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APPLIED RESEARCH

Acceleration-Based Collision Criticality Metric for Holistic Online Safety Assessment in Automated Driving

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ABSTRACT Criticality metrics are not only essential for collision avoidance systems but also play a vital role for verification and validation of automated vehicles. With respect to the first application, criticality metrics should be real-time capable and applicable in various traffic situations. For the second application, holistic safety evaluation by criticality metrics is desired. However, existing criticality metrics hardly meet these two requirements. They are either only applicable in post-processing or only assess the safety of maneuvers in longitudinal direction. Therefore, we propose a new acceleration-based criticality metric, which is real-time capable and applicable in both longitudinal and lateral directions. The theory of the proposed criticality metric is introduced and the definition is explained according to different scenarios. A simulation platform is established to validate the criticality metric. The simulation results demonstrate that the proposed criticality metric takes all possible maneuvers into account when meeting a critical situation. Apart from the longitudinal behavior, the lateral behavior of automated vehicles can also be evaluated in real-time. Consequently, it has a wider application scope than other criticality metrics. To demonstrate its contribution to verification and validation of automated vehicles, we apply the criticality metric to a naturalistic driving dataset. The results prove that our criticality metric has a higher precision and recall than Time to Collision. Additionally, it combines the abilities of Time to Collision and Time Head Way to assess the safety of automated vehicles in the longitudinal direction. The proposed criticality metric is real-time capable and is suitable for different situations.

INDEX TERMS Autonomous vehicles, vehicle safety, road safety, collision avoidance, performance evaluation.

I. INTRODUCTION

One of the major goals in automated driving is to increase safety in road traffic. Driver assistance systems like lane keeping assist or automatic emergency brake systems are already widely used and help to reduce the number of accidents [1]. While these assistance systems only actively support the human driver in specific situations, an automated driving system (ADS) of SAE level 4 [2] and higher must be able to handle every possible situation within its operational design domain by itself. Based on the existence and dynamic behavior of other objects, the ADS must decide if they pose an unreasonable risk for the subject vehicle. Thus, the ADS must assess the criticality of each object in order to draw conclusions for the own safe behavior planning. To do this, criticality metrics are used that quantify object criticality. Especially in case of suddenly and unexpectedly appearing objects, those criticality metrics are needed by emergency functions. Apart from the role of criticality metric in decision-making systems, the criticality metrics are also useful in verification and validation (V&V) of automated vehicles (AVs). First, they can provide safety argumentations on the performance of AVs, since the safety of AVs in situations could be quantified. Second, they can be used to identify

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safety-relevant scenarios so that irrelevant data is filtered out. As a result, the identified scenarios as a vital source for the scenario-based testing [3], [4] of AVs can further be utilized to generate more similar relevant scenarios.

II. STATE OF THE ART

In the following, the currently common used criticality metrics are introduced. According to the definitions, we classify four categories: time-based, distance-based, intensity-based and other criticality metrics.

A. TIME-BASED CRITICALITY METRICS

The time to collision (TTC) [5] is a representative in the first category. It describes the remaining time until two vehicles collide with each other. The true TTC is only known retrospectively after a collision really happened, because the future movement of the colliding vehicles is not exactly known in advance. In order to assess the criticality of a situation right in time, the vehicle movements must be predicted using specific assumptions. For the $t_{\mathrm{tc},v}$, constant relative velocity between both vehicles is assumed.

$$t_{\mathrm{tc},v} = \frac{d_x}{v_{\mathrm{rel},x}} \tag{1}$$

In order to evaluate the criticality of a scenario, the minimum TTC can be calculated, which is also only applicable retrospectively.

$$t_{\mathrm{tc},v,\min} = \min\left(\left\{\frac{d_x}{v_{\mathrm{rel},x}}, v_{\mathrm{rel},x} > 0\right\}, \infty\right) \tag{2}$$

Here, $v_{\mathrm{rel},x}$ denotes the relative velocity and d_x denotes the gap between the subject vehicle and the leading object in x-direction. The longitudinal distance d_x is zero if both vehicles' bodies overlap in x-direction. The $t_{\mathrm{tc},v,\min}$ describes the smallest value in the course of the TTC, i.e., no collision happens in the considered scenario if $t_{\mathrm{tc},v,\min} > 0$. Since a minimum $t_{\mathrm{tc},v}$ can only exist if a relative acceleration or deceleration is present, this actually contradicts the assumption of constant $v_{\mathrm{rel},x}$. When $t_{\mathrm{tc},v,\min}$ is reached, the most critical part of the scenario is already over. Further explanation of this fact is given in the appendix.

Time Exposed TTC (TET) and Time Integrated TTC (TIT) [6] as two variants of TTC evaluate the safety of vehicles over a certain space and time. Thus, they could provide a more complete and comprehensive safety level in the overserved space and time. They are defined as:

$$\mathrm{TET} = \int_{t=0}^{T} \delta\left(t_{\mathrm{tc},v}, t^{*}_{\mathrm{tc},v}\right) \mathrm{d}t \tag{3}$$

$$\mathrm{TIT} = \int_{t=0}^{T} \left(t^{*}_{\mathrm{tc},v} - t_{\mathrm{tc},v}\right) \mathrm{d}t, \forall 0 \le t_{\mathrm{tc},v} \le t^{*}_{\mathrm{tc},v} \tag{4}$$

in which $\delta(t_{\mathrm{tc},v}, t^{*}_{\mathrm{tc},v})$ is 1 if $0 \le t_{\mathrm{tc},v} < t^{*}_{\mathrm{tc},v}$. Otherwise, it is 0. $t^{*}_{\mathrm{tc},v}$ denotes the threshold of $t_{\mathrm{tc},v}$. T is the observed period.

Additionally, the relative acceleration of two vehicles is not considered in TTC, which further impairs the earlier recognition of a critical scenario. Thus, the enhanced TTC [7] is motivated, which uses the relative acceleration of two vehicles directly in the definition for the prediction, as expressed by

$$t_{\mathrm{tc,e}} = \frac{\sqrt{v^2_{\mathrm{rel},x} + 2D_{\mathrm{rel}}d_x} - v_{\mathrm{rel},x}}{D_{\mathrm{rel}}} \quad \forall\, v^2_{\mathrm{rel},x} + 2D_{\mathrm{rel}}d_x \ge 0 \tag{5}$$

where D_{rel} is the relative deceleration, so the negative relative acceleration.[1] This model leads to implausible collision predictions if the object vehicle reaches standstill before the collision. Then, according to enhanced TTC, the object vehicle would still have a negative acceleration and thus start driving backwards.

Different from TTC, time head-way (THW) [8] describes the time remained that the subject vehicle reaches the position of the leading vehicle, as shown in (6). In this equation, v_{sub} is the velocity of the subject vehicle. Since small TTC values can only appear when small THW occurs, THW can quantify potential critical situations, while TTC measures the actual occurrences of critical situations (with predicted constant velocity for the subject vehicle):

$$\mathrm{THW} = \frac{d_x}{v_{\mathrm{sub}}} \tag{6}$$

Furthermore, the validity of TTC reduce greatly in lateral directions. Hence, post encroachment time (PET) [9] is proposed, which is more appropriate for intersection situations. The time gap between one vehicle entering and another vehicle leaving a conflicted area, as defined in (7), assuming $t_{\mathrm{sub/obj,entry/exit}}$ is the time that the subject or object vehicle enters or leaves the conflicted area.

$$\mathrm{PET} = \max\left(\begin{matrix}\left(t_{\mathrm{obj,entry}} - t_{\mathrm{sub,exit}}\right), \\ \left(t_{\mathrm{sub,entry}} - t_{\mathrm{obj,exit}}\right)\end{matrix}\right) \tag{7}$$

If PET is negative according to this definition, a collision happens between subject and object vehicle. Since only the time to reach a defined location is required, PET is easy to measure and intuitive to understand. However, it is only suitable for post analysis of traffic situations. Therefore, the predictive encroachment time (PrET) [10] is motivated, which predicts the motion of involved objects with a constant velocity point model. The states $\boldsymbol{p}_1(t+t_1)$ and $\boldsymbol{p}_2(t+t_2)$ represent the predicted positions of two objects. t_1 and t_2 are the remaining time for the two objects to reach the collision position.

$$\begin{aligned}\mathrm{PrET} &= \min\left(\left|t_1 - t_2\right|, \infty\right) \\ \text{with } \boldsymbol{p}_1&\left(t+t_1\right) = \boldsymbol{p}_2\left(t+t_2\right), t_1, t_2 \ge 0\end{aligned} \tag{8}$$

Worst-time-to-collision (WTTC) [11] is theoretically applicable in all directions by considering the worst situation,

[1] Deceleration D is introduced to avoid misunderstanding of minimum and maximum, which could occur in case of negative accelerations.

in which the actions of two vehicles towards a collision are selected. It is obtained by finding the $t_{\min}$ that meets the inequality (9). The size of vehicles are simplified as cycles with radius r_{sub} and r_{obj} and the accelerations of subject and object vehicle are given by a_{sub} and a_{obj}, respectively. The Euclidean distance of the two vehicles is represented by $d\left(\boldsymbol{p}_{\text{sub}}, \boldsymbol{p}_{\text{obj}}\right)$. Since the worst situation is selected, the criticality is usually overestimated. However, it is suitable to filter out irrelevant data.

$$\frac{1}{2}a_{\text{sub}}t^2 + r_{\text{sub}} + \frac{1}{2}a_{\text{obj}}t^2 + r_{\text{obj}} \geq d\left(\boldsymbol{p}_{\text{sub}}, \boldsymbol{p}_{\text{obj}}\right) \tag{9}$$

The time-based metrics described above do not take into account the possible actions of a driver. Hence, time to react (TTR) [12] including time to steer (TTS), time to brake (TTB) and time to kickdown (TTK) are motivated. They represent the remaining time for the driver to begin a maneuver that avoids a collision with the object. Generally, TTR is expressed as

$$\max_{t^*}\left\{t^* \geq 0 \mid \exists u(t) : \boldsymbol{p}_{\text{sub}}(t) \cap \boldsymbol{p}_{\text{obj}}(t) = \emptyset\right\} \ \forall t \in (0, t_{\max}] \tag{10}$$

The positions of subject and object vehicle at time t are described by $\boldsymbol{p}_{\text{sub}}(t)$ and $\boldsymbol{p}_{\text{obj}}(t)$, respectively. The action of a driver is represented by $u(t)$ and consists of steering and braking. The time until the subject vehicle passes by the object is given by $t_{\max}$. The variable t^* to be maximized is TTR.

From the definition in (10), we know that the difficulty to prevent a collision is expressed. The less time left, the more difficult it is to prevent a collision. However, no difficulty of the maneuver executed by the driver is included.

B. DISTANCE-BASED CRITICALITY METRICS

The distance-based criticality metrics quantify the criticality of scenarios based on the distance remaining before a collision occurs. A simple example in this category is the distance head-way (DHW) [8], which describes intuitively the distance remaining to the leading vehicle. In other words, the DHW equals d_x to the currently leading vehicle. The difference of space distance and stopping distance (DSS) [13] and the proportion of stopping distance (PSD) [9] are two most common used ones in this category.

$$\text{DSS} = \left(\frac{v_{\text{obj}}^2}{2\mu g} + d_x\right) - \left(v_{\text{sub}}\tau + \frac{v_{\text{sub}}^2}{2\mu g}\right) \tag{11}$$

$$\text{PSD} = \frac{d_x}{\text{MSD}}, \text{ with MSD} = \frac{v_{\text{sub}}^2}{2a_{\max,\text{acc}}} \tag{12}$$

where μ is the friction coefficient, τ is the reaction time, d_x represents the remaining distance to the potential point of collision, MSD is minimum acceptable stopping distance and $a_{\max,\text{acc}}$ denotes the maximum acceptable deceleration.

DSS measures the remaining distance by subtracting the stopping distance from the space distance, as can be seen in Fig. 1. Therefore, if DSS is negative, a collision is not avoidable. PSD defines the ratio between the remaining

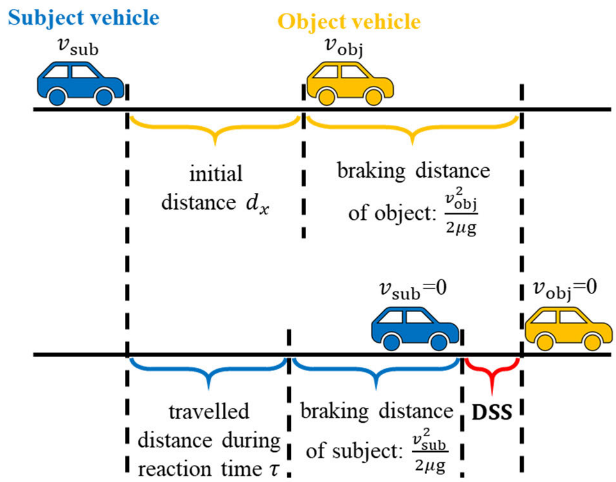

FIGURE 1. Explanation of DSS: the upper part depicts the initial distance between subject and object vehicle as well as the braking distance of the object. The lower part shows the stopping distance of the subject and the remaining DSS.

clearances of two objects to the minimum acceptable stopping distance. Therefore, if the PSD value is less than one, a situation is regarded as critical and unacceptable. Based on its definition, PSD would be suitable to be applied at intersections, where the subject vehicle should stop before the traffic lights or stop signs.

C. INTENSITY-BASED CRITICALITY METRICS

The intensity-based criticality metrics include the required longitudinal deceleration and the required lateral acceleration. Different from the above two categories, this category shows directly how difficult it is to avoid a collision by considering their possible actions and in terms of force proportional acceleration. The most common used one is deceleration rate to avoid the crash (DRAC) [14], which describes the required deceleration for the subject vehicle to avoid collision with the leading vehicle. If an evasion maneuver is also possible, the required lateral acceleration [15] is proposed. However, DRAC does not include the maximum braking performance of a vehicle, which can underestimate the criticality in some situations, since the maximum braking performance is different under different conditions. Therefore, brake threat number (BTN) [15] is proposed, which is the DRAC divided by the maximum available longitudinal deceleration as shown in (13). Similar to BTN, steer threat number (STN) [15] takes maximum available lateral acceleration into account for the required lateral acceleration. Thus, if BTN or STN larger is than one, a collision is unavoidable. Based on the above literatures, it is obvious that the current intensity-based metrics have not considered braking and steering simultaneously.

$$\text{BTN} = \frac{a_{\text{req},x}}{a_{x,\max}} \text{ and STN} = \frac{a_{\text{req},y}}{a_{y,\max}} \tag{13}$$

D. VELOCITY-BASED CRITICALITY METRICS

In addition, there are also some metrics for quantifying the crash severity. The delta-v (Δv) [16] belongs to this category and gives the probability of a severe injury or even a fatality. It is defined as

$$\Delta v_{\text{sub}} = \frac{m_{\text{obj}}}{m_{\text{sub}} + m_{\text{obj}}} \sqrt{v_{\text{sub}}^2 + v_{\text{obj}}^2 - 2v_{\text{sub}}v_{\text{obj}}\cos\alpha} \tag{14}$$

with the mass of the subject and object vehicle m_{sub} and m_{obj} and the approach angle α. A value of $\alpha = 180°$ represents a head-on collision. Similarly, Δv_{obj} can be obtained as well. Thus, the severity is max $(\Delta v_{\text{obj}}, \Delta v_{\text{sub}})$. Δv is further extended to conflict severity (CS) [17] by considering the braking maneuver of a driver. It supposes that the severity decreases if the braking maneuver is involved.

$$\text{CS} = \Delta v_{\text{sub}} - \frac{m_{\text{obj}}}{m_{\text{sub}} + m_{\text{obj}}} (\text{TA} \cdot a_x) \tag{15}$$

TA represents the available braking time. $\boldsymbol{a_x}$ is the estimated deceleration of the subject vehicle during the braking maneuver.

E. OTHER CRITICALITY METRICS

There are still some other criticality metrics, which do not fall into the above mentioned three categories. For instance, the responsibility sensitive safety (RSS) [18] defines a safe lateral and longitudinal distances. By comparing the current lateral distance d_y and longitudinal distance d_x with the safe distances $(d_{x,\min}, d_{y,\min})$, a critical situation can be determined.

$$\text{RSS}_{\text{critical}} = \begin{cases} 1, & \text{if} d_x > d_{x,\min} \cap d_y > d_{y,\min} \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

Another example is the trajectory criticality index (TCI) [19], which is derived from the optimization of trajectory planning. The cost of the optimal trajectory is regarded as the TCI in the current situation. As defined in (17), the minimum a_x and a_y quantify the situation criticality.

$$\begin{aligned} \text{TCI} = \min_{a_x, a_y} \sum_{k=1}^{N-1} & w_x R_x(k) + w_y R_y^2(k) \\ & + \frac{w_{ax} a_x^2(k) + w_{ay} a_y^2(k)}{(\mu_{\max} g)^2} \end{aligned} \tag{17}$$

where N is the prediction horizon, k is one step in N and $\mu_{\max}$ is the maximum friction coefficient. Further parameters are the gravitational constant g, the weights$w$ as well as R_x and R_y, that represent the reserve for corrections in speed and course angle, respectively. R_x and R_y are dependent on a_x and a_y.

The reachability analysis (RA) proposed by [20] measures the safety of an AV whether its occupancy has no intersection with that of other objects all the time. Therefore, if a specific time step exists at which the occupancy of the subject vehicle intersects with that of the object, the safety cannot be guaranteed. $t_{\max}$ is the last time step of the intended trajectory.

$$\text{RA}_{\text{critical}} = \begin{cases} 1, & p_{\text{sub}}(t) \cap p_{\text{obj}}(t) \neq \emptyset, \exists t(0, t_{\max}] \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

F. SUMMARY STATE-OF-THE-ART

Five different categories of criticality metrics are presented. The intensity-based category reflects the difficulty of a maneuver, while the time-based category shows the difficulty to avoid a collision [21]. The velocity-based criticality metric defines the potential crash severity, which is not considered in the first two categories. Since most metrics focus on rear-end collision, metrics to quantify risk arising from the lateral direction are less studied, some optimization technique-based metrics are recently studied, e.g., TCI. However, the criticality problem has non-linear properties that require a high computational effort. Thus, an effortless and holistic criticality metric is desired.

In this paper, we focus on the microscopic risk of AVs, which means the risk in a single scene [21]. Macroscopic risk, so the average risk of a vehicle is out of scope, i.e., the occurrence rate of fatal accidents of a vehicle with and without an ADS will not be discussed. However, the macroscopic risk is vital for the release of AVs and is a key issue in the acceptance of this technology by stakeholders. In addition to the social aspect, the economic benefit is also a concerning aspect when introducing a new technology. The P.E.A.R.S. consortium [22] is, for example, a group aiming at developing a harmonized approach to assess the effectiveness of road safety technologies.

III. METHODOLOGY

After we presented the state of the art in the previous section, we see that various approaches to quantify collision criticality already exist. Still, there is lack of application for some of these metrics. Especially the intensity-based metrics only seem to cover a few specific traffic scenarios, since they focus on solely the criticality assessment in longitudinal direction. Thus, we follow up on these intensity-based criticality metrics and describe them more general in order to make them applicable to more scenarios. Our contributions in this paper are the following:

- An intensity-based criticality metric for both longitudinal and lateral directions is proposed. Its theory is explained and modelling is given. As a result, its application scope is extended.
- The performance of the proposed criticality metric is revealed and compared with other common criticality metrics to demonstrate its strength. The proposed criticality metric is capable of assessing the safety of AVs during the driving, i.e., it is real-time capable of identifying critical scenarios. Additionally, it can be involved in the decision-making of AVs to make safe decisions.

In the next section, we identify the relevant parameters for criticality assessment and analyze in theory what traffic scenarios are collision-critical. Based on that, we derive an acceleration-based criticality metric that covers these critical scenarios and provides the safest maneuver option in each situation. Afterwards, we test the metric in two steps. First, we apply it to a simulated AV in the defined critical scenarios.

Second, we apply it to real traffic by using the HighD [23] dataset. After evaluating the test results, we finally conclude the findings of our work and give an outlook on future research.

IV. THEORY

The basic concept of our new acceleration-based criticality metric is to determine the smallest required longitudinal or lateral acceleration for collision avoidance. Therefore, different maneuver options like e.g. braking or evading are analyzed. The smallest required acceleration of all considered maneuver options defines the criticality of the current traffic situation. In this section, we first introduce each maneuver option that is considered by the criticality metric step by step. Corresponding definitions and equations for each of them are given. Then, the final criticality metric, which is composed of the previously introduced components, is explained in detail. In different scenarios, we illustrate which component plays a role in the final criticality metric. Finally, the theory to calculate the proposed criticality metric is established.

A. DEFINITION OF D_{req} AND $a_{req,eva}$

When the subject vehicle meets a leading slow or static object, it is motivated to brake or change its lane if possible. The deceleration or acceleration required to perform these two possible maneuvers to avoid the collision with the leading object can be quantified as

$$D_{\mathrm{req,imm}} = D_{\mathrm{obj}} - \frac{v_{\mathrm{rel},x}\left|v_{\mathrm{rel},x}\right|}{2d_x} \tag{19}$$

$$v_{\mathrm{rel},x} = v_{\mathrm{obj},x} - v_{\mathrm{sub},x} \tag{20}$$

$$a_{\mathrm{req,eva},v,\mathrm{imm}} = \frac{2\left(y_{\mathrm{eva}} - v_{\mathrm{rel},y}t_{\mathrm{tc},v,x}\right)}{t^2_{\mathrm{tc},v,x}} \tag{21}$$

where $D_{\mathrm{req,imm}}$ represents the longitudinal deceleration required to avoid the collision with the front vehicle in the current lane, while $a_{\mathrm{req,eva},v,\mathrm{imm}}$ represents the lateral acceleration required to evade to avoid the collision under the assumption of constant velocity. Both assume that the deceleration or acceleration is immediately applied. The parameter D_{obj} is the deceleration of the leading object and $v_{\mathrm{rel},y}$ is the relative lateral velocity towards the evasion direction. The time $t_{\mathrm{tc},v,x}$ depicts the TTC in longitudinal direction with the prediction of constant velocity and y_{eva} is the lateral space required for evasion, which is expressed by

$$y_{\mathrm{eva}} = \begin{cases} \frac{w_{\mathrm{obj}}+w_{\mathrm{sub}}}{2} + d_y, & \text{for } y_{\mathrm{eva_r}} \\ \frac{w_{\mathrm{obj}}+w_{\mathrm{sub}}}{2} - d_y, & \text{for } y_{\mathrm{eva_l}} \end{cases} \tag{22}$$

with w_{obj} and w_{sub} being the width of the leading object and the subject vehicle, respectively. The distance d_y is the lateral gap between the subject vehicle and leading object, which can be obtained by calculating the perpendicular distances of the leading object and the subject vehicle to a reference line, as shown in Fig. 2. The index l and r stand for left and right, respectively.

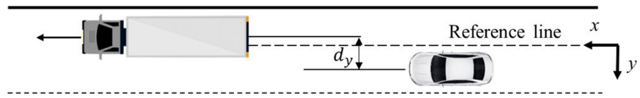

FIGURE 2. Explanation of y_{eva} and the map coordination system.

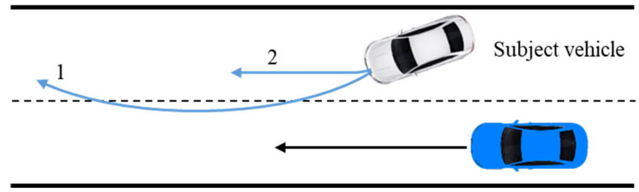

FIGURE 3. Two options of collision avoidance with object on an adjacent lane. Option 1: entering the adjacent lane and steering back to subject lane before object reaches subject vehicle. Option 2: stay in subject lane.

The presented equations are valid for the assumption that $D_{\mathrm{req,imm}}$ or $a_{\mathrm{req,eva},v,\mathrm{imm}}$ are applied immediately without delay. However, considering computation time and the delays of actuators like build-up time of brake pressure requires a modification of these equations. We assume that during the delay time τ, all vehicles keep their current accelerations and thus, D_{rel} is constant as well. After τ, the subject vehicle instantly applies the required reaction by braking or steering. Then, in (19) $v_{\mathrm{rel},x}$ is to be replaced by $v_{\mathrm{rel},x} + D_{\mathrm{rel}}\tau$ and d_x is to be replaced by $d_x - v_{\mathrm{rel},x}\tau - 0.5D_{\mathrm{rel}}\tau^2$. Furthermore, $a_{\mathrm{req,eva}}$ is then defined using $t_{\mathrm{tc,e}}$ instead of $t_{\mathrm{tc},v}$ and y_{eva} is replaced by $y_{\mathrm{eva}} - v_{\mathrm{rel},y}\tau$. This leads to the following equations:

$$D_{\mathrm{req}} = D_{\mathrm{obj}} + \frac{\left(v_{\mathrm{rel},x} + D_{\mathrm{rel}}\tau\right)\left|v_{\mathrm{rel},x} + D_{\mathrm{rel}}\tau\right|}{2\left(d_x - v_{\mathrm{rel},x}\tau - 0.5D_{\mathrm{rel}}\tau^2\right)} \tag{23}$$

$$a_{\mathrm{req,eva}} = \frac{2\left(y_{\mathrm{eva}} - v_{\mathrm{rel},y}\left(t_{\mathrm{tc,e}} + \tau\right)\right)}{t^2_{\mathrm{tc,e}}} \tag{24}$$

If the evasion is possible only to one side, e.g., on a two-lane road, the acceleration required to the other side will be infinite. As a result, impossible evasion is excluded when we use $\min(D_{\mathrm{req}}, \left|a_{\mathrm{req,eva_l}}\right|, \left|a_{\mathrm{req,eva_r}}\right|)$ to quantify the criticality of the situations.

B. DEFINITION OF $a_{req,ste,v}$

The above introduced D_{req} and $a_{\mathrm{req,eva}}$ are only applicable in the longitudinal direction, i.e., there is only a leading object in front. However, this is not always the case. When taking an evasion maneuver, the rear left or rear right object matters. It decides whether the evasion maneuver is safe or not. When the subject vehicle intends to change its lane to the left and already moves laterally, but suddenly notices that there is a critical approaching rear left object, there are two maneuver options. The first one is to temporarily enter the left lane and leave it again before the object longitudinally reaches the subject vehicle. The second option is to steer to the right and stay in the current lane, as depicted in Fig. 3.

In order to determine whether the subject vehicle will collide with the rear adjacent object, we need $t_{\mathrm{tc},v,x}$ and $t_{\mathrm{tc},v,y}$ in longitudinal and lateral direction, respectively. If the time difference $\Delta t_{\mathrm{tc},v}$ between $t_{\mathrm{tc},v,x}$ and $t_{\mathrm{tc},v,y}$ is small enough, the subject vehicle will collide with the rear adjacent object, since they will reach the same place at the same time. Therefore, whether the subject vehicle should steer back depends on if

$$\Delta t_{\mathrm{tc},v} = |t_{\mathrm{tc},v,x} - t_{\mathrm{tc},v,y}| \tag{25}$$

is smaller than a threshold $\Delta t_{\mathrm{tc},v,\mathrm{crit}}$, which is deduced later in this section. If $\Delta t_{\mathrm{tc},v} < \Delta t_{\mathrm{tc},v,\mathrm{crit}} \wedge \max\left(t_{\mathrm{tc},v,x}, t_{\mathrm{tc},v,y}\right) > 0$, the acceleration $a_{\mathrm{req,ste},v}$ required for steering back is defined as the minimum of the required accelerations for the two maneuver options in Fig. 3. The first argument in the minimum function corresponds to maneuver option 1, the second argument to option 2.

$$a_{\mathrm{req,ste},v} = \min\left(\begin{array}{c} \frac{2\left|y_{\mathrm{ste}} - v_{\mathrm{rel},y}\left(t_{\mathrm{tc},v,\max} + \tau\right)\right|}{t^2_{\mathrm{tc},v,\max}}, \\ \frac{v^2_{\mathrm{rel},y}}{2\max\left(y_{\mathrm{ste}} - v_{\mathrm{rel},y}\tau, 0\right)} \end{array}\right) \tag{26}$$

$$t_{\mathrm{tc},v,\max} = \max\left(t_{\mathrm{tc},v,x}, t_{\mathrm{tc},v,y}\right) \tag{27}$$

Here, $v_{\mathrm{rel},y}$ denotes the relative lateral velocity between the subject vehicle and the rear object in adjacent lane. Since the collision will not occur before both $t_{\mathrm{tc},v,x}$ and $t_{\mathrm{tc},v,y}$ are reached, we use the maximum of both in this equation. y_{ste} represents the lateral distance until the subject vehicle enters the neighbouring lane and is defined in (28). With the distance $d_{y,\mathrm{marking}}$ from the middle of the subject vehicle to the lane marking, where the subject vehicle is heading to, we define

$$y_{\mathrm{ste}} = \begin{cases} d_{y,\mathrm{marking}} - \frac{w_{\mathrm{sub}}}{2}, & \text{for } y_{\mathrm{ste_r}} \\ d_{y,\mathrm{marking}} + \frac{w_{\mathrm{sub}}}{2}, & \text{for } y_{\mathrm{ste_l}} \end{cases} \tag{28}$$

If $y_{\mathrm{ste}} < 0$, the second maneuver option is not applicable anymore. Thus, in (26) the second argument is infinite.

In order to derive $\Delta t_{\mathrm{tc},v,\mathrm{crit}}$, we consider two cases. The subject vehicle can either change the lane after the object on the left lane has passed or before the object passes. In the first case, $t_{\mathrm{tc},v,x} < t_{\mathrm{tc},v,y}$ and $\Delta t_{\mathrm{tc},v,\mathrm{crit}} = \frac{l_{\mathrm{sub}}}{v_{\mathrm{rel},x}}$, l_{sub} depicts the length of the subject vehicle. Because then, the front of the subject vehicle just touches the back of the object when entering the left lane. If $\Delta t_{\mathrm{tc},v} < \frac{l_{\mathrm{sub}}}{v_{\mathrm{rel},x}}$, there will be a longitudinal overlap between the subject vehicle and the object when the subject vehicle enters the left lane.

In the second case, $\Delta t_{\mathrm{tc},v,\mathrm{crit}}$ is determined as the time that the object needs to brake to reach the velocity of v_{sub} with braking distance of d_x. As a result, when the object reaches the back of the subject vehicle, $v_{\mathrm{rel},x} = 0$ and a crash is avoided. In this case, $t_{\mathrm{tc},v,y} < t_{\mathrm{tc},v,x}$. Starting from the time when the subject vehicle enters the left lane, the required time until $v_{\mathrm{rel},x} = 0$ is $\frac{v_{\mathrm{rel},x}}{a_{x,\max}} + \tau$. The maximum acceptable deceleration that the object vehicle has to apply is given by $a_{x,\max}$ and τ is the reaction time until $a_{x,\max}$ is applied.

$$\Delta t_{\mathrm{tc},v,\mathrm{crit}} = \begin{cases} \frac{l_{\mathrm{sub}}}{v_{\mathrm{rel},x}}, & \text{if } 0 < t_{\mathrm{tc},v,x} < t_{\mathrm{tc},v,y} \\ \frac{v_{\mathrm{rel},x}}{a_{x,\max}} + \tau, & \text{if } 0 < t_{\mathrm{tc},v,y} < t_{\mathrm{tc},v,x} \end{cases} \tag{29}$$

C. DEFINITION OF THE CRITICALITY METRIC C_a

D_{req}, $a_{\mathrm{req,eva}}$ and $a_{\mathrm{req,ste},v}$ introduced above are designed to avoid collisions in three specific simple traffic situations. By combining these different situations, we derive the final criticality metric C_{a}. In order to properly combine longitudinal and lateral accelerations, we use vectors for $\boldsymbol{D}$ and $\boldsymbol{a}$ in the following. The metric C_{a} is expressed by a minimum function, which contains an argument for each available combination of the iterators δ and α:

$$\min\left(\begin{array}{l} \big\| \delta\left(\boldsymbol{D}_{\mathrm{req}} + \lambda \boldsymbol{a}_{\mathrm{req,ste}_\alpha,v}\right) \\ + (1-\delta)\left(\boldsymbol{a}_{\mathrm{req,eva}_\alpha} + \boldsymbol{D}_{\mathrm{req}_\alpha}\right)\big\|_{\mathrm{foreach}\delta,\alpha} \end{array}\right) \tag{30}$$

where $\delta = \{0, 1\}$ is a switch factor. It decides that the subject vehicle can either stay in the current lane or evade to an adjacent lane. For the maneuvers to maintain the current lane, a further distinction is made between pure braking and simultaneous braking and steering, which is realized by a Boolean function λ. It is defined as:

$$\lambda = \begin{cases} 1, & \text{if } \Delta t_{\mathrm{tc},v} < \Delta t_{\mathrm{tc},v,\mathrm{crit}} \\ 0, & \text{otherwise} \end{cases} \tag{31}$$

As mentioned in the definition of $\boldsymbol{a}_{\mathrm{req,ste},v}$, if $\Delta t_{\mathrm{tc}} < \Delta t_{\mathrm{tc,crit}}$, a collision with an object on an adjacent lane will happen if the vehicles do not change their current movement states. Thus, we determine the required lateral acceleration $\mathbf{a}_{\mathrm{req,ste},v}$ for steering back to avoid the collision. Furthermore, we use a direction index α to determine which steering direction should be analyzed. It is expressed by:

$$\alpha = U \setminus O, \text{ with } U = \{\text{left}, \text{right}\}, \quad O \subseteq U \tag{32}$$

where U depicts the universal set of lateral directions, while O is a subset of U and means the lateral direction in which a critical situation would arise due to a lateral movement of the subject vehicle. Hence, O defines the adjacent lanes that the subject vehicle is not allowed to enter and α is the complementary set of O. λ affects O since it quantifies if the subject vehicle during lane changing will collide with the approaching object in an adjacent lane. However, the situations in which $t_{\mathrm{tc},v,x}$ or $t_{\mathrm{tc},v,y}$ are infinite or negative is not covered by λ. If, e.g., the object at an adjacent lane has the same velocity as the subject vehicle but a small longitudinal distance, the subject vehicle is also not allowed to enter the adjacent lane. These situations are quantified by:

$$\beta = \begin{cases} 1, & \text{if } d_x < d_{\mathrm{crit}} \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

According to [24], we define d_{crit} as:

$$d_{\mathrm{crit}} = \frac{v_{\mathrm{I}}^2}{2D_{\max,\mathrm{I}}} + v_{\mathrm{I}}\tau - \frac{v_{\mathrm{II}}^2}{2D_{\max,\mathrm{II}}} \tag{34}$$

where I is the following vehicle and II is the leading vehicle.

Therefore, both λ and β define O. If $\lambda = 1 \vee \beta = 1$ for an object in an adjacent lane of the subject vehicle, the lateral direction of this object from the perspective of the subject

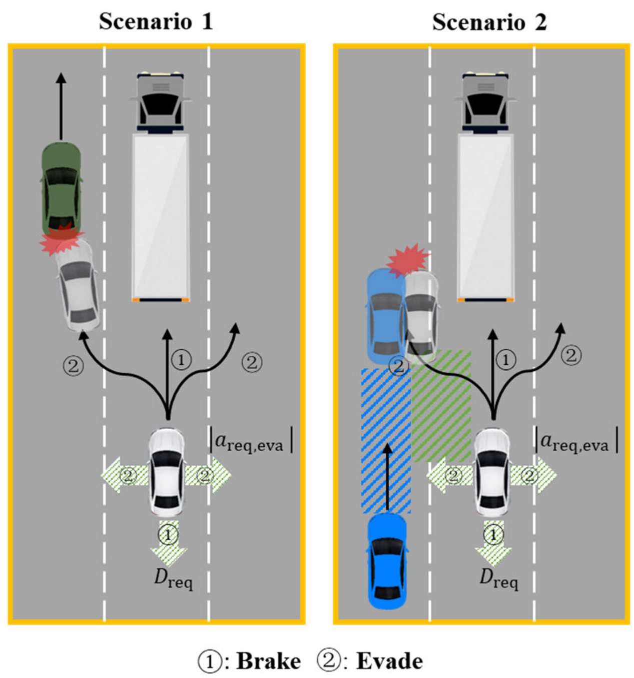

FIGURE 4. Collision-critical scenarios and different reaction options for the subject vehicle.

vehicle is an element of O. If $\lambda = \beta = 0$ for all objects in adjacent lanes, O is empty.

In order to give a clear explanation of the defined C_{a}, we use the following two scenarios to demonstrate how C_{a} concretely looks like, as shown in Fig. 4.

Scenario 1: a slow front object and a front left object exist. In this case, λ and β are always zero, because no objects are approaching from behind on adjacent lanes. Therefore, $O = \emptyset$ and we iterate through $\delta = \{0, 1\}$ and $\alpha = \{\text{left}, \text{right}\}$. The metric from (30) can be rewritten as:

$$C_{\text{a}} = \min \left(\begin{array}{c} \|\boldsymbol{D}_{\text{req}}\|, \\ \|\boldsymbol{a}_{\text{req,eva}_\text{l}} + \boldsymbol{D}_{\text{req}_\text{l}}\|, \\ \|\boldsymbol{a}_{\text{req,eva}_\text{r}}\| \end{array} \right) \tag{35}$$

Due to the slow front object, the subject vehicle is motivated to brake or evade to the adjacent lane. Braking in the middle lane and evading to the right lane are quantified by $\left\|\boldsymbol{D}_{\text{req}}\right\|$ and $\left\|\boldsymbol{a}_{\text{req,eva}_\text{r}} + \boldsymbol{D}_{\text{req}_\text{r}}\right\|$, respectively. Since there is no object in the right lane, $\boldsymbol{D}_{\text{req}_\text{r}}$ is zero and can be ignored. If the subject vehicle decides to evade to the left, it may need to brake as well due to the front left object. Consequently, $\boldsymbol{D}_{\text{req}_\text{l}}$ and $\boldsymbol{a}_{\text{req,eva}_\text{l}}$ are added in vector in this maneuver. The minimum value of the acceleration or deceleration required in those three maneuvers is defined as the final criticality value in this scenario.

Scenario 2: except from the slow front object, an object at the rear left approaches the subject vehicle. If the subject vehicle has the potential to collide with this object, i.e., $\Delta t_{\text{tc}} < \Delta t_{\text{tc,crit}}$ is fulfilled, then $\lambda = 1$. This also implies that the subject and object vehicle are approaching laterally.

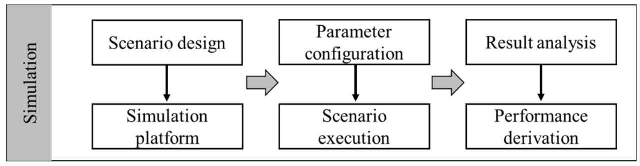

FIGURE 5. Verification process in simulations.

If this is not yet the case but moving to the left lane would still lead to a critical situation, then $\beta = 1$. In both cases, $O = \{\text{left}\}$ and therefore $\alpha = \{\text{right}\}$. If both cases are not true if the approaching object has no influence on the decisions of the subject vehicle even in case of entering the left lane, i.e. $\lambda = \beta = 0$, then $O = \emptyset$ and $\alpha = \{\text{left}, \text{right}\}$. In any case, $\boldsymbol{D}_{\text{req}_\text{l}} = \boldsymbol{D}_{\text{req}_\text{r}} = \boldsymbol{0}$, because there are no front objects on adjacent lanes. Therefore, C_{a} in scenario 2 is defined as:
If $\lambda = 1$:

$$C_{\text{a}} = \min \left(\|\boldsymbol{D}_{\text{req}} + \boldsymbol{a}_{\text{req,ste}_\text{r},v}\|, \|\boldsymbol{a}_{\text{req,eva}_\text{r}}\| \right) \tag{36.a}$$

If $\lambda = 0$, $\beta = 1$:

$$C_{\text{a}} = \min \left(\left\|\boldsymbol{D}_{\text{req}}\right\|, \left\|\boldsymbol{a}_{\text{req,eva}_\text{r}}\right\| \right) \tag{36.b}$$

If $\lambda = \beta = 0$:

$$C_{\text{a}} = \min \left(\left\|\boldsymbol{D}_{\text{req}}\right\|, \left\|\boldsymbol{a}_{\text{req,eva}_\text{l}}\right\|, \left\|\boldsymbol{a}_{\text{req,eva}_\text{r}}\right\| \right) \tag{36.c}$$

In the first case of (36), the subject vehicle already moves to the left and risks to collide with the approaching object. The subject vehicle can either steer back to the middle lane and brake behind the front vehicle or evade to the right lane. In the second case, the subject vehicle moves straight and can either brake in the middle lane or evade to the right lane. Evasion to the left lane is not allowed because a critical object is approaching from behind. In the third case, the subject vehicle has the same options like in the second case. Additionally, it can evade to the left lane because it would not collide with the rear left object.

As we showed by two examples, the defined C_{a} quantifies the criticality of all possible maneuvers in scenarios and always gives the maneuver with minimum criticality. This principle is quite appropriate for the safety evaluation of AVs, since their trajectory planning modules are actually also cost optimization problems. Generally, the defined C_{a} in (30) is applicable in various scenarios. With respect to intersection scenarios, C_{a} is still applicable if the objects can be predicted well.

V. SIMULATION

Fig. 5 shows the verification process of the proposed criticality metric in simulation. First, we design suitable scenarios for tailored testing of the criticality metric. Second, we determine the simulation platform in order to execute different scenarios. For each simulated scenario, the criticality metric is calculated. Based on the results, we derive the performance as well as the limitations of the criticality metric.

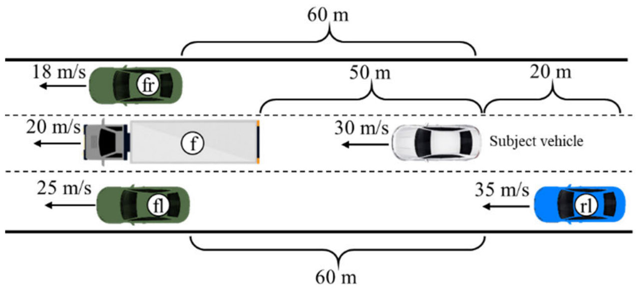

FIGURE 6. Description of the designed scenario to demonstrate the criticality metric.

A. TEST SCENARIOS

In order to verify the proposed criticality metric, we first define test scenarios. Generally, there are several ways to obtain test scenarios. For instance, the test scenarios can come from traffic accidents, field operation data of AVs and expert knowledge, etc. However, these scenarios are usually functional scenarios with a rather high level of abstraction, for simulative testing we need concrete scenarios that are defined in more detail [25]. We set specific parameters for all vehicles in the scenarios in order to create collision-critical scenarios where the subject vehicle has to find a proper reaction. The goal of the designed scenarios is to demonstrate the criticality metric as well as important parameters. Consequently, the scenario as shown in Fig. 6 is developed.

The subject vehicle is trying to change to the left due to the front slow truck. During the lane changing process, the subject vehicle notices that it is no longer safe to continue changing due to the rear left fast object. Since this scenario covers all possible maneuvers together including evasion to the left, steering and evasion to the right and braking, it is suitable for demonstrating the criticality metric.

B. TEST PLATFORM

With respect to the execution of the designed test scenarios, suitable test platforms are necessary. Therefore, several requirements are first defined:

- The test platform should have sensor models in order to perceive the surroundings.
- The driver model should be parametrizable so that different driving styles can be applied.
- The platform should have an interface to read the dynamic state of the perceived objects as well as the state of the subject vehicle.
- Different kinds of scenarios including urban and highway scenarios should be able to be simulated.

Based on the requirements listed above, the simulation environment IPG CarMaker is a good option. IPG CarMaker provides different sensor models, e.g., ideal sensor models, high fidelity sensor models and raw signal interface models. Since different sensor effects, such as false positive and false negative object detections, can be simulated by simply deleting or adding objects in the perceived object list, ideal models are chosen. Subsequently, the designed scenarios are established. Based on the perceived objects and the status of the subject vehicle, the criticality metric can be calculated.

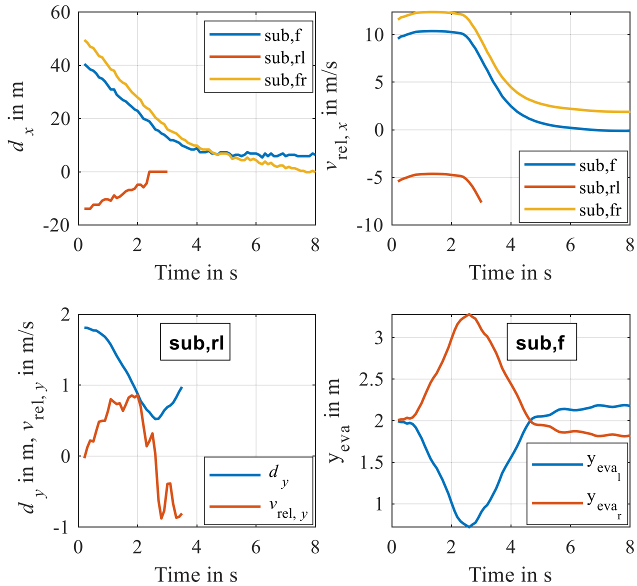

FIGURE 7. Necessary parameters for the criticality metric in the simulated scenario. The diagrams on top display the relative longitudinal distance and velocity to the front, front right and rear left vehicle over time. The bottom left diagram shows the relative lateral distance and velocity to the rear left vehicle, the bottom right diagram depicts the subject vehicle's lateral evasion distance in both directions to pass the front vehicle collision-free.

C. RESULTS AND DISCUSSION

In order to show the evolution of the scenario, necessary parameters for the criticality metrics are drawn in Fig. 7.

The longitudinal distance d_x between subject vehicle and front object (sub, f) decreases slowly. This is because the subject vehicle decelerates until it has the same speed as the front object, which can be observed in the $v_{\text{rel},x}$ plot. Since the subject vehicle changes the lane (with positive $v_{\text{rel},y}$) and then steers back (with negative $v_{\text{rel},y}$) due to the approaching rear left object, d_y reduces firstly and afterwards increases again. Consequently, y_{eva_l} increases and then decreases when the subject vehicle steers back to the previous lane. The evasion distance y_{eva_r} is opposite to y_{eva_l}. When the rear left object is alongside the subject vehicle, d_x is zero and when it drives in front of the subject vehicle, no rear left object exists. Hence, no information of rear left object is available.

In this process, we have quantified the required acceleration or deceleration for each possible maneuver, as shown in Fig. 8. The final criticality metric C_a is always the smallest value among all possible required accelerations or decelerations. In order to show when C_a equals to which maneuver, we have divided the entire scenario into five phases. The time span, in which the line is broken, means that the given maneuver is not applicable, e.g., a collision will occur if the maneuver is executed, as the red line $a_{\text{req},\text{eva}_\text{l}}$ illustrated in the upper plot in Fig. 8.

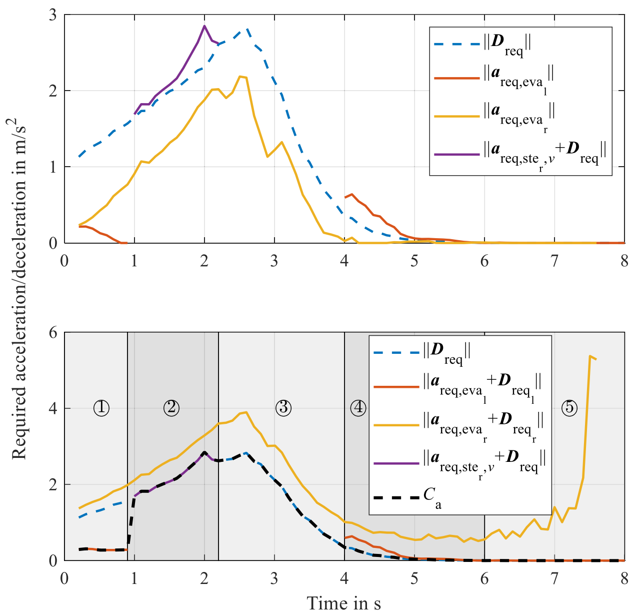

FIGURE 8. The upper figure shows the criticality of possible maneuvers and the lower figure shows the criticality of the maneuvers to determine the final criticality metric. The scenario is divided into five zones. The preferred maneuver in each zone is ①: left evasion; ②: steering back and braking; ③: braking; ④: braking; ⑤: neither deceleration nor acceleration.

In phase ①, evasion to the left is the optimal maneuver. Since the front left object is faster than the subject vehicle, no deceleration of the subject vehicle is required in this case. The rear left object poses no threat to the subject vehicle yet. The front right object has lower velocity than the front left, which means larger deceleration is required for evasion to the right.

In phase ②, the rear left object is getting too close to the subject vehicle, so evasion to the left is not safe anymore. The subject vehicle can either steer back and brake behind the front object or evade directly to the right and brake behind the front right object. Since the latter is expensive because of the low velocity of the front right object, steering back and braking behind the front object is the best choice.

In phase ③, the braking maneuver is preferred. Since the rear left object is still too close to the subject vehicle, evasion to the left is still not possible. The acceleration required to evade to the right is larger than the required deceleration for braking. Thus, the braking maneuver is the cheapest one in terms of acceleration effort.

In phase ④, braking is still preferred. Evasion to the left is possible again since the rear object turns into the front left object and is no longer critical to the subject vehicle. However, the evasion maneuvers become expensive because the subject vehicle is already too close to the front object.

In phase ⑤, the subject vehicle has the same velocity as the front object. No acceleration or deceleration is required. The fact that the TTC between the front right object and the subject vehicle is small results in a large $D_{\mathrm{req_r}}$. When the right object is laterally next to the subject vehicle, evasion to the right is critical and the line of $a_{\mathrm{req,eva_r}}$ is broken at this time. In the rest of the paper, we define TTC to be equivalent to $t_{\mathrm{tc},v}$.

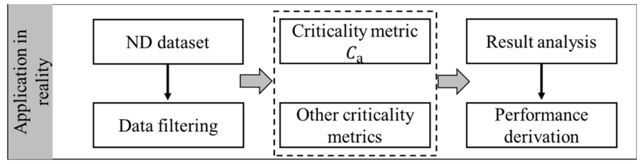

FIGURE 9. The process to evaluate the performance of the proposed criticality metric in a dataset.

VI. APPLICATION TO REAL SCENARIOS

Apart from demonstrating the online capability of the proposed criticality metric in simulation, its ability to assess the safety of AVs in reality is also deserved to explore. Fig. 9 shows the process to derive the performance of the proposed criticality metric in a dataset. The data in a naturalistic driving (ND) dataset is filtered to abandon irrelevant data. Subsequently, our defined criticality metric as well as other common criticality metrics such as TTC are applied in the filtered data. In the result analysis, the performance of different criticality metrics are compared. The strength and weakness of the proposed criticality metric are derived.

A. DATASET INTRODUCTION

To test the newly developed criticality metric with real traffic data, we use the highD dataset recorded by a drone on six different German highway sections of 420 m length each. It contains vehicle trajectories as well as the corresponding vehicle type, size, maneuvers or kinematics like e.g., speed or acceleration. Additionally, information regarding surrounding traffic participants like distance or time to collision are provided for each vehicle. More than 110500 recorded vehicles drive 44500 km in total and have a typical positioning error of less than 10 cm. [23]

Since many vehicles driven by human drivers are available in the dataset, we first filter the dataset to abandon the uninteresting scenarios by using the warning TTC, which is defined as

$$t_{\mathrm{tc,warn}} = \tau - \frac{v_{\mathrm{rel},x}}{2D_{\max}} \tag{37}$$

where τ includes the reaction time and the braking delay time. $D_{\max}$ is the maximum available deceleration.

After filtering there are 111 scenarios left. In the next step we apply our criticality metric to those identified scenarios and also observe DHW, THW and TTC in each of them. Since only $\mathrm{DHW_{min}}$, $\mathrm{THW_{min}}$ and $\mathrm{TTC_{min}}$ in a scenario can quantify how critical the scenario is, we focus on their minimum values. For our criticality metric C_a the maximum value in each scenario is of interest.

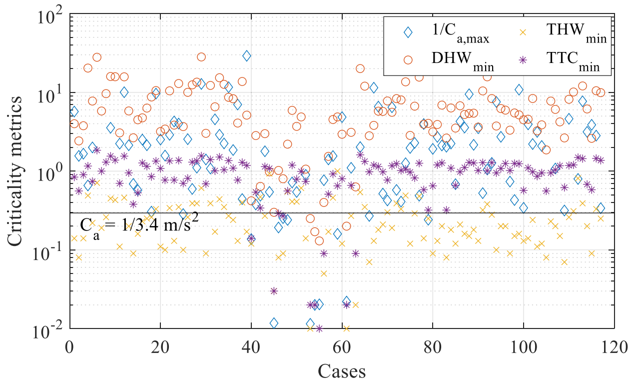

FIGURE 10. ***DHW*** $_{min}$**,** ***THW*** $_{min}$**,** ***TTC*** $_{min}$ **and** $1/C_{a,max}$ **in all 111 scenarios.**

B. RESULTS AND DISCUSSION

The $\mathrm{DHW_{min}}$, $\mathrm{THW_{min}}$ and $\mathrm{TTC_{min}}$ as well as $1/C_{\mathrm{a,max}}$ in all 111 scenarios are illustrated in Fig. 10. $1/C_{\mathrm{a,max}}$ is utilized so that all metrics have same monotonicity to the criticality of situations. The acceleration value of 3.4 m/s^2 recommended by [26]–[28] to differentiate vehicles involved with high risks is also drawn in this figure. There are totally 7 scenarios in which C_{a} gets larger than the given threshold. Critical scenarios resulted from incorrect information in the dataset are removed. The $\mathrm{TTC_{min}}$ in most scenarios are around 1 s, whereas the average $\mathrm{THW_{min}}$ is smaller than that. It indicates that those scenarios are potentially critical in longitudinal direction as pointed out by [29]. However, when we observe $1/C_{\mathrm{a,max}}$, they are mostly large. The reason is that more possible maneuvers to avoid potential critical situations are considered in our criticality metric, while $\mathrm{TTC_{min}}$ and $\mathrm{THW_{min}}$ focus solely on longitudinal direction. With respect to the 7 critical scenarios identified by C_{a}, it is necessary to figure out what the reasons are to cause large C_{a} and how other criticality metrics, e.g., TTC perform in those scenarios. The results can be found in Table 1 in the Appendix, where the reasons for criticality in these scenarios are shown.

As mentioned before, $\mathrm{TTC_{min}}$ is small in some scenarios where $1/C_{\mathrm{a,max}}$ is large, i.e., the scenarios are rated as critical according to $\mathrm{TTC_{min}}$, but classified as uncritical based on $C_{\mathrm{a,max}}$. We utilize the following case 1 to explain why this happens at times.

Case 1: $\mathrm{TTC_{min}}$ *is small, but* $1/\mathrm{C_{a,max}}$ *is large.*

Fig. 11 shows the scenario that we have selected. At the beginning, the subject vehicle can only brake due to the occupancy of the left lane. Thus, C_{a} equals to the required deceleration D_{req}. Afterwards, the truck at the left lane drives away and leaves its lane free for the subject vehicle to evade. Therefore, evading to the left is now the better choice compared to braking. Evasion to the right is impossible because of the lane boundary. The required acceleration to evade to the left defines the C_{a} in this timespan. From frame 7745 to 7770, the required acceleration is zero. On the one hand, the front truck in the middle lane is faster than the subject vehicle. On the other hand, the lateral velocity

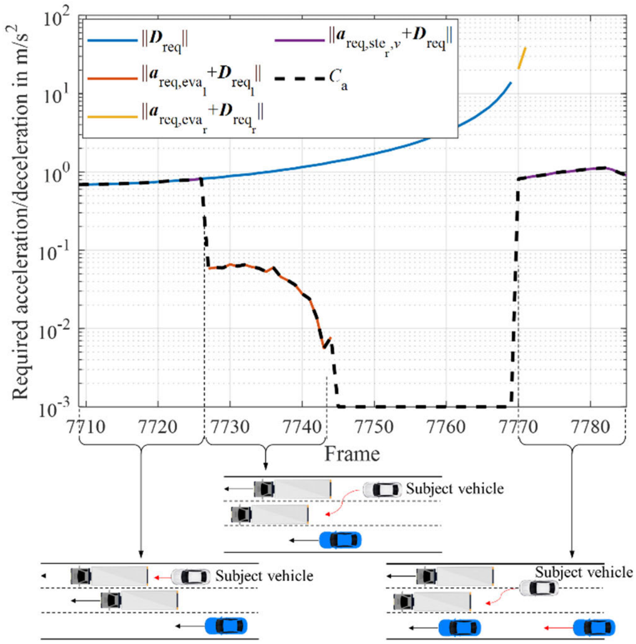

FIGURE 11. **Evolution of the scenario and** C_a **in case 1.**

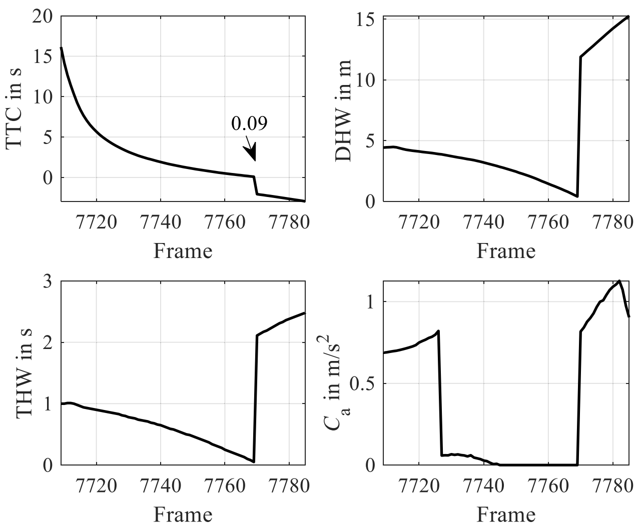

FIGURE 12. **TTC, DHW, THW and** C_a **in case 1.**

is high enough and lateral evasion offset $y_{\mathrm{eva_l}}$ decreases continuously so that the evasion can be done without any lateral acceleration. From frame 7770 the evasion is finished and the subject vehicle is in the middle lane. Due to the high lateral velocity, it is possible that the subject vehicle collides with the rear object at the leftmost lane. As a result, $a_{\mathrm{req,ste_r}}$ is positive to make sure that the subject vehicle will stay in the middle lane and not drive to the leftmost lane. Evasion to the right is expensive due to small TTC to the truck at the rightmost lane and becomes critical when the subject vehicle is laterally next to it, as the yellow line shows.

To compare C_{a} with other criticality metrics, we also illustrate the TTC, DHW, THW and C_{a} in this scenario in

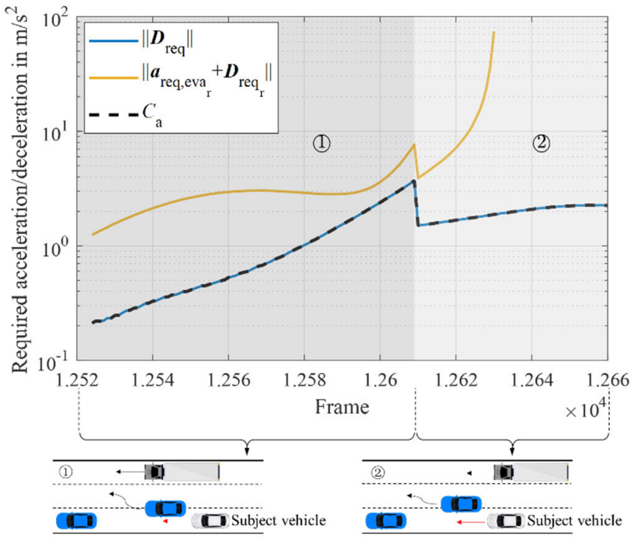

FIGURE 13. Evolution of the scenario and C_a in case 2.

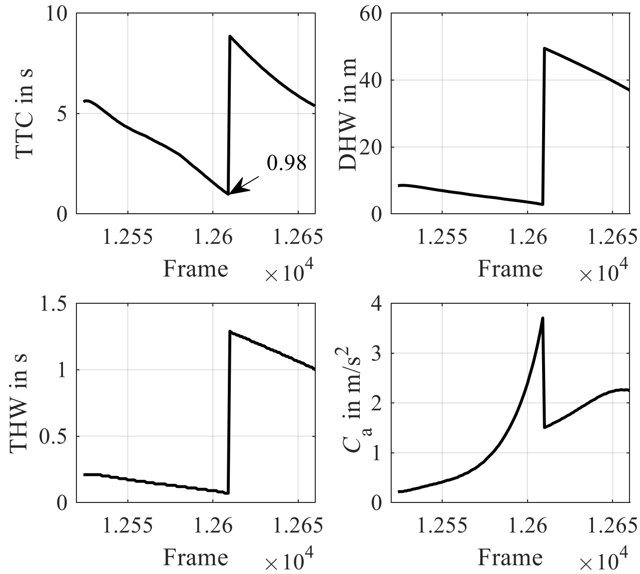

FIGURE 14. TTC, DHW, THW and C_a in case 2.

Fig. 12. As can be seen, the TTC_{min} (0.09 s) is rather small. When the evasion to the left is finished, TTC turns into negative due to the faster truck in front in the middle lane. The THW_{min} is also very small and has a value of 0.05 s.

Nevertheless, our $\text{C}_{\text{a,max}}$ also has a small value of $0.82\ \text{m/s}^2$. It indicates that the scenarios is actually uncritical since the evasion maneuver can be executed without strong action, i.e., our criticality metric is more reasonable and can evaluate the criticality scenarios more accurately than TTC and THW.

In contrast to case 1, it is also worth exploring whether C_{a} is more capable of identifying critical scenarios than TTC and THW, i.e., an actually critical scenario is missed by TTC and THW, but identified by the C_{a}. Therefore, we utilize case 2 to demonstrate this point.

Case 2: TTC_{min} *is large, but* $1/\text{C}_{\text{a,max}}$ *is small.*

This case is about a cut-out maneuver of the front object and an acceleration maneuver of the subject vehicle, as shown in Fig. 13. Braking and evasion to the right are only two possible maneuvers. Due to the large lateral evasion offset y_{eva_r}, the braking maneuver is preferred. At the frame 12610, the cut-out maneuver of the front object is finished. Due to the small TTC to the front object in the middle lane, evasion is still not a better choice and becomes critical when the subject vehicle is driving alongside it. Therefore, the yellow line increases dramatically and is vanished afterwards.

Fig. 14 shows the changing of the studied criticality metrics in this scenario. The TTC_{min} in this scenario is 0.98 s, whereas the THW_{min} is 0.07 s. This is because the relative longitudinal velocity between the subject vehicle and the front object is small. The $C_{\text{a,max}}$ has reached $3.71\ \text{m/s}^2$, which means that the scenario is critical when using the threshold of $3.4\ \text{m/s}^2$ and the critical scenario could not be identified by TTC_{min}. By means of THW_{min} the critical scenario can be discovered. Case 2 proves that the criticality metric C_{a} combines the capabilities of TTC_{min} and THW_{min} in longitudinal direction.

VII. CONCLUSION

In this paper, we proposed a novel acceleration-based criticality metric. This criticality metric can not only be applied in real-time for collision avoidance systems but also assess the safety of AVs in both longitudinal and lateral direction. We elaborated the theory of the criticality metric and derived its expression. In order to validate its performance, special scenarios were designed to validate it first in simulations. Subsequently, its performance is further demonstrated by comparing it with TTC and THW in a naturalistic driving dataset.

Based on the results in the simulations and the dataset, we derived several conclusions:

1) The proposed criticality metric is applicable to assess the safety of AVs in lateral direction. As a result, error from motion prediction can be discovered, since a not well predicted rear object at an adjacent lane could motivate lateral movement of the subject vehicle.
2) All possible maneuvers in a situation are quantified by the criticality metric. Thus, compared to existing criticality metrics, our approach is more general and reasonable to assess the safety of AVs.
3) Compared to TTC_{min}, our proposed criticality metric rates less uncritical situations as critical, since other maneuvers but longitudinal action are not considered by TTC_{min}.
4) The proposed criticality metric is more capable of discovering critical situations than TTC_{min} and combines the abilities of TTC_{min} and THW_{min}.

Since the criticality metric takes the reaction into account, the difficulty to avoid a collision is expressed intuitively. Thus, the criticality of a situation is more understandable, which facilitates its application for collision avoidance systems. Additionally, it is powerful to discover critical scenarios due to its usability in various situations. Then, the identified scenarios can be used to test AVs and assist the V&V of AVs.

With respect to future work, the performance of the criticality metric in intersection scenarios could be further studied. Additionally, the threshold should be further researched, since the threshold of 3.4 m/s^2 was derived in 2004. The driving behavior of AVs varies and road conditions change, so new analysis is necessary.

APPENDIX

A. EXPLANATION OF $t_{tc,v,min}$

The time when $t_{\mathrm{tc},v,\min}$ is reached can be determined by equating the time derivative of the TTC with zero. In this moment, the relative deceleration between the leading object and the subject vehicle $D_{\mathrm{rel},x} = D_{\mathrm{obj},x} - D_{\mathrm{sub},x}$ is twice as high as the actually required deceleration $D_{\mathrm{req},x}$ for collision avoidance, as can be seen in (39) and (40). Here, we assume that the object's deceleration $D_{\mathrm{obj},x}$ is zero. Therefore, the most critical part of the scenario is already over when $t_{\mathrm{tc},v,\min}$ is reached.

$$\frac{\mathrm{d}}{\mathrm{d}t} t_{\mathrm{tc},v} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{d_x}{v_{\mathrm{rel},x}}\right) = \frac{v_{\mathrm{rel},x}^2 - d_x D_{\mathrm{rel},x}}{v_{\mathrm{rel},x}^2} = 0 \quad (38)$$

$$D_{\mathrm{rel},x}\left(t_{\mathrm{tc},v,\min}\right) = \frac{v_{\mathrm{rel},x}^2}{d_x} \quad (39)$$

$$D_{\mathrm{req},x}\left(t_{\mathrm{tc},v,\min}\right) = \frac{v_{\mathrm{rel},x}^2}{2d_x} = \frac{D_{\mathrm{rel},x}\left(t_{\mathrm{tc},v,\min}\right)}{2}$$
$$\text{with } D_{\mathrm{obj},x} = 0 \quad (40)$$

B. SCENARIO OVERVIEW

We also attach the critical scenarios identified by our criticality metric, which can serve as a useful source to test AVs.

TABLE 1. Identified criticality scenarios by C_a in the highd dataset.

(File, Vehicle)	DHW_{min} in m	THW_{min} in s	TTC_{min} in s	$C_{a,max}$ in m/s²	Reasons
(16,523)	3.68	0.10	0.70	3.51	Critical braking
(25,186)	0.42	0.12	0.14	7.30	Critical braking
(25,1101)	1.02	0.08	0.29	5.20	Critical acceleration during cut-out of the front vehicle
(25,1380)	0.80	0.09	0.27	3.95	Critical left evasion
(25,1654)	2.20	0.55	0.56	4.17	Critical braking
(31,1011)	2.81	0.07	0.98	3.71	Critical acceleration during cut-out of the front vehicle
(36,2389)	4.04	0.25	0.32	4.14	Critical left evasion

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