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Hybrid of "Intersection" Algorithm for Multi-Objective Optimization with Response Surface Methodology and its Application

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Abstract: Recently, a new "intersection" method for multi-objective optimization was developed in the points of view set theory and probability theory, which introduces a new idea of favorable probability to reflect the favorable degree of the utility of performance indicator in multi-objective optimization, and the product of all partial favorable probabilities of entire utilities of performance indicators makes the overall / total favorable probability of the candidate. Here, in this paper, the new "intersection" algorithm for multi-objective optimization is combined effectively with response surface methodology (RSM) by taking each response as one objective, which transfers the multi-response optimization problem into a single response one with the help of the overall / total favorable probability of each scheme. The overall / total favorable probability is the uniquely decisive index of the scheme in the optimization. Applications of the hybrid approach with two examples in material technology are given, proper predictions are obtained.

Keywords: favorable probability; "intersection" method; hybrid; multi-object optimization; response surface methodology

1 INTRODUCTION

Experimentation is an indispensable part in scientific research and technical development [1-3]. A well-designed experiment is very powerful route to get informative achievement. In general, a well-designed experiment is of representative in limited number of tests. The special technique called "experimental design" aims to effectively reach to the goal. Excellent experimental design could give more effective consequences with limited time and cost.

Response surface methodology (RSM) is a collection of statistical and mathematical technique, which is useful for optimizing processes. It has been widely used in the design, development, and formulation of new products, as well as improvement of designs of existing product [2].

The most extensive applications of RSM are in the industrial world, particularly in situations where multiple input variables potentially affect quality characteristics or performance of product or process. These quality characteristics or performance of product or process are usually called the ***response*** in the treatment. In most actual cases, the RSM faces more than one response, i.e., multiple response problems. The input variables are subjected to adjust to appropriate status for purposes of optimizing the ***responses*** to their ideal position.

While, as to optimizing the ***responses*** to their ideal position, it is uneasy problem since several responses might be with competing relationship sometimes. So, a compromised technique is usually employed in the treatment. Till now, some algorithms has been employed as compromised technique for multi-objective optimizations [1], such as the so called "comprehensive balance method", "comprehensive scoring method", "grey relational analysis" [4], signal to noise ratio analysis [5], or signal to noise ratio (SNR) together with the technique for order preference by similarity to ideal solution (TOPSIS) [6], Pareto front optimization [2], etc. However, these algorithms involve many inherent defects, such as, personal factors, setting the beneficial index and unbeneficial index at unequal positions. So, these algorithms cannot be considered as full quantitative, but empirical ones instead. Particularly, the "additive" algorithm is frequently used in the compromised treatments of the previous multi-objective optimizations, which diverges from the spirit of "simultaneous optimization" of multiple responses in the viewpoint of probability theory [7].

Recently, a new "intersection" method for multi-objective optimization was developed in the points of view set theory and probability theory [7], which aims to solve the inherent problems of personal factors and diverging from the spirit of "simultaneous optimization" of multiple responses in previous multi-objective optimization. A new concept of favorable probability was proposed to reflect the favorable degree of the candidate in the optimization, each performance utility indicator of the candidate contributes to one partial favorable probability quantitatively, and the overall / total favorable probability of a candidate is the product of all partial favorable probabilities in the viewpoint of probability theory; the total favorable probability is the uniquely decisive index in the competitive selection process, which thus transfers the multi-objective optimization problem into a single objective one.

As a further development to the newly proposed "intersection" method for multi-objective optimization, here in this paper, the new "intersection" algorithm for multi-objective optimization is combined hybridly with response surface methodology (RSM) to extend the experimental design and get an impersonal prediction. Applications of the hybrid approach with two examples in material technology are given to show the procedure of the approach.

2 COMBINATION OF THE "INTERSECTION" METHOD FOR MULTI-OBJECTIVE OPTIMIZATION ALGORITHM WITH RESPONSE SURFACE METHODOLOGY

2.1 Transformation of Multi-response Optimization Problem into a Single Response One

As to simultaneous optimization of multiple responses, several responses are involved. Lets' take each response of the scheme as one objective of new "intersection" method for multi-objective optimization [7]. Then some utilities of response might have the characteristics of "the higher the better", i.e., beneficial type, but other utilities of response might have the characteristics of "the lower the better", i.e., unbeneficial type. Thus, each utility of response contributes to one partial favorable probability in linear manner according to its type individually [7].

Furthermore, as the scheme is an integral body of both beneficial and unbeneficial indicators of utility, the overall / total favorable probability of a scheme can be obtained by the product of all partial favorable probabilities according to probability theory for simultaneous optimization of multi-response. Thus, the overall / total favorable probability transfers the multi-response optimization problem into a single response one, which is the uniquely decisive index of the scheme in the optimization.

2.2 Regression Analysis

Moreover, regression analysis is conducted for the overall / total favorable probabilities of all schemes of the designed experiment to get a regressed function of the overall / total favorable probability. Then, the maximum value of the overall / total favorable probability and values of corresponding specific input variables are gained by common algorithm of mathematics. The next step is to regress each response to get its regressed function, and then substitute the values of corresponding specific input variables into each regressed function of the response to get its compromised result.

By far, the procedure of hybrid of "Intersection" algorithm for multi-objective optimization with response surface methodology is developed.

3 EXAMPLES OF APPLICATIONS

3.1 Optimal Design of PP / EPDM / GnPs / GF Composites

Niyaraki, et al employed RSM to optimize the mechanical properties of impact strength and elastic modulus of polypropylene (PP) / ethylene propylene dine monomer (EPDM) / grapheme Nano sheets (GnPs) / glass fiber (GF) hybrid nano-composites with Box-Behnken method of RSM [8]. The impact strength and elastic modulus are the two responses of the optimization of the nano-composite.

In literature [8], there are three levels as the input variables for the three parameters, i.e., EPDM (5, 10 and 15 wt.%), GnPs (0, 1 and 2 wt.%) and glass fiber (10, 20 and 30 wt.%). It was discovered that GnPs, glass fiber and EPDM played important roles in the impact strength and elastic modulus of the nano-composites.

Here the hybrid of "Intersection" algorithm for multi-objective optimization with response surface methodology is utilized to deal with this problem.

Tab. 1 shows the experimental results of impact strength and elastic modulus, together with their partial favorable probabilities and total probabilities; both impact strength and elastic modulus are beneficial performance utility indicators in the hybrid approach [7].

From Tab. 1, it can be seen that the scheme No. 12 exhibits the maximum of the overall / total favorable probability P_i, so it could be primarily chosen as one of the optimal scheme of the multi-response RSM at the first glance directly.

Furthermore, regression analysis is conducted to the total favorable probability so as to get more accurate prediction. The regressed function for the total favorable probability is,

$$\begin{aligned}&f_{P_{t1}}\times 10^3 = 0.5751+259.6483X_1+13.0371X_3-\\&-5592.7607X_1^2+46.5832X_3^2-476.762X_1X_2+\\&+7.0492X_2X_3-167.1209X_1X_3,\\&R^2=0.9931.\end{aligned}\tag{1}$$

The function $f_{Pt1}\times10^3$ reaches to its maximum value of 6.2262 at specific values of the input variables X_1 = 0.0082 wt.%, X_2 = 0.3 wt.% and X_3 = 0.15 wt.%.

The predicted values for impact strength and elastic modulus could be obtained by substituting the above specific values of input variables X_1, X_2 and X_3 into the regression functions of impact strength and elastic modulus, respectively.

The regressed function of impact strength is,

$$\begin{aligned}&f_{I_m}=18.5833+6333.333X_1+601.6667X_2+\\&+178.3333X_3-154167X_1^2-691.667X_2^2+\\&+1233.333X_3^2-12500X_1X_2-100X_2X_3-2000X_1X_3,\\&R^2=0.9958.\end{aligned}\tag{2}$$

The predicted value for impact strength is 195.19 J/m at X_1 = 0.0082 wt.%, X_2 = 0.3 wt.% and X_3 = 0.15 wt.% from Eq. (2).

The regression function of elastic modulus is,

$$\begin{aligned}&f_{E_l}=579.875+14025X_1-155X_2-710X_3-\\&-171250X_1^2+2087.5X_2^2+2350X_3^2-23750X_1X_2+\\&+650X_2X_3-23750X_1X_3,\\&R^2=0.9961.\end{aligned}\tag{3}$$

The predicted value for elastic modulus is 712.73 MPa at X_1 = 0.0082 wt.%, X_2 = 0.3 wt.% and X_3 = 0.15 wt.% from Eq. (3).

The tested result is 195.17 J/m for impact strength, and 713.08 MPa for elastic modulus [8], which agrees with the predicted data very well and not far from the experimental results of the test 12 of Tab. 1.

3.2 Maximizing Yield and Minimizing Molecular Weight with Desired Viscosity

Myers et al. one presented a simultaneous optimal problem of maximizing yield and minimizing molecular weight with desired viscosity through two input variables reaction time x_1 and temperature x_2 [2]. The three responses variables, i.e., the yield y_1 (%), the viscosity y_2 (cSt) and the molecular weight y_3 (Mr.) of the product, and the input variables x_1 and x_2 are shown in Tab. 2.

The desired value for viscosity y_2 (cSt) is 65 cSt [2], so the utility u_2 of response y_2 might be reflected by the deviation from 65 cst, i.e., $u_2 = |y_2 - 65|$. The utility u_2 has the characteristics of "the lower the better", which pertains to unbeneficial performance index. Therefore, the assessment of partial favourable probability for desired yield y_2 is conducted by its utility u_2 as an unbeneficial index.

Table 1 Test results of impact strength and elastic modulus, and their partial favorable probabilities and total probabilities

Scheme	X_1 GnPs (wt.%)	X_2 Glass fiber (wt.%)	X_3 EPDM (wt.%)	Elastic Modulus Values (MPa)	Impact Strength Values (J/m)	Favorable probability		
						P for Elastic Modulus	*P* for Impact Strength	$P_t \times 10^3$
1	0	0.1	0.1	540	103	0.0550	0.0458	2.5144
2	0.02	0.1	0.1	660	138	0.0672	0.0613	4.1174
3	0	0.3	0.1	695	163	0.0707	0.0724	5.1212
4	0.02	0.3	0.1	720	148	0.0733	0.0657	4.8172
5	0	0.2	0.05	612	122	0.0623	0.0542	3.3753
6	0.02	0.2	0.05	703	136	0.0715	0.0604	4.3221
7	0	0.2	0.15	598	162	0.0609	0.0720	4.3795
8	0.02	0.2	0.15	642	172	0.0653	0.0764	4.9919
9	0.01	0.1	0.05	648	116	0.0659	0.0515	3.3981
10	0.01	0.3	0.05	737	158	0.0750	0.0702	5.2641
11	0.01	0.1	0.15	610	156	0.0621	0.0693	4.3019
12	0.01	0.3	0.15	712	196	0.0725	0.0871	6.3087
13	0.01	0.2	0.1	650	158	0.0661	0.0702	4.6427
14	0.01	0.2	0.1	645	160	0.0656	0.0711	4.6653
15	0.01	0.2	0.1	655	163	0.0667	0.0724	4.8265

The assessments of partial favourable probability for maximizing yield y_1 and minimizing molecular weight y_3 are performed according to the usual procedures of the "Intersection" algorithm for multi-objective optimization [7].

Table 2 Designed experiment and results of maximizing yield and minimizing molecular weight with desired viscosity

No.	Reaction time, x_1 /min	Temperature, x_2 /°C	Yield, y_1 /%	Viscosity, y_2 /cSt	Molecular weight, y_3 /Mr.
1	80	76.67	76.5	62	2940
2	80	82.22	77	60	3470
3	90	76.67	78	66	3680
4	90	82.22	79.5	59	3890
5	85	79.44	79.9	72	3480
6	85	79.44	80.3	69	3200
7	85	79.44	80	68	3410
8	85	79.44	79.7	70	3290
9	85	79.44	79.8	71	3500
10	92.07	79.44	78.4	68	3360
11	77.93	79.44	75.6	71	3020
12	85	83.37	78.5	58	3630
13	85	75.52	77	57	3150

Table 3 Consequences of assessments for partial and total favourable probabilities of the desired experiment

	Response Variables			Favourable Probability				
No.	y_1 /%	y_2 /cSt	y_3 /Mr	P_{y1}	P_{y2}	P_{y3}	$P_t \times 10^3$	Rank
1	76.5	62	2940	0.0750	0.1132	0.0869	0.7376	2
2	77	60	3470	0.0755	0.0755	0.0751	0.4275	7
3	78	66	3680	0.0765	0.1509	0.0704	0.8120	1
4	79.5	59	3890	0.0779	0.0566	0.0657	0.2897	10
5	79.9	72	3480	0.0783	0.0377	0.0748	0.2211	11
6	80.3	69	3200	0.0787	0.0943	0.0811	0.6021	5
7	80	68	3410	0.0784	0.1132	0.0764	0.6781	3
8	79.7	70	3290	0.0781	0.0755	0.0791	0.4662	6
9	79.8	71	3500	0.0782	0.0566	0.0744	0.3293	9
10	78.4	68	3360	0.0768	0.1132	0.0775	0.6743	4
11	75.6	71	3020	0.0741	0.0566	0.0851	0.3570	8
12	78.5	58	3630	0.0769	0.0377	0.0715	0.2075	12
13	77	57	3150	0.0755	0.0189	0.0822	0.1171	13

The consequences of the assessments for partial and total favourable probabilities P_{y1}, P_{y2}, P_{y3} and P_t of this product experiment are shown in Tab. 3.

From Tab. 3, it can be seen that the test No. 3 is with the maximum total favourable probability, followed by No. 1, No. 7 and No. 10.

Furthermore, regression of the total favourable probability can be done to get more accurate optimization. The fitted result for the total favourable probability is

$$\begin{aligned} P_t \times 10^3 &= -203375.1310 - 2654.9450 x_1 + 215.4526 x_2 + \\ &+15.5210 x_1^2 - 2.6907 x_2^2 - 0.0038 x_1 x_2 + \\ &+75610.091 \cdot \ln(x_1) - 0.0403 x_1^3 + 0.0112 x_2^3, \\ R^2 &= 0.7675. \end{aligned} \tag{4}$$

$P_t \times 10^3$ gets its maximum value $P_{t\,\max} \times 10^3 = 0.9334$ at $x_1 = 91.0622$ minutes, and $x_2 = 77.6053$ °C.

Meanwhile, the fitted result for the yield y_1 is

$$\begin{aligned} y_1 &= -326843.3710 - 4410.0970 x_1 + 48.0746 x_2 + \\ &+0.0180 x_1 x_2 + 25.9987 x_1^2 - 0.4803 x_2^2 + \\ &+12747.8522 \cdot \ln(x_1) + 0.0682 x_1^3 + 0.0014 x_2^3, \\ R^2 &= 0.9926. \end{aligned} \tag{5}$$

The yield y_1 gets its proper value of $y_{1\mathrm{Opt}} = 78.2722$ % at $x_1 = 91.0622$ minutes, and $x_2 = 77.6053$ °C.

Simultaneously, the fitted result for viscosity y_2 is

$$y_2 = 1454310.2050 + 20242.9592x_1 + 2436.0490x_2 - \\ -0.0900x_1x_2 + 117.4347x_1^2 - 29.7790x_2^2 - \\ -580304 \cdot \ln(x_1) + 0.3025x_1^3 + 0.1215x_2^3, \quad (6)$$

$$R^2 = 0.9723.$$

The viscosity y_2 gets its appropriate value $y_{2\mathrm{Opt}} = 68.8928$ cSt at $x_1 = 91.0622$ minutes, and $x_2 = 77.6053$ °C.

Subsequently, the fitted result for molecular weight y_3 is

$$y_3 = -176217474.000 - 2438309.5500x_1 - \\ -13078.9964x_2 - 5.7600x_1x_2 + 14549.0282x_1^2 + \\ +170.7936x_2^2 + 68063495.1500 \cdot \ln(x_1) - \quad (7) \\ -8.5337x_1^3 - 0.7128x_2^3,$$

$$R^2 = 0.9238.$$

The optimal molecular weight y_3 gets it proper value $y_{3\mathrm{Opt}} = 3590.0681$ Mr. at $x_1 = 91.0622$ minutes, and $x_2 = 77.6053$ °C.

Obviously, the optimal status for this problem is close to test No. 3 of Tab. 2.

4 DISCUSSION

From above study, the hybrid of "Intersection" algorithm for multi-objective optimization with response surface methodology is developed. Through the procedure, the multi-response optimization problem is transformed into a single response one with the help of favorable probability favorable probability; the overall / total favorable probability is the uniquely decisive index of the scheme in the optimization. Furthermore, regression analysis is conducted for the overall / total favorable probabilities of all schemes to complete the entire optimization.

5 CONCLUSION

The hybrid of "Intersection" algorithm for multi-objective optimization with response surface methodology is an effective approach in the designed experiment, which makes it possible to obtain more accurate prediction for the multi-response optimization. Considering the advantages and physical essence of the newly proposed "intersection" method for multi-objective optimization, this hybrid could supply a novel and simply way for multi-objective experimental designs.

Conflict Statement

There is no conflict of interest.

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