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Quantum Computation through Entangling Single Photons in Multipath Interferometers

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Single-photon interferometry has been used to simulate quantum computations. Its use has been limited to studying few-bit applications due to rapid growth in physical size with numbers of bits. We propose a hybrid approach that employs n photons, each having L degrees of freedom yielding L^n basis states. The photons are entangled via a quantum nondemolition measurement. This approach introduces the essential element of quantum computing, that is, entanglement into the interferometry. Using these techniques, we demonstrate a controlled-NOT gate and a Grover's search circuit. These ideas are also applicable to the study of nonlocal correlations in many dimensions.

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One of the simplest systems for studying quantum computation is based upon single photons and linear optics. The unitary evolution of single photons in linear networks has been used to simulate the evolution of typical quantum computers [1–6]. It has been shown that linear optics can realize any unitary transformation on single photons [2]. Each degree of freedom of the single photon is labeled as an eigenvector in the Hilbert space. The degrees of freedom of the single photon correspond to the basis states of a typical binary quantum computer. For example, in an n-qubit binary quantum computer there are $2^n = N$ basis states. To model an n-bit quantum computer using single photons requires 2^n degrees of freedom. One can consider the single photon to be a 1-qubit N basis state quantum computer, which can simulate an n-qubit N basis state binary quantum computer.

The difficulty of 1-qubit devices, regardless of the number of basis states that the qubit may have, is that the apparatus size and complexity scales with the number of basis states [7]. For example, if one wishes to double the number of states using a 1-qubit device, then one must double the number of degrees of freedom. For few-bit applications this is relatively simple. The difficulty arises when there are more than a few qubits (3 to 5). For example, with a binary n-qubit computer, simply adding another bit will double the number of basis states in the system.

In this Letter, we propose a hybrid approach to quantum computing. Single photons having L degrees of freedom are entangled via quantum nondemolition measurements (QND) [8–12]. Single-photon multipath interferometry is employed for each photon. The photons will operate in spatially separated single-photon subcircuits, which we will refer to as S_b, where b labels the subcircuit in the total circuit. The various subcircuits "communicate" via appropriately chosen QND measurements [8,9]. This Letter assumes that it is possible to have a π cross-phase modulation (during the QND measurement) at the single-photon level [13–17]. Any implementation which allows such cross-phase modulations can be used to realize these ideas (Ref. [9] is a review of several current QND measurement schemes). Such an entanglement allows n photons with L degrees of freedom to represent L^n basis states in the calculation. This approach has several important features. First, there is a large number of basis states for relatively few qubits. Second, the fiber networks provide excellent isolation from the environment and can be well stabilized yielding small decoherence. Third, only a few QND measurements are needed in order to perform nontrivial calculations.

The photons must have carefully chosen properties. First, the photons must be time synchronized so that they interact according to design in the QND device. Second, the frequency of the photons must be set according to the device used. In some cases, depending upon the QND device, the frequency of the photons will be different. Also, in many cases, the polarization of the photons determines the strength of the QND interaction. Several single-photon sources or photon "turnstiles" have been proposed and studied [18–20] that would allow the separate sources to be synchronized. Alternatively, one could consider using weak coherent pulses having on average much less than one photon per pulse such that there is a small probability of having multiple photons in a single pulse [21]. Then coincidence detection schemes may be used to establish the synchronization. The expense is a reduced repetition rate.

A convenient method for implementing multipath single-photon interferometry employs linear integrated optics [5]. We will make use of this approach to describe the realization of the many path operations. For example, a single symmetric $M \times M$ fiber coupler will perform the discrete Fourier transform on M spatial modes of the single photon [22]. One concern with using fiber optics networks is the nonunitary operations associated with loss in the networks. For example, in [22] the loss is modeled using a diagonal matrix with each of the diagonal elements having a value set by the loss in each path of the interferometer. Approximately equal loss in each of the arms of the interferometer will not perform a "which-path" measurement in the interferometer [23]. Hence, the interference visibility will remain high. In this situation, all of the diagonal elements are roughly equal. The loss

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can then be modeled as a unity matrix multiplied by some constant. For the most part, integrated devices follow this type of behavior. In this paper, we assume a lossless system for simplicity. The primary advantage of using fiber optics is that many-path devices exist and alignment is constrained to a one-dimensional fiber.

The purpose of the QND measurements is to give a state-specific phase modulation [14]. This single-state phase shift is referred to as a quantum phase gate [13] and is one of the basic quantum gates. The cross-phase modulation entangles the photons [13]. Consider the interaction of two optical field modes in a Kerr medium [8] having a $\chi^{(3)}$ nonlinear susceptibility. The interaction Hamiltonian in the Kerr medium is given by

$$\hat{H} = 2\hbar\chi\hat{n}_{1\gamma}\hat{n}_{2\sigma}, \tag{1}$$

where the subscripts of the number operators denote which subcircuit and which spatial mode the number operator is operating on. For example, $n_{1\gamma}$ denotes that it is operating on S_1 in the γ spatial mode. We assume no self-phase modulation. Also, χ is a function of frequency and intensity and can be adjusted. The number operator is a constant of the motion. By complementarity the phase of the photons will be changed. The QND operation in the Fock state basis is then given by

$$\hat{q} = e^{i\delta\hat{n}_{1\gamma}\hat{n}_{2\sigma}}, \tag{2}$$

where δ is the net phase shift [11]. Hence, the only state that gets a δ phase shift is the one in which photon 1, in S_1, is in the γ spatial mode and photon 2, in S_2, is in the σ spatial mode. This QND operation corresponds to a quantum phase gate [13].

One of the fundamental gates is the controlled-NOT gate [24]. To implement the controlled-NOT gate, we employ the quantum phase gate. A phase shift of $\delta = \pi$ is necessary to construct the controlled-NOT gate. The schematic for the controlled-NOT gate is shown in Fig. 1. In this figure, the control bit is photon 1 and the target bit is photon 2. There are two spatial modes or paths for each photon. Photon 2 passes through a Mach-Zehnder interferometer consisting of two symmetric single-mode 2×2 fiber couplers. Without the QND measurement, photon 2 would, after leaving the Mach-Zehnder interferometer, remain in the same path. With the inclusion of the QND measurement, assuming that photon 1 is in the 1 path, photon 2 exits in the other path. If photon 1 is in the 0 path then photon 2 exits in the same initial path. This is the controlled-NOT transformation.

The transformation can be observed by looking at the evolution of the state vector. For example, let photon 1 be in the 1 spatial mode and photon 2 be in the 0 spatial mode. Then, for this example, the initial state vector is given by

$$|\Psi_0\rangle = |1\rangle_{11}|0\rangle_{10}|0\rangle_{21}|1\rangle_{20}, \tag{3}$$

where $|0\rangle$ is the vacuum state and $|1\rangle$ is the single-photon number state. The first 2×2 fiber coupler creates an

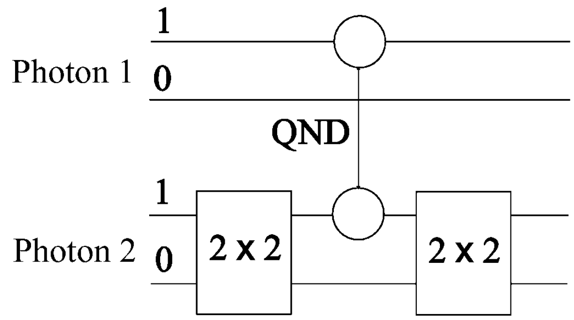

FIG. 1. Controlled-NOT gate using QND and linear integrated optics.

equal amplitude superposition of spatial modes for photon 2. The state vector is then given by

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|1\rangle_{11}|0\rangle_{10}(|0\rangle_{21}|1\rangle_{20} + |1\rangle_{21}|0\rangle_{20}), \tag{4}$$

the QND operator yields

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|1\rangle_{11}|0\rangle_{10}(|0\rangle_{21}|1\rangle_{20} + e^{i\pi}|1\rangle_{21}|0\rangle_{20}), \tag{5}$$

and finally the second beam splitter yields

$$|\Psi_3\rangle = |1\rangle_{11}|0\rangle_{10}|1\rangle_{21}|0\rangle_{20}. \tag{6}$$

Hence, photon 2 has switched spatial modes. A simpler notation is commonly used in quantum computing [1]. The basis states are written in terms of spatial modes and the subcircuits correspond to positions in the register. For example, the initial wave function for the controlled-NOT gate example would have been written as $|10\rangle$. The first number in the ket represents the spatial mode of photon 1 and the second position represents the spatial mode of photon 2. Also, for simplicity, the operators will be defined in terms of matrices. For example, the 2×2 fiber coupler in the $|00\rangle \ldots |11\rangle$ basis has the matrix form

$$B = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \tag{7}$$

and the QND phase shift has the matrix form

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{8}$$

Then the matrix form of the total transformation has the form

$$BQB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{9}$$

which is the controlled-NOT transformation matrix.

The controlled-NOT gate example demonstrates that quantum logic can be realized using QND. However, the example does not display the advantages of using linear integrated optics in this hybrid approach. The primary reason for using integrated optics and QND is that many spatial modes for each photon are possible. For example, consider commercially available 16-path devices (e.g., 16×16 fiber couplers). For a 1-photon setup, this is the equivalent number of states as a standard 4-qubit quantum computer. Then, for n photons there would be 2^{4n} basis states. In addition, it is not necessary to have the same number of spatial modes for each photon.

We have demonstrated some of the basic quantum gates. We can also implement any of the currently proposed quantum algorithms using this hybrid approach. At present it is difficult to realize a π cross-phase modulation. Therefore, we examine algorithms which require few QND operations. One example is a Grover's search algorithm [3,5,6,25,26]. The primary function of Grover's search algorithm is to take some initial state $|i\rangle$ and transform it into some target state $|t\rangle$. We desire the flexibility of starting from any initial eigenvector in the Hilbert space and evolving to any arbitrarily chosen eigenvector. With this condition, it is necessary to choose a unitary search matrix U which has equal magnitude for each of its elements. For a unitary matrix having N basis states, the magnitude of each matrix element is then equal to $1/\sqrt{N}$.

In our example, we consider a 2-photon Grover's search algorithm with each photon having M spatial modes, as shown in Fig. 2. This figure shows one iteration of an $N = M^2$ basis state search. The initial state is given by $|M-1, M-1\rangle$ and the target state is $|0,0\rangle$. Grover showed that the operator $Q = -I_i U^{-1} I_t U$ could be used to search for a desired state [26].

Consider the transformation effected by the $M \times M$ fiber coupler in each subcircuit. The symmetric fiber coupler performs the discrete Fourier transform (DFT) in the subcircuits [5,6] and will be denoted F_a where the subscript a denotes which subcircuit. The matrix elements of the DFT are given by

$$(F_a)_{jk} = \frac{1}{\sqrt{M}} e^{i2\pi jk/M}, \tag{10}$$

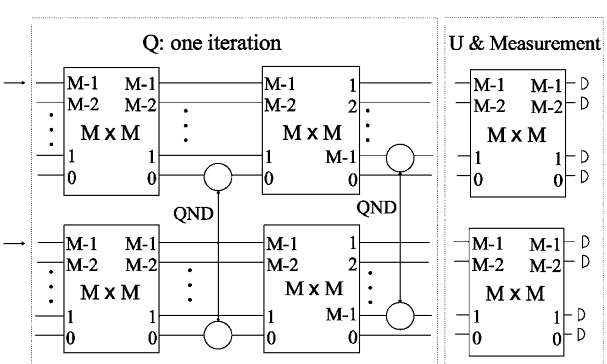

FIG. 2. Grover's search employing single-photon interferometry and a quantum nondemolition measuring device.

where j, k have values $0, 1 \ldots M-1$. The U operator is obtained by taking the tensor product of the DFT in each subcircuit $U = F_1 \otimes F_2$. The symmetric fiber couplers satisfy the condition for having equal magnitudes for each matrix element. The inverse transformation U^{-1} is obtained by taking the tensor product of the inverse DFT in each subcircuit: $U^{-1} = F_1^{-1} \otimes F_2^{-1}$. The inverse DFT can also be generated by using a symmetric $M \times M$ fiber coupler followed by a relabeling of the output paths, as shown in Fig. 2. The relabeling of the paths goes as $M-1 \leftrightarrow 1$, $M-2 \leftrightarrow 2$, ..., $M/2 \leftrightarrow M/2$ (for a specific example, see Ref. [4]).

The I_t transformation is a diagonal matrix with all of the diagonal elements equal to 1, except for the tt element, which is equal to -1. The I_i transformation is similar except that the ii element is equal to -1. These two transformations are realized by a QND cross-phase modulation in the appropriate paths. They are represented by two connected bubbles. For example, the I_{00} transformation is obtained by having a QND operation in the 0 path of S_1 and the 0 path of S_2, and in matrix form is given by

$$I_{00} = \begin{pmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

One iteration is shown in the dashed box labeled Q. In order to complete the search, the appropriate number of iterations needs to be performed. For the particular example, i.e., for 2 photons, the number of iterations goes as $O(M)$. There are several possible techniques for iteration. For example, one could use the polarization of the photons and a fast electro-optic switch as a means of extracting the photons after the desired number of iterations are completed. The photons are then taken to be measured. A final U transformation is needed and is followed by detection [26].

Thus one of the standard quantum computation algorithms may be implemented by this hybrid approach. Note that Grover's search algorithm can serve as a starting point for other quantum circuits. For example, we have shown in [6] that a slightly modified version of Grover's search can realize a quantum associative memory.

Also of key interest is the realization of a time-regulated source of nonlocally correlated photons. If time-regulated single-photon sources are used to generate the input photons, then time-regulated sources of spatially entangled photons in many dimensions [27] will be generated. These ideas can be generalized to many photons and still maintain the production rates. A source that could realize such correlations would be of significant interest fundamentally and would surpass any current technique for generating entangled photons.

The examples in this Letter have been limited to 2-photon setups. However, circuits having many photons are possible. In order to entangle n photons, $n - 1$ QND devices are required. The realization of such large circuits would be of great fundamental and computational interest.

A serious concern is the difficulty of obtaining π cross-phase modulations during QND measurements. Our ability to obtain large cross-phase modulations has increased by many orders of magnitude in just a few years. However, it is still very difficult to obtain appreciable modulations for single photons. There are two approaches that have significant promise for realizing such large nonlinear phase shifts—cavity quantum electrodynamics [13,15,16] and electromagnetically induced transparency (EIT) [14,17,28]. For example, recently Lukin and Imamoğlu [17] proposed π shifts at the single-photon level using a novel EIT scheme. Two photons with equal, slow-group velocities can interact in a transparent, nonlinear mixture of isotopes of alkali atoms. The results in their paper are based on exact overlap of the photon wave packets. Since the free-space coherence length of the single photons is many meters, it should be relatively simple to achieve good overlap of the photon wave packets. Lukin and Imamoğlu's proposed results suggest that π phase shifts may be attainable at the single-photon level.

As stated earlier, another concern is loss [22]. In this Letter we have considered the fiber networks to be lossless. However, as the number of paths increases, the loss dramatically increases. This sets a practical limit on the number of spatial modes that can be used per photon.

These ideas are a natural extension of the use of single-photon interferometry for simulating quantum logic. By adding entanglement to the system, a true quantum computing system has been proposed. Hence, it no longer just "simulates" quantum logic, but actually performs quantum computations. The addition of entanglement between photons via the QND measurements also addresses the problem of the rapid growth in complexity of these circuits as the Hilbert space grows. This hybrid approach should allow the realization of many-state optical quantum computing networks.

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