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# 3D Equivariant Graph Implicit Functions

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**Abstract.** In recent years, neural implicit representations have made remarkable progress in modeling of 3D shapes with arbitrary topology. In this work, we address two key limitations of such representations, in failing to capture local 3D geometric fine details, and to learn from and generalize to shapes with unseen 3D transformations. To this end, we introduce a novel family of graph implicit functions with equivariant layers that facilitates modeling fine local details and guaranteed robustness to various groups of geometric transformations, through local  $k$ -NN graph embeddings with sparse point set observations at multiple resolutions. Our method improves over the existing rotation-equivariant implicit function from 0.69 to 0.89 (IoU) on the ShapeNet reconstruction task. We also show that our equivariant implicit function can be extended to other types of similarity transformations and generalizes to unseen translations and scaling.

**Keywords:** Implicit neural representations; equivariance; graph neural networks; 3D reconstruction; transformation.

## 1 Introduction

Neural implicit representations are effective at encoding 3D shapes of arbitrary topology [32,30,10]. Their key idea is to represent a shape by a given latent code in the learned manifold and for each point in space, the neural implicit function checks whether a given coordinate location is occupied within the shape or not. In contrast to traditional discrete 3D representations such as triangle meshes or point clouds, this new paradigm of implicit neural representations has gained significant popularity due to the advantages such as being continuous, grid-free, and the ability to handle various topologies.

Despite their success, latent-code-conditioned implicit representations have two key limitations. First, the latent code of the shape captures coarse high-level shape details (i.e., the global structure) without any explicit local spatial information, hence it is not possible to learn correlations between the latent code and local 3D structural details of the shape. As a result, the surface reconstruction from latent-code-conditioned implicit functions tends to be over-smoothed and they are not good at capturing local

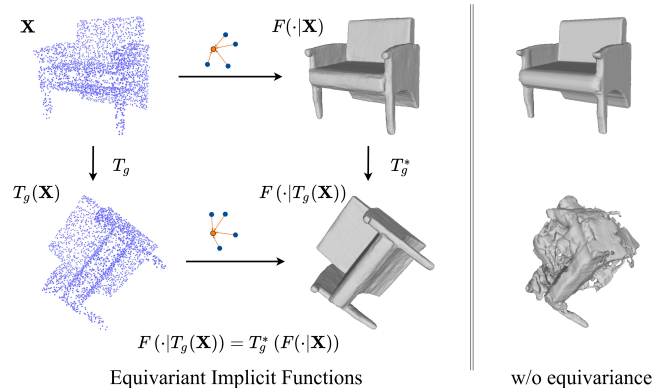


Fig. 1: **Our equivariant graph implicit function** infers the implicit field  $F(\cdot|\mathbf{X})$  for a 3D shape, given a sparse point cloud observation  $\mathbf{X}$ . When a transformation  $T_g$  (rotation, translation, or/and scaling) is applied to the observation  $\mathbf{X}$ , the resulting implicit field  $F(\cdot|T_g(\mathbf{X}))$  is *guaranteed* to be the same as applying a corresponding transformation  $T_g^*$  to the inferred implicit field from the untransformed input (*middle*). The property of equivariance enables generalization to unseen transformations, under which existing models such as ConvONet [34] often struggle (*right*).

surface detail [34,11,25,21,18]. Second, implicit representations are sensitive to various geometric transformations, in particular to rotations [16]. The performance of the implicit representations heavily relies on the assumption that shape instances in the same category are required to be in the same canonical orientation such that shape structures of planes and edges are in line with the coordinate axes. While data augmentation loosely addresses this second issue to some degree, a principled approach is to enable the representations to be inherently aware of common geometric operations such as rotations, translations, and scaling, which are found commonly in real-world 3D objects.

To address the first challenge of modeling local spatial information, recent methods [34,11] first discretize 3D space into local 2D or 3D grids and then store implicit codes locally in the respective grid cells. However, these methods are still sensitive to transformations as the grid structure is constructed in line with the chosen coordinate axes. This results in deteriorated performance under transformations as shown in Fig. 1. In addition, the grid discretization often has to trade fine details of the shape, hence the quality of shape reconstruction, for better computational efficiency through a low resolution grid. Deng *et al.* [17] propose VN-ONet to tackle the second challenge of pose-sensitivity with a novel vector neuron formulation, which enables the network architecture to have a rotation equivariant representation. Nevertheless, similar to implicit representations, the VN-ONet encodes each shape with a global representation and hence fails to capture local details. As grid discretization is not robust to transformations, this solution is not compatible with the grid-based local implicit methods. Thus, integrating VN-ONet with grid-approach is not a feasible solution.

In this work, our goal is to simultaneously address both challenges of encoding local details in latent representations and dealing with the sensitivity to geometric transfor-

mations such as rotations, translations, and scaling. To this end, we propose a novel equivariant graph-based local implicit function that unifies both of these properties. In particular, we use graph convolutions to capture local 3D information in a non-Euclidean manner, with a multi-scale sampling design in the architecture to aggregate global and local context at different sampling levels. We further integrate equivariant layers to facilitate generalization to unseen geometric transformations. Unlike the grid-based methods [34,11] that requires discretization of 3D space into local grids, our graph-based implicit function uses point features from the input point cloud observation directly without interpolation from grid features. Our graph mechanism allows the model to attend detailed information from fine areas of the shape surface points, while the regularly-spanned grid frame may place computations to less important areas. In addition, our graph structure is not biased towards the canonical axis directions of the given Cartesian coordinate frame, hence less sensitive than the grid-local representations [34,11]. Therefore, our graph representation is maximally capable of realizing an equivariant architecture for 3D shape representation. In summary, our contributions are as follows:

- We propose a novel graph-based implicit representation network that enables effective encoding of local 3D information in a multi-scale sampling architecture, and thus modeling of high-fidelity local 3D geometric detail. Our model features a non-Euclidean graph representation that naturally adapts with geometric transformations.
- We incorporate equivariant graph layers in order to facilitate inherent robustness against geometric transformations. Together with the graph embedding, our equivariant implicit model significantly improves the reconstruction quality from the existing rotation equivariant implicit method [17].
- We extend our implicit method to achieve a stronger equivariant model that handles more types of similarity transformations simultaneously with guaranteed perfect generalization, including rotation, translation and scaling.

## 2 Related Work

**Implicit 3D representations.** Neural implicits have been shown to be highly effective for encoding continuous 3D signals of varying topology [32,30,10]. Its variants have been used in order to reconstruct shapes from a single image [38,52,53], or use weaker supervision for raw point clouds [1,2,3] and 2D views [29,31,26].

**Local latent implicit embeddings.** ConvONet [34] and IF-Net [11] concurrently propose to learn multi-scale local grid features with convolution layers to improve upon global latent implicit representations. Other variants of grid methods [25,5] takes no global cues, hence restricted by requiring additional priors during inference such as normals [25] or the partial implicit field [5]. The paradigm is extended to adaptive grids or octrees [44,45,47]. Inspired by non-grid local embeddings in point cloud networks [36,48,28,20], we aim for non-grid local implicit methods, which are less explored. The existing non-grid local implicits [22,18] are limited without multi-scale hierarchical designs, and thus restricted to single objects. The pose-sensitivity problem is not addressed in all these local methods. In contrast, we use a hierarchical graph embedding that effectively encodes multi-scale context, as the first local implicit method to address the pose-sensitivity problem.

**Pose-sensitivity in implicit functions.** As generalization becomes a concern for 3D vision [42,4,9,8], Davies *et al.* [16] first point out that the implicit 3D representations are biased towards canonical orientations. Deng *et al.* [17] introduce a rotation equivariant implicit network VN-ONet that generalises to random unseen rotations, but yet with the restrictions from the global latent. Concurrent to our work, [41,7,54] extend equivariance of implicit reconstruction to SE(3) group for reconstruction and robotic tasks, while our method further handles scaling transformation, and recovers significantly better local details with the graph local embedding design. As grid embeddings are sensitive to rotation, seeking a compatible local latent embedding is a non-trivial problem.

**Rotation equivariance with 3D vector features.** Equivariance has drawn attention in deep learning models with inductive priors of physical symmetries, e.g., the success of ConvNets are attributed to translation equivariance. Advanced techniques are developed for equivariance to rotation [14,51,46], scale [43,56] and permutation [55,35]. Recently, a new paradigm for rotation equivariance uses 3D vectors as neural features [17,40] with improved effectiveness and efficiency upon methods based on spherical harmonics [51,46,50,49,19]. Shen *et al.* [40] first introduced pure quaternion features that are equivalent to 3D vectors. Deng *et al.* [17] proposed a similar design with improved nonlinear layers. Satorras *et al.* [39] proposed to aggregate vector inputs in graph message passing, but without vector nonlinearities involved. Leveraging on existing work [40,17,39], we introduce hybrid vector and scalar neural features for better performance and efficiency. We also adapt the paradigm for scale equivariance, for the first time in literature.

### 3 A Definition of Equivariance for Implicit Representations

While equivariance to common geometric transformations is widely studied for explicit representations of 2D and 3D data [13,51,46], the property for implicit representations that encode signals in a function space is more challenging since continuous queries are involved, yet an important problem for 3D reconstruction. We discuss standard 3D implicit functions and then define equivariance for the representation.

**3D implicit representations.** We build our model on neural 3D occupancy field functions [30], widely used as a shared implicit representation for a collection of 3D shapes. Given an observation  $\mathbf{X} \in \mathcal{X}$  of a 3D shape, the conditional implicit representation of the shape,  $F(\cdot|\mathbf{X}) : \mathbb{R}^3 \rightarrow [0, 1]$ , is a 3D scalar field that maps the 3D Euclidean domain to occupancy probabilities, indicating whether there is a surface point at the coordinate. In this work, we consider  $\mathbf{X}$  as a sparse 3D point cloud, such that the information from the observation is indifferent under an arbitrary global transformation, as required for equivariance. For each 3D query coordinate  $\vec{p} \in \mathbb{R}^3$ , the conditioned implicit representation is in the form of

$$F(\vec{p}|\mathbf{X}) = \Psi(\vec{p}, \vec{z}) = \Psi(\vec{p}, \Phi(\mathbf{X})), \quad (1)$$

where  $\Phi(\mathbf{X}) = \vec{z} \in \mathbb{R}^{C_z}$  is the latent code in the form of a  $C_z$ -dimensional vector from the observation  $\mathbf{X}$ , and  $\Phi$  is the *latent feature extractor* that encodes the observation data  $\mathbf{X}$ .  $\Psi$  is the implicit decoder implemented using a multi-layered perceptron with

ReLU activations. Occupancy probabilities are obtained by the final sigmoid activation function of the implicit decoder. Following [30,34], the model is trained with binary cross-entropy loss supervised by ground truth occupancy. The underlying shape surface is the 2-manifold  $\{\vec{p}' | F(\vec{p}' | \mathbf{X}) = \tau\}$ , where  $\tau \in (0, 1)$  is the surface decision boundary.

**Preliminary of equivariance.** Consider a set of transformations  $T_g : \mathcal{X} \rightarrow \mathcal{X}$  on a vector space  $\mathcal{X}$  for  $g \in G$ , where  $G$  is an abstract group. Formally,  $T_g = T(g)$  where  $T$  is a representation of group  $G$ , such that  $\forall g, g' \in G, T(gg') = T(g)T(g')$ . In the case that  $G$  is the 3D rotation group  $\text{SO}(3)$ ,  $T_g$  instantiates a 3D rotation matrix for a rotation denoted by  $g$ . We say a function  $\Xi : \mathcal{X} \rightarrow \mathcal{Y}$  is equivariant with regard to group  $G$  if there exists  $T_g^* : \mathcal{Y} \rightarrow \mathcal{Y}$  such that for all  $g \in G: T_g^* \circ \Xi = \Xi \circ T_g$ . We refer to [13,46] for more detailed background theory. Next, we define equivariance for implicit representations as follows:

**Definition 1 (Equivariant 3D implicit functions).** Given a group  $G$  and the 3D transformations  $T_g$  with  $g \in G$ , the conditioned implicit function  $F(\cdot | \mathbf{X})$  is equivariant with regard to  $G$ , if

$$F(\cdot | T_g(\mathbf{X})) = T_g^*(F(\cdot | \mathbf{X})), \quad \text{for all } g \in G, \mathbf{X} \in \mathcal{X}. \quad (2)$$

where the transformation  $T_g^*$  applied on the implicit function associated to  $T_g$  is applying the inverse coordinate transform on query coordinates  $T_g^*(F(\cdot | \mathbf{X})) \equiv F(T_g^{-1}(\cdot) | \mathbf{X})$ .

**Remark 1** Eq. (2) can be reformulated more intuitively as:

$$F(\cdot | \mathbf{X}) = F(T_g(\cdot) | T_g(\mathbf{X})), \quad \text{for all } g \in G, \mathbf{X} \in \mathcal{X}. \quad (3)$$

Eq. (3) indicates that the equivariance is satisfied if for any observation  $\mathbf{X}$  and query  $\vec{p}$ , the implicit output of  $F(\vec{p} | \mathbf{X})$  is *locally* invariant to any  $T_g$  applied jointly to  $\mathbf{X}$  and  $\vec{p}$  in the implicit model.

## 4 Transformation-robust Graph Local Implicit Representations

Our goal is to design an equivariant implicit function model using local feature embeddings to capture fine details of the 3D geometry. However, existing grid-based local implicit functions are sensitive to geometric transformations such as rotations, thus not suitable for equivariant implicit representations. To address this limitation, we propose a graph-based local embedding which is robust to geometric transformations.

**Background: grid local implicit representations.** To overcome the limitation of a global latent feature, recent methods, such as ConvONet [34] and IF-Net [11], propose to learn spatially-varying latent features  $\mathbf{z}_{\mathbf{p}} = \Phi_{\text{grid}}(\mathbf{p}; \mathbf{X})$ . The main idea is to partition the 3D space into a grid and compute latent codes locally. Specifically, these methods formulate the local latent implicit function on the grid as  $F_{\text{grid}}(\mathbf{p} | \mathbf{X}) = \Psi(\mathbf{p}, \mathbf{z}_{\mathbf{p}}) = \Psi(\mathbf{p}, \Phi_{\text{grid}}(\mathbf{p}; \mathbf{X}))$ , where the *grid local latent extractor*  $\Phi_{\text{grid}}$  is further decomposed as  $\Phi_{\text{grid}}(\mathbf{p}; \mathbf{X}) = \psi_{\text{grid}}(\mathbf{p}, \phi_{\text{grid}}(\mathbf{X}))$ . The function  $\phi_{\text{grid}}$  is the *grid feature encoder* that learns to generate a 2D or 3D grid-based feature tensor  $\mathbf{M}$  from the entirety of point observations  $\mathbf{X}$ . For each grid location, point features are aggregated for all  $\mathbf{x}_i \in \mathbf{X}$  in

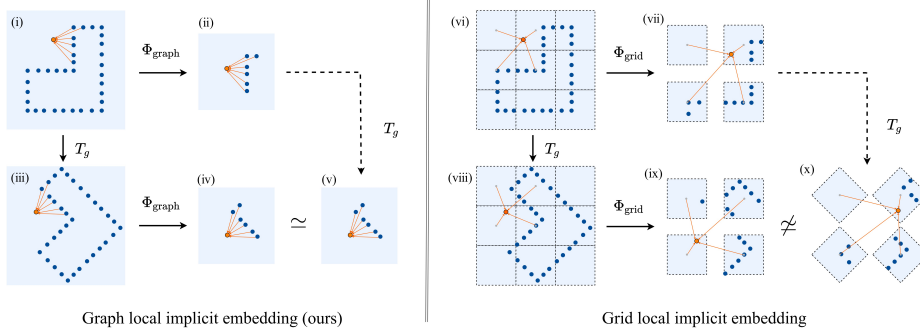


Fig. 2: **Graph (left) vs. grid (right) local implicit feature embeddings under rotation.**  $\Phi_{\text{graph}}$  extracts  $k$ -NN graphs and applies graph convolutions;  $\Phi_{\text{grid}}$  partitions points into regular grids and applies regular convolutions. *Left*: given a point cloud observation  $\mathbf{X}$  (navy) (i), our method aggregates the local latent feature at any query coordinate  $\mathbf{p}$  (orange) from a local  $k$ -NN graph connecting its neighbours in  $\mathbf{X}$  (ii). Moreover, when a transformation  $T_g$ , e.g. rotation, is applied to the shape, (iv) the constructed local graph is in the same structure as (v) applying  $T_g$  to the graph from untransformed data. *Right*: in contrast, (vi) visualizes discretized grid features. (vii) The off-the-grid query location  $\mathbf{p}$  interpolates the neighboring on-grid features. However, (viii) with  $T_g$  applied to the raw observation, often (ix) the sub-grid point patterns for the local features are different from (x) applying  $T_g$  to the untransformed local grids. This makes the grid local implicit models sensitive to transformations such as rotation.

the corresponding bin. Convolutional layers are applied to the grid-based  $\mathbf{M}$  to capture multi-scale information and maintain translation equivariance. Given the local 3D feature tensor  $\mathbf{M}$ , the *local latent aggregator*  $\psi_{\text{grid}}$  computes local latent feature  $\mathbf{z}_{\mathbf{p}}$  on any off-the-grid query coordinate  $\mathbf{p}$  using a simple trilinear interpolation. We refer to [25,5] for other variants of grid-based representations using purely local information without global cues, while restricted by requiring additional priors during inference such as the normals. Overall, grid partitioning is not robust to general transformations such as rotations, especially when the sub-grid structure is considered for the resolution-free implicit reconstruction. Fig. 2 (right) shows an illustration of this limitation, which we will address in our method.

#### 4.1 Graph-structured local implicit feature embeddings

We propose to use graphs as a non-regular representation, such that our local latent feature function is robust to these transformations and free from feature grid resolutions. The graph-local implicit function extends the standard form of Eq. (1) to

$$F_{\text{graph}}(\mathbf{p}|\mathbf{X}) = \Psi(\mathbf{p}, \mathbf{z}_{\mathbf{p}}) = \Psi(\mathbf{p}, \Phi_{\text{graph}}(\mathbf{p}; \mathbf{X})) = \Psi(\mathbf{p}, \psi(\mathbf{p}, \phi(\mathbf{X}))). \quad (4)$$

$\Phi_{\text{graph}}$  is a deep network that extracts *local* latent features  $\mathbf{z}_{\mathbf{p}}$  on the graph, composed of two sub-networks  $\phi$  and  $\psi$ . The *point feature encoder*  $\phi$  maps the point set  $\mathbf{X} =$



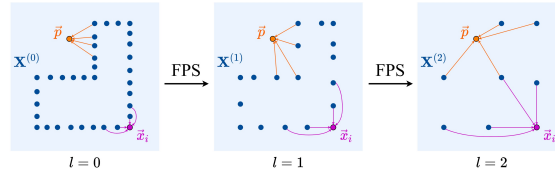


Fig. 3: **Multi-scale design**, with enlarged receptive fields of local  $k$ -NN graphs for the graph point encoder  $\phi$  (orange) and the graph local latent aggregator  $\psi$  (violet).

$\{\mathbf{x}_i\}$  to the associated features  $\{\mathbf{h}_i\}$ , and is invariant to the sampled query location  $\mathbf{p}$ . The *graph local latent feature aggregator*  $\psi$  propagates the input point feature to the query coordinate  $\mathbf{p}$ . Unlike grid-based methods, we directly aggregate local information from the point cloud feature  $\{(\mathbf{x}_i, \mathbf{h}_i)\}$  without an intermediate grid feature tensor. In particular, we construct a local  $k$ -nearest neighbor ( $k$ -NN) graph  $(\mathcal{V}, \mathcal{E})$  for every query point  $\mathbf{p}$ , where the vertices  $\mathcal{V} = \mathbf{X} \cup \{\mathbf{p}\}$  include the point set elements and the query coordinate. The edges  $\mathcal{E} = \{(\mathbf{p}, \mathbf{x}_{i'})\}$  are between the query point  $\mathbf{p}$  and its  $k$ -NN points from the observation point set  $\mathbf{x}_{i'} \in \mathcal{N}_k(\mathbf{p}, \mathbf{X})$ , with  $\mathcal{N}_k(\mathbf{p}, \mathbf{X})$  denoting the set of the  $k$ -NN points of  $\mathbf{p}$  from  $\mathbf{X}$ . Last, with graph convolutions, we aggregate into the local feature vector  $\mathbf{z}_{\mathbf{p}}$  the point features of the neighbors of  $\mathbf{p}$ . We adopt a simple spatial graph convolution design in the style of Message Passing Neural Network (MPNN) [23], which is widely used for 3D shape analysis [48,24]. For each neighboring point  $\mathbf{x}_{i'}$  from the query  $\mathbf{p}$ , messages are passed through a function  $\eta$  as a shared two-layer ReLU-MLP, where the inputs are the point features  $\mathbf{h}_{i'}$  and the query coordinate  $\mathbf{p}$  as node feature as well as the displacement vector  $\mathbf{x}_{i'} - \mathbf{p}$  as the edge feature, followed by a permutation-invariant aggregation AGGRE over all neighboring nodes, e.g., max- or mean-pooling:

$$\mathbf{z}_{\mathbf{p}} = \text{AGGRE}_{i'} \eta(\mathbf{p}, \mathbf{h}_{i'}, \mathbf{x}_{i'} - \mathbf{p}). \quad (5)$$

For each neighboring point as a graph node, the edge function  $\eta$  take as inputs, the query coordinate  $\mathbf{p}$ , the node point feature  $\mathbf{h}_{i'}$ , and the relative position  $\mathbf{x}_{i'} - \mathbf{p}$ .

As all the graph connections are relative between vertices, the local latent feature aggregation is robust to transformations like rotations, as illustrated in Fig. 2 (left).

## 4.2 Learning multi-scale local graph latent features

To capture the context of the 3D geometry at multiple scales, both ConvONet [34] and IF-Net [11] rely on a convolutional U-Net [37], with progressively downsampled and then upsampled feature grid resolutions to share neighboring information at different scales. Our graph model enables learning at multiple scales by farthest point sampling (FPS). That is, we downsample the point set  $\mathbf{X}$  to  $\mathbf{X}^{(l)}$  at sampling levels  $l = 1, \dots, L$  with a progressively smaller cardinality  $|\mathbf{X}^{(l)}| < |\mathbf{X}^{(l-1)}|$ , where  $\mathbf{X}^{(0)} = \mathbf{X}$  is the original set.

Moreover, we use a graph encoder for the point encoder  $\phi$ , instead of PointNet [35]. This way, without involving regular grid convolutions, we can still model local features and facilitate a translation-equivariant encoder, which is beneficial in many scenarios,



especially for learning scene-level implicit surfaces [34]. Next, we sketch the multi-scale graph point encoder and latent feature aggregator, with Fig. 3 as a conceptual illustration.

The *graph point encoder*  $\phi$  learns point features  $\{\mathbf{h}_i^{(l)}\}$  for the corresponding points  $x_i \in \mathbf{X}^{(l)}$  at each sampling level  $l = 0 \dots, L$ . The encoder starts from the initial sampling level  $l = 0$  where the input features are the raw coordinates. At each sampling level, a graph convolution is applied to each point to aggregate message from its local  $k$ -nearest neighbor point features, followed by an FPS operation to the downsampled level  $l + 1$ . The graph convolution is similar to that in Eq. (5), with the point features from both sides of the edge and the relative position as inputs. The graph convolutions and FPS downsampling are applied until the coarsest sampling level  $l = L$ . Then the point features are sequentially upsampled back from  $l = L$  to  $l = L - 1$ , until  $l = 0$ . At each sampling level  $l$ , the upsampling layer is simply one linear layer followed by ReLU activation. For each point, the input of the upsampling layer is the nearest point feature from the last sampling level  $l + 1$ , and the skip-connected feature of the same point at the same sampling level from the downsampling stage. Thus far, we obtain multi-scale point features  $\{(\mathbf{x}_i, \mathbf{h}_i^{(l)})\}$  for  $\mathbf{x}_i \in \mathbf{X}^{(l)}$  at sampling levels  $l = 0, \dots, L$ , as the output from the graph point encoder  $\phi$ .

For the *graph latent feature aggregator*  $\psi$ , at each query coordinate  $\mathbf{p}$ , we use graph convolutions to aggregate the  $k$ -neighboring features  $\{(\mathbf{x}_i, \mathbf{h}_i^{(l)})\}$  at different sampling levels  $l$ , as described in Sec 4.1. The aggregated features from all sampling levels  $l$  are concatenated to yield the local latent vector  $\mathbf{z}_{\mathbf{p}}$  as output. The detailed formulations of  $\phi$  and  $\psi$  are provided in the supplementary material.

## 5 Equivariant Graph Implicit Functions

The local graph structure of the proposed implicit function, with  $\mathbf{X} \cup \{\mathbf{p}\}$  as the set of vertices, is in line with the requirement of equivariance in Sec. 3 and Eq. (3). As a result, the local graph implicit embedding can be used for an equivariant model to achieve theoretically guaranteed generalization to unseen transformations. To do this, we further require all the graph layers in the latent extractor  $\mathcal{F}_{\text{graph}}$  to be equivariant in order to obtain equivariant local latent feature. We present the equivariant layers for 3D rotation group  $G = SO(3)$ , a difficult case for implicit functions [16].

To build the equivariant model from equivariant layers, we additionally remove the query coordinate input  $\mathbf{p}$  to ensure the implicit decoder  $\mathcal{V}$  spatially invariant in Eq. (4), as the local spatial information is already included in the latent  $\mathbf{z}_{\mathbf{p}}$ . See Appendix A.1 for details. We also extend the method to other similarity transformations, such as translation and scaling. See Appendix A.2.

### 5.1 Hybrid feature equivariant layers

Our equivariant layers for the graph convolution operations is inspired by recent methods [17,40] that lift from a regular neuron feature  $h \in \mathbb{R}$  to a vector  $\mathbf{v} \in \mathbb{R}^3$  to encode rotation, and the list of 3D vector features  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{C_v}]^T \in \mathbb{R}^{C_v \times 3}$  that substitutes the regular features  $\mathbf{h} \in \mathbb{R}^{C_h}$ . However, using only vector features in the

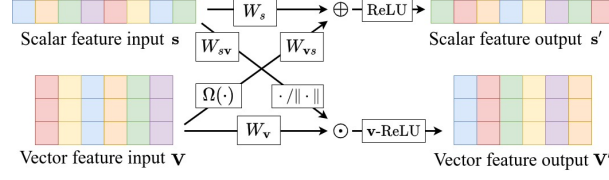


Fig. 4: **Hybrid feature equivariant layers.** Visualization of how vector and scalar features share information in linear and nonlinear layers. Vector features go through an invariant function  $\Omega$  that is added to the scalar part. Scalar features are transformed with normalizing  $\cdot / \|\cdot\|$  to scale the vector feature channels.

network is non-optimal for both effectiveness and efficiency, with highly regularized linear layers and computation-demanding nonlinearity projections.

To this end, we extend the method and propose hybrid features  $\{s, \mathbf{V}\}$  to replace the regular feature  $\mathbf{h}$ , where vector features  $\mathbf{V}$  encode rotation equivariance, and scalar features  $s \in \mathbb{R}^{C_s}$  are rotation-invariant. In practice, hybrid features show improved performance and computation efficiency by transferring some learning responsibility to the scalar features through more powerful and efficient standard neural layers.

**Linear layers.** For input hybrid hidden feature  $\{s, \mathbf{V}\}$  with  $s \in \mathbb{R}^{C_s}$ , and  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{C_v}]^T \in \mathbb{R}^{C_v \times 3}$ , we define a set of weight matrices,  $\mathbf{W}_s \in \mathbb{R}^{C'_s \times C_s}$ ,  $\mathbf{W}_v \in \mathbb{R}^{C'_v \times C_v}$ ,  $\mathbf{W}_{sv} \in \mathbb{R}^{C'_s \times C_s}$ , and  $\mathbf{W}_{vs} \in \mathbb{R}^{C'_v \times C_v}$  for the linear transformation, with information shared between scalar and vector features in the inputs. The resulting output features  $\{s', \mathbf{V}'\}$  become:

$$s' = \mathbf{W}_s s + \mathbf{W}_{vs} \Omega(\mathbf{V}) \quad (6)$$

$$\mathbf{V}' = \mathbf{W}_v \mathbf{V} \odot (\mathbf{W}_{sv} s / \|\mathbf{W}_{sv} s\|) \quad (7)$$

where  $\odot$  is channel-wise multiplication between  $\mathbf{W}_v \mathbf{V} \in \mathbb{R}^{C'_v \times 3}$  and  $\mathbf{W}_{sv} s \in \mathbb{R}^{C'_v \times 1}$ , and the normalized transformed scalar feature  $\mathbf{W}_{sv} s / \|\mathbf{W}_{sv} s\|$  learns to scale the output vectors in each channel;  $\Omega(\cdot)$  is the invariance function that maps the rotation equivariant vector feature  $\mathbf{V} \in \mathbb{R}^{C_v \times 3}$  to the rotation-invariant scalar feature  $\Omega(\mathbf{V}) \in \mathbb{R}^{C_v}$ , to be added on the output scalar feature. The design of the invariance function  $\Omega(\cdot)$  is introduced later in Eq. (9).

**Nonlinearities.** Nonlinearities apply separately to scalar and vector features. For scalar features, it is simply a  $\text{ReLU}(\cdot)$ . For vector features, the nonlinearity  $\mathbf{v}\text{-ReLU}(\cdot)$  adopts the design from Vector Neurons [17]: the vector feature  $\mathbf{v}_c$  at each channel  $c$  takes an inner-product with a learnt direction  $\mathbf{q} = [\mathbf{W}_q \mathbf{V}]^T \in \mathbb{R}^3$  from a linear layer  $\mathbf{W}_q \in \mathbb{R}^{1 \times C_v}$ . If the inner-product is negative,  $\mathbf{v}_c$  is projected to the plane perpendicular to  $\mathbf{q}$ .

$$[\mathbf{v}\text{-ReLU}(\mathbf{V})]_c = \begin{cases} \mathbf{v}_c & \text{if } \langle \mathbf{v}_c, \frac{\mathbf{q}}{\|\mathbf{q}\|} \rangle \geq 0, \\ \mathbf{v}_c - \langle \mathbf{v}_c, \frac{\mathbf{q}}{\|\mathbf{q}\|} \rangle \frac{\mathbf{q}}{\|\mathbf{q}\|}. & \text{otherwise.} \end{cases} \quad (8)$$

**Invariance layer.** The invariance function  $\Omega(\cdot)$  maps the rotation equivariant vector feature  $\mathbf{V} \in \mathbb{R}^{C_v \times 3}$  to a rotation-invariant scalar feature  $\Omega(\mathbf{V}) \in \mathbb{R}^{C_v}$  with the same

Table 1: **ShapeNet implicit reconstruction from sparse noisy point clouds.** The grid resolutions are marked for ConvONet and IF-Net.

	SO(3) equiv.	Mean IoU $\uparrow$	Chamfer- $\ell_1$ $\downarrow$	Normal consist. $\uparrow$
ONet	×	0.736	0.098	0.878
ConvONet-2D ( $3 \times 64^2$ )	×	0.884	0.044	0.938
ConvONet-3D ( $32^3$ )	×	0.870	0.048	0.937
IF-Net ( $128^3$ )	×	0.887	0.042	0.941
GraphONet (ours)	×	<b>0.904</b>	<b>0.038</b>	<b>0.946</b>
VN-ONet	✓	0.694	0.125	0.866
E-GraphONet-SO(3) (ours)	✓	0.890	0.041	0.936

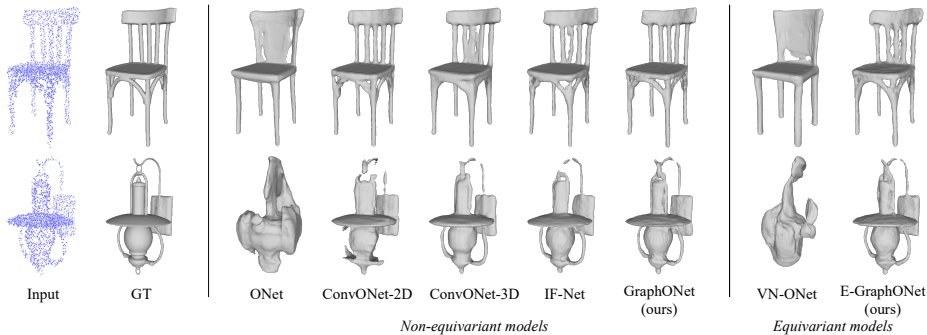


Fig. 5: **ShapeNet object reconstruction in canonical space.** GraphONet is among the best performing non-equivariant implicit representation methods; E-GraphONet significantly improves on the existing equivariant method VN-ONet [17].

channel dimension  $C_v$ . At each layer  $c = 1, \dots, C_v$ ,  $[\Omega(\mathbf{V})]_c \in \mathbb{R}$  takes the inner product of  $\mathbf{v}_c$  with the channel-averaged direction:

$$[\Omega(\mathbf{V})]_c = \left\langle \mathbf{v}_c, \frac{\bar{\mathbf{v}}}{\|\bar{\mathbf{v}}\|} \right\rangle, \quad (9)$$

where  $\bar{\mathbf{v}} = \frac{1}{C_v} \sum_{c'=1}^{C_v} \mathbf{v}_{c'}$  is the channel-averaged vector feature. One can verify that applying any rotation on the vector feature does not change the inner product. Our  $\Omega$  adopts from [17] with modification. Ours is parameter-free using averaged direction, while [17] learns this vector.

The invariance function is applied in each linear layer in Eq. (7) to share the information between vector and scalar parts of the feature. In addition, at the end of equivariant graph feature network  $\Phi_{\text{graph}}$ ,  $\Omega(\mathbf{V})$  is concatenated with the scalar feature as the final invariant local latent feature  $\mathbf{z}_p = \Omega(\mathbf{V}) \parallel \mathbf{s}$ , as to be locally invariant with transformed  $\mathbf{p}$  and  $\mathbf{X}$  in line with Eq. (3).

## 6 Experiments

We experiment on implicit surface reconstruction from sparse and noisy point observations. In addition, we evaluate the implicit reconstruction performance under random

Table 2: **Implicit surface reconstruction with random rotation, IoU $\uparrow$** . The *left* table evaluates non-equivariant methods, and the *right* one compares the best performing model with equivariant methods.  $I$  denotes canonical pose and  $SO(3)$  random rotation.  $I/SO(3)$  denotes training with canonical pose and test with random rotation, and so on. \*: models not re-trained with augmentation due to guaranteed equivariance.

<i>training / test</i>	$I/I$	$I/SO(3)$	$SO(3)/SO(3)$	<i>training / test</i>	equiv.	$I/I$	$I/SO(3)$	$SO(3)/SO(3)$
ONet	0.742	0.271 [-0.471]	0.592 [-0.150]	GraphONet (ours)	×	<b>0.904</b>	0.846 [-0.058]	0.887 [-0.017]
ConvONet-2D	0.884	0.568 [-0.316]	0.791 [-0.093]	VN-ONet	SO(3)	0.694	0.694 [-0.000]	0.694* [-0.000]
ConvONet-3D	0.870	0.761 [-0.109]	0.838 [-0.032]	E-GraphONet (ours)	SO(3)	0.890	<b>0.890</b> [-0.000]	<b>0.890*</b> [-0.000]
GraphONet (ours)	<b>0.904</b>	<b>0.846</b> [-0.058]	<b>0.887</b> [-0.017]					

transformations of rotation, translation and scaling. Our method is referred to as Graph Occupancy networks, or GraphONet, while E-GraphONet is the equivariance model with 3 variants: SO(3), SE(3) and similarity transformations (Sim.).

**Implementation details.** We implement our method using PyTorch [33]. The number of neighbours in  $k$ -NN graph is set as 20. For the multi-scale graph implicit encoder, we take  $L = 2$ , *i.e.*, the point set is downsampled twice with farthest point sampling (FPS) to 20% and 5% of the original cardinality respectively. The permutation invariant function AGGRE adopts mean-pooling for vector features and max-pooling for scalar features. We use an Adam optimizer [27] with  $\gamma = 10^{-3}$ ,  $\beta_1 = 0.9$  and  $\beta_2 = 0.999$ . Main experiments are conducted on the ShapeNet [6] dataset with human designed objects, where the train/val/test splits follow prior work [12,34] with 13 categories. Following [34], we sample 3000 surface points per shape and apply Gaussian noise with 0.005 standard deviation. More details are provided in the Appendix. Code is available at <https://github.com/yunlu-chen/equivariant-graph-implicit>.

## 6.1 Canonical-posed object reconstruction

We experiment on ShapeNet object reconstruction following the setups in [34]. For quantitative evaluation in Table 1, we evaluate the IoU, Chamfer- $\ell_1$  distance and normal consistency, following [30,34]. The qualitative results are shown in Fig. 5. Our GraphONet outperforms the state-of-the-arts methods ConvONet [34] and IF-Net [11], as our graph-based method aggregates local feature free of spatial grid resolution and captures better local details. IF-Net is better than ConvONet but at large memory cost per batch with  $128^3$  resolution grid feature. For rotation equivariant models, our E-GraphONet significantly outperforms VN-ONet [17], benefiting from the graph local features.

## 6.2 Evaluation under geometric transformations

**Rotation.** First, we investigate how implicit models perform under random rotations, which is challenging for neural implicits [16]. In Table 2 left, GraphONet shows the smallest performance drop among all non-equivariant methods under rotations, either with ( $SO(3)/SO(3)$ ) or without augmentation ( $I/SO(3)$ ) during training, since the graph structure is more robust to rotations. ConvONet [34]-2D is more sensitive than the 3D

Table 3: **Implicit surface reconstruction performance under various types of seen/unseen transformations, mIoU $\uparrow$ .**

Transformation(s) Training augmentation	equiv.	translation				scale		rot. & transl.		rot. & scale		transl. & scale		all	
		-	-	×	✓	×	✓	×	✓	×	✓	×	✓	×	✓
ONet	×	0.738	0.221	0.716	0.423	0.685	0.154	0.585	0.235	0.591	0.202	0.713	0.121	0.573	
ConvONet-2D ( $3 \times 128^2$ )	×	0.882	0.791	0.878	0.812	0.850	0.532	0.771	0.542	0.789	0.723	0.838	0.481	0.728	
ConvONet-3D ( $64^3$ )	×	0.861	0.849	0.856	0.797	0.837	0.759	0.836	0.742	0.832	0.771	0.835	0.721	0.803	
GraphONet (ours)	×	<b>0.901</b>	<b>0.884</b>	<b>0.898</b>	0.857	<b>0.893</b>	0.837	0.881	0.798	0.880	0.852	<b>0.888</b>	0.798	0.874	
VN-ONet	SO(3)	0.682	0.354	0.667	0.516	0.662	0.357	0.658	0.511	0.666	0.360	0.638	0.309	0.615	
E-GraphONet (ours)	SO(3) <sup>†</sup>	0.887	0.823	0.876	0.824	0.880	0.825	0.877	0.825	<b>0.882</b>	0.726	0.872	0.729	0.870	
E-GraphONet (ours)	SE(3)	0.884	<b>0.884</b>	0.884*	0.840	0.880	<b>0.884</b>	<b>0.884</b> *	0.838	0.880	0.841	0.878	0.840	0.878	
E-GraphONet (ours)	Sim. <sup>†</sup>	0.882	0.882	0.882*	<b>0.882</b>	0.882*	0.882	0.882*	<b>0.882</b>	<b>0.882</b> *	<b>0.882</b>	0.882*	<b>0.882</b>	<b>0.882</b> *	

<sup>†</sup> Similarity transformation group. <sup>‡</sup> The (graph) convolution subnetwork is translation equivariant. \* with no augmentation due to guaranteed equivariance.

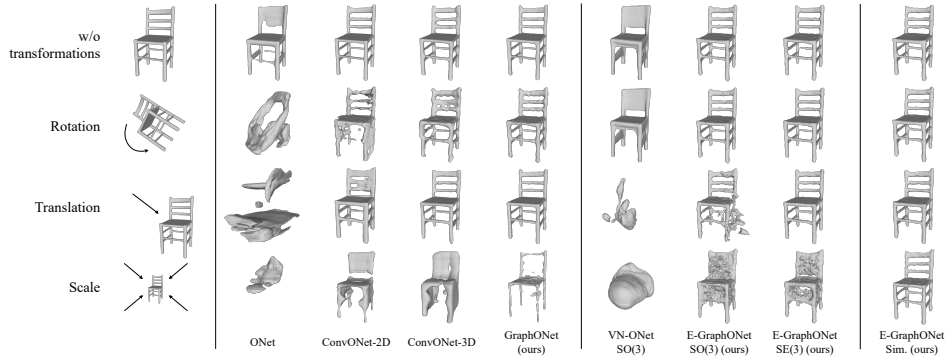


Fig. 6: **Implicit surface reconstruction under *unseen* transformations.** Visualizing back-transformed shapes. Shapes are scaled by a factor of 0.25, and translation is from the unit cube center to the corner. E-GraphONet-Sim is robust to all similarity transformations. See the appendix video for reconstructions under different poses and scales.

version, as 3D rotation would lead to highly distinct 2D projections. In Table 2 right, E-GraphONet, equipped with equivariant layers, achieves better performance under random rotations, even when the non-equivariant methods are trained with augmentation. It also outperforms the previous equivariant method VN-ONet [17] by a large margin.

**Scale, translation and combinations.** We evaluate how implicit methods perform under various similarity transformations besides SO(3) rotation, including scale, translation and combinations. We apply random scales and rotations in a bounded unit cube  $[-0.5, 0.5]^3$ , as assumed by grid methods, and set the canonical scale to be half of the cube. ConvONet resolutions are doubled to keep the effective resolution. Random scaling and translation are added under the constraint of the unit bound, with the minimum scaling factor of 0.2.

From the results in Table 3, GraphONet is more robust to transformations than other non-equivariant models. For equivariant models, VN-ONet and the SO(3) E-GraphONet models perform poorly on other types of transformations, as they are optimized towards the rotation around origin only. Similarly, the SE(3) E-GraphONet does not generalize to scaling. Our model with full equivariance performs well on all similarity transformations

Table 4: **Parameter- and data-efficiency.** Evaluated on the full test set with 5 runs of 130 randomly sampled training examples.

	#param.	IoU $\uparrow$
ConvONet-2D	$2.0 \times 10^6$	$0.727 \pm 0.009$
ConvONet-3D	$1.1 \times 10^6$	$0.722 \pm 0.008$
GraphONet (ours)	$1.9 \times 10^5$	$0.867 \pm 0.004$
E-GraphONet (ours)	$6.5 \times 10^4$	<b><math>0.873 \pm 0.002</math></b>

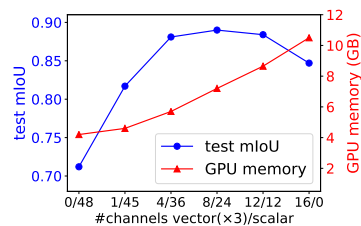


Fig. 7: **Ablation on hybrid feature channels.** Mixed vectors and scalars are more effective and efficient than pure vectors.

with numerically the same performance. Fig 6 shows qualitative examples, where our E-GraphONet-Sim handles all types of unseen similarity transformations.

### 6.3 Analysis

We show some ablation experiments while more results are provided in the Appendix.

**Learning from very few training examples.** We show that our graph method is both parameter- and data-efficient, and the transform-robust modeling inherently benefits generalization. As reported in Table 4, we use less than 10% of the parameters of ConvONet as the graph conv kernel shares parameters for all directions. We evaluate the test set performance when training on only 130 examples - 10 per class - instead of the full training set size of 30661. While ConvONets fail to achieve good performance, GraphONets does not drop by far from the many-shot results in Table 1. The E-GraphONet demonstrates even better performance, with more parameter-sharing from the equivariance modeling, indicating better power of generalization.

**Ablation on vector and scalar feature channels.** We validate our design of hybrid features by experimenting different ratio of vector and scalar channels. We constrain in total 48 effective channels, with one vector channel counted as three scalars. In Fig. 7, using both vectors and scalars with a close-to-equal ratio of effective channels obtains higher performance with less memory cost than using pure vectors, i.e., in the Vector Neurons [17]. This indicates the expressive power of the scalar neuron functions. We provide additional results for non-graph-based equivariant models in the Appendix.

### 6.4 Scene-level reconstructions

In addition to the ability of handling object shape modeling under transformations, our graph implicit functions also scale to scene-level reconstruction. We experiment on two datasets: (i) *Synthetic Rooms* [34], a dataset provided by [34], with rooms constructed with walls, floors, and ShapeNet objects from five classes: chair, sofa, lamp, cabinet and table.

Table 5: **Indoor scene reconstructions.**

Dataset	Synthetic room			ScanNet
	IoU $\uparrow$	Chamfer $\downarrow$	Normal $\uparrow$	Chamfer $\downarrow$
ONet	0.514	0.135	0.856	0.546
ConvONet-2D ( $3 \times 128^2$ )	0.802	0.038	0.934	0.162
ConvONet-3D ( $64^3$ )	0.847	0.035	0.943	0.067
GraphONet (ours)	<b>0.883</b>	<b>0.032</b>	<b>0.944</b>	<b>0.061</b>
E-GraphONet (ours)	0.851	0.035	0.934	0.069

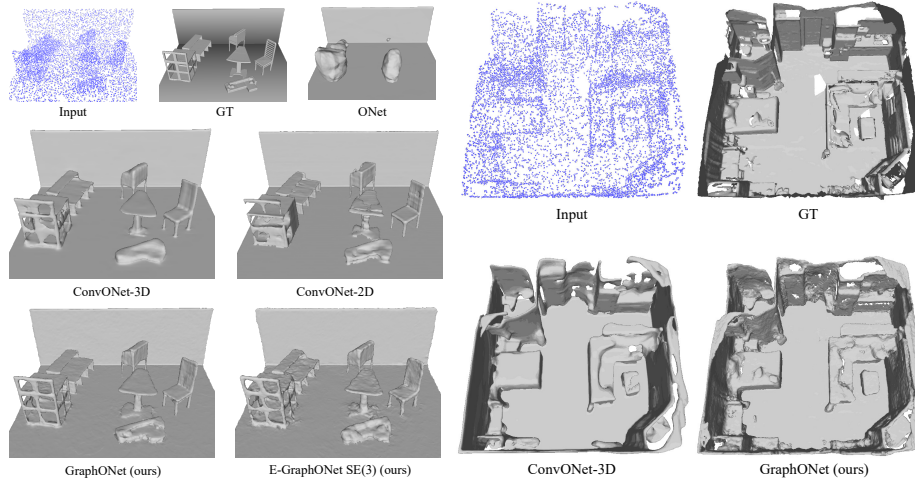


Fig. 8: **Indoor scene reconstructions on Synthetic rooms (left) and ScanNet (right).** Our GraphONet produces sharper edges and better finer details.

(ii) *ScanNet* [15], a dataset of RGB-D scans of real-world rooms for testing synthetic-to-real transfer performance.

We train and evaluate our model on Synthetic room dataset [34] using 10,000 sampled points as input. The quantitative results are shown in Table 5 and qualitative results in Fig. 8 (left). Our GraphONet performs better than ConvONets [34] at recovering detailed structures. We evaluate the SE(3) variant of our equivariance model, and it performs generally well, but less smooth at flat regions. In addition, we evaluate the model transfer ability of our method on ScanNet [15] with the model trained on synthetic data, for which we report the Chamfer measure in Table 5. Our GraphONet outperforms other methods. Fig. 8 (right) shows a qualitative example of our reconstructions.

## 7 Conclusion

In this paper, we introduce graph implicit functions, which learn local latent features from  $k$ -NN graph on sparse point set observations, enabling reconstruction of 3D shapes and scenes in fine detail. By nature of graphs and in contrast to regular grid representations, the proposed graph representations are robust to geometric transformations. What is more, we extend the proposed graph implicit functions with hybrid feature equivariant layers, thus guarantee theoretical equivariance under various similarity transformations, including rotations, translations and scales, and obtain models that generalize to arbitrary and unseen transformations.

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## References

1. Atzmon, M., Haim, N., Yariv, L., Israelov, O., Maron, H., Lipman, Y.: Controlling neural level sets. arXiv preprint arXiv:1905.11911 (2019) [3](#)
2. Atzmon, M., Lipman, Y.: Sal: Sign agnostic learning of shapes from raw data. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 2565–2574 (2020) [3](#)
3. Atzmon, M., Lipman, Y.: Sal++: Sign agnostic learning with derivatives. arXiv preprint arXiv:2006.05400 (2020) [3](#)
4. Bautista, M.A., Talbott, W., Zhai, S., Srivastava, N., Susskind, J.M.: On the generalization of learning-based 3d reconstruction. In: Proceedings of the IEEE/CVF Winter Conference on Applications of Computer Vision. pp. 2180–2189 (2021) [4](#)
5. Chabra, R., Lenssen, J.E., Ilg, E., Schmidt, T., Straub, J., Lovegrove, S., Newcombe, R.: Deep local shapes: Learning local sdf priors for detailed 3d reconstruction. In: European Conference on Computer Vision. pp. 608–625. Springer (2020) [3](#), [6](#)
6. Chang, A.X., Funkhouser, T., Guibas, L., Hanrahan, P., Huang, Q., Li, Z., Savarese, S., Savva, M., Song, S., Su, H., et al.: Shapenet: An information-rich 3d model repository. arXiv preprint arXiv:1512.03012 (2015) [11](#)
7. Chatzipantazis, E., Pertigkiozoglou, S., Dobriban, E., Daniilidis, K.: Se (3)-equivariant attention networks for shape reconstruction in function space. arXiv preprint arXiv:2204.02394 (2022) [4](#)
8. Chen, Y., Fernando, B., Bilen, H., Mensink, T., Gavves, E.: Neural feature matching in implicit 3d representations. In: International Conference on Machine Learning. pp. 1582–1593. PMLR (2021) [4](#)
9. Chen, Y., Hu, V.T., Gavves, E., Mensink, T., Mettes, P., Yang, P., Snoek, C.G.: Pointmixup: Augmentation for point clouds. arXiv preprint arXiv:2008.06374 (2020) [4](#)
10. Chen, Z., Zhang, H.: Learning implicit fields for generative shape modeling. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 5939–5948 (2019) [1](#), [3](#)
11. Chibane, J., Alldieck, T., Pons-Moll, G.: Implicit functions in feature space for 3d shape reconstruction and completion. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 6970–6981 (2020) [2](#), [3](#), [5](#), [7](#), [11](#)
12. Choy, C.B., Gwak, J., Savarese, S., Chandraker, M.: Universal correspondence network. arXiv preprint arXiv:1606.03558 (2016) [11](#)
13. Cohen, T., Welling, M.: Group equivariant convolutional networks. In: International conference on machine learning. pp. 2990–2999. PMLR (2016) [4](#), [5](#)
14. Cohen, T.S., Welling, M.: Steerable cnns. arXiv preprint arXiv:1612.08498 (2016) [4](#)
15. Dai, A., Chang, A.X., Savva, M., Halber, M., Funkhouser, T., Nießner, M.: Scannet: Richly-annotated 3d reconstructions of indoor scenes. In: Proceedings of the IEEE conference on computer vision and pattern recognition. pp. 5828–5839 (2017) [14](#)
16. Davies, T., Nowrouzezahrai, D., Jacobson, A.: On the effectiveness of weight-encoded neural implicit 3d shapes. arXiv preprint arXiv:2009.09808 (2020) [2](#), [4](#), [8](#), [11](#)
17. Deng, C., Litany, O., Duan, Y., Poulencard, A., Tagliasacchi, A., Guibas, L.: Vector neurons: A general framework for so (3)-equivariant networks. arXiv preprint arXiv:2104.12229 (2021) [2](#), [3](#), [4](#), [8](#), [9](#), [10](#), [11](#), [12](#), [13](#)
18. Erler, P., Guerrero, P., Ohrhallinger, S., Mitra, N.J., Wimmer, M.: Points2surf learning implicit surfaces from point clouds. In: European Conference on Computer Vision. pp. 108–124. Springer (2020) [2](#), [3](#)
19. Fuchs, F., Worrall, D., Fischer, V., Welling, M.: Se (3)-transformers: 3d roto-translation equivariant attention networks. Advances in Neural Information Processing Systems **33** (2020) [4](#)

20. Fujiwara, K., Hashimoto, T.: Neural implicit embedding for point cloud analysis. In: Proceedings of the IEEE/CVF conference on computer vision and pattern recognition. pp. 11734–11743 (2020) [3](#)
21. Genova, K., Cole, F., Sud, A., Sarna, A., Funkhouser, T.: Deep structured implicit functions. arXiv preprint arXiv:1912.06126 (2019) [2](#)
22. Genova, K., Cole, F., Sud, A., Sarna, A., Funkhouser, T.: Local deep implicit functions for 3d shape. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 4857–4866 (2020) [3](#)
23. Gilmer, J., Schoenholz, S.S., Riley, P.F., Vinyals, O., Dahl, G.E.: Neural message passing for quantum chemistry. In: International conference on machine learning. pp. 1263–1272. PMLR (2017) [7](#)
24. Hanocka, R., Hertz, A., Fish, N., Giryes, R., Fleishman, S., Cohen-Or, D.: Meshcnn: a network with an edge. ACM Transactions on Graphics (TOG) **38**(4), 1–12 (2019) [7](#)
25. Jiang, C., Sud, A., Makadia, A., Huang, J., Nießner, M., Funkhouser, T.: Local implicit grid representations for 3d scenes. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (2020) [2](#), [3](#), [6](#)
26. Jiang, Y., Ji, D., Han, Z., Zwicker, M.: Sdfdiff: Differentiable rendering of signed distance fields for 3d shape optimization. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 1251–1261 (2020) [3](#)
27. Kingma, D.P., Ba, J.: Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980 (2014) [11](#)
28. Li, J., Chen, B.M., Lee, G.H.: So-net: Self-organizing network for point cloud analysis. In: Proceedings of the IEEE conference on computer vision and pattern recognition. pp. 9397–9406 (2018) [3](#)
29. Liu, S., Saito, S., Chen, W., Li, H.: Learning to infer implicit surfaces without 3d supervision. In: Advances in Neural Information Processing Systems. pp. 8295–8306 (2019) [3](#)
30. Mescheder, L., Oechsle, M., Niemeyer, M., Nowozin, S., Geiger, A.: Occupancy networks: Learning 3d reconstruction in function space. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 4460–4470 (2019) [1](#), [3](#), [4](#), [5](#), [11](#)
31. Niemeyer, M., Mescheder, L., Oechsle, M., Geiger, A.: Differentiable volumetric rendering: Learning implicit 3d representations without 3d supervision. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 3504–3515 (2020) [3](#)
32. Park, J.J., Florence, P., Straub, J., Newcombe, R., Lovegrove, S.: Deepsdf: Learning continuous signed distance functions for shape representation. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 165–174 (2019) [1](#), [3](#)
33. Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., et al.: Pytorch: An imperative style, high-performance deep learning library. arXiv preprint arXiv:1912.01703 (2019) [11](#)
34. Peng, S., Niemeyer, M., Mescheder, L., Pollefeys, M., Geiger, A.: Convolutional occupancy networks. In: Computer Vision–ECCV 2020: 16th European Conference, Glasgow, UK, August 23–28, 2020, Proceedings, Part III 16. pp. 523–540. Springer (2020) [2](#), [3](#), [5](#), [7](#), [8](#), [11](#), [13](#), [14](#)
35. Qi, C.R., Su, H., Mo, K., Guibas, L.J.: Pointnet: Deep learning on point sets for 3d classification and segmentation. In: Proceedings of the IEEE conference on computer vision and pattern recognition. pp. 652–660 (2017) [4](#), [7](#)
36. Qi, C.R., Yi, L., Su, H., Guibas, L.J.: Pointnet++: Deep hierarchical feature learning on point sets in a metric space. Advances in neural information processing systems **30** (2017) [3](#)
37. Ronneberger, O., Fischer, P., Brox, T.: U-net: Convolutional networks for biomedical image segmentation. In: International Conference on Medical image computing and computer-assisted intervention. pp. 234–241. Springer (2015) [7](#)

38. Saito, S., Huang, Z., Natsume, R., Morishima, S., Kanazawa, A., Li, H.: Pifu: Pixel-aligned implicit function for high-resolution clothed human digitization. In: Proceedings of the IEEE International Conference on Computer Vision. pp. 2304–2314 (2019) [3](#)
39. Satorras, V.G., Hooeboom, E., Welling, M.: E(n) equivariant graph neural networks. arXiv preprint arXiv:2102.09844 (2021) [4](#)
40. Shen, W., Zhang, B., Huang, S., Wei, Z., Zhang, Q.: 3d-rotation-equivariant quaternion neural networks. In: Computer Vision—ECCV 2020: 16th European Conference, Glasgow, UK, August 23–28, 2020, Proceedings, Part XX 16. pp. 531–547. Springer (2020) [4](#), [8](#)
41. Simeonov, A., Du, Y., Tagliasacchi, A., Tenenbaum, J.B., Rodriguez, A., Agrawal, P., Sitzmann, V.: Neural descriptor fields: Se (3)-equivariant object representations for manipulation. In: 2022 International Conference on Robotics and Automation (ICRA). pp. 6394–6400. IEEE (2022) [4](#)
42. Sitzmann, V., Chan, E., Tucker, R., Snavely, N., Wetzstein, G.: Metasdf: Meta-learning signed distance functions. Advances in Neural Information Processing Systems **33**, 10136–10147 (2020) [4](#)
43. Sosnovik, I., Szmaja, M., Smeulders, A.: Scale-equivariant steerable networks. In: International Conference on Learning Representations (2019) [4](#)
44. Takikawa, T., Litalien, J., Yin, K., Kreis, K., Loop, C., Nowrouzezahrai, D., Jacobson, A., McGuire, M., Fidler, S.: Neural geometric level of detail: Real-time rendering with implicit 3d shapes. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. pp. 11358–11367 (2021) [3](#)
45. Tang, J.H., Chen, W., Wang, B., Liu, S., Yang, B., Gao, L., et al.: Octfield: Hierarchical implicit functions for 3d modeling. Advances in Neural Information Processing Systems **34**, 12648–12660 (2021) [3](#)
46. Thomas, N., Smidt, T., Kearnes, S., Yang, L., Li, L., Kohlhoff, K., Riley, P.: Tensor field networks: Rotation-and translation-equivariant neural networks for 3d point clouds. arXiv preprint arXiv:1802.08219 (2018) [4](#), [5](#)
47. Wang, P.S., Liu, Y., Tong, X.: Dual octree graph networks for learning adaptive volumetric shape representations. arXiv preprint arXiv:2205.02825 (2022) [3](#)
48. Wang, Y., Sun, Y., Liu, Z., Sarma, S.E., Bronstein, M.M., Solomon, J.M.: Dynamic graph cnn for learning on point clouds. *Acm Transactions On Graphics (tog)* **38**(5), 1–12 (2019) [3](#), [7](#)
49. Weiler, M., Geiger, M., Welling, M., Boomsma, W., Cohen, T.: 3d steerable cnns: Learning rotationally equivariant features in volumetric data. In: NeurIPS (2018) [4](#)
50. Weiler, M., Hamprecht, F.A., Storath, M.: Learning steerable filters for rotation equivariant cnns. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 849–858 (2018) [4](#)
51. Worrall, D.E., Garbin, S.J., Turmukhambetov, D., Brostow, G.J.: Harmonic networks: Deep translation and rotation equivariance. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 5028–5037 (2017) [4](#)
52. Xu, Q., Wang, W., Ceylan, D., Mech, R., Neumann, U.: Disn: Deep implicit surface network for high-quality single-view 3d reconstruction. In: Advances in Neural Information Processing Systems. pp. 492–502 (2019) [3](#)
53. Xu, Y., Fan, T., Yuan, Y., Singh, G.: Ladybird: Quasi-monte carlo sampling for deep implicit field based 3d reconstruction with symmetry. In: European Conference on Computer Vision. pp. 248–263. Springer (2020) [3](#)
54. Yuan, Y., Nüchter, A.: An algorithm for the se (3)-transformation on neural implicit maps for remapping functions. *IEEE Robotics and Automation Letters* (2022) [4](#)
55. Zaheer, M., Kottur, S., Ravanbakhsh, S., Poczos, B., Salakhutdinov, R., Smola, A.: Deep sets. arXiv preprint arXiv:1703.06114 (2017) [4](#)
56. Zhu, W., Qiu, Q., Calderbank, R., Sapiro, G., Cheng, X.: Scale-equivariant neural networks with decomposed convolutional filters. arXiv preprint arXiv:1909.11193 (2019) [4](#)