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Original scientific paper

Analysis of Reliability Correlation Degree of Rolling Bearings Based on Zero-Failure Data

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Abstract: Modern equipment has higher requirements for the reliability of rolling bearings. The time and economic cost of obtaining bearing failure data through test methods are getting higher. Usually, truncation time tests of small sample are used to obtain zero-failure data of bearings. Based on the zero-failure data model and multi-layer Bayesian theory, this paper improves the reliability evaluation method of rolling bearings by changing the values of hyperparameters, and calculates the estimated value of failure probability at each truncation time to obtain the reliability of the bearing. This paper adopts the theory of grey relational degree to analyze the relationship and change law of bearing reliability at each truncation time, to understand the reliability change trend of rolling bearing more comprehensively. Experiments show that the method is reasonable.

Keywords: correlation degree; reliability; rolling bearing; zero-failure data

1 INTRODUCTION

Bearing reliability is very high, and the test to study its reliability is expensive, which takes a long time. It is very difficult to obtain the failure data of the service process by means of the full life test of the bearing. Therefore, the research on the service reliability of rolling bearings is mainly based on the reliability research of zero- failure data during the service process of rolling bearings.

In the study on the reliability of the bearing's zero-failure data, Wang Wei et al. [1] combined the actual bearing problems in production and used the distribution curve to analyze the reliability of the bearing's zero-failure data. Liu Tengteng et al. [2] used the shape and scale parameters of the Weibull distribution to obtain the reliability of the bearing for a small sample of zero-failure data of the bearing. Zhang Wei et al. [3] used sequential test method to deal with the small sample failure or zero-failure data of bearing life, which can quickly obtain test results and greatly improve the efficiency of the test. Dan Zhaojiang et al. [4] transformed the Weibull distribution into an exponential distribution, combined with the zero failure data of the bearing, fitted the probability distribution of the shape parameter, and obtained the characteristic life estimation of the Weibull distribution. Lou Hongliang et al. [5] proposed a method of adding the zero- failure data of the previous time cut-off point to the reliability estimation of each truncation time point to form virtual failure information. In this paper, the characteristic life and shape parameter estimates obtained by this method have the smallest fluctuations and have better stability. Xia Xintao et al. [6] used the zero- failure data to study the reliability of the bearing, the maximum entropy method for data processing on the collected zero- failure data, and established the function of bearing failure to realize the reliability evaluation of the bearing. Xu Lingtian et al. [7] studied the performance degradation of the bearing during the working process. In the absence of information, the bearing self-sampling of the zero- failure data was carried out to obtain a large amount of data, and the reliability of the bearing performance degradation was evaluated. Li Shuang et al. [8] used Bayesian theory to modify the prior distribution of failure probability in the case of Weibull distribution without failure data. The revised failure probability estimate has strong robustness, and the method is feasible for bearing test data.

In the research of grey system theory, Xia Xintao et al. [9] avoided statistical principles and used fuzzy forecasting technology to study the reliability of bearings. This method can solve the problem of bearing dynamic performance prediction with unknown probability under the condition of less sample data. Wang Yahong et al. [10] simulated the unknown distribution under the condition of a small sample, used the intrinsic fusion technology in the lack of information to analyze the characteristics of the research object from different aspects. In the analysis process, varieties of mathematical methods were used to derive the entire probability distribution. Xu Yongzhi et al. [11] took the vibration sequence of the bearing as the research object, and used the qualitative fusion theory in the gray system to expand the amount of information of the bearing, and more reasonably infer the factors that affect the reliability of the bearing.

In the research of bearing gray Correlation degree, Xia Xintao et al. [12] used the Correlation degree theory to explore the bearing vibration problem, combed and summarized the relationship between the various factors that affect the incidental vibration, and provided a new way to reduce the working vibration of the bearing. Yang Chao et al. [13] obtained the data of the entire life cycle of bearing operation, grouped them, calculated the correlation between the data of other groups and the first group of data, and judged the reliability of the bearing according to the degree of correlation. Meng Zong et al. [14] proposed a rolling bearing fault diagnosis based on the combination of local mean decomposition multi-scale fuzzy entropy and gray similarity correlation. The gray relationship analysis method is used to analyze the relationship between the vibration and temperature of the rolling bearing under different working conditions. The research results provide a new idea for the analysis of the performance of the bearing [15]. The reliability analysis of the roughness influence in bearing processing is very complicated. According to the difficulty of the analysis, the gray Correlation degree is used for analysis [16].

At present, the main method for the reliability analysis of rolling bearings is to study according to statistical principles, and Bayesian theory is widely used in reliability evaluation. The maximum entropy theory and fuzzy technology in grey theory are used more and more in reliability assessment. However, there are few pieces of literature on the reliability evaluation of bearings combined with Bayesian theory and gray theory. This article is based on the bearing's zero- failure data to carry out the research, using the multi-layer Bayesian theory to calculate the parameter c to take different values, according to the multi-layer Bayes formula to explore the reliability of the rolling bearing at each truncation time.

2 ZERO-FAILURE DATA MODEL

As the key supporting component of mechanical equipment, rolling bearing is the prerequisite to ensure the normal operation of equipment and mechanical equipment. The reliability life test for high-reliability products is generally a timed truncation test. The current test data for bearing reliability mainly adopts zero-failure data. In the reliability test, it is assumed that k sets of timing truncation tests are performed on the tested bearings, and the sequence of the truncation moments is $t_1 < t_2 < \cdots < t_k$, among $t_1 < t_2 < \cdots < t_k$. The number of bearings put into each group is $n_1, n_2, \cdots, n_k$. After the test, if there is no bearing failure, the test data is zero- failure data. In order to study bearing reliability, the number of test bearings must reach a certain number, and the number of bearings in each group, the number of groups of test bearings, and the timing of each group's timing truncation have a certain impact on the reliability evaluation results. For the above several influencing parameters, they are mainly combined with the actual application of the bearing and the requirements for the reliability of the bearing to make a reasonable plan.

3 RELIABILITY ASSESSMENT METHOD

3.1 Multi-Layer Bayesian Estimation

For the failure probability of the bearing at time t_i is p_i, the corresponding estimated value is $\hat{p}_i$, and the prior density function of the failure probability p_i is as follows.

$$\pi(p_i \mid a,b) = \frac{p_i^{a-1}(1-p_i)^{b-1}}{B(a,b)} \tag{1}$$

Among them, $0 < p_i < 1$, $a > 0$, $b > 0$, a, b are hyper parameters, and the Beta function is as follows.

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,\mathrm{d}t$$

According to the nature of the failure probability, the smaller the value of the probability is large, so it should be a decreasing function. According to the nature of the derivative, $\pi(p_i \mid a,b)$ is the decreasing function of p_i, the condition is $0 < a \le 1,\ b > 1$.

According to Bayesian theory and the nature of Beta function distribution, comprehensively considering the robustness of Bayesian estimation, a uniform distribution is adopted for the distribution of hyper parameters a, b:

$$\pi_1(a) = 1 \tag{2}$$

$$\pi_2(b) = \frac{1}{c-1},\ 1 < b < c \tag{3}$$

Hyperparameter c is constant. If it is too large, it will affect the robustness, so it should not be too large, generally the most appropriate value in ref. [2, 8, 18]. For different working environments and working conditions, analysis and judgments should be made according to the specific types of rolling bearings.

When a, and b are uniformly distributed according to Eq. (2) and Eq. (3), the prior density function of failure probability p_i is as follows.

$$\pi(p_i \mid b) = b(1-p_i)^{b-1},\ 0 < p_i < 1 \tag{4}$$

According to the above assumptions, under the squared loss, the multi-level *Bayes* estimation is as follows.

$$p_{ihB} = \frac{(s_i+1)\ln\left(\dfrac{s_i+c+1}{s_i+2}\right) - s_i \ln\left(\dfrac{s_i+c}{s_i+1}\right)}{c-1-s_i \ln\left(\dfrac{s_i+c}{s_i+1}\right)} \tag{5}$$

3.2 Correlation Degree Analysis

The gray Correlation degree belongs to the category of lack of information, which is mainly used to describe the tightness of the data changes between the reference sequence and the observation sequence. According to the similarity of the geometric shape of the curve, it is judged whether the connection of different sequences is close.

For the two sequences X_0 and X_i, the length is the same, and the initial value is not 0.

$$X_0 = (x_0(1),\ x_0(2),\ ...,\ x_0(n))$$
$$X_i = (x_i(1),\ x_i(2),\ ...,\ x_i(n))$$

The absolute Correlation degree of the two sequences is as follows.

$$\varepsilon_{0i} = \frac{1+|s_0|+|s_i|}{1+|s_0|+|s_i|+|s_i-s_0|} \tag{6}$$

$$|s_0| = \left|\sum_{k=2}^{n-1} x_0^0(k) + \frac{1}{2}x_0^0(n)\right|,\ |s_i| = \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right|$$

X'_0, X'_I are the initial value images of X_0 and X_i, respectively, then the relative degree of association is as follows.

$$r_{0i} = \frac{1+|s'_0|+|s'_i|}{1+|s'_0|+|s'_i|+|s'_i-s'_0|} \tag{7}$$

Among them,

$$|s'_0| = \left|\sum_{k=2}^{n-1} x_0^{0'}(k) + \frac{1}{2}x_0^{0'}(n)\right|,\ |s'_i| = \left|\sum_{k=2}^{n-1} x_i^{0'}(k) + \frac{1}{2}x_i^{0'}(n)\right|$$

The gray relative correlation describes the change of the two sequences, mainly reflecting the change rate of each data relative to the first data at the beginning. If the change rate of the reference sequence and the observation sequence relative to their respective starting points are closer, the gray relative degree of correlation will be greater, and vice versa, the gray relative degree of correlation between the two sequences is smaller. The value of gray relative Correlation degree is between (0, 1]. Since the rate of change of any two series has a certain relationship, the value of gray relative Correlation degree cannot be zero. The gray absolute Correlation degree is mainly to examine the reference sequence. The problem of the value change rate of the two series and the observation sequence has nothing to do with the value of the observation. Similarly, the value of the gray absolute Correlation degree is between (0, 1]. Since any two sequences are correlated, therefore the gray absolute Correlation degree is all greater than zero. Because the size of the gray absolute Correlation degree is only related to the geometry of the two sequences, the reference sequence and the observation sequence are translated in the coordinate system, and the result will not affect the gray absolute correlation degree. For some data in the reference sequence and the observation sequence under study, some changes have occurred, and the result will affect the size of the gray absolute Correlation degree. Therefore, for the two sequences that need to be analyzed for the Correlation degree, the data variation or the change of the data length will affect the analysis of the results. The gray relative Correlation degree and the gray absolute Correlation degree are integrated as the comprehensive Correlation degree. Therefore, the size of gray comprehensive relevance is restricted by two relevance degrees. The gray comprehensive correlation degree not only reflects the similar characteristics of the absolute correlation degree, but also reflects the change rate characteristics of the relative correlation degree. It can fully reflect the relationship between the reference sequence and the observation sequence. If the similarity of the two sequences is more important, then the value θ should be selected, which is larger than 0.5. If you are very concerned about the rate of change of the two sequences relative to the starting point, then the value θ should be less than 0.5. If you value both at the same time, θ takes 0.5 as shown in Eq. (8), $\theta \in [0,1]$.

$$\rho_{0i} = \theta\varepsilon_{0i} + (1-\theta)r_{0i} \tag{8}$$

3.3 New Method of Bearing Reliability Evaluation

For the zero-failure data of rolling bearings, the research is carried out according to the multi-layer Bayesian theory, and the reliability of the rolling bearings at each truncation time is obtained, and then the reliability research is carried out according to the related theory of gray correlation. The bearing test data used in this paper is zero- failure data. Using the multi-layer Bayesian theory, according to the different values of parameter c, the reliability of the rolling bearing at truncation time is evaluated, and the reliability of the rolling bearing is calculated according to the multi-layer Bayes. The specific value is the reliability of the rolling bearing. At this time, the calculated value is calculated under the condition of different parameter c, which has a good generality. It is suitable for the evaluation of the reliability of the bearing under different conditions. The specific evaluation steps are as follows:

(1) Time truncation test is carried out on the bearing under study, and zero- failure data is obtained;

(2) The multi-layer Bayesian formula is used to calculate the zero- failure data, according to the different values of the parameter c, a series of reliability estimates can be obtained at each truncation time to form a sequence.

(3) The number of truncation moments determines the number of sequences. The sequence at the intermediate moment is used as the reference sequence, and the remaining sequences are observation sequences. In this paper, the grey correlation degree is used to analyze and obtain the changing trend of the reliability evaluation results of rolling bearings at different truncation moments.

4 NUMERICAL ANALYSIS

In order to obtain the bearing failure-free test data, according to the data provided in the literature [17], the bearing is selected as the product of a certain bearing factory, and the number of samples taken is 20 sets. Reliability test of timing truncation was carried out on 20 sets of bearings selected at random. During the test, at each stop time point, the scrolling did not appear to be in a failure state, and it was able to perform normal work.

Table 1 Bearing timing truncation test data

Truncation times i	Truncation time t_i / h	Sample size n_i	Total zero-failure S_i
1	422	2	20
2	539	4	18
3	602	2	14
4	770	4	12
5	847	4	8
6	924	4	4

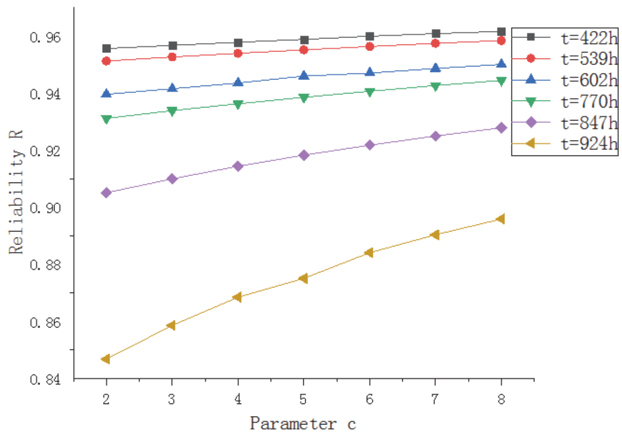

Figure 1 The relationship between multi-layer Bayesian reliability estimates and parameter c

According to the zero-failure data in Tab. 1, the reliability estimates calculated by the different parameters c at each truncation time are calculated, as shown in Fig. 1. According to Fig. 1, it can be seen that the smaller the truncation time, the reliability estimate is less affected by the parameter c. As the truncation time increases, the reliability estimate is more affected by the parameter c. In particular, the evaluation results and the two censored moments t = 847 h and t = 924 h are most affected by the parameters.

According to the correlation formula, the reliability estimate calculated by taking the middle truncation time t = 602 h is the reference sequence, and the reliability estimates at the remaining truncation time are the observation sequence for correlation analysis. The calculated data is shown in Tab. 2. It can be seen from Tab. 2 that at each truncation time, the absolute Correlation degree of the reliability estimation value of the rolling bearing is always greater than the relative Correlation degree, which indicates that the reliability of the rolling bearing changes consistently during the entire test process, and the reliability is linear and more obvious. The relative Correlation degree reflects the linear relationship between the change of subsequent reliability and the reliability value of the starting point is weaker than the linear relationship of the absolute Correlation degree. When the comprehensive relevance θ = 0.5 is used, and the absolute relevance and relative relevance of the rolling bearing reliability at each truncation time are valued. From the data in Tab. 2, we can see that the farther away from the reference sequence t = 602 h, the worse the relevance and the lower the comprehensive relevance. The order is as follows, t = 770 h, t = 539 h, t =422 h, t = 847, t = 924 h.

Table 2 Multi-layer Bayesian reliability grey correlation degree

Truncation time t / h	Absolute relevance	Relative relevance	Comprehensive relevance
422	0.9857	0.9845	0.9851
539	0.9894	0.9884	0.9889
602	1	1	1
770	0.9921	0.9913	0.9917
847	0.9642	0.9598	0.9620
924	0.9003	0.8846	0.8925

According to the zero-failure data in Tab. 1, the reliability of each truncation time is calculated by Eq. (5). According to the method of parameter selection provided by literature [17], the value of parameter c is 5, as shown in Tab. 3.

Table 3 Bearing reliability at truncation time

Truncation No. / i	Truncation time t_i / h	Sample size n_i	Reliability R
1	422	2	0.9589
2	539	4	0.9552
3	602	2	0.9460
4	770	4	0.9386
5	847	4	0.9183
6	924	4	0.8751

It can be seen from Tab. 3 that the relationship of the reliability changes of the rolling bearing at each truncation time is consistent with the change law of Tab. 2. According to the numerical changes of reliability in Tab. 2, the reliability continues to decrease with the increase of the truncation time. According to the data shown in Tab. 3, it can be seen that the closer the truncation time is to the truncation time t = 602 h, the closer the absolute Correlation degree, relative Correlation degree, and comprehensive Correlation degree of reliability are. The absolute and relative degree of correlation between the truncation time t = 770 h and the reference sequence is the largest.

5 ALGORITHM VERIFICATION

The life of the rolling bearing conforms to the two-parameter Weibull distribution, and the least square method has been performed on the data in the literature [18] to calculate the shape parameter and the scale parameter $\hat{m} = 1.7771$, $\hat{\eta} = 11272$, respectively. Therefore, the reliability point of the rolling bearing at any time t can be estimated as follows.

$$\hat{R}(t) = \exp\left(-\left(\frac{t}{11272} \right)^{1.7771} \right) \tag{9}$$

According to Eq. (5), the estimated value of the reliability of the bearing at each truncation time in Tab. 1 can be calculated, as shown in Tab. 4.

Table 4 Reliability estimates at truncation time of rolling bearings

Truncation No. / i	Truncation time / h	Sample size	Reliability R
1	422	2	0.9971
2	539	4	0.9955
3	602	2	0.9945
4	770	4	0.9936
5	847	4	0.9900
6	924	4	0.9883

This paper takes the third truncation time t = 602 h as the reference sequence, and the truncation moments with similar reliability are t = 770 h, t = 539 h, t = 422 h, t = 847 h, t = 924 h, and the reliability of multi-layer Bayesian calculations is compared and analyzed by the gray Correlation degree, and the numerical closeness is t = 770 h, t = 539 h, t = 422 h, t = 847, t = 924 h, respectively. There is a certain error between the reliability of the multi-layer Bayesian calculation and the reliability of the least square method. However, the change rule of the reliability at each truncation time is consistent with Tab. 3, which verifies the rationality of the gray Correlation degree in reliability evaluation.

6 CONCLUSION

(1) The research on reliability mainly adopts Bayesian theory. When reliability estimation is performed on multi-layer Bayes, the reliability gradually decreases as the truncation time increases.

(2) The value of parameter c will affect the evaluation result of multi-layer Bayesian estimation, and it needs to be selected reasonably according to actual reliability requirements.

(3) This paper analyzes the influence of the multi-layer Bayesian parameter c on the reliability by using the gray Correlation degree, and we can see the relationship between the reliability estimates at each censored moment. The difference is greater between the reference sequence and the observation sequence corresponding to the rolling bearing truncation time, the correlation is smaller.

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