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1 Decomposing phenotypic skew and its effects on the  
2 predicted response to strong selection

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9 **The major frameworks for predicting evolutionary change assume that a phenotype's**  
10 **underlying genetic and environmental components are normally distributed. However,**  
11 **the predictions of these frameworks may no longer hold if distributions are skewed.**  
12 **Despite this, phenotypic skew has never been decomposed, meaning the fundamental**  
13 **assumptions of quantitative genetics remain untested. Here, we demonstrate that**  
14 **the substantial phenotypic skew in the body size of juvenile blue tits (*Cyanistes***  
15 **caeruleus) is driven by environmental factors. Although skew had little impact on**  
16 **our predictions of selection response in this case, our results highlight the impact**  
17 **of skew on the estimation of inheritance and selection. Specifically, the non-linear**  
18 **parent-offspring regressions induced by skew, alongside selective disappearance, can**  
19 **strongly bias estimates of heritability. The ubiquity of skew and strong directional**  
20 **selection on juvenile body size implies that heritability is commonly overestimated,**  
21 **which may in part explain the discrepancy between predicted and observed trait**  
22 **evolution.**

23 Quantitative genetics describes how traits respond to selection in terms of selection and  
24 inheritance. Typically we use two equations to describe this, the breeder's equation (<sup>1</sup>  
25 Chapter 12) and Lande's gradient equation (<sup>2</sup> Eq 7). The breeder's equation gives the  
26 predicted response to selection as the heritability ( $h^2$ ) multiplied by the selection differential  
27 ( $S$ ), whereas Lande's gradient equation describes the response to selection as the additive  
28 genetic variance ( $V_A$ ) of the trait multiplied by the selection gradient ( $\beta$ ). Although these  
29 frameworks are generally thought to be interchangeable, they only converge when phenotypes  
30 (and their genetic and environmental components) are normally distributed or fitness functions  
31 (the relationship between a trait and fitness) are linear (<sup>3</sup> Chapter 29). Given that fitness  
32 functions are highly unlikely to be linear in practice<sup>4;5</sup>, any deviation from normality can  
33 lead to problems with the application of these equations. Consequently, normality is seen  
34 as a fundamental assumption in quantitative genetics<sup>6-8</sup>, yet to our knowledge has not been  
35 directly tested, despite the major consequences it has for how traits are predicted to respond  
36 to selection<sup>9-17</sup>.

37 The most natural interpretation of heritability in the context of the breeder's equation is the  
38 slope of a *linear* parent-offspring (PO) regression<sup>12-14;18;19</sup>, whilst  $S$  (the covariance between  
39 a trait and fitness) describes the *linear* relationship between a phenotype and fitness. The  
40 accuracy of the breeder's equation relies heavily on the linearity of both of these functions -  
41 if both are non-linear, the residuals from the linear functions may be correlated, creating a  
42 'spurious response to selection'<sup>14</sup>. The linearity of the parent-offspring relationship breaks  
43 down when the amount of skew (asymmetry) differs between genetic and environmental  
44 components<sup>20;21</sup>, with genetic and environmental skew causing curvature in opposite directions  
45 (Figure 1). Formally, a distribution is skewed when it has a non-zero third central moment.

46 Whilst the gradient equation is robust to environmental skew, it doesn't correctly describe the  
47 response to selection in the presence of genetic skew if the fitness function is non-linear (<sup>11</sup> Eq  
48 42). Environmental skew, through its contribution to phenotypic skew, can, however, impact  
49 the estimation of  $\beta$  when it is approximated using Lande-Arnold regression<sup>5;17;22</sup>.

50 Although extensions to these two equations have been derived that allow for the non-linearity  
51 of the PO-regression<sup>12</sup> and the non-normality of genetic values<sup>11</sup>, the majority of the work  
52 in this area remains theoretical. Non-linearity in PO-regressions has been demonstrated in

53 the lab<sup>12;23–28</sup> and ad-hoc methods have been used to test for skew at the genetic level<sup>29;30</sup>.  
54 Nevertheless, to our knowledge, no study has 1) relaxed the normality assumptions when  
55 making statistical inferences to examine the origin and extent of skew at different levels, and  
56 2) explored how observed patterns of natural selection interact with skew to determine how  
57 well these two equations predict selection response in the wild.

58 Juvenile body size is under strong, persistent, directional selection across taxa<sup>31</sup>, yet is known  
59 to show little response to this selection<sup>32</sup>. We show that juvenile body size is highly negatively  
60 skewed (long tail of small individuals) across bird species, but the origin of this skew is  
61 unknown. To determine this, we developed statistical methods to decompose the phenotypic  
62 distribution into a set of skew-t distributions, and predict the shape of PO-regression based  
63 on the estimated skew. We applied these methods to data from a long-term cross-fostering  
64 experiment of a wild bird population. By estimating survival selection acting on juvenile body  
65 size, we tested the robustness of the predicted response to selection from the breeder's and  
66 gradient equations.

## 67 **Results**

### 68 **Prevalence of Phenotypic Skew**

69 Across 27 species of birds, tarsus length (a common measure of structural size) was substantially  
70 negatively skewed (long tail of small individuals) in juveniles (coefficient of skew: -1.054  
71 [-1.394, -0.686], pMCMC<0.001), but not adults (-0.302 [-0.641, 0.052], pMCMC=0.086),  
72 with tarsus length being significantly more skewed in juveniles than adults (difference = -0.752  
73 [-1.124, -0.366], pMCMC<0.001; Figure 2).

### 74 **Decomposing Phenotypic Skew**

75 Using data on four juvenile body size traits (tarsus length, head-bill length, mass and wing  
76 length), measured on 15 day old chicks from a long-term cross-fostering experiment on a wild  
77 population of blue tits, we decomposed phenotypic skew into genetic, between- and within-nest  
78 environmental components. We used a mixed model approach with skew-t distributed random  
79 effects which allowed the extent and direction of skew to vary between these levels. There  
80 was considerable phenotypic skew in all four traits, with the coefficient of skew ranging from  
81 -0.51 to -1.60 (Figure 3). There was little evidence of genetic skew in any trait (Figure 3,  
82 Tables S5, S8, S11 and S12 and further discussion in supplementary methods). Phenotypic  
83 skew was instead driven by considerable environmental skew at both between- and within-nest  
84 levels, with the relative magnitude of this skew varying between traits (Figure 3, Tables S6,  
85 S9, S12 and S15).

86 Given the environmental origin of the negative phenotypic skew, we would expect a convex  
87 PO-regression for all traits<sup>20</sup> (Figure 1c). By deriving a method to compute this non-linear  
88 PO-regression (Equation 1), we can show that for all traits the slope in the lower tail of the  
89 distributions is close to zero, but becomes steeper with increasing body size (Figure 3).

## 90 Selection on Juvenile Body Size

91 To quantify selection acting on body size, we estimated the linear and quadratic effects of body  
92 size on survival from both day 15 to fledging and fledging to local recruitment in a bivariate  
93 probit event-history model. As expected, all traits showed significant positive linear effects of  
94 body size on survival at both stages, with survival increasing at larger body sizes (Figure 4,  
95 Tables S16-19). Interestingly, all quadratic effects of juvenile size on survival between day 15  
96 and fledging were positive, with these effects being suggestive and significant for mass and  
97 wing length, respectively (Figure 4, Tables S16-19), indicating an accelerating effect of size  
98 on offspring survival. In contrast, negative quadratic effects were typical for survival from  
99 fledging to recruitment although this effect was only suggestive in the case of tarsus length  
100 (Figure 4, Tables S16-19). The fitness functions over both events were generally concave  
101 (Figure 4), which would indicate stabilising selection, but the hypothesis that the optimal  
102 trait value lay outside of the observed phenotypic range for any trait could not be rejected  
103 (proportion of iterations with an internal optimum: tarsus 0.853; head-bill 0.543; mass 0.757;  
104 wing 0.017).

105 Using these fitness functions, we were able to estimate selection gradients ( $\beta$ ) for each  
106 trait by taking the partial derivative of the individual relative fitness function with respect  
107 to the trait and averaging it over the trait's distribution. However,  $\beta$  is more frequently  
108 approximated using a Lande-Arnold regression of fitness on a trait<sup>22</sup> and phenotypic skew can  
109 bias this approximation when the fitness function is not linear or quadratic (as is the case  
110 for survival functions)<sup>22</sup>. To test this, we calculated the expected estimates of  $\beta$  that would  
111 be obtained from the Lande-Arnold approach without ( $\beta_1$ ) and with ( $\beta_2$ ) a quadratic term  
112 fitted<sup>22;33;34</sup>, over the posterior distribution of the survival models (Equations 10 and 11).  
113 Figure 4 shows that generally there is little meaningful difference between estimates, with the  
114 exception of wing length, where there is suggestive evidence that  $\beta_1$  would underestimate  $\beta$   
115 by approximately 30% ( $\beta_1/\beta$ : 0.702 [0.494, 0.881], pMCMC=0.010).

## 116 Predicted Response to Selection

117 In the absence of genetic skew, the correct response to selection is given by Lande's gradient  
118 equation ( $V_A\beta$ ), which for these traits gives: tarsus: 0.085mm [0.034, 0.127]; head-bill:  
119 0.069mm [0.037, 0.102]; mass: 0.094g [0.052, 0.139]; wing: 0.175mm [0.077, 0.280]. The  
120 breeder's equation is equal to the gradient equation when the Lande-Arnold regression without  
121 the quadratic term gives good estimates of the selection gradient, irrespective of whether  
122 the PO-regression is linear or not (i.e if  $\beta_1 = \beta$  then  $h^2S = V_A\beta$ ;<sup>3</sup> Chapter 29). Given the  
123 similarity between  $\beta$  and  $\beta_1$  for tarsus, head-bill and mass, the breeder's equation will therefore  
124 give accurate predictions of the selection response for these traits. However, it underestimates  
125 the response to selection in wing length by approximately 30%, as the proportional change in  
126 the predicted response to selection is equal to  $\beta_1/\beta$  (shown above).

## 127 Selection Bias and Heritability Estimation

128 The heritability in the breeder's equation is the heritability *before* selection ( $h_b^2$ ) which can be  
129 interpreted as the slope of the PO-regression averaged over all individuals irrespective of their  
130 fitness. However, direct estimates of the PO-regression can only be obtained from individuals

131 that survive to become parents and so to some extent measure the heritability *after* selection  
 132 ( $h_a^2$ ; note the terms heritability before and after selection are used in a broader sense than  
 133 in<sup>14</sup>, and capture a different bias; see<sup>3</sup> p171 for a clear explanation of Heywood's usage).  
 134 Since larger individuals are more likely to survive, and the PO-regression is steeper for these  
 135 individuals, direct estimates of the PO-regression are likely to be upwardly biased estimates  
 136 of heritability. To demonstrate this, we obtained direct estimates of the PO-regression from  
 137 the 182 individuals (118 male and 64 female) that were measured as chicks and survived to  
 138 produce offspring that were also measured. Although the estimated linear regression (blue line  
 139 in Figure 5) is similar to the predicted non-linear PO-regression (red line in Figure 5) for the  
 140 large surviving individuals (the linear and non-linear regressions fit the data equally well for all  
 141 traits; tarsus  $p = 0.195$ , head-bill  $p = 0.087$ , mass  $p = 0.060$  and wing  $p = 0.052$ ), the two  
 142 diverge substantially at small body sizes (Figure 5). In order to directly compare  $h_a^2$  and  $h_b^2$ ,  
 143 we used the parameters of the quantitative genetic and survival models described above to  
 144 calculate  $h_a^2$  as the linear PO-regression weighted by the fitness of the parents (Equation 16)  
 145 and  $h_b^2$  as  $V_A/V_P$ . For tarsus, head-bill and mass,  $h_a^2$  was substantially and significantly higher  
 146 than  $h_b^2$ , with a proportional increase in  $h_a^2$  of over 60% for head-bill and mass ( $h_a^2/h_b^2$ : tarsus  
 147 1.223 [1.137, 1.333], pMCMC=0.002; head-bill 1.664 [1.421, 1.951], pMCMC<0.001; mass  
 148 1.645 [1.325, 2.046], pMCMC<0.001; wing 1.584 [0.373, 2.551], pMCMC=0.372).

149 Estimates of  $h_b^2$  will only be accurate if they do not suffer from the same selection bias  
 150 present in PO-regression. Our experimental cross-fostering design means that the majority  
 151 of information used to estimate  $V_A$  in our analysis comes from the comparison of siblings  
 152 (569 nests have chicks from at least 2 clutches), rather than parents and offspring (182  
 153 parent-offspring comparisons). Sibling comparisons are made before selection, and so should  
 154 not suffer from the same selection bias as parent-offspring comparisons. However, many  
 155 wild bird pedigrees rely largely on information from parent-offspring relationships to estimate  
 156 genetic effects - without partial cross-fostering and using social pedigrees (no within-nest  
 157 variation in relatedness), sibling comparisons provide little information on genetic effects  
 158 because they are confounded with common environment (nest) effects. As both PO-regression  
 159 and the animal model assume that the relationship between offspring and parental phenotypes  
 160 is linear, animal models relying mainly on the information from parent-offspring comparisons  
 161 may also be biased. To test this, we simulated data using the parameters from our quantitative  
 162 genetic and selection models for mass, assuming social and genetic monogamy, with and  
 163 without skew and with and without partial cross-fostering. As expected, environmental skew  
 164 caused  $h^2$  estimated from PO-regressions to be consistently and substantially upwardly biased  
 165 by a similar amount as we observed in our data, regardless of cross-fostering (estimated/simulated:  
 166 no cross-fostering 1.609; cross-fostering 1.616). Without cross-fostering (information mainly  
 167 from parent-offspring comparisons), estimates of  $V_A$ , and so heritability, from animal models  
 168 were upwardly biased, although less than in the PO-regressions (estimated/simulated:  $V_A$   
 169 1.226,  $h^2$  1.228), whereas cross-fostering (information mainly from sibling comparisons) led  
 170 to the correct estimation of  $V_A$  and  $h^2$  (estimated/simulated: 1.012 and 1.015 respectively;  
 171 Table 1).

## Discussion

A common assumption in quantitative genetics is that phenotypes, and their underlying genetic and environmental components, are normally distributed. Here we demonstrate that this assumption is commonly violated, and in four morphological traits the observed negative phenotypic skew is driven by environmental, rather than genetic, skew. There was strong directional viability selection acting on all four traits, with non-linear fitness functions. Under these conditions the breeder's equation may give inaccurate predictions for the response to selection, but Lande's gradient equation - which only assumes genetic values are normally distributed - is expected to be accurate<sup>11</sup>. However, this assumes that the methods used to obtain estimates of  $\beta$  and  $V_A$  are robust to deviations from normality. Here we empirically demonstrate that common methods used to estimate both metrics can produce biased estimates in the presence of environmental skew.

Perhaps the most striking result is the apparent absence of genetic skew. Theory shows that directional selection can generate genetic skew, but the direction of the skew differs between models. Under the infinitesimal (Gaussian descendants<sup>35</sup>) model (assumed in our analyses), directional selection can drive a Gaussian distribution of breeding values to be skewed in the direction of selection through the build up of linkage disequilibrium<sup>11;36;37</sup>. However, stabilising selection may mitigate this (<sup>11</sup> Eq 46) and the breeding value distribution quickly returns to normality if selection ceases (the skew quarters each generation for unlinked loci;<sup>36</sup> p149). Finite allele models also generate genetic skew through changes in allele frequency. Under the rare-alleles model, directional selection after a long period of stabilising selection generates skew in the direction of selection<sup>10;11</sup> but sustained long term directional selection (with new mutations, on average, having effects in the opposite direction) is expected to drive skew in the opposite direction to selection<sup>38;39</sup>. Given juvenile body size appears to be under sustained positive directional selection<sup>31</sup> and gene knockout studies in mice show that loss-of-function mutations reduce size more often than increase it<sup>40</sup>, we would predict negative genetic skew in our system. However, these models predict that the amount of skew generated through selection should be small, consistent with our finding of no or negligible genetic skew. Other processes, such as few loci, alleles of large effect, extreme allele frequencies or substantial non-additive gene action, particularly directional dominance, could generate greater levels of skew<sup>21;38;41;42</sup>. This seems unlikely for body size, which appears to be highly polygenic<sup>43;44</sup>, although the finding that inbred individuals are on average smaller does suggest some directional dominance<sup>45-48</sup> which would also generate skew in the opposite direction to selection. Two other studies have looked at the distribution of breeding values (indirectly through estimating the skew of breeding values estimated in a Gaussian model) and while one also found little evidence of skew<sup>30</sup>, the other found skew in the opposite direction to selection<sup>29</sup>. Lack of genetic skew would also be a consequence of selection acting on an environmentally correlated trait, rather than acting directly on size<sup>49;50</sup> (discussed further below). More widespread assessments of the prevalence of genetic skew are needed to assess the generality of these results.

Environmental skew has received little attention from theoreticians, with most studies assuming that environmental effects are normally distributed<sup>11;12;14</sup>. There are, however, several biological processes that are known to induce environmental skew. As far as we are aware, these processes are all predicted to generate negative environmental skew, which fits with our

216 general observation of negative skew in juvenile body size across species (Figure 2). For  
217 example, asymmetric competition, when larger individuals have a disproportionate negative  
218 competitive effect on others, can drive negative skew<sup>51-54</sup>. Blue tits have moderate levels of  
219 hatching asynchrony (hatching spread is approximately 2 days; see<sup>55</sup> for distribution across  
220 bird species) which is expected to generate asymmetries in competitive ability<sup>56</sup> and therefore  
221 skew at the within-nest level. However, the dominant source of phenotypic skew is at the  
222 between-nest level (contribution to phenotypic skew relates to standardised skew and variance)  
223 and so if asymmetric competition was the main driver of phenotypic skew, it would require  
224 parental ability to be driven by asymmetric adult competition, perhaps through differences  
225 in condition and/or territory quality. An alternative explanation is that (some) chicks have  
226 yet to reach their asymptotic size by the time of measurement and so variation in their size  
227 at this time is driven by variation in growth rate and asymptotic size. If variation in growth  
228 rate is largely at the between-nest level and variation in the asymptote is largely genetic, as  
229 has been suggested in great tits<sup>57</sup>, then the non-linearity of growth functions could result in  
230 skew that is primarily environmental in origin (see<sup>58</sup> for a related result). This skew would be  
231 expected to disappear further into development as all chicks reach their asymptotic size, but  
232 due to the strong selective disappearance of small chicks this may not necessarily manifest  
233 itself (see below).

234 The strong, negative environmental skew led the PO-regression in all traits to be convex. This  
235 occurs because the long tail of small individuals are primarily small because of environmental  
236 factors and so resemble their parents less than larger individuals. Most discussions of the  
237 linearity of the PO-regression focus on how, in combination with a non-linear fitness function,  
238 a non-linear PO-regression leads the breeder's equation to be inaccurate, through generating a  
239 covariance between the residuals from a linear fitness function and the linear PO-regression<sup>3;14</sup>  
240 (see also Figure S18). This 'spurious response to selection'<sup>14</sup> will be largest when the  
241 non-linear fitness function and the PO-regression either have the same non-linear shape (e.g.  
242 both concave) causing a positive covariance between residuals, leading the breeder's equation  
243 to under-estimate the response to selection or opposite shapes (e.g. one concave and one  
244 convex), creating negative covariance between residuals and so over-estimation of selection  
245 response. Skew generates quite predictable and simple non-linearity in the PO-regression  
246 (Figure 1), and so generally accelerating or decelerating fitness functions will be more likely  
247 to generate a spurious response to selection, as is seen with wing length (Figure S18).

248 We additionally show that the selective disappearance of small individuals alongside a non-linear  
249 PO-regression leads to  $h^2$  estimates that are biased towards the slope of the surviving large  
250 individuals. This selection bias is particularly striking in estimates from PO-regression (approx  
251 65% increase in  $h^2$  for mass and head-bill length; Figure 5) but importantly also occurs  
252 in animal models applied to pedigrees where information about the genetic variance comes  
253 primarily from parent-offspring comparisons (e.g. typical bird pedigrees without cross-fostering),  
254 although to a lesser degree (23% increase in animal models compared to a 61% increase in  
255 PO-regression; Table 1). This bias occurs because both PO-regression and the animal model  
256 assume that the relationship between offspring and parental phenotypes is linear, and so  
257 assumes the missing parent-offspring comparisons would follow the same slope. It is worth  
258 noting that we simulated closed populations and so a higher relatedness structure than in  
259 most wild bird populations, which are characterised by low recruitment and high immigration.  
260 Thus, our simulations likely underestimated the possible bias in animal models. We also



261 demonstrated that cross fostering eliminated this bias in animal models. This occurs because  
262 cross-fostering shifts the majority of the information for estimating  $V_A$  from parent-offspring  
263 comparisons, to sibling comparisons, and sibling comparisons are made before selection whilst  
264 parent-offspring comparisons are made after.

265 Previous work in this system has shown that selection differentially eliminates negative environmental,  
266 but not genetic, deviations for mass over the course of development<sup>59</sup>. This was interpreted  
267 as mass being an environmentally correlated target of selection rather than the true target  
268 (i.e. no causal relationship between size and survival)<sup>49</sup>. However, incorporating skew into  
269 our models challenges this interpretation as, under our model, size is the true target of  
270 selection. As the long tail of small individuals are small for environmental reasons, the selective  
271 disappearance of these individuals drives the observed decrease in environmental variance and  
272 skew though ontogeny. Given the selective disappearance previously observed was prior to the  
273 measurements analysed here<sup>59</sup> it seems likely that the environmental skew we observe is an  
274 underestimation of the true skew, meaning we are likely underestimating the true non-linearity  
275 of the PO-regression. Multivariate methods would account for this selective disappearance<sup>60</sup>,  
276 however, these proved too complex to implement in this instance.

277 Given the consistent negative environmental skew we see across the four traits, and the  
278 conserved nature of negative phenotypic skew in juvenile (but not adult) size across bird  
279 species, we believe a concave PO-regression for juvenile size traits might be a general finding.  
280 As found here, juvenile body size is also generally under strong viability selection across  
281 taxa<sup>31</sup>. Together, this suggests that previous heritability estimates of juvenile size are likely  
282 to have been systematically over-estimated, especially as a large proportion are based on  
283 PO-regressions<sup>61</sup>. Indeed, tarsus length heritability estimates from PO-regressions have been  
284 shown to be consistently larger than those from animal models<sup>61</sup>. Juvenile size is a hallmark  
285 trait of evolutionary stasis, whereby traits that should respond to selection in the wild appear  
286 not to. Although these results do not fully explain this stasis, they do show that the predicted  
287 response to selection may be being substantially overestimated in traits with non-Gaussian  
288 phenotypic distributions.

289 Lande-Arnold regression is by far the most common method for estimating  $\beta$ <sup>5;33;62</sup> and is  
290 known to be unbiased in the presence of phenotypic skew only if the fitness function is linear  
291 or quadratic *and* this quadratic term is modelled<sup>22</sup>. Although the estimated survival functions  
292 deviated from a quadratic for all traits, estimates of  $\beta$  were close to those that would have  
293 been obtained under Lande-Arnold regression including the quadratic term ( $\beta_2$ ) for all traits,  
294 and without the quadratic term ( $\beta_1$ ) for three traits. The near equivalence of these different  
295 estimates seems at odds with the conclusions of Bonamour *et al.*<sup>17</sup>, who demonstrate that  
296 selection gradients approximated with Lande-Arnold regression are biased in the presence of  
297 phenotypic skew. However, Bonamour *et al.* only modelled the linear term in the Lande-Arnold  
298 regression ( $\beta_1$ ) whilst assuming a quadratic fitness function - had the quadratic term also been  
299 included, the linear term in the Lande-Arnold regression ( $\beta_2$ ) would have been unbiased (<sup>22</sup>,<sup>3</sup>  
300 Chapter 29), in correspondence with our wing length results ( $\beta_1$  underestimated  $\beta$ , but  $\beta_2$   
301 did not). However, there is no reason to believe including a quadratic term in a Lande-Arnold  
302 regression will generally result in a good approximation of  $\beta$ . Indeed, Morrissey & Sakrejda<sup>5</sup>  
303 compared  $\beta$  with that approximated from a quadratic Lande-Arnold regression and found quite  
304 large proportional differences (approx. 30%), although small differences in absolute terms.

305 We therefore urge caution in assuming that our results are a general statement about the  
306 accuracy of Lande-Arnold regression under non-normality.

307 Quantitative genetics uses two main frameworks to predict how traits will respond to selection.  
308 Here we demonstrate how both of these frameworks are affected by skew at the environmental  
309 and genetic levels. Genetic skew can lead both the breeder's equation and Lande's gradient  
310 equation to be inaccurate. Although little or no genetic skew has been found in the few  
311 studies that have tried to quantify it, it remains unknown to what extent this is a generality,  
312 and will be highly dependent on the genetic architecture of specific traits. In the absence of  
313 genetic skew, the gradient equation presents an accurate prediction of selection response<sup>11</sup>,  
314 although environmental skew provides challenges to the accurate estimation of both  $\beta$  and  $V_A$ .  
315 Whilst the breeder's equation may provide a more intuitive way of thinking about selection  
316 response, the extensions to this framework that allow for non-linearity<sup>12</sup> are complex and  
317 computationally expensive. We therefore recommend a focus on the gradient equation (and  
318 its extensions<sup>11</sup>) in wild systems, where fitness functions are highly likely to be non-linear and  
319 trait distributions are commonly skewed.

## 320 **Methods**

321 This study was preregistered (see <https://osf.io/7qyp4/>). We have highlighted in the following  
322 sections where our methods deviate from those planned.

### 323 **Meta-analysis of Skew**

324 We collected raw data on juvenile and adult tarsus length from several sources: we used a  
325 mailing list to request data, we searched the dryad repository for 'tarsus', we emailed groups  
326 with known long-term avian datasets that were not represented in these sources and included  
327 any tarsus length data that we otherwise encountered. When datasets from different studies of  
328 the same population overlapped in time, we use the largest single dataset available. Datasets  
329 were taken from [44:63–100](#).

Sample standardised skew was estimated from raw data  $z$  as

$$\frac{\frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^3}{\left[\frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^2\right]^{3/2}} \frac{\sqrt{n(n-1)}}{n-2}$$

with sampling variance as

$$\frac{6n(n-1)}{(n-2)(n+1)(n+3)}$$

330 where  $n$  is sample size and  $\hat{\mu}$  the estimate of the trait mean [101](#).

331 Using these data, we ran a random-effect meta-analytic model in MCMCglmm with age  
332 (juvenile or adult) as a fixed factor and random effects of species and study. Models were run  
333 for 65000 iterations, with a burnin of 15000 and a thinning intervals of 50. The priors for the  
334 random-effect variances were scaled (by 100)  $F_{1,1}$  and the prior for the residual variance was  
335 inverse-gamma with a shape and scale of 0.001. The fixed effects had a diffuse normal prior  
336 (mean=0, variance= $10^{10}$ ).

### 337 **Study population**

338 We used data from a nest-box population of blue tits (*Cyanistes caeruleus*), on the Dalmeny  
339 estate, Edinburgh, United Kingdom, collected from 2011 to 2018, with 253 nest-boxes over  
340 two sites. Detailed methods are described in [59:102](#). Briefly, all nests were visited regularly until  
341 the discovery of the first egg, and then daily for egg cross-fostering, when eggs were weighed.  
342 From 2011-2013 and 2016-2018 a partial egg cross-fostering design was used to enable additive  
343 genetic and nest-of-rearing effects on offspring size to be separated [59](#). In 2014-2015 a mixture  
344 of full and partial cross-fostering was used as part of a separate experiment. Full details of  
345 cross-fostering can be found in [103](#). After egg laying was complete, nests were left undisturbed  
346 for 11 days and then checked daily for hatching. At hatching (day 0), all chicks were uniquely  
347 marked (within a nest). The chicks had blood samples taken at day 3 and were given a unique  
348 metal ring at day 9. At day 15, chick's tarsus, wing and head-bill lengths were measured and  
349 they were weighed. For the morphometric measurements, one chick from each nest was  
350 measured twice in order to account for measurement error [59](#). From day 10, adults were

351 caught at the nest in order to identify them; blood samples and morphometric measurements  
352 were taken and the birds were uniquely ringed. At the end of the season we checked all  
353 nests and recorded any dead chicks left in the nest. From this we could infer which chicks  
354 fledged. Chicks were considered recruited if they were recaptured as breeders in subsequent  
355 years. Permission to monitor, catch and ring the birds was given by Scottish Natural Heritage  
356 and the British Trust for Ornithology and permission to take blood samples was granted by  
357 the UK Government's Home Office. All permission and licenses were granted to JDH.

358 Social parentage was assigned through catching parents at the nest. When no female was  
359 caught, the social female was assigned a dummy mother identity. When no male was caught,  
360 the social father was assigned as the genetic sire with the largest proportion of paternity in  
361 a nest, either a male caught at a different nest that year, or an unsampled male assigned a  
362 dummy identity.

363 For the assignment of genetic parentage and chick sex, genotypes were obtained using blood  
364 and tissue samples from adults and chicks. Genotyping and pedigree reconstruction largely  
365 followed protocols outlined in<sup>59</sup> and<sup>102</sup>. However, adults not caught in the focal year but  
366 that were known to be alive (because they were caught in subsequent years and were aged 2  
367 years or over) were allowed to be parents of chicks in the focal year. The distance between  
368 the nest-of-origin of the chicks and the nest at which these candidate parents were caught in  
369 the subsequent year was fitted as a covariate. Mothers were allowed to be polygamous when  
370 (half) sib-ships were assigned to chicks with unknown fathers (see Supplementary Methods).  
371 When assigning chick sex, we used morphological sexing of recruits over molecular sexing from  
372 chicks (sexing didn't match for 5 chicks).

373 For our analysis we included data on chick size measured at day 15 post-hatching, collected on  
374 this project from 2011-2018, and additionally chick recruitment data from 2019 and 2020. We  
375 included all nests for which hatching date was known. Although similar morphological data  
376 was collected in 2010, we excluded all records from this year as egg size was not measured.  
377 Egg size was used to account for nest-of-origin effects in our models (see below). We also  
378 excluded data from an additional two nests where egg size was not measured, from chicks for  
379 which molecular sexing was not successful (n=20 chicks) and where we did not have one of  
380 the day 15 measurements (n=11 chicks). In total, we had records of 5123 day 15 chicks in  
381 715 nests, with 642 chicks repeatedly measured.

## 382 **Statistical analysis**

383 All models were run in a Bayesian framework. From all models posterior means and 95%  
384 credible intervals are presented. A p-value for the fixed effects and covariances in these  
385 models was approximated (pMCMC) as two times the smaller number of iterations where the  
386 parameter value is either less than zero or greater than zero<sup>104</sup>. We use a threshold of 0.005  
387 to refer to results as significant and those between 0.05 and 0.005 as suggestive<sup>105</sup>.

## 388 **Decomposing phenotypic skew using hierarchical models**

389 We modelled the four traits (tarsus length, head-bill length, mass and wing length) measured  
390 at day 15 using linear mixed effects models with sex (2 level factor), year (8 level factor), time  
391 of day (continuous - hours from midnight) and egg size (continuous) as fixed. Additive genetic

392 and nest-of-rearing effects were modelled as random. Because we have repeated measurements  
393 of tarsus, wing and head-bill lengths, we additionally modelled measurement error effects in  
394 these traits, by including bird identity effects, which are equivalent to the residuals in a model  
395 without repeat measures, and the residuals are measurement error effects<sup>59</sup>. In contrast to  
396 past analyses<sup>59;102</sup>, we do not model nest-of-origin effects but rather include egg size as a  
397 covariate to account for these effects (see<sup>59</sup> and Supplementary materials). As estimating  
398 skew-t distributed random effects (see below) is parameter heavy, including a covariate rather  
399 than a random effect is preferable, especially as nest-of-origin effects are very small for these  
400 traits<sup>59;102</sup>.

401 Skew due to the fixed effects was obtained by multiplying the fixed effect design matrix by the  
402 fixed effects and estimating the parameters for the skew-t distribution of the resulting variable.  
403 These were used when calculating the non-linear parent offspring regression and when plotting  
404 the sample skew. This method assumes that the joint distribution of the covariates is equal to  
405 the empirical distribution we observe. In combination with a diffuse prior on the fixed effects,  
406 this assumption probably leads to a small inflation in the estimated (absolute) skew. Time  
407 of day was excluded from this estimate as any skew induced by this is due to our sampling  
408 design rather than being biologically relevant.

409 In order to estimate skew in the random effects, we fitted random effects with skew-t  
410 distributions. The residuals for the repeat measured traits were treated as Gaussian as these  
411 represent measurement errors. As with the normal distribution, the skew-t distribution<sup>106–109</sup>  
412 has a location  $\xi$  and scale  $\omega$  parameter, but also parameters  $\delta$  and  $\nu$  which modify the skew and  
413 tailness, respectively. The distribution converges on a normal distribution when  $\delta = 0$  and  $\nu$   
414 approaches infinity. As  $\delta$  moves away from 0 and  $\nu$  decreases the (absolute) skew in a variable  
415 increases, with the sign of  $\delta$  signifying the direction of the skew. The skew-t distribution is  
416 unbounded and readily allows for considerable amounts of positive and negative skew. The  
417 reasons for the use of this distribution are further discussed in the supplementary materials.  
418 Our approach to modelling the additive genetic effects is to extend standard quantitative  
419 genetic models by allowing the base population breeding values to have a skew-t distribution,  
420 with normally distributed Mendelian sampling deviations in the descendants (with variance  
421  $\omega^2(1 - \bar{F})/2$  where  $\bar{F}$  is the average inbreeding coefficient of the individual's parents). This  
422 assumes that inheritance occurs under the Gaussian descendants infinitesimal model<sup>35;110</sup>; i.e.  
423 the Mendelian sampling deviations are normally distributed within families, and any genetic  
424 skew results from selection. In practice, however, the Mendelian sampling deviations are largely  
425 confounded with residual effects in our data because there are few parent-offspring comparisons  
426 (due to high migration and low recruitment) and so inferences are probably quite robust to  
427 any violation of the Gaussian descendants assumption. Initially we tried to fit this model in an  
428 animal model framework, but due to poor mixing we chose to approximate the model using  
429 a dam-sire model. This model discards information about the Mendelian-sampling deviations  
430 and subsumes them in the residual effects which then come from a mixture distribution<sup>111</sup>.  
431 Given there is little information in our data about the Mendelian-sampling deviations the  
432 dam-sire and animal models are expected to give almost identical answers (see Supplementary  
433 Materials). Although this method allows us to directly estimate skew in breeding values, when  
434 the environmental residuals are skew-t, as assumed here, the mixture distribution does not  
435 have standard form. Here, we approximate the mixture distribution as skew-t and although  
436 we cannot derive the full distribution of the environmental residuals we are able to obtain their

437 variance and skew. These models provided little evidence for genetic skew in any trait and so  
438 we reverted to an animal model with normally distributed breeding values - the animal model  
439 approach having the advantage that the environmental residual skew can then be directly  
440 estimated. The dam and sire effects were modelled in a multi-membership model where the  
441 two sets of effects were constrained to having the same skew-t distribution.

442 Initially, we intended to model chick mass over ontogeny in a multivariate framework (see  
443 preregistration), as in previous studies of this population<sup>59;102</sup>. However, implementing the  
444 required multivariate skew-t models proved too challenging. Since there is strong directional  
445 selection on chick body mass throughout ontogeny<sup>59;102</sup>, our estimates of skew at day 15  
446 are likely underestimates as the univariate analysis used will fail to account for selective  
447 disappearance prior to day 15<sup>59;102</sup>. We also planned to have a global box-cox parameter  
448 in case there was a single transformation that would make everything linear and additive.  
449 However, given the problems we had with implementing more complex models, we chose not  
450 to include this additional complexity.

451 It should also be noted that estimates from these skew-t models seem to be more sensitive  
452 to unmodelled heteroskedasticity than standard Gaussian mixed effects models, even when  
453 skew exists, and this can lead to biased fixed effect and variance estimates. This led us  
454 to fit a reduced set of fixed effects compared with previous analyses<sup>59;102</sup> and outlined in  
455 our pre-registration (see Supplementary materials). To partly address this issue we also ran  
456 equivalent Gaussian models for all skew-t models, and present the results in the Supplementary  
457 materials. There were small differences between models but the results remain qualitatively  
458 the same (see SM; Figure S17, Tables S4-15).

459 These models were run using Stan (version 2.21.0)<sup>112</sup> using the cmdstanr package (Stan  
460 Development Team, 2019) in R (version 4). Four chains were run for each model with  
461 a warmup of 4000 iterations and 6000 iterations post-warmup, with the exception of the  
462 dam-sire wing length model which was run with a warmup of 5000 iterations and 10000  
463 iterations post-warmup. Convergence of individual chains was visually assessed, as well as  
464 ensuring that the Gelman–Rubin diagnostic (R-hat) across chains was less than 1.1<sup>113</sup>. We  
465 used diffuse normal priors for fixed effects (mean=0 and standard deviation=100), half-Cauchy  
466 priors (mean=0 and standard deviation=10) for standard deviations and uniform priors from  
467 -1 to 1 for  $\delta$  and 4 to 40 on  $\nu$ . The choice of priors is discussed further in the Supplementary  
468 materials.

## 469 **Non-Linear Parent-Offspring Regression**

470 The PO-regression function is defined as  $E[z_o|z]$  where  $z_o$  is the phenotype of offspring from a  
471 parent with phenotype  $z$ . Assuming random mating and environmental values in the offspring  
472 ( $e_o$ ) are independent of parental phenotypes this becomes  $\frac{1}{2}E[g|z] + \frac{1}{2}E[g] + E[e_o]$  under  
473 the Gaussian descendants assumption, where  $g$  is breeding value. Have  $\theta_g$  be the parameters  
474 of the breeding value distribution and  $\theta_e$  the parameters of the environmental distribution.  
475 Then,

$$E[g|z] = \frac{\int (z-e)p(z-e|\theta_g)p(e|\theta_e)de}{\int p(z-e|\theta_g)p(e|\theta_e)de} \quad (1)$$

476 The integrals have to be evaluated numerically, which is time consuming, and so the regression  
 477 function was evaluated at the posterior mean of the parameters from the skew-t animal models  
 478 to give  $E[z_o|z]$  for each trait (Figure 5). Also, note that in the presence of pre-breeding survival  
 479 selection, the term  $\frac{1}{2}E[g]$  in the intercept of the regression function should be replaced by  
 480  $\frac{1}{2}(E[g] + \Delta g)$  where  $\Delta g$  is the change in mean breeding value due to selection such that  
 481  $E[g] + \Delta g$  is the expected breeding value of the other parent<sup>14</sup>.

## 482 Selection on chick body mass

483 Given that we were not able to model chick body mass in a multivariate framework, we did  
 484 not model survival throughout ontogeny as originally planned (see preregistration), but rather  
 485 modelled survival from day 15 to fledging and fledging to recruitment. We modelled this as  
 486 an event history in a probit regression (binomial error distribution and probit link function)  
 487 including a quadratic effect of chick size at day 15 on both events, allowing us to model the  
 488 stabilising component of selection. These models accounted for measurement error in tarsus,  
 489 head-bill and wing lengths, using the repeated measurements of these traits. Originally we  
 490 planned to correct our measurements for time of day effects (see preregistration). However,  
 491 these effects proved to be very small and for most traits non-significant (see Supplementary  
 492 Results). We therefore decided not to add this extra complexity into our models.

493 Sex, day of hatching within the nest, year, clutch size, male presence, nest hatch date were  
 494 also included as fixed effects. All fixed effects were allowed to differ between the two events.  
 495 Finally we modelled the 2x2 covariance matrix of nest-of-rearing effects. This model was  
 496 run using Stan. Four chains were run for each model with 5000 iterations and a warmup of  
 497 2500 iterations with a thinning interval of 10. Convergence of chains was assessed as above.  
 498 Diffuse priors for fixed effects (mean=0 and standard deviation=100), half-Cauchy priors for  
 499 all standard deviations (mean=0 and standard deviation=10) and LKJ priors on correlations  
 500 with shape=2<sup>14</sup> were used.

## 501 The Individual Relative Fitness Function

502 Partitioning the linear predictors for each survival event (1: day 15 to fledging, 2: fledging to  
 503 recruitment) into a part due to the trait and a part due to remaining terms (denoted  $\eta$ ), and  
 504 assuming that the distribution of  $\eta^{(1)}$  and  $\eta^{(2)}$  are bivariate normal conditional on the trait  $z$ ,  
 505 then the absolute fitness function has the form:

$$W(z) = F_{MVN}(s|\Sigma) \quad (2)$$

506 where  $F_{MVN}$  is the multivariate normal cumulative density function in which the first argument  
 507 is the quantile to be evaluated and the second argument is the (co)variance of the variates  
 508 (the means are zero and are therefore not given). For event  $i$

$$s^{(i)} = E[\eta^{(i)}] + \frac{COV(\eta^{(i)}, z)}{\mu_2}(z - \mu) + \beta^{(i)}z + \frac{1}{2}\gamma^{(i)}z^2 \quad (3)$$

509 where  $\beta^{(i)}$  and  $\frac{1}{2}\gamma^{(i)}$  are the linear and quadratic effect of the trait on event  $i$ ,  $\mu$  is the trait  
 510 mean and  $\mu_i$  the  $i^{th}$  central moment of the phenotypic distribution.

$$\Sigma^{(i,j)} = COV(\eta^{(i)}, \eta^{(j)}) - \frac{COV(\eta^{(i)}, z)COV(\eta^{(j)}, z)}{\mu_2} + COV(u^{(i)}, u^{(j)}) + \delta^{(i,j)} \quad (4)$$

511 where  $u^{(i)}$  are the nest effects for event  $i$  and  $\delta^{(i,j)} = 1$  when  $i = j$  and represents the residual  
 512 variance.

513 The partial derivative of  $W(z)$  with respect to  $z$  is given by

$$\begin{aligned} \frac{\partial W(z)}{\partial z} = & f_N(s^{(1|2)} | \Sigma^{(1|2)}) \left( \frac{COV(\eta^{(1)}, z)^2}{\mu_2} + \beta^{(1)} + \gamma^{(1)}z - \frac{\Sigma^{(1,2)}}{\Sigma^{(2)}} \left( \frac{COV(\eta^{(2)}, z)^2}{\mu_2} + \beta^{(2)} + \gamma^{(2)}z \right) \right) \\ & F_N(s^{(2)} | \Sigma^{(2)}) + f_N(s^{(2)} | \Sigma^{(2)}) \left( \frac{COV(\eta^{(2)}, z)^2}{\mu_2} + \beta^{(2)} + \gamma^{(2)}z \right) F_N(s^{(1|2)} | \Sigma^{(1|2)}) \end{aligned} \quad (5)$$

514 where  $f_N$  and  $F_N$  are the density and cumulative density functions for a centred normal  
 515 distribution, and

$$s^{(1|2)} = s^{(1)} - \frac{\Sigma^{(1,2)}}{\Sigma^{(2)}} s^{(2)} \quad \Sigma^{(1|2)} = \Sigma^{(1)} - \frac{(\Sigma^{(1,2)})^2}{\Sigma^{(2)}} \quad (6)$$

516 Solving Equation 5 to find the stationary point(s), and therefore the optimal trait value, is  
 517 difficult. Instead we evaluated the derivative of Equation 5 at the minimum and maximum  
 518 observed trait value and assessed whether the derivative at the minimum is positive and  
 519 negative at the maximum. This condition implies an optimal trait value within the range of  
 520 observed trait values.

## 521 Selection Gradients

522 The Lande-Arnold method<sup>22</sup> for estimating the selection gradient is only robust to phenotypic  
 523 skew if the fitness function is quadratic and both the mean-centered trait value and its square  
 524 are fitted in the regression<sup>3;22</sup>. We therefore computed three selection gradients. Using the  
 525 notation in<sup>34</sup>, we calculated our best estimate of it<sup>115</sup>,

$$\beta = E \left[ \frac{\partial w(z)}{\partial z} \right] = \int \frac{\partial w(z)}{\partial z} p(z) dz \approx \frac{1}{n} \sum_{i=1}^n \frac{\partial w(z)}{\partial z} \Big|_{z_i} \quad (7)$$

526 where  $p(z)$  is the probability density function for  $z$ ,  $w(z)$  is the relative fitness function  
 527 obtained by dividing  $W(z)$  by mean fitness ( $E[W] = \int W(z)p(z)dz$ ) and  $z_i$  are the observed  
 528 trait values. Put simply, we calculated the mean partial derivative of individual fitness function  
 529 (from Equation 5) across our observed phenotypic distributions, divided by mean fitness.

530 The linear selection differential is defined as

$$S = \int zw(z)p(z)dz - \mu \approx \frac{1}{n} \sum_{i=1}^n z_i w(z_i) - \hat{\mu} \quad (8)$$



531 and the quadratic selection differential as

$$C = \int (z - \mu)^2 p(z) w(z) dz - \mu_2 \approx \frac{1}{n} \sum_{i=1}^n (z_i - \hat{\mu})^2 w(z_i) - \hat{\mu}_2 \quad (9)$$

532 From these we can calculate the expected linear regression coefficient from the Lande-Arnold  
533 method when only the linear term was fitted:

$$\hat{\beta}_1 = \frac{\hat{S}}{\hat{\mu}_2} \quad (10)$$

534 and the linear regression coefficient from the Lande-Arnold method when both the linear and  
535 quadratic term are fitted (Eq. 29.28a from<sup>3</sup>):

$$\hat{\beta}_2 = \frac{(\hat{\mu}_4 - \hat{\mu}_2^2)\hat{S} - \hat{\mu}_3\hat{C}}{\hat{\mu}_2(\hat{\mu}_4 - \hat{\mu}_2^2) - \hat{\mu}_3^2} \quad (11)$$

536 Selection cannot operate on between-sex differences in trait values (the average fitness of  
537 the two sexes is constrained to be equal) and we assume that selection does not operate on  
538 between-year differences in trait values (which might occur if juvenile size impacts on adult  
539 survival). We therefore estimated each  $\beta$  as the average of each sex by year combination  
540 (Figure 4 e-h), calculated across the posterior distribution of the survival model.

#### 541 **Response to Selection**

542 The extension of Lande's gradient equation to a non-normal distribution of genetic effects is  
543 (combining Equations 26 and 42 from<sup>11</sup>):

$$\Delta\mu = \sum_{j=1}^{\infty} K^{j+1}(g) \frac{1}{j!} \int \frac{\partial^j w(z)}{\partial z^j} p(z) dz \quad (12)$$

544 where  $K^j(x)$  denotes the  $j^{\text{th}}$  cumulant of  $x$ , which up to the third cumulant (skew) is

$$\Delta\mu = V_A E \left[ \frac{\partial w(z)}{\partial z} \right] + \frac{S_A}{2} E \left[ \frac{\partial^2 w(z)}{\partial z^2} \right] \quad (13)$$

545 where  $S_A$  is the skew in the additive genetic effects. When the distribution of additive genetic  
546 values is normal and/or the fitness function is linear, Equation 12 reduces to Lande's gradient  
547 equation

$$\Delta\mu = V_A E \left[ \frac{\partial w(z)}{\partial z} \right] = V_A \beta \quad (14)$$

548 since all cumulants  $> 2$  of the genetic distribution are zero.

## 549 Heritability

550 We compared how well our inferred non-linear PO-regression (Equation 1) performed at  
551 predicting offspring phenotype compared to linear single-parent mid-offspring regression. Using  
552 the 182 individuals (118 male and 64 female) that were measured as chicks at day 15 and  
553 survived to produce offspring that were also measured at day 15, we fitted a weighted (by  
554 family size) regression with our inferred non-linear PO-regression fitted as an offset. We then  
555 compared the fit of this model to an identical model but where the raw parental phenotype  
556 was also fitted as a covariate with a free parameter.

557 We then compared estimates of the heritability before and after selection ( $h_b^2$  and  $h_a^2$ , respectively).  
558 The heritability can be defined as the regression coefficient of a linear mid-PO-regression, and  
559 can be calculated before selection

$$h_b^2 = 2 \frac{COV(z_o, z)}{\mu_2} = \frac{V_A}{V_P} \quad (15)$$

560 or after selection

$$h_a^2 = 2 \frac{E[w(z)z_o z] - E[w(z)z_o]E[w(z)z]}{E[w(z)z^2] - E[w(z)z]^2} \quad (16)$$

561 The posterior distribution of  $h_b^2$  was evaluated directly, but the  $i^{th}$  posterior sample of  $h_a^2$   
562 was obtained by simulating  $10^4$  values of  $z$  and  $z_o$  using the parameters sampled at the  
563  $i^{th}$  iteration of the trait model, calculating expected fitness for each sampled  $z$  using the  
564 parameters sampled at the  $i^{th}$  iteration of the fitness model, and then evaluating the relevant  
565 expectations.

## 566 Simulations

567 To test how different sampling designs and standard estimation procedures (PO-regression  
568 and Gaussian animal model) impact estimates of heritability in the presence of skew and  
569 selection, we simulated data according to the posterior mean of the parameters from our  
570 skew-t quantitative genetic and selection models for mass. A closed population with 1000  
571 breeding pairs was simulated over three generations, with 10 measured full-sib offspring per  
572 pair. Four scenarios were simulated: either nests were not cross-fostered or they were paired  
573 and five offspring reciprocally crossed, and the random effects were either skew t-distributed  
574 (with  $\omega$ ,  $\delta$  and  $\nu$  parameters set to their posterior means) or they were normally distributed but  
575 with matching variance. The probability of a chick recruiting to be a parent was obtained by  
576 applying the estimated survival model for chick mass to the simulated phenotype. Each of the  
577 four scenarios were simulated 2000 times and for each data set the heritability was estimated  
578 directly using PO-regression and as the estimate of the additive genetic variance over the sum  
579 of all variances estimated from a Gaussian animal model fitted in ASReml-R<sup>116</sup>.

## 580 Data availability

581 All data and code can be found at <https://doi.org/10.5281/zenodo.5794316>.

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## 589 **Author contributions**

590 JLP and JDH conceived and designed the project. JLP, HEL, CET and JDH generated the  
591 data. JLP and JDH analysed the data and wrote the paper. All authors have read and  
592 approved the paper.

## 593 **Competing interests**

594 The authors declare no competing interests

## 595 **References**

- 596 [1] Lush, J. L. *Animal Breeding Plans* (Iowa State College Press, Ames, Iowa, 1937).
- 597 [2] Lande, R. Natural selection and random genetic drift in phenotypic evolution. *Evolution*  
598 **30**, 314–334 (1976).
- 599 [3] Walsh, B. & Lynch, M. *Evolution and Selection of Quantitative Traits* (Oxford  
600 University Press, Oxford, UK, 2018).
- 601 [4] Schluter, D. Estimating the Form of Natural Selection on a Quantitative Trait. *Evolution*  
602 **42**, 849–861 (1988).
- 603 [5] Morrissey, M. B. & Sakrejda, K. Unification Of Regression-Based Methods For The  
604 Analysis Of Natural Selection. *Evolution* **67**, 2094–2100 (2013).
- 605 [6] Falconer, D. & Mackay, T. F. *Introduction to Quantitative Genetics* (Longman, New  
606 York, 1996), 4th edn.
- 607 [7] Lynch, M. & Walsh, B. *Genetics and Analysis of Quantitative Traits* (Sinauer  
608 Associates, Inc., Sunderland, MA, 1998).
- 609 [8] Roff, D. A. *Evolutionary quantitative genetics* (Springer Science & Business Media,  
610 2012).
- 611 [9] Nishida, A. & Abe, T. Non-Linear Heritability and Asymmetrical Selection Responses  
612 caused by Skewed Distribution of Breeding Value in Selected Population. *Japanese*  
613 *Journal of Zootechnical Science* **51**, 495–500 (1980).

- 614 [10] Barton, N. H. & Turelli, M. Adaptive landscapes, genetic distance and the evolution of  
615 quantitative characters. *Genetical Research* **49**, 157–173 (1987).
- 616 [11] Turelli, M. & Barton, N. H. Genetic and statistical analyses of strong selection on  
617 polygenic traits: What, me normal? *Genetics* **138**, 913–941 (1994).
- 618 [12] Gimelfarb, A. & Willis, J. H. Linearity versus nonlinearity of offspring-parent regression:  
619 An experimental study of *Drosophila melanogaster*. *Genetics* **138**, 343–352 (1994).
- 620 [13] Rice, S. H. *Evolutionary theory: mathematical and conceptual foundations* (Sinauer  
621 Associates, 2004).
- 622 [14] Heywood, J. S. An exact form of the breeder's equation for the evolution of a  
623 quantitative trait under natural selection. *Evolution* **59**, 2287–2298 (2005).
- 624 [15] Jones, A. G., Bürger, R., Arnold, S. J., Hohenlohe, P. A. & Uyeda, J. C. The effects  
625 of stochastic and episodic movement of the optimum on the evolution of the G-matrix  
626 and the response of the trait mean to selection. *Journal of Evolutionary Biology* **25**,  
627 2210–2231 (2012).
- 628 [16] Urban, M. C., Bürger, R. & Bolnick, D. I. Asymmetric selection and the evolution of  
629 extraordinary defences. *Nature Communications* **4** (2013).
- 630 [17] Bonamour, S., Teplitsky, C., Charmantier, A., Crochet, P. A. & Chevin, L. M. Selection  
631 on skewed characters and the paradox of stasis. *Evolution* **71**, 2703–2713 (2017).
- 632 [18] Jacquard, A. Heritability: one word, three concepts. *Biometrics* 465–477 (1983).
- 633 [19] Charlesworth, B. The Heritability of Fitness. In Bradbury, J. & Andersson, M. (eds.)  
634 *Sexual Selection: Testing the Alternatives*, 21–40 (John Wiley & Sons Limited, 1987).
- 635 [20] Nishida, A. & Abe, T. Distribution of Genetic and Environmental Effects and Linearity  
636 of Heritability. *Canadian Journal of Genetics and Cytology* **16**, 3–10 (1974).
- 637 [21] Robertson, A. The non-linearity of offspring-parent regression. In Pollak, E.,  
638 Kempthorne, . & Bailey Jr, T. B. (eds.) *Proceedings of the International Conference*  
639 *on Quantitative Genetics*, 297–304 (Iowa State University Press, Ames, 1977).
- 640 [22] Lande, R. & Arnold, S. J. . The Measurement of Selection on Correlated Characters.  
641 *Evolution* **37**, 1210–1226 (1983).
- 642 [23] Beardsley, J. P., Bratton, R. & Salisbury, G. The Curvilinearity of Heritability of  
643 Butterfat Production. *Journal of Dairy Science* **33**, 93–97 (1950).
- 644 [24] Nishida, A. Some Characteristics of Parent-Offspring Regression in Body-Weight of  
645 Mus-Musculus at Different Ages. *Canadian Journal of Genetics and Cytology* **14**,  
646 293–303 (1972).
- 647 [25] Mäki-Tanila, A. *The Validity of the Heritability Concept in Quantitative Genetics*. Ph.D.  
648 thesis, University of Edinburgh (1982).
- 649 [26] Gifford, D. R. & Barker, J. S. The nonlinearity of offspring-parent regression for  
650 total sternopleural bristle number of *Drosophila melanogaster*. *Theoretical and Applied*  
651 *Genetics* **82**, 217–220 (1991).

- 652 [27] Koerhuis, A. N. Non-normality of egg production distributions in poultry and the  
653 effects of outlier elimination and transformation on size and curvilinearity of heritability.  
654 *Livestock Production Science* **45**, 69–85 (1996).
- 655 [28] Mbagha, S. H. & Hill, W. G. Linear versus nonlinear offspring-parent regression in  
656 unselected random-bred mice. *Journal of Animal Breeding and Genetics* **114**, 299–307  
657 (1997). URL <http://www.ncbi.nlm.nih.gov/pubmed/21395825>.
- 658 [29] McGuigan, K., Van Homrigh, A. & Blows, M. W. Genetic analysis of female preference  
659 functions as function-valued traits. *The American Naturalist* **172**, 194–202 (2008).
- 660 [30] Reid, J. M. *et al.* Immigration counter-acts local micro-evolution of a major fitness  
661 component: Migration-selection balance in free-living song sparrows. *Evolution Letters*  
662 **5**, 48–60 (2021).
- 663 [31] Rollinson, N. & Rowe, L. Persistent directional selection on body size and a resolution  
664 to the paradox of stasis. *Evolution* **69**, 2441–2451 (2015).
- 665 [32] Merilä, J., Sheldon, B. & Kruuk, L. Explaining stasis: microevolutionary studies in  
666 natural populations. *Genetica* **112**, 199–222 (2001).
- 667 [33] Kingsolver, J. G. *et al.* The strength of phenotypic selection in natural populations.  
668 *The American Naturalist* **157**, 245–261 (2001).
- 669 [34] Geyer, C. J. & Shaw, R. G. Commentary on Lande-Arnold analysis. technical report no.  
670 670 (2008).
- 671 [35] Turelli, M. Commentary: Fisher's infinitesimal model: A story for the ages. *Theoretical*  
672 *Population Biology* **118**, 46–49 (2017).
- 673 [36] Bulmer, M. G. *The Mathematical Theory of Quantitative Genetics* (Oxford University  
674 Press, Oxford, UK, 1980).
- 675 [37] Turelli, M. & Barton, N. H. Dynamics of polygenic characters under selection.  
676 *Theoretical Population Biology* **38**, 1–57 (1990).
- 677 [38] Zeng, Z. B. Genotypic distribution at the limits to natural and artificial selection with  
678 mutation. *Theoretical Population Biology* **32**, 90–113 (1987).
- 679 [39] Keightley, P. D. & Hill, W. G. Directional selection and variation in finite populations.  
680 *Genetics* **117**, 573–582 (1987).
- 681 [40] Reed, D. R., Lawler, M. P. & Tordoff, M. G. Reduced body weight is a common effect  
682 of gene knockout in mice. *BMC Genetics* **9** (2008).
- 683 [41] Fisher, R. A., Immer, F. R. & Tedin, O. The genetical interpretation of statistics of the  
684 third degree in the study of quantitative inheritance. *Genetics* **17**, 107–124 (1932).
- 685 [42] Gimelfarb, A. Offspring-parent genotypic regression: how linear is it? *Biometrics* **42**,  
686 67–71 (1986).
- 687 [43] Santure, A. W. *et al.* Replicated analysis of the genetic architecture of quantitative  
688 traits in two wild great tit populations. *Molecular Ecology* **24**, 6148–6162 (2015). URL  
689 <https://onlinelibrary.wiley.com/doi/abs/10.1111/mec.13452>.

- 690 [44] Silva, C. N. S. *et al.* Insights into the genetic architecture of morphological traits in two  
691 passerine bird species. *Heredity* **119**, 197–205 (2017). URL <https://www.nature.com/articles/hdy201729>.  
692
- 693 [45] Becker, P. J., Hegelbach, J., Keller, L. F. & Postma, E. Phenotype-associated  
694 inbreeding biases estimates of inbreeding depression in a wild bird population. *Journal*  
695 *of Evolutionary Biology* **29**, 35–46 (2016).
- 696 [46] Huisman, J., Kruuk, L. E., Ellisa, P. A., Clutton-Brock, T. & Pemberton, J. M.  
697 Inbreeding depression across the lifespan in a wild mammal population. *Proceedings of*  
698 *the National Academy of Sciences of the United States of America* **113**, 3585–3590  
699 (2016).
- 700 [47] Pemberton, J. M., Ellis, P. E., Pilkington, J. G. & Bérénos, C. Inbreeding depression  
701 by environment interactions in a free-living mammal population. *Heredity* **118**, 64–77  
702 (2017).
- 703 [48] Hajduk, G. K. *et al.* Inbreeding, inbreeding depression, and infidelity in a cooperatively  
704 breeding bird\*. *Evolution* **72**, 1500–1514 (2018).
- 705 [49] Rausher, M. D. The measurement of selection on quantitative traits: biases due to  
706 environmental covariances between traits and fitness. *Evolution* **46**, 616–626 (1992).
- 707 [50] Alatalo, R. V., Gustafsson, L. & Lundberg, A. Phenotypic Selection on Heritable  
708 Size Traits: Environmental Variance and Genetic Response. *The American Naturalist*  
709 **135**, 464–471 (1990). URL [https://www.journals.uchicago.edu/doi/10.1086/](https://www.journals.uchicago.edu/doi/10.1086/285056)  
710 [285056](https://www.journals.uchicago.edu/doi/10.1086/285056).
- 711 [51] Koyama, H. Intraspecific competition among higher plants. VIII. Frequency distribution  
712 of individual plant weight as affected by the interaction between plants. *J Inst Polytech*  
713 *Osaka Cy University* **7**, 73–94 (1956).
- 714 [52] Mock, D. W. & Parker, G. A. *The Evolution of Sibling Rivalry* (Oxford University Press,  
715 Oxford, UK, 1997).
- 716 [53] Weiner, J. Asymmetric competition in plant populations (1990).
- 717 [54] Bassar, R. D. *et al.* The effects of asymmetric competition on the life history of  
718 Trinidadian guppies. *Ecology Letters* **19**, 268–278 (2016).
- 719 [55] Muller, M. & Groothuis, T. G. Within-clutch variation in yolk testosterone as an  
720 adaptive maternal effect to modulate avian sibling competition: evidence from a  
721 comparative study. *The American Naturalist* **181**, 125–136 (2013).
- 722 [56] Nilsson, J.-A. & Svensson, M. Sibling competition affects nestling growth strategies in  
723 marsh tits. *Journal of Animal Ecology* 825–836 (1996).
- 724 [57] Gebhardt-Henrich, S. & Van Noordwijk, A. The genetical ecology of nestling growth  
725 in the great tit. environmental influences on the expression of genetic variances during  
726 growth. *Functional ecology* 469–476 (1994).
- 727 [58] Gebhardt-Henrich, S. Heritability of growth curve parameters and heritability of final  
728 size: a simulation study. *Growth, development, and aging: GDA* **56**, 23–33 (1992).

- 729 [59] Hadfield, J. D., Heap, E. A., Bayer, F., Mittell, E. A. & Crouch, N. M. A. Disentangling  
730 genetic and prenatal sources of familial resemblance across ontogeny in a wild passerine.  
731 *Evolution* **67**, 2701–13 (2013).
- 732 [60] Hadfield, J. D. Estimating evolutionary parameters when viability selection is operating.  
733 *Proceedings of the Royal Society B: Biological Sciences* **275**, 723–734 (2008).
- 734 [61] Postma, E. Four decades of estimating heritabilities in wild vertebrate populations:  
735 Improved methods, more data, better estimates? In Charmantier, A., Garant, D. &  
736 Kruuk, L. E. B. (eds.) *Quantitative Genetics in the Wild*, 16–33 (Oxford University  
737 Press, Oxford, 2014).
- 738 [62] Dingemanse, N. J., Araya-Ajoy, Y. G. & Westneat, D. F. Most published selection  
739 gradients are underestimated: Why this is and how to fix it. *Evolution* 1–13 (2021).
- 740 [63] Arct, A., Drobniak, S., Mellinger, S., Gustafsson, L. & Cichon, M. Data from:  
741 Parental genetic similarity and offspring performance in blue tits in relation to brood  
742 size manipulation (2020). URL <https://doi.org/10.5061/dryad.v6r0758>.
- 743 [64] Bebbington, K. *et al.* Data from: Consequences of sibling rivalry vary across life in a  
744 passerine bird (2016). URL <https://doi.org/10.5061/dryad.12np0>.
- 745 [65] Bebbington, K. *et al.* Data from: Telomere length reveals cumulative individual and  
746 transgenerational inbreeding effects in a passerine bird (2016). URL <https://doi.org/10.5061/dryad.52fp4>.
- 747
- 748 [66] Becker, P. J. J. *et al.* Data from: Mother-offspring and nest mate resemblance but no  
749 heritability in early-life telomere length in white-throated dippers (2015). URL <https://doi.org/10.5061/dryad.b2v37>.
- 750
- 751 [67] Berzins, L. L., Gilchrist, H. G. & Burness, G. Data from: No assortative mating  
752 based on size in black guillemots breeding in the Canadian Arctic (2015). URL <https://doi.org/10.5061/dryad.1bm5t>.
- 753
- 754 [68] Caizergues, A. E., Gregoire, A. & Charmantier, A. Data from: Urban versus forest  
755 ecotypes are not explained by divergent reproductive selection (2018). URL <https://doi.org/10.5061/dryad.tv45802>.
- 756
- 757 [69] Camacho, C., Canal, D. & Potti, J. Data from: Nonrandom dispersal drives phenotypic  
758 divergence within a bird population (2014). URL <https://doi.org/10.5061/dryad.h22n9>.
- 759
- 760 [70] Class, B. & Brommer, J. Data from: Can dominance genetic variance be ignored in  
761 evolutionary quantitative genetic analyses of wild populations? (2020). URL <https://doi.org/10.5061/dryad.zpc866t6d>.
- 762
- 763 [71] Cornell, A., Gibson, K. F. & Williams, T. D. Data from: Physiological maturity at a  
764 critical life-history transition and flight ability at fledging (2017). URL <https://doi.org/10.5061/dryad.c2n66>.
- 765
- 766 [72] Cox, A. R., Robertson, R. J., Lendvai, A. Z., Everitt, K. & Bonier, F. Data from: Rainy

- 767 springs linked to poor nestling growth in a declining avian aerial insectivore (*Tachycineta*  
768 *bicolor*) (2019). URL <https://doi.org/10.5061/dryad.7m41jd8>.
- 769 [73] DeSimone, J. G., Clotfelter, E. D., Black, E. C. & Knutie, S. A. Data from: Avoidance,  
770 tolerance, and resistance to ectoparasites in nestling and adult tree swallows (2017).  
771 URL <https://doi.org/10.5061/dryad.9bb60>.
- 772 [74] Dubuc-Messier, G. *et al.* Data from: Gene flow does not prevent personality and  
773 morphological differentiation between two blue tit populations (2018). URL <https://doi.org/10.5061/dryad.31tc3s8>.  
774
- 775 [75] Grunst, M. L., Raap, T., Grunst, A. S., Pinxten, R. & Eens, M. Data from: Artificial  
776 light at night does not affect telomere shortening in a developing free-living songbird:  
777 a field experiment (2019). URL <https://doi.org/10.5061/dryad.8216g63>.
- 778 [76] Husby, A., Schielzeth, H., Forstmeier, W., Gustafsson, L. & Qvarnström, A. Data  
779 from: Sex chromosome linked genetic variance and the evolution of sexual dimorphism  
780 of quantitative traits (2012). URL <https://doi.org/10.5061/dryad.451n7>.
- 781 [77] Ihle, M. *et al.* Data from: Rearing Success Does Not Improve With Apparent Pair  
782 Coordination in Offspring Provisioning (2019). URL [https://zenodo.org/record/  
783 3459642](https://zenodo.org/record/3459642).
- 784 [78] Jacob, S. *et al.* Data from: Microbiome affects egg carotenoid investment, nestling  
785 development and adult oxidative costs of reproduction in Great tits (2015). URL <https://doi.org/10.5061/dryad.9n741>.  
786
- 787 [79] Krause, E. T., Krüger, O. & Schielzeth, H. Data from: Long-term effects of early  
788 nutrition and environmental matching on developmental and personality traits in zebra  
789 finches (2018). URL <https://doi.org/10.5061/dryad.6j700>.
- 790 [80] Krist, M., Janča, M., Edme, A. & Dzuro, R. Data from: Are prenatal maternal resources  
791 more important in competitive than in benign postnatal environments? (2016). URL  
792 <https://doi.org/10.5061/dryad.823f0>.
- 793 [81] Krist, M., Remeš, V., Uvírová, L., Nádvorník, P. & Bureš, S. Data from: Egg size and  
794 offspring performance in the collared flycatcher (*Ficedula albicollis*): a within-clutch  
795 approach (2010). URL <https://doi.org/10.5061/dryad.1758>.
- 796 [82] Kvalnes, T. *et al.* Data from: Offspring fitness and the optimal propagule size in a  
797 fluctuating environment (2018). URL <https://doi.org/10.5061/dryad.m74c7m9>.
- 798 [83] Kvalnes, T. *et al.* Data from: Reversal of response to artificial selection on body size  
799 in a wild passerine (2017). URL <https://doi.org/10.5061/dryad.v50r8>.
- 800 [84] Moiron, M. *et al.* Data from: Functional relations between body mass and risk-taking  
801 behavior in wild great tits (2018). URL <https://doi.org/10.5061/dryad.14cn58v>.
- 802 [85] Nishida, Y. & Takagi, M. Data from: Song performance is a condition-dependent  
803 dynamic trait honestly indicating the quality of paternal care in the Bull-headed Shrike  
804 (2018). URL <https://doi.org/10.5061/dryad.c84f7c4>.



- 805 [86] Nord, A. & Nilsson, J.-A. Data from: Incubation temperature affects growth and  
806 energy metabolism in blue tit nestlings (2011). URL [https://doi.org/10.5061/  
807 dryad.jb314](https://doi.org/10.5061/dryad.jb314).
- 808 [87] Pap, P. L. *et al.* Data from: Selection on multiple sexual signals in two Central- and  
809 Eastern-European populations of the barn swallow (2019). URL [https://doi.org/  
810 10.5061/dryad.64p7k2f](https://doi.org/10.5061/dryad.64p7k2f).
- 811 [88] Perrier, C., Delahaie, B. & Charmantier, A. Data from: Heritability estimates from  
812 genome wide relatedness matrices in wild populations: application to a passerine, using  
813 a small sample size (2018). URL <https://doi.org/10.5061/dryad.k6r1mk8>.
- 814 [89] Podofillini, S. *et al.* Data from: Benefits of extra food to reproduction depend on  
815 maternal condition (2020). URL <https://doi.org/10.5061/dryad.5db0168>.
- 816 [90] Poissant, J., Morrissey, M. B., Gosler, A. G., Slate, J. & Sheldon, B. C. Data from:  
817 Multivariate selection and intersexual genetic constraints in a wild bird population  
818 (2016). URL <https://doi.org/10.5061/dryad.qt745>.
- 819 [91] Poorboy, D. *et al.* Data from: Experimental cross-fostering of eggs reveals effects of  
820 territory quality on reproductive allocation (2018). URL [https://doi.org/10.5061/  
821 dryad.h8v8157](https://doi.org/10.5061/dryad.h8v8157).
- 822 [92] Rioux Paquette, S., Pelletier, F., Garant, D. & Bélisle, M. Data from: Severe recent  
823 decrease of adult body mass in a declining insectivorous bird population (2014). URL  
824 <https://doi.org/10.5061/dryad.67t23>.
- 825 [93] Sakaluk, S. K. *et al.* Data from: Genetic and environmental variation in condition,  
826 cutaneous immunity, and haematocrit in house wrens (2014). URL [https://doi.  
827 org/10.5061/dryad.jk2m0](https://doi.org/10.5061/dryad.jk2m0).
- 828 [94] Simpson, R. K. & McGraw, K. J. Data from: Multiple signaling in a variable  
829 environment: expression of song and color traits as a function of ambient sound and  
830 light (2017). URL <https://doi.org/10.5061/dryad.1j81k>.
- 831 [95] Song, Z. *et al.* Data from: Silver spoon effects of hatching order in an asynchronous  
832 hatching bird (2018). URL <https://doi.org/10.5061/dryad.184c1dj>.
- 833 [96] Torres, R., Chin, E., Rampton, R. & Williams, T. D. Data from: Are there synergistic  
834 or antagonistic effects of multiple maternally-derived egg components (antibodies and  
835 testosterone) on offspring phenotype? (2019). URL [https://doi.org/10.5061/  
836 dryad.j348s75](https://doi.org/10.5061/dryad.j348s75).
- 837 [97] Vermeulen, A., Müller, W. & Eens, M. Data from: Vitally important – does early  
838 innate immunity predict recruitment and adult innate immunity? (2016). URL [https://  
839 //doi.org/10.5061/dryad.p0s3g](https://doi.org/10.5061/dryad.p0s3g).
- 840 [98] Weber, B. M. *et al.* Data from: Pre- and post-natal effects of experimentally  
841 manipulated maternal corticosterone on growth, stress reactivity, and survival of nestling  
842 house wrens (2019). URL <https://doi.org/10.5061/dryad.16049f4>.
- 843 [99] Santiago-Alarcon, D. & Parker, P. G. Sexual Size Dimorphism and Morphological

- 844 Evidence Supporting the Recognition of two Subspecies in the Galápagos Dove. *The*  
845 *Condor* **109**, 132–141 (2007).
- 846 [100] Santos, E. S. A. & Nakagawa, S. Breeding Biology and Variable Mating System of a  
847 Population of Introduced Dunnocks (*Prunella modularis*) in New Zealand. *PLOS ONE*  
848 **8**, e69329 (2013).
- 849 [101] Joanes, D. N. & Gill, C. A. Comparing measures of sample skewness and kurtosis.  
850 *Journal of the Royal Statistical Society: Series D (The Statistician)* **47**, 183–189 (1998).  
851 URL <https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-9884.00122>.
- 852 [102] Thomson, C. E. *et al.* Selection on parental performance opposes selection for larger  
853 body size in a wild population of blue tits. *Evolution* **71**, 716–732 (2017).
- 854 [103] Thomson, C. E. & Hadfield, J. D. No evidence for sibling or parent–offspring  
855 coadaptation in a wild population of blue tits, despite high power. *Evolution* **73**, 28–41  
856 (2019).
- 857 [104] Baayen, R. H., Davidson, D. J. & Bates, D. M. Mixed-effects modeling with crossed  
858 random effects for subjects and items. *Journal of memory and language* **59**, 390–412  
859 (2008).
- 860 [105] Benjamin, D. J. *et al.* Redefine statistical significance. *Nature Human Behaviour* **2**,  
861 6–10 (2018).
- 862 [106] Branco, M. D. & Dey, D. K. A general class of multivariate skew-elliptical distributions.  
863 *Journal of Multivariate Analysis* **79**, 99–113 (2001).
- 864 [107] Azzalini, A. & Capitanio, A. Distributions generated by perturbation of symmetry with  
865 emphasis on a multivariate skew t-distribution. *Journal of the Royal Statistical Society.*  
866 *Series B: Statistical Methodology* **65**, 367–389 (2003).
- 867 [108] Arellano-Valle, R. B. & Azzalini, A. On the Unification of Families of Skew-normal  
868 Distributions. *Scandinavian Journal of Statistics* **33**, 561–574 (2006).
- 869 [109] Azzalini, A. *The Skew-Normal and Related Families*. Institute of Mathematical Statistics  
870 Monographs (Cambridge University Press, 2013).
- 871 [110] Barton, N. H., Etheridge, A. M. & Véber, A. The infinitesimal model: Definition,  
872 derivation, and implications. *Theoretical Population Biology* **118**, 50–73 (2017).
- 873 [111] Quaas, R. L. & Pollak, E. J. Mixed model methodology for farm and ranch beef cattle  
874 testing programs. *journal of Animal Science* **51**, 1277–1287 (1980).
- 875 [112] Carpenter, B. *et al.* Stan: A probabilistic programming language. *Journal of Statistical*  
876 *Software* **76** (2017).
- 877 [113] Gelman, A., Rubin, D. B. *et al.* Inference from iterative simulation using multiple  
878 sequences. *Statistical science* **7**, 457–472 (1992).
- 879 [114] Lewandowski, D., Kurowicka, D. & Joe, H. Generating random correlation matrices  
880 based on vines and extended onion method. *Journal of multivariate analysis* **100**,  
881 1989–2001 (2009).

- 882 [115] Janzen, F. J. & Stern, H. S. Logistic regression for empirical studies of multivariate  
883 selection. *Evolution* **52**, 1564–1571 (1998).
- 884 [116] Butler, D., Cullis, B. R., Gilmour, A. R., Gogel, B. J. & Thompson, R. *ASReml-R*  
885 *Reference Manual Version 4* (VSN International Ltd, Hemel Hempstead, UK, 2017).

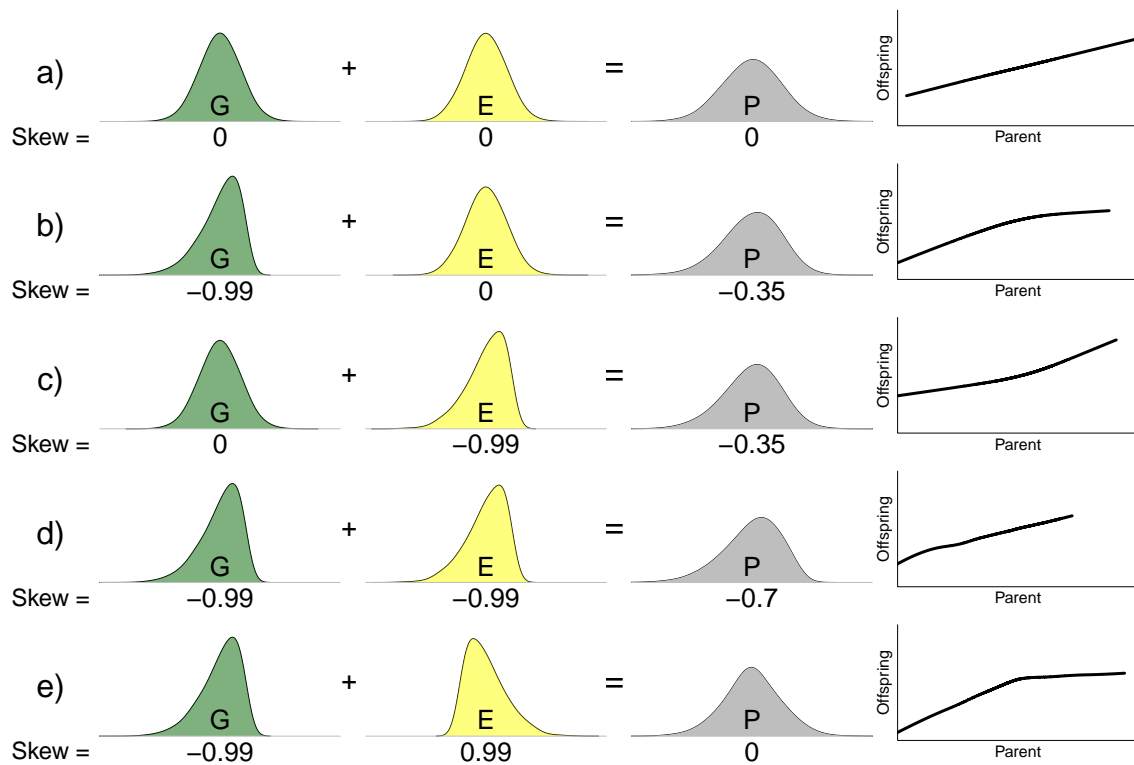


Figure 1: The effects of different distributions of breeding values (G) and environmental values (E) on the distribution of phenotypes (P) and the shape of the PO-regression. When both genetic and environmental values are normally distributed (a), as typically assumed, there is a linear PO-regression. Negative genetic (b) and environmental (c) skew affect the shape of the parent-offspring relationship in opposite directions, whilst inducing the same phenotypic skew. If genetic and environmental distributions are skewed in the same direction (d) their effects on the parent-offspring relationship can cancel each other out, giving a linear parent-offspring relationship, despite considerable phenotypic skew. If genetic and environmental are skewed in opposite directions (e), although they may cancel each other out at the phenotypic level, they induce a highly non-linear parent-offspring relationship. 1-5) are all simulated with a heritability ( $V_A/V_P$ ) of 0.5.

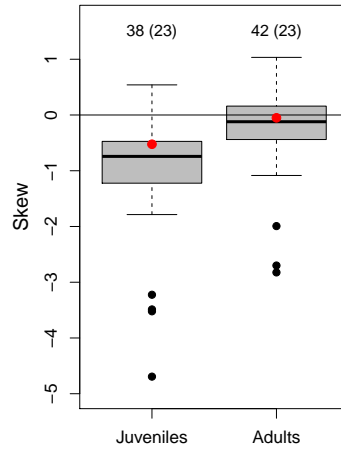


Figure 2: Skew in the distribution of avian tarsus lengths across different species, measured as the coefficient of skew. In the boxplots, the center line shows the median; box limits show upper and lower quartiles; whiskers show 1.5x interquartile range; points show outliers. Numbers above the plots show the number of estimates, and species in parenthesis. The red points show the skew in our blue tit data.

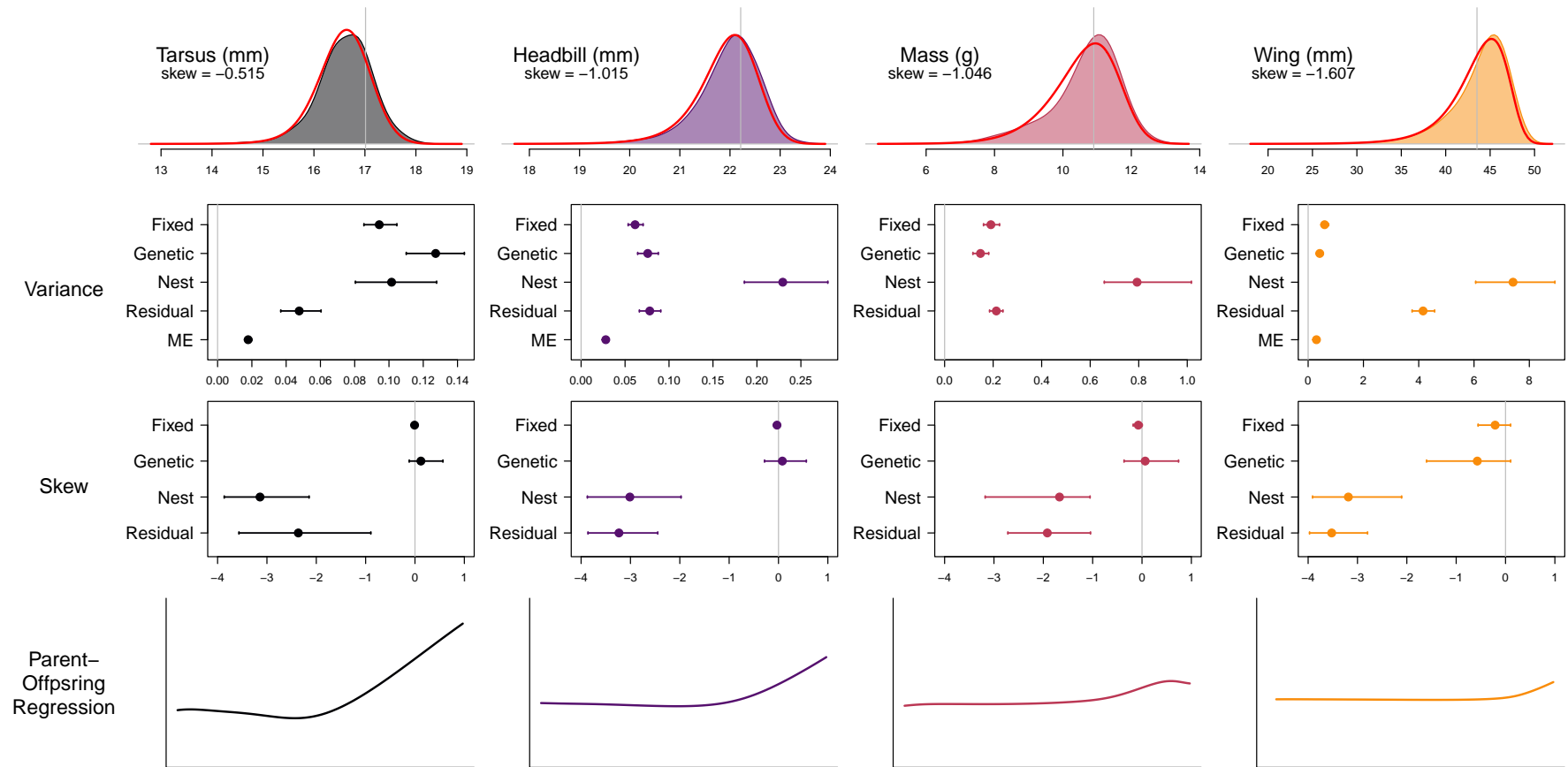


Figure 3: Decomposition of variance and skew in juvenile body size traits in blue tits. Top plots shows the phenotypic distribution of the traits, with the red line showing the distribution predicted from the skew models. The middle rows show the variance and skew (top and bottom, respectively) for each component for all four traits, with all model estimates coming from the skew-t animal model, except the genetic skew which was estimated in the skew-t dam-sire model (see methods). ME stands for measurement error. The bottom row shows the predicted shape of the PO-regression based on the model estimates.

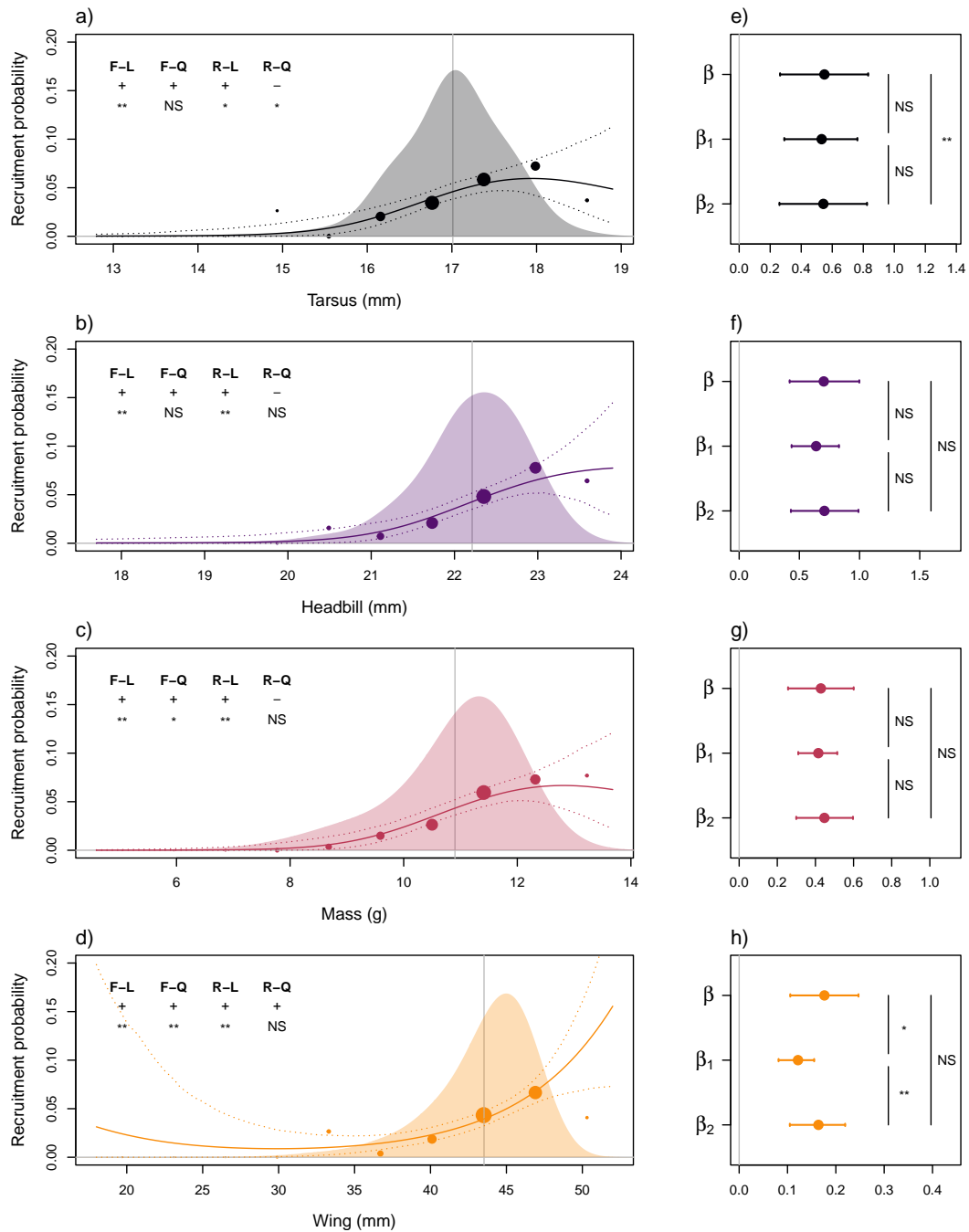


Figure 4: Average (over years and sexes) fitness functions (a-d) and selection gradients (e-h) for tarsus length, head-bill length, mass and wing length, respectively, from day 15 to recruitment. In plots a-d, solid lines show the posterior mean fitness functions, dotted lines show the 95% credible intervals, and points show the average survival of measured individuals from day 15 to recruitment in equally spaced intervals. The size of the points is proportional to the square root of the sample size. The phenotypic distribution of the traits is shown, with the grey vertical line showing the phenotypic mean. The direction and significance of the effect of the trait on fitness is also shown, 'F' and 'R' are survival from day 15 to fledging and from fledging to recruitment respectively, and 'L' and 'Q' and linear and quadratic effects. In plots e-h,  $\beta$  refers to the selection gradient derived from this fitness function,  $\beta_1$  and  $\beta_2$  refer to the approximations from the Lande-Arnold regression excluding and including a quadratic term, respectively. In all plots 'NS' indicates  $p > 0.05$ , '\*' indicates  $0.05 > p > 0.005$  and '\*\*'  $p < 0.005$ .

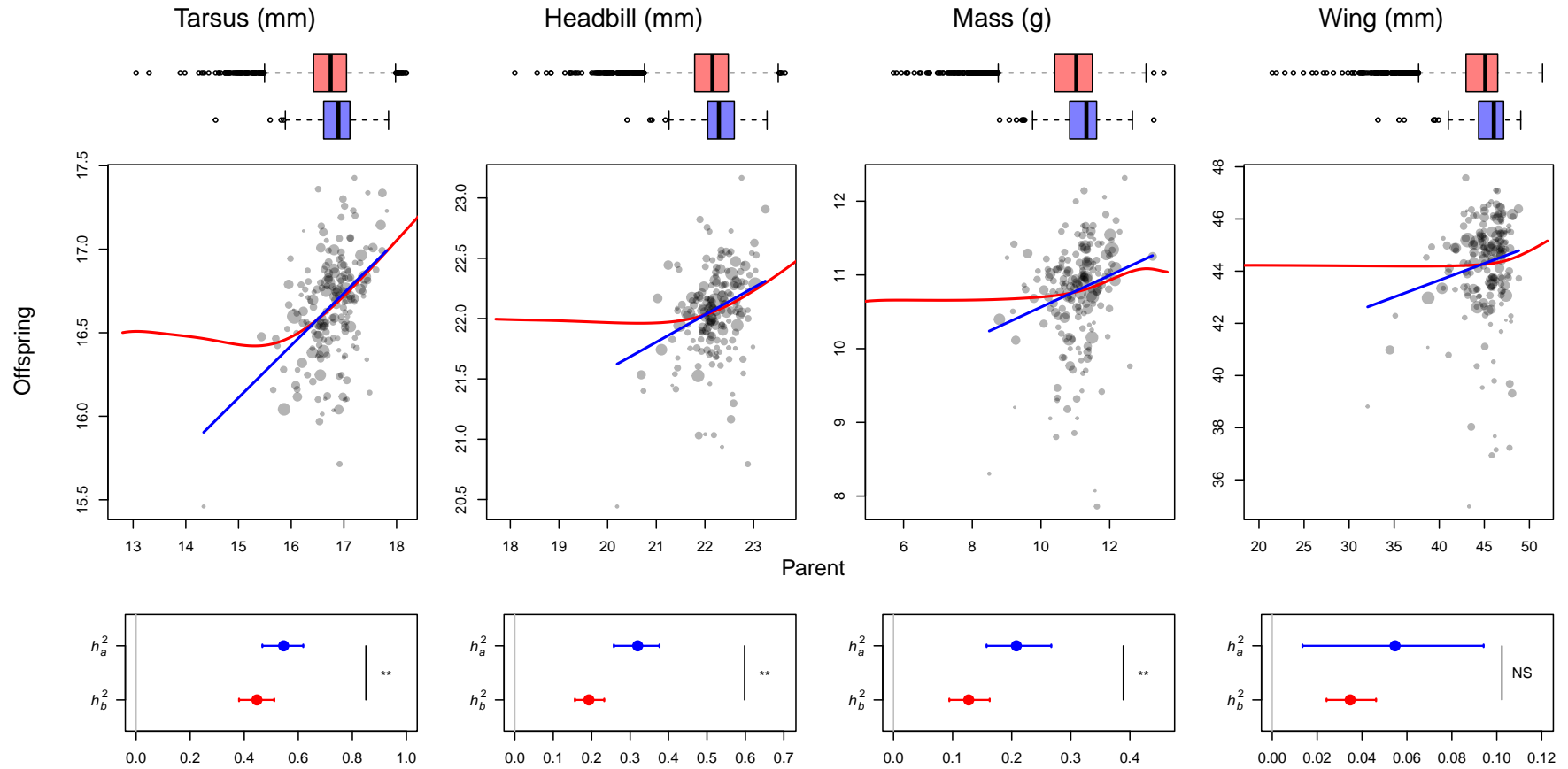


Figure 5: PO-regressions for four body size traits. Top panels show distribution of all chicks (red) and those that survived to recruit (blue), representing the distribution of potential parents before and after selection, respectively. Scatter plots show mid-offspring versus single parental traits. Values are corrected for year, sex and time of day at which they were measured, and the size of the points is proportional to the square root of the family size. The red line is the predicted non-linear PO-regression based on the posterior means of the parameters from the skew-t quantitative genetic model and the blue line is the fit of a weighted (by family size) linear regression to the actual data. These are not corrected for measurement error. Lower panels show the comparison between heritabilities calculated before ( $h_b^2$ ) and after ( $h_a^2$ ) selection, calculated across the posterior distribution of the skew-t animal model trait models. In these lower plots all heritabilities account for measurement error. In all plots 'NS' indicates  $p > 0.05$ , '\*' indicates  $0.05 > p > 0.005$  and '\*\*'  $p < 0.005$ .



Table 1: Estimates (mean $\pm$  SE) of heritability and additive genetic variance from PO-regression and Gaussian animal models (AM) across 2000 simulated data sets. Three-generation simulations were set up with either no cross-fostering (N) or with nests paired and half of each nest's offspring reciprocally crossed (X). Phenotypes were simulated according to the model estimated for chick mass exactly (skewed) or as Gaussian with matching variance. The probability of a chick recruiting to be a parent was obtained by applying the estimated survival model for chick mass.

	Simulated	Gaussian		Skewed	
		N	X	N	X
$h^2$ PO	0.138	0.139 $\pm$ 0.001	0.140 $\pm$ 0.001	0.223 $\pm$ 0.001	0.224 $\pm$ 0.001
$h^2$ AM	0.138	0.137 $\pm$ 0.001	0.135 $\pm$ 0.000	0.170 $\pm$ 0.001	0.141 $\pm$ 0.000
$V_A$ AM	0.148	0.146 $\pm$ 0.001	0.144 $\pm$ 0.000	0.181 $\pm$ 0.001	0.150 $\pm$ 0.000