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Experimental studies and theoretical analysis of the residual properties of three-span small-scale continuous concrete slabs after a fire

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Abstract

This study investigated the effect of travelling fire scenarios, span–thickness ratios, and recurring ages on the post-fire residual behaviour of small-scale continuous reinforced concrete slabs. The mechanical performance of five fire-damaged continuous slabs and one reference slab were investigated, including load–deflection curves, concrete and reinforcement strains, cracking patterns, and failure modes, and the observations were compared with those discussed in companion papers. In addition to considering the boundary restraint and failure criteria, this study proposed a simple ellipse equation to determine the tensile membrane action region and residual ultimate loads of concrete slabs at the limit state. The results indicated that the travelling fire scenario, including the fire direction and time delay, had a slight effect on the residual behaviour of fire-damaged slabs with larger span–thickness ratios. For fire-damaged slabs with larger span–thickness ratios, flexural failure easily occurred with sufficient development of the tensile membrane action. The proposed ellipse method can be used to accurately determine the residual ultimate loads of fire-damaged continuous slabs with large span–thickness ratios.

Keywords: continuous concrete slab; post fire; residual strength; recurring age; span–thickness ratio; ellipse equation

Nomenclature

r	aspect ratio (L/l)
b	parameter defining the magnitude of the membrane force (Plate ①)
b'	parameter defining the magnitude of the membrane force (Plate ③)
k	parameter defining the magnitude of the membrane force (Plate ①)
k'	parameter defining the magnitude of the membrane force (Plate ③)
K	ratio of yield force in the reinforcing steel along the short span to the yield force in the reinforcing steel along the long span
T_0	yield force in reinforcing steel per unit width in the long span (kN/m)
d	average effective depth of reinforcement (mm)
d_1	effective depth of reinforcement in the short span (mm)
d_2	effective depth of reinforcement in the long span (mm)
e	overall enhancement of theoretical yield-line load due to membrane action
e_1	net enhancement for Element 1
e_2	net enhancement for Element 2
e_3	net enhancement for Element 3
e_{1b}	enhancement due to bending action for Element 1
e_{2b}	enhancement due to bending action for Element 2
e_{3b}	enhancement due to bending action for Element 3
e_{1m}	enhancement due to membrane forces for Element 1
e_{2m}	enhancement due to membrane forces for Element 2
e_{3m}	enhancement due to membrane forces for Element 3
g_1	parameter defining the compressive stress block in flexural action along the short span
g_2	parameter defining the compressive stress block in flexural action along the long span
w	vertical central deflection of the slab (mm)
P_y	uniformly distributed theoretical yield-line load at ambient temperature
P_{limit}	predicted load corresponding to maximum allowable displacement w

1. Introduction

Recently, the structural performance of reinforced concrete (RC) slabs exposed to fire has been of significant interest for researchers. Several experimental and numerical studies have been conducted on the fire performance of RC slabs [1–6]. However, in addition to the fire behaviour of RC slabs, assessing the post-fire load-carrying capacities of RC slabs has been of considerable interest [7–9] as the residual performance should be assessed to determine whether the residual load-bearing capacity remains sufficient to ensure a safe level.

Several studies have been conducted on the post-fire mechanical performance of concrete slabs, including five two-span continuous concrete slabs [10], unbonded prestressed concrete continuous slabs [11], concrete slabs with polypropylene fibres [12], and glass fibre reinforced polymer RC slabs [13-14]. However, the uniform fire scenario was often used in these studies, and inadequate focus was given to the travelling fire scenario or different compartment fires, particularly slabs with larger span–thickness ratios. Thus, Wang et al. [15] investigated the effect of the number and position of heated spans on the post-fire residual behaviour of five continuous concrete slabs (4700 mm × 2100 mm × 80 mm). The results indicated that concrete spalling significantly affects the ultimate load and failure mode of fire-damaged continuous slabs. In addition to the flexural failure mode, punching shear failure also occurred in the fire-damaged continuous slab, particularly in the span with considerable explosive concrete spalling. In addition, Wang et al. [16] further investigated the effect of different factors on the residual performance of four concrete continuous slabs (4700 mm × 2100 mm × 100 mm), including different compartment fire scenarios, reinforcement ratios, and bar arrangements. The results indicated that the residual structural stiffness and ultimate loads were significantly enhanced with increasing reinforcing ratio, but brittle punching failure readily occurred. The span–thickness ratios of the above two-series continuous slabs were 14.5 [15] and 18.1 [16], respectively. Meanwhile, the average mid-span failure deflection of each span was approximately $l/50$; thus, the tensile membrane action could not be sufficiently developed. In such scenario, the conventional yield line and ACI punching shear methods can be used to determine the residual ultimate load of each span in continuous slabs. However, in a normal engineering practice, the span–thickness ratio of the concrete slab is often larger than 20; thus, the residual tests of thinner continuous slabs should be further investigated, particularly after travelling fire scenarios and longer recurring ages (the time between the fire test and the post-fire loading).

In addition to experiments, analytical methods must be developed to assess the residual strength of post-fire RC slabs. Bailey et al. [1, 17], Omer et al. [18-19], Li et al. [20], Wang et al. [21], Herraiz and Vogel [22], and Burgess [23] proposed different methods of determining the ultimate load capacity of a simply supported concrete slab. Note that for among existing methods, no simple method of determining the tensile membrane action region of the two-way slab at the limit state exists. Therefore, an ellipse equation to determine the ultimate loads of two-way simply supported slabs at large deflections as well as the tensile (compressive) membrane action region was proposed [24]. With this ellipse method, the intersecting points of the three yield lines in the middle region were assumed to be two focuses of the elliptic equation. However, the effect of the boundary condition was not considered, particularly the negative moment along the edges of continuous slab over the supports. Thus, in this study, the ellipse method was further developed to consider the beneficial effect of the boundary condition, and the experimental results of the fire-damaged continuous slabs were used to verify its effectiveness.

Therefore, the objectives of this research were as follows: (1) To investigate the residual load-carrying capacities of post-fire continuous RC slabs with large span–thickness ratios (approximately 30); (2) observe their cracking patterns, failure characteristics (brittle or ductile failure), and failure mode; (3) propose a simple method of determining the tensile membrane action region and predicting the residual ultimate load of fire-damaged continuous slabs with different boundary conditions; and (4) establish the reasonable failure criteria of the post-fire continuous RC slabs with larger span–thickness ratios. This method can be easily modified to predict the ultimate limit loads of concrete slabs at ambient and elevated temperatures.

In this study, residual strength tests were conducted on the post-fire behaviour of five three-span small-scale continuous RC slabs after various travelling fires. One reference slab was tested without exposure to fire. First, the furnace temperatures and concrete and steel temperatures of the slabs were briefly investigated. Second, five fire-damaged slabs and one reference slab were loaded to failure at ambient temperature. For each slab, the vertical and horizontal deflections, concrete or reinforcement strains, cracking patterns, and failure modes were investigated. Finally, the residual ultimate limit loads predicted using the proposed method were compared with the experimental and other theoretical results.

2. Test setup

2.1 Test slabs

Six three-span two-way continuous RC slabs (each with dimensions of 4700 mm × 2100 mm × 50 mm) with the same reinforcement ratio and arrangement were tested. One slab (Slab CS0) was the reference slab without fire exposure, while the other five slabs (Slabs CS1, CS3 to CS6) were subjected to different travelling fire scenarios, and the residual strength tests (Slabs CS1-PF, CS3-PF to CS6-PF) were then conducted. After the fire test, the slabs were stored outside the laboratory (natural environment: 518 to 830 days), and the concrete age of the reference slab was 897 days. The details of each slab are presented in Fig. 1(a) and Table 1. Note that the residual test of Slab CS2-PF was not conducted owing to the previous failure (due to transportation and hoisting).

2.2 Test procedure

2.2.1 Fire tests

In this study, five slabs were tested under different travelling fires, and the details of the fire tests are available in Ref. [25]. The uniform applied loads (iron bricks) of each tested slab during the fire test are listed in Table 1. As shown in Fig. 1(b), six thermocouple trees were used to measure the temperature of each heated compartment. As depicted in Fig. 1(c), each thermocouple tree consisted of six thermocouples (1–6) for concrete and four thermocouples in Points R-1 to R-4 for the reinforcement. Other details of the fire tests are available in Ref. [25].

2.2.2 Post-fire strength tests

After the fire tests, the residual load-carrying capacities of continuous Slabs CS1-PF and CS3-PF to CS6-PF, as well as for Slab CS0, were investigated. For the residual tests, each slab was tested using a reaction steel frame (Fig. 2(a)). One steel plate (160 mm × 300 mm) was placed at each loading point, and the load was applied using three hydraulic jacks. For each span, the load increment was 8 kN (initial and middle stages) or 4 kN (near the failure or later stage), and the applied load at each step was maintained for 5 min. The failure criteria for each slab included concrete crushing, reinforcement fracture, and punching shear failure. If the failure criteria was reached in a span of the slab, the test of that span was stopped, but the loads in other spans of the slab continued to increase until the corresponding failure occurred in those spans. Note that because of the severe deterioration of the concrete (rehydration of CaO: volume increase) [26-27], one hole appeared in Span CS1-A during the re-curing stage; thus, its residual strength was zero.

As shown in Figs. 2(b)–2(d), four corners were anchored by the four steel beams, and the reaction force at each corner, denoted as Points P-1 to P-4, was measured using pressure transducers. Strain gauges were used to measure the concrete and reinforcement strains (Fig. 3(a)). In addition, Fig. 3(b) shows the positions of vertical points V-A, V-B, and V-C and horizontal points H-1 to H-4.

3. Results of the fire tests

The temperature variations in the furnace and concrete and steel with time during the heating phases for the five slabs are shown in Figs. 4(a)–4(e). Note that the maximum furnace temperatures of two unheated Spans CS1-B and CS1-C were lower than 200 °C, indicating that they had higher residual strengths.

The maximum furnace temperatures of the heated spans in Slabs CS3 to CS6 [25] ranged from 830 to 1102 °C, with an average value of 980 °C (Table 1). In contrast, for Slabs CS3 to CS6, the maximum concrete temperatures on the bottom (top) surfaces of the heated spans ranged from 640 (220) to 851 (355) °C, with an average value of 764 (274) °C. Meanwhile, the bottom steel temperatures of Slabs CS3 to CS6 ranged from 518 to 721 °C, with an average value of 629 °C. Note that as discussed in Ref. [15], the maximum temperatures for the bottom concrete (steel) ranged from 671 (529) to 1130 (718) °C, with an average value of 893 (645) °C. Similarly, as discussed in Ref. [16], the average concrete (steel) temperatures on the bottom and top surfaces of the heated spans were 828 (781) and 254 (497) °C, respectively. As expected, because of the longer heating duration (approximately 180 min) and larger heat capacities, the maximum temperatures of Slabs S1 to S5 [15] and B1 to B4 [16] were higher (100–200 °C) than those of the tested slabs presented here. Other details of the tested slabs in Refs. [15-16] were shown in Table 2. Overall, compared with the fire travelling direction and time delay, the heating duration and slab thickness considerably affected the maximum material temperatures of the slabs, particularly small-scale slabs.

Similar to Refs. [15-16], post-cooling concrete spalling (falling of concrete pieces) occurred because of the moisture absorbed by the calcareous aggregate (rehydration). As discussed above, a severe failure occurred in Span CS1-PF-A before the residual strength tests owing to the post-fire spalling and higher experienced temperatures (Fig. 4(a)). However, for fire-damaged slabs, in Refs. [15-16], no holes appeared before the residual tests, although the experienced maximum temperatures were relatively higher. Thus, a concrete slab with a minimum thickness is required to prevent severe spalling after the cooling stage. In other words, increasing the slab thickness is an effective method of reducing the detrimental effect of post-spalling on the residual behaviour of the slab.

4. Results of the post-fire tests

This section discusses the post-fire experimental results for each slab and provides a brief explanation of the observed behaviour, including the load–deflection curves, concrete and reinforcement strains, cracking pattern, and failure mode. In addition, the residual behaviour of the tested slabs was compared with those of other fire-damaged slabs in the companion papers [15-16].

4.1 Failure behaviour

Figs. 5(a)–10(d) show the cracking patterns on the top and bottom surfaces of each span in the continuous slabs. As shown in Figs. 6(a)-10(d), for each fire-damaged slab, the red and dark lines indicate new and original cracks, respectively, and the blue elliptic line and blue rectangular dash line indicate the tensile membrane action region and bottom reinforcing steel yield region, respectively.

- Reference slab CS0 (unheated)

For Slab CS0, before 28 kN, no cracks appeared on the top surface. Between 28 and 36 kN, cracks appeared in the middle region of Span B as well as at the corners. At approximately 50 kN, small new cracks first appeared near the two internal supports. Subsequently, cracks appeared through two internal supports at approximately 70 kN, and many arc cracks appeared at the corners of each span. Between 80 and 100 kN, mid-span cracks appeared at the two edge spans A and C. At approximately 104 kN, concrete crushing occurred at one corner of Span A. Here, the load at Span A did not increase and remained constant. At 106 kN, concrete crushing appeared at Span C, and then its load remained constant. Finally, at 110 kN, concrete crushing occurred at Span B, and the test was stopped.

As shown in Figs. 5(a) and 5(b), as expected, the cracking patterns at Spans A and C were similar, and the top arc cracks appeared near the edge region. For Span B, its top cracking pattern differed slightly from those of Spans A and C owing to its higher boundary restraint. Note that some cracks appeared in the middle region of each span owing to the tensile membrane action (large deflection). Moreover, the conventional yield-line failure mode or rectangular cracking pattern occurred on the bottom surface of each span (Figs. 5(c) and 5(d)). However, in contrast to this observation, concrete crushing easily occurred along the bottom surface of the internal supports because of the larger concrete stresses [15-16]. This comparison indicates that the span–thickness ratio has a considerable effect on the brittle failure mode of the internal supports, such as concrete crushing or local punching shear failure mode.

- Fire-damaged Slab CS1-PF (heated span A)

For this slab, only Span A was heated during the fire test (180 min). As shown in Fig. 6(a), Span A was not loaded because of the hole, and Spans B and C were simultaneously loaded during the residual test. Before 20 kN, the width of the original top cracks gradually increased, and no new cracks appeared. Subsequently, many new top cracks first appeared near the two internal supports, and then arc cracks appeared at the corners of each span. At approximately 96 kN, concrete crushing appeared at the two corners of Span C, and its load remained constant until the end of the test. At 108 kN, the cracks across Span B appeared in its middle region, and the test stopped.

Figs. 6(a)–6(d) show the cracking pattern on the top and bottom surfaces of Slab CS1-PF. On one hand, the bottom failure mode of two spans were similar to those of Slab CS0, and no concrete crushing appeared on the bottom surface of two internal supports. On the other hand, owing to the lower experienced temperatures (Table 1), the ultimate loads (108 and 96 kN) of Spans B and C in Slab CS1-PF were similar to those (110 kN and 106 kN) of Slab CS0, and the reduction factors of Spans B and C were 1.8% and 9.4%, respectively. Note that this observation differs from those in Ref. [15]. For instance, compared with the Slab S0, the maximum reduction factors of the residual strengths in Slabs S1-PF and S2-PF were 18.1% and 30%, respectively. This comparison further indicates that as the span–thickness ratio decreases, the difference in the residual carrying capacity among different spans tends to increase owing to various failure modes [15-16].

- Fire-damaged slabs CS3-PF to CS6-PF (three heated spans)

As discussed above, for each span, the direct heating time was approximately 90 min, and the maximum concrete and steel temperatures were similar (Table 1). During the residual strength test, the cracking development and failure mode of each fire-damaged slab were similar.

During the initial stage, the original cracks gradually widened, and no new cracks appeared on the top surface before approximately 20 kN. Subsequently, many new cracks primarily appeared near the internal supports and edges. At the limit state, concrete crushing at the corners easily occurred on one span. Note that, for any span–thickness ratio, the original crack distribution of the slab in fire was different from the new crack distribution of the fire-damaged slabs, owing to different mechanical mechanisms.

Figs. 7(a)–10(d) show the cracking patterns and failure modes on the top and bottom surfaces of Slabs CS3-PF and CS6-PF. On one hand, on the top surface of each span, arc and large cracks appeared on the edge and two internal supports, respectively. Meanwhile, in addition to the punching shear failure (severe spalling) of Span CS4-PF-A, concrete crushing primarily occurred on the top surface of all other spans. However, as discussed in Ref. [16], the punching shear failure or flexural-punching combined failure occurred in six spans of Slabs B1-PF to B4-PF (total 12 spans: traveling fire); the main reasons were the cross shape (+) of the original cracks, lower span–thickness ratio, and concentrated loading system. In addition, four spans in Slabs S1-PF to S5-PF (total 15 spans: uniform fire) experienced punching shear failure [15]. As discussed above, for the tested slabs, only one span (total 16 spans) in the present fire-damaged slabs experienced punching shear failure, and all other spans experienced flexural failure. In addition to the four-point loading system and slight spalling, another important reason is that there were fewer network or map original cracks on the top surface of each span, and many original arc cracks appeared at the edge of each span. No doubt, the network original cracks led to the low bond strength and dowel actions between the concrete and steel, i.e., the punching shear strength [5-6]. Therefore, for any span–thickness ratio, compared with the boundary restraint, severe spalling has significant adverse effects on the residual strength, bond, stress or strain concentration, and insufficient tensile membrane action, and it easily results in the punching shear failure of the concrete slabs, such as for Span CS4-PF-B.

- Discussion

Based on the above observation, as the span–thickness ratio increases, the travelling fire scenarios have slight effects on the failure mode of a concrete slab, and the flexural failure mode often occurs in each span owing to better rotation capacity. In other words, the support of the slabs with lower span-thickness ratio is more flexible to rotate and more ductile. In this case, the smooth deflected shape (double curved) of the two-way slab easily formed, particularly under the uniform load. Thus, the bottom reinforcement gradually stretched with increasing deflection, and the tensile field in its central region sufficiently developed. An important premise is that the punching shear capacity of the fire-damaged slab is higher than the flexural capacity. As the span–thickness ratio decreased, the punching shear failure easily occurred owing to the lower rotation capacity. The results indicated that the span–thickness ratio, fire scenario, and loading system should be considered to determine the reasonable failure modes of fire-damaged slabs.

Moreover, the conventional yield-line failure mode occurred on the bottom surface of each slab, including diagonal and central cracks parallel to the short span. In contrast to the observations in Refs. [15-16], the cracks of the tested slabs were sufficiently developed, indicating that the tensile membrane action appeared at a large deflection ($l/20$). Thus, as the span–thickness ratio increases (such as ≥30), the beneficial effect of the tensile membrane action can be considered; otherwise, the limit carrying capacities predicted using the conventional yield-line theory will be underestimated, as discussed later. In contrast, as the span–thickness ratio decreases (such as ≤20), the tensile membrane action can be neglected; otherwise, the limit carrying capacities predicted using the tensile membrane action theory will be overestimated [15-16].

Overall, in addition to the fire scenario (uniform or travelling fire scenario), the span–thickness ratio (20 or 30) and loading system (concentrated load or uniform load) have significant effects on the failure mode of fire-damaged continuous slabs. For any span–thickness ratio, local punching shear failure easily occurs if severe spalling occurs during the fire or in the post-cooling stage.

4.2 Load vs. displacement responses

This section discusses the vertical and horizontal deflections observed in each tested slab. For the vertical deflections, the positive displacement is downward, while for the horizontal displacement, positive values indicate outward and negative values of inward movement. For some spans, the data were not measured because of holes or previous failures.

4.2.1 Load vs. vertical deflection responses

Figs. 11(a)–11(e) show the load–deflection curves of each slab, and the load–deflection curve trend of different spans were noticeably similar. In addition, the initial structural stiffness and the energy ductility (μ_E) of each span are listed in Table 2. The energy ductility (μ_E) is ($E_{total}/(2E_{el})+0.5$), where E_{total} and E_{el} are the elastic and total energies (the areas under the load-deflection curve) of the fire-damaged slab, respectively, details of μ_E are available in Ref. [15].

(1) Initial structural stiffness and energy ductility

The initial structural stiffness (K_0) of the heated edge spans ranged from 1.55 to 3.59 kN/mm, with an average value of 2.47 kN/mm (Table 2). In addition, the average K_0 of the heated middle spans was 3.04 kN/mm. Similar to the observation for Slab CS0, the K_0 of the middle span was slightly higher than that of the edge span. In addition, the original cracks mainly concentrated on the internal supports and edge of the tested slabs during the fire test, and less original cracks appeared at the middle region of each span. Thus, the travelling fire scenario and internal original cracks hardly affected the residual initial structural stiffness of the fire-damaged slabs with larger span–thickness ratios. However, as discussed in Refs. [15-16], larger differences were observed among different spans for the initial residual structural stiffness. For instance, the initial residual structural stiffness of Slabs B1-PF to B4-PF [16] (S1-PF to S5-PF [15]) ranged from 6.0 (4.3) to 20.4 (110.5) kN/mm, with an average value of 13.1 (26.9) kN/mm. Thus, as the thickness increased, the initial residual structural stiffness was more sensitive to the fire scenarios since it was primarily dependent on the initial elastic modulus (E) (or original cracks) and the thickness (h), i.e. $Eh^3/[12(1-\mu^2)]$, particularly the concrete.

Table 2 lists the energy ductility values of each span in the tested slabs. The ductility of the fire-damaged slab was slightly higher than that of the reference slab. For instance, the μ_E of five fire-damaged slabs (reference slab) ranged from 1.09 (1.29) to 3.01 (1.80), with the average value of 1.80 (1.52). Compared with the reference slab, the average increase in the ductility of the fire-damaged slabs was approximately 18.4%. In addition, for the thinner fire-damaged slabs, the ductility difference among these spans was small owing to the lower boundary restraint or rotation restraint. However, for thicker fire-damaged slabs [15-16], the μ_E values of the concrete slabs fluctuated significantly, particularly in the uniform fire scenario. For instance, for the fire-damaged slabs subjected to uniform (traveling) fire scenarios, μ_E ranged from 1.13 (1.06) to 19.91 (4.80), with the average value of 5.15 (2.14). In other words, as the span–thickness ratio increased, the effect of the boundary restraint on the energy ductility gradually decreased.

In summary, as the span-thickness ratio increased, the initial residual structural stiffness and energy ductility tended to decrease, and the effect of the spreading direction of travelling fire and delay time could be neglected. However, as the span-thickness ratio decreased, the initial residual structural stiffness and energy ductility of the fire-damaged slabs were more sensitive to the fire scenario, particularly to the position and number of heated spans.

(2) Ultimate load-carrying capacities

Table 2 lists the ultimate loads (P_u) and ultimate deflections (δ_u) of the fire-damaged slabs. Except for Spans CS1-PF-A and CS5-PF-C, the minimum ultimate load within two or three spans was considered as the actual ultimate load of each slab. Thus, the residual ultimate loads of Slabs CS1-PF and CS3-PF to CS6-PF were 96 kN (Span C), 80 kN (Span B), 84 kN (Span A), 92 kN (Span A), and 88 kN (Span A), respectively, with an average value of 88 kN. For the ultimate load, the ratio for the reference slab (104 kN) and the fire-damaged slabs ranged from 76.9% to 92.3%, with an average value of 84.6%. Note that, compared with the limit loads of the reference Slab S0, this ratio of Slabs S1-PF to S5-PF [15] ranged from 58.4% to 100%, with an average value of 79.3%. This comparison indicated that as the span–thickness ratio of the continuous slabs increased, fewer limit load fluctuations occurred among different spans. In other words, for a slab with a larger span–thickness ratio, the effect of the traveling fire scenario, including the time delay and fire traveling direction, on the residual limit loads can be neglected.

As the span–thickness ratio decreased, different types of failure modes verified this observation, including the flexural failure mode, punching shear failure, flexural punching failure, and interior support failure, as discussed in Refs. [15-16]. The flexural failure mode is primarily dependent on the residual properties of steel, but the punching shear failure mode is dependent on the residual strength, cracking pattern, and spalling of concrete. Thus, for the same fire scenario and reinforcement layout (ratio), the thermal gradient gradually increased with increasing thickness, and larger residual material property differences existed across the thickness. In addition, different original cracking patterns easily occurred in the thicker slabs during the heating stage, including the original crossshaped cracks, many cracks parallel to the short spans, and spalling region or depth. In contrast, for the tested fire-damaged slabs, the above differences among different spans could be neglected, as discussed above. Therefore, the loading system (uniform load or concentrated load) has significant effects on the failure mode and residual ultimate loads of fire-damaged slabs.

As shown in Table 2, for the fire-damaged slabs, the limit deflection (δ_u) ranged from 34.0 to 79.43 mm with an average ultimate deflection of 64.3 mm. For the thinner slabs, the ultimate deflection was approximately $l/20$ (72.5 mm). Note that this observation differed from the average limit deflection $l/50$ (29 mm) of Slabs S1-PF to S5-PF [15] and Slabs B1-PF to B4-PF [16]. Thus, for fire-damaged slabs, the effect of the span–thickness ratio should be considered to establish a reasonable deflection failure criterion.

Overall, for the fire-damaged continuous slabs, the residual performance was dependent on several factors, including the furnace temperature, heating time, boundary condition, reinforcement ratio, reinforcement layout (top continuous or discontinuous), thickness, span–thickness ratio, original cracks, spalling, travelling fire, and uniform fire. Owing to many uncertainties related to the above key factors, a simple and effective method should be established to predict the accurate residual limit loads of fire-damaged slabs, particularly the reasonable failure modes and failure criteria.

4.2.2 Load vs. horizontal deflection responses

Figs. 12(a)–12(e) show the measured horizontal displacement vs. load curves for each slab. As expected, during the early stage of loading, the horizontal deflection of each measured point was small owing to the small vertical deflection. After approximately 40 kN, the horizontal deflection rapidly increased until the end of the test. At the end of the residual strength test, the maximum horizontal deflection of these slabs ranged from 1 to 4 mm. Overall, the maximum horizontal deflection of the tested slabs were basically similar to the observations in Refs. [15-16], indicating that the thickness or span–thickness ratio has minimal effect on the residual horizontal deflection of the fire-damaged slabs.

4.3 Load–concrete and steel strain curves

The concrete and reinforcement strains measured for all slabs are shown in Figs. 13(a)–13(f), and the concrete peak strain and reinforcement yield strain were identified according to Ref. [28]. A positive value represents the tensile strain, whereas a negative value indicates compressive strain.

As indicated in Figs. 13(a)–13(f), as expected, the concrete compressive strain at each corner gradually increased with the load. After approximately 40 kN, the concrete strain rapidly increased, particularly during the later stages. Compared with those of the reference slab, the concrete strains of the fire-damaged slabs were relatively larger, indicating that they had better ductility. This observation was consistent with the experimental results in Refs. [15-16]. On one hand, the maximum concrete strain of Span CS0-C was approximately 3773×10^{-6}, which coincided with the conventional concrete crushing strain, such as 3500×10^{-6} or 3800×10^{-6} [29]. On the other hand, for the fire-damaged slabs at the limit state, the maximum concrete strains of several measured points were larger than 4000×10^{-6}, such as Spans CS6-PF-A and CS6-PF-B. Meanwhile, owing to the smaller thickness, higher experienced temperatures, and larger deflections, the concrete strains of the tested slabs at the limit states were larger than those in Refs. [15-16]. In summary, according to the above experimental results, the concrete crushing strain of the fire-damaged slab can be considered as 4500×10^{-6} in this paper. Note that, the concrete crushing strain was basically conformed to the experimental observation [33].

Figs. 13(a)–13(f) also show the reinforcement strain at different measured points of each span in the tested slabs. As expected, the reinforcement strains increased with the load until the end of each test. In addition, the load–strain trend basically coincided with the load–deflection curves (Figs. 11(a)–11(e)). Note that, similar to Refs. [15-16], large differences were observed between different measured points owing to the stress or strain concentration.

Overall, for the fire-damaged slabs, concrete or steel strains are not suitable for the failure criteria owing to data scatter. According to the companion papers, the mid-span deflection, i.e. $l/50$ or $l/20$, is suggested to determine the residual limit loads of fire-damaged slabs with lower (≤20) or larger span–thickness ratios (≥30).

5. Proposed method

In this paper, equations for predicting the strength of rectangular two-way concrete (RC) slabs with different edge support conditions and under uniformly distributed loading are further developed from the ellipse equation theory [24]. As shown in Fig. 14, the tested continuous slabs had two types of boundary conditions, including three simply supported edges and one fixed long edge (or the edge span), and two edges fixed and two edges simply supported (or the middle span). Thus, two types of membrane force distribution patterns are defined in this paper, namely, stress patterns I and II (Figs.15(a)–15(f) and 16(a)–16(d)). Note that, for each stress pattern, the intersecting points of three yield lines in the middle region are assumed to be the two foci of the elliptic equation. However, different from the existing elliptic equation approach [24], the present method mainly considered the effect of the boundary condition (the negative moment) on the position of the intersecting points of the concrete slabs, as discussed later.

Similar to the yield line theory, Bailey method [17] and the reinforcement strain difference method [21] and the elliptic equation method [24], the residual deflection or the material residual strain (such as the concrete transient strain [34]) was also not considered in this paper, and it only focused on the residual strength of the bottom steel and top surface concrete.

5.1 Stress pattern I

5.1.1 Membrane forces

- For Plate ①

As shown in Figs. 15(a)–15(b), for Plate ①, angle θ is defined as [30]

$$\sin\theta = \alpha L / [\sqrt{(\alpha L)^2 + (\beta l)^2}], \quad \alpha = \frac{1}{5.8282 r^2}\left[\sqrt{1+8.7417 r^2} - 1\right], \quad \beta = 0.4142 , \tag{1}$$

where α and β are two factors, L (l) is the length (width) of the slab, and r is the aspect ratio (L/l). Note that, α and β were dependent on the boundary conditions [30]. In other words, the effect of the boundary conditions was considered according to the two parameters.

According to the in-plane membrane force equilibrium (Figs. 15(b)–15(d)), the following equations can be obtained:

$$(T_1/2)\sin\alpha = C_1 - T_2 , \quad (T_1/2)\cos\alpha = S , \quad T_1 = bKT_0(L - 2\alpha L) \tag{2a}$$

$$C_1 = \frac{kbKT_0}{2}\left(\frac{k}{1+k}\right)\sqrt{(\alpha L)^2 + (\beta l)^2} , \quad T_2 = \frac{bKT_0}{2}\left(\frac{1}{1+k}\right)\sqrt{(\alpha L)^2 + (\beta l)^2} , \tag{2b}$$

$$k = \frac{\alpha r^2 (1\text{-}2\alpha)}{\alpha^2 r^2 + \beta^2} + 1, \tag{2c}$$

where k is the parameter defining the magnitude of the membrane force, T_0 is the yield force in the reinforcing steel per unit width (kN/m), T_1 (T_2) is the resultant in-plane tension forces in the x- (y-aligned) rebar at the yield line, b is the parameter defining the magnitude of the membrane force, K is the ratio of the yield force in the reinforcing steel along the short span to the yield force in the reinforcing steel along the long span, S is the in-plane shear force along a diagonal yield line; and C_1 and C_2 are the compressive forces.

- Plate ③

As shown in Figs. 15(e)–15(f), for Plate ③, θ' is defined as [30]

$$\sin\theta' = \alpha L / [\sqrt{(\alpha L)^2 + (l - \beta l)^2}], \quad \alpha = \frac{1}{5.8282 r^2}\left[\sqrt{1 + 8.7417 r^2} - 1\right], \quad \beta = 0.4142 . \tag{3}$$

According to the membrane force equilibrium, for Plates ③ and ③', the following equations can be obtained:

$$(T_1'/2)\sin\theta' = C_1' - T_2', \quad (T_1'/2)\cos\theta' = S', \quad T_1' = b'KT_0(L - 2\alpha L) \tag{4a}$$

$$C_1' = \frac{k'b'KT_0}{2}\left(\frac{k'}{1+k'}\right)\sqrt{(\alpha L)^2 + (l - \beta l)^2}, \quad T_2' = \frac{b'KT_0}{2}\left(\frac{1}{1+k'}\right)\sqrt{(\alpha L)^2 + (l - \beta l)^2}, \tag{4b}$$

$$k' = \frac{\alpha r^2 (1 - 2\alpha)}{\alpha^2 r^2 + (1 - \beta)^2} + 1 \tag{4c}$$

where k' is the parameter defining the magnitude of the membrane force, T_1' (T_2') is the resultant in-plane tension forces in the x- (y-aligned) rebar at the yield line, b' is the parameter defining the magnitude of the membrane force, S' is the in-plane shear force along a diagonal yield line, and C_1' and C_2' are the compressive forces. Note that, because the effect of the in-plane force (interior support) is neglected, T_1' is not equal to T_1. In other words, the difference between T_1' and T_1 is equal to $\boldsymbol{\Delta} T$.

5.1.2 Membrane action region

As shown in Fig. 15(a), according to the geometric equation, the ellipse equation (tensile membrane action region) is

$$\frac{\left(x+A_x\right)^2}{a_R^2}+\frac{\left(y+B_y\right)^2}{b_V^2}=1,\quad A_x=0 \tag{5a}$$

$$a_R=\sqrt{{b_V}^2+\left(\frac{L}{2}-\alpha L\right)^2},\quad B_y=b_V-\frac{\sqrt{\frac{(\alpha L)^2+(\beta l)^2}{(1+k)^2}}+\sqrt{\left(L-\alpha L-\frac{\alpha kl}{1+k}\right)^2+\left(\frac{\beta l}{1+k}\right)^2}}{2} \tag{5b}$$

$$b_V=\frac{\sqrt{\frac{(\alpha L)^2+(\beta l)^2}{(1+k)^2}}+\sqrt{\frac{(\alpha L)^2+(l-\beta l)^2}{(1+k')^2}}+\sqrt{\left(L-\alpha L-\frac{\alpha kl}{1+k}\right)^2+\left(\frac{\beta l}{1+k}\right)^2}+\sqrt{\left(L-\alpha L-\frac{\alpha kl}{1+k'}\right)^2+\left(\frac{l-\beta l}{1+k'}\right)^2}}{4} \tag{5c}$$

$$x_c=\beta l-\frac{\sqrt{\frac{(\alpha L)^2+(\beta l)^2}{(1+k)^2}}+\sqrt{\left(L-\alpha L-\frac{\alpha kl}{1+k}\right)^2+\left(\frac{\beta l}{1+k}\right)^2}}{2} \tag{5d}$$

$$x_c'=(l-\beta l)-\frac{\sqrt{\frac{(\alpha L)^2+(l-\beta l)^2}{(1+k')^2}}+\sqrt{\left(L-\alpha L-\frac{\alpha kl}{1+k'}\right)^2+\left(\frac{l-\beta l}{1+k'}\right)^2}}{2} \tag{5e}$$

where the width of the compressive membrane action at the slab edges (L_{EG} and $L_{\mathrm{E'G'}}$) are defined as x_{c} and x_{c}', respectively.

5.1.3 Key parameters

- Plates ①'

As shown in Fig. 15(d), for Plate ①', the compressive membrane action distribution at the line *EG* is triangular; thus, the equilibrium equation is defined as

$$C_2=KT_0\left(\beta l-x_{\mathrm{c}}\right)+C_1\cos\theta-T_2\cos\theta-S\sin\theta, \tag{6a}$$

or

$$C_2=\frac{KT_0}{2}\left(2\beta l-2x_{\mathrm{c}}+(k-1)b\beta l-\frac{\alpha\beta blL^2(1-2\alpha)}{(\alpha L)^2+(\beta l)^2}\right). \tag{6b}$$

For Plate ①', the moment equilibrium (about Point *E*) is

$$T_2\left[\left(\frac{\cos\theta\times L}{2}-\frac{\frac{L}{2}-\alpha L}{\cos\theta}\right)\frac{1}{\tan\theta}-\frac{\sqrt{(\alpha L)^2+(\beta l)^2}}{3(1+k)}\right]-KT_0(\beta l-x_c)\left(\frac{\beta l}{2}+\frac{x_c}{2}\right)$$
$$+\frac{1}{3}C_2x_c-\frac{T_1}{4}\left(\frac{L}{2}-\alpha L\right)+C_1\left[\frac{\sin\theta L}{2}-\frac{k\sqrt{(\alpha L)^2+(\beta l)^2}}{3(l+k)}\right]+S\frac{L}{2}\cos\theta=0 \quad (7)$$

By substituting T_1 (Eq. (2a)), T_2 (Eq. (2b)), C_2 (Eq. (6b)) and S (Eq. (2a)) into Eq. (7), b can be obtained as

$$b=\frac{(\beta l-x_c)\left(\frac{\beta l}{2}+\frac{x_c}{2}\right)-\frac{x_c(\beta l-x_c)}{3}}{A-B+C+D+E}, \quad (8)$$

where

$$A=\frac{x_c\beta l(k^2-1)}{6(1+k)}-\frac{x_c\alpha\beta lL^2(1-2\alpha)}{6\left[(\alpha L)^2+(\beta l)^2\right]},\quad B=\frac{L^2}{8}(1-2\alpha)^2$$

$$C=\frac{1}{2(1+k)}\left[\frac{(\beta l)^2}{2\alpha}-\left(\frac{1-2\alpha}{2\alpha}+\frac{1}{3(1+k)}\right)\left((\alpha L)^2+(\beta l)^2\right)\right],\quad D=\frac{k^2}{2(1+k)}\left[\frac{\alpha L^2}{2}-\frac{k}{3(1+k)}\left((\alpha L)^2+(\beta l)^2\right)\right],$$

$$E=\frac{\beta^2l^2L^2(1-2\alpha)}{4\left[(\alpha L)^2+(\beta l)^2\right]}.$$

As discussed in Ref. [17], the moments M_{01} and M_{02} are defined as

$$M_{01}=T_0d_1\left(\frac{3+g_1}{4}\right),\quad M_{02}=KT_0d_2\left(\frac{3+g_2}{4}\right),\quad g_1=\left[\frac{d_1}{2}-\frac{T_0}{f_{cu}}\right]\Big/\frac{d_1}{2},\quad g_2=\left[\frac{d_2}{2}-\frac{KT_0}{f_{cu}}\right]\Big/\frac{d_2}{2}, \quad (9)$$

where M_{01} and M_{02} are the moments of resistance (no axial force) in the short and long spans, respectively, d_1 and d_2 are the effective depths of reinforcement in the short and long spans, respectively, f_{cu} is the compressive cube strength of concrete, and g_1 and g_2 are parameters defining the compressive stress block in flexural action in the short and long spans, respectively.

- Plates ③'

As shown in Figs. 15(e)–15(f), for Plate ③', the compressive membrane action distribution at the line $E'G'$ is triangular; thus, the equilibrium equation is defined as

$$C_2'=KT_0\left(l-\beta l-x_c'\right)+C_1'\cos\theta'-T_2'\cos\theta'-S'\sin\theta', \quad (10a)$$

or

$$C_2' = \frac{KT_0}{2}\left(2\left(l-\beta l\right)-2x_c'+\left(k'-1\right)b'\left(l-\beta l\right)-\frac{\alpha\left(1-\beta\right)b'lL^2\left(1-2\alpha\right)}{\left(\alpha L\right)^2+\left(l-\beta l\right)^2}\right). \tag{10b}$$

As shown in Fig. 15(f), for Plate ③', the moment equilibrium (approximately Point E') is

$$\begin{gathered}T_2'\left[\left(\frac{\cos\theta'\times L}{2}-\frac{\frac{L}{2}-\alpha L}{\cos\theta'}\right)\frac{1}{\tan\theta'}-\frac{\sqrt{\left(\alpha L\right)^2+\left(l-\beta l\right)^2}}{3\left(1+k'\right)}\right]-KT_0\left(l-\beta l-x_c\right)\left(\frac{\left(l-\beta l\right)}{2}+\frac{x_c'}{2}\right)\\ +\frac{1}{3}C_2'x_c'-\frac{T_1'}{4}\left(\frac{L}{2}-\alpha L\right)+C_1'\left[\frac{\sin\theta' L}{2}-\frac{k'\sqrt{\left(\alpha L\right)^2+\left(l-\beta l\right)^2}}{3\left(l+k'\right)}\right]+S'\frac{L}{2}\cos\theta'=0\end{gathered}. \tag{11}$$

By substituting T'_1 (Eq. 4a), T'_2 (Eq. 4b), C'_2 (Eq. 10b), and S' (Eq. 4a) into Eq. (11), b' can be obtained as

$$b' = \frac{\left(l-\beta l-x_c'\right)\left(\frac{\left(l-\beta l\right)}{2}+\frac{x_c'}{2}\right)-\frac{x_c'\left(l-\beta l\text{-}x_c'\right)}{3}}{A-B+C+D+E}, \tag{12}$$

where

$$A=\frac{x_c'\left(1-\beta\right)l\left(k'^2-1\right)}{6\left(1+k'\right)}-\frac{x_c'\alpha\left(1-\beta\right)lL^2\left(1-2\alpha\right)}{6\left[\left(\alpha L\right)^2+\left(l-\beta l\right)^2\right]},\quad B=\frac{L^2}{8}\left(1-2\alpha\right)^2$$

$$C=\frac{1}{2\left(1+k'\right)}\left[\frac{\left(l-\beta l\right)^2}{2\alpha}-\left(\frac{1-2\alpha}{2\alpha}+\frac{1}{3\left(1+k'\right)}\right)\left(\left(\alpha L\right)^2+\left(l-\beta l\right)^2\right)\right],$$

$$D=\frac{k'^2}{2\left(1+k'\right)}\left[\frac{\alpha L^2}{2}-\frac{k'}{3\left(1+k'\right)}\left(\left(\alpha L\right)^2+\left(l-\beta l\right)^2\right)\right],\quad E=\frac{\left(1-\beta\right)^2l^2L^2\left(1-2\alpha\right)}{4\left[\left(\alpha L\right)^2+\left(l-\beta l\right)^2\right]}.$$

5.1.4 Enhancement factors

- ***e_{1m}, e_{2m}, and e_{3m}***

As shown in Figs. 17(a)–17(f), for Plates ① and ②, at the limit state (maximum displacement: w), the moments (M_{1m} and M_{2m}) about the supports owing to the membrane forces are expressed as

$$M_{1m}=T_1w+2T_2\sin\theta w\left(1-\frac{1}{3\left(k+1\right)}\right)-2C_1\sin\theta w\frac{k}{3\left(k+1\right)}-2S\cos\theta\frac{1}{2}w \tag{13a}$$

$$M_{2\mathrm{m}}=\left[\left(T_2\cos\theta\right)\left(1-\frac{1}{3(k+1)}\right)+\left(T_2'\cos\theta'\right)\left(1-\frac{1}{3(k'+1)}\right)\right]w+\left(S\sin\theta+S'\sin\theta'\right)\frac{1}{2}w$$
$$-\left[\left(C_1\cos\theta\right)\frac{k}{3(k+1)}+\left(C_1'\cos\theta'\right)\frac{k'}{3(k'+1)}\right]w \tag{13b}$$

$$M_{3\mathrm{m}}=T_1'w+2T_2'\sin\theta'w\left(1-\frac{1}{3(k'+1)}\right)-2C_1'\sin\theta'w\frac{k'}{3(k'+1)}-2S'\cos\theta'\frac{1}{2}w+M_uL. \tag{13c}$$

$M_{1\mathrm{m}}$ (Eq. 13a), $M_{2\mathrm{m}}$ (Eq. 13b), and $M_{3\mathrm{m}}$ (Eq. 13c) are divided by $M_{01}L$, $M_{02}l$, and $M_{01}L$, respectively, and the enhancement factors $e_{1\mathrm{m}}$, $e_{2\mathrm{m}}$, and $e_{3\mathrm{m}}$ are defined as

$$e_{1\mathrm{m}}=\frac{M_{1m}}{M_{01}L},\quad e_{2\mathrm{m}}=\frac{M_{2m}}{M_{02}l},\quad e_{3\mathrm{m}}=\frac{M_{3m}}{M_{01}L}. \tag{14}$$

- **$e_{1\mathrm{b}}$, $e_{2\mathrm{b}}$, and $e_{3\mathrm{b}}$**

For Plates ① and ②, if the axial compressive force N is present, the moment capacity M is defined as

$$\frac{M}{M_0}=1+\alpha_0\left(\frac{N}{T_0}\right)-\beta_0\left(\frac{N}{T_0}\right)^2,\quad \alpha_0=\frac{2\times g_0}{3+g_0},\quad \beta_0=\frac{1-g_0}{3+g_0}, \tag{15}$$

where g_0 is the parameter fixing depth of the compressive stress block when no membrane force is present [17].

As shown in Fig. 18(a), for Plate ①, for the yield line AB, the distance between B and the projection (x-axis) is x', and the membrane force $N_{x'}$ is

$$N_{x'}=bKT_0\left(\frac{x'(k+1)}{\alpha L}-1\right). \tag{16}$$

Thus, the moment contribution Z for yield lines AB and CD (Fig. 15(a)) is

$$Z_1=2\int_0^{\alpha L}\frac{M}{M_0}dx'=2\alpha L\left[1+\frac{\alpha_1 b}{2}(k-1)-\frac{\beta_1 b^2}{3}\left(k^2-k+1\right)\right],\quad \alpha_1=\frac{2\times g_1}{3+g_1},\quad \beta_1=\frac{1-g_1}{3+g_1}. \tag{17}$$

Similarly, for the yield line BC in Fig. 15(b), the membrane force is constant, $N=-bKT_0$, and we obtain

$$Z_2=\int_0^{L-2\alpha L}\frac{M}{M_0}\,dx=(L-2\alpha L)(1-\alpha_1 b-\beta_1 b^2). \tag{18}$$

For the yield line GF in Fig. 15(d), the membrane force is constant, $N=-KT_0$, and we obtain

$$Z_3 = 2\int_0^{\frac{l}{2}-x_c} \frac{M}{M_0}\, dy = 2(\beta l - x_c)(1 - K\alpha_2 - \beta_2 K^2)\,,\quad \alpha_2 = \frac{2\times g_2}{3+g_2},\quad \beta_2 = \frac{1-g_2}{3+g_2}. \tag{19}$$

Thus, according to Eqs. (17), (18), and (19), the enhancement factor e_{1b} is defined as

$$e_{1b} = \frac{Z_1}{L} + \frac{Z_2}{L} + \frac{Z_3}{l}. \tag{20}$$

For Plate ②, across the yield line *AB* in Fig. 18(b), at a distance of y' from *A*, the membrane force $N_{y'}$ is

$$N_{y'} = bKT_0\left(\frac{y'(k+1)}{\beta l} - 1\right) \quad , \quad N'_{y'} = bKT_0\left(\frac{y'(k'+1)}{(1-\beta)l} - 1\right). \tag{21}$$

Similarly, for Plate ②, the moment contribution for yield lines *A'B* and *AB* is

$$Y = \int_0^{\beta l} \frac{M}{M_0} \mathrm{d}y' = \beta l\left[1 + \frac{\alpha_2 bK}{2}(k-1) - \frac{\beta_2 b^2 K^2}{3}(k^2 - k + 1)\right] \tag{22a}$$

$$Y' = \int_0^{(1-\beta)l} \frac{M'}{M_0} \mathrm{d}y' = (1-\beta)l\left[1 + \frac{\alpha_2 b' K}{2}(k'-1) - \frac{\beta_2 b'^2 K^2}{3}(k'^2 - k' + 1)\right]. \tag{22b}$$

Thus, according to Eqs. (22a) and (22b), the enhancement factor e_{2b} is

$$e_{2b} = \frac{Y + Y'}{l}. \tag{23}$$

Similarly, as shown in Fig. 18(a), for Plate ③, for the yield line *AB*, the distance between *B* and the projection (*x-axis*) is x', and the membrane force $N_{x'}$ is

$$N_{x'} = b'KT_0\left(\frac{x'(k'+1)}{\alpha L} - 1\right). \tag{24}$$

Thus, the moment contribution *Z* for yield lines *AB* and *CD* (Fig. 15(e)) is

$$Z_1' = 2\int_0^{\alpha L} \frac{M}{M_0} dx' = 2\alpha L\left[1 + \frac{\alpha_1 b'}{2}(k'-1) - \frac{\beta_1 b'^2}{3}(k'^2 - k' + 1)\right]. \tag{25}$$

Similarly, for the yield line *BC* in Fig. 15(e), the membrane force is constant, N=-bKT_0, and we obtain

$$Z_2' = \int_0^{L-2\alpha L} \frac{M}{M_0}\, dx = (L - 2\alpha L)(1 - \alpha_1 b' - \beta_1 b'^2). \tag{26}$$

For the yield line *G'F'* in Fig. 15(f), the membrane force is constant, N=-KT_0, and we obtain

$$Z'_3 = 2\int_0^{\frac{l}{2}-x'_c} \frac{M}{M_0}\, dy = 2(l-\beta l - x'_c)(1-K\alpha'_2-\beta'_2 K^2)\,. \tag{27}$$

Thus, according to Eqs. (25), (26), and (27), the enhancement factor e_{3b} is defined as

$$e_{3b} = \frac{Z_1'}{L} + \frac{Z_2'}{L} + \frac{Z_3'}{l}\,. \tag{28}$$

5.1.5 Ultimate limit loads

For each plate, the enhancement factor is defined as

$$e_1 = e_{1m} + e_{1b}\,,\quad e_2 = e_{2m} + e_{2b}\,,\quad e_3 = e_{3m} + e_{3b}\,. \tag{29}$$

For the edge span, the limit load $P_{y,\,edge}$ based on Ref. [30] is

$$P_{y,edge} = \frac{6M_{01}}{3-2\alpha}\left[\frac{1}{\beta l^2} + \frac{1}{(1-\beta)l^2} + \frac{2\lambda}{\alpha L^2}\right],\ \lambda = M_{02}/M_{01} \tag{30}$$

where M_1 and M_2 are the positive bending moments of the short and long spans per unit, respectively.

For the edge span, the ultimate limit load $P_{\text{limit, edge}}$ is defined as

$$P_{\text{limit, edge}} = \frac{P_{y,\text{edge}}\left(e_1 L + 2e_2 l + e_3 L\right)}{\left(2L + 2l\right)}\,. \tag{31}$$

5.2 Second stress pattern II

For the middle span, the angle θ of Plate ① in Figs. 16(a)–16(b) is defined as [30]

$$\sin\theta = \alpha L / [\sqrt{(\alpha L)^2 + \left(\frac{l}{2}\right)^2}\,],\quad \alpha = (\sqrt{1+6r^2}-1)/(4r^2),\quad \beta = 0.5\,. \tag{32}$$

Owing to the symmetrical property, the ellipse equation is defined as

$$\frac{(x+A_x)^2}{a_R^2} + \frac{(y+B_y)^2}{b_V^2} = 1,\quad A_x = 0,\quad B_y = 0 \tag{33a}$$

$$a_R = \sqrt{b_V^2 + \left(\frac{L}{2} - \alpha L\right)^2},\quad b_V = \frac{\sqrt{\frac{(\alpha L)^2 + (l/2)^2}{(1+k)^2}} + \sqrt{\left(L - \alpha L - \frac{\alpha k l}{1+k}\right)^2 + \left(\frac{l}{2(1+k)}\right)^2}}{2}\,. \tag{33b}$$

Similarly, two parameters k and b are defined as

$$k=\frac{4\alpha r^2\left(1-2\alpha\right)}{4\alpha^2r^2+1}+1,\quad b=\frac{\left(\frac{l}{2}-x_c\right)\left(\frac{l}{4}+\frac{x_c}{2}\right)-\frac{x_c\left(l-2x_c\right)}{6}}{A-B+C+D+E}, \tag{34}$$

where

$$A=\frac{x_c\left(k^2l-l\right)}{12\left(1+k\right)}-\frac{x_c\alpha lL^2(1-2\alpha)}{12\left[\left(\alpha L\right)^2+(l/2)^2\right]},\quad B=\frac{1}{2}\left(\frac{L}{2}-\alpha L\right)^2,\quad E=\frac{l^2L^2(1-2\alpha)}{16\left[\left(\alpha L\right)^2+(l/2)^2\right]}$$

$$C=\frac{1}{2\left(1+k\right)}\left[\frac{l^2}{8\alpha}-\frac{1-2\alpha}{2\alpha}\left(\left(\alpha L\right)^2+(l/2)^2\right)-\frac{\left(\alpha L\right)^2+(l/2)^2}{3\left(1+k\right)}\right],\quad D=\frac{k^2}{2\left(1+k\right)}\left[\frac{\alpha L^2}{2}-\frac{k}{3\left(1+k\right)}\left(\left(\alpha L\right)^2+\frac{l^2}{4}\right)\right].$$

As shown in Figs.19(a)–19(b), for Plates ① and ②, the enhancement factors (e_{1m} and e_{2m}) owing to the moment are defined as

$$e_{1m}=\frac{M_{1m}}{M_{01}L}=\frac{4Kb}{3+g_1}\left(\frac{\omega}{d_1}\right)\left(1-2\alpha+\frac{\alpha\left(2-k\right)}{3}-\frac{l^2(1-2\alpha)}{8\left[\left(\alpha L\right)^2+(l/2)^2\right]}\right)+\frac{M_u}{M_{01}} \tag{35a}$$

$$e_{2m}=\frac{M_{2m}}{M_{02}l}=\frac{4b}{3+g_2}\left(\frac{\omega}{d_2}\right)\left(\frac{2-k}{6}+\frac{\alpha L^2(1-2\alpha)}{4\left[\left(\alpha L\right)^2+(l/2)^2\right]}\right). \tag{35b}$$

Meanwhile, for Plates ① and ②, Z_1, Z_2 and Z_3 are defined as

$$2\int_0^{\alpha L}\frac{M}{M_0}dx=2\alpha L\left[1+\frac{\alpha_1b}{2}(k-1)-\frac{1}{3}\beta_1b^2\left(k^2-k+1\right)\right]=Z_1 \tag{36a}$$

$$\int_0^{L-2\alpha L}\frac{M}{M_0}dx=(L-2\alpha L)(1-\alpha_1b-\beta_1b^2)=Z_2,\quad 2\int_0^{\frac{l}{2}-x_c}\frac{M}{M_0}dy=2(\frac{l}{2}-x_c)(1-K\alpha_2-\beta_2K^2)=Z_3\cdot \tag{36b}$$

For Plates ① and ②, the enhancement factors (e_1 and e_2) are defined as

$$e_{1b}=\frac{Z_1}{L}+\frac{Z_2}{L}+\frac{Z_3}{l},\quad e_{2b}=1+\frac{\alpha_2bK}{2}\left(k-1\right)-\frac{\beta_2b^2K^2}{3}\left(k^2-k+1\right) \tag{37}$$

$$e_1=e_{1m}+e_{1b},\quad e_2=e_{2m}+e_{2b}.$$

For the middle span, the ultimate limit load $P_{\text{limit, middle}}$ is

$$P_{\text{limit, middle}}=\frac{P_{y,\,middle}\times\left(2e_1L+2e_2l\right)}{\left(2L+2l\right)}=\frac{P_{y,\,middle}\times\left(e_1L+e_2l\right)}{\left(L+l\right)},$$
$$P_{y,middle}=\frac{12M_0}{3-2\alpha}\left[\frac{2}{l^2}+\frac{\lambda}{\alpha L^2}\right],\ \lambda=M_{02}/M_{01} \tag{38}$$

where $P_{y,\text{ middle}}$ is the yield-line load, based on Ref. [30].

Fig. 20 shows the flow chart for analysing the ultimate loads of concrete slabs based on the above equations.

5.3 Comparison analysis

In this study, the proposed ellipse method (P_e), conventional yield-line method (P_y), Bailey's method (P_b), and the steel strain difference method (P_s) were used to predict the residual ultimate loads of the tested slabs. The details of other methods are available in Refs. [17, 21, 31]. The residual mechanical properties of fire-damaged slabs are listed in Table 4.

As depicted in Figs. 5(a)–10(d), the tensile membrane action (blue ellipse region) and bottom steel yielding region (blue rectangular region) were predicted using the proposed method and reinforcement strain difference method, respectively, and they were compared with the cracks obtained from the tests. Note that the punching shear capacity of the fire-damaged slab should also be checked at the limit state, but this is beyond the scope of this paper. In addition, to be conservative, the concrete strength recovery was neglected in this study [32].

- Yield-line method

As indicated in Table 5, because the tensile membrane action was neglected, the P_y of the fire-damaged slabs was relatively conservative, and the ratio (P_y/P_u) ranged from 0.38 to 0.56, with an average value of 0.46. However, as discussed in Refs. [15-16], the P_y/P_u ratio ranged from 0.43 (0.73) to 0.86 (1.39), with an average value of 0.61 (1.07). As the span–thickness ratio decreased, the average P_y/P_u ratio gradually increased, indicating that the beneficial effect of the tensile membrane action gradually decreased. However, we can conclude that for a fire-damaged slab with any span–thickness ratio, the yield-line theory can be used to predict the conservative ultimate loads.

- Bailey method and steel strain difference method

As shown in Table 4, P_b and P_s were in good agreement with the experimental results. For instance, the P_b (P_s) $/P_u$ ratio ranged from 0.39 (0.52) to 0.63 (0.82), with the average value of 0.49 (0.65). Similarly, the predicted results were significantly conservative, particularly for Bailey's method.

On one hand, for Bailey's method, the conservative predictions were due to the inaccurate failure mode assumption and underestimated ultimate deflection (average value: $l/40$). For instance, the compressive force was assumed to be concentrated over a very small area near the edge of the slab [17]. Our experimental results indicated that a greater proportion of the slab was in compression near the edge of the slab, i.e. x_c (Figs. 5(a)–10(d)). In addition, the maximum deflection δ_b proposed by

Bailey was significantly conservative (Table 4).

On the other hand, for the steel strain difference method, the reinforcement yielding region (blue rectangular region) on the bottom surface of the fire-damaged slabs was noticeably smaller than the cracking region (Figs. 5(a)–10(d)), which resulted in conservative predictions.

- Ellipse method

As shown in Table 4, compared with the experimental results, the predicted results based on the proposed method were relatively better, including the residual limit loads (P_e) and limit deflection (δ_e). For instance, the P_e/P_u (δ_e/δ_u) ratio ranged from 0.81 (0.87) to 1.19 (2.13), with the average value of 0.97 (1.15).

As shown in Figs. 5(a)–10(d), the predicted tensile membrane action region (blue ellipse region) was in agreement with the mid-span cracking region. Meanwhile, a remarkable observation was that, for any span, the concrete crushing region at the corners of each span was outside the ellipse region edge, which indicated that the proposed ellipse equation is reasonable and effective.

Table 6 shows the key parameters (x_0 and y_0) predicted using the steel strain difference method and the proposed method. For each span, the membrane action region predicted using the proposed method was larger than that predicted using the steel strain difference method. As expected, for the latter, the smaller membrane action region resulted in lower limit loads (Table 4). In addition, for the proposed method, x_c (x'_c) is also provided in Table 6, indicating that the compressive membrane action region at the edge had a certain length. The experiment verified this observation, and no concrete crushing appeared at the middle region of the edge in each span (Figs. 5(a)–10(d)).

In summary, according to the companion papers [15-16], for fire-damaged slabs with lower span–thickness ratios (≤15), the yield-line method is suggested to predict the residual limit loads because the tensile membrane action is not sufficiently developed. In other words, the tensile membrane action method tends to overestimate the residual limit loads of fire-damaged slabs. In contrast, as the span–thickness ratio was larger than 30, the tensile membrane action method was suggested to predict the residual limit loads of the fire-damaged slabs, particularly for uniform loads, because it can sufficiently develop. More importantly, the effect of the negative moment should be considered when predicting the limit loads of the slab. Otherwise, the limit loads predicted using the tensile membrane action method (simply supported slab) were too conservative, such as for the reinforcement steel difference and Bailey methods.

6. Conclusion

This paper presents the experimental results of the ultimate capacity of five fire-damaged continuous RC slabs and one reference slab. In addition, considering the effect of the boundary condition, an ellipse equation method is proposed to determine the tensile membrane action region and ultimate loads of fire-damaged continuous slabs. The results obtained from the tested slabs were compared with those of other companion fire-damaged slabs and the theoretical results. Based on the investigation, the following conclusions were drawn:

(1) Compared with the travelling fire direction and delay time, the span–thickness ratio had a greater effect on the failure mode of the fire-damaged continuous slabs. As the span–thickness ratio increased, several flexural failure modes easily occurred in the fire-damaged slab, that is, concrete crushing at the corners, reinforcement fracture, larger top cracks near the interior support, and interior support dislocation between the middle and edge spans. In contrast, the flexural failure mode occurred easily in the fire-damaged slabs.

(2) For the continuous slabs with large span–thickness ratio (≥30), the initial structural stiffness, ductility, and ultimate loads of different spans were similar; this is because of the similar flexural mechanism of each span.

(3) The deflection failure criterion should be established by considering the effect of the span–thickness ratio. For a span–thickness ratio larger than 30 (or less than 20), the mid-span deflection $l/20$ ($l/50$) can be considered as the deflection failure criterion.

(4) The span–thickness ratio and effect of the boundary conditions should be considered to establish reasonable methods of predicting the ultimate loads of fire-damaged slabs. For a span–thickness ratio of the slab larger than 30 (or less than 20), the tensile membrane action method (the yield-line theory) is suggested to analyse the ultimate load of the fire-damaged slabs.

(5) The proposed ellipse equation method can be used to predict the tensile membrane action region, ultimate loads, limit mid-span deflections, and failure modes of fire-damaged continuous slabs with larger span–thickness ratios (≥30).

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Captions

Fig. 1. Details of the tested slabs (all dimensions in mm): (a) Reinforcement details; (b) typical layout of thermocouples in each slab; (c) thermocouples across the full-depth of each slab.

Fig. 2. Details of the test setup (all dimensions in mm): (a) Photograph of the test setup; (b) photograph of the support; (c) plan view of the test setup; (d) cross section 1-1 of the test setup.

Fig. 3. Details and instrument layout of the tested slabs (all dimensions in mm): (a) Layout of reinforcement and concrete strain gauges; (b) layout of vertical and horizontal displacement transducers.

Fig. 4. Average furnace temperature and concrete and steel temperature–time curves of five slabs: (a) Slab CS1-PF, (b) Slab CS3-PF, (c) Slab CS4-PF, (d) Slab CS5-PF and (e) Slab CS6-PF.

Fig. 5. Failure modes of Slab CS0 (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) crack pattern on the bottom surface.

Fig. 6. Failure modes of Slab CS1-PF (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) crack pattern on the bottom surface.

Fig. 7. Failure modes of Slab CS3-PF (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) crack pattern on the bottom surface.

Fig. 8. Failure modes of Slab CS4-PF (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) Crack pattern on the bottom surface.

Fig. 9. Failure modes of Slab CS5-PF (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) Crack pattern on the bottom surface.

Fig. 10. Failure modes of Slab CS6-PF (all dimensions in mm): (a) Photograph of cracks on the top surface; (b) crack pattern on the top surface; (c) photograph of cracks on the bottom surface; (d) crack pattern on the bottom surface.

Fig. 11. Mid-span vertical deflection-load curves of five tested slabs: (a) Slab CS0; (b) Slab CS1-PF; (c) Slab CS3-PF; (d) Slab CS5-PF; (e) Slab CS6-PF.

Fig. 12. Horizontal deflection-load curves of five slabs: (a) Slab CS0; (b) Slab CS1-PF; (c) Slab CS3-PF; (d) Slab CS5-PF; (e) Slab CS6-PF.

Fig. 13. Concrete and reinforcement strain-load curves of six slabs: (a) Slab CS0; (b) Slab CS1-PF; (c) Slab CS3-PF; (d) Slab CS4-PF; (e) Slab CS5-PF; (f) Slab CS6-PF.

Fig. 14. Stress patterns I and II of the tested slabs.

Fig. 15. Ellipse region, plates, and internal force distribution in the edge span of the concrete continuous slab (Stress pattern I) (a) Ellipse region; (b) Plate ①; (c) Plate ②; (d) Plate ①'; (e) Plate ③; (f) Plate ③'.

Fig. 16. Ellipse region, plates, and internal force distribution in the middle span of the concrete continuous slab (Stress pattern II) (a) Ellipse region; (b) Plate ①; (c) Plate ②; and (d) Plate ①'.

Fig. 17. Internal forces on the plates of the concrete slab (Stress pattern I) (a) Plate □; (b) Plate ②-Side AB; (c) Plate ②-Side AB'; and (d) Plate ③′.

Fig. 18. Two distances proposed in the model: (a) Horizontal distance x' (from Point B) and (b) vertical distance y' (from Point A).

Fig. 19. Internal forces distribution in the middle span of the concrete continuous slab (a) Plate ① and (b) Plate ②

Fig. 20. Flow chart for analysing the ultimate loads of concrete slabs.