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Copula Stochastic Volatility in oil returns: Approximate Bayesian Computation with volatility prediction

Audronė Virbickaitė [†] M. Concepción Ausín [‡] Pedro Galeano [§]

Abstract

Modeling the volatility of energy commodity returns has become a topic of increased interest in recent years, because of the important role it plays in today's economy. In this paper we propose a novel copula-based stochastic volatility model for energy commodity returns that allows for asymmetric volatility persistence. We employ Approximate Bayesian Computation (ABC), a powerful tool to make inferences and predictions for such highly-nonlinear model. We carry out two simulation studies to illustrate that ABC is an appropriate alternative to standard MCMC-based methods when the state transition process is challenging to implement. Finally, we model the volatility of WTI and Brent oil futures' returns with the proposed copula-based stochastic volatility model and show that such model outperforms symmetric alternatives in terms of in- and out-of-sample volatility prediction accuracy.

Keywords: ABC; Bayesian inference; Energy commodity returns; MCMC; Realized volatility.

Corresponding author

[†]Department of Quantitative Methods, Colegio Universitario de Estudios Financieros (CUNEF), Calle de Leonardo Prieto Castro, 2, 28040 Madrid, Spain, audrone.virbickaite@cunef.edu

[‡]Department of Statistics and UC3M-Santander Big Data Institute, Universidad Carlos III de Madrid, Getafe (Madrid), Spain, 28903, concepcion.ausin@uc3m.es

[§]Department of Statistics and UC3M-Santander Big Data Institute, Universidad Carlos III de Madrid, Getafe (Madrid), Spain, 28903, pedro.galeano@uc3m.es

1 Introduction

Crude oil is among the most actively traded commodities, and it is a major input in industrial production and transportation, among other industries. Therefore fluctuations of oil prices, as well as other energy commodity prices, deserve a special attention from the market participants and policymakers, since they extensively affect the development of the economy (Papapetrou, 2001). For that, many authors have used Generalized Autoregressive Conditional Heteroschedasticity (GARCH) models proposed by Engle (1982) and Bollerslev (1986) to model the volatility of commodity returns. For instance, Cheng and Hung (2011) used the conventional GARCH model coupled with flexible distributions for modeling petroleum and metal log returns. Also, Hou and Suardi (2012) used a non-parametric GARCH to model and forecast the volatility of crude oil prices and show that the out-of-sample volatility forecast of the proposed non-parametric model results into superior performance as compared to standard parametric GARCH models. Additionally, Efimova and Serletis (2014) used various univariate and multivariate GARCH models for modeling oil, natural gas, and electricity price volatilities in the United States. More recently, Billio et al. (2018) have proposed a new Bayesian multi-chain Markov switching GARCH model for hedging crude oil risk. GARCH-type models for crude oil volatility have been also considered by Sadorsky (2006), Wei et al. (2010) and Klein and Walther (2016), among many others.

Alternatively, other authors have used Stochastic Volatility (SV) models, proposed by Taylor (1982), for modeling energy commodity returns since these models provide more flexibility than GARCH-type specifications, see Kim et al. (1998), Yu (2002) and Broto and Ruiz (2004). Indeed, Chan and Grant (2016) compared numerous GARCH and SV models using nine series of commodity spot prices (oil, petroleum products and natural gas) using Bayes factors and found that the SV models generally compare favorably to their GARCH counterparts. Also, Baum and Zerilli (2016) considered several SV models to analyze the volatility of crude oil futures returns and found that SV models with jumps are more effective than SV models without jumps. Recently, Chen et al. (2019) used multiple SV models for modeling spot crude oil returns and showed that the best out-of-sample prediction results for measuring risk, namely Value-at-Risk, are produced by the traditional SV models with Normally distributed errors. SV-type models for crude oil volatility have also been considered by Sadorsky (2005) and Trolle and Schwartz (2009), among many others.

In early 2000's, high frequency trading data became available to practitioners and researchers alike, marking a shift in paradigm in volatility modeling. Andersen and Bollerslev (1998) introduced the realized volatility to accurately measure volatility, see Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004) for general overviews. Realized volatility is an actually observable volatility measure (unlike SV) and does not rely on any model assumptions (unlike GARCH), albeit in many cases is highly contaminated by market microstructure noise and is not always readily available. For modeling and forecasting purposes, Corsi (2009) proposed the HAR model that assumes that the log realized volatility follows an AR(22) process with Normal errors. Although other alternatives have been proposed, the HAR model remains very popular due to its simplicity and ability to forecast multiple steps ahead. RV measures can also be used for estimating conventional volatility models, such as GARCH or SV. For example, Baum and Zerilli (2016) used a conditional moment estimator for SV-type models based on matching the sample moments of RV with population moments of integrated volatility for crude oil futures price data.

The key assumption in these three conventional volatility models - GARCH, SV and RV - is the linear dependence structure. Call $r_t \quad h_t \; _t \quad F$ the de-meaned log returns with some distribution F that depends on parameters . Here h_t^2 is the volatility of the log returns, which can be modeled as one of the following:

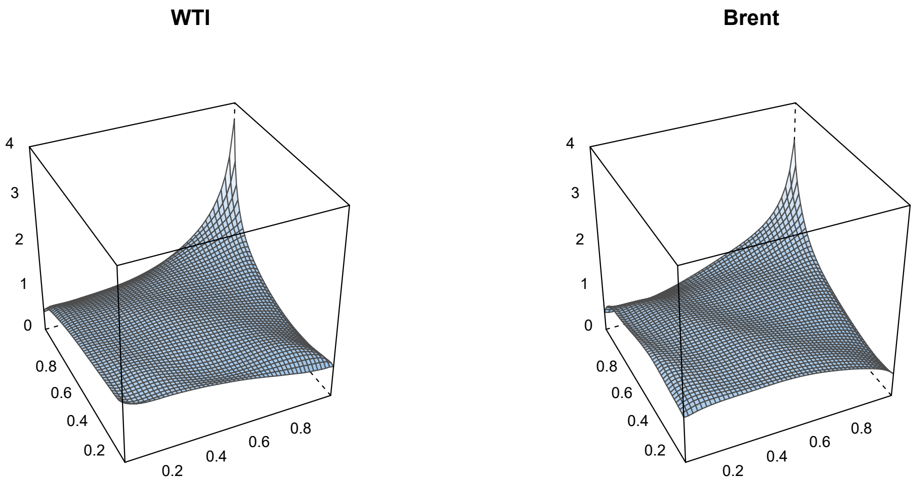
$$\text{GARCH:} \quad h_t^2 \quad _0 \quad _1 r_{t\ 1}^2 \quad _2 h_{t\ 1}^2, \tag{1}$$

$$\text{SV:} \quad \log h_t^2 \quad _0 \quad _1 \log h_{t\ 1}^2 \quad _0 \quad _2 \ _t, \quad _t \quad N\ 0,1\ , \tag{2}$$

$$\text{HAR:} \quad \log h_t^2 \quad _0 \quad _1 \log h_{t\ 1}^2 \quad _2 \log \overline{h}_{t\ 1,5}^2 \quad _3 \log \overline{h}_{t\ 1,22}^2 \quad _4 \ _t, \quad _t \quad N\ 0,1\ , \tag{3}$$

where $\overline{h}_{t\ 1,i}^2$ is the average volatility over the last i days. However, the assumption of linear temporal dependence in the above models is not in line with the actually observed behavior of the volatilities, since in practice many financial time series exhibit asymmetry not only in the mean but also often in the variance, see Chen et al. (2009). As an illustration of non-linear dependence in the variances, consider 1500 observations

Figure 1: Kernel density estimates between probability integral transforms of squared returns and lagged squared returns.



of de-meaned log returns for two oil futures - WTI and Brent - that are analysed in real data application section. Since for the return data the volatility is not observed, the most commonly used proxy for volatilities is the squared returns. Figure 1 plots the kernel density estimators of dependence between u_t and u_{t-1}, where u_t is the empirical probability integral transform of squared returns. In other words, the marginals of the squared returns are modeled non-parametrically, and then we fit a kernel density estimator on the uniformly distributed data and its first lag. As seen from the plots, there is positive dependence between the volatility proxies. This is in line with the financial econometrics literature, where the estimated volatility persistence in models (1)-(3) is always positive. However, we can also observe increased dependence in the upper tail, a feature that cannot be captured using conventional linear models. This stylized feature is characteristic not only for oil futures returns, but for most of the publicly traded assets and index returns.

In order to incorporate such asymmetry, the benchmark linear model (SV, GARCH or RV) can be extended by allowing for structural breaks, regime-switches or jumps, but then the model quickly loses its parsimonious representation. For example, Fong and See (2002) found strong evidence of regime shifts in GARCH volatility of the WTI daily oil futures prices. Vo (2009) used regime switching SV model for crude oil prices and found clear evidence of regime switching in the oil market; similar findings were also present in Chen et al. (2019) for crude oil spot price data. Larsson and Nossman (2011) and Brooks and Prokopczuk (2013) both found the jump-in-volatility parameter to be statistically significant for the crude oil returns in the SV with jumps model. Sokolinskiy and van Dijk (2011) proposed to use copula based time series models to capture the non-linear dependence structure observed in RV data. The authors found that their proposed model outperforms the HAR model in 1-day-ahead volatility forecasts. Moreover, the Gumbel copula model achieved the best forecast performance, meaning that the volatility persistence is asymmetric. This article is motivated by the model in Sokolinskiy and van Dijk (2011), however, we allow for even more flexibility by considering the SV model, where the volatilities are actually not observed, giving rise to a copula stochastic volatility (CSV) model. As mentioned before, the RV measure has a very attractive property of being an actually observed volatility measure although it is not always readily available.

The use of copulas in stochastic volatility setting is only recently gaining popularity. Loaiza-Maya et al. (2018) considered mixture copulas to capture serial dependence in heteroskedastic time series and derived new measures of volatility persistence. These authors applied their proposed model to daily foreign exchange returns and compared it to the conventional GARCH specifications. Smith and Maneesoonthorn (2018) proposed a new class of the so called inversion copulas that are constructed by inverting parametric nonlinear state space models, such as the stochastic volatility model for example. Their approach allows to combine the same serial dependence as a stochastic volatility model, but with arbitrary margins that can be asymmetric. The authors illustrate their methodology to model and forecast the U.S. quarterly inflation. Kreuzer and Czado (2018) considered modeling multivariate time series by using a factor copula with stochastic volatility models for the univariate marginal distributions. Additionally, Ibragimov and Lentzas (2017) investigated the persistence properties of copula-based time series. They showed via simulations that stationary Markov processes generated by Clayton copulas may exhibit a spurious long memory-like behavior. This finding advocates the use of copulas for volatility modeling, since the realized volatility measures might exhibit long-memory like behavior. Also, the authors applied the survival Clayton (rotated Clayton) copula to a GARCH-type model using financial time series data and showed that it performs better than Clayton copula and the simple GARCH model, indicating an important finding: the volatility persistence has a stronger upper-tail dependence.

As for estimation, we employ a novel Bayesian estimation technique called Approximate Bayesian Computation (ABC), see Marin et al. (2012) for a general introduction. ABC is especially well suited for models with intractable likelihoods or when the state transition process is challenging to implement within an exact algorithm, and when the model is easy to simulate from. As noted by some authors, ABC can be seen as the Bayesian version of the indirect inference (Gourieroux et al., 1993). Since the model proposed in this paper is highly non-linear and involves latent states, ABC seems like a good choice. This paper makes use of a special ABC variant, namely, ABC based on the scores of the auxiliary model, a methodology proposed in Martin et al. (2019). This approach is computationally less demanding than other ABC alternatives, and, since the ABC algorithm does not involve loops, we are able to parallelise the algorithm reducing the computational cost dramatically as compared to the possible MCMC-based alternatives. Important to mention, that ABC is not the only possible estimation strategy and particle MCMC (Andrieu et al., 2010) or SMC^2 (Chopin et al., 2013) are perfectly valid alternatives.

The rest of the paper is organized as follows. Section 2 introduces the CSV model and the estimation algorithm. Section 3 contains two simulation examples to illustrate the ABC's ability to recover the true model parameters for a conventional SV model and for a newly proposed CSV model. Section 4 presents the modeling of the volatility of WTI and Brent oil futures' returns with the proposed copula-based stochastic volatility model and evaluates model performance in volatility prediction. Finally, Section 5 concludes.

2 Model and estimation

In this section we shortly introduce copula-based time-series models and their main uses. We then propose a more general version of the conventional SV model that is able to capture non-linear volatility dependence via such copula time series models. Finally, we describe in detail the use of ABC in our specific setting.

2.1 Copula based time series

The construction of flexible multivariate distributions using copulas has started with the seminal work of Sklar (1959). For a formal introduction and details on copulas the reader is referred to the books of Nelsen (2006) and Joe (2015), and for applications of copulas in the context of financial time series, see Patton (2009), among others.

Nelsen (2006) defines copulas in the following manner. Consider a collection of random variables $Y_1, \ldots, Y_d$ with corresponding distribution functions $F_i \; y_i \quad P \; Y_i \quad y_i$ for $i \quad 1 \ldots, d$ and a joint distribution function $H \; y_1, \ldots, y_d \quad P \; Y_1 \quad y_1, \ldots, Y_d \quad y_d$. Then, according to a theorem by Sklar (1959), there exists a copula C such that

$$H \; y_1, \ldots, y_d \quad C \; F_1 \; y_1 \; , \ldots, F_d \; y_d \quad .$$

In other words, it is possible to model the univariate marginals and the dependence structure separately. Copulas are defined in the unit hypercube $0, 1 \; ^d$, where d is the dimension of the data, and all univariate marginals are uniformly distributed.

The majority of the copula-related literature focuses on modeling contemporaneous dependence between multiple time series. For example, Liu et al. (2017) use time-varying copula models to investigate the dependence between oil returns and crude oil volatility index, meanwhile Ji et al. (2019) investigate the dynamic dependence between crude oil and the exchange rates of the United States and China also using time-varying copula models. Ho et al. (2019), on the other hand, use non-parametric copulas to analyze tail dependence of crude oil price returns between four major markets.

Nonetheless, copulas also permit to model the temporal dependence of a univariate time series, as noted in Chen and Fan (2006). The use of copulas in modeling temporal dependence of univariate time series relates to Markov processes and have been described in Darsow et al. (1992), and Joe (2015), for example. Depending on the copula family, it is possible to model non-linear temporal dependencies, as opposed to the standard linear regression type models. Chen and Fan (2006), Ibragimov (2009), Ibragimov and Lentzas (2017) explore the relationship between Markov processes and copula functions for univariate time series.

As described in Chen and Fan (2006), let Y_t be a stationary first order Markov process whereas its probabilistic behavior is completely defined by the joint distribution function H between $Y_{t\ \ 1}$ and Y_t. On the other hand, as seen above, using Sklar's theorem, this joint can be expressed using a copula representation $H\ \ y_t, y_{t\ \ 1} \quad C\ \ F\ \ y_t\ \ , F\ \ y_{t\ \ 1}\ \ ,$ where F is a marginal cumulative distribution function (CDF) of Y_t and is a set of copula parameters. This formulation allows to model a stationary Markov process using a copula, where the transition probability is constant over time. Let h be the joint density of Y_t and $Y_{t\ \ 1}$, and f the corresponding marginal probability density function (PDF) of Y_t. Then h can be expressed as a product of the marginals and a copula density, which defines the dependence structure:

$$h\ \ y_t, y_{t\ \ 1} \quad c\ \ F\ \ y_t\ \ , F\ \ y_{t\ \ 1} \quad f\ \ y_t \quad f\ \ y_{t\ \ 1}\ \ ,$$

and the conditional distribution of y_t given $y_{t\ \ 1}$ is

$$f\ \ y_t\ y_{t\ \ 1} \quad h\ \ y_t, y_{t\ \ 1}\ \ / f\ \ y_{t\ \ 1} \quad c\ \ F\ \ y_t\ \ , F\ \ y_{t\ \ 1} \quad f\ \ y_t\ \ .$$

The parameter completely determines the dependence structure which is constant over time. Then the collection of Y_t follows a stationary first order Markov process with constant transition probabilities.

As mentioned before, Sokolinskiy and van Dijk (2011) employ the described copula time series approach for modeling the RV, where RV is an observable volatility measure. In the following section we introduce a copula stochastic volatility model, in which, differently than in Sokolinskiy and van Dijk (2011), the volatilities are unobserved.

2.2 Copula Stochastic Volatility model

Define r_t as the de-meaned log returns (in %) for day t:

$$r_t \quad 100 \quad \left(\log \frac{P_t}{P_{t\ \ 1}} \quad \mathrm{E}\left[\log \frac{P_t}{P_{t\ \ 1}}\right]\right), \qquad t \quad 1, \ldots, T,$$

where $P_{t\ \ 1}$ and P_t are the prices at the beginning and at the end of the period, respectively. Then, the complete model for the log returns has the following form, where the dynamics of the standardized log volatility x_t can be modeled via a copula specification:

$$
\begin{aligned}
r_t \quad h_t \;{}_t, \quad {}_t \quad F \;, \\
\log h_t^2 \qquad\quad x_t, \quad x_t \quad N\; 0, 1 \\
f\; x_t\; x_{t\;\;1} \qquad c \qquad x_t\;, \quad x_{t\;\;1} \qquad x_t\;.
\end{aligned}
\tag{4}
$$

Here only r_t is observed, and are the parameters for the Normal marginals for the log volatility, meanwhile are the copula-related parameters. Call , , the complete set of model parameters. Also, define Kendalls' tau, also known as rank correlation coefficient, $0, 1$, such that there is a one-to-one copula-specific transformation between and (at least for the copulas we consider in this work). Here stands for 'Kendall' and we use the subscript in order to differentiate from the standard deviation parameter . As shown in Chen and Fan (2006), if both marginals are Normal and the copula is bivariate Gaussian, the above copula time series model reduces to the well-known AR(1) process, i.e., the standard SV model. The model in (4) is a non-linear state-space model.

Throughout the paper, we have considered that ${}_t$ is Normally distributed, i.e. F $N\; 0, 1$. At first glance, the assumption of Normality can be seen as being rather restrictive. Nonetheless, there is evidence in the literature that modeling volatility as a latent, instead of a deterministic, process can capture excess kurtosis in the distribution of the returns, even if ${}_t$ is Normally distributed. As noted in Brooks and Prokopczuk (2013), higher values of the variance of the log-volatility (called in our model) can capture higher levels of kurtosis in the returns. Chan and Grant (2016) fitted GARCH and SV models with t-distributed errors on the WTI data. The authors found that degrees of freedom parameter increased significantly in the SV model (from 11 in GARCH-t model to 56 in SV-t model), indicating that the tails of the t-distributed returns are thin and similar to those of the Gaussian distribution. Their explanation is that SV-type models are inherently more flexible, making the necessity of the t-distributed errors less apparent; in other words, SV-type models are less sensitive to misspecification. Finally, Chen et al. (2019) used daily spot returns of the crude oil markets (Brent and WTI) and found that conventional SV models with Normally distributed errors perform the best according to several out-of-sample metrics, even compared to the more flexible t and asymmetric

Laplace distributed errors.

Therefore, we argue that given the main objective of the paper - volatility forecasting[1], and the ability of the SV-type models to capture at least some of the kurtosis, the distributional assumption of the returns becomes secondary. However, if the ultimate goal were to model and forecast the entire distribution of the returns rather than just the volatility, a more flexible specification should definitely be considered.

2.3 Approximate Bayesian Computation

Note that in the model in (4) the latent log volatilities x_t are not observed, thus the model has a complicated likelihood function involving multidimensional integrals. Even though in principle the likelihood-based inference is feasible, it is, however, complicated. Thus we choose to employ ABC as one of possible estimation techniques, especially since we are able to simulate from the model with relative ease. We make use of the procedure described in Martin et al. (2019), which is based on the score of an auxiliary model. These authors illustrate the proposed method by using three stochastic volatility models that are challenging to estimate via standard MCMC or sequential Monte Carlo (SMC) methods. The method selects a simple auxiliary model that approximates the features of the true data generating process (DGP). Then the sufficient statistic, a key ingredient in the ABC methods, is simply the score of the auxiliary model. The auxiliary likelihood-based ABC, as described in Martin et al. (2019), is as follows:

1. Obtain $\hat{}_{MLE}\ r_{1:T}$ from the simple auxiliary model using observed data $r_{1:T}$ $r_1, \ldots, r_T$.
2. Simulate M values of the parameters of interest from the priors: ^{m} , , ^{m} , where m $1, \ldots, M$. Here M is the size of the ABC sampler.
3. Given ^{m} , simulate M datasets $z^{m}_{1:T}$ of size T each from our proposed model.
4. Evaluate each dataset at the score of the auxiliary model given the MLE parameters $S\ z^{m}_{1:T};\ \hat{}_{MLE}\ r_{1:T}$, the closer the score is to zero, the closer the simulated data are to the true data.

[1]Precise volatility forecasts are essential ingredients in option pricing and risk management models.

5. Calculate the distances of M scores:

$$d^{m} \quad \sqrt{S \; z_{1:T}^{m} ; \hat{\ }_{MLE} \; r_{1:T} \qquad S \; z_{1:T}^{m} ; \hat{\ }_{MLE} \; r_{1:T}} \; ,$$

   where is a weighting matrix, for example, the variance covariance matrix of $\hat{\ }_{MLE} \; r_{1:T}$.

6. Select those parameter values ^{m} , , ^{m} that give the smallest P distances d^{m} .

Step 1 is done only once at the beginning of the estimation procedure. Step 2 is straightforward. In Step 3 the conditional density from the CSV model in (4) is of non-standard form:

$$f \; x_t \; x_{t \;\; 1} \qquad c \qquad x_t \; , \quad x_{t \;\; 1} \qquad x_t \; ,$$

$$F_x \; x_t \; x_{t \;\; 1} \qquad \int_{\infty}^{x_t} f \qquad x_{t \;\; 1} \; d \quad .$$

Evaluating $F_x \; x_t \; x_{t \;\; 1}$ requires numerical integration and then in order to sample from this conditional distribution we would need to solve this integral, which is computationally costly. Luckily, there is a more efficient way of how to draw samples from the copulas belonging to the Archimedean family by using the so called h-functions, see Aas et al. (2009) for more details. Step 4 involves the evaluation of S for a simple auxiliary model at the simulated data, most likely using a numerical differentiation when S is not known in closed form (Martin et al., 2019). Steps 5 and 6 are also straightforward. The auxiliary model has to approximate the features of the true DGP and can be estimated with relative ease. Same as in Martin et al. (2019), we consider some simple auxiliary models, such as N-GARCH, t-GARCH or GJR-GARCH with Normal errors.

Finally, in order to calculate the size of the ABC sample, M, we first need to choose a certain tolerance level. For example, Marin et al. (2012) consider tolerance level equal to the 0.1% quantile of the sample of the distances. On the other hand, Martin et al. (2019) choose the tolerance level that is a function of the sample size: $50/T^{3/2}$. Same as in Martin et al. (2019), we fix a sample of size P 250 from the posterior to be retained. For example, in a sample of T 1000 observations the tolerance level turns out to be

0.158%, and the corresponding ABC sample size is $M \quad 158114$. Overall, the choice of the tolerance level presents a trade off: when it goes to zero, the ABC algorithm becomes exact, however, smaller tolerance levels are associated with higher computational costs (Marin et al., 2012).

2.4 Bootstrap filter

As noted in Martin et al. (2019), the proposed ABC technique focuses on estimation of the static parameters only, and marginal inference on the latent states, if necessary, can be carried out at a second stage. In particular, once we have the accepted draws from the posterior $p \quad , \quad , \quad r_{1:T}$ we can make use of existing filtering and smoothing methods to obtain draws from the posterior distributions of the latent states x_t in (4). Since the model of interest contains extreme non-linearities, exact Kalman-type filters are not available. Thus we rely on Sequential Monte Carlo (SMC) methods, also known as particle filters, to produce draws that approximate the posterior latent states. For a general review of the SMC filters with illustrations refer to Lopes and Tsay (2011), among others.

In particular, we rely on a bootstrap filter (BF), also known as the sequential importance sampling with resampling (SISR) filter, which was introduced by Gordon et al. (1993). BF is a propagate-resample type filter: first it propagates the latent states from $t \quad 1$ to t by sampling from a transition density and then it resamples the propagated particles with weights proportional to the predictive density at time t. The model in (4) can be re-written as a generic state-space model:

$$r_t \; x_t \quad p \; r_t \; x_t \; , \tag{5}$$

$$x_t \; x_{t \quad 1} \quad p \; x_t \; x_{t \quad 1} \; . \tag{6}$$

Here (5) is an observation equation, oil spot log returns in our case, and (6) is the transition equation, latent log volatilities in our case. All static parameters, governing the dynamics of observation and state transition equations are known (estimated via ABC previously). Then, the BF, for each particle j, such that $j \quad 1, \ldots, P$, iterates through the following steps:

Step 1: Propagate $x_{t\ 1}^{j}\ {}_{j\ 1}^{P}$ to $\tilde{x}_t^{j}\ {}_{j\ 1}^{P}$ through $p\ x_t\ x_{t\ 1}$.

Step 2: Resample $x_t^{j}\ {}_{j\ 1}^{P}$ from $\tilde{x}_t^{j}\ {}_{j\ 1}^{P}$ with weights proportional to $p\ r_t\ \tilde{x}_t^{j}$.

Here we set the number of particles equal to P which is equal to the sample size of the posterior ABC output.

3 Simulation studies

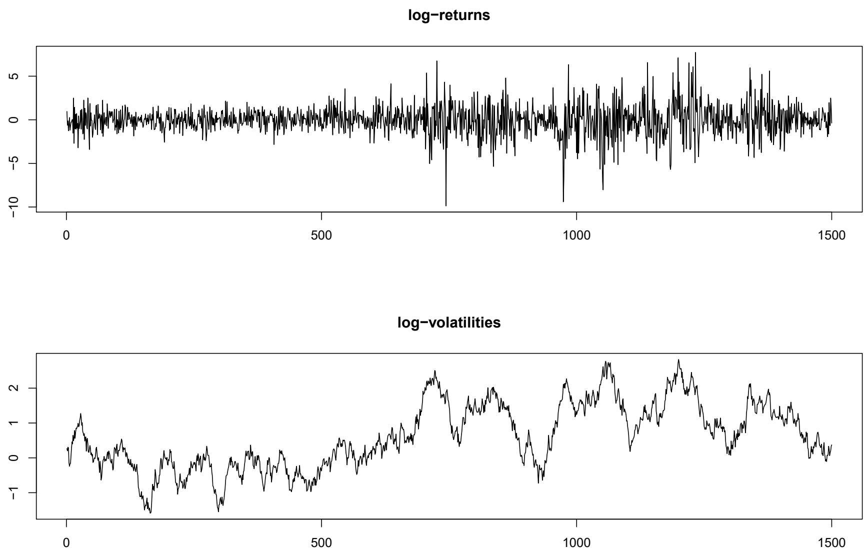
This section contains two simulation studies. The objective of the first simulation study is to investigate the performance of the ABC estimation technique for a simple SV model and compare the posterior output with the MCMC. The goal of the second simulation study is to investigate if the ABC estimation technique is able to recover the true parameter values using the simulated data from the new proposed model, where MCMC estimation procedure is computationally demanding and there are no readily available routines.

3.1 Simulation study I: SV model

Since the seminal paper of Jacquier et al. (1994), numerous MCMC schemes have been proposed for estimating SV-type models. In this simulation study we investigate the performance of the ABC estimation technique for a simple SV model and compare the posterior output with the MCMC. In particular, given the exact same priors, we compare the posterior densities for the parameters and filtered volatility states for both alternative estimation methods, MCMC and ABC. This simulation exercise also allows us to compare the three different ABC variants, that are based on three different auxiliary models (N-GARCH, t-GARCH and GJR-GARCH). For this purpose, we simulate T 1500 observations from a volatility-linear SV model in (2) with Normal errors. We choose some realistic parameter values: $_0$ 0.35, $_1$ 0.99, $_2$ 0.14. The priors are as in Kastner and Frühwirth-Schnatter (2014): $_0$ $N\ 0, 10$, $_1$ 1 /2

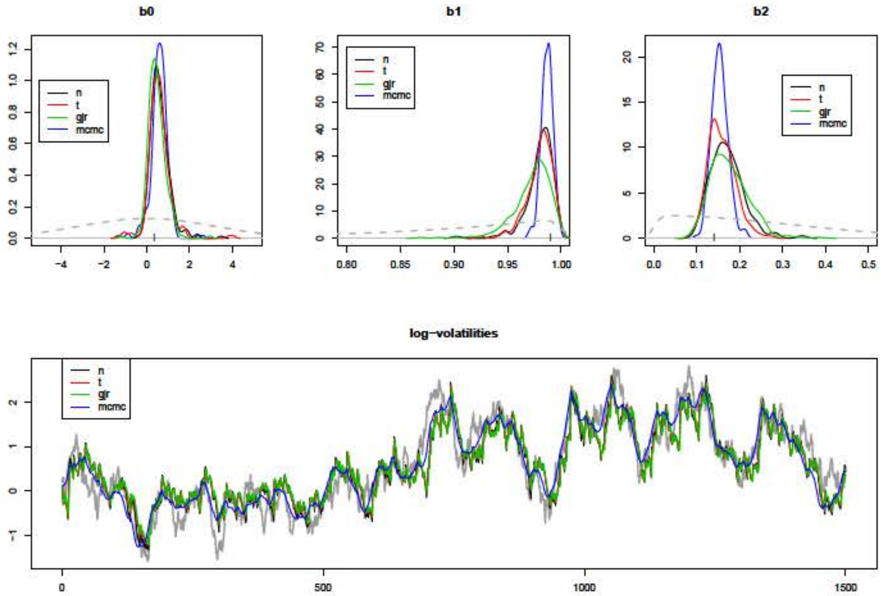
B 15,1 , $^{2}_{2}$ G 0.5,2.5 . The prior for the mean is very uninformative. Kastner and Frühwirth-Schnatter (2014) propose using such hyperparameter values for the prior on the persistence parameter that the prior mean is E $_{1}$ 0.99 and prior standard deviation is SD $_{1}$ 0.022. We opt for a less informative prior such that E $_{1}$ 0.87 and SD $_{1}$ 0.2. Simulated data can be seen in Figure 2. As we can see, the simulated data resembles the actually observed return series and exhibits such stylized features as volatility clustering.

Figure 2: Simulated data from the volatility-linear Normal SV model, T 1500.



For ABC estimation we consider N-GARCH, as seen in (1), t-GARCH and GJR-GARCH as auxiliary models. ABC sample size was chosen using the tolerance level of 0.086%, retaining 250 parameter sets that yield the smallest distances. The MCMC estimation was carried out using the `stochvol` package in `R`, see Kastner (2017), which is based on the ancillarity-sufficiency interweaving approach of Kastner and Frühwirth-Schnatter (2014). Figure 3 contains the estimation results. In general, all three auxiliary models produced very similar posterior densities for model parameters, that are also very similar to the MCMC output. Same goes for the filtered log volatilities. Note that the ABC posterior densities for persistence parameter are wider, but, as seen in Martin et al. (2019), MCMC and ABC dot not necessary produce the exact posteriors. Important to note that MCMC outputs present smoothed volatility estimates, meanwhile the BF procedure provides the filtered volatility states. Nonetheless, if necessary, smoothing can be performed for BF as well. Thus in the subsequent simulation study and real data application we will use N-GARCH as an auxiliary model because there are no apparent differences in the posterior output and it has the smallest computational cost.

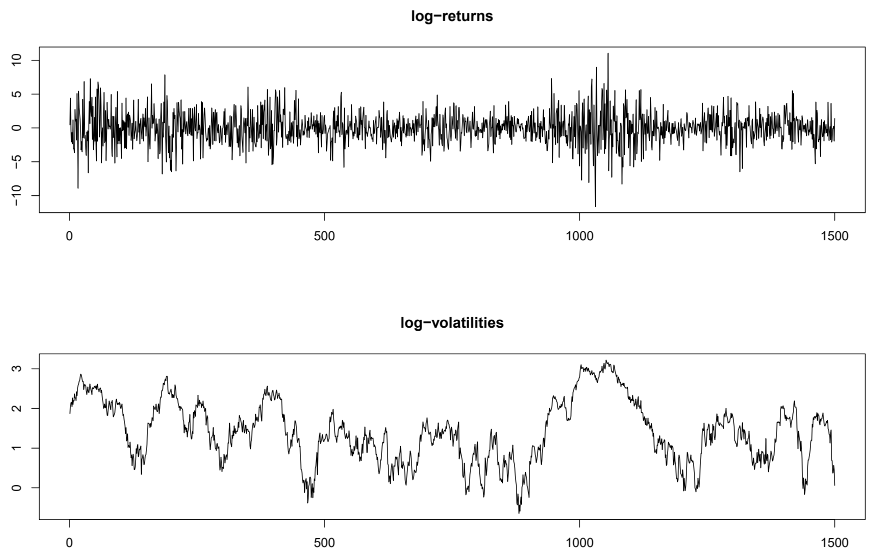
Figure 3: Simulated data from the volatility-linear Normal SV model, $T = 1500$, ABC estimation results. Top plots: MCMC is in blue and priors are in grey, bottom plot: MCMC is in blue and the true latent log volatility is in grey. Letters n, t and gjr present ABC schemes based on three different auxiliary GARCH models.



3.2 Simulation study II: CSV model

Here we investigate the performance of the ABC estimation approach using the synthetic data from the newly proposed copula stochastic volatility model. In this case MCMC estimation scheme is not readily available and is computationally demanding given extreme non-linearities in both, transition and observation, equations. We show that the ABC estimation method is able to recover the true parameter values and the BF is capable of filtering out the latent states. For that purpose, we simulate $T = 1500$ observations from a model in (4) with $\mu = 1.5$, $\tau = 1$ and $\tau_\kappa = 0.9$ (corresponding $\psi = 10$) from a Gumbel copula. The parameter values are chosen to be similar to the ones produced by fitting real data to the CSV model. Compared to the simple SV model, Gumbel copula based model allows for asymmetric serial dependence in the latent volatility process, at the same time maintaining parsimonious model representation. Here we estimate rather than the copula parameter , since always has the support in 1, 1 and there is a copula-specific one-to-one transformation between and . The priors for the mean and persistence are the same as in the conventional SV model, and the prior for the variance is ^{2} G 0.5, 0.5 , which approximately corresponds to the induced prior on the unconditional variance of the SV model. Figure 4 presents the simulated dataset, meanwhile Figure 5 presents the estimation results. As we can see, the N-GARCH-based auxiliary ABC is able of accurately estimating the parameters and BF is able to filter out the latent volatility states.

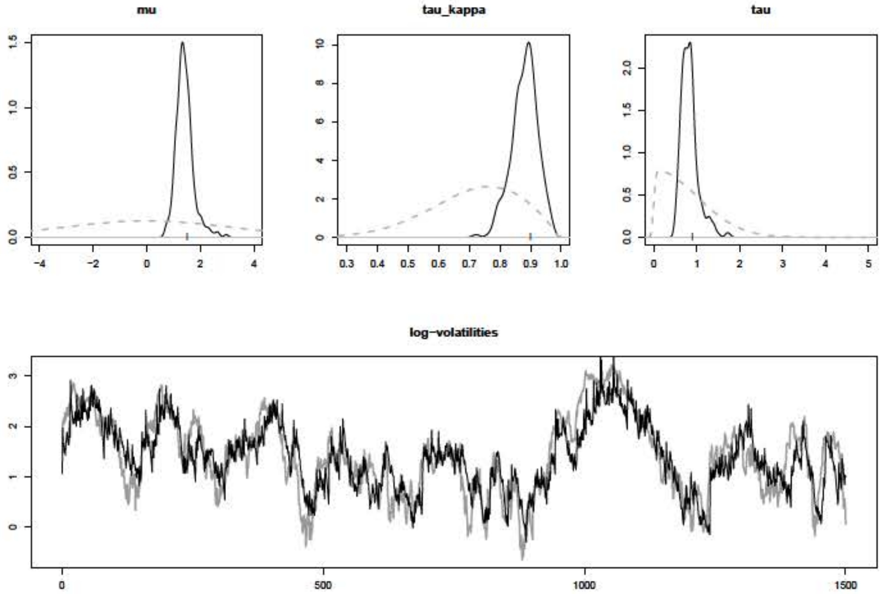
Figure 4: Simulated data from the CSV-Gumbel model, T 1500.



4 Real data application

In real data study we consider daily NYMEX Light Sweet Crude Oil (WTI - West Texas Intermediate) Electronic Energy Future Continuation and ICE Brent Crude Electronic Energy Future Continuation obtained from Thomson Reuters Eikon Database. WTI data is from 2013-04-03 till 2019-03-14, resulting into a sample size of 1500 daily log returns, meanwhile Brent data is from 2013-05-27 till 2019-03-19, which makes it 1500 returns,

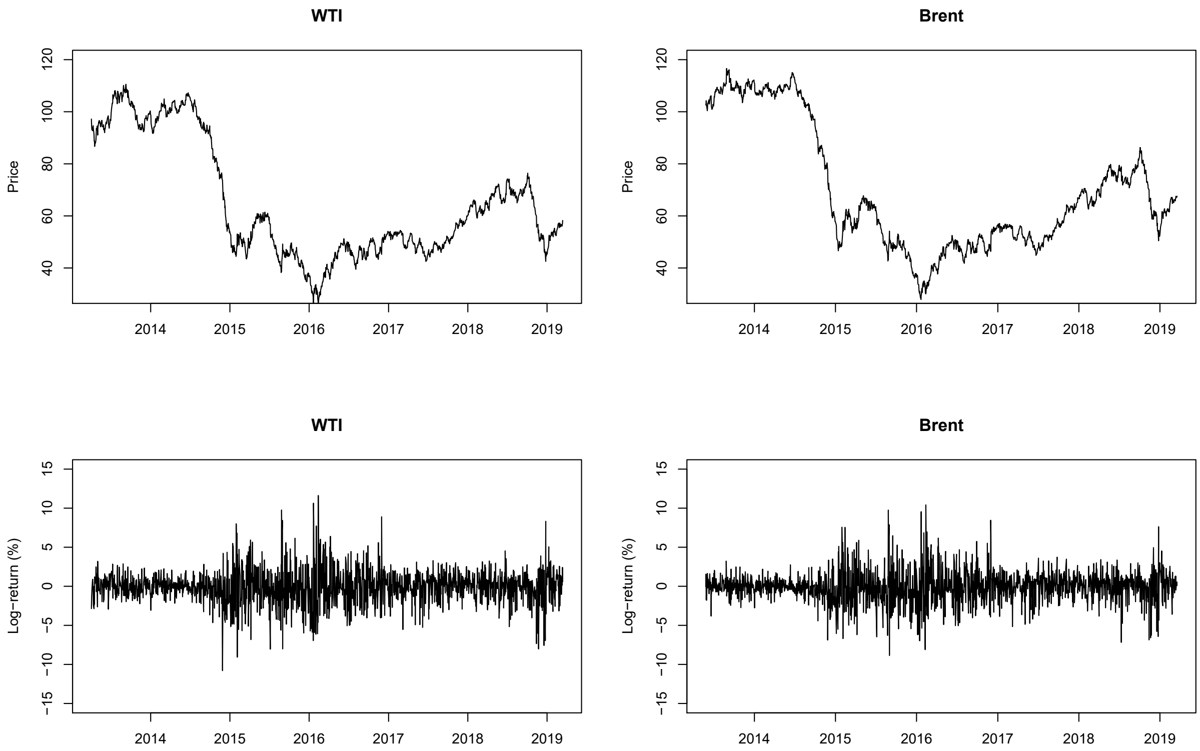
Figure 5: Simulated data from the CSV-Gumbel model, $T = 1500$, ABC with N-GARCH auxiliary model estimation results. Top plots: priors are in grey and posteriors in black, bottom plot: the true latent log volatility is in grey and filtered mean log volatility in black.



see Figure 6. We fit the model in (4) with $F_\lambda \equiv N(0,1)$ and we consider four copula specifications: Normal, Gumbel, Joe and rotated Clayton, see Table 1. Normal copula corresponds to the conventional variance-linear Normal SV model and we use it as a benchmark. ABC estimation scheme is the same as described before, including the priors.

Estimation results. Figures 7 and 8 draw the prior and posterior densities for models parameters. Overall, the posterior densities and posterior means for all copulas parameters are rather similar to each other. We apply a bootstrap filter to filter out the latent log volatilities, see Figures 9 and 10. The grey lines are filtered latent volatilities, meanwhile the black lines draw the empirical 30-day rolling window variance. As expected, the estimated volatilities exhibit more abrupt changes as compared to the rolling window estimate, since the latter presents a smoothed version of volatility. Also, Joe copula estimated volatilities are more smooth. Finally, Figures 11 and 12 draw the estimated copulas for the latent log volatilities for both datasets and both models. Because of the unobservable nature of the underlying volatility, we cannot compare these estimated

Figure 6: WTI and Brent daily closing prices and de-meaned log returns (in %), 1500 observations each.



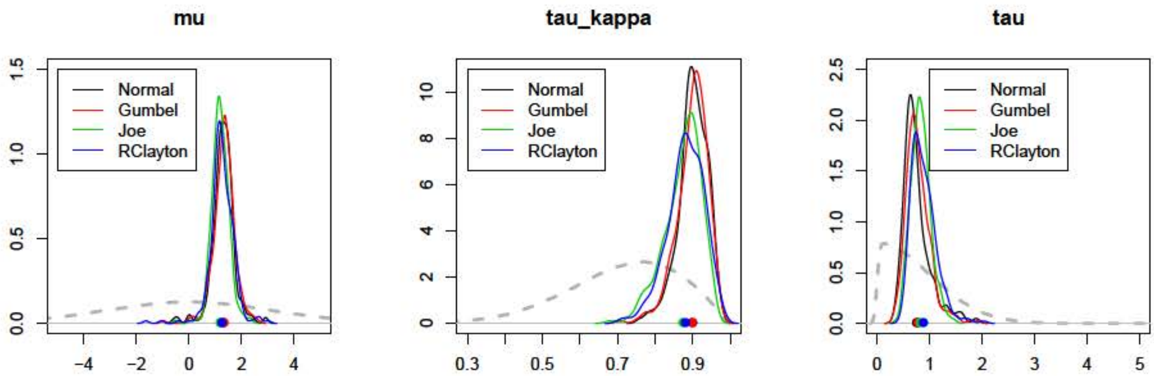
copulas to some observed data. However, these estimated surfaces should resemble the plots in Figure 1, which draw the copula surfaces for the closest proxy of actual volatility - squared returns. Just by eye-balling the plots it is obvious that the Normal copula does not resemble the actual observed data, because the dependence structure is symmetric, as in standard SV models. The copulas that resemble the most the actual observed features of the squared returns are Joe and rotated Clayton. In order to examine in more detail which is the most adequate model specification, we evaluate the in- and out-of-sample model fits.

In-sample model performance. As mentioned before, the most commonly used proxy for the volatility of the returns are the squared returns (Ghysels et al., 2006). We also consider the 15, 30 and 45-day rolling window variances as another possible proxy. Therefore, for this model comparison exercise we filter out the volatilities for the entire dataset (1500 observations) and calculate the Root Mean Squared Error (RMSE) with respect to

Table 1: Gaussian, Gumbel, Joe and Clayton copulas: their CDFs and Kendall's τs.

Gaussian:	$C(u,v;\psi) = \Phi_2(\Phi^{-1}(u), \Phi^{-1}(v); \psi),\ \psi \in [0,1]$ $\tau_\kappa = 2\arcsin(\psi)/\pi,\ \psi = \sin(\pi\tau_\kappa/2)$
Gumbel:	$C(u,v;\psi) = \exp\left\{-\left[(-\log u)^\psi + (-\log v)^\psi\right]^{1/\psi}\right\},\ \psi \in [1,\infty)$ $\tau_\kappa = (\psi-1)/\psi,\ \psi = 1/(1-\tau_\kappa)$
Joe:	$C(u,v;\psi) = 1 - \left[(1-u)^\psi + (1-v)^\psi - (1-u)^\psi(1-v)^\psi\right]^{1/\psi},\ \psi \in [1,\infty)$ No closed form expression between τ_κ and ψ (numerical inversion)
Clayton:	$C(u,v;\psi) = (u^{-\psi} + v^{-\psi} - 1)^{-1/\psi},\ \psi \in [0,\infty)$ $\tau_\kappa = \psi/(\psi+2),\ \psi = 2\tau_\kappa/(1-\tau_\kappa)$

Figure 7: WTI dataset: ABC posterior parameter densities, priors are in grey.



the two proxies:

$$\text{RMSE}_{rw}^{k} = \frac{1}{P}\sum_{i=1}^{P}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma_t^{2rw} - h_t^{2(i)})^2},$$

$$\text{RMSE}_{r^2} = \frac{1}{P}\sum_{i=1}^{P}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t^2 - h_t^{2(i)})^2}.$$

Figure 8: Brent dataset: ABC posterior parameter densities, priors are in grey.

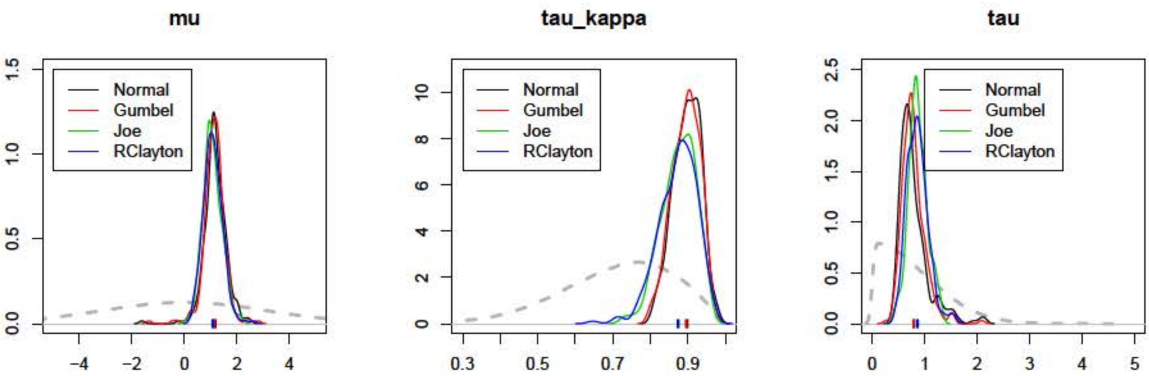
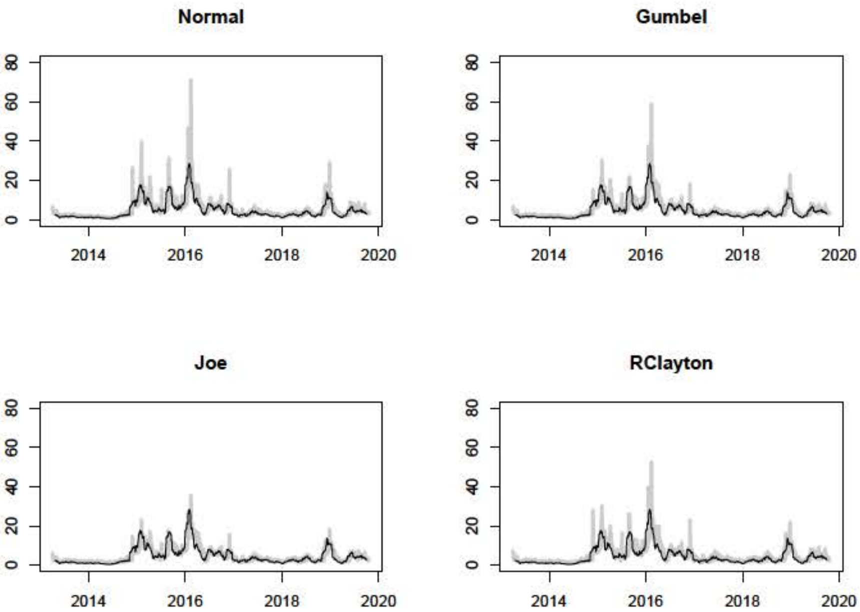
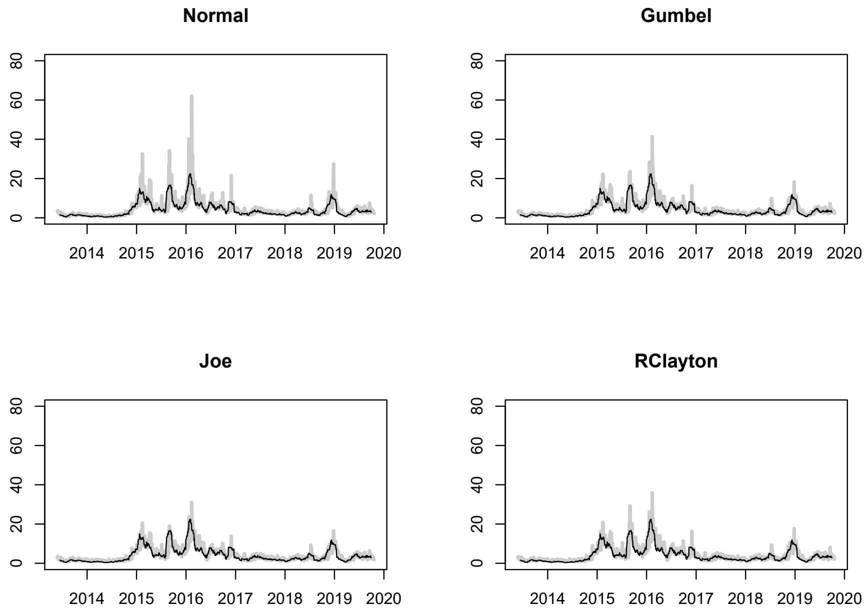


Figure 9: WTI data, mean of the estimated volatilities for four copula specifications in grey and 30-day rolling window empirical variance.



Here σ_t^{2rw} is the k-day rolling window estimate of the variance with $k = \{15, 30, 45\}$ and r_t^2 are the squared returns. We also calculate Mean Absolute Errors (MAE), which, as

Figure 10: Brent data, mean of the estimated volatilities for four copula specifications in grey and 30-day rolling window empirical variance.



compared to RMSE, do not put so much weight on extreme errors:

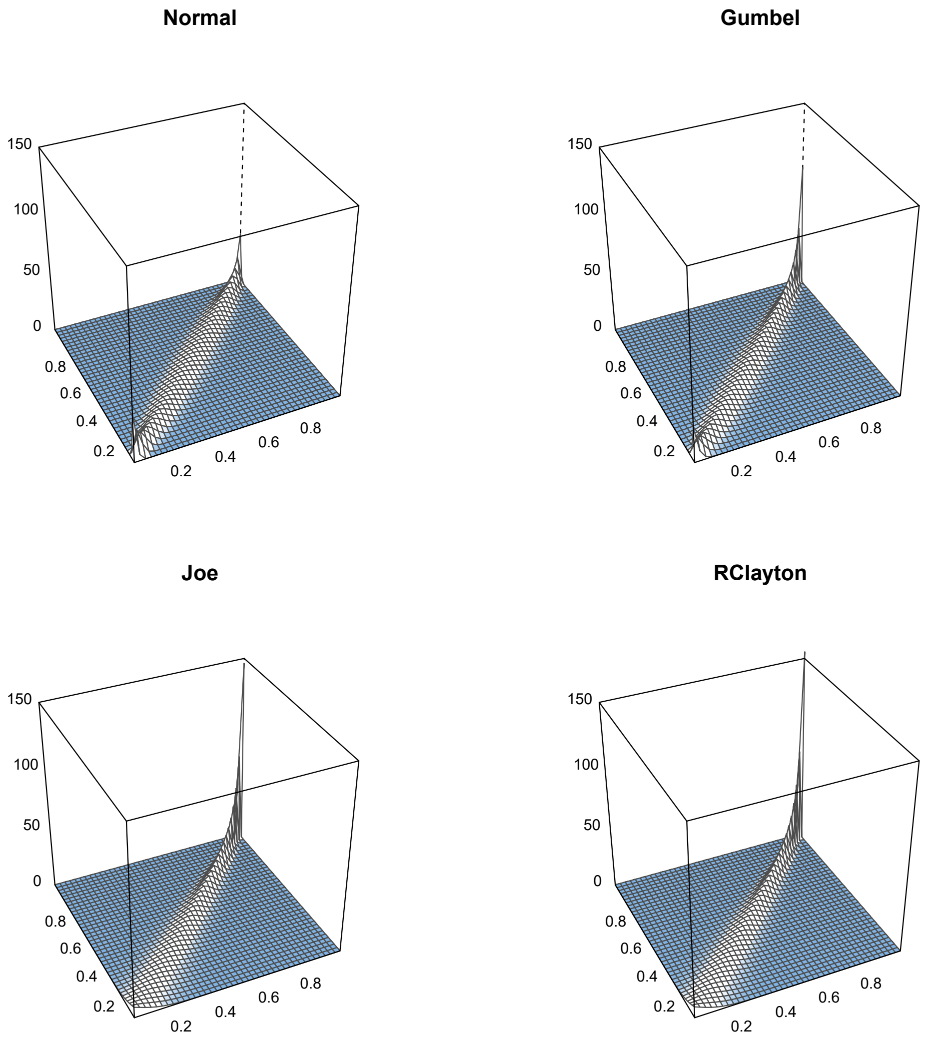
$$\text{MAE}_{rw}^{k} = \frac{1}{P}\sum_{i=1}^{P}\frac{1}{T}\sum_{t=1}^{T}\left|\sigma_t^{2rw} - h_t^{2(i)}\right|,$$

$$\text{MAE}_{r^2} = \frac{1}{P}\sum_{i=1}^{P}\frac{1}{T}\sum_{t=1}^{T}\left|r_t^2 - h_t^{2(i)}\right|.$$

Tables 2 and 3 present the ratios of RMSE and MAE with respect to the Normal copula (i.e. standard SV specification). Numbers lower than one indicate better model performance with respect to the benchmark model. The smaller the number, the better the model. For both datasets for all metrics Joe copula consistently provides the best model performance.

Out-of-sample model performance. In order to evaluate out-of-sample predictive model performance we employ a realized volatility measure extracted from intraday prices. Datastream database provides information for a limited period of high frequency data, therefore, the out of sample evaluation period consists of more than 7 months of daily

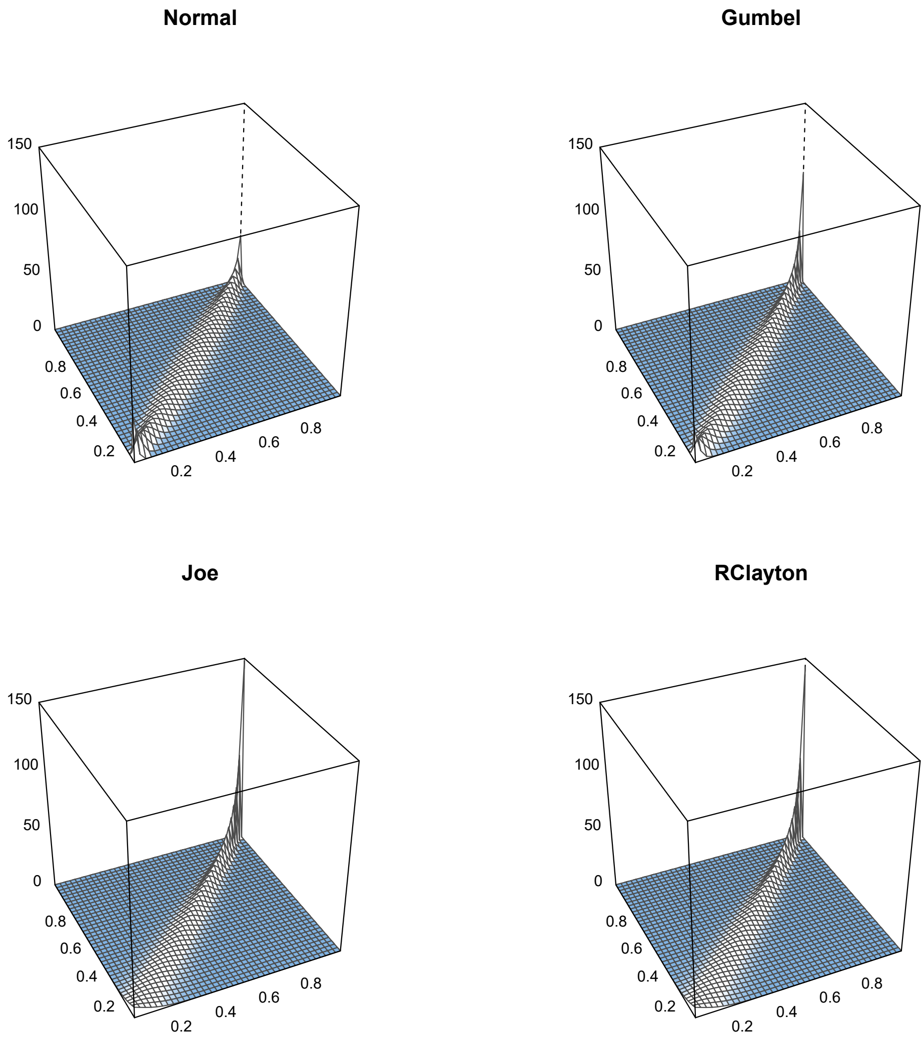
Figure 11: WTI data, estimated copulas for latent volatilities.



observations.

Define $RV_t = \sum_{j=1}^{J} \tilde{r}_{t,j}^2$ as a realized *ex post* volatility measure, where $\tilde{r}_{t,j}$ is a l-minute log-return for day t and J is the number of l-minute intervals in a trading day (Barndorff-Nielsen and Shephard, 2002, Andersen et al., 2003, Barndorff-Nielsen and Shephard, 2004). For an excellent review of realized volatility refer to McAleer and Medeiros (2008). As noted by Andersen et al. (2003), one has to choose such sampling frequency as to balance between the accuracy of the realized volatility measure by increasing J, and the negative effects of the market microstructure noise, which are more pronounced for larger J. Thus for empirical applications we employ 15-minute sampling frequency, re-

Figure 12: Brent data, estimated copulas for latent volatilities.



sulting into J 129 intraday prices for each day t for WTI and Brent data (the data for these futures are available for 24 hours). We also employ 30-minute sampling frequency as a robustness check. The out-of-sample evaluation period for WTI data lasts from 2019-03-15 till 2019-10-21 resulting into 158 out-of-sample observations, and for Brent from 2019-03-20 till 2019-10-21 resulting into 154 out of sample observations.

Figures 13 and 14 draw the mean and 95% credible intervals for the estimated out-of-sample volatilities for four copula specifications in grey and 15 and 30-minute realized volatilities in black. The realized volatilities measures look very similar to each other, indicating the minimum effect of the market microstructure noise. Just by eye-balling

Table 2: In-sample model performance: ratios of RMSE and MSE for WTI with Normal copula as benchmark.

	$RMSE_{rw}^{15}$	MAE_{rw}^{15}	$RMSE_{rw}^{30}$	MAE_{rw}^{30}	$RMSE_{rw}^{45}$	MAE_{rw}^{45}	$RMSE_{r^2}$	MAE_{r^2}
Normal	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Gumbel	0.7815	0.9507	0.7598	0.9303	0.7574	0.9244	0.9212	0.9975
Joe	0.5657	0.8764	0.5179	0.8351	0.5075	0.8212	0.8566	0.9827
RClayton	0.8285	1.0118	0.8045	0.9923	0.7958	0.9790	0.9144	0.9900

Table 3: In-sample model performance: ratios of RMSE and MSE for Brent with Normal copula as benchmark.

	$RMSE_{rw}^{15}$	MAE_{rw}^{15}	$RMSE_{rw}^{30}$	MAE_{rw}^{30}	$RMSE_{rw}^{45}$	MAE_{rw}^{45}	$RMSE_{r^2}$	MAE_{r^2}
Normal	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Gumbel	0.6738	0.9084	0.6545	0.8897	0.6542	0.8882	0.8625	0.9828
Joe	0.5639	0.8996	0.5293	0.8722	0.5232	0.8604	0.8317	0.9740
RClayton	0.6835	0.9264	0.6638	0.9055	0.6619	0.9005	0.8736	0.9887

the plots we can clearly see that Normal and rotated Clayton copula produced credible intervals are wider resulting in more uncertainty when it comes to volatility estimation and prediction.

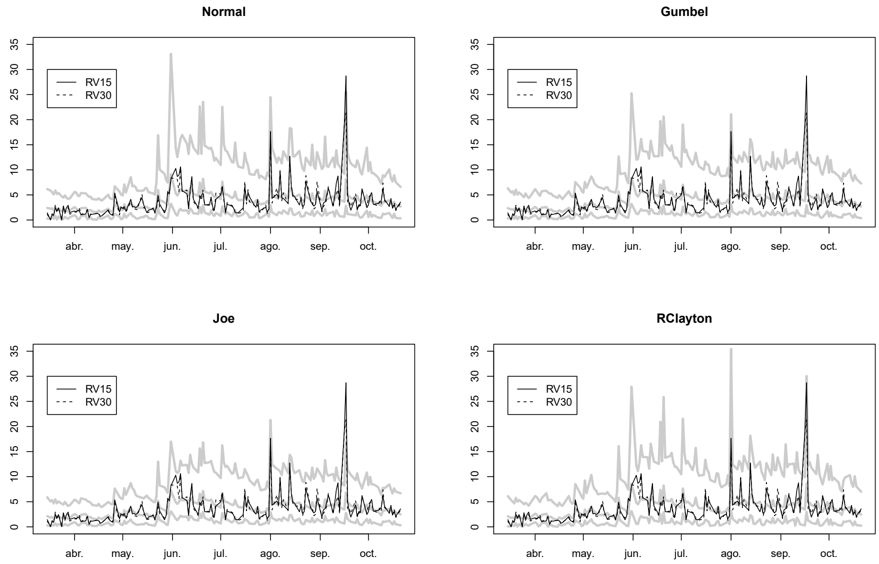
Next, using the estimated realized volatility measure (15 and 30 minutes sampling) and the predicted volatilities h_t^2 for the four models we calculate the RMSE and MAE:

$$\text{RMSE}_{RVm} = \frac{1}{P}\sum_{i=1}^{P}\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(RVm_t - h_t^{2(i)}\right)^2},$$

$$\text{MAE}_{RVm} = \frac{1}{P}\sum_{i=1}^{P}\frac{1}{T}\sum_{t=1}^{T}\left|RVm_t - h_t^{2(i)}\right|,$$

where T is the out-of-sample evaluation period (158 for WTI and 154 for Brent), $P = 250$ is the sample size of the posterior distribution of parameters and RV is either 15 or 30-minute sampling frequency based realized volatility measure with $m \in \{15, 30\}$. Finally, in order to evaluate the estimation accuracy of the mean and variance parameters $(, ^2)$ we rely on the assumption that the latent volatility is marginally normally distributed, see (4). We calculate the average log score using the estimated realized volatil-

Figure 13: WTI data, mean and 95% credible intervals of the estimated out-of-sample volatilities for four copula specifications in grey and 15 and 30-minute realized volatilities in black.



ity measure:

$$LS \quad \frac{1}{P}\sum_{i\;\;1}^{P}\frac{1}{T}\sum_{t\;\;1}^{T} f_N \;\; \log RV_t; \quad ^{i} , \quad ^{2\,i} \quad .$$

Here f_N is the PDF for the Normal distribution. The results for the out-of-sample model performance are in Tables 4 and 5, with Normal copula as benchmark. Because the log scores are negative, the ratio of two log scores becomes positive and we prefer the model with the smallest ratio as compared to the benchmark specification. Again, Joe copula provides the best model fit for the three metrics and two realized volatility measures.

Table 4: Out-of-sample model performance: ratios of RMSE and MSE for WTI with Normal copula as benchmark.

	RMSE_{RV15}	MAE_{RV15}	RMSE_{RV30}	MAE_{RV30}	LS_{RV15}	LS_{RV30}
Normal	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Gumbel	0.9415	1.0005	0.9330	1.0002	0.9878	0.9732
Joe	0.8852	0.9542	0.8650	0.9510	0.8718	0.8376
RClayton	0.9990	1.0262	1.0001	1.0248	0.8962	0.8437

These findings closely relate to the results in the existing time-varying volatility liter-

Figure 14: Brent data, mean and 95% credible intervals of the estimated out-of-sample volatilities for four copula specifications in grey and 15 and 30-minute realized volatilities in black.

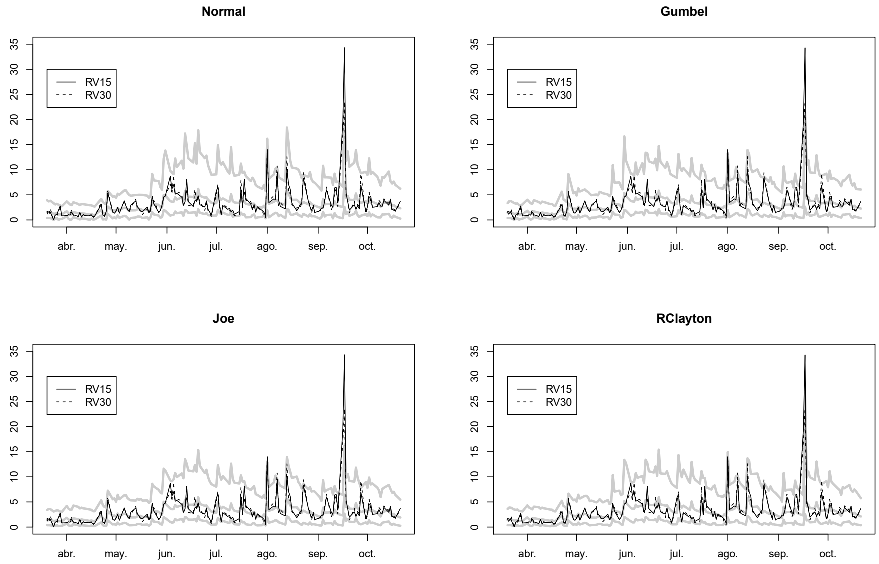


Table 5: Out-of-sample model performance: ratios of RMSE and MSE for Brent with Normal copula as benchmark.

	RMSE$_{RV15}$	MAE$_{RV15}$	RMSE$_{RV30}$	MAE$_{RV30}$	LS$_{RV15}$	LS$_{RV30}$
Normal	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Gumbel	0.9716	0.9821	0.9637	0.9824	0.9779	0.9739
Joe	0.9534	0.9815	0.9356	0.9791	0.9170	0.8883
RClayton	0.9801	0.9976	0.9773	0.9965	0.9243	0.8985

ature in energy economics, especially when we consider the non-linearity of the volatility process. Volatility models with jumps allow for deviations from the linear volatility process by modeling such jumps as a discrete Poisson variable (Duffie et al., 2000). Such models were considered by Larsson and Nossman (2011), Brooks and Prokopczuk (2013), Chan and Grant (2016), among others, who found that jump component is always statistically significant when modeling the crude oil prices. Also, models that account for such jumps in the volatility process outperform the models that do not, in- and out-of-sample. Similarly, the regime switching specification allows for changes in volatility persistence in different market conditions via a discrete Markov switching process. Such models were considered by Fong and See (2002), Vo (2009), for example, who found clear evidence of regime shifts in oil price volatility (either in SV or

GARCH framework), resulting into better in- and out-of-sample forecasts. The presence of jumps or regime shifts indicates non-linear volatility response to its own past. Finally, we model the volatility non-linearity by allowing for a gradual (continuous) transition, instead of some discrete process (Poisson or Markov switching). This transition must happen accordingly to the assumed copula model and different copulas allow for different pre-defined transition pathways. Overall, the empirical implications are clear: our findings, in accordance with the previous studies, once more confirm the existence of severe non-linearities in the volatility process of the crude oil returns. Capturing these non-linearities, whether via discrete or continuous processes, results into superior in-sample model fit and more precise out-of-sample volatility forecasts. As noted by Larsson and Nossman (2011), conventional (i.e. AR-type) SV-type models cannot generate the high levels of volatility seen during the turbulent periods, meanwhile jump, Markov-Switching, and copula stochastic volatility models - can.

5 Conclusions and discussion

In this paper we have proposed a novel copula-based stochastic volatility model. The proposed model allows for asymmetric volatility persistence, often observed in empirical applications. In other words, when the markets are in turmoil the volatility persistence increases and when the markets are in calm state the volatility persistence decreases. The proposed model is a highly non-linear state-space model, therefore we employ a novel ABC estimation technique called auxiliary-likelihood based ABC. For latent state filtering we apply the bootstrap filter. We carry out two simulation studies and show that ABC is a comparable alternative to standard MCMC based methods in estimating the unknown model parameters. We also present a real data application using WTI and Brent returns. The in- and out-of-sample model comparison results always favor the asymmetric specification, Joe copula in particular.

The drawbacks of the methodology developed in this paper also should be noted. For the sake of simplicity and because the focus of the paper is the volatility prediction we have considered Normally distributed returns. However, if the objective is to forecast the entire distribution of the returns and the related measures, such as Value-at-Risk, it would be appropriate to consider a more flexible specification for the distribution of the returns. In particular, one could assume an -stable distribution for example, which can be easily estimated using ABC.

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