end{aligned}
$$

where

$$
\begin{aligned}
& Q = \text{diag}(q), \quad c = Q(h^0 - s), \quad \ell = s - \bar{h}, \\
& u = s - \underline{h}, \quad E = -I_n \otimes \mathbb{1}_K^T, \quad d = E(s - h^0).
\end{aligned} \tag{5}
$$

Here, $\otimes$ denotes the Kronecker product.

The prices $x_i^+(k)$ and $x_i^-(k)$ are proposed by the aggregator. In this work, the aggregator acts as a self-interested agent which has the ability to anticipate the reaction of the prosumers. Therefore, knowing the reaction of the prosumers, the aggregator sets the prices to maximize its revenue as an intermediary player between the grid and the prosumers. In the next subsection, we elaborate on the aggregator's problem as a bilevel optimization problem.

2.2. Aggregator's problem

The aggregator receives two prices from the grid for each time-step. The price $p^+(k)$ is the price for selling energy to grid and the price $p^-(k)$ is the price for buying energy from the grid at kth time-step. Having these prices and the ability of the aggregator to anticipate the reaction of the prosumers allow the aggregator to propose prices $x_i^+(k)$ and $x_i^-(k)$ to the prosumers in an optimal way. The bilevel optimization below models this problem for the aggregator:

$$
\begin{aligned}
\text{AP} : \max_{x^+, x^-, y^+, y^-} \quad & (p^+ - x^+)^T y^+ - (p^- - x^-)^T y^- && \text{(6a)} \\
\text{subject to} \quad & x^+, x^- \geq \underline{\rho} \mathbb{1}_m && \text{(6b)} \\
& x^+, x^- \leq \bar{\rho} \mathbb{1}_m && \text{(6c)} \\
& (y^+, y^-) \in \text{MIN(PP)} && \text{(6d)}
\end{aligned}
$$

where p^+ is defined as in (3) by taking $p_i^+(k) = p^+(k)$ for all $i \in \{1, \ldots, n\}$ and p^- is defined in the same way. The first term in (6a) corresponds to the aggregator's revenue from selling energy to the grid whereas the second term models the aggregator's cost for buying energy from the grid. A lower bound and upper bound for the prices x^+ and x^- are considered in (6b) and (6c) to guarantee a minimum revenue for each prosumer and also to prevent a high aggregator profit. Note that this lower and upper bounds are determined by a regulatory agency. We assume that $\bar{\rho} \geq \underline{\rho} \geq 0$.

In this paper, we consider a scenario where $p^+ = p^- = p$ and the aggregator proposes prices x^+ and x^- such that $x^+ = x^- = x$. Therefore, we can rewrite the optimization problems AP and PP based on the new decision variables x and $y = y^+ - y^-$ as the minimization problems BLP and LLP, respectively.

$$
\begin{aligned}
\text{BLP} : \min_{x, y} \quad & \phi(x, y) = (x - p)^T y && \text{(7a)} \\
\text{subject to} \quad & \underline{\rho} \mathbb{1}_m \leq x \leq \bar{\rho} \mathbb{1}_m && \text{(7b)} \\
& y = \text{MIN(LLP)} && \text{(7c)}
\end{aligned}
$$

Here the decision vector $x \in \mathbb{R}^m$ is the proposed prices of the aggregator and the parameter vector $p \in \mathbb{R}^m$ is the prices of selling to and buying from the grid. The prosumers' reactions y to the proposed prices are the solution of the optimization problem LLP.

$$
\begin{aligned}
\text{LLP} : \min_{y} \quad & \frac{1}{2} y^T Q y + (c - x)^T y && \text{(8a)} \\
\text{subject to} \quad & \ell \leq y \leq u && \text{(8b)} \\
& E y = d && \text{(8c)}
\end{aligned}
$$

The vector $y \in \mathbb{R}^m$ is the decision variable for LLP. The vectors and matrices $c, \ell, u \in \mathbb{R}^m$, $d \in \mathbb{R}^n$, $Q \in \mathbb{R}^{m \times m}$, $E \in \mathbb{R}^{n \times m}$ are parameters for LLP as defined in (5). Moreover, $x \in \mathbb{R}^m$ is the decision variable for the aggregator and the prosumers has no control over it. It should be noted that $m = nK$ and $\text{rank}\, E = n \leq m$. We assume that there exists $\bar{y}$ which satisfies (8b)-(8c). Since $Q = \text{diag}(q)$ is positive definite, LLP is a strictly convex quadratic optimization problem and hence has always a unique optimal solution, i.e., the set MIN(LLP) is a singleton.

Bilevel optimization problems are in general nonconvex and have combinatorial nature. Many algorithms and approaches have been developed to solve different classes of bilevel problems. Recent surveys on bilevel optimization can be found in [6] and [16]. In contrast to existing methods that deal with rather more general bilevel optimization problems, our focus here is to exploit the particular structure of (7) in order to introduce a convex optimization problem which has the same global optimum as the bilevel one. The next section investigates the conditions under which the global optimal solution of the optimization (7) can be found by solving a convex problem.

3. Main results

In this section, we will show that the set of global optima for a specific convex optimization problem is a subset of the set of global optima for the optimization BLP, under some assumptions on the parameters of the problem. First, we rewrite the optimization problem BLP as a Mathematical Program with Complementarity Constraints (MPCC). We elaborate on the special structure and properties of this MPCC. Then, we introduce a convex optimization problem which can be used to find a subset of MIN(BLP). Finally, we comment on the restrictions of the proposed convex optimization for the RTM platform.

3.1. From bilevel optimization to MPCC

Consider the optimization problem (7). Since the lower level optimization problem (8) is convex, we can replace it by the necessary and sufficient KKT conditions

$$Qy + c - x + E^T\lambda - \mu + \nu = 0, \tag{9}$$

$$Ey = d, \tag{10}$$

$$0 \leq \mu \perp y - \ell \geq 0 \tag{11}$$

$$0 \leq \nu \perp u - y \geq 0 \tag{12}$$

where $\mu \in \mathbb{R}^m$, $\nu \in \mathbb{R}^m$ are dual variables for the lower bound and upper bound constraints in (8b), respectively. Also, $\lambda \in \mathbb{R}^n$ is the dual variable for the constraint (8c). The dual variable λ can be eliminated from KKT conditions (9)-(12). First, we solve y from (9) as

$$y = Q^{-1}(x + \mu - \nu - c - E^T\lambda), \tag{13}$$

and then substitute y in (10):

$$EQ^{-1}(x + \mu - \nu - c - E^T\lambda) = d. \tag{14}$$

Since E has full row rank and Q is positive definite, $EQ^{-1}E^T$ is nonsingular. Therefore, we obtain

$$\lambda = (EQ^{-1}E^T)^{-1}(EQ^{-1}(x + \mu - \nu - c) - d). \tag{15}$$

By substituting λ in (13), we can write (9)-(12) as

$$y = M(x + \mu - \nu) + r,$$

$$0 \leq \mu \perp y - \ell \geq 0,$$

$$0 \leq \nu \perp u - y \geq 0$$

where

$$M = Q^{-1} - Q^{-1}E^T(EQ^{-1}E^T)^{-1}EQ^{-1}, \tag{16}$$

$$r = Q^{-1}E^T(EQ^{-1}E^T)^{-1}d - Mc = s - h^0. \tag{17}$$

Consequently, we can rewrite the bilevel optimization problem BLP as the following single-level optimization problem.

$$\text{SLP}: \quad \min_{x,y} \quad \phi(x, y) \tag{18a}$$

$$\text{subject to} \quad (x, y) \in S \tag{18b}$$

where

$$S = \{(x, y) \mid \underline{\rho}\mathbb{1}_m \leq x \leq \bar{\rho}\mathbb{1}_m,\ y = M(x + \mu - \nu) + r,\ \ell \leq y \leq u,$$
$$\mu^T(y - \ell) = 0,\ \nu^T(u - y) = 0 \text{ for some } \mu \geq 0 \text{ and } \nu \geq 0\}.$$

The problem SLP is an instance of an MPCC where the set S and the matrix M characterize the linear complementarity constraints. In what follows, we will explore the properties of the matrix M.

Lemma 2. *The matrix M is a positive semidefinite matrix.*

Proof. Clearly M is the Schur complement of

$$X = \begin{bmatrix} Q^{-1} & Q^{-1}E^T \\ EQ^{-1} & EQ^{-1}E^T \end{bmatrix} \tag{19}$$

with respect to $EQ^{-1}E^T$. Since $X = \begin{bmatrix} Q^{-\frac{1}{2}} \\ EQ^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} Q^{-\frac{1}{2}} & Q^{-\frac{1}{2}}E^T \end{bmatrix}$, X is positive semidefinite. It follows from [25, Theorem 1.12] that M is also positive semidefinite. □

Remark 3. The vector $\mathbb{1}_m$ can be written as $\mathbb{1}_m = -E^T\mathbb{1}_n$. It is clear form (16) that $ME^T = 0$. Then, it follows immediately that $\mathbb{1}_m \in \ker M$.

Also, it turns out that M has a special structure. The following definitions elaborate on this specific structure.

Definition 4. A matrix $N \in \mathbb{R}^{k \times k}$ is called

- a Z-matrix if its off-diagonal entries are nonpositive.
- an M-matrix if it is a Z-matrix and the real parts of its eigenvalues are nonnegative.

Remark 5. In particular, a positive semidefinite matrix is an M-matrix if its off-diagonal entries are nonpositive.

Lemma 6. *The matrix M is an* M*-matrix.*

Proof. The matrix M is the Schur complement of matrix X given by (19). Since E is a matrix with orthogonal rows and Q is positive definite and diagonal, $EQ^{-1}E^T$ and hence $(EQ^{-1}E^T)^{-1}$ are also positive definite diagonal matrices. Moreover, EQ^{-1} is nonpositive which makes X an M-matrix. Consequently, it follows from [9, Theorem 5.13] that M is also an M-matrix. □

In the next subsection, we show that M being positive semidefinite and M-matrix let us to find a subset of the global optima set of the MPCC (18) by solving a convex optimization problem.

3.2. From MPCC to convex optimization

The theorem below asserts that there exists a convex optimization problem which can capture a subset of the set of global optima for SLP.

Theorem 7. *Consider the optimization problem*

$$\text{CVX}: \quad \min_{x,y} \quad \phi(x, y)$$

$$\text{subject to} \quad (x, y) \in C$$

where

$$C = \{(x, y) \mid \underline{\rho}\mathbb{1}_m \le x \le \bar{\rho}\mathbb{1}_m, \quad y = Mx + r, \quad \ell \le y \le u\}.$$

Suppose that $\ell \le 0$, $u \ge 0$ and $\ell < r < u$. Then, MIN(CVX) $\subseteq$ MIN(SLP). Furthermore, the optimization problem CVX is convex.

To prove this theorem, we need the following auxiliary results.

Lemma 8. *Consider the optimization problem*

$$\text{SLP}': \quad \min_{x,y} \quad \phi(x, y) \qquad \text{subject to} \quad (x, y) \in S'$$

where

$$S' = \{(x, y) \mid \underline{\rho}\mathbb{1}_m \le x \le \bar{\rho}\mathbb{1}_m,\ y = M(x + \mu - \nu) + r,\ \ell \le y \le u,\ \mu^T(y - \ell) = 0,\ \nu^T(y - u) = 0,\ \mu^T(x - \bar{\rho}\mathbb{1}_m) = 0,\ \nu^T(x - \underline{\rho}\mathbb{1}_m) = 0, \text{ for some } \mu, \nu \ge 0\}.$$

Suppose that $\ell \le 0$ and $u \ge 0$. Then, for any $(\bar{x}, \bar{y}) \in S$ there exists $(\hat{x}, \bar{y}) \in S'$ such that $\phi(\hat{x}, \bar{y}) \le \phi(\bar{x}, \bar{y})$.

Proof. Let $(\bar{x}, \bar{y}) \in S$. Therefore, there exist $\bar{\mu} \ge 0$ and $\bar{\nu} \ge 0$ such that

$$\underline{\rho}\mathbb{1}_m \le \bar{x} \le \bar{\rho}\mathbb{1}_m, \quad \bar{y} = M(\bar{x} + \bar{\mu} - \bar{\nu}) + r, \quad \ell \le \bar{y} \le u, \quad \bar{\mu}^T(\bar{y} - \ell) = 0, \quad \bar{\nu}^T(u - \bar{y}) = 0. \tag{20}$$

We define three disjoint index sets $\alpha_1 \subseteq \{1, 2, \dots, m\}$, $\alpha_2 \subseteq \{1, 2, \dots, m\}$ and $\alpha_3 \subseteq \{1, 2, \dots, m\}$ such that $\alpha_1 \cup \alpha_2 \cup \alpha_3 = \{1, 2, \dots, m\}$ and

$$\begin{aligned} &\bar{\mu}_{\alpha_1} = 0, && \bar{\mu}_{\alpha_2} > 0, && \bar{\mu}_{\alpha_3} = 0, \\ &\bar{\nu}_{\alpha_1} = 0, && \bar{\nu}_{\alpha_2} = 0, && \bar{\nu}_{\alpha_3} > 0. \end{aligned} \tag{21}$$

Then, (20) can be written as

$$\begin{gathered} \underline{\rho}\mathbb{1}_{\alpha_1} \le \bar{x}_{\alpha_1} \le \bar{\rho}\mathbb{1}_{\alpha_1}, \ \ \underline{\rho}\mathbb{1}_{\alpha_2} \le \bar{x}_{\alpha_2} \le \bar{\rho}\mathbb{1}_{\alpha_2}, \ \ \underline{\rho}\mathbb{1}_{\alpha_3} \le \bar{x}_{\alpha_3} \le \bar{\rho}\mathbb{1}_{\alpha_3}, \\ \ell_{\alpha_1} \le \bar{y}_{\alpha_1} = M_{\alpha_1\alpha_1}\bar{x}_{\alpha_1} + M_{\alpha_1\alpha_2}(\bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_1\alpha_3}(\bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_1} \le u_{\alpha_1}, \\ \ell_{\alpha_2} = \bar{y}_{\alpha_2} = M_{\alpha_2\alpha_1}\bar{x}_{\alpha_1} + M_{\alpha_2\alpha_2}(\bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_2\alpha_3}(\bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_2} \le u_{\alpha_2}, \\ \ell_{\alpha_3} \le \bar{y}_{\alpha_3} = M_{\alpha_3\alpha_1}\bar{x}_{\alpha_1} + M_{\alpha_3\alpha_2}(\bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_3\alpha_3}(\bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_3} = u_{\alpha_3}, \\ \bar{\mu}_{\alpha_1} = \bar{\nu}_{\alpha_1} = 0, \ \bar{\mu}_{\alpha_2} > 0, \bar{\nu}_{\alpha_2} = 0, \ \bar{\mu}_{\alpha_3} = 0, \bar{\nu}_{\alpha_3} > 0 \end{gathered}$$

We choose $\hat{x}$, $\hat{\mu}$ and $\hat{\nu}$ based on $\bar{x}$, $\bar{\mu}$ and $\bar{\nu}$ as

$$\hat{x}_{\alpha_1} = \bar{x}_{\alpha_1}, \ \hat{\mu}_{\alpha_1} = \bar{\mu}_{\alpha_1} = 0, \ \hat{\nu}_{\alpha_1} = \bar{\nu}_{\alpha_1} = 0,$$

$$\begin{bmatrix} \hat{x}_{\alpha_2} \\ \hat{\mu}_{\alpha_2} \\ \hat{\nu}_{\alpha_2} \end{bmatrix} = \begin{cases} \begin{bmatrix} \bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2} \\ 0 \\ 0 \end{bmatrix} & \text{if} \quad \bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2} \le \bar{\rho}\mathbb{1}_{\alpha_2}, \\ \begin{bmatrix} \bar{\rho}\mathbb{1}_{\alpha_2} \\ \bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2} - \bar{\rho}\mathbb{1}_{\alpha_2} \\ 0 \end{bmatrix} & \text{if} \quad \bar{x}_{\alpha_2} + \bar{\mu}_{\alpha_2} > \bar{\rho}\mathbb{1}_{\alpha_2}, \end{cases}$$

$$\begin{bmatrix} \hat{x}_{\alpha_3} \\ \hat{\mu}_{\alpha_3} \\ \hat{\nu}_{\alpha_3} \end{bmatrix} = \begin{cases} \begin{bmatrix} \bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3} \\ 0 \\ 0 \end{bmatrix} & \text{if} \quad \bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3} \ge \underline{\rho}\mathbb{1}_{\alpha_3}, \\ \begin{bmatrix} \underline{\rho}\mathbb{1}_{\alpha_3} \\ 0 \\ \bar{\nu}_{\alpha_3} - \bar{x}_{\alpha_3} + \underline{\rho}\mathbb{1}_{\alpha_3} \end{bmatrix} & \text{if} \quad \bar{x}_{\alpha_3} - \bar{\nu}_{\alpha_3} < \underline{\rho}\mathbb{1}_{\alpha_3}. \end{cases}$$

Note that with these choices $M(\hat{x} + \hat{\mu} - \hat{\nu}) + r = \bar{y}$ and $(\hat{x}, \bar{y}) \in S'$. Moreover, these choices imply $\hat{x}_{\alpha_1} = \bar{x}_{\alpha_1}$, $\hat{x}_{\alpha_2} > \bar{x}_{\alpha_2}$ and $\hat{x}_{\alpha_3} < \bar{x}_{\alpha_3}$. Since $\ell \le 0$ and $u \ge 0$, we have the following implications:

$$\begin{gathered} \hat{x}_{\alpha_1} = \bar{x}_{\alpha_1} \Longrightarrow (\hat{x}_{\alpha_1} - p_{\alpha_1})^T \bar{y}_{\alpha_1} = (\bar{x}_{\alpha_1} - p_{\alpha_1})^T \bar{y}_{\alpha_1}, \\ \hat{x}_{\alpha_2} > \bar{x}_{\alpha_2}, \ \bar{y}_{\alpha_2} = \ell_{\alpha_2} \le 0 \Longrightarrow (\hat{x}_{\alpha_2} - p_{\alpha_2})^T \bar{y}_{\alpha_2} \le (\bar{x}_{\alpha_2} - p_{\alpha_2})^T \bar{y}_{\alpha_2}, \\ \hat{x}_{\alpha_3} < \bar{x}_{\alpha_3}, \ \bar{y}_{\alpha_3} = u_{\alpha_3} \ge 0 \Longrightarrow (\hat{x}_{\alpha_3} - p_{\alpha_3})^T \bar{y}_{\alpha_3} \le (\bar{x}_{\alpha_3} - p_{\alpha_3})^T \bar{y}_{\alpha_3} \end{gathered}$$

which conclude that $\phi(\hat{x}, \bar{y}) \le \phi(\bar{x}, \bar{y})$. □

In general, the set S' is a nonconvex set due to complementarity relations. However, it can be shown that it coincides with the set C in CVX under certain conditions on ℓ, u and r.

Lemma 9. *Suppose that $\ell < r < u$. Then, $\emptyset \ne C = S'$.*

Proof. Let $(\bar{x}, \bar{y}) \in S'$. Therefore, there exist nonnegative $\bar{\mu}$ and $\bar{\nu}$ such that

$$\begin{gathered} \underline{\rho}\mathbb{1}_m \le \bar{x} \le \bar{\rho}\mathbb{1}_m, \ \bar{y} = M(\bar{x} + \bar{\mu} - \bar{\nu}) + r, \\ \ell \le \bar{y} \le u, \ \bar{\mu}^T(\bar{y} - \ell) = 0, \ \bar{\nu}^T(\bar{y} - u) = 0, \\ \bar{\mu}^T(\bar{x} - \bar{\rho}\mathbb{1}_m) = 0, \ \bar{\nu}^T(\bar{x} - \underline{\rho}\mathbb{1}_m) = 0. \end{gathered} \tag{22}$$

We define three disjoint index sets $\alpha_1 \subseteq \{1, 2, \dots, m\}$, $\alpha_2 \subseteq \{1, 2, \dots, m\}$ and $\alpha_3 \subseteq \{1, 2, \dots, m\}$ such that $\alpha_1 \cup \alpha_2 \cup \alpha_3 = \{1, 2, \dots, m\}$ and (21) holds. Suppose that $\alpha_2 \cup \alpha_3 \ne \emptyset$. Then, (23) follows from (22).

$$\begin{gathered} \underline{\rho}\mathbb{1}_{\alpha_1} \le \bar{x}_{\alpha_1} \le \bar{\rho}\mathbb{1}_{\alpha_1}, \quad \bar{x}_{\alpha_2} = \bar{\rho}\mathbb{1}_{\alpha_2}, \quad \bar{x}_{\alpha_3} = \underline{\rho}\mathbb{1}_{\alpha_3}, \quad \bar{\mu}_{\alpha_2} > 0, \quad \bar{\nu}_{\alpha_3} > 0, \\ \ell_{\alpha_2} = M_{\alpha_2\alpha_1}\bar{x}_{\alpha_1} + M_{\alpha_2\alpha_2}(\bar{\rho}\mathbb{1}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_2\alpha_3}(\underline{\rho}\mathbb{1}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_2}, \\ M_{\alpha_3\alpha_1}\bar{x}_{\alpha_1} + M_{\alpha_3\alpha_2}(\bar{\rho}\mathbb{1}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_3\alpha_3}(\underline{\rho}\mathbb{1}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_3} = u_{\alpha_3} \end{gathered} \tag{23}$$

Since $M_{\alpha_2\alpha_1}$, $M_{\alpha_2\alpha_3}$ and $M_{\alpha_3\alpha_1}$ are nonpositive matrices (Lemma 6) and $\bar{\rho} \ge \underline{\rho} \ge 0$, (23) implies (24).

$$\begin{gathered} \bar{\mu}_{\alpha_2} > 0, \quad \bar{\nu}_{\alpha_3} > 0, \\ \ell_{\alpha_2} \ge M_{\alpha_2\alpha_1}\bar{\rho}\mathbb{1}_{\alpha_1} + M_{\alpha_2\alpha_2}(\bar{\rho}\mathbb{1}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_2\alpha_3}(\bar{\rho}\mathbb{1}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_2}, \\ M_{\alpha_3\alpha_1}\underline{\rho}\mathbb{1}_{\alpha_1} + M_{\alpha_3\alpha_2}(\underline{\rho}\mathbb{1}_{\alpha_2} + \bar{\mu}_{\alpha_2}) + M_{\alpha_3\alpha_3}(\underline{\rho}\mathbb{1}_{\alpha_3} - \bar{\nu}_{\alpha_3}) + r_{\alpha_3} \ge u_{\alpha_3} \end{gathered} \tag{24}$$

As $\mathbb{1}_m \in \ker M$, $(\ell - r) < 0$ and $(r - u) < 0$, (24) implies

$$\begin{bmatrix} M_{\alpha_2\alpha_2} & -M_{\alpha_2\alpha_3} \\ -M_{\alpha_3\alpha_2} & M_{\alpha_3\alpha_3} \end{bmatrix} \begin{bmatrix} \bar{\mu}_{\alpha_2} \\ \bar{\nu}_{\alpha_3} \end{bmatrix} < 0, \quad \begin{bmatrix} \bar{\mu}_{\alpha_2} \\ \bar{\nu}_{\alpha_3} \end{bmatrix} > 0$$

which is infeasible due to positive semidefiniteness of M. Therefore, we reach a contradiction. This means that $\alpha_2 = \alpha_3 = \emptyset$, $\bar{\mu} = \bar{\nu} = 0$ and $S' = C$. Moreover, since $\ell < r < u$ and $\mathbb{1}_m \in \ker M$ (Remark 3), we see that $(\theta\mathbb{1}_m, r) \in C$ for all $\theta \in \left[\underline{\rho}, \bar{\rho}\right]$. As such, $C \ne \emptyset$. □

After these preparations, we are in a position to prove Theorem 7.

Proof of Theorem 7. It follows from Lemma 9 that $S' = C$ and hence the optimization problems SLP′ and CVX are the same. Therefore, MIN(CVX) = MIN(SLP′). Thus, it suffices to prove MIN(SLP′) $\subseteq$ MIN(SLP).

Let $(x^*, y^*) \in$ MIN(SLP′). Then,

$$\phi(x^*, y^*) \le \phi(x, y) \quad \forall\ (x, y) \in S'.$$

Let $(\bar{x}, \bar{y}) \in S$. Then, it follows from Lemma 8 that there exists $(\hat{x}, \bar{y}) \in S'$ such that $\phi(\hat{x}, \bar{y}) \le \phi(\bar{x}, \bar{y})$. Consequently, we obtain

$\phi(x^*, y^*) \le \phi(\hat{x}, \bar{y}) \le \phi(\bar{x}, \bar{y})$. This means that $\phi(x^*, y^*) \le \phi(\bar{x}, \bar{y})$ for all $(\bar{x}, \bar{y}) \in S$. It is clear that $S' \subseteq S$. Therefore, $(x^*, y^*) \in S$ and hence $(x^*, y^*) \in \text{MIN(SLP)}$. Thus, we can conclude MIN(CVX) = MIN(SLP′) ⊆ MIN(SLP). Furthermore, C is a polyhedron and hence convex. Also, ϕ is convex on C since M is positive semidefinite due to Lemma 2. Therefore, CVX is a convex optimization problem. □

3.3. Interpretation of Theorem 7 for RTM

Theorem 7 has certain hypotheses on the parameters ℓ, u, and r. Here, we discuss the implications of these hypotheses for the proposed RTM platform. It follows from Theorem 7 that the vectors $\ell = s - \bar{h}$ and $u = s - \underline{h}$ should be nonpositive and nonnegative, respectively. The vector s is the generated energy by RESs of the prosumers, the vector $\underline{h}$ is the lower bound of the prosumers' demand and the vector $\bar{h}$ is the upper bound for their demand. To have $\ell \le 0$ and $u \ge 0$, the aggregator should ask the prosumers to set the upper bound of their demands $\bar{h}$ greater than or equal to their RESs' capacity and also the lower bound for their demands $\underline{h}$ less than or equal to s. Moreover, Theorem 7 states that r should be strictly between ℓ and u. Considering Assumption 1, we can show that

$$\underline{h} < h^0 < \bar{h} \Longrightarrow \ell < r < u.$$

4. Conclusions

The problem of participation of the prosumers in the wholesale market through the aggregator has been widely studied in the literature. To represent the intrinsic hierarchy of this problem, we developed a market platform based on a bilevel optimization problem. Bilevel optimization are generally highly nonconvex and current approaches to deal with these problems are computationally expensive. To implement this market platform in real-time, we proposed a specific convex optimization problem and showed that each global minimizer of this convex problem are also a global minimizer for the original bilevel problem under some assumptions on the parameters.

While the proposed convex approach can reduce the computational time significantly in contrast to the state-of-the-art methods (e.g., MIP), the assumption that the aggregator has a centralized control over the prosumers may limit the applicability of the proposed method to large scale networks. An interesting important area of future research could be design of a decentralized or distributed control mechanism using the convex problem to tackle this issue.

Data availability

No data was used for the research described in the article.

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